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## Capacity Of Automated Highway Systems: Effect Of Platooning And Barriers

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CALIFORNIA PATH PROGRAM  
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## **Capacity of Automated Highway Systems: Effect of Platooning and Barriers**

**H.-S. Jacob Tsao**  
**Randolph W. Hall**  
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# CAPACITY OF AUTOMATED HIGHWAY SYSTEMS: EFFECT OF PLATOONING AND BARRIERS

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## EXECUTIVE SUMMARY

The concept of Automated Highway Systems (AHS) is based on the belief that an appropriate integration of control and communication technologies placed on the vehicle and on the highway can lead to a large improvement in capacity and safety without requiring a significant amount of additional highway right-of-way. Stemming from this belief are various conceptual scenarios for vehicle/highway automation.

An AHS consists of two major components: vehicle/highway automation technology and highway operating strategy. In this paper, we study the capacity of key AHS operating scenarios. The definition of capacity and its calculation for AHS with traffic needing no lane changes is relatively straightforward. In such a case, capacity of a lane can be defined and measured as the maximum achievable flow in vehicles per lane per hour, which does not reflect at all the *lateral flow* - the flow of vehicles between lanes. In this report, we define *lateral capacity between two adjacent lanes* as the maximum rate at which vehicles can change from one lane to the other given the lanes' longitudinal flows. While most literature on AHS performance focuses on the longitudinal traffic flow, we will emphasize the lateral flow equally. In our opinion, it is the ability to reach the desired destination efficiently (through lane changes) that the users of highways want most. Therefore, the performance objective should be to maximize the AHS flow subject to the stringent constraint that all or nearly all users exit the AHS at their desired exits.

A major configuration option is the erection of lane barriers and the concomitant openings, called *gates*, for lane changes. A major AHS operating rule that has extensive effect on all aspects of AHS operation is the lane-flow rule, the rule that governs the behavior of vehicles in a common lane (e.g. the

spacing between any two adjacent vehicles). Two basic lane-flow rules are the *platooning rule* and the *free-agent rule*. The platooning **rule** was first proposed and studied by Shladover in the late 70's [8] and received renewed interest in the last few years.

We tackled the capacity estimation problem with two parallel but coordinated efforts: analytical modeling and computer simulation. This dual approach, analysis combined with simulation, has been adopted to best utilize the strengths of each. We developed analytical models to study the vehicle/platoon and gap distributions on individual AHS lanes. Based on these models, we provided estimates for the time required for a complete lane change. We also modified **SmartPath**, an existing simulator developed by Eskafi and Varaiya [1], to study the effect of platooning and lane barriers on AHS capacity.

Among the analytical models developed is the model for predicting the platoon size distribution. Based on this model, we obtained the probability distribution under four representative traffic conditions (see figure 5). The average and the standard deviation of the platoon size distributions associated with a complete range of traffic conditions are also plotted (see Figure 6).

In most of the simulation test cases, the exit success percentage is well below 100%, which poses a major challenge to designing AHS operating strategies. Compared to the platooning lane-flow rule, the free-agent rule results in lower exit success percentages. The main reason is the lack of gaps sufficiently large for safe lane changes. A major challenge to improving the lateral capacity of **AHSs** under the free-agent rule is to manage gaps more efficiently. According to our simulation study, the presence of lane barriers results in lower longitudinal flow but makes little difference in exit success rate. This is because the lane changes are initiated well before the desired exits and, to accommodate the lane changes, the traffic has to slow down. Note that neither the analytical models nor the simulation models studied in this paper adequately represent how a future AHS would actually be operated. More sophisticated operating strategies and models are required to optimize the capacity of an AHS.

Both the analytical models and the simulation results indicate a direct trade-off between the longitudinal and lateral capacities of an AHS. According to our study, the average lane-change completion time increase with the flow in the destination lane and, at high **flow** levels, the increase is at a higher

rate than the flow. Therefore, in predicting the maximum achievable flow of the AHS, the amount of lane changes assumed and the lateral capacity required to accommodate the lane changes must be explicitly considered. In particular, using the longitudinal flow, e.g. the number of vehicles per lane per hour, as the only measure for AHS capacity without any reference to the requirement of lateral flow is misleading. We suggest more study to accurately define the concept and measures of AHS capacity.

Most of the fundamental AHS concepts, e.g. shortening the longitudinal spacing between vehicles, have the potential of increasing only the longitudinal capacity of AHS. While the short spacing increases the longitudinal capacity, it may decrease the lateral capacity to such a degree that the lateral capacity becomes the bottleneck. Since exiting vehicles at the desired off-ramps without sufficient lateral capacity will lead to traffic slowdown, the longitudinal flow suffers as a result. Therefore, the issue of how to optimize the longitudinal flow subject to the requirement of lateral flow is an important issue to be resolved.



# CAPACITY OF AUTOMATED HIGHWAY SYSTEMS: EFFECT OF PLATOONING AND BARRIERS

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## **(1) INTRODUCTION**

### MOTIVATION OF AUTOMATED HIGHWAY SYSTEMS (AHS)

Highway congestion has in recent years become a pervasive problem for urban and suburban areas alike. Lost time, highway fatalities and injuries and air pollution have all risen by such an extent that they are no longer acceptable. The traditional approach to meeting the demand for automobile travel is to expand existing highways and build more highways. However, given the saturation of land dedicated to the existing highways and the difficulty in acquiring private land for highway expansion and construction, this traditional approach is becoming obsolete. The concept of Automated Highway Systems (AHS) is based on the belief that an appropriate integration of control and communication technologies placed on the vehicle and on the highway can lead to a large improvement in capacity and safety without requiring a significant amount of additional highway right-of-way. Stemming from this belief are various conceptual scenarios for vehicle/highway automation.

### AHS CAPACITY

An AHS consists of two major components: vehicle/highway automation technology and highway operating strategy. This paper concentrates on the operating strategy and assumes the feasibility of the automation technology that supports it. The desirability of an AHS hinges upon its performance and crucial performance categories of AHS operation include safety, capacity and human factors. Further assuming driver/public acceptability of the operating strategies considered in this paper, we study the capacity of key AHS operating scenarios. The achievable levels of capacity and safety are heavily interdependent. As pointed out by Shladover [II], different investigators made radically different

assumptions about acceptable safety levels and derived radically different capacity estimates for automated highways. We adopt a robust approach in which key determinants of both the capacity and safety are treated as parameters.

