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Binary Multilevel Convolutional Codes with Unequal Error Protection Capabilities

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***Abstract*— Binary multilevel convolutional codes (CC's) with unequal error protection (UEP) capabilities are studied. These codes belong to the class of generalized concatenated (GC) codes [1]. Binary CC's are used as outer codes. Binary linear block codes of short length, and selected subcodes in their two-way subcode partition chain, are used as inner codes. Multistage decodings are presented that use Viterbi decoders operating on trellises with similar structure to that of the constituent binary CC's. Simulation results of example binary two-level CC's are also reported.**

***Index Terms*—Multilevel codes, multistage decoding, punctured convolutional codes, unequal error protection.**

I. Introduction

GENERALIZED concatenated (GC) codes [1] are a powerful family of error correcting codes based on multiple outer codes, and an inner code and its partition into subcodes. Due to their multilevel structure [2], GC codes can be designed as unequal error protection (UEP) codes. The need for UEP arises in communications systems where part of the source messages are more important, or error sensitive, than others. Specific examples include practically all digital speech and image transmission systems.

Convolutional UEP codes based on the GC code construction are generalizations of the two-level convolutional codes (CC's) introduced independently by Pellizzoni and Spalvieri [3] and by Hattori and Saitoh [4], and then studied in [5] and [6]. Moreover, multistage decoding procedures can be devised for convolutional UEP codes as a natural generalization of the two-stage decoding of $|\overline{u}|\overline{u}+\overline{v}|$ CC's [3]–[6].

In [3] and [4] multilevel coset (or "superimposed") codes with outer CC's are presented. In [3] two-level CC's were constructed and shown to offer improved error performance, with reduced decoding complexity, compared to previously known codes with the same number of states. A construction using punctured convolutional (PC) codes as component codes in the $|\overline{u}|\overline{u}+\overline{v}|$ construction was proposed in [4].

In this letter it is shown that two-level CC's based on the $|\overline{u}|\overline{u}+\overline{v}|$ construction are special cases of GC codes. This gives binary multilevel CC's a rich algebraic structure that is useful in explaining their UEP capabilities and in devising low-complexity multistage decoding algorithms.

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II. Binary Multilevel Convolutional Codes

As usual, let (n, k, d) denote a binary linear block code of length n, dimension k, and minimum distance d. The construction of a binary multilevel CC C starts with a binary (n_I, k_1, d_1) code C_1 and its two-way partition as a chain of M (n_I, k_i, d_i) subcodes $C_i, i = 2, 3, \cdots, M+1$, such that $C_1 \supset C_2 \supset \cdots \supset C_{M+1}$, where, for convenience, we define $C_{M+1} \triangleq \{\overline{0}\}$ and $d_{M+1} \triangleq \infty$. Let $C_{Ii} = [C_i/C_{i+1}]$ denote an (n_I, k_{Ii}, δ_i) binary subcode of C_i, which is a set of coset representatives of C_{i+1} in C_i, of dimension $k_{Ii} = k_i - k_{i+1} = 1$ and minimum Hamming distance $\delta_i \geq d_i, 1 \leq i \leq M$. Then C_1 is the direct sum $C_1 = C_{I1} + C_{I2} + \cdots + C_{IM}$. For $1 \leq i \leq M$, let C_{Oi} denote a rate-k_{Oi}/n_O memory-m_i binary CC with minimum distance d_{Oi}. Then a binary multilevel CC C is defined as the direct sum

$$C \triangleq C_{O1} * C_{I1} + C_{O2} * C_{I2} + \cdots + C_{OM} * C_{IM} \qquad (1)$$

where $C_{Oi} * C_{Ii}$ denotes a product (or concatenated) code with C_{Oi} as outer code and C_{Ii} as inner code, $1 \leq i \leq M$. C is a binary M-level CC of rate $R = k/n_I n_O$, memory $m_1 + m_2 + \cdots + m_M$, and minimum distance d, where k is given by $k = \Sigma_{i=1}^{M} k_{Oi}$ and $d \geq \min\{\delta_1 d_{O1}, \cdots, \delta_M d_{OM}\}$.

