1994

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BLOCK QPSK MODULATION CODES WITH TWO LEVELS OF ERROR PROTECTION

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Abstract: A class of block QPSK modulation codes for unequal error protection (UEP) is presented. These codes are particularly suitable either for broadcast channels or for communication systems where parts of the information messages are more important than others. An example of the latter is coded speech transmission. Not much is known on the application of block UEP codes in combined coding and modulation schemes. We exhibit a method to combine binary linear UEP (LUEP) block codes of even length, using a Gray mapping, with a QPSK signal set to construct efficient block QPSK modulation codes with nonuniform error protection capabilities for bandwidth efficient transmission over AWGN (additive white Gaussian noise) and Rayleigh fading channels.

I. INTRODUCTION

In recent years, coded modulation schemes that offer nonuniform error protection have received considerable attention. Application examples of these schemes are broadcast of digital high-definition television signals [1][2], and transmission of coded speech and image [3][4][5][6]. In the former application, good receiver quality is required for the important data under bad channel conditions (e.g., distant receivers), while in the latter some of the source information bits are more sensitive to errors than the other bits. A code that offers different levels of error protection is called an *unequal error protection* (UEP) code. Linear UEP codes, or LUEP codes, were introduced by Masnik and Wolf [7].

In this work, we use binary LUEP block codes in conjunction with QPSK signal constellations, to obtain new efficient block modulation codes for unequal error protection. The purpose is to obtain code sequences associated with the most important message bits separated by a distance greater than the minimum distance of the modulation code. By *distance* we mean (1) *squared Euclidean distance (SED)* when transmission is over an AWGN channel, or (2) *symbol and product distances* for transmission over a Rayleigh fading channel. We show that as a result of accomplishing the above objective, with transmission over an AWGN channel or a Rayleigh fading channel, the most important (or more sensitive to errors) message bits have a *probability of a bit error* lower than the minimum probability of a bit error guaranteed by the modulation code. Several examples of block QPSK modulation codes with two levels of error protection, having the same minimum squared Euclidean distance (MSED) as that of optimal block QPSK modulation codes for the AWGN channel of the same rate and length [8], are presented. Because a Gray mapped QPSK signal set is used, maximizing the minimum Hamming distance of the underlying binary LUEP code maximizes both the MSED for an AWGN channel and the minimum symbol and product distances for a Rayleigh fading channel.

II. BINARY TWO-LEVEL LUEP CODES

Let C be an (n, k) binary linear block code. As usual, an element $\bar{m}$ from $\{0,1\}^k$ is called a *message*, and an element $\bar{c}(\bar{m})$ from C is called a *codeword*. Let the *message space* $\{0,1\}^k$ be decomposed into the direct product of two disjoint *message subspaces*, $\{0,1\}^{k_i}$, $i = 1,2$, such that $\{0,1\}^k = \{0,1\}^{k_1} \times \{0,1\}^{k_2}$. Then a message can be written as $\bar{m} = (\bar{m}_1, \bar{m}_2)$, $\bar{m}_i \in \{0,1\}^{k_i}$, $i = 1,2$. The *separation vector* of C is defined as the two-tuple $\bar{s} = (s_1, s_2)$, where

$$s_i \triangleq \min\{\mathrm{wt}(\bar{c}(\bar{m})) : \bar{m}_i \neq \bar{0}, \quad \bar{m}_i \in \{0,1\}^{k_i}\},$$

$i = 1,2$, where $\mathrm{wt}(\bar{x})$ denotes the Hamming weight (number of nonzero entries) of vector $\bar{x}$. We assume that code C has both components of its separation vector distinct and arranged in decreasing order, $s_1 > s_2$, and call $\bar{m}_1$ the *most important* message part (or MSB) and $\bar{m}_2$ the *least important* message part (or LSB).

Code C is said to be an (n, k) binary *two-level LUEP code* with separation vector $\bar{s} = (s_1, s_2)$, for the message space $\{0,1\}^{k_1} \times \{0,1\}^{k_2}$. In terms of levels of error correction, k_i information bits can be successfully decoded in the presence of up to $\lfloor (s_i - 1)/2 \rfloor$ random errors,

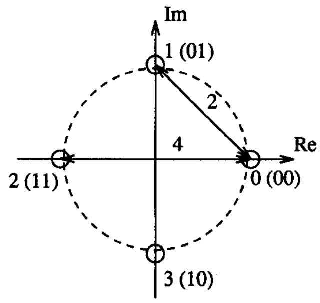

FIGURE 1: A GRAY MAPPED QPSK SIGNAL SET.