The definition of capacity and its calculation for AHS with traffic needing no lane changes is relatively straightforward. In such a case, capacity of a lane can be defined and measured as the maximum achievable flow in vehicles per lane per hour, which does not reflect at all the *lateral flow* - the flow of vehicles between lanes. However, when lane changes are required, this definition no longer suffices. Furthermore, the whole concept of longitudinal capacity may no longer be meaningful. In fact, the control of highway traffic has multiple objectives. Therefore, the capacity performance of a controlled highway is a multi-dimensional quantity and, moreover, such a highway will have multiple capacities. Note that longitudinal flow can be defined for an isolated lane but this is not true for lateral flow because it involves more than one lane. Therefore, the precise definition of capacity of an automated highway depends on its configuration. In particular, its definition depends on the type and the number of flows that are of interest to the traffic engineer.

In this report, we define *lateral capacity from one lane to the other*, given the two longitudinal flows, as the maximum rate at which vehicles can change from one lane to the other. We use a specific metric as a surrogate for the lateral capacity. The metric is a function of the flows on the two adjacent lanes and is defined to be, given the two flows, the time required for a successful lane change, from the time of lane change preparation to completion without any interference by the other lane change maneuvers. (There are also other types of maneuvers that may interfere with a lane change maneuver. Consideration of such maneuver types are beyond the scope of this paper.) As will be seen, this metric is an increasing function of the longitudinal flows, which exemplifies the antagonistic relationship between longitudinal and lateral flows.

#### AHS OPERATION - THE PRINCIPAL DETERMINANT OF CAPACITY

An operating strategy consists of a collection of operating rules, such as access, lane flow, lane selection, lane change and egress rules [13]. All of these rules will have an impact on AHS capacity. The capacity calculation without detailed consideration of capacity loss due to lane changes has been



reported in the literature [e.g. 10]. Recognizing that entrance and egress of vehicles are the primary cause of traffic stream disturbance and will ultimately dictate the sustainable flow, Rao et al. [8] recently investigated three different strategies for allowing vehicles to enter and leave automated lanes. In an effort of a different direction, Rao and Varaiya [6] studied the achievable capacity and traffic stream stability when only a portion of vehicles on a highway use the Autonomous Intelligent Cruise Control technology - a partial automation technology. A common focus of these efforts is the achievable longitudinal flow. With minimum interaction between traffic entering the automated lane and the traffic exiting it, exiting success rates were also simulated for a few different combinations of highway configuration and traffic demand.

The sustainable throughput of an AHS hinges on how it is configured and operated. Major design decisions include (i) the degree of segregation (or mixing) of automated vehicles from (with) **manually**-driven vehicles, (ii) if and how to separate two neighboring lanes by physical barriers, and (iii) if and what close-following vehicle formation, in particular *platoon*, should be adopted [13]. All three decisions will have a significant impact on capacity and safety. We will define the scope of our investigation with respect to these three AHS attributes.

Vehicle uniformity makes control of automated vehicles simpler and likely safer. It should also lead to a higher capacity. In a physically isolated AHS dedicated to automated vehicles, unequipped vehicles can be prevented from entering. This is much more difficult when manual and automated lanes are adjacent.

## TWO MAJOR AHS OPERATIONAL ISSUES - VEHICLE CLUSTERING AND LANE BARRIERS

A major configuration option is the erection of lane barriers and the concomitant openings, called *gates*, for lane changes. Erection of barriers is motivated by the desire to prevent catastrophic collisions. Since any AHS that increases highway capacity would operate with reduced headways, a vehicle failure has the potential of causing catastrophic multiple collisions. After a failure of the lateral control, a vehicle may stray into a neighboring lane, very likely to have dense traffic, and cause a collision. If vehicles or debris spin or sway into a neighboring lane after a minor collision, a major collision may result. Motivated by these two concerns, Hitchcock [2] proposed the erection of barriers between lanes.

A major AHS operating rule that has extensive effect on all aspects of AHS operation is the lane-flow rule, the rule that governs the behavior of vehicles in a common lane (e.g. the spacing between any two adjacent vehicles). Two basic lane-flow rules are the *platooning rule* and the *free-agent rule*. The platooning rule was first proposed and studied by Shladover in the late 70's [10] and received renewed interest in the last few years. Under this rule, adjacent vehicles in the same lane either travel very close to or very far from each other. As a result, vehicles are organized in a clustered formation. Each cluster of vehicles is called a platoon. The large inter-platoon spacing can minimize the probability of any collision between platoons in the same lane and the short intra-platoon spacing ensures that any collision within a platoon will initially have a small relative speed and, presumably, low severity. Under the free-agent rule, vehicles move without any clustered formation and the minimum longitudinal spacing is significantly longer than typical intra-platoon spacings, but significantly shorter than typical inter-platoon spacings. Relative to the platooning rule, the free-agent rule reduces the overall frequency of collisions, but potentially increases the frequency of severe ones. Also, AHS with the free-agent rule might be easier to operate, with potentially higher lateral capacity. For a detailed comparison of safety between these two lane-flow rules, refer to [12].

#### SCOPE OF PAPER

We limit the scope of our study to the effect of the lane-flow rule, platooning or free-agent, as well as the lane barriers, on AHS capacity. Since it is the need to change lanes that makes the capacity estimation difficult, we will pay special attention to the interaction between the lane-flow rule and the lane change requirement in our study.

This paper consists of two major components, analytical models and AHS simulation. After a brief introduction of AHS operating strategies, analytical models are developed for general AHS. However, for simulation, we focus on a segregated AHS that has one automated lane and one transition lane. In other words, all vehicles that enter the AHS are equipped with automation equipment and they access the automated lane from the transition lane. Equipped vehicles switch into automated driving mode on the transition lane before they move into the automated lane. Similarly, they switch back to

the manual driving mode in the transition lane. This focus is motivated by its simplicity and the fact that the insights gained will facilitate the study of more complicated AHS, e.g. non-segregated AHS. Figure 1 shows the AHS under simulation study, with the option of vehicle clustering and lane barriers.

In this report, the capacity of an AHS is measured by the following metrics:

(M1) the rate at which vehicles can change lanes (and the success probability of a lane-change attempt),

(M2) traffic flow in each lane,

(M3) speed distribution, in particular the average speed and the standard deviation.

(M1) measures the lateral flow between two lanes while (M2) the longitudinal flow. In addition, (M3) measures the stability of longitudinal flow. While most existing literature on AHS performance focuses on the longitudinal traffic flow, we will emphasize the lateral flow equally. In our opinion, it is the ability to reach the desired destination efficiently (through lane changes) that the users of highways want most. Therefore, the performance objective should be to maximize the AHS flow subject to the stringent constraint that all or nearly all users exit the AHS at their desired exits.

The result of this study, in particular in terms of the three metrics, can be used as input to the lane-selection models for the capacity optimization of an AHS corridor or network.