A. UEP Capabilities

Because of their structure, multilevel CC's can be designed as UEP CC's. The UEP capability of a linear CC follows from a necessary and sufficient condition for a linear code to be a UEP code [7]: *the set of minimum-weight codewords does not span a linear UEP code*. From this condition and the definitions above, it follows that if

$$\delta_1 d_{O1} \geq \delta_2 d_{O2} \geq \cdots \geq \delta_M d_{OM} \qquad (2)$$

then those coded sequences in correspondence with k_{Oi} information bits have a Hamming distance between them of at least $\delta_i d_{Oi}, 1 \leq i \leq M$. In the language of UEP codes we say that a binary multilevel CC is an M-level UEP code with *separation vector*

$$\overline{\boldsymbol{s}} = (\delta_1 d_{O1}, \delta_2 d_{O2}, \cdots, \delta_M d_{OM})$$

and *message space*

$$\{0,1\}^{k_{O1}} \times \{0,1\}^{k_{O2}} \times \cdots \times \{0,1\}^{k_{OM}}.$$

For simplicity, in this letter only *two-level UEP* CC's are dealt with. It is assumed that the M-component concatenated codes are chosen in such a way that the separation vector is $\bar{\boldsymbol{s}} = (s_1, s_2)$, where $s_1 = \delta_1 d_{O1} = \delta_2 d_{O2} = \cdots = \delta_L d_{OL}, s_2 = \delta_{L+1} d_{O(L+1)} = \delta_{L+2} d_{O(L+2)} = \cdots = \delta_M d_{OM}$, with $s_1 > s_2$ and $1 \le L < M$.

III. Two Constructions of Binary Multilevel Convolutional Codes

In this section two specific constructions of binary UEP CC's, based on generalized concatenation, are presented. For simplicity, we only consider the cases where the inner block code is of length $n_I = 2$ or $n_I = 3$, and two or three binary CC's are used as outer codes, although other choices of n_I and number of component codes are possible.

A. Construction I

For $i = 1, 2$, let C_{Oi} be a rate-k_{Oi}/n_O, memory-m_i, and minimum distance d_{Oi} CC. Let C_1 be a binary block (2, 2, 1) code with generator matrix

$$G_1 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \tag{3}$$

so that $C_{I1} = \{00, 01\}$ and $C_{I2} = \{00, 11\}$. Then it follows from the results of Section II that the code

$$C = C_{O1} * C_{I1} + C_{O2} * C_{I2}$$

is a two-level CC of rate $(k_{O1} + k_{O2})/2n_O$, memory $(m_1 + m_2)$, and minimum distance

$$d \ge \min\{d_{O1}, 2d_{O2}\}.$$

Suppose that $d_{O1} > 2d_{O2}$. Then it follows from (2) that C is a *two-level UEP CC* with separation vector $\bar{\boldsymbol{s}} = (d_{O1}, 2d_{O2})$ and message space $\{0,1\}^{k_{O1}} \times \{0,1\}^{k_{O2}}$.

Note that two-level CC's constructed in this fashion are, in fact, equivalent to $|\overline{u}|\overline{u} + \overline{v}|$-constructed codes.

Example 1: Let C_{O1} be a CC of rate 1/4, memory 2, and minimum distance 10 (from [9, Table A.5]). Let C_{O2} be a PC code of rate 3/4, memory 3, and minimum distance 4 (from [9, Table A.8]). Construction I results in a two-level CC C_{GC} of rate 4/8, memory 5, and minimum distance 8. From condition (2), it follows that C_{GC} is a *two-level UEP CC* with separation vector $\bar{\boldsymbol{s}} = (10, 8)$ and message space $\{0,1\}^1 \times \{0,1\}^3$. Also, note that the minimum distance of the best rate-1/2 memory-3 CC is equal to 8. $\Delta\Delta$

A Two-Stage Decoding Procedure: For practical applications, in order to reduce the complexity of Viterbi decoding (at the expense of an increased error coefficient or degradation in coding gain [3], [6]), suboptimal two-stage decoding (TSD) may be adopted. Coded bits are Binary phase-shift keying (BPSK)-modulated and transmitted over an additive white Gaussian noise (AWGN) channel. The following TSD procedure is considered.

