

11-30-2010

# Error Correcting Codes for Cooperative Broadcasting

Robert H. Morelos-Zaragoza

San Jose State University, robert.morelos-zaragoza@sjsu.edu

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Robert H. Morelos-Zaragoza. "Error Correcting Codes for Cooperative Broadcasting" *Faculty Publications* (2010).

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# ***Error Correcting Codes for Cooperative Broadcasting***

Robert Morelos-Zaragoza  
Electrical Engineering Department  
San José State University

Presented at the ISL Symposium  
Stanford University  
November 30, 2010

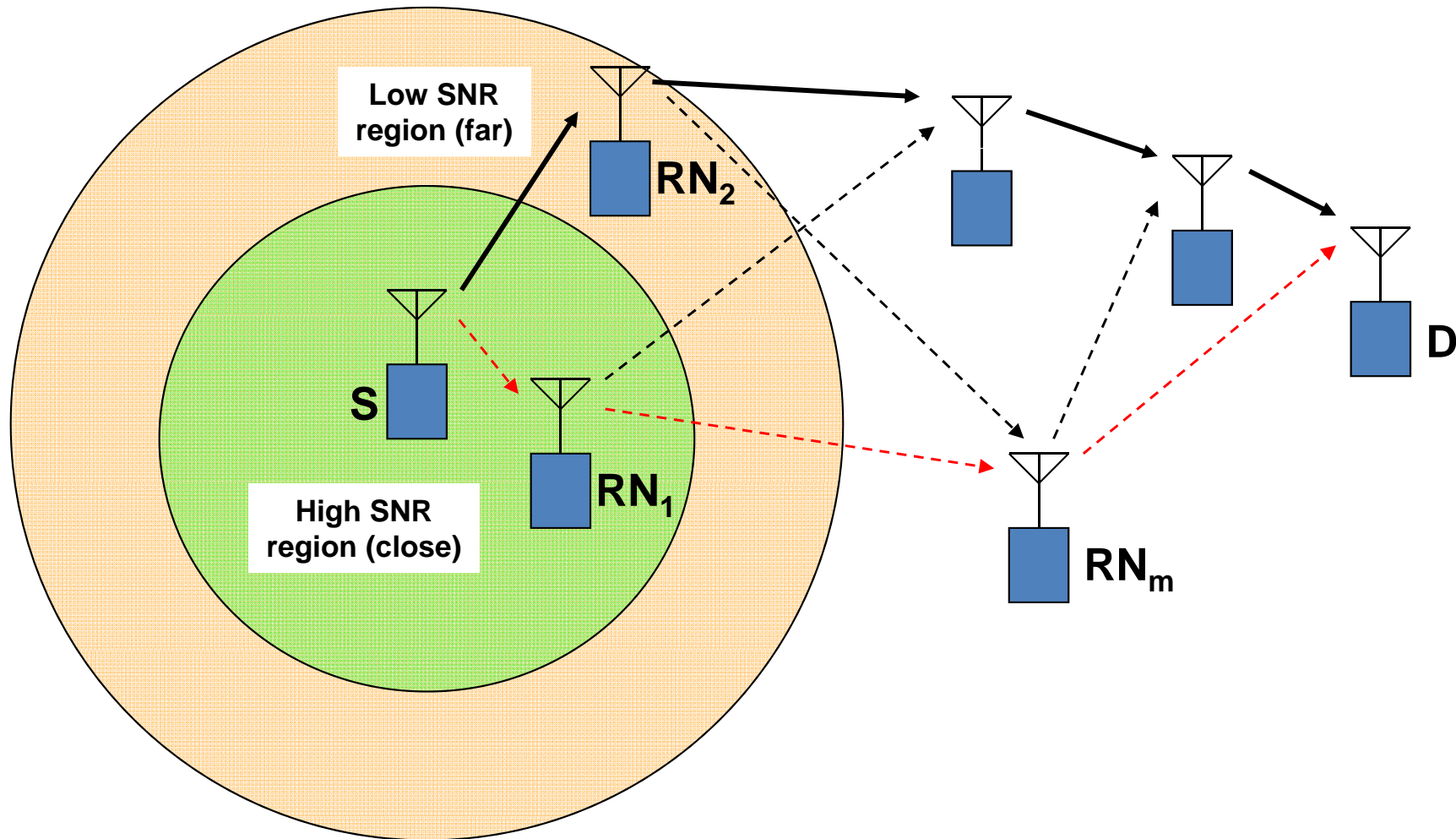
# Outline

1. Motivation
2. Related work
3. Broadcast channels
4. Multilevel codes and the  $|u|u+v|$  construction
  - Time sharing  $|u|v|$  versus  $|u|u+v|$  construction
  - Decoding for  $|u|u+v|$  construction: Two stages versus single state
5. Demapping and decoding performance
  - BPSK and QPSK modulations
  - 4-PAM modulation: Hierarchical mapping
6. Conclusions and future work

# 1. Motivation

- Background on ***unequal error protection (UEP) codes***:  
*Ph.D. thesis on multilevel error-correcting codes*
- UEP codes are based on the idea of ***superposition coding*** proposed in Cover's paper (1972) on broadcast channels
- The nodes of a wireless network (cooperative or not) always broadcast information (i.e., every node, in principle, receives this information)
- ***Cooperative broadcasting*** paper (Bergmans and Cover ,1974)  
*"Superposition coding always outperforms orthogonal (time-division or frequency-division) schemes"*

# Broadcasting in a Wireless Network



1. Relay  $RN_1$  has higher SNR compared to  $RN_2$   $\longrightarrow C_1 > C_2$
2. “Shortest path” (smallest number of hops) not always most reliable

## 2. Related work

- Cover (1972): “Broadcast Channels”
  - Cloud structure of capacity-achieving superposition codes
- Bergmans and Cover (1974): “Cooperative Broadcasting”
  - Superposition codes always outperform orthogonal assignment
- Laneman, Tse and Wornell (2004): “Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior”
  - Amplify-and-forward, decode-and-forward, adaptive relaying, incremental relaying
- Stefanov and Erkip (2004): “Cooperative Coding for Wireless Networks”
  - Propose superposition coding. Do not refer to Cover!!?
- Yi, Azimi, Kalyanaraman and Subramanian (2005): “Error Control Code Combining Techniques in Cluster-based Cooperative Wireless Networks”
  - Use Chase code-combining with hybrid ARQ

# Related work (cont.)

- S. Katti, H. Rahul, W. Hu, D. Katabi, M. Medard, and J. Crowcroft,: "XORs in The Air: Practical Wireless Network Coding," (2006)
  - Opportunistic coding
- Chen and Ahmed (2008): Throughput Enhancement in Cooperative Diversity Wireless Networks using Adaptive Modulation
  - Feedback CSI and adapt constellation accordingly
- Li, Ge, Tang and Xiong (2008): Cooperative Diversity Based on Alamouti Space-Time Code
  - Multiple-access stage of relay nodes achieved with Alamouti's scheme
- L. Xiao, T.E. Fuja, J. Klierer and D.J. Costello (2009): Error Performance Analysis of Signal Superposition Coded Cooperative Diversity
  - Consider superposition coding performed at a single source, metrics
- H.J. Yang, Y. Choi and J. Chun (2010): Modified High-Order PAMs for Binary Coded Physical-Layer Network Coding
  - Constellation design for superposition (physical layer) network coding

# 3. Broadcast Channels

- Thomas Cover (1972) – Gaussian broadcast channel with one source broadcasting information to two users: User 1 has a larger signal-to-noise ratio (SNR) than User 2 ( $N_1 < N_2$ ):

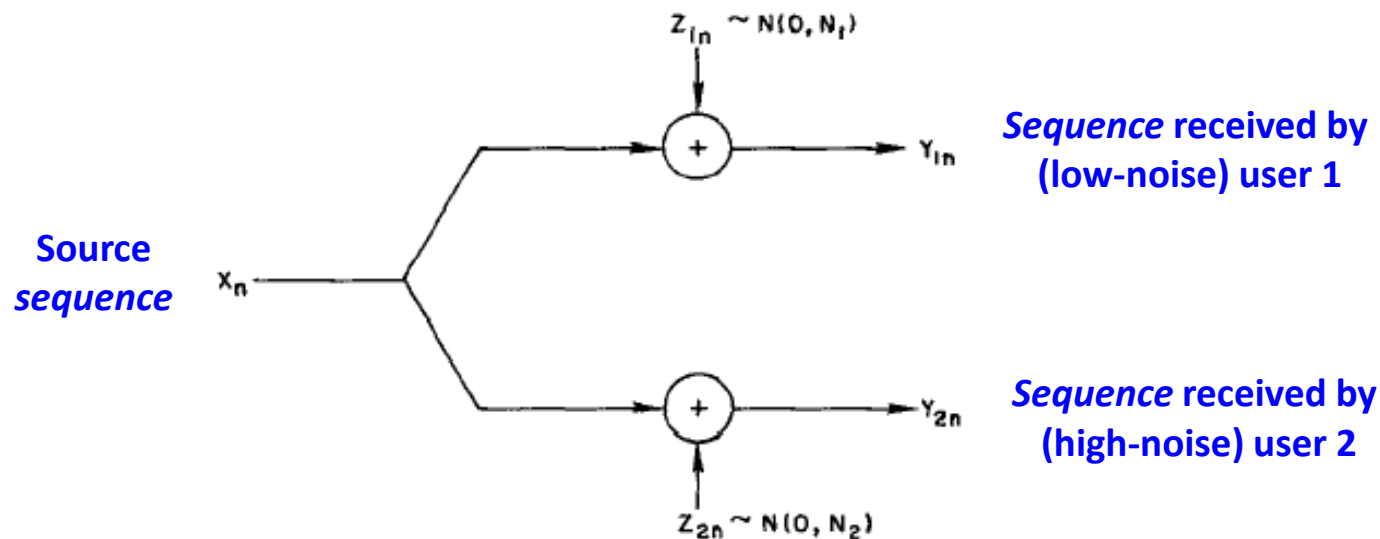


Fig. 16. Gaussian broadcast channel.

From: T. Cover, "Broadcast Channels," *IEEE Trans. Info. Theory*, vol. IT-18, no. 1, pp. 2-14, Jan. 1972



# Coding for broadcast channels: Cloud concept

- Cover showed that a channel code achieving capacity has a **cloud structure**, shown below for a binary symmetric broadcast channel:

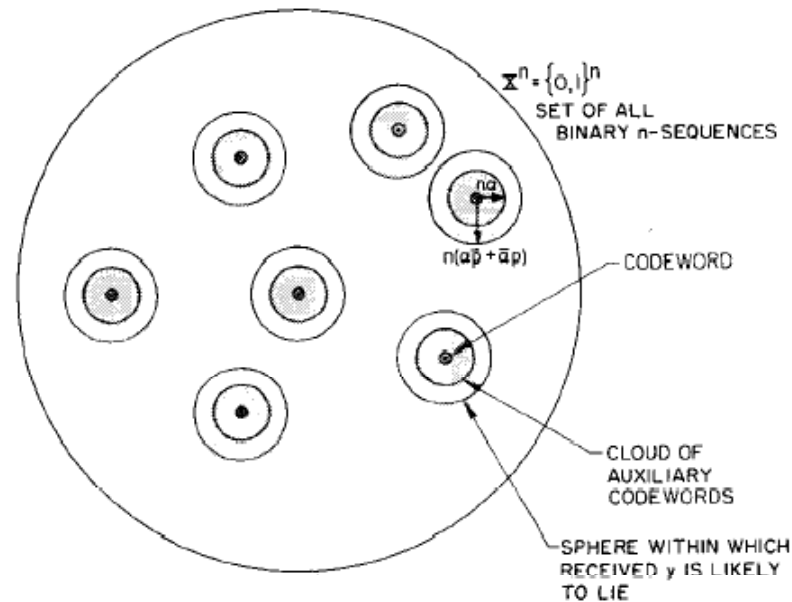


Fig. 4. Space of codewords for BSC.