#### APPROACH ANALYTICAL AND SIMULATION

We tackle the capacity estimation problem with two parallel but coordinated efforts: analytical modeling and computer simulation. The former does not account for the details of AHS operation but provides insight into AHS throughput as a function of a variety of AHS parameters. The latter provides detailed performance statistics but requires a large computing power. This dual approach, analysis combined with simulation, has been adopted to best utilize the strengths of each.

In the analytical effort, we develop mathematical models for vehicle/platoon and gap distributions to understand the traffic patterns on individual AHS lanes. Based on the models developed and insights obtained, we provide estimates for a key measure of lane-change efficiency of AHS operating strategy.

These analytical models when coupled with models for lane selection and access/egress, being developed but to be reported separately, can provide analytical estimates for AHS capacity under some operating strategies.

In the simulation effort, we study the throughput by simulating an AHS segment with existing highway traffic flowing in at the beginning of the segment and also with traffic **in-flow** and out-flow at the entrances and exits respectively.

After the analytical models for lane selection are developed and the simulation tool extended to incorporate intelligent lane selection rules, the success rate of vehicles reaching the desired exits may be used as another important metric for gauging the capacity of an AHS.

## ORGANIZATION OF PAPER

Section 2 provides the theoretical foundation of this capacity study: operating concepts and mathematical models. Section 3 briefly describes the simulator **SmartPath**. Section 4 presents the result of our simulation study. Concluding remarks are given in Section 5.

## **(2) THEORY**

This section consists of two subsections. In the first subsection, we define the AHS under study, introduce the major categories of operating rules, discuss the lane-flow and lane-change rules and their interaction. In the second, we develop analytical models for headway distribution under the free-agent longitudinal-separation (vehicle-following) rule, and the platoon-size and interplatoon-spacing distributions under platooning. For both longitudinal-separation rules, we develop models to estimate the time required for a lane change. Numerical examples are given to illustrate the theory.

## (2.1) Operating Strategies

### MAJOR AHS ATTRIBUTES AND ASSUMPTIONS

We deal with only those operating rules pertaining to normal AI-IS operation. The detail of these rules depends heavily on whether automated vehicles are mixed with the manual vehicles and, if so, how they are mixed. They also depend on the types of vehicles to be automated. We assume a segregated AHS dedicated to automated vehicles. In such an AI-IS, there may be a transition lane through which automated vehicles switch into automated driving or back into the manual driving mode. We assume that this lane is dedicated to the use of transition. We further assume that only one type of vehicle is accommodated on the AHS.

### MAJOR CATEGORIES OF OPERATING RULES AND FOCUS

Major categories of rules for normal AHS operation include: access, egress, lane flow, lane selection, and lane change. We concentrate on the lane-flow and lane-change rules and their interaction.

### LANE-FLOW RULES

Lane-flow rules stipulate how vehicles should move along a lane, in particular their grouping (if any) spacing, speed, and deceleration/acceleration. The platooning and the free-agent rules, differing in vehicle grouping, are the two most prominent classes of lane **flow** rules. Besides the capacity issues, these two rules have safety and human factors implications.

### LANE-CHANGE RULES

#### Cooperative vs. Non-Cooperative Lane Changes

A major design issue for AHS operation is whether and the degree to which vehicles cooperate with one another for a lane-change maneuver. Cooperation for a lane change refers to the accommodating movements made by the vehicles nearby the lane-change vehicle *after* a lane-change request. In a non-cooperative design, a lane change can be completed only when the lane-change vehicle encounters

a gap of sufficient length in the receiving lane. However, in a cooperative design, the vehicles nearby the lane-change vehicles may coordinate, after the lane-change request, their movement in order to facilitate the lane change, possibly through creating a gap of sufficient length. Vehicles can cooperate in many different ways to facilitate a lane change with different “degrees of cooperation”. Note that vehicles can cooperate for lane-change facilitation whether the lane flow is under the platooning or free-agent rule.

#### Gap Management

The roadside control system may manage - e.g. maintain, create, lengthen, shorten and close - gaps by moving vehicles, independent of the specific lane-change requests and regardless of the lane-flow rule adopted.

#### Lane-Change Scheduler

Starting the lane change process for an exiting vehicle early would maximize the probability of successful exiting. However, starting the lane change procedure too early may clog the slower lane (i.e., the outer lane, which is expected to have a slower average speed than the inner lanes). Such clogging may cause lane-change congestion to simply migrate upstream. A primary goal of a good lane-change rule is to seek an appropriate balance between these two conflicting objectives, successful exiting and high longitudinal flow in the receiving lane. Since demand may vary over length of highway, lane changes may also be needed for balancing traffic flows on different lanes (for maximization of overall longitudinal flow). Given the identification of the vehicles needing lane changes, their current position, their destination lanes or exits and the traffic condition, an algorithm is needed to determine the timing of lane-change initiation. This algorithm will be referred to as lane-change scheduler. This scheduling function is needed regardless of the degree of lane-change cooperation and the lane-flow rule.

### The Difference Between Lane-Change and Lane-Selection Categories

If the AHS tracks the vehicle/gap movement precisely and accurately, then the lane changes can be, at least in theory, done in such a way that optimizes the longitudinal lane flow with a high probability of successful exiting for every vehicle. The distinction between the realms of lane-change rules and lane-selection rules lies in the focus of such optimization. The former focuses on the vehicles/gaps assignment in a short AHS segment based on local traffic conditions while the latter looks across segments and plans for vehicle trips from origin to destination. Balancing the flows among different lanes is achieved through moving some vehicles from one lane to another, which involves (i) the identification of lane-change vehicles, (ii) the designation of their destination lane and (iii) their lane-change maneuvers. Since (i) and (ii) should take into consideration the vehicles' destination off-ramp, they are under the jurisdiction of the lane-selection rules. Once (i) and (ii) have been determined, the lane-change scheduler then determines the timing of the actual lane-change maneuvers, i.e. (iii). In this paper, we focus on modeling for lane-change rules.

Rao and Varaiya [7] proposed a roadside controller design to optimize traffic flow along a stretch of automated highway. Each controller operates over a few-kilometer segment of automated highway and requires only simple information about traffic conditions in its vicinity and a small amount of information from the next controller downstream. Using simple policies, it prescribes and regulates the lane-changing activities. Due to the local nature of this controller, it provides local path planning but does not assign a full path from a vehicle's entry on-ramp to its desired off-ramp traversing through different lanes at different times. In other words, their local path planning is different from our concept of lane selection defined above. It is also different from our concept of lane-change scheduling in that it does not time or sequence the multiple lane change maneuvers to be made in the segment. In fact, they modeled the traffic as a compressible fluid in their simulator and deal with the macroscopic behavior of traffic flow, instead of the microscopic movement of individual vehicles.

