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Recommended Citation
Robert H. Morelos-Zaragoza and Shu Lin. "Unequal Error Protection QPSK Modulation Codes" *Faculty Publications* (1992): 1121-1125. doi:10.1109/ICCS.1992.255087

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Unequal Error Protection QPSK Modulation Codes [1]

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Abstract

Unequal error protection (UEP) codes find applications in broadcast channels, as well as in other digital communication systems, where messages have different degrees of importance. In this paper, we use binary linear UEP (LUEP) codes, in combination with a QPSK signal set and Gray mapping, to obtain new efficient block QPSK modulation codes with *unequal minimum squared Euclidean distances.* We give several examples of QPSK modulation codes that have the same minimum squared Euclidean distance (MSED) as the best QPSK modulation codes of the same rate and length. In the new constructions of QPSK modulation codes, even-length binary LUEP codes are used. Good LUEP codes are obtained when we combine shorter linear codes using either the well known $|\bar{u}|\bar{u}+\bar{v}|$-construction or the so-called construction X. Both constructions have the advantage of yielding optimal or near optimal binary LUEP codes of short to moderate lengths, using very simple constituent codes, and may be used as constituent codes in the new constructions. LUEP codes lend themselves quite naturally to *multi-stage decodings*, using the decodings of component subcodes. In this paper, we present a new suboptimal two-stage soft-decision decoding of LUEP codes and apply it to LUEP QPSK modulation codes.

1 Introduction

There are many practical applications in which it is required to design a code that protects messages against different levels of noise, or messages with different levels of importance over a noisy channel with the same level of noise power. Examples of such situations are: broadcast channels, multi-user channels, computer networks, pulse coded modulation (PCM) systems and source coding systems, among others. Such a code is usually said to be an *unequal error protection* (UEP) code. In this paper, we propose to use binary linear UEP (LUEP) codes [1], in combination with QPSK signal constellations, to obtain new efficient block QPSK modulation codes with *unequal minimum squared Euclidean distances.* That is, code sequences associated with the most important message bits are separated by a squared Euclidean distance (SED) larger than the SED between code sequences associated with less important message bits. In this paper two types of messages are considered, one more important than the other. Several examples of block LUEP QPSK modulation codes, having the same minimum squared Euclidean distance (MSED) as that of optimal QPSK modulation codes of the same rate and length [2-3], are presented. The paper is organized as follows. In section 2, basic concepts and two constructions of LUEP codes based on specifying the generator matrix are presented. Section 3 deals with constructions of QPSK modulation codes and a suboptimal two-stage *soft-decision* (TSSD) decoding of LUEP codes. An example is given which illustrates TSSD decoding of QPSK modulation codes. Finally, in section 4, conclusions on the results are presented.

2 Basic concepts of LUEP codes

When a code is used to provide multiple levels of error protection, the conventional definition of minimum distance must be generalized. Since different levels of error protection are possible with a UEP code, a vector of minimum distances, one for each level of error protection, needs to be defined. Let C be an (n,k) block code (not necessarily linear) over a finite alphabet A, $n \geq k$. That is, C is a one-to-one mapping from A^k to A^n, i.e.,

$$\bar{m} \in A^k \quad \overset{C}{\mapsto} \quad \bar{c}(\bar{m}) \in A^n,$$

[1] This research was supported by the NSF under Grant NCR-88813480 and by NASA under Grant NAG 5-931.

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where

$$A^k = \underbrace{A \times A \times \cdots \times A}_{k \text{ times}}.$$

As usual, an element $\bar{\mathbf{m}}$ from A^k is called a *message*, and an element $\bar{\mathbf{c}}(\bar{\mathbf{m}})$ from C is called a *codeword*. A^k is known as the *message set*. Let A^k be decomposed into the direct product of ℓ disjoint *message subsets*, A^{k_i}, $1 \le i \le \ell$, such that

$$A^k = A^{k_1} \times A^{k_2} \times \cdots \times A^{k_\ell},$$

and a message $\bar{\mathbf{m}} \in A^k$ can be expressed as

$$\bar{\mathbf{m}} = (\bar{\mathbf{m}}_1, \bar{\mathbf{m}}_2, \cdots, \bar{\mathbf{m}}_\ell), \quad \bar{\mathbf{m}}_i \in A^{k_i}, \quad 1 \le i \le \ell,$$