$i = 1, 2$ [7], where $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.

For $i = 1, 2$, let C_i be an (n, k_i, d_i) binary linear code. For two binary vectors $\bar{\mathbf{u}} = (u_0, u_1, \cdots, u_{n-1})$ and $\bar{\mathbf{v}} = (v_0, v_1, \cdots, v_{n-1})$, define the *concatenation* operation $\bar{\mathbf{u}} \circ \bar{\mathbf{v}}$ as

$$\bar{\mathbf{u}} \circ \bar{\mathbf{v}} \stackrel{\triangle}{=} (u_0, u_1, \cdots, u_{n-1}, v_0, v_1, \cdots, v_{n-1}).$$

Then the following code, based on C_1 and C_2,

$$\mu(C_1, C_2) = \{\bar{\mathbf{w}} \ : \ \bar{\mathbf{w}} = \bar{\mathbf{v}} \circ (\bar{\mathbf{u}} + \bar{\mathbf{v}}), \quad \bar{\mathbf{u}} \in C_1, \bar{\mathbf{v}} \in C_2\},$$

is a $(2n, k_1 + k_2)$ binary linear code. This combination of linear codes is a modified version of the well known $|\bar{\mathbf{u}}|\bar{\mathbf{u}} + \bar{\mathbf{v}}|$ construction [9], and it can be shown (see [9]) that the minimum distance of code $\mu(C_1, C_2)$ is $d = \min\{2d_2, \max\{d_1, d_2\}\}$. The following result is known [10]:

Theorem: Suppose $d_1 > 2d_2$. Then $\mu(C_1, C_2)$ is a binary $(2n, k_1 + k_2)$ two-level LUEP code with separation vector $\bar{\mathbf{s}} = (\mathbf{s}_1, \mathbf{s}_2)$, for the message space $\{0,1\}^{k_1} \times \{0,1\}^{k_2}$, where

$$\mathbf{s}_1 = \min\{\max\{d_1, d_2\}, d_1\} = d_1, \text{ and}$$

$$\mathbf{s}_2 = \min\{2d_2, \max\{d_1, d_2\}\} = 2d_2.$$

III. LUEP QPSK MODULATION CODES

In a (unit energy) QPSK signal constellation with *Gray mapping* between 2-bit labels and signal points, as illustrated in Figure 1, the squared Euclidean distance (SED) between signal points is *proportional to* the Hamming distance between the corresponding labels. For example, in Figure 1, the Hamming distance between the label (00) of signal point "0" and the label (01) of signal point "1" is equal to 1, while the SED between these signal points is 2. All adjacent signal points have labels separated by a Hamming distance of 1 and are separated by an SED of 2, while opposite signal points (e.g., "0" and "2") have labels separated by a Hamming distance of 2 and are at an SED of 4 from each other. This QPSK signal constellation is said to form a *second-order Hamming space* [11]. By mapping 2-bit symbols onto signal points in a QPSK signal set, via Gray mapping, we may combine $(2n, k_1 + k_2)$ binary 2-level LUEP codes with QPSK modulation to construct a block coded modulation system with two levels of error protection as follows:

Let C_b be a $(2n, k_1 + k_2)$ binary LUEP code with separation vector $\bar{\mathbf{s}} = (\mathbf{s}_1, \mathbf{s}_2)$ for the message space $\{0,1\}^{k_1} \times \{0,1\}^{k_2}$. Let S denote the QPSK signal set depicted in Figure 1 and use the following (Gray) mapping $\mathbf{M}$ between 2-bit symbols and S: $M(00) = 0$, $M(01) = 1$, $M(11) = 2$ and $M(10) = 3$. Let

$$\begin{aligned} \mathbf{M}(C_b) \ \stackrel{\triangle}{=} \ \{ & (s_0, s_1, \cdots, s_{n-1}) : s_i \in S, \\ & s_i = M(c_{2i}c_{2i+1}), \text{ and} \\ & (c_0, c_1, \cdots, c_{2n-1}) \in C_b\}. \end{aligned}$$