From: T. Cover, "Broadcast Channels," *IEEE Trans. Info. Theory*, vol. IT-18, no. 1, pp. 2-14, Jan. 1972

- A cloud is a set of codewords (or sequences) that is selected with the information bits (most important or MSB) to be transmitted to the high-noise user

# Cooperative Broadcasting (1974)

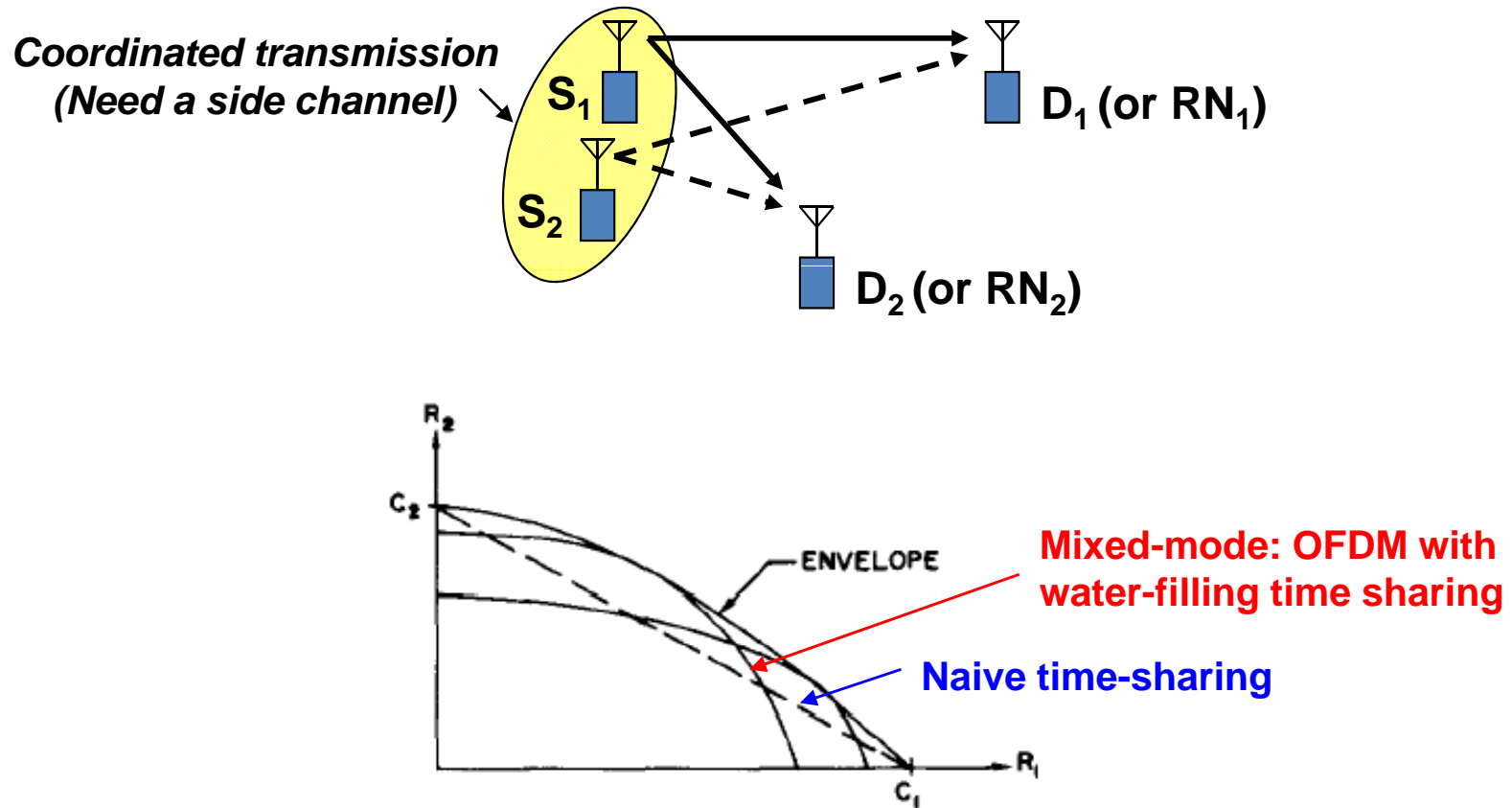


Fig. 3. Rates achievable by frequency division and variable power time sharing.

From: P. Bergmans and T. Cover, "Cooperative Broadcasting," *IEEE Trans. Info. Theory*, vol. IT-20, no. 3, pp. 317-324, May 1974.

# 4. Multilevel LUEP codes and $|u|u+v|$ construction

- An LUEP code has subcode partition-chain  $C \supset C_1 \supset \dots \supset C_L$  with

$$G = \left( \begin{array}{c} G_1 \\ G'_2 \\ \vdots \\ G_L \end{array} \right), \quad G_2 = \left( \begin{array}{c} G'_2 \\ G'_3 \\ \vdots \\ G_L \end{array} \right), \quad \dots, \quad G_L,$$

and  $d_1 > d_2 > \dots > d_L$ .

- Practical *two-level LUEP codes* can be constructed based on *block, convolutional or LDPC codes* and Plotkin's (or  $|u|u+v|$ ) construction:

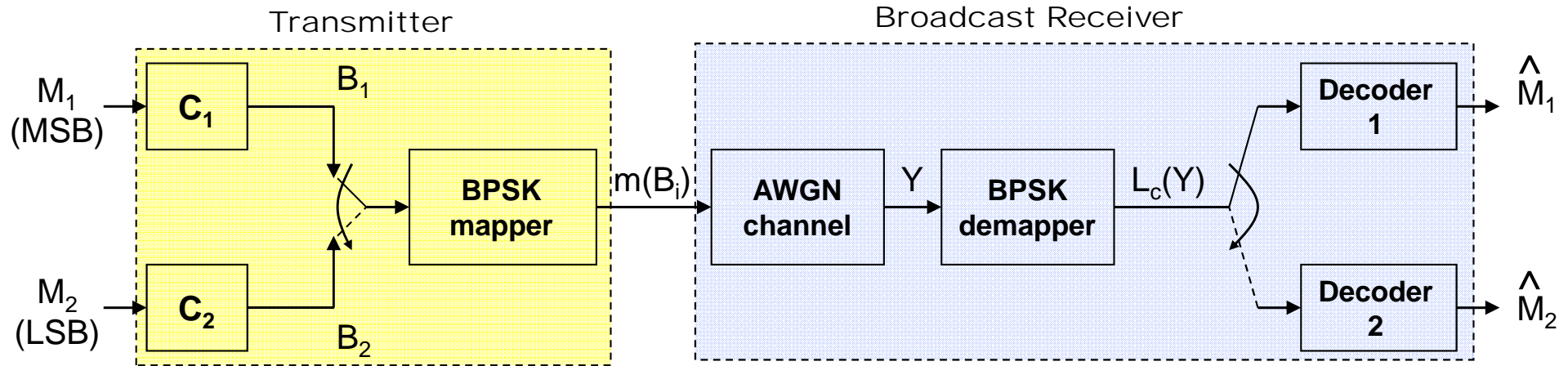
$$G = \left( \begin{array}{cc} 0 & G_1 \\ G'_2 & G'_2 \end{array} \right),$$

← Cloud centers (coset leaders)  
← Cloud (subcode) codewords

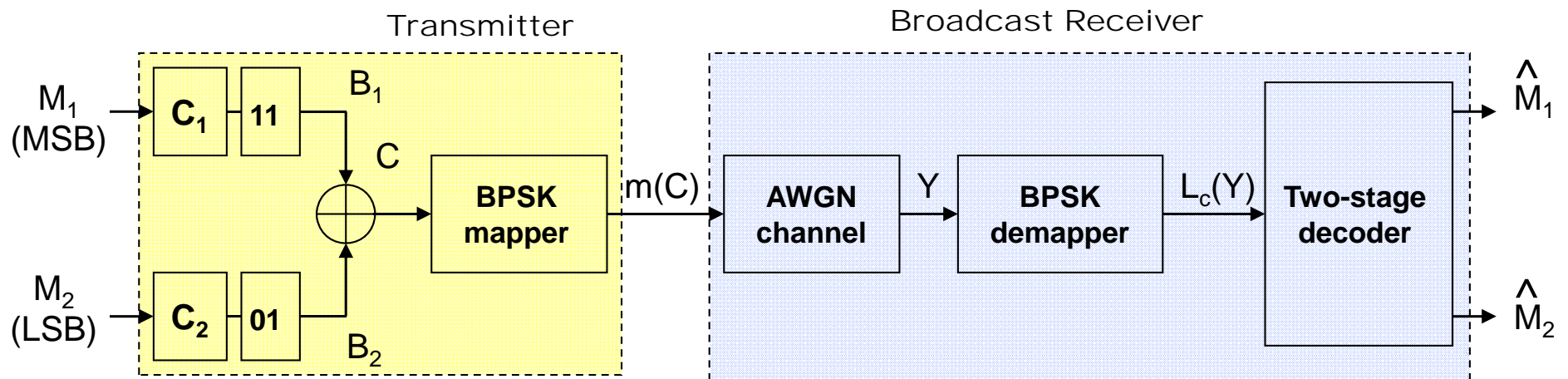
where  $d_2 = 2d'_2 < d_1$ .

# Time-sharing ( $|u|v|$ ) versus Plotkin ( $|u|u+v|$ )

- Time-sharing:

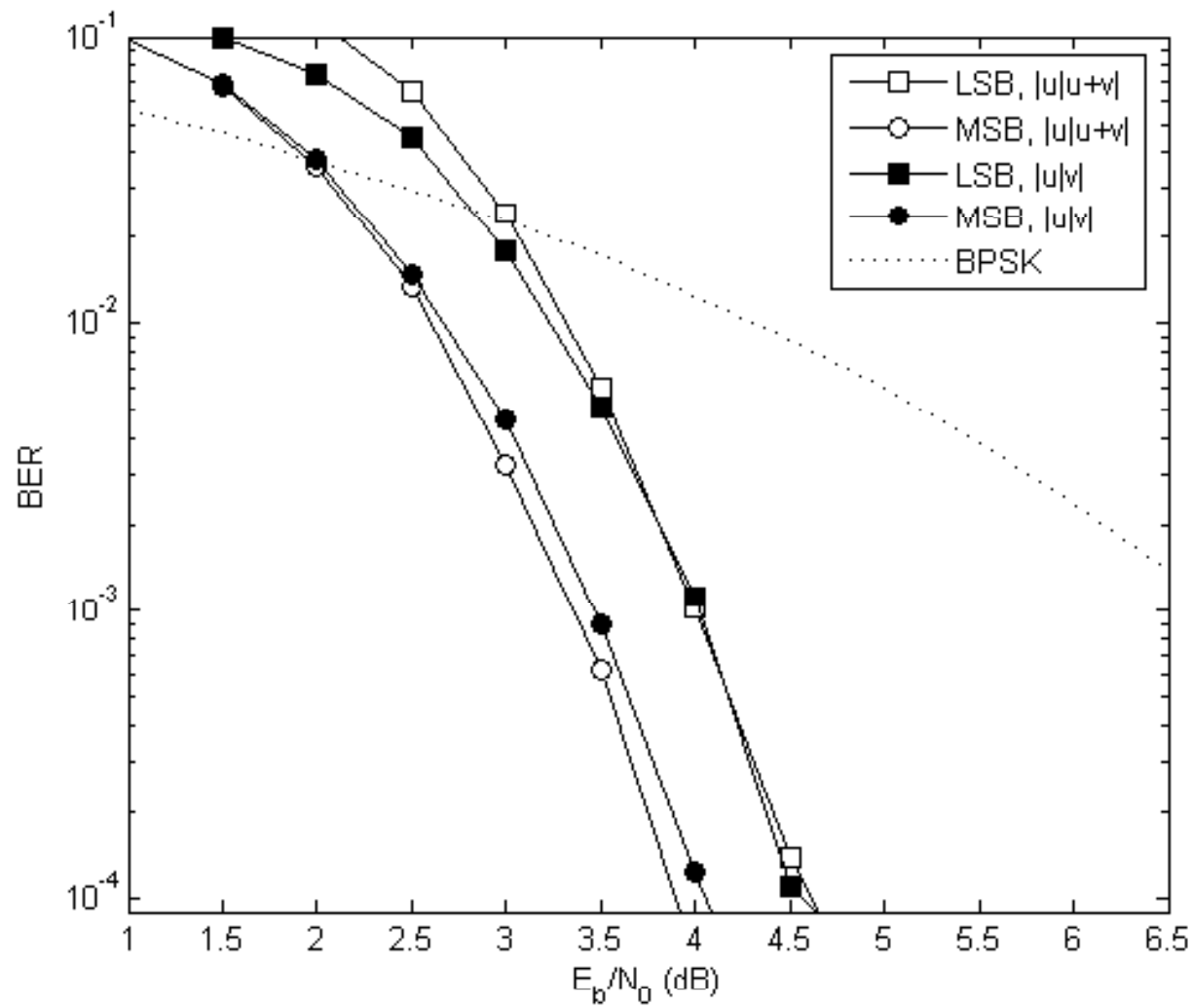


- Plotkin:



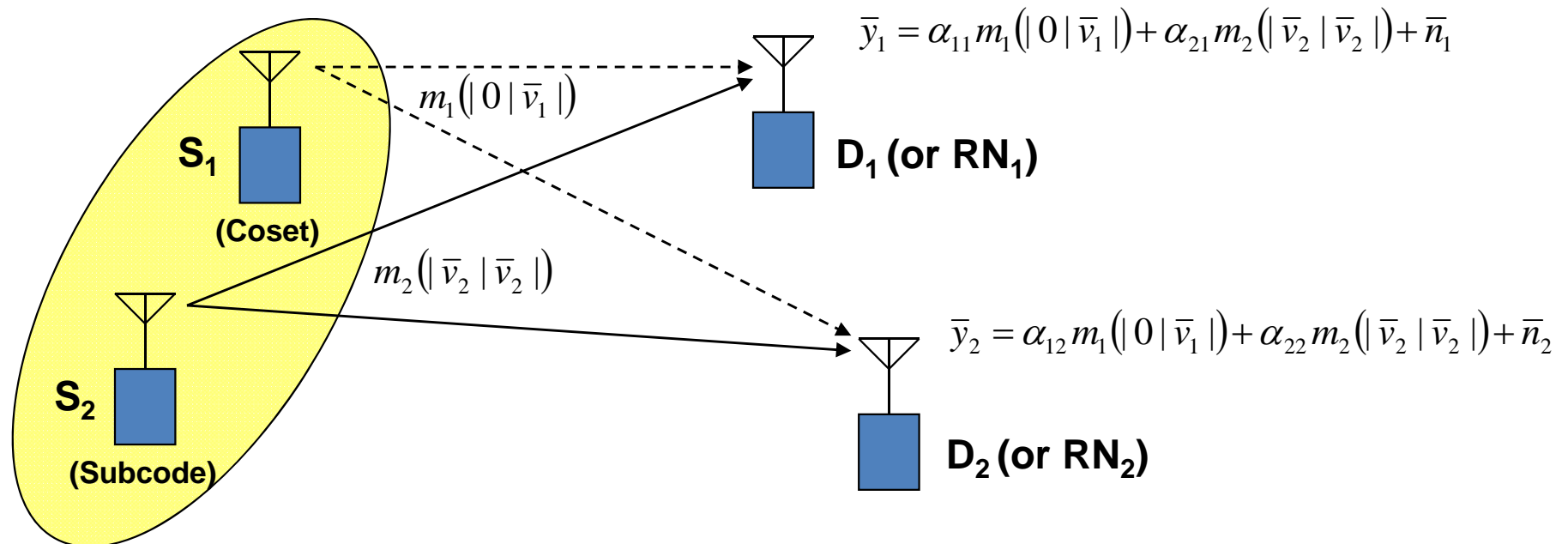
# Performance of $|u|v|$ vs. $|u|u+v|$ : short LDPC codes