Stage 1: A decoder for a CC $C_1' \supset C_1$ is used in this stage, where

$$C_1' = C_1 \oplus \{0,1\}^{n_O} * \{00, 11\} = C_1 \oplus \{00, 11\}^{n_O}.$$

The trellis structure T_1' of code C_1' is isomorphic (up to connections between states) to that of the trellis T_1 of the first-level code $C_1 = C_{O1} * C_{I1}$. The only difference between T_1' and T_1 is that T_1' contains two parallel subbranches per original two-bit symbol in a branch of T_1. That is, {00, 11} (resp. {01, 10}) for 00 (resp. 01). Once an information sequence $\overline{m}_1$ is decoded in this first stage, the k_{O1} most important bits (MIB's) are recovered. Sequence $\overline{m}_1$ is then reencoded by C_1 to obtain a coded sequence $\overline{u}_1$. From this sequence, a modified received sequence $\overline{r}'$ is obtained, where $r_j' = (-1)^{u_{1j}} r_j$, and passed on to the second stage.

Stage 2: Using a trellis T_2 for the second component code C_2, the modified received sequence $\overline{r}'$ is decoded and the k_{O2} less important bits (LIB's) extracted.

Note that iterative decoding may be used, as proposed in [8]. Also, it is possible to use two-stage decoding with interleaving [6].

B. Construction II

In this section a new construction method that combines three CC's is presented. For $i = 1, 2, 3$, let C_{Oi} be a rate-k_{Oi}/n_O, memory-m_i, and minimum distance d_{Oi} CC. Let C_1 be a binary block (3, 3, 1) code with generator matrix

$$G_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \tag{4}$$

and subcodes $C_{I1} = \{000, 001\}$, $C_{I2} = \{000, 010\}$, and $C_{I3} = \{000, 111\}$. Then it follows from the results of Section II that the code

$$C = C_{O1} * C_{I1} + C_{O2} * C_{I2} + C_{O3} * C_{I3}$$

is a CC of rate $(k_{O1} + k_{O2} + k_{O3})/3n_O$, memory $(m_1 + m_2 + m_3)$, and minimum distance

$$d \ge \min\{d_{O1}, d_{O2}, 3d_{O3}\}.$$

This construction is a permuted version of a "$|\overline{u}|\overline{u} + \overline{v}|\overline{u} + \overline{w}|$ construction."

Suppose that $d_{O1} = d_{O2}$ and $d_{O2} > 3d_{O3}$. Then, from condition (2), C is a *two-level UEP CC* with separation vector $\bar{\boldsymbol{s}} = (d_{O1}, 3d_{O3})$ and message space $\{0,1\}^{k_{O1}+k_{O2}} \times \{0,1\}^{k_{O3}}$.

A Three-Stage Decoding Procedure: A decoding procedure for codes obtained from Contruction II is similar to the two-stage decoder in the previous section—decoding proceeds in three stages, passing decoded information from one stage to the next, with Viterbi decoders for codes C_1', C_2', and C_3, where $C_1' = C_1 \oplus \{0,1\}^{n_O} * \{000, 010, 101, 111\} = C_1 \oplus \{000, 010, 101, 111\}^{n_O}$ with $C_1' \supset C_1$, and $C_2' = C_2 \oplus \{0,1\}^{n_O} * \{000, 111\} = C_2 \oplus \{000, 111\}^{n_O}$ with $C_2' \supset C_2$.

IV. Tables of Codes

By means of a computer search, good binary two-level CC's were found based on the construction methods presented in the previous section. The goal of the search was to find codes with minimum multistage decoding complexity (number

TABLE I
PARAMETERS OF THE COMPONENT CODES OF BEST TWO-LEVEL CC'S OBTAINED IN THE COMPUTER SEARCH. EACH ROW LISTS THE PARAMETERS OF A MEMORY-m, RATE-k/n, AND FREE DISTANCE d_f CC. LABELS $Gi.j$ REFER TO PC CODES. Ci AND Di REFER TO RATE-1/3 AND RATE-1/4 CODES, RESPECTIVELY

