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This paper is dedicated to the memory of Gus Solomon

Abstract

In this paper, the error performance of bandwidth-efficient concatenated multilevel coded modulation (MCM) schemes for digital satellite broadcasting is analyzed. Nonstandard partitioning, multistage decoding, and outer Reed-Solomon (RS) codes are employed to provided unequal error protection capabilities.

1 Introduction

Multilevel codes, in combination with digital modulation, nonstandard partitioning and multistage decoding, provide unequal error protection capabilities needed in the transmission of information over broadcast channels [1, 2]. In the special case of digital high-definition television (HDTV) broadcasting, this approach achieves a so-called "graceful degradation" of the transmitted signals. This refers to a system in which the bit stream carrying the basic resolution of the TV signal is received always at a low bit error rate (BER), even under adverse weather conditions, i.e., heavy rain. As channel conditions improve, the bit streams carrying progressively higher resolutions are received at a low BER as well.

Concatenated systems are studied in this paper for two main reasons: (1) Most of digital broadcasting systems include powerful Reed-Solomon (RS) codes and thus the analysis of their error performance is of practical interest; (2) outer RS codes can further improve the error performance of multilevel coded modulation (MCM).

Another plausible reason for selecting RS codes is that their weight distribution is completely known [3]. This can facilitate the analysis of the performance of a concatenated system based on RS codes. For both simplicity and the nature of digital satellite broadcasting, focus is on three-level coded 8-PSK modulation. However, the results presented in this paper can be extended to other coded M-ary modulation systems (e.g., QAM modulation for terrestrial digital HDTV broadcasting).

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In Ref. [2], it was shown that, with multistage decoding, Ungerboeck partitioning is inappropriate to provide unequal error protection (UEP), due to a large number of nearest neighbors (NN), particularly at the first decoding stage. An unconventional partitioning was proposed that results in a reduction of the effective NN with multistage decoding. The purpose of this paper is to analyze the improvement in performance of multilevel coded modulation for UEP, with unconventional partitioning and multistage decoding, achieved by the use of outer RS codes.

2 Concatenated coded 8-PSK modulation

Two basic encoding structures of a concatenated multilevel coded 8-PSK modulation system are possible. One uses a single RS code over $GF(2^m)$ in the outer stage and an interleaver. The output of the interleaver consists of three bit streams which are used to drive the inputs of the MCM in the inner stage. Another structure consists of three separate RS codes in the outer stage and an interleaver at each level. For practical reasons, a single outer RS code is considered. A block diagram of the encoder of the proposed concatenated system is shown in Fig. 1.

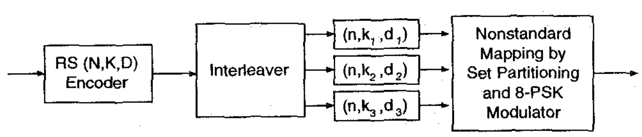

Fig. 1. Encoder structure of a concatenated coded modulation scheme for satellite broadcasting.

2.1 Encoding

The encoding process of the proposed concatenated system is as follows: Let k_i/n denote the rate of the i-th level binary code C_{Ii} used in the MCM system, $i = 1, 2, 3$. Note that C_{Ii} may be a binary linear block or convolutional code. Let C_O denote an (N, K, D) RS code over $GF(2^m)$ used in the outer stage.

The interleaver used in the intermediate stage stores the binary representation of $(k_1 + k_2 + k_3)$ codewords in C_O as the rows of a binary $(k_1 + k_2 + k_3) \times Nm$ array. For $i = 1, 2, 3$, k_i bits of each column are encoded by C_{Ii}. This produces a $3n \times Nm$ array, shown in Fig. 2 which is then mapped, via nonstandard partitioning, onto a sequence of nNm 8-PSK signals.

It is easy to verify that the rate of this concatenated system is $R = (k_1 + k_2 + k_3)K/nN$ bits/symbol. By proper selection of the outer RS code and the inner binary codes, the rate R can be made equal to approximately 2 bits/symbol, the same as uncoded QPSK modulation.