## (2.2) Analytical Models

We develop several lane-change models for AHS scenarios without lane barriers and without lane-change cooperation among vehicles. We discuss these models in two subsections, one for the free-agent rule and the other for platooning. Since, under operating strategies with lane barriers, a successful lane change requires the alignment of the lane-changing vehicle, the gate and the receiving gap at the time of lane change, more intelligent vehicle control as well as roadside control would be required. If so, the operation would be more complicated and may not lend itself to tractable analytical modeling. Also, since lane-change cooperation could make AHS operation more complicated, analytical modeling would be more difficult.

### (2.2.1) Lane-Change Models Under the Free-Agent Lane-Flow Rule:

#### OVERVIEW OF MODELS AND MODELING APPROACH

##### Lane-Change Completion Time Model Based on Physical Vehicle/Gap

###### Distribution Model

We first model the vehicle/gap distribution in a lane. Based on this distribution, we model the time from the initiation of lane-change request into this lane until the lane-change attempt is completed successfully, without any interference from any other lane-change maneuvers. (We will refer to this time as *the lane-change completion time*, which consists of the *waiting time* and the *maneuvering time*.) These models can be used to estimate the lateral capacity of different operating scenarios.

###### Axiomatic Approach

For all the models, we adopt an axiomatic approach. In other words, we first state the assumptions and derive the resulting models as necessary consequences of the assumptions. Therefore, the justifiability of the models is precisely that of the assumptions.

In practice, there may be interference among different lane-change maneuvers, e.g. interference caused by near-by lane-change maneuvers, in the same or opposite direction. Therefore, the probability



distribution of completion time obtained this way can be used as an upper bound on the actual completion time distribution.

Assumption of No Cooperation, and Positive and Constant Speed Differential

As indicated earlier, we assume no cooperation among vehicles for lane changes. Therefore, the only way to complete a lane change is for the lane-change vehicle to encounter a sufficiently large gap. To increase the likelihood of encountering such a gap, we assume that the speed differential,  $\Delta v$ , between the two lanes is strictly positive. (This assumption is relaxed in our simulation study.)

If, during a lane-change maneuver, the longitudinal acceleration/deceleration is constant and the lateral velocity is also constant, the maneuvering time,  $t_m$ , is simply

$$t_m = \max \left\{ \frac{\Delta v}{d}, \frac{w_l}{v_l} \right\},$$

where  $\Delta v$  = lane speed differential ,

$d$  = acceleration/deceleration rate during maneuver ,

$w_l$  = lane width (center line to center line),

$v_l$  = lateral velocity during lane change.

Note that if (i) the acceleration rate is the same as the deceleration rate in a lane change from the faster lane to a slower lane and (ii) the lateral velocity is the same regardless of direction, then the maneuvering time is independent of the direction of the lane change. Since the waiting time until the arrival of a gap of a sufficient length depends only on the relative traffic speed and the traffic conditions on the receiving lane but *not the direction itself*, there exists, for modeling purposes, a directional symmetry. Therefore, we treat only the case where a vehicle on a faster lane tries to change into a neighboring slower lane.

## MODEL INPUT/OUTPUT

### Physical Vehicle/Gap Distribution

For the vehicle/gap distribution, the major input is the vehicle density in a lane and the output is the joint distribution of all the gap lengths on the lane. (Under the assumptions to be itemized, the gap length distributions are independent.)

### Time Till Lane-Change Completion

Major inputs to the lane-change time model consist of the relative speed between two lanes, the minimum length of a gap required for a safe lane change and a measure for the vehicle density in the receiving lane, which is also the major input to the vehicle/gap distribution model.

The following two subsections describe briefly the vehicle/gap distribution models and the lane-change completion time models.

#### **(2.2.1.1) Gap Distribution Between Two Free-Agents**

We develop two models for the probability distribution of gap length. In either model, a vehicle occupies a slot consisting of (i) the space physically occupied by the vehicle and (ii) two safety spacings “padded” on the two ends of the vehicle; a gap is defined to be the space between the slots occupied by two adjacent vehicles in the same lane. The first model is based on the assumption that a lane is partitioned into a number of moving vehicle slots and each slot is either occupied by a vehicle or empty. Therefore, the gap length can only be a non-negative integer multiple of the slot length. The second model relaxes this assumption and allows the gap length to be any non-negative real number. In these two models, velocity will play no role.

#### **(2.2.1.2) Gap Length Distribution: Under the Slot Assumption**

In this model, a segment of highway is divided into a number of slots and a gap length is simply the number of empty slots between two longitudinally adjacent vehicles. Consider a segment of a traffic lane with the following assumptions:

ASSUMPTIONS

- (AG1) The segment is partitioned into  $s$  equi-length slots, whose length, denoted by  $l_s$ , is the maximum of (i) length of a vehicle plus the minimum inter-vehicle safety spacing and (ii) the length of a gap which a lane-changing vehicle can safely move into from a neighboring lane without disturbing the speed of traffic on either lane †.
- (AG2) There are  $v$  vehicles (occupied slots) in the lane.
- (AG3) Any distribution of the  $v$  vehicles in the  $s$  slots is as likely as any other distribution.

These concepts and some of the assumptions are illustrated in Figure 2.

INDEPENDENT GEOMETRIC GAP LENGTH DISTRIBUTIONS

Under these assumptions, one can calculate the joint distributions of the  $v+1$  gaps as well as the marginal distributions of the individual gaps. Furthermore, when the occupancy ratio, i.e.  $v/s$ , is kept constant while the length of the segment is approaching infinity, it can be shown that (i) all the gap distributions are independent and identically distributed and (ii) the identical distribution is a Geometric Distribution with a success probability of  $v/s$ . Let  $N_i^s$  denote the gap length, i.e. the number of slots between two adjacent vehicles. The gap length distribution is simply:

$$p(N_i^s = i) = \frac{v}{s} \left[ 1 - \frac{v}{s} \right]^i, \quad i=0,1,2, \dots$$

Let  $N_c^s$  denote the cluster size, i.e. the number of occupied slots between two adjacent vacant slots. The cluster size distribution is simply:

$$p(N_c^s = i) = \left[ 1 - \frac{v}{s} \right] \left[ \frac{v}{s} \right]^i, \quad i=0,1,2, \dots$$

Appendix A provides a simple derivation to illustrate these facts.

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† By definition, the spacing is dictated by the maximum because split and join operations, for creating (or lengthening) and closing (or shortening) a gap, are not allowed in the moving-slot system. The length of such a gap depends on the speed differentials between lanes. Under the assumption that all vehicles on a lane maintain a constant speed, although low speed differential would require a shorter gap, the time and hence the distance needed to encounter such a gap may become longer.

### (2.2.1.3) Gap Length Distribution: Continuous Length

Consider again a segment of a traffic lane. In this model, a gap length is defined to be the distance between two longitudinally adjacent vehicles minus the minimum safety spacing between them.