$C_1, C_2$ : LDPC (96,50) codes of degrees (3,6) and (4,8)



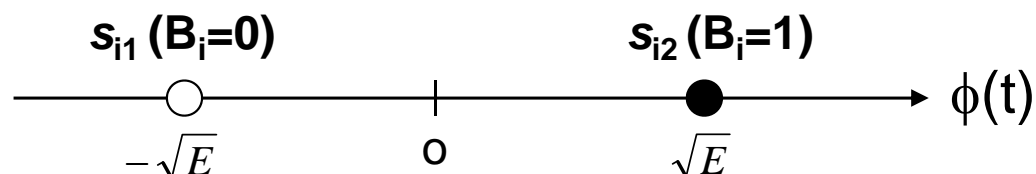
# A Plotkin $|u|u+v|$ coding scheme

- Multilevel codes *always improve the throughput* over any orthogonal (time- or frequency-division) approach
- Follow Bergmans' and Cover's idea: Design an “*over-the-air*”  $|u|u+v|$  (Plotkin) coding scheme:

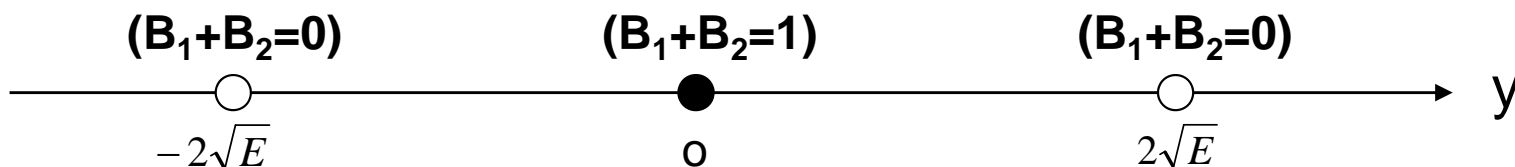


# Two sources with BPSK mapping

- **BPSK Mapping**  $m_i$  from a bit to a signal set  $M_i$ ,  $i=1,2$ . Assume  $\alpha_{i1}=\alpha_{i2}$
- $M_1=\{s_{11},s_{12}\}$  and  $M_2=\{s_{21},s_{22}\}$ :

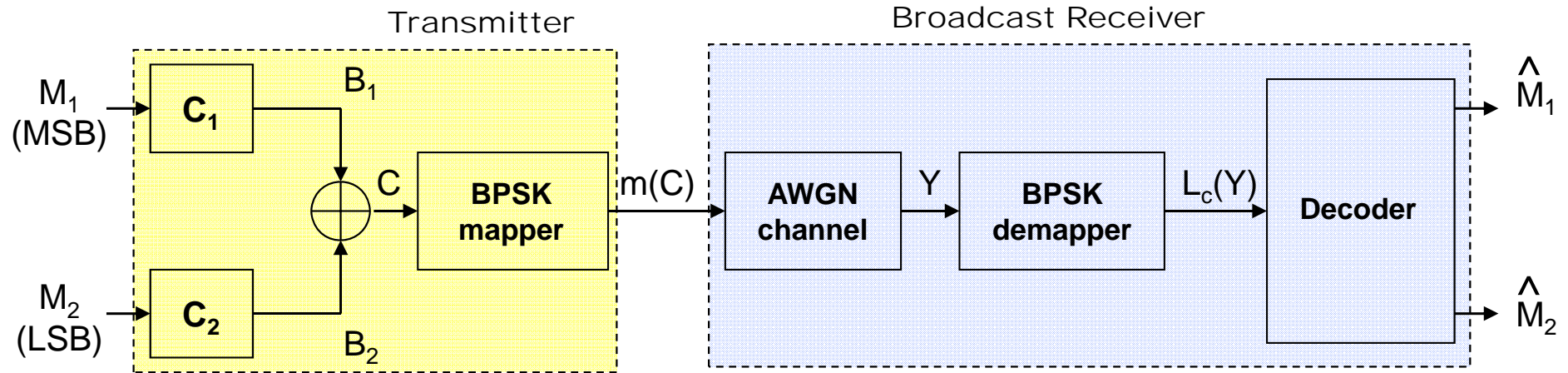


- At the receiver, the **direct-sum**  $M_1+M_2$  is equal to a ternary signal set:

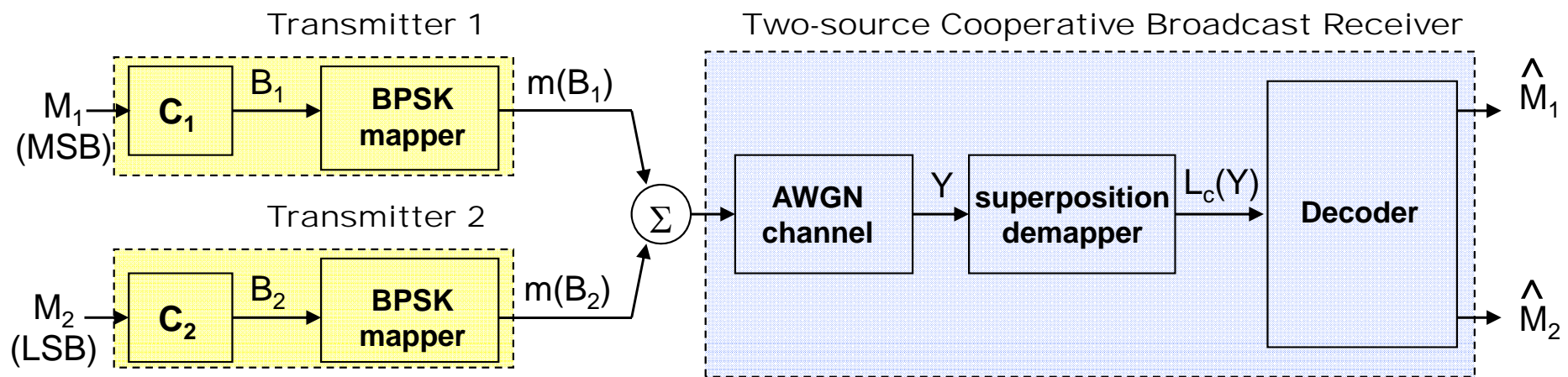


# Single-source and cooperative broadcasting

- Single source:



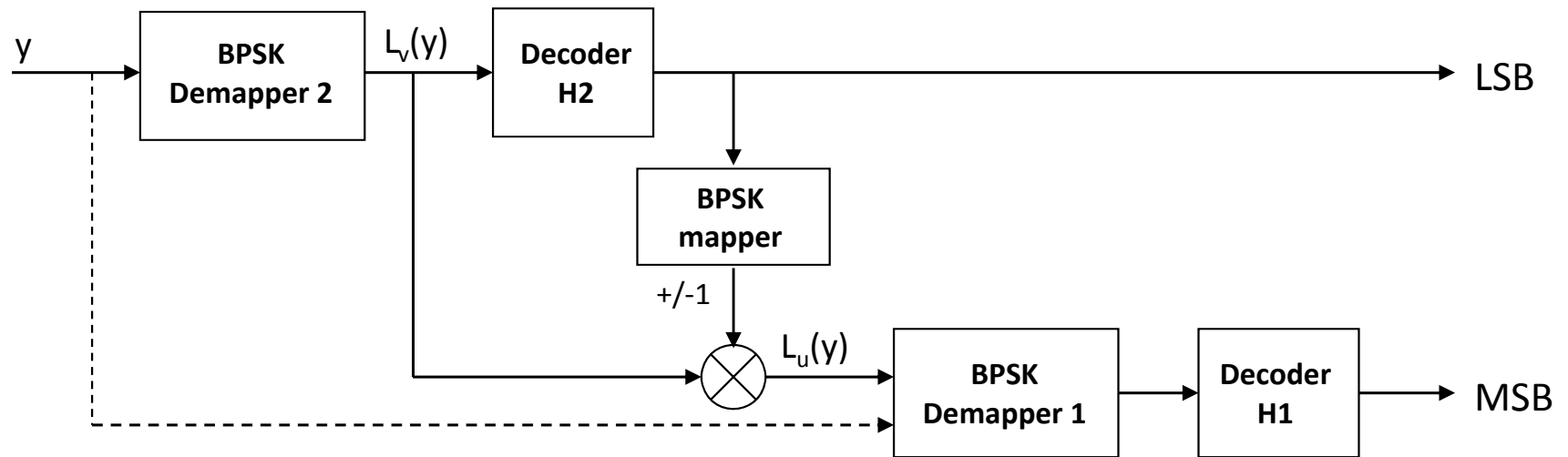
- Cooperative (two sources):