Label	m	k	n	d_f	c_f	$\overline{G}$	P
G1.1	2	2	3	3	1	(5,7)	$\begin{pmatrix}1&0\\1&1\end{pmatrix}$
G1.2	3	2	3	4	10	(15,17)	$\begin{pmatrix}1&1\\1&0\end{pmatrix}$
G2.1	2	3	4	3	15	(5,7)	$\begin{pmatrix}1&0&1\\1&1&0\end{pmatrix}$
G2.2	3	3	4	4	124	(15,17)	$\begin{pmatrix}1&1&0\\1&0&1\end{pmatrix}$
C2	3	1	3	10	6	(13,15,17)	—
C3	4	1	3	12	12	(45,33,37)	—
C4	6	1	3	15	11	(133,145,175)	—
D1	2	1	4	10	2	(5,7,7,7)	—
D2	3	1	4	13	4	(13,15,15,17)	—
D3	4	1	4	16	8	(45,27,33,37)	—
D4	5	1	4	18	6	(53,67,71,75)	—
D5	6	1	4	20	37	(135,135,147,163)	—
D6	7	1	4	22	2	(235,275,313,357)	—
D7	8	1	4	24	4	(463,535,733,745)	—
D8	9	1	4	27	10	(1117,1365,1633,1653)	—

TABLE II
TWO-LEVEL CC'S OBTAINED FROM CONSTRUCTION I

C_{O2}	C_{O1}	m	k	n	k_2	k_1	$2d_{O2}$	d_{O1}
G1.1	C2	5	3	6	2	1	6	10
G1.1	C3	6	3	6	2	1	6	12
G1.2	D2	5	4	8	3	1	6	13
G1.1	C4	8	3	6	2	1	6	15
G1.2	D3	6	4	8	3	1	6	16
G1.2	D4	7	4	8	3	1	6	18
G1.2	D5	8	4	8	3	1	6	20
G2.2	D1	5	4	8	3	1	8	10
G2.1	C2	6	3	6	2	1	8	10
G2.1	C3	7	3	6	2	1	8	12
G2.2	D2	6	4	8	3	1	8	13
G2.2	D3	7	4	8	3	1	8	16
G2.2	D4	8	4	8	3	1	8	18
G2.2	D5	9	4	8	3	1	8	20
G2.2	D6	10	4	8	3	1	8	22
G2.2	D7	11	4	8	3	1	8	24
G2.2	D8	12	4	8	3	1	8	27

of additions and comparisons in a Viterbi decoder). The search was performed using the parameters (rate, memory, and minimum distance) of codes from tables of best convolutional and PC codes [9].

Table I shows the parameters of memory-m rate-k/n CC's found in the search to be components of the best codes. The notation used in Table I is as follows: d_f denotes the free distance, c_f is the path multiplicity, or number of code sequences at distance d_f, $\overline{G}$ is the polynomial generator matrix of the code, with polynomials represented as octal numbers, and P is the puncturing matrix, in the case of a PC code.

Tables II and III list the parameters of codes found in the search. For simplicity, in the three-level constructions the component first- and second-level CC's were chosen such that $C_{O1} = C_{O2}$ in order to obtain *two-level UEP codes*. In addition, all of the binary component codes were selected to have equal number of output bits, i.e., $n_{O1} = n_{O2}$ or $n_{O1} = n_{O2} = n_{O3}$.

TABLE III
TWO-LEVEL CC'S OBTAINED FROM CONSTRUCTION II

C_{O3}	C_{O2}	C_{O1}	m	k	n	k_3	k_2	k_1	$3d_{O3}$	d_{O2}	d_{O1}
G1.2	D4	D4	12	5	12	3	1	1	9	18	18
G1.2	D5	D5	14	5	12	3	1	1	9	20	20
G1.2	D6	D6	16	5	12	3	1	1	9	22	22
G1.2	D7	D7	18	5	12	3	1	1	9	24	24
G1.2	D8	D8	20	5	12	3	1	1	9	27	27
G2.2	D3	D3	11	5	12	3	1	1	12	16	16
G2.2	D4	D4	13	5	12	3	1	1	12	18	18
G2.2	D5	D5	15	5	12	3	1	1	12	20	20

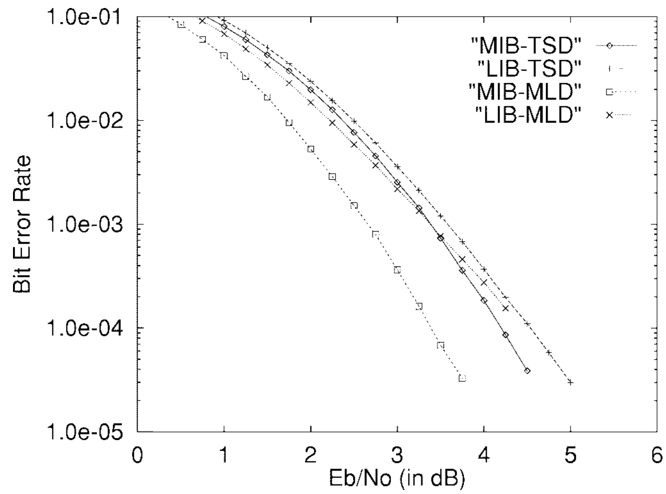

Fig. 1. Error performance of code S1: MLD and TSD of the MIB's and LIB's.