#### ASSUMPTIONS:

- (AE1) There are  $v$  equi-length vehicles, with a length of  $l$ , on a lane segment of length  $s$ .
- (AE2) The minimum safety distance between any two vehicles is  $d$ . Together with (AE1), a vehicle occupies a length of  $l + d$ .
- (AE3) The  $v$  vehicles are at random positions on the lane in the following sense: After removing the space occupied by the  $v$  vehicles and consolidating the gaps, the segment length becomes  $s - v(l + d)$  and each vehicle is represented as a point on the segment.
- (AE4) The counting process  $\{N(x), x \geq 0 \mid N(x) = \text{the number of vehicles (points) between the beginning of the segment and length } x \text{ into the segment}\}$  is a Poisson Process.

The definition of a gap and some of the assumptions are illustrated in Figure 3.1 and Figure 3.2.

#### INDEPENDENT EXPONENTIAL GAP DISTRIBUTIONS

Based on these assumptions, all gap lengths are independent and identically distributed with an exponential distribution and a rate of  $v/(s - v(l + d))$ . The probability density function,  $f(x)$ , of the distribution of gap length, denoted by  $N_l^c$ , is:

$$f(x) = \lambda e^{-\lambda x}, \text{ where } \lambda = \frac{v}{s - v(l + d)}.$$

To illustrate the idea in a more intuitive way, an alternative approach with an identical result is provided in Appendix B.

### (2.2.1.4) Lane-Change Time/Distance Requirement

We propose two models, which are based on the models proposed earlier for vehicle/gap distribution.

### (2.2.1.5) Lane-Change Time/Distance Requirement: Under the Slot Assumption

We adopt the terminology and assumptions made in Section (2.2.1.2), and make two further assumptions:

(AG4) The lane change is initiated at a point that is random with respect to the destination lane. In other words, the distribution of the initiation point is independent of the vehicle/gap distribution in the destination lane.

(AG5) When a vehicle initiates a lane change, it must be properly positioned next to a slot in the destination lane so that it can move into the slot safely if the slot is empty.

We first find the distribution of the number of slots passed by the lane-change vehicle, denoted by  $N_w^s$ , before the success, which is simply:

$$p(N_w^s = i) = \left(1 - \frac{v}{s}\right) \left(\frac{v}{s}\right)^i, \text{ where } i=0,1,2, \dots$$

The total time  $T$  from the initiation of a lane-change preparation until a successful lane change is the sum of the waiting time, denoted by  $T_w$ , and the time  $t_m$  required for the lane-change maneuver itself.

By the assumption of constant speed difference,

$$T_w = \frac{N_w^s \times l_s}{\Delta v},$$

and the probability function of  $T$  is

$$p\left(T = t_m + \frac{i \times l_s}{\Delta v}\right) = \left(1 - \frac{v}{s}\right) \left(\frac{v}{s}\right)^i.$$

Therefore, the expected value of  $T$  is

$$E(T) = \sum_{i=1}^{\infty} \left(1 - \frac{v}{s}\right) \left(\frac{v}{s}\right)^{i-1} \left[ t_m + \frac{i \times l_s}{\Delta v} \right].$$

Note that, contrary to Assumption (AG5), when a vehicle initiates a lane change, its position relative to the neighboring slot in the destination lane may not allow a safe lane change. Therefore,  $T$  defined above may underestimate the total amount of time till the lane-change completion. However, adding the time  $l_s/\Delta v$  required for the lane-change vehicle to catch up with the length of one slot to  $T$  would produce an upper bound for the the total time.

### (2.2.1.6) Lane-Change Time/Distance Requirement: Continuous Length

We adopt the terminology and assumptions made in Section (2.2.1.3) and further assume:

- (AE5) The lane change is initiated at a point that is random with respect to the traffic on the destination lane.
- (AE6) When a vehicle initiates a lane change, it is properly positioned next to a gap in the destination lane so that it can move into the gap safely if the gap is sufficiently large.

The distribution of the number of gaps passed by the lane-change vehicle, denoted by  $N_g$ , before the success is a geometric distribution, due to the memoryless property of the exponential distribution:

$$p(N_g = i) = (1-q)q^i,$$

where  $q$  is the probability of a gap length less than  $l_g$ , the minimum gap length required for a safe lane change †, and

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† This could include the distance needed to complete the speed change. In fact, a question about controlling AHS traffic is when to change the speed of the lane-changing vehicle. Three major timing options are (i) before the lane change maneuver can begin, (ii) after the maneuver has completed, and (iii) while the vehicle is changing lane as a part of the lane-change maneuver. If the **speed** change is completed before the maneuver begins, then the minimum gap length could be quite small. However, if the destination lane has a lower speed, then the speed change may slow down the traffic on the origin lane.

$$q = 1 - e^{-\lambda l_g} \text{ and } \lambda = \frac{v}{s - v \times (l+d)}$$

Given  $N_g = n_g$ , the waiting time, denoted by  $T_{w, n_g}$ , is a random variable and

$$T_{w, n_g} = \frac{n_g \times (l + d) + \sum_{i=1}^{n_g} V_i}{Av},$$

where  $V_i, i=1,2,\dots,n_g$ , are independent and identically distributed with a probability density function of

$$g(x) = \frac{\lambda e^{-\lambda x}}{1 - \lambda e^{-\lambda l_g}} \text{ if } 0 \leq x \leq l_g,$$

$$0 \quad \text{otherwise}.$$

Weighing the conditional distributions of  $T_{w, n_g}$  by that of  $N_g$  gives the distribution of total waiting time. To obtain the distribution of the total amount of time  $T$  from the initiation of lane-change preparation till its completion, the waiting time distribution should be shifted by  $t_m$ , the amount of time required for the maneuver itself.

Note that, contrary to Assumption (AE6), when a vehicle initiates a lane change, its position relative to the neighboring gap, if there is any, in the destination lane may not allow a safe lane change. Therefore,  $T$  defined above may slightly underestimate the total amount of time till the lane-change completion. However, adding the time  $(l+d)/\Delta v$ , the time required for the lane-change vehicle to catch up the length of the space occupied by a vehicle one slot, to  $T$  would slightly overestimate the total time.

We close this subsection with the following remarks. To maximize the probability of successful exit and lane change, the lane-flow rules do play an important role. For example, in a free-agent scenario, the gap between two vehicles should better be a multiple of the length of a *slot*, a minimum space which a lane-change vehicle in a neighboring lane can safely move into. Any gap shorter than the slot length will be a waste of space. Also, if the vehicles needing to change lane are randomly **dis-**

tributed in the AHS, then randomly distributed empty slots may shorten the time or distance requirement for a successful lane change.