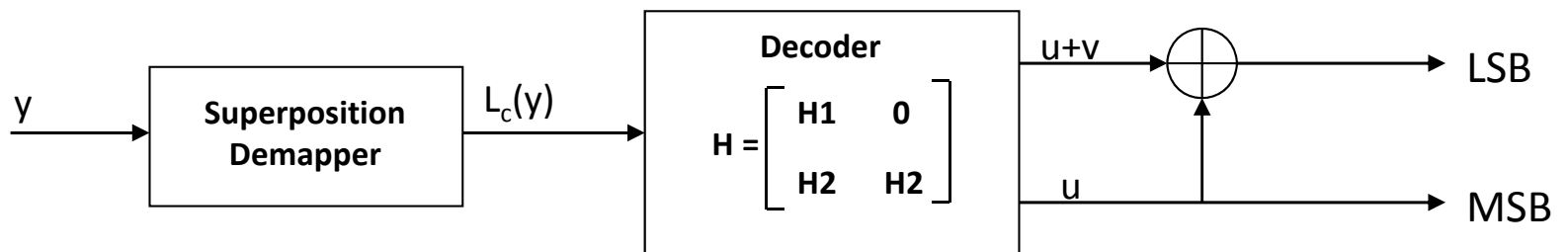


# Decoding for $|u|u+v|$ construction

## Two stages (Kumar-Milenkovic, 2006)

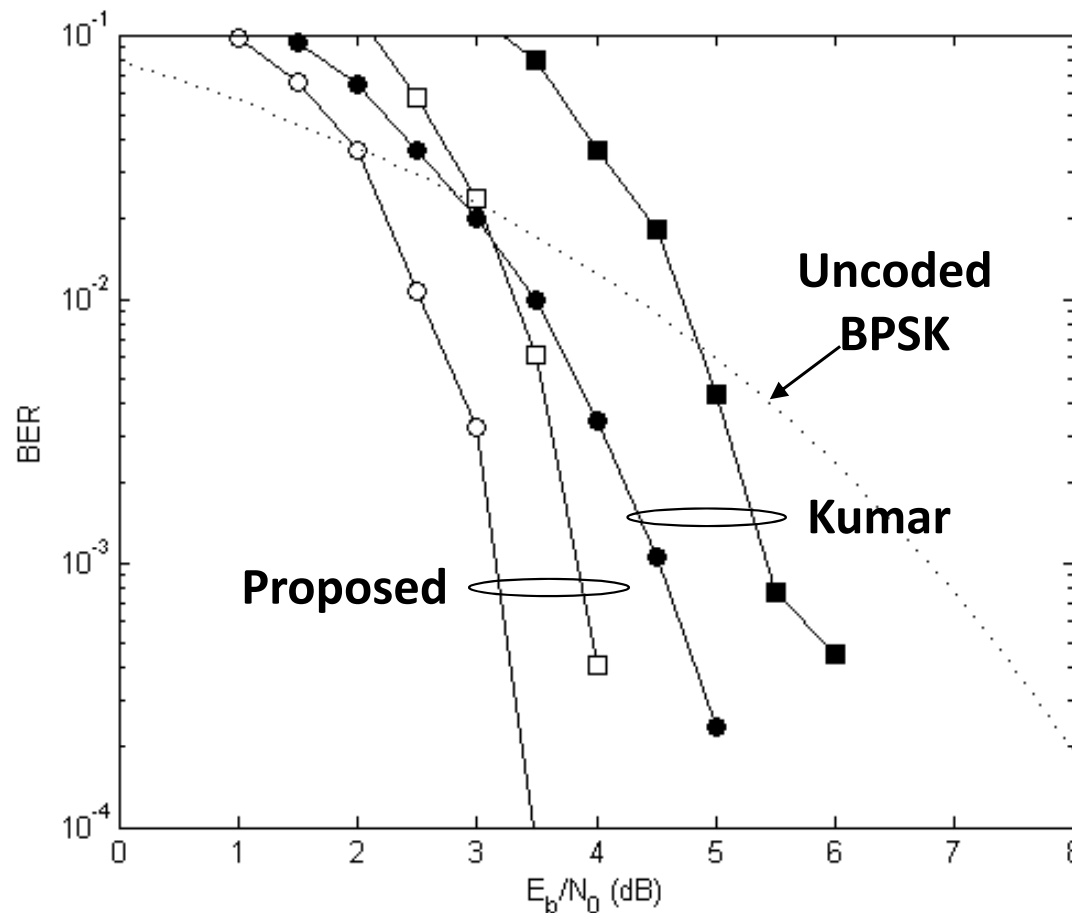


## Single stage (SJSU, 2009)

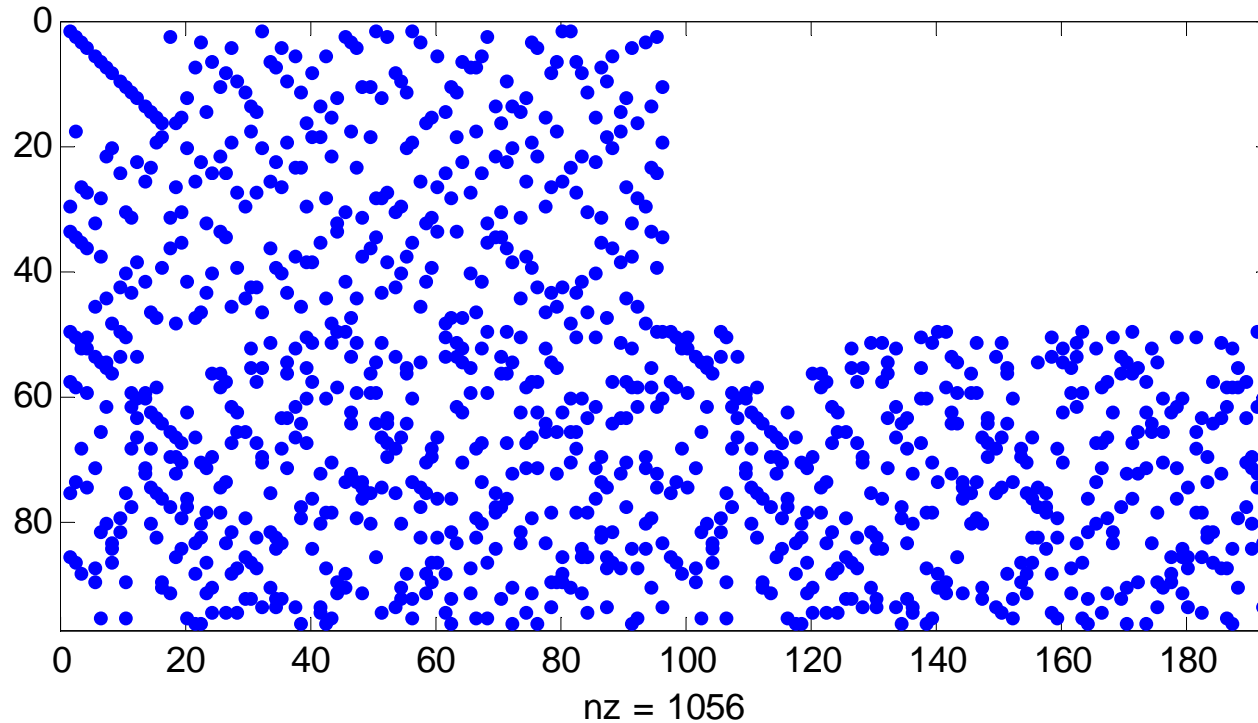


# $|u|u+v|$ decoding: Simulation results

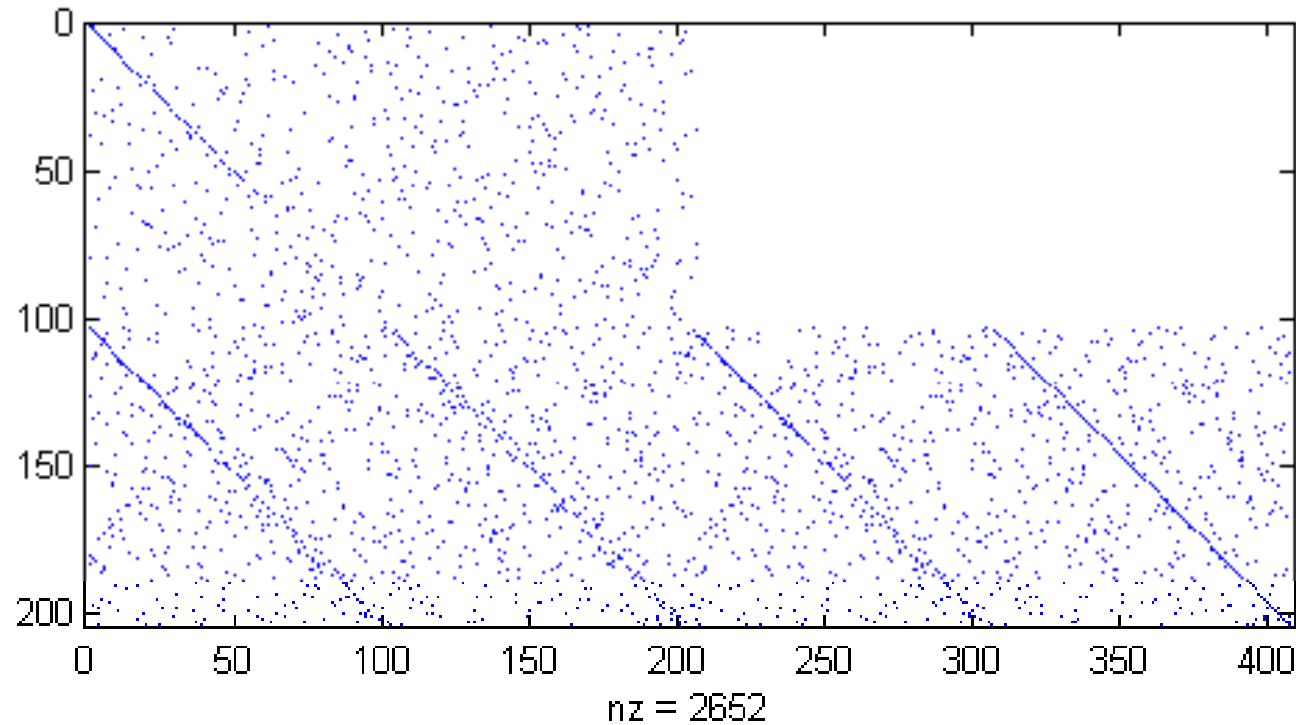
$C_1$ : Regular (96,50) LDPC code with degrees (3,6);  $C_2$ : Regular (96,49) LDPC codes with degrees (4,8)



# H matrix used in $|u|u+v|$ construction with LDPC codes of length 96



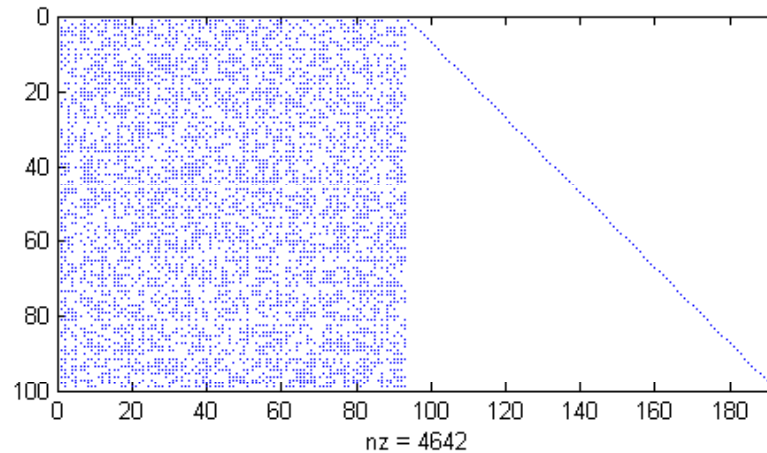
# H matrix used in $|u|u+v|$ construction with LDPC codes of length 204



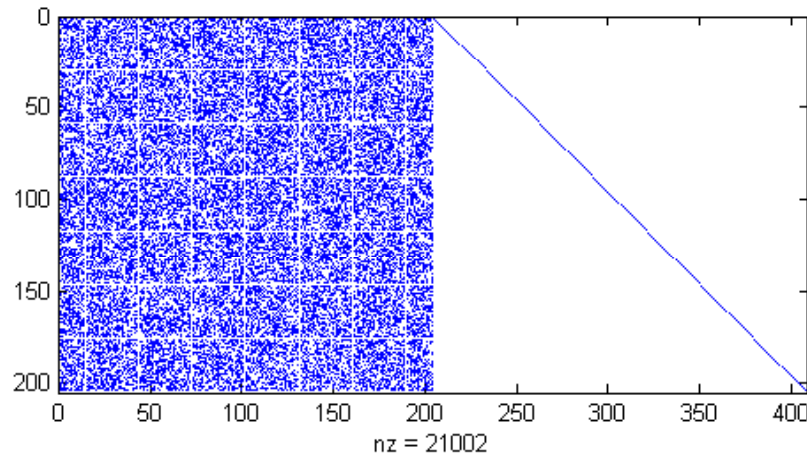
# Systematic encoding

- Use Gaussian elimination to produce systematic generator matrices and a permutation:

Length 96:



Length 204:



# Single-source versus cooperative: BPSK Metrics

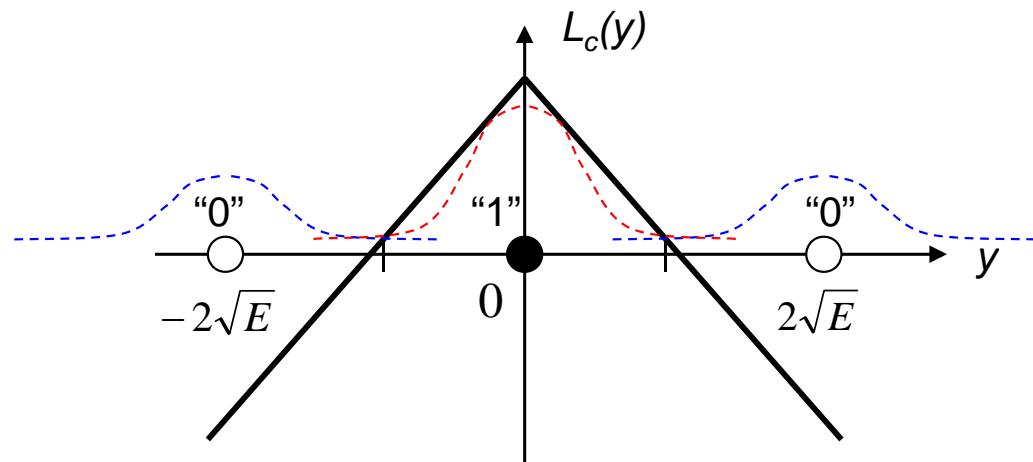
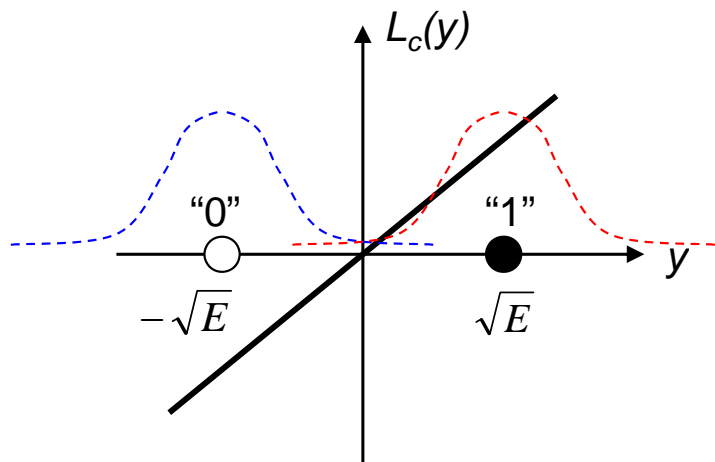
Single source

$B_1$	$B_2$	$C=B_1+B_2$	$y$
0	0	0	-1
0	1	1	+1
1	0	1	+1
1	1	0	-1

Cooperative (dual source)

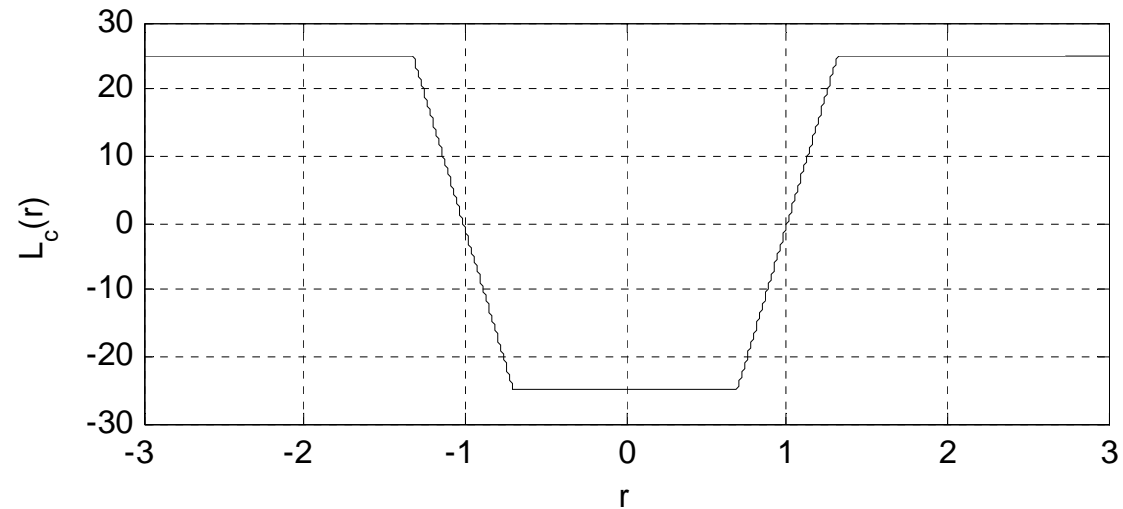
$B_1$	$B_2$	$y$
0	0	-2
0	1	0
1	0	0
1	1	-2