Then $C = \mathbf{M}(C_b)$ is a 2-level LUEP QPSK block modulation code of length n, dimension k, rate $R = k/2n$ (bits/dimension), and *squared Euclidean separation vector* [12]

$$\bar{\mathbf{S}}_{SED} = (2\mathbf{s}_1, 2\mathbf{s}_2).$$

In conventional coded modulation for an AWGN channel, the *asymptotic coding gain* G is a function of the minimum squared Euclidean distance (MSED) and the rate of a modulation code. For high signal-to-noise ratios (SNR), G equals the ratio of the MSED of the coded system to the MSED of an uncoded system transmitting at the same rate (number of bits per signal). Accordingly, for each component of $\bar{\mathbf{S}}_{SED}$ above, we may define an asymptotic coding gain component. For QPSK modulation over AWGN channels at high SNR, we define the *asymptotic coding gain vector* of C as

$$\bar{G} = (G_1, G_2),$$

where, for $i = 1, 2$,

$$G_i = 10 \log_{10} \left[\frac{2\mathbf{s}_i}{4 \sin^2 (\pi/2^R)} \right] \quad \text{(dB)}.$$

We note that, as in the case of conventional uniform error protection coded modulation systems, the above asymptotic coding gains can only be reached with *maximum-likelihood soft-decision decoding.*

In Table 1 we list some block QPSK modulation codes with two levels of error protection. Some of the

TABLE 1
SOME LUEP QPSK MODULATION CODES

$2n$	k	k_1	k_2	s_1	s_2	R	G_1	G_2	
10	5	1	4	5	4	1/2	3.98	3.01	⋆
10	8	1	7	3	2	4/5	3.06	1.30	
12	6	1	5	6	4	1/2	4.77	3.01	⋆
12	6	2	4	5	4	1/2	3.98	3.01	
12	9	1	8	4	2	3/4	3.96	0.95	
12	10	1	9	3	2	5/6	3.32	1.56	
14	7	1	6	7	4	1/2	5.44	3.01	⋆
14	7	4	3	5	4	1/2	3.98	3.01	
14	8	1	7	5	4	4/7	4.07	3.10	
14	11	1	10	4	2	11/14	4.21	1.20	
14	11	4	7	3	2	11/14	2.96	1.20	

codes in Table 1 have the same minimum squared Euclidean distance as that of *optimal* block QPSK modulation codes for an AWGN channel with the same rate and length [13], and provide additional coding gain (smaller probability of bit error) for the k_1 most important message bits. These codes are labeled ⋆ in Table 1 and are obtained from the modified version of the $|\bar{u}|\bar{u}+\bar{v}|$ construction discussed in section II. Other codes in Table 1 are taken from [14]. It is interesting to note [15] that optimal block QPSK modulation codes with the same parameters as those found by Sayegh [8] [13], lengths $n=5$ to $n=10$, can be obtained from the modified $|\bar{u}|\bar{u}+\bar{v}|$ construction combined with Gray labeled QPSK signal sets.

In a Rayleigh fading channel, the error performance of a modulation code at high SNR is dominated by its minimum symbol and product distances as well as its number of nearest neighbors [16][17]. (At low SNR, the MSED also plays a role in the error performance.) For $i=1,2$, let s_i denote the i-th separation vector component of the underlying binary LUEP code, C_b, used in this section. With a Gray mapped QPSK signal set, an LUEP QPSK modulation code $C=M(C_b)$ has minimum symbol distance between code sequences associanted with k_i message bits equal to $\delta_{H,i}[C]=s_i$ and minimum product distance $\Delta_{p,i}[C]=a^{s_i}$, where a is the minimum Euclidean distance between ponts in the QPSK signal set. (For the signal set depicted in Figure 1, $a=\sqrt{2}$.) Therefore, good binary LUEP codes designed for the Hamming metric map onto good LUEP QPSK modulation codes for a fading channel.

Example: Let C_1 be a $(8,1,8)$ repetition code and C_2 be a $(8,7,2)$ parity check code. Then applying the modified version of the $|\bar{u}|\bar{u}+\bar{v}|$ construction explained in section II, we obtain a $(16,8)$ LUEP code

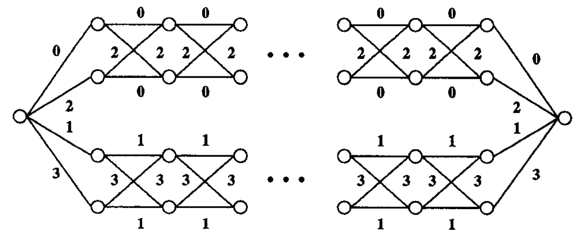

FIGURE 2: TRELLIS DIAGRAM FOR AN LUEP QPSK MODULATION CODE.