## (2.2.2) Models for Platooning

In studying the capacity associated with any operating scenario **with** the platooning lane-flow rule, the platoon size distribution is needed for a variety of reasons. For example, a possible lane-change strategy may be to allow a vehicle to join a platoon only at the front of the platoon and, in this case, the platoon length distribution is needed to estimate the time for a lane-change vehicle to catch up with the front of the platoon.

A lane can be thought of as an alternating cycle of platoon and gap. Assuming probabilistic independence among all different cycles and **the** independence between the gap and platoon size distributions, all one needs to know about the traffic in the target neighboring lane are the gap length distribution and the platoon size distribution.

### (2.2.2.1) Gap Distribution

#### DEFINITION OF GAP LENGTH

A gap between two platoons is defined as follows and depicted in Figure 4. Each vehicle requires a length of  $l+s_1$  within a platoon, including the vehicle length  $l$  and the safety spacing  $s_1$  between two vehicles. In other words, each of the vehicles in a platoon occupies a slot of length  $l+s_1$ . For ease of discussion, assume that  $s_1/2$  is allocated in the front and the other  $s_1/2$  is allocated in the rear of the vehicle, regardless of whether there is an adjacent vehicle to keep a safety spacing from. There is a minimum safety spacing  $s_2$  required for any pair of adjacent platoons. For ease of discussion and as assumed for the intra-platoon spacing  $s_1$ , assume that  $s_2/2$  is allocated in the front and the other  $s_2/2$  is allocated in the rear of the platoon, regardless of whether there is a platoon to keep a safety spacing from. Therefore, a platoon of size  $n$  occupies a length of  $s_2+n(l+s_1)$ .

Given a fixed number of platoons per unit distance of AHS, one can use the argument employed in Section (2.2.1.3) to justify the use of an exponential distribution for **the** gap length. However, unlike



the free-agent scenarios where a highway segment can be justifiably partitioned into slots and the number of gaps (possibly having 0 slots in the gap) is simply the number of vehicles in the segment plus 1; the number of platoons is uncertain. This makes the use of a single exponential distribution theoretically unjustified. However, the gap length distribution can be modeled as a mixture of exponential distributions. In practice, when the average number of platoons per segment length is known and the variation of the number of platoons across different segments is small, the use of an exponential distribution may be a good approximation. We will use this family of distributions in estimating the time/distance elapsed until a successful lane change. We now turn to the calculation of the platoon size distribution.

### (2.2.2.2) Platoon Size Distribution

The dynamic nature of highway traffic adds the dimension of time to the definition of the platoon size distribution. The probability of a particular size may be interpreted as the long-term proportion of time any platoon, out of the many or in theory the infinite number of platoons on a lane, has a particular size.

#### WHAT CHANGES THE PLATOON SIZE?

Before developing a model for the distribution, we first identify the important factors that affect the platoon size. We make **the** following assumptions:

Vehicles change lane as individuals, not in platoons.

When a vehicle is joining or leaving a platoon, no **other** vehicles can join or leave the platoon.

In an AHS with multiple automated lanes, a vehicle, to reach a target lane, may have to pass through a number of intermediate lanes and in the process join and leave various platoons. We assume that such temporary stays with a platoon do not have a significant effect on the distribution of platoon size.

Therefore, the important factors affecting **the** size of a particular platoon are:

- (C1) a vehicle, upon entering the AHS, *joins* the platoon and stays until its time for exiting the AHS,
- (C2) a vehicle, after an extended stay, leaves the platoon for exiting,
- (C3) a vehicle, for the purpose of balancing traffic flow in different lanes, *joins* the platoon,
- (C4) a vehicle, for flow balancing, leaves the platoon,
- (C5) two platoons *join* to become one, and
- (C6) a platoon *splits* and becomes two.

We do not model the last two factors because (i) not all platooning scenarios allow such joins and splits and (ii) they would require a different modeling approach from the one to be described in the rest of the section. Therefore, we only deal with size changes resulting from lane changes. As a result, the size of a platoon can change only by 1. Note that the first four factors involve a vehicle either entering a platoon *for an extended stay* or departing a platoon *after an extended stay*. Since the entry and departure rates depend on the routing of traffic from their entrance to their exits via different lanes in different sections, we will simply develop a parametric model. We focus on a single lane and assume that the flow on the lane remains constant throughout **the** highway segment and throughout the time horizon. Also, we assume a known entry rate and, consequently, an identical departure rate, resulting from lane changes, to ensure a constant flow.

#### MAXIMUM PLATOON SIZE

The size of a platoon may be constrained for a variety of reasons, e.g., the need to be able to complete a cycle of message relay from the platoon leader to all its followers in the platoon and back. We will assume an upper limit on the platoon size.

#### A SPECIAL FEATURE OF SIZE EVOLUTION

The size of a platoon changes if and only if a new member vehicle arrives or an existing member vehicle departs. The arrival rate and the departure rate are the two principal determinants of the platoon size distribution. Suppose a vehicle will join a platoon if and only if it is adjacent to **the** space occu-

pied by a platoon at the lane-change initiation time. Then the larger the platoon size, the more space it occupies and the higher the arrival rate. Also suppose each vehicle in a common lane has an identical probability of departing the lane. Then the departure rate also increases with the platoon size. In our opinion, any credible model should definitely **account** for the maximum size and the dependence of size-change rates on the platoon size.

### MODELING APPROACH

Assuming that all platoon sizes are identically distributed, we provide a dynamic treatment for calculating the platoon size distribution. In other words, instead of conducting a static combinatorial analysis for the vehicle distribution on a segment as in deriving the gap distribution between two **free-**agents in Section (2.2.1.2), we concentrate on the size evolution of a platoon through time. We will use the technique of Continuous Time Markov Chains to model the evolution [9].

By an argument similar to the one used in deriving the exponential gap-length distribution for the free-agent scenario in Section (2.2.1.3), we can justify the use of an exponential distribution to model the inter-arrival time and inter-departure time. Since these processes can be safely assumed to be independent, we can use the Birth and Death Process, a special Continuous Time Markov Chain, to model platoon size evolution.

We first study the Markov Chain embedded in the Birth and Death Process. Let the size of the platoon be the state. The embedded Markov Chain clearly has only  $n_{\max}$ , the maximum allowable size, **states**. However, since a platoon disappears from a lane after a vehicle, as a single-vehicle platoon, changes lane, we augment the state space of the embedded Markov Chain to include 0. Note that a complete specification of this augmented Markov Chain requires the knowledge of the unknown birth rate at state zero. However, it turns out that, to obtain the platoon size distribution, this birth rate is not required. This is because the platoon size distribution is the *conditional* distribution of states 1 through  $n_{\max}$ , *given* that the state is not 0. As will become clear later, this conditional distribution is *independent of* the birth rate at state 0.