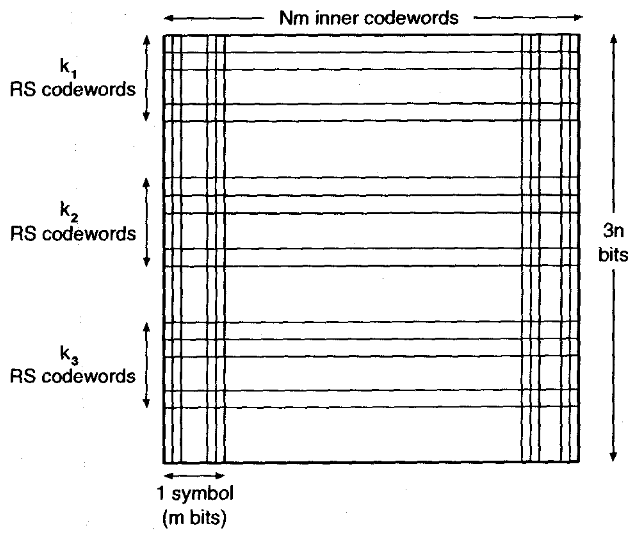

Fig. 2. Interleaver for the concatenated scheme.

2.2 Decoding

The inner MCM is decoded using a multistage decoder. At each stage, maximum likelihood decoding is performed by assuming that the sequences at the lower stages are random. Although suboptimal, this approach is known to give a good trade-off between error performance and decoding complexity. The unconventional labeling of an 8-PSK signal set, shown in Fig. 3, is used. It is induced by a non-standard set partitioning. The partitioning is such that it reduces the effective number of nearest neighbor sequences (NN) at the first and second decoding stages, compared to the NN for conventional Ungerboeck partitioning.

Although a relatively poor average performance is achieved with non-standard partitioning, proper selection of inner codes makes the most important bits encoded at the first level have a very high error performance. In addition, over a satellite channel with coherent demodulation, the first and second stage decoders can operate in parallel [2].

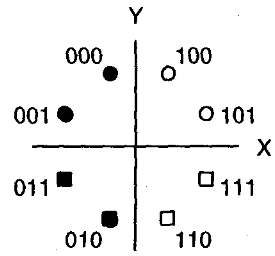

Fig. 3. Unconventional labeling of an 8-PSK signal set.

The outer RS code is decoded by the Berlekamp-Massey algorithm for errors-only correction, without erasures or any other reliability measure on the decoded m-bit symbols from the inner multistage decoder.

3 Error performance

In this section, tight upper bounds on the bit error performance of the inner MCM derived in [2], and on the probability of a decoding error for RS codes [4, 5], the probabilities of a symbol error and a bit error of the proposed concatenated systems are estimated. Expressions are evaluated and compared with computer simulation results, for various example schemes. It should be noted that, although concatenated systems with inner block codes are examined, both block and convolutional codes may be used as component codes in the inner MCM.

3.1 Inner three-level block coded 8-PSK modulation

Let E_b/N_0 denote the energy-per-bit to noise ratio, R the overall rate in bits/symbol, $A_w^{(i)}$ and d_i the weight distribution and minimum Hamming distance of the error correcting code used at the i-th stage of the inner MCM, $i = 1, 2, 3$. The probability of a bit error p_i at the i-th decoding stage may be obtained from a conventional union bound argument, as shown in [2]:

(I) First and second decoding stages ($i = 1, 2$)

$$p_i \leq \sum_{w=d_i}^{n} \frac{w}{n} A_w^{(i)} 2^{-w} \sum_{j=0}^{w} \binom{w}{j} Q\left(\sqrt{\frac{RE_b}{2N_0}\, d_P^2(j)}\right), \tag{1}$$

where

$$d_P^2(i) = \frac{1}{w}\Big[i \sin(\pi/8) + (w-i)\cos(\pi/8)\Big]^2, \tag{2}$$

and

$$Q(x) = \frac{1}{\sqrt{\pi N_0}} \int_x^{\infty} e^{-n^2/N_0} dn. \tag{3}$$

(II) Third decoding stage ($i = 3$)

$$p_3 \leq \sum_{w=d_3}^{n} \frac{w}{n} A_w^{(3)} Q\left(\sqrt{\frac{RE_b}{2N_0}\, w\Delta^2}\right), \tag{4}$$

where $\Delta = \sqrt{2}/2$.

3.2 Outer Reed-Solomon code

In the following, the i-th level probabilities of a symbol error, P_{E_i}, and of a bit error, P_{b_i}, for an errors-only Reed-Solomon decoder are estimated. In the expressions below, p_i denotes the probability of a bit error at the i-th decoding stage of the inner MCM, for $i = 1,2,3$, given by expressions (1) and (4) in the previous section.

Let C be a T-error-correcting RS(N,K,D) code over $GF(q)$, $q = 2^m$. The probability of a symbol error at the output of an *errors-only* decoder is [4]

$$P_{E_i} = \frac{1}{N}\sum_{h=D}^{N} h P_{ICD}(h), \quad (5)$$

with $P_{ICD}(h)$ defined as the probability of incorrect decoding, which can be expressed as

$$P_{ICD}(h) = A_h \sum_{s=0}^{T}\sum_{k=h-s}^{h+s} n(h,k;s)P(k), \quad (6)$$

with A_h denoting the number of codewords of weight h in the RS code,

$$P(k) = \frac{P_{CE_i}^k (1-P_{CE_i})^{N-k}}{(q-1)^k}, \quad (7)$$

P_{CE_i} is the channel symbol error probability,

$$P_{CE_i} = 1 - (1-p_i)^m, \quad (8)$$

and

$$n(h,k;s) = \sum_{r=r_1}^{r_2} \binom{h}{h-s+r}\binom{s-r}{k-h+s-2r} \binom{N-h}{r}(q-2)^{k-h+s-2r}(q-1)^r, \quad (9)$$

with $r_1 = \max\{0, k-h\}$ and $r_2 = \lceil (k-h+s)/2 \rceil$. It is well known (e.g., p.321 of [3]) that, for $D \le h \le N$,

$$A_h = \binom{N}{h}(q-1)\sum_{i=0}^{h-D}(-1)^i\binom{h-1}{i}q^{h-D-i}. \quad (10)$$

Since decoding is performed with an errors-only decoder, the following approximated bound may be used for the i-th level probability of a bit error [5],

$$P_{b_i} < \frac{2^{m-1}}{2^m-1}\sum_{j=T+1}^{N}\frac{j+T}{N}\binom{N}{j}P_{CE_i}^j(1-P_{CE_i})^{N-j}, \quad (11)$$

where P_{CE_i} is given by (8). Bound (11) is particularly useful to estimate the performance of codes over $GF(2^m), m \ge 8$, for which expression (5) becomes difficult, if not impossible, to evaluate.

TABLE I OUTER REED-SOLOMON CODES

(N,K,D)	Overall rate, R
(63,57,7)	2.064
(127,121,7)	2.173
(255,239,17)	2.138
(255,223,33)	1.995

3.3 Simulation results

Several examples of concatenated schemes were simulated. At the inner coding stage, a simple three-level coded 8-PSK modulation system for UEP, denoted Λ, was chosen with extended BCH (eBCH) codes, eBCH(32,16,8), eBCH(32,26,4) and eBCH(32,31,2), in the first, second and third encoding stages, respectively. The overall rate of Λ is 2.28125 bits/symbol. As a reference, Fig. 4 shows plots of analytical bounds and simulation results of Λ. The notation is as follows: Sim$(n,k)-i$ and UB$(n,k)-i$ indicate simulations and upper bounds on the bit error rate of the eBCH(n,k,d) code used at the i-th stage, $i = 1,2,3$.