where each $\bar{\mathbf{m}}_i$ is called the *i-th message part*, $1 \le i \le \ell$. The *separation vector* of C is defined as the ℓ-tuple $\bar{\mathbf{s}} = (\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_\ell)$, where

$$\mathbf{s}_i \triangleq \min\{d(\bar{\mathbf{c}}(\bar{\mathbf{m}}), \bar{\mathbf{c}}'(\bar{\mathbf{m}}')) : \bar{\mathbf{m}}_i \ne \bar{\mathbf{m}}'_i, \quad \bar{\mathbf{m}}_i, \bar{\mathbf{m}}'_i \in A^{k_i}\}, \quad 1 \le i \le \ell,$$

where $d(\bar{\mathbf{x}}, \bar{\mathbf{x}}')$ denotes the Hamming distance between $\bar{\mathbf{x}}$ and $\bar{\mathbf{x}}'$ in A^n. Assume that C has all the components of its separation vector distinct and arranged in decreasing order, i.e.,

$$\mathbf{s}_1 > \mathbf{s}_2 > \cdots > \mathbf{s}_\ell,$$

so that C is an (n, k) block code of minimum distance $\mathbf{s}_\ell$. Code C is said to be an (n, k) *ℓ-level UEP code* with separation vector

$$\bar{\mathbf{s}} = (\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_\ell),$$

for the message set $A^{k_1} \times A^{k_2} \times \cdots \times A^{k_\ell}$. This work concentrates on *linear* block multi-level error correcting codes. That is, A is taken to be the Galois field $GF(q)$ of q elements, where q is a power of a prime number. For a *linear* multi-level error correcting code, or LUEP code, denoted C, each element of the separation vector is given by

$$\mathbf{s}_i \triangleq \min\{\mathrm{wt}(\bar{\mathbf{c}}(\bar{\mathbf{m}})) : \bar{\mathbf{m}}_i \ne \bar{0}, \ \bar{\mathbf{m}}_i \in GF(q)^{k_i}\}, \quad 1 \le i \le \ell. \tag{1}$$

C is said to be an (n, k) ℓ-level LUEP code, with separation vector $\bar{\mathbf{s}} = (\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_\ell)$, for the *message space*

$$GF(q)^{k_1} \times GF(q)^{k_2} \times \cdots \times GF(q)^{k_\ell}.$$

2.1 LUEP codes specified by their generator matrix

In this section, constructions of LUEP codes by appending (or time sharing) cosets of subcodes in linear codes are presented. These constructions may be used to obtain constituent binary LUEP codes which, in conjunction with QPSK signals, yield efficient LUEP QPSK modulation codes (see section 3).

2.1.1 The $|\bar{\mathbf{u}}|\bar{\mathbf{u}} + \bar{\mathbf{v}}|$ construction

For $i = 1, 2$, let C_i be an (n, k_i, d_i) linear code with generator matrix G_i. Define the *append* operation between two vectors

$$\bar{\mathbf{u}} = (u_0, u_1, \cdots, u_{n-1}) \text{ and } \bar{\mathbf{v}} = (v_0, v_1, \cdots, v_{n-1})$$

as

$$\bar{\mathbf{u}} \circ \bar{\mathbf{v}} \triangleq (u_0, u_1, \cdots, u_{n-1}, v_0, v_1, \cdots, v_{n-1}).$$

Based on C_1 and C_2 construct the following code,

$$\mu(C_1, C_2) = \{\bar{\mathbf{w}} | \ \bar{\mathbf{w}} = \bar{\mathbf{u}} \circ (\bar{\mathbf{u}} + \bar{\mathbf{v}}), \bar{\mathbf{u}} \in C_1, \bar{\mathbf{v}} \in C_2\}$$