C_b with separation vector $\bar{s}=(8,4)$, for the message space $\{0,1\}^1\times\{0,1\}^7$. Gray mapping 2-bit symbols onto QPSK signals results in a block QPSK modulation code with two levels of error protection, $M(C_b)$, of length 8, rate $R=1/2$ (bits/dim) and squared Euclidean separation vector $\bar{\mathbf{S}}_{SED}=(16,8)$. The reference uncoded system is BPSK, which has an MSED of 4. It follows that the asymptotic coding gain vector for this two-level LUEP QPSK block modulation code is $\bar{G}=(6.02,3.01)$. In other words, 12.5% of the information is transmitted practically error free, while the remaining 87.5% of the information is provided with a coding gain of 3 dB with respect to uncoded BPSK. A trellis diagram for $M(C_b)$ has 4 states and 8 sections, with the structure indicated in Figure 2. (See also [15]).

This LUEP QPSK block modulation code compares well with a uniform error protection trellis modulation (TCM) code based on a binary *convolutional code* of the same rate and number of trellis diagram states: A rate 1/2 TCM code with constraint length 2 (4-state trellis diagram) and Gray mapped QPSK, achieves an asymptotic coding gain of 3.97 dB over uncoded BPSK. Code $M(C_b)$ also compares favorably with a *time-sharing* coding scheme that uses two separate binary linear block codes to provide the same levels of error protection: To provide an asymptotic coding gain of 6 dB for 1 bit and of 3 dB for 7 bits, an $(8,1,8)$ repetition code (or 4 QPSK signal transmissions) and a $(12,7,4)$ linear code (or 6 QPSK signal transmissions) may be used. This results in a $(20,8)$ binary LUEP code with the same separation vector and message space that requires 4 more redundant bits (or 2 more QPSK signal transmissions). $\Delta\Delta$

The above example can be generalized as follows: Let C_1 be a binary $(n,1,n)$ repetition code and C_2 be a binary $(n,n-1,2)$ parity check code. Applying the construction method outlined in section II we obtain a $(2n,n)$ binary two-level LUEP code

TABLE 2
EXPECTED CODING GAINS OF SOME LUEP QPSK MODULATION CODES OVER AN AWGN CHANNEL

n	8	16	32
MSB	4.62	6.03	5.84
LSB	2.05	1.63	1.22

$C_b = \mu(C_1, C_2)$ with separation vector $\bar{s} = (n, 4)$, for the message space $\{0,1\}^1 \times \{0,1\}^{n-1}$. Using a Gray mapped QPSK signal set we obtain an LUEP QPSK modulation code $M(C_b)$ of length n, rate $R = 1/2$ (bits/dim), and squared Euclidean separation vector $\bar{S}_{SED} = (2n, 8)$. Thus the asymptotic coding gain vector is $\bar{G} = (10\log_{10} n - 3.01, 3.01)$.

Figures 3 and 4 show computer simulation results on the error performance of LUEP QPSK modulation codes $M(C_b)$ of lengths 8, 16 and 32, based on the above construction. The vertical scale is the probability of a bit error, P_e, while the horizontal scale is the energy per bit-to-noise ratio, E_b/N_o. Simulations were performed using the Viterbi algorithm with soft decisions and a trellis diagram for $M(C_b)$ having the structure shown in Figure 2. As can be seen from these graphs, the construction improves from length $n = 8$ to $n = 16$, but then deteriorates at length $n = 32$. This is because of a larger number of nearest neighbors, or *error coefficient*, for the most important message part: The error coefficient (also called path multiplicity) for codewords associated with the most important message part (MSB) is $N_1 = 2^{n-1}$, while the error coefficient for codewords associated with the least important message part (LSB) is $N_2 = \binom{n}{2}$. As a result, the expected coding gain for the most important message bits will be reduced considerably as the length n increases. For short lengths ($5 \leq n \leq 10$) however, these codes are optimal block QPSK modulation codes, as pointed out before.