The approach is, intuitively and simply put, that when a platoon has formed, we zero in on it and

observe the size evolution. When a platoon has just vanished from the lane, we look elsewhere in the lane for a newly-formed single-vehicle platoon to continue the observation.

### ASSUMPTIONS

We now list the major assumptions, some of which have been discussed earlier:

- (AP1) Vehicles change lanes as individuals, not in platoons, and when a vehicle is entering or departing a platoon, no other vehicles may do so.
- (AP2) The platoon size is affected by only factors (C1) - (C4). In other words, platoon merges and splits are not allowed. Together with (AP1), the platoon size may change only by one.
- (AP3) A lane-change vehicle will join the platoon if its longitudinal position, at the initiation of the lane-change, is within  $s_2$  beyond either end of the platoon.
- (AP4) The platoon size behaves according to a Birth and Death Process in which (i) the arrival rate is proportional to the length of space occupied by the platoon (except size 0 and the maximum size) and (ii) the departure rate is proportional to the size of the platoon. The per-unit-distance rate (number/(time×length)) of vehicles entering a lane is  $r_e$ . The per-vehicle rate (number/time×vehicle) of vehicles leaving the lane is  $r_l$ .

### THE SOLUTION

Let  $\lambda_i, i=0,1,2, \dots, n_{\max}$ , denote the birth rate, i.e. the exponential rate at which a new vehicle joins the platoon, when the platoon consists of  $i$  vehicles. Similarly, let  $\mu_i, i=1,2, \dots, n_{\max}$ , denote the death rate when the platoon has  $i$  vehicles. In terms of the rates defined in (AP4),

$$\lambda_i = r_e \times (i \times (l + s_1) + 2s_2), \text{ for } i=1,2, \dots, n_{\max}-1, \text{ and } \lambda_{n_{\max}} = 0;$$

$$\mu_i = r_l \times i, \text{ for } i=1,2, \dots, n_{\max}, \text{ and } \mu_0 = 0.$$

Note that  $\lambda_0$  is unknown. Denote the limiting probability that the platoon is of size  $i$  by  $p_i$ . By equating the transition rates in and out of each size, we obtain the following set of equations:

$$\lambda_0 p_0 = \mu_1 p_1$$

$$(\lambda_i + \mu_i) p_i = \mu_{i+1} p_{i+1} + \lambda_{i-1} p_{i-1}, \quad i=1, 2, \dots, n_{\max}-1, \quad \text{and}$$

$$\mu_{n_{\max}} p_{n_{\max}} = \lambda_{n_{\max}-1} p_{n_{\max}-1}.$$

These equations can easily be solved in terms of  $p_0$  and

$$p_i = \frac{\prod_{k=0}^{i-1} \lambda_k}{\prod_{k=1}^i \mu_k} p_0, \quad i=1, 2, \dots, n_{\max}.$$

Since we are only interested in the conditional probabilities, denoted by  $p'_i, i=1, 2, \dots, n_{\max}$ , of positive platoon length given the existence of the platoon, regardless of the value of  $\lambda_0$ , we obtain the probability distribution of platoon size as follows:

$$p'_i = \frac{\prod_{k=1}^{i-1} \lambda_k}{\prod_{k=2}^i \mu_k} p'_1, \quad i=2, \dots, n_{\max}, \quad \text{and}$$

$$p_1 = \frac{1}{1 + \sum_{i=2}^{n_{\max}} \frac{\prod_{k=1}^{i-1} \lambda_k}{\prod_{k=2}^i \mu_k}}.$$

The gap and platoon size distributions can be used to estimate the time/distance till the completion of a lane change. The usage depends on the lane change rule. In those rules in which vehicles can join a platoon anywhere in the platoon with little preparation, e.g. a minor splitting and the subsequent joining, the waiting time and hence the time till lane-change completion should be relatively short. In this case, the platoon size distribution is of little use. † For lane-change rules that are more restrictive,

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† Note that the preparation not only takes time but also increase the probability of interference between separate lane-change attempts. If preparation requires the platoon to perform a full split into two separate platoons and then sub-

e.g. one that only allows lane-change vehicles to join in the front or at the end of the platoon, this distribution can be very useful, as illustrated in the next subsection. We close this subsection with an example.

### EXAMPLES

The key parameter to the platoon size distribution is the traffic density. We consider four different densities and compare the resulting distributions. The four densities and the corresponding approximate traffic flows at the speed of 60 miles/hour are:

(C1) 5 vehicles/kilometer (500 vehicles per hour)

(C2) 20 vehicles/kilometer

(C3) 40 vehicles/kilometer

(C4) 80 vehicles/kilometer

Note that condition (C2) represents the typical lane capacity of a conventional highway while Conditions (C1) and (C4) represent very light and very heavy AHS traffic respectively.

### Other Parameters

Consider a hypothetical scenario with

(P1) a maximum platoon size of 10,

(P2) a highway segment of length 1 kilometer,

(P3) 4 vehicles entering and leaving the segment per minute,

(P4) an interplatoon safety spacing of 50 meters,

(P5) a common vehicle length of 5 meters, and

(P6) an intra-platoon spacing of 1 meter.

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sequently merge back into one, not only the lateral flow but also the stability of longitudinal flow suffer. Because of the apparent advantages of a minor split followed by a merge for a lane change, its feasibility, in terms of control technology and safety, should be investigated.

### Numerical Solution

Given the maximum platoon size of 10, the mean platoon size should be very small under Condition 1 but very large under Condition 4. Based on the common parameter values in (P1) through (P6) and the four different traffic densities, the four platoon size distributions are calculated and plotted in Figure 5. The expected value and the standard deviation of platoon size distributions are plotted against 16 different traffic conditions in Figure 6.

### **(2.2.2.3) Time/Distance Till Lane Change Success: An Application of the Platoon Size Distribution**

Consider an AHS operating scenario with the following rules.

- (R1) Vehicles move along a lane in platoons,
- (R2) Vehicles change lane individually,
- (R3) When changing lane from a faster lane to a slower lane, a vehicle is allowed to join a platoon in the destination lane only at the front of the platoon.
- (R4) When changing lane from a slower lane to a faster lane, a vehicle is allowed to join a platoon in the destination lane only at the rear of the platoon.