Figs. 5 to 8 show simulation results and the bound (11) for several concatenated systems with outer RS codes of length N, $N = 63, 127$ and 255, compared with uncoded QPSK modulation. The parameters and overall rate for each selection of outer RS code are shown in Table I. In the figures, Si and UBi represent, respectively, the simulations and bound (11) on the bit error rate at the i-th stage, for $i = 1,2,3$.

In all cases, there is a gap between the simulations and the aproximated upper bound (11), that can be explained by the fact that interleaving is present between the outer RS code and the inner MCM. It is also interesting to note that the difference between the performance of concatenated schemes using the RS(63,57,7) code and the RS (255,223,33) is about 1.5 dB for a BER of 10^{-10} at each decoding stage. It is surprising the fact that the RS(63,57,7) and the RS(127,121,7) codes have practically the same error performance.

In the simulations, RS codes of length $N = 255$ were chosen for practical reasons: A shortened version of the RS(255,239,17) code has been proposed for digital satellte broadcasting, while the RS(255,223,33) code is the well known "NASA-standard" code. Also, an extended RS (128,122,7) code is used in a concatenated system for digital video over cable in the U.S. standard.

4 Conclusions

Numerical and analytical results were presented on the error performance of concatenated multilevel coded modulation systems, using a single outer RS code. These results are useful in estimating the performance of concatenated systems at very low BER, needed in the transmission of digital HDTV.

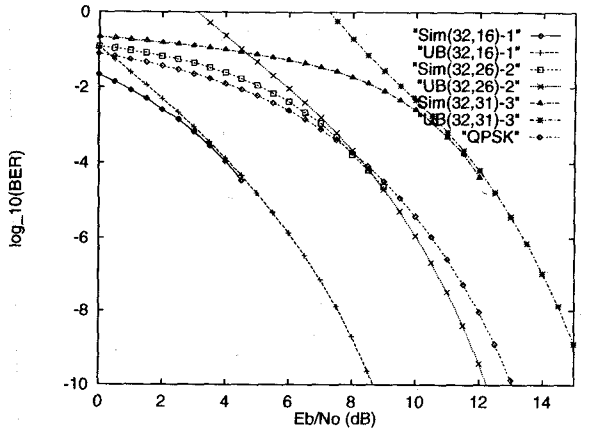

Fig. 4. Bounds and simulations of Λ, a coded 8-PSK modulation with three levels of error protection, unequal error protection and three-stage decoding.

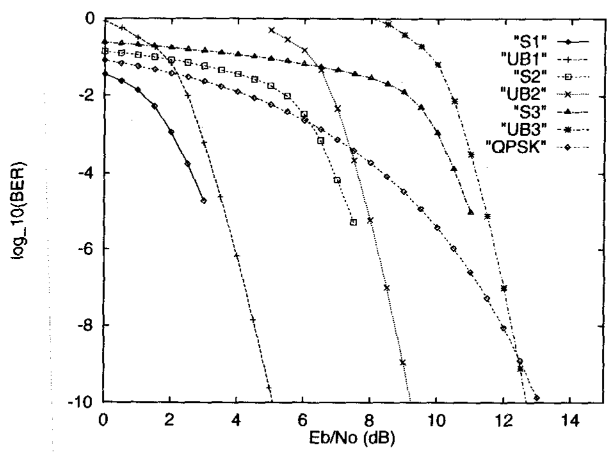

Fig. 5. Bounds and simulations of a concatenated three-level coded modulation with an outer RS (63,57,7) code.