**Log-likelihood ratio (LLR) metric:** 
$$L_c(y) = \frac{\Pr\{c = 1|y\}}{\Pr\{c = 0|y\}}$$

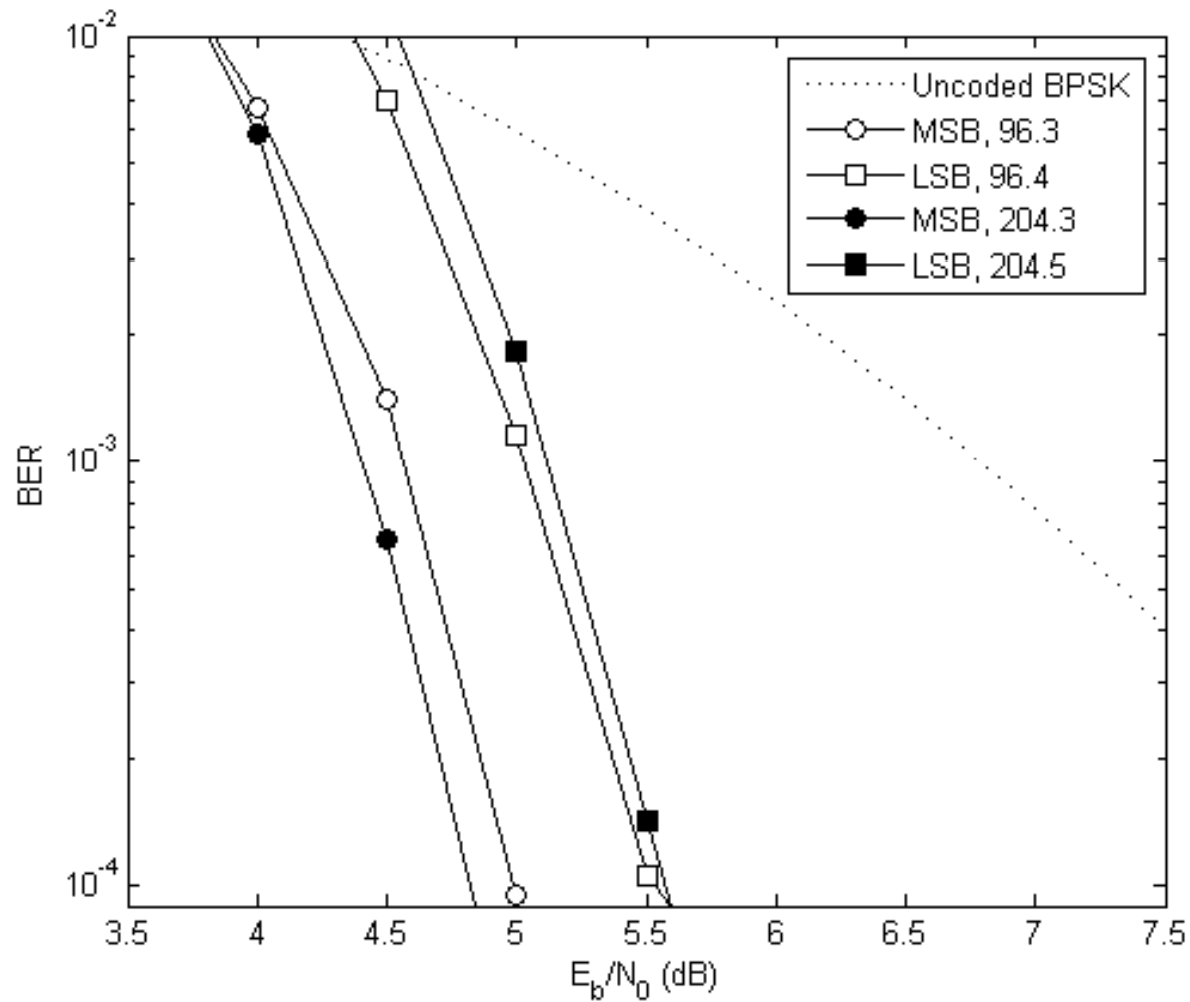


# LLR metrics for cooperative broadcasting and BPSK modulation ( $E/N_0=10$ dB)

B	m(B)
0	-1
1	1



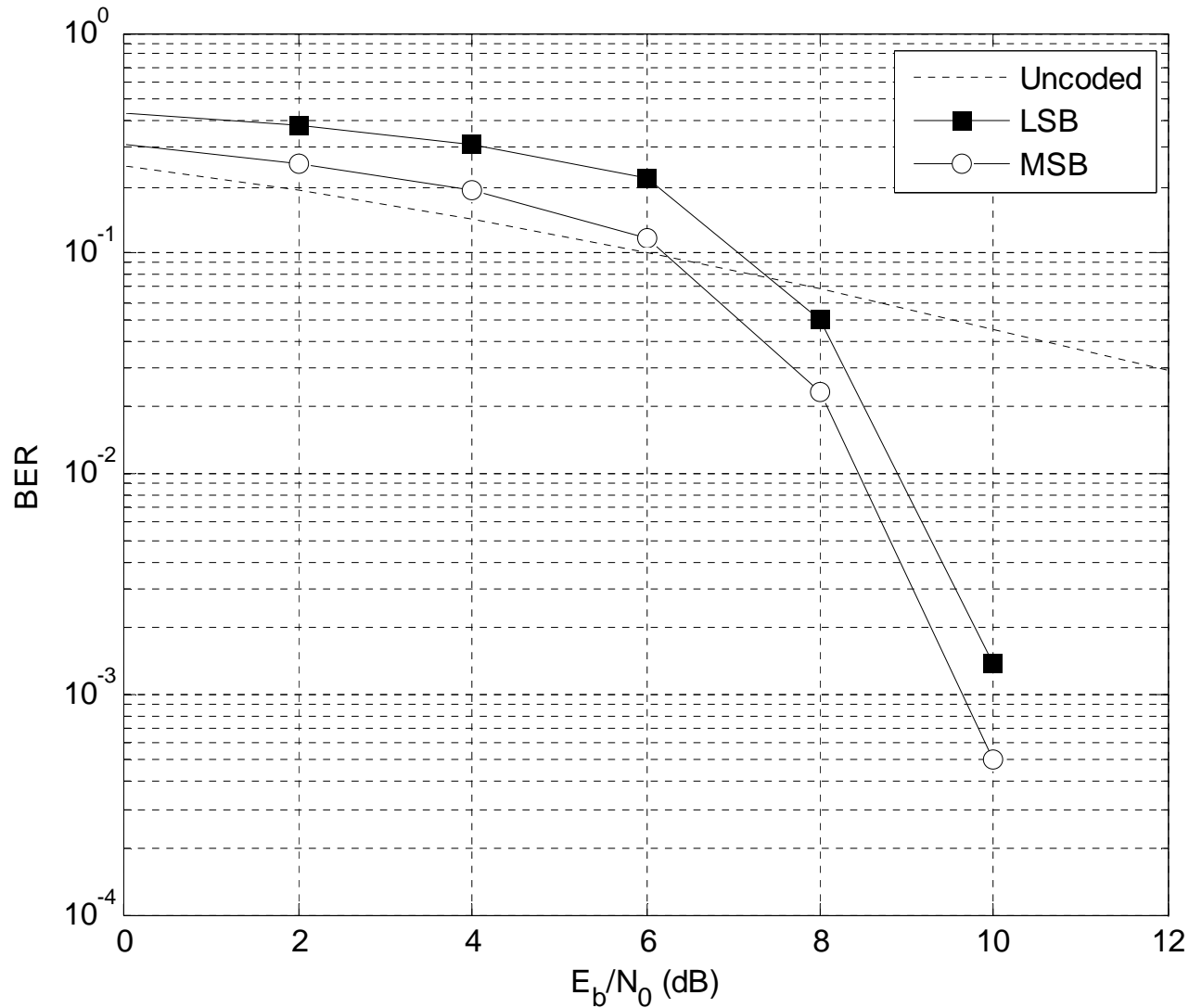
# Cooperative $|u|u+v|$ : Lengths 96 and 204 AWGN channel