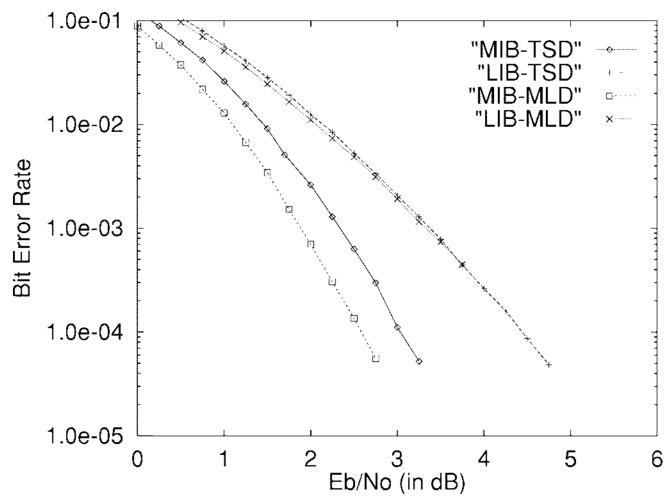

Fig. 2. Error performance of code S3: MLD and TSD of the MIB's and LIB's.

V. COMPUTER SIMULATION RESULTS

To illustrate the error performance of binary multilevel CC's, two codes from Construction I were simulated. In Figs. 1 and 2, MIB-MLD and LIB-MLD are used to denote the bit-error rates of the MIB's and LIB's, with maximum-likelihood decoding (MLD), respectively. Similarly, MIB-TSD and LIB-TSD refer to the bit-error rates with TSD.

The label S1 denotes the example code presented in Section III-A. The code labeled S3 is obtained from code S1 by replacing the rate-1/4 component code D1 of Table I by a more powerful rate-1/4 code D3. It can be seen from Figs. 1 and 2 that the performance of the MIB with MLD improves drastically when increasing the number of states from code S1 to code S3 (component code D3 needs 16 states as opposed to four states of D1). The improvement in performance for the MIB is gained by, in addition to increasing the number of states, reducing the coding gain for the LIB. A discussion of the effect of error coefficients in two-stage decoding can be found in [6] and is not addressed here.

REFERENCES

[1] V. A. Zinoviev, "Generalized cascated codes," *Probl. Peredachi Inf.*, vol. 12, no. 1, pp. 5–15, 1976.

[2] H. Imai and S. Hirakawa, "A new multilevel coding method using error-correcting codes," *IEEE Trans. Inform. Theory*, vol. IT-23, pp. 371–377, May 1977.

[3] R. Pellizzoni and A. Spalvieri, "Binary multilevel coset codes based on Reed-Muller codes," *IEEE Trans. Commun.*, vol. 42, pp. 2357–2360, July 1994.

[4] M. Hattori and Y. Saitoh, "A coding technique based on punctured convolutional codes," *Electron. Lett.*, vol. 30, no. 13, pp. 1042–1043, June 1994.

[5] M. P. C. Fossorier, S. Lin, and D. Stojanovic, "Research report: Decomposable convolutional codes and multi-stage decoding," Dept. Elec. Eng., Univ. Hawaii, Honolulu, Mar. 1995.

[6] J.-F. Cheng, "Hyperimposed convolutional codes," in *Proc. ICC'96*, Dallas, TX, June 1996, pp. 979–983.

[7] I. M. Boyarinov and G. L. Katsman, "Linear unequal error protection codes," *IEEE Trans. Inform. Theory*, vol. IT-27, pp. 168–175, Mar. 1981.

[8] K. Fazel, "Iterative decoding of generalized concatenated Blokh-Zyablov codes," in *Proc. ICC'96*, Dallas, TX, June 1996, pp. 96–101.

[9] A. Dholakia, *Introduction to Convolutional Codes With Applications*. Norwell, MA: Kluwer, 1994.