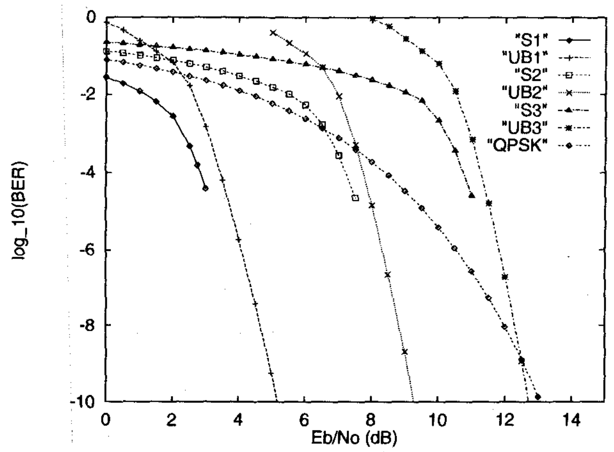

Fig. 6. Bounds and simulations of a concatenated three-level coded modulation with outer RS (127,121,7) code.

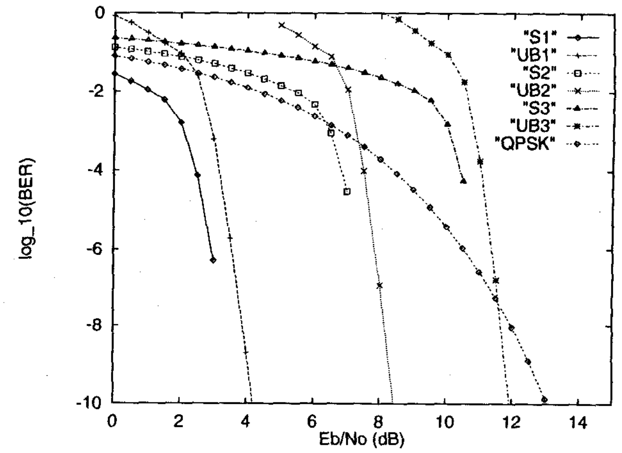

Fig. 7. Bounds and simulations of a concatenated three-level coded modulation with outer RS (255,239,17) code.

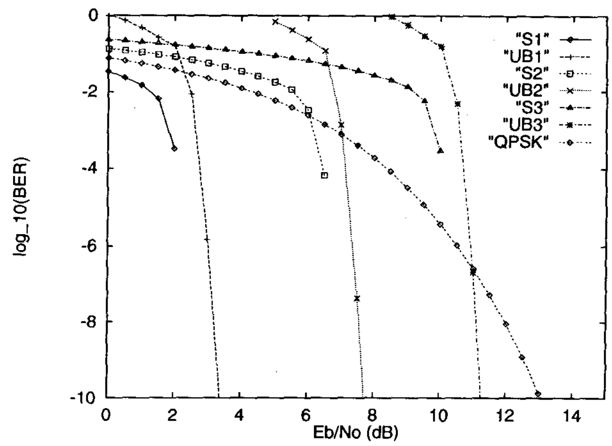

Fig. 8. Bounds and simulations of a concatenated three-level coded modulation with outer RS (255,223,33) code.

References

[1] T. Cover, "Broadcast channels," *IEEE Trans. Info. Theory*, vol. IT-18, no. 1, pp. 2-14, January 1972.

[2] R.H. Morelos-Zaragoza, M.P.C. Fossorier, S. Lin and H. Imai, "Multilevel Block Coded Modulation With Unequal Error Protection," in *Proceedings of the 1997 IEEE Intl. Symp. on Info. Theory (ISIT'97)*, Ulm, Germany, July 4, 1997, and submitted to the *IEEE Trans. Info. Theory*, March 1997.

[3] F.J. Macwilliams and N.J.A. Sloane, *The Theory of Error-Correcting Codes*, North-Holland, Amsterdam, 1977.

[4] Z.M. Huntoon and A.M. Michelson, "On the Computation of the Probability of Post-Decoding Error Events for Block Codes," *IEEE Trans. Info. Theory*, vol. IT-23, no. 5, May 1977.

[5] G.C. Clark, Jr. and J.B. Cain, *Error Correcting Coding for Digital Communications*, Plenum Press, New York, 1981.