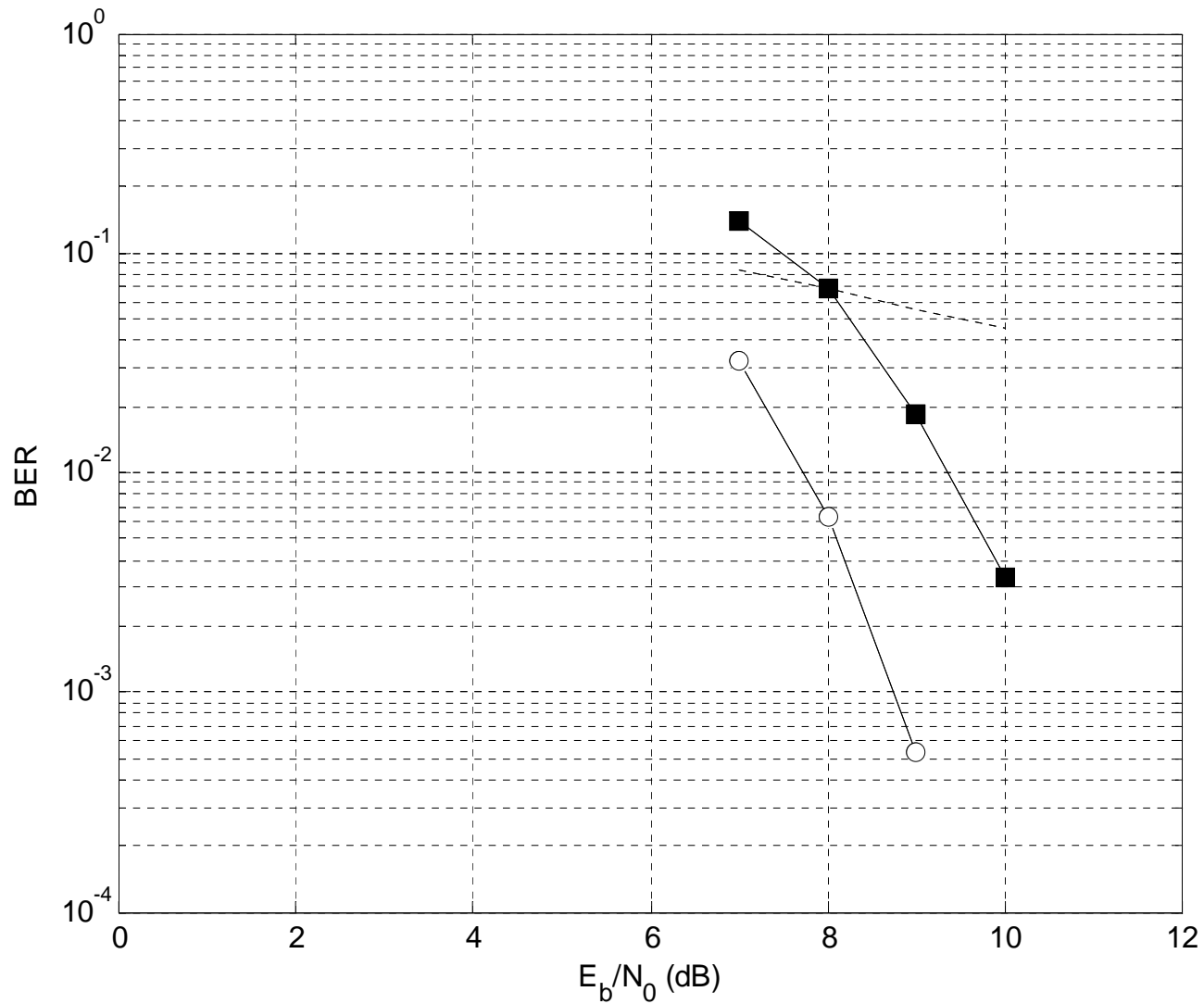
# Cooperative $|u|u+v|$ : Length 96

## Flat Rayleigh fading channel

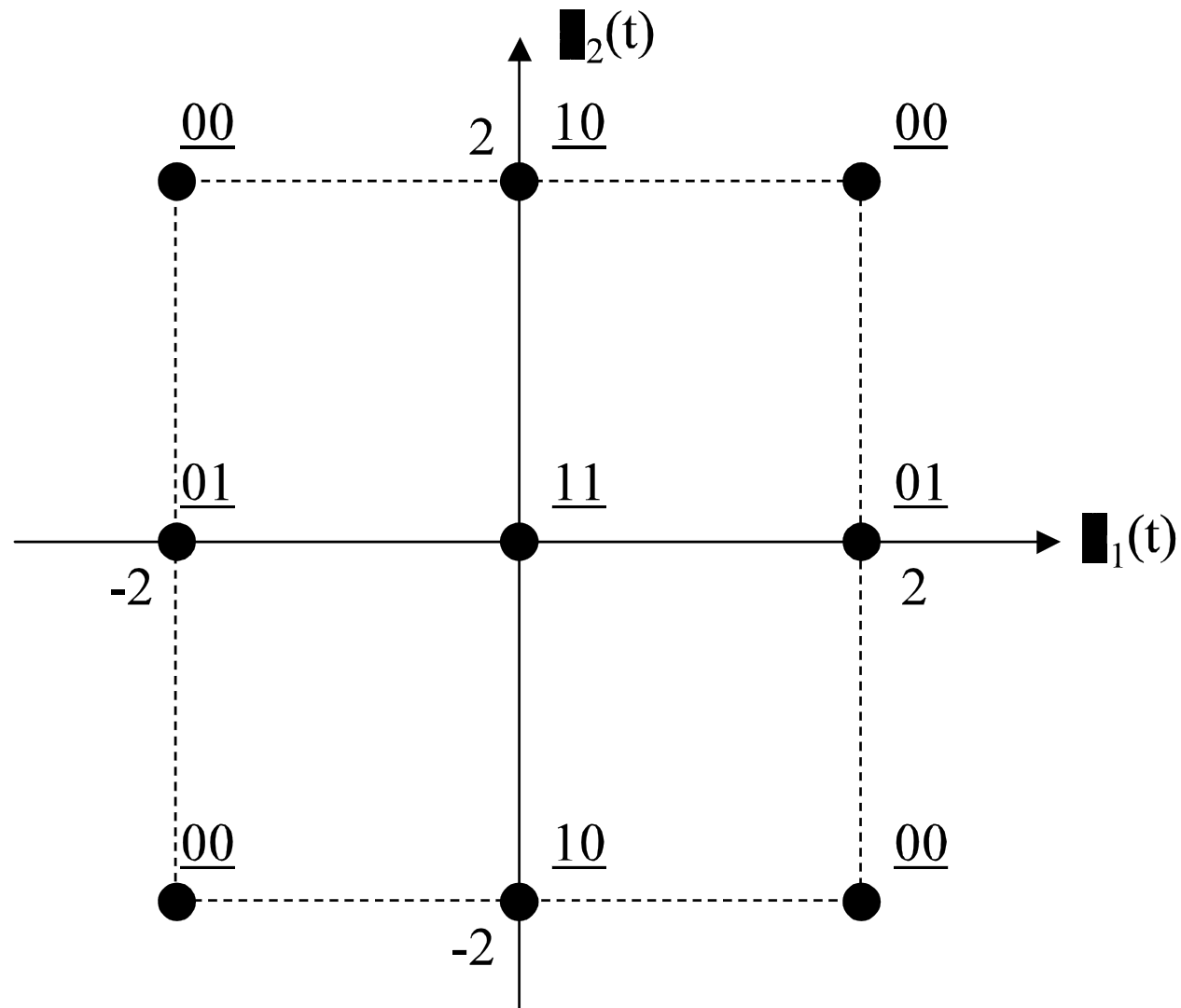


# Cooperative $|u|u+v|$ : Length 204

## Flat Rayleigh fading channel

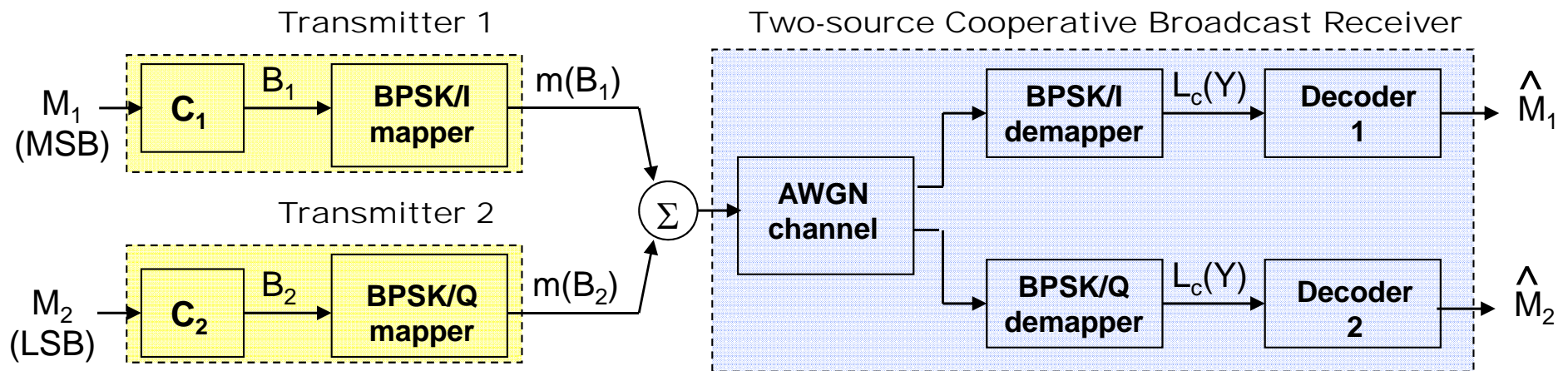


# Results are applicable to QPSK modulation



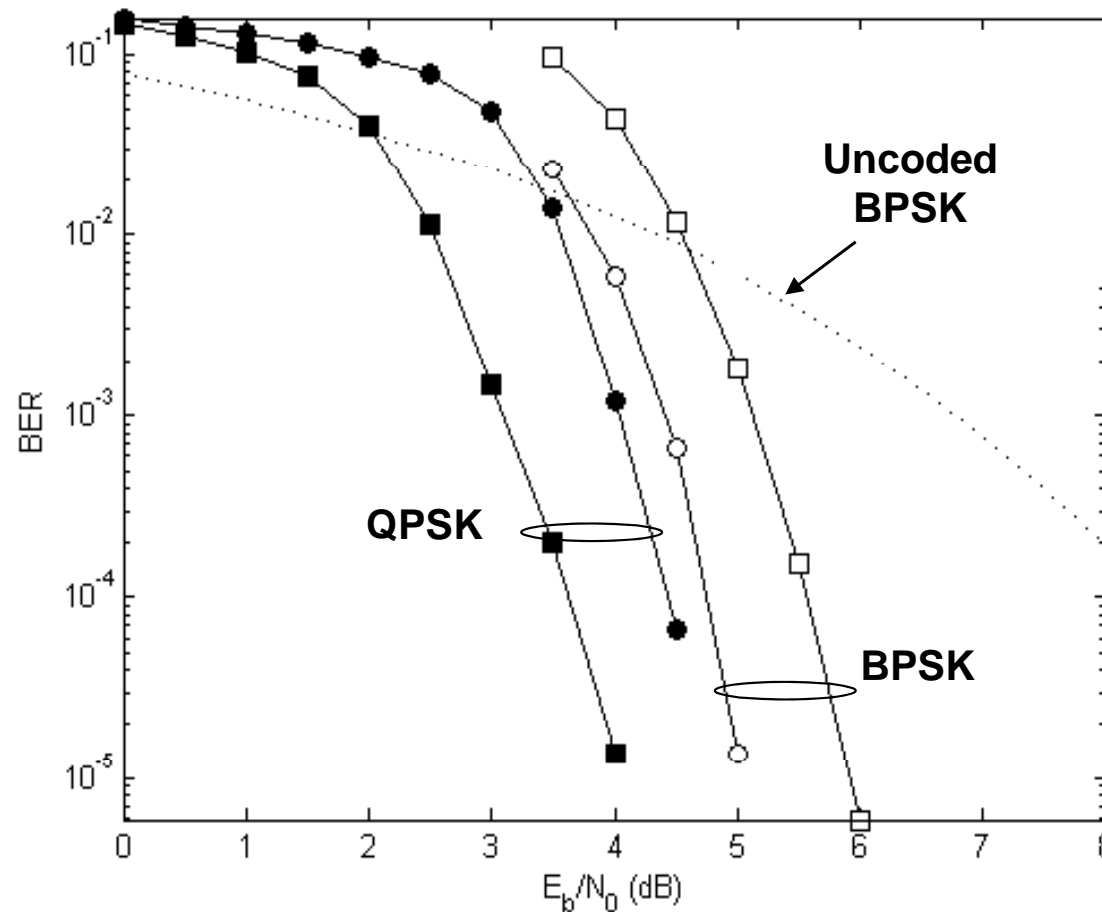
# QPSK = BPSK x BPSK: Subset mapping

- Two basis functions:  $\phi_1(t), \phi_2(t)$ .
- Basic idea: Quadrature multiplexing
- Source  $i$ : BPSK mapping with  $\phi_i(t), i=1,2$
- Receiver processes two BPSK sequences in parallel branches



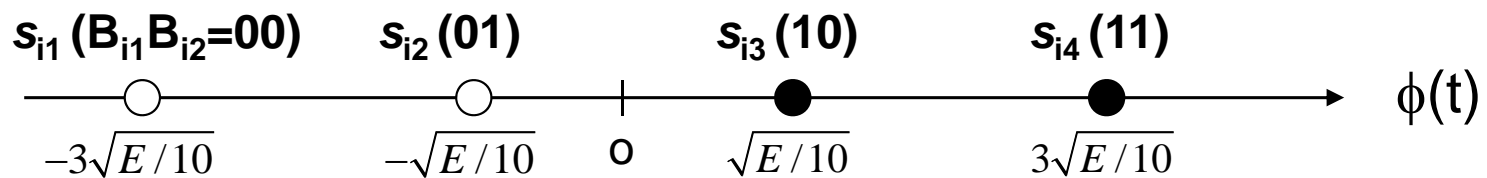
# QPSK subset vs Cooperative $|u|u+v|$ BPSK

$C_1, C_2$ : LDPC (204,102) codes of node degrees (3,6) and (5,10)

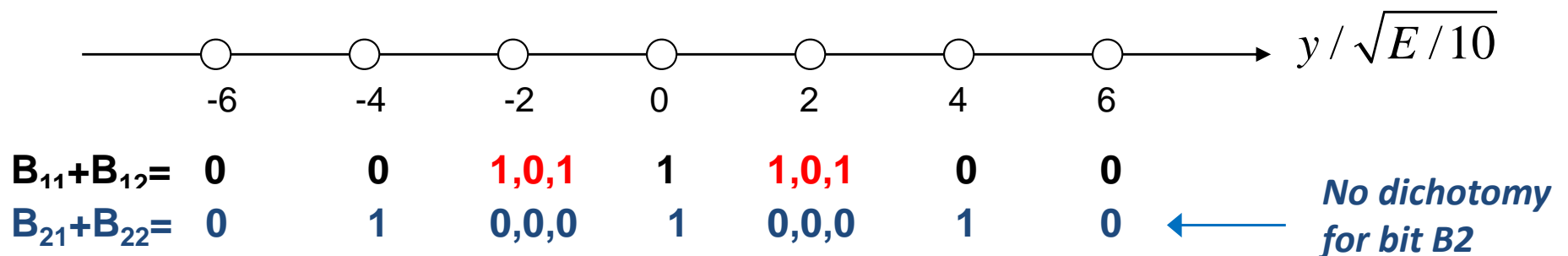


# Cooperative $|u|u+v|$ with 4-PAM and natural mapping

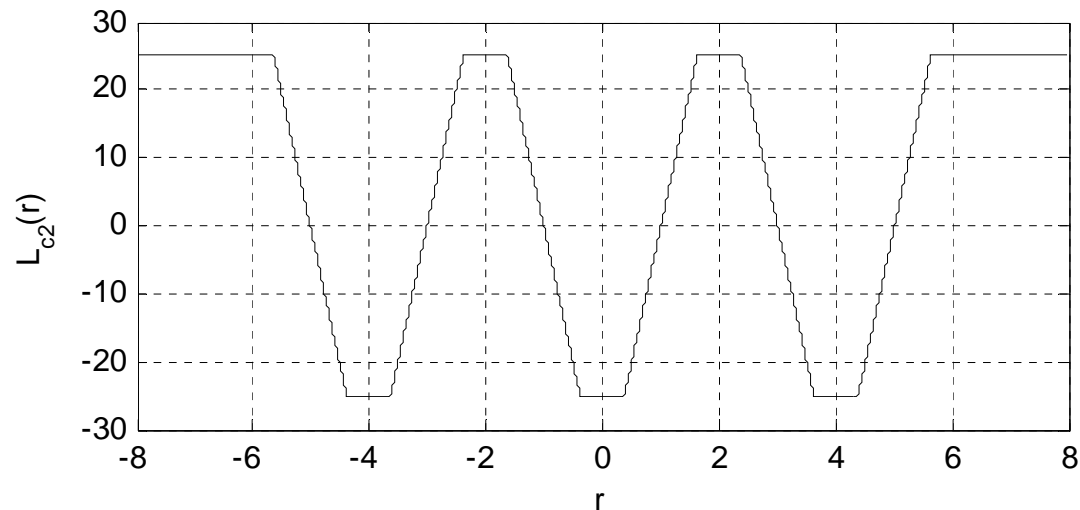
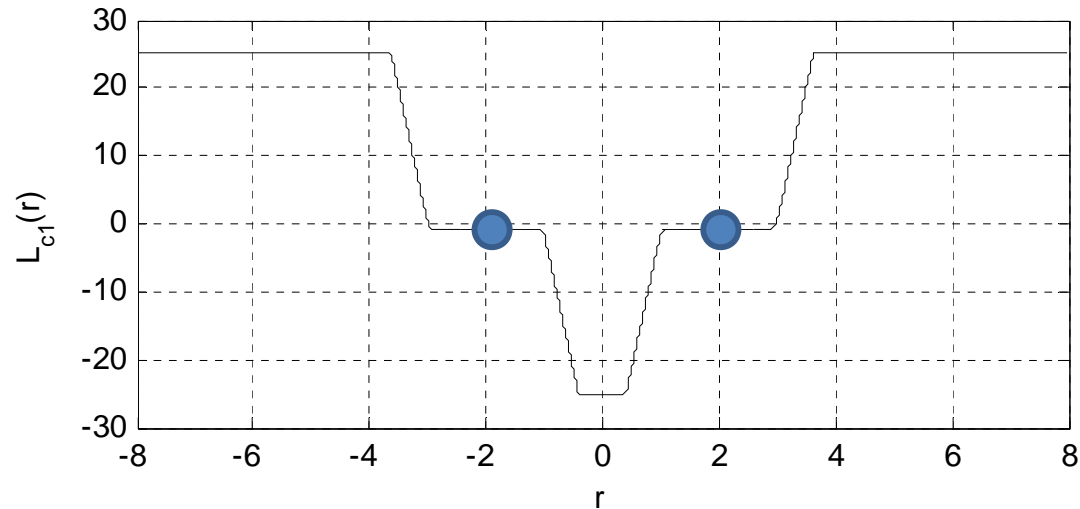
- **4-PAM Mapping**  $m_i$  from two bits to a signal set  $M_i$ ,  $i=1,2$ . Assume  $\alpha_{i1}=\alpha_{i2}$
- $M_1=\{s_{11},s_{12},s_{13},s_{14}\}$  and  $M_2=\{s_{21},s_{22},s_{23},s_{24}\}$  :