Clearly, $\mu(C_1, C_2)$ is a $(2n, k_1 + k_2)$ linear code with generator matrix,

$$G = \begin{pmatrix} G_1 & G_1 \\ 0 & G_2 \end{pmatrix}$$

The minimum distance of $\mu(C_1, C_2)$ is [8]

$$d = \min\{2d_1, \max\{d_1, d_2\}\}.$$

Theorem 1: $\mu(C_1, C_2)$ is a 2-level LUEP code with separation vector $\bar{\mathbf{s}} = (\mathbf{s}_1, \mathbf{s}_2)$, for the message space $\{0,1\}^{k_1} \times \{0,1\}^{k_2}$, where

$$\mathbf{s}_1 \ge \min\{2d_1, \max\{d_1, d_2\}\}, \text{ and}$$
$$\mathbf{s}_2 \ge \min\{\max\{d_1, d_2\}, d_2\} = d_2.$$

Proof: See [7]. ΔΔ

2.1.2 Construction X [4]

For $i = 1, 2, 3$, let C_i denote a linear (n_i, k_i, d_i) binary code. Assume $C_3 \subseteq C_2$, so that $k_3 \le k_2$ and $d_3 \ge d_2$. Let C_X be the linear code whose generator matrix is

$$G_X = \begin{pmatrix} G_1 & G_2 \\ 0 & G_3 \end{pmatrix}$$

where G_1, $[G_2^T G_3^T]^T$ and G_3 are the generator matrices of C_1, C_2 and C_3 respectively (Note that it is required that $k_1 = k_2 - k_3$). Then C_X is an $(n_1 + n_3, k_1 + k_3)$ linear code with minimum distance $d_X = \min\{d_3, d_1 + d_2\}$ [8]. This method of combining shorter linear codes to obtain a linear code with increased length and minimum distance is known as *Construction X*, and can be viewed as a generalization of the $|\bar{\mathbf{u}}|\bar{\mathbf{u}} + \bar{\mathbf{v}}|$ construction. By an argument similar to that used to prove Theorem 1, we can prove the following result:

Theorem 2: C_X is a 2-level LUEP code with separation vector $\bar{\mathbf{s}} = (\mathbf{s}_1, \mathbf{s}_2)$, for the message space $M = \{0,1\}^{k_1} \times \{0,1\}^{k_3}$, where

$$\mathbf{s}_1 \ge d_1 + d_2, \text{ and}$$
$$\mathbf{s}_2 \ge \min\{d_3, d_1 + d_2\}.$$

3 LUEP QPSK modulation codes

In this section, we present a method of combining binary two-level LUEP codes and QPSK modulation to achieve coded modulation schemes that offer *two values of minimum squared Euclidean distance*, one for each message part to be protected. In this way, the most important message part will be mapped into code sequences with a larger squared Euclidean distance between them than that corresponding to the less important message part. If data transmission is performed over an additive white Gaussian noise (AWGN) channel, and the channel code is selected properly (i.e., efficient soft-decision decoding and small number of

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nearest neighbors), then at high signal-to-noise ratios, we will have a *smaller* probability of bit error for the most important message part than for the rest of the message. To achieve the performance promised by a given minimum squared Euclidean distance, we present a new suboptimal *two-stage soft-decision decoding* of two-level LUEP codes, that uses their trellis structure.

3.1 Constructions via Gray mapping

In a QPSK signal constellation with *Gray mapping* between labels and signal points, the squared Euclidean distance between signal points is *twice* the Hamming distance between the corresponding labels. We say that this QPSK signal constellation forms a *second-order Hamming space* [6]. By mapping 2-bit symbols into signal points in a QPSK signal set, via Gray mapping, we can combine $(2n, k_1 + k_2)$ 2-level LUEP codes and QPSK modulation to achieve a block coded modulation system that offers two values of minimum squared Euclidean distances, one for each message part. Some of the resulting LUEP QPSK block modulation codes have the same minimum squared Euclidean distance as optimal QPSK block modulation codes with the same rate and length [2-3], while offering in addition a larger minimum squared Euclidean distance between code sequences associated with most important message parts. The proposed construction is as follows:

Let C_b be a $(2n, k_1 + k_2)$ binary LUEP code with separation vector $\bar{\mathbf{s}} = (\mathbf{s}_1, \mathbf{s}_2)$ for the message space $\{0,1\}^{k_1} \times \{0,1\}^{k_2}$. Consider a QPSK signal set S with the Gray mapping $\mathbf{M}$ between 2-bit symbols and S,

$$\begin{aligned} 00 &\mapsto 0 \\ 01 &\mapsto 1 \\ 11 &\mapsto 2 \\ 10 &\mapsto 3 \end{aligned}$$

Then $C = \mathbf{M}(C_b)$ is a 2-level LUEP QPSK block modulation code of length n, dimension k, rate $R = k/2n$ (bits/dimension), and *squared Euclidean separation vector*

$$\bar{\mathbf{S}}_{SED} = (2\mathbf{s}_1, 2\mathbf{s}_2).$$

In conventional coded modulation, given the minimum squared Euclidean distance (MSED) and rate of a modulation code, an *asymptotic coding gain* G is defined for an AWGN channel. For high signal-to-noise ratios, G equals the ratio of the MSED of the coded system to the MSED of an uncoded system transmitting at the same rate (number of bits per signal). Accordingly, for each component of $\bar{\mathbf{S}}_{SED}$ we may define an asymptotic coding gain. In this work, we define an *asymptotic coding gain vector* as

$$\bar{G} = (G_1, G_2),$$

where, for $i = 1, 2$,

$$G_i = 10 \log_{10} \left[\frac{2\mathbf{s}_i}{4 \sin^2 (\pi/2^R)} \right] \quad \text{(dB)}$$

Note that, as in the case of conventional coded modulation systems, these asymptotic coding gains can only be reached if *maximum-likelihood soft-decision* decoding is available. To illustrate this construction method, in Table 1 some LUEP BPSK block modulation codes are listed. Some codes in Table 1 have the same minimum squared Euclidean distance as that of *optimal* block QPSK modulation codes with the same rate and length [2], and provide additional coding gain (or, equivalently, smaller probability of bit error) for the k_1 most important message bits. In Table 1, codes labeled with $\star$ are LUEP QPSK modulation codes obtained from the $|\bar{\mathbf{u}}|\bar{\mathbf{u}} + \bar{\mathbf{v}}|$ construction. Other codes are taken from [9]. It is interesting to note that *all* optimal block QPSK modulation codes found by Sayegh [2-3], of lengths 5 to 10, can be obtained from the $|\bar{\mathbf{u}}|\bar{\mathbf{u}} + \bar{\mathbf{v}}|$ construction combined with Gray mapped QPSK signal sets. All of these codes are actually LUEP QPSK modulation codes, and this appears to be the first time that this fact has been pointed out.

3.2 Two-stage soft-decision decoding

Let C be an (n, k) two-level LUEP code with separation vector $(\mathbf{s}_1, \mathbf{s}_2)$ for the message space $\{0,1\}^{k_1} \times \{0,1\}^{k_2}$. Then C can be represented as the direct sum of subcodes C_1 and C_2, $C = C_1 \oplus C_2$, i.e,

$$C = \{\bar{\mathbf{c}} = \bar{\mathbf{c}}_1 + \bar{\mathbf{c}}_2 : \bar{\mathbf{c}}_1 \in C_1 \quad \text{and} \quad \bar{\mathbf{c}}_2 \in C_2\},$$

where C_2 is an $(n, k_2, \mathbf{s}_2)$ subcode which contains all codewords of minimum weight of C, and C_1 is an $(n, k_1, \mathbf{s}_1)$ subcode spanned by a system of coset representatives of C_2 in C. Let T_i be a trellis diagram for subcode C_i of C, $i = 1, 2$. Then a trellis diagram of C can be expressed as the direct product of T_1 and T_2, $T = T_1 \otimes T_2$. That is, states in T are pairs (s_1, s_2), where s_i is a state in T_i, for $i = 1, 2$. The pair (s_1, s_2) is joined to all pairs (s_1', s_2'), in such a way that, for $i = 1, 2$, s_i is joined to s_i' in T_i [11]. Viterbi maximum likelihood decoding algorithm can then be applied to T to estimate the most likely codeword of C using soft decisions. To reduce the number of computations in soft-decision decoding of a modulation code, a technique called *multi-stage decoding* is usually employed. The proposed suboptimal two-stage soft-decision decoding for two-level LUEP codes is as follows:

1. Using soft-decisions (squared Euclidean distance) and the Viterbi algorithm, determine the closest path $\hat{c}_1$ in T_1 to the received sequence. At this decoding stage, the most significant message part is decoded.
2. Using soft-decisions and the Viterbi algorithm, determine the closest path $\hat{c}_2$ in $\hat{c}_1 + T_2$ to the received sequence, to estimate the less significant message part. Here $\hat{c}_1 + T_2$ indicates that the branch metrics of $\hat{c}_1$, obtained in the first decoding stage, are used at each decoding step of the Viterbi algorithm operating on trellis T_2.

This two-stage soft-decision decoding is well known [2], [5], [10], [11], [12]. However, this appears to be the first

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time that multi-stage soft-decision decoding has been used for unequal error protection purposes. Although at each stage the decoding is maximum-likelihood, the multi-stage soft-decision decoding method described above is suboptimal. At each decoding stage, the most likely path is estimated using only part (T_i) of the trellis T of C. This suboptimal multi-stage soft-decision decoding also increases the effective number of nearest neighbors, but this results in only a fraction of a dB in overall coding gain degradation [2], [10], [11], [12].

4 Conclusions

In this paper, we have introduced a new construction of QPSK block modulation codes for unequal error protection. These codes offer two values of minimum squared Euclidean distance (MSED), one for each message part. That is, code sequences associated with the most important message part are separated by a larger MSED. When these sequences are transmitted over an AWGN channel, a larger MSED translates into having a lower probability of error for the most important message symbols.

We used Gray mapping on a QPSK signal set to obtain a second order Hamming space in which $(2n, k)$ LUEP codes with separation vector $\bar{\mathbf{s}} = (\mathbf{s}_1, \mathbf{s}_2)$ are mapped into (n, k) LUEP QPSK modulation codes with squared Euclidean separation $\bar{\mathbf{S}}_{SED} = (2\mathbf{s}_1, 2\mathbf{s}_2)$. We have introduced a new suboptimal two-stage soft-decision decoding for LUEP codes and shown its application in decoding LUEP QPSK modulation codes. Numerical results indicate that besides the well known penalty of a few tenths of a dB in overall coding gain, there is a larger cost to pay in coding gain for the most important message part, although this coding gain is still considerably larger than the overall coding gain.

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Table 1: Some LUEP QPSK block modulation codes

$2n$	k	k_1	k_2	s_1	s_2	R (bits/dim)	G_1 (dB)	G_2 (dB)	
4	2	1	1	3	2	1/2	1.76	0.00	
8	5	1	4	4	2	5/8	3.28	0.27	
8	5	4	1	3	2	5/8	2.03	0.27	
8	6	1	5	3	2	3/4	2.71	0.95	
10	5	1	4	5	4	1/2	3.98	3.01	$\star$
10	7	1	6	4	2	7/10	3.65	0.64	
10	7	4	3	3	2	7/10	2.40	0.64	
10	8	1	7	3	2	4/5	3.06	1.30	
12	6	1	5	6	4	1/2	4.77	3.01	$\star$
12	6	2	4	5	4	1/2	3.98	3.01	
12	9	1	8	4	2	3/4	3.96	0.95	
12	9	4	5	3	2	3/4	2.71	0.95	
12	10	1	9	3	2	5/6	3.32	1.56	
14	7	1	6	7	4	1/2	5.44	3.01	$\star$
14	7	4	3	5	4	1/2	3.98	3.01	
14	8	1	7	5	4	4/7	4.07	3.10	
14	11	1	10	4	2	11/14	4.21	1.20	
14	11	4	7	3	2	11/14	2.96	1.20	
14	12	1	11	3	2	6/7	3.51	1.75	

$\star$ = LUEP QPSK code based on the $|\bar{\mathbf{u}}|\bar{\mathbf{u}} + \bar{\mathbf{v}}|$ construction.

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