Table 2 lists the values of expected coding gains for $n = 8, 16, 32$, using the well known Forney's rule [18] which states that the coding loss at a bit error probability of 10^{-5} over an AWGN channel is $0.2\log_2(N_c/N_u)$, where N_c is the error coefficient of the modulation code and N_u is the error coefficient of the uncoded system, which in this case is BPSK, with $N_u = 1$. As an example, for $n = 32$, the asymptotic coding gain vector is $\bar{G} = (12.04, 3.01)$, while the error coefficients are $N_1 = 2^{31}$ and $N_2 = 496$. The coding loss due to these error coefficients is 6.2 dB for the MSB and 1.79 dB for the LSB, which accounts for the computer simulation results shown in Figure 3. Also note that because

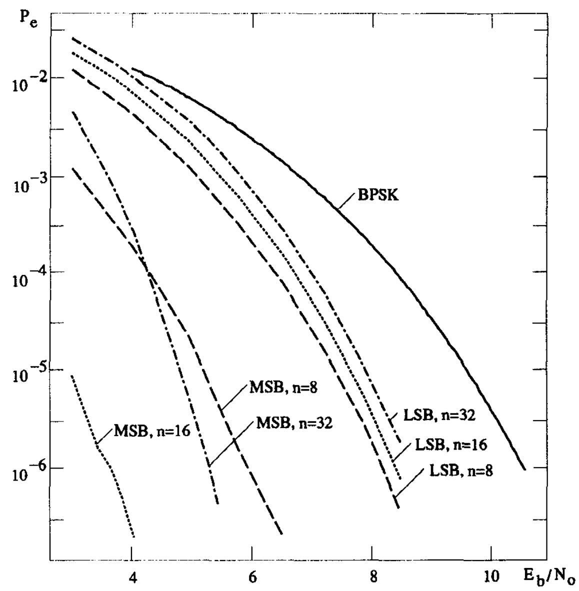

FIGURE 3: ERROR PERFORMANCE OF LUEP QPSK MODULATION CODES OVER AN AWGN CHANNEL.

a Gray mapped QPSK signal constellation is used, the effect of the error coefficients on the error performance for a Rayleigh fading channel is similar, as shown in Figure 4.

IV. CONCLUSIONS

We presented block QPSK modulation codes with two levels of error protection. We used Gray labeling of QPSK signals to map binary $(2n, k)$ LUEP codes, with separation vector $\bar{s} = (s_1, s_2)$, onto two-level LUEP QPSK block modulation codes of length n, rate $k/2n$ (bits/dimension) and squared Euclidean separation $\bar{S}_{SED} = (2s_1, 2s_2)$. These codes have two values of minimum squared Euclidean distance, or minimum symbol and product distances, between code sequences of QPSK signals. For short lengths, the resulting two-level LUEP QPSK block modulation codes for the AWGN channel are optimal in the sense of having the same parameters as the best block QPSK modulation codes [13]. Simulation results show that these codes achieve good error performance on a Rayleigh fading channel, while at the same time have an extremely simple trellis structure and thus low decoding complexity. We expect these codes to be used in applications where an embedded QPSK signal set is used, and a simple yet efficient block coded modulation system with two values of error protection is desired.

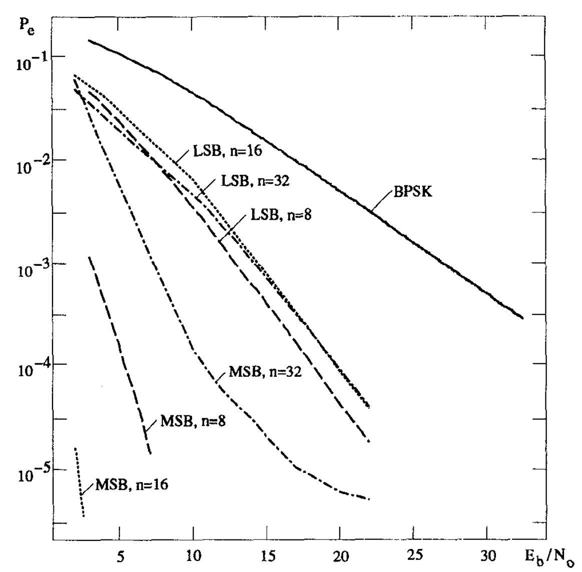

FIGURE 4: ERROR PERFORMANCE OF LUEP QPSK MODULATION CODES OVER A RAYLEIGH FADING CHANNEL.

V. ACKNOWLEDGEMENTS

This work was supported in part by NASA under Grant NAG 5-931, by the NSF under Grants NCR-88813480 and NCR-9115400, and by the Japanese Society for the Promotion of Science (JSPS) under Postdoctoral Fellowship ID No. 93157. A portion of this paper was presented at the 1993 IEEE International Symposium on Information Theory and submitted to the IEEE Transactions on Information Theory.

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