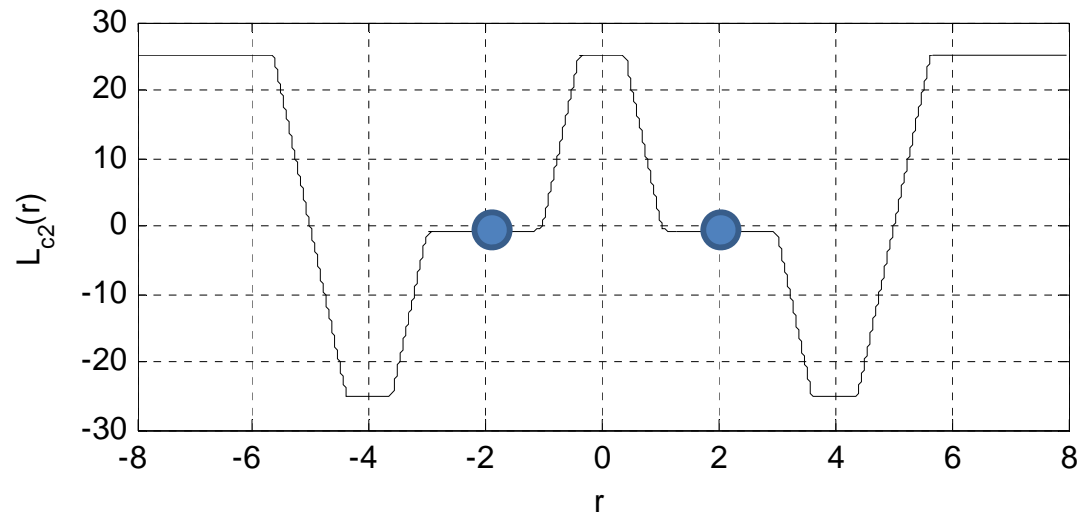
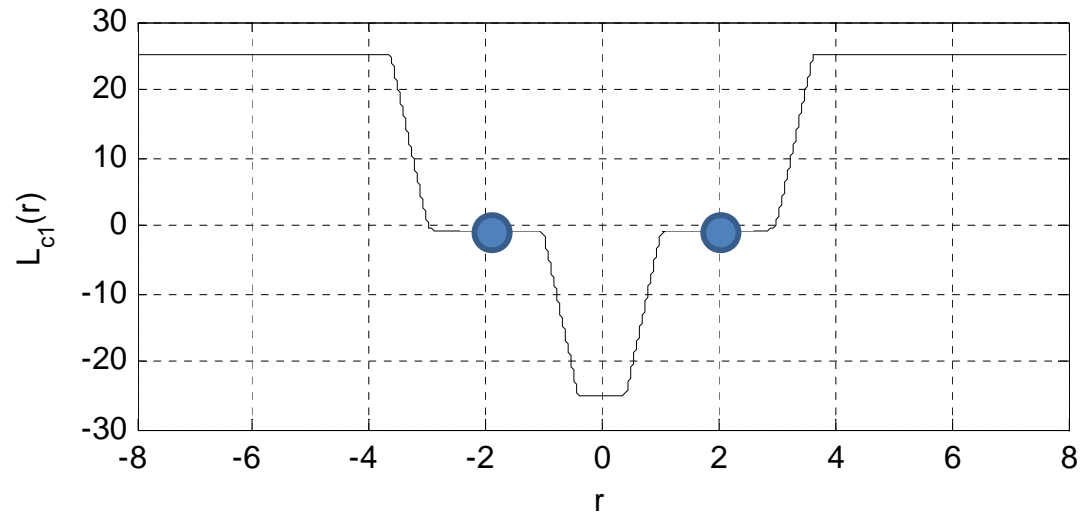
- At the receiver, the **direct-sum**  $M_1+M_2$  is equal to a 7-ary signal set:



# LLR metrics for cooperative broadcasting and 4-PAM modulation with natural mapping ( $E/N_0=10$ )



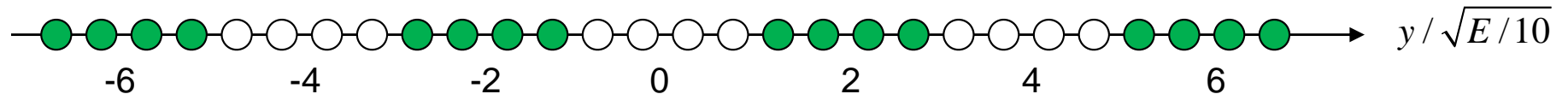
# Metrics for cooperative broadcasting and 4-PAM modulation with Gray mapping ( $E/N_0=10$ )





# Cooperative $|u|u+v|$ with 4-PAM and hierarchical mapping

- **4-PAM Mapping**  $m_i$  from two bits to a signal set  $M_i$ ,  $i=1,2$ . Assume  $\alpha_{i1}=\alpha_{i2}$
- Again, two sets:  $M_1=\{s_{11},s_{12},s_{13},s_{14}\}$  and  $M_2=\{s_{21},s_{22},s_{23},s_{24}\}$  with **two power levels**
- At the receiver, the **direct-sum**  $M_1+M_2$  is equal to a 28-ary signal set:

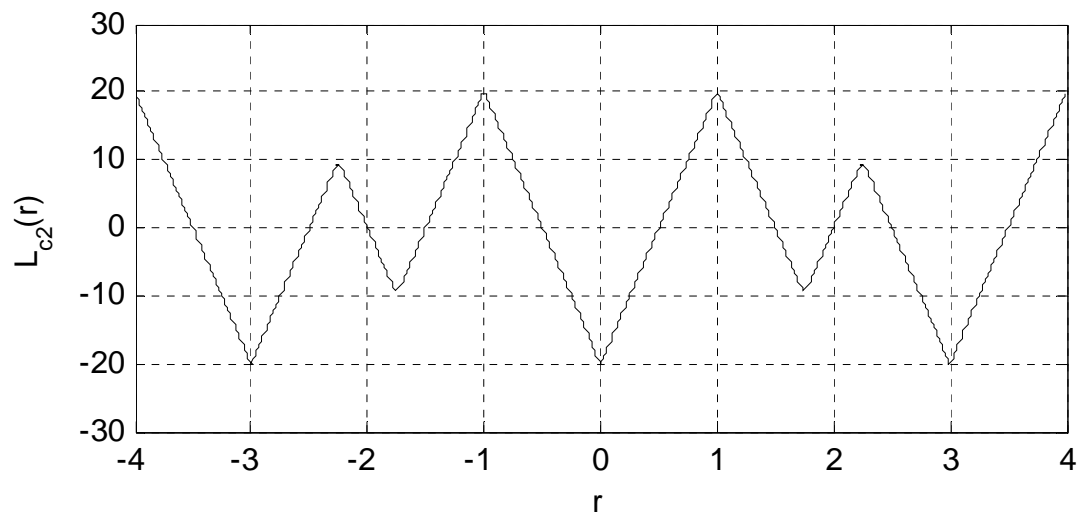
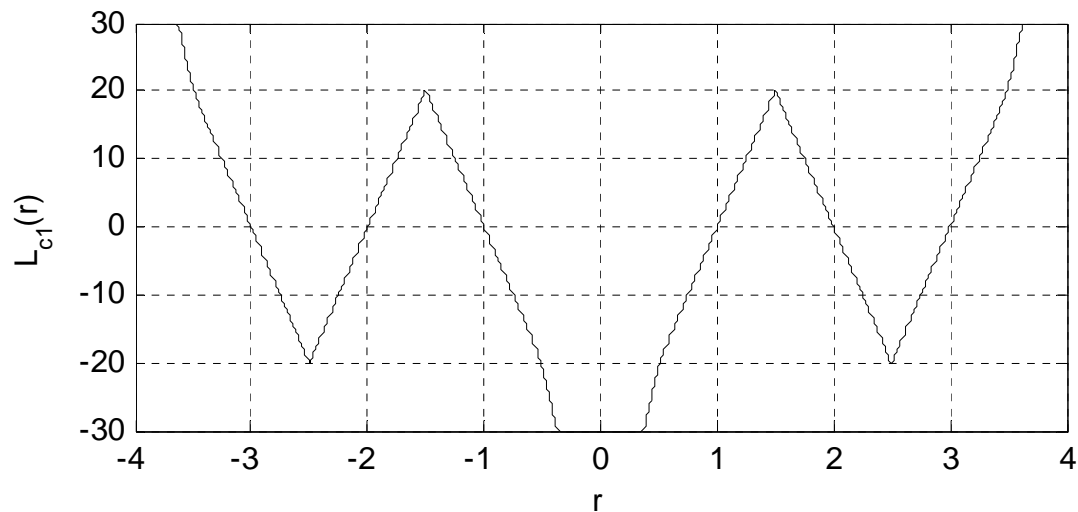


This idea was proposed by L. Xiao, T.E. Fuja, J. Kliewer and D.J. Costello (2009)

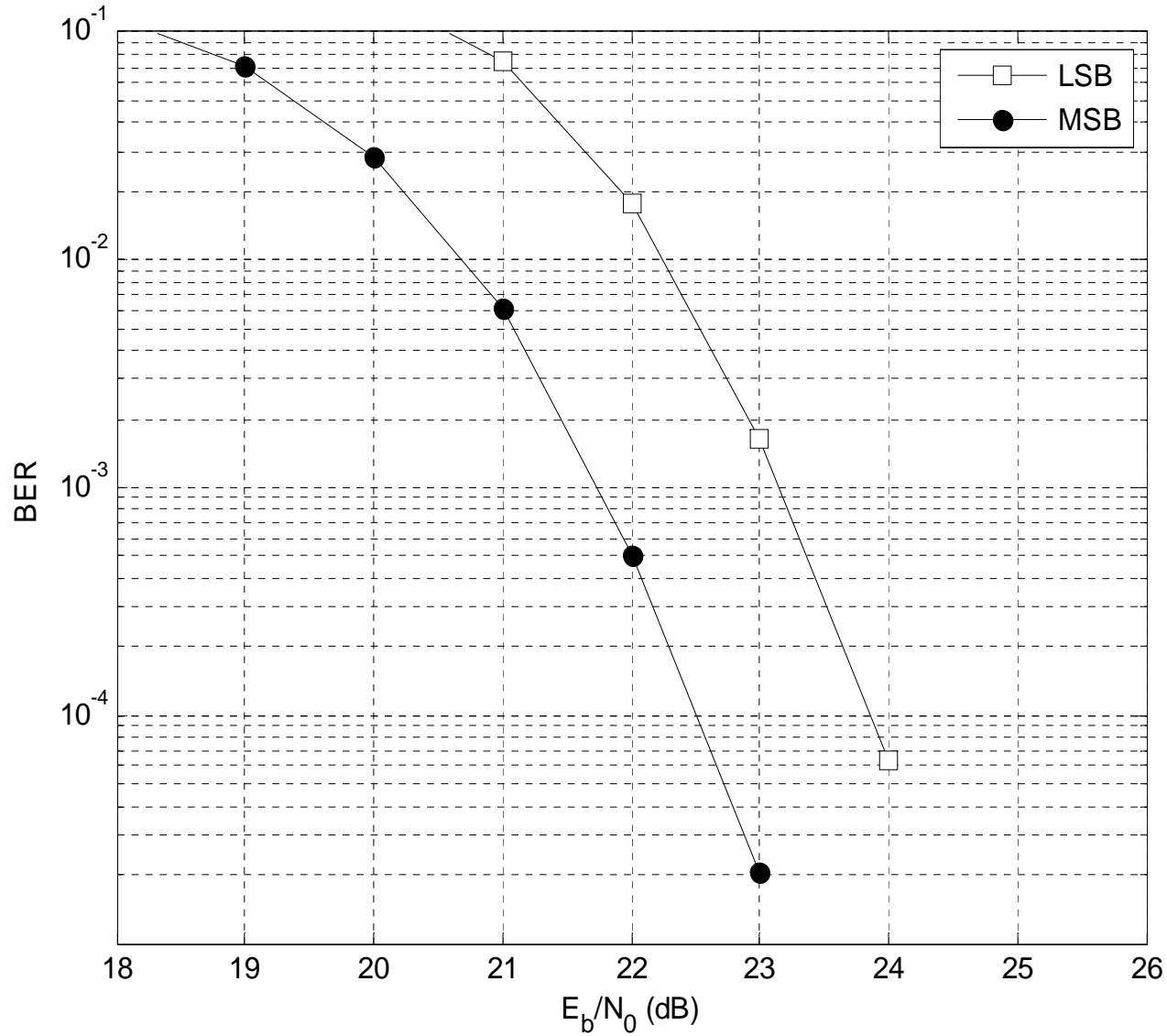
Note: In their scheme, superposition takes place at the transmitter ...

- All dichotomies (i.e., metric  $L_c=0$ ) are removed, in exchange for decreased error performance
- **Results are applicable to 16-QAM modulation**

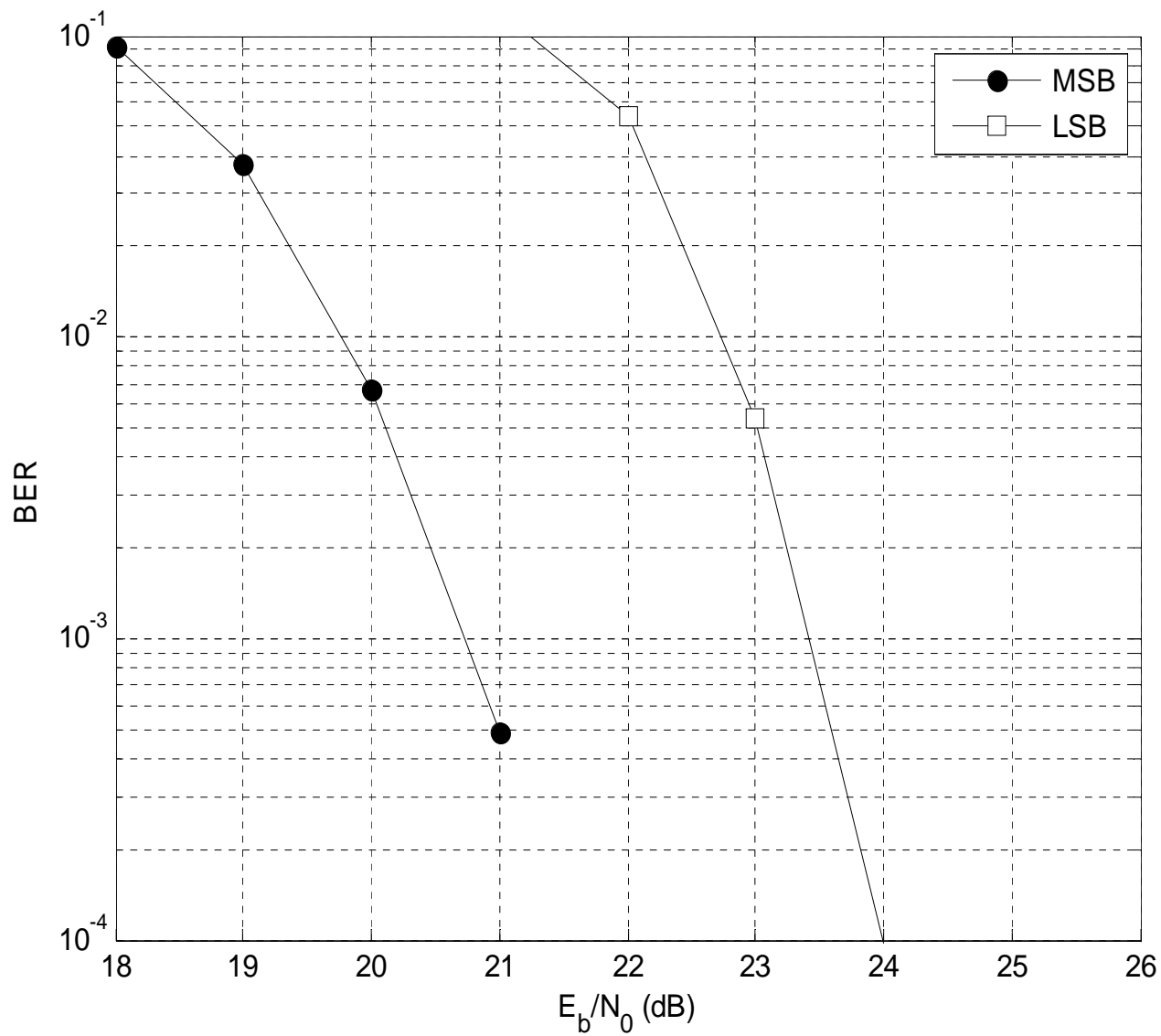
# Metrics for cooperative broadcasting with 4-PAM modulation and hierarchical mapping



# Performance of 4-PAM with hierarchical mapping. Length 96 LDPC codes



# Performance of 4-PAM with hierarchical mapping. Length 204 LDPC codes

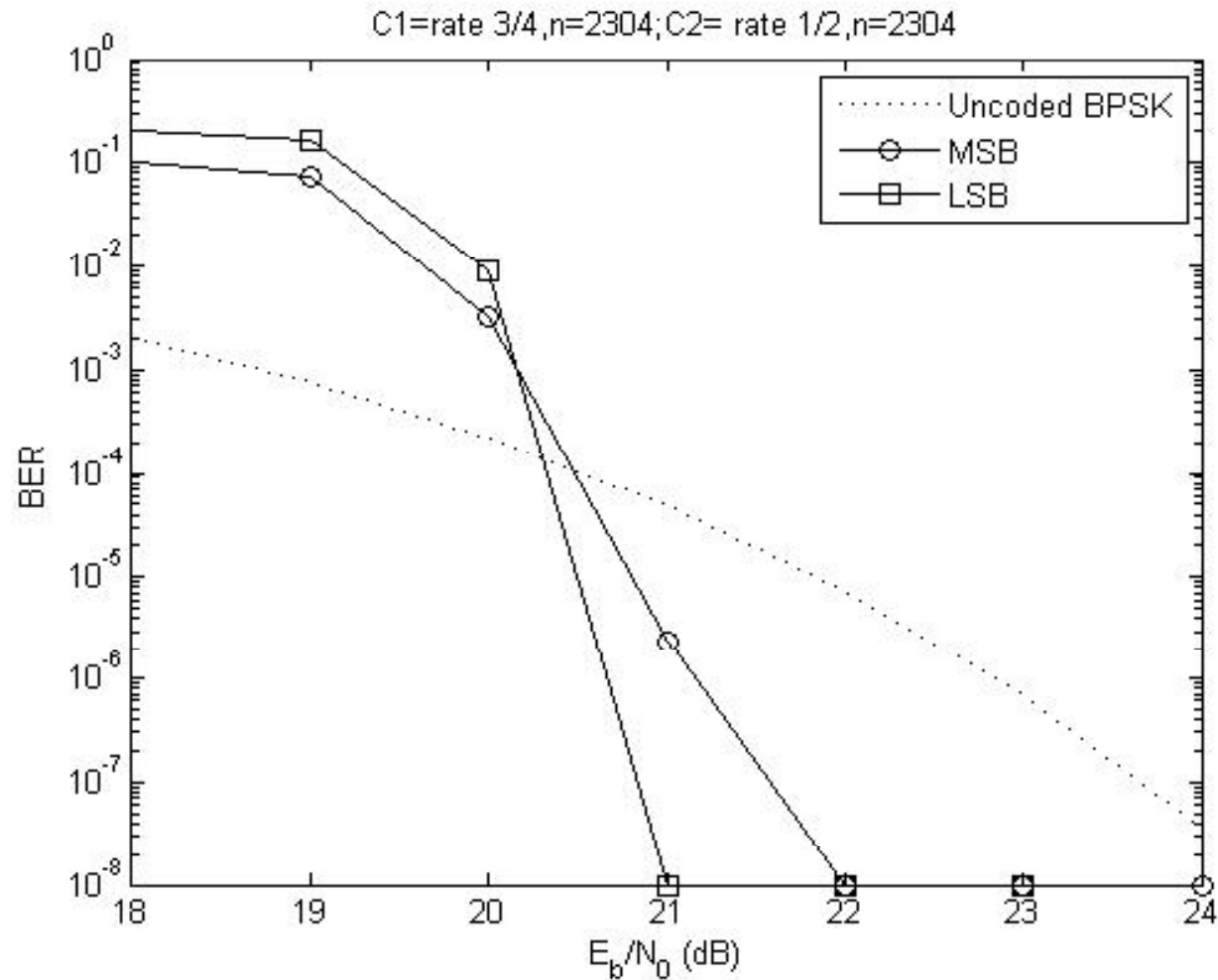


# Conclusions and future work

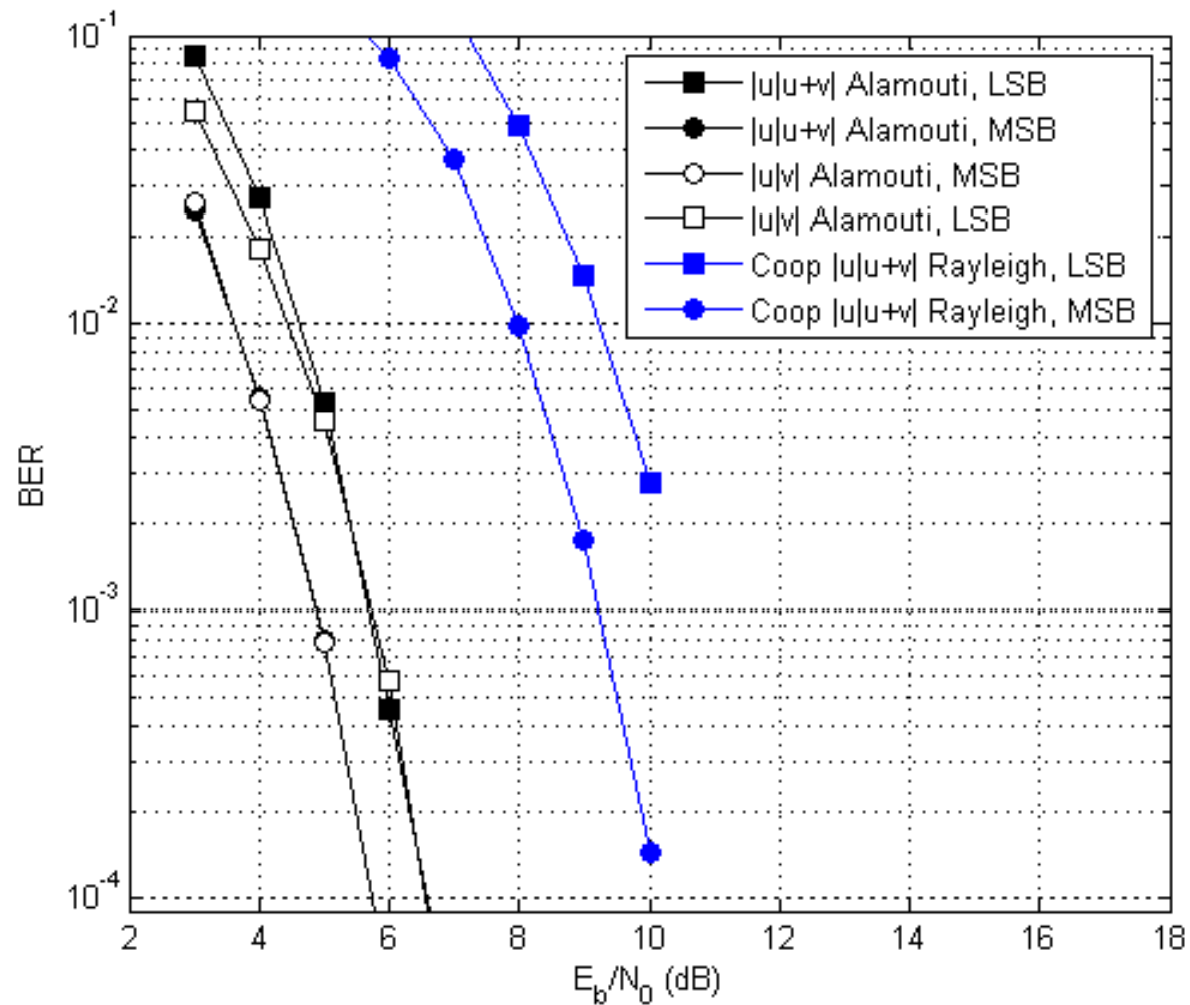
- Proposed a coding scheme for two-user cooperative broadcasting (“over-the-air mixing”), based on Plotkin’s  $|u|u+v|$  construction using BPSK, QPSK, 4-PAM and 16-QAM modulations
- ***Cooperative broadcasting = Network coding over physical layer***  
MILCOM 2010 presentation, comment from Matthew C. Valenti: (His paper was on “Receiver Design for Noncoherent *Digital Network Coding*”)
- Future directions
  - Performance with ***longer LDPC codes*** (such as those used in WiMax)
  - Design rules based on LDPC code parameters (minimum distance, node distributions) versus proportion of MSB and performance
  - Use ***a software radio*** platform to study
    - Synchronization techniques for over-the-air mixing
    - Effect of channel estimation errors
    - Error performance over realistic (frequency-selective) wireless channels
    - Noncoherent modulation: Differential encoding, FSK (as in Valenti’s paper)

# Supporting slides

# Performance of WiMax codes



# Combining cooperative $|u|u+v|$ with Alamouti: Length 96 codes





# Combining cooperative $|u|u+v|$ with Alamouti: Length 204 codes