We now use the platoon size distribution to approximate the elapsed time for a lane change from a *faster* lane to a *slower* lane in this scenario. Under the assumptions made in Section (2.2.2.2), we further assume:

- (AP5) The position of the lane-change vehicle can be represented as a point.
- (AP6) If, at the lane-change initiation time, the vehicle is adjacent to a platoon in the destination lane, we overestimate the lane-change time by requiring that the lane-change vehicle catch up with the *full* length of the platoon.
- (AP7) The traffic on the segment of the destination lane consists of recurrent cycles of platoon and gap and any platoon/gap cycle is partitioned into 3 sections: (i) a section of length  $s_2$ , the minimum inter-platoon safety spacing, (ii) a section consisting of a sequence of vehicle slots each of which is of length  $l_s = l + s_1$ , and (iii) a section of

empty space between the front of the platoon and the beginning of the next cycle, as depicted in Figure 7. If the lane-change preparation is initiated when the vehicle is adjacent to section 1 or 2 in the destination lane, it will have to wait until it catches up with the front of the platoon. Otherwise, it can make the change with no waiting time at all. †

(AP8) The length of section 3 has an exponential distribution.

(AP9) Lane change initiation time is random, i.e. no scheduling or sequencing for individual lane change maneuvers.

### APPROACH

We first find the probability that the vehicle, at the time of lane-change initiation, is adjacent to each of the 3 sections. Given the section, we calculate the conditional distribution of the total elapsed time. The unconditional elapsed time distribution is then obtained by weighting the three conditional distributions by the three position probabilities.

Denote the length of section  $i$ ,  $i=1,2,3$ , by  $L_i$ . Since this “cyclic” process can be modeled as an “extended” alternating process, the three probabilities are simply:

$$q_i = \frac{E(L_i)}{E(L_1)+E(L_2)+E(L_3)}.$$

where

$$L_1=s_2 \text{ w.p. } 1 \text{ and } E(L_1)=s_2;$$

$$E(L_2)=l_s \times E(S) \text{ and } E(S) \equiv \sum_{i=1}^{n_{\max}} ip_i'.$$

Note that  $E(S)$  is the expected platoon size. To approximate  $E(L_3)$ , we need to approximate the

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† If the lane-change preparation is initiated when the vehicle is in section 1, allowing the vehicle to slow down to enter a gap or join the platoon in the previous cycle may slow down the traffic in the origin lane. This particular strategy is studied only as an example application of the platoon size distribution and is not being advocated by the authors. Other strategies will be studied in the future.



number of platoons per unit length. Given a traffic density  $c$ , i.e. the ratio of the number of vehicles over the segment length, we can approximate the number of platoons per unit length by  $c/E(S)$ . Therefore,

$$E(L_3) = \frac{E(S)}{c} - s_2 - l_3 \times E(S)$$

By denoting, as before, the time required for the lane-change maneuver itself by  $t_m$ , we now calculate the three conditional probability distributions for the total time till lane-change completion. Denote the corresponding random variables by  $T_i^c, i=1,2,3$ . Since  $p_i'$  is the limiting probability that the platoon is of size  $i$ ,

$$p(T_2^c = i \times \frac{l_s}{\Delta v} + t_m) = p_i', \quad i=1,2,\dots,n_{max} \text{ and}$$

$$T_1^c = T_2^c + \frac{U_{[0,s_2]}}{\Delta v},$$

where  $U_{[0,s_2]}$  is the uniform random variable over the interval  $[0, s_2]$ , which is independent of  $T_2^c$ . Finally,

$$T_3^c = t_m.$$

Note that  $T_1^c$  has a mixed probability distribution, i.e. a mixture of discrete probability distributions and absolutely continuous distributions. Further weighting the distributions of  $T_i^c$  according to  $q_i, i=1,2,3$ , gives the unconditional distribution of total time till a successful lane change. This mixed distribution can be obtained numerically.

## EXAMPLES

We continue the examples given in Section (2.2.2.2).

### Additional Parameters

(P7) a speed difference of 3 meters/second between the origin and destination lanes and

(P8) a maneuvering time of 5 seconds for a lane change.

With these additional common parameter values, the four lane-change completion time distributions are calculated and plotted in Figure 8. The expected value and the standard deviation of lane-change completion times are plotted against 16 different traffic conditions in Figure 9.

As shown in Figure 9, the average lane-change completion time increases with the flow on the destination lane while the standard deviation tends to be smaller at very low or very high flow levels. At high flow rates on the destination lane, the average grows at a faster rate than the flow. This demonstrates the trade-off between the longitudinal flow and lateral flow. From Figure 8 and 9, it is apparent that both the average (especially for the cases with high longitudinal flow in the destination lane) and the standard deviation of the lane-change completion time are large. Consider an AHS with two automated lanes and automated on/off-ramps. Because of the long completion time required, to make sure that all exiting vehicles currently on the inner lane get to the outer lane before the desired off-ramps, the lane changes need to be initiated early. However, because of the large variability, many exiting vehicles would change lane successfully well before the off-ramp and would have to stay on the outer lane for an extended period of time before exiting. This may clog the outer lane and prevent efficient access to the automated lanes by the entering vehicles. A perhaps more serious problem occurs when the AHS has no automated on/off-ramps and all vehicles can access the automated lanes only through the transition lane. In such a case, the transition lane may become a bottleneck. To support the large amount of automated traffic, the stay by the automation-equipped vehicles on the transition lane should be as brief as possible. The long waiting time required for entering automated lanes with high longitudinal flows from the transition lane may limit the number of entries and hence cast doubt about the achievability of such high flows.

We now point out a way to improve this lane-change completion time model. In the current model, if the lane-change vehicle is adjacent to a platoon in the destination lane at the initiation time, it overestimates the waiting time by assuming that the vehicle is at the very end of the platoon. Also, it underestimates the waiting time by not requiring the vehicle to keep waiting when the adjacent platoon is at the maximum size already. Two improvements can be made accordingly. Note that when the adjacent platoon cannot accept the vehicle, the vehicle has to wait until a large enough gap or a small enough platoon (smaller than the maximum size) appears. It is conjectured that the improved model will show a much longer and much more variable lane-change completion time for the cases with high longitudinal flows. The trade-off between the longitudinal capacity and the lateral capacity is expected to become clearer and more drastic.

We close this subsection with the following final remarks. First of all, the above observations apply only to the specific example AHS operating scenario. Other scenarios will be studied in the future. In platooning scenarios, it may be better to operate in such a way that a platoon can receive a lane-change vehicle into any part of it with only a minor split at the receiving position. These types of lane-change considerations should be reflected in the detailed definition of lane-flow rules.

### **(3) THE SIMULATOR**

In Section 2, we developed several lane-change models for AHS operating scenarios without lane barriers. Due to the complexity of analytical modeling, we study the effect of lane barriers on AHS capacity only through simulation. We also simulate AHS operating scenarios without lane barriers for a more realistic and detailed study of the impacts of the platooning rule and the barriers on AHS capacity..

We adopted and made major modifications to an existing simulator, SmartPath [1], to study the capacity impacts of different combinations of lane-flow rules and barrier options: (i) platooning without barriers, (ii) platooning with barriers, (iii) free-agent lane-flow without barriers, and (iv) free-agent lane-flow with barriers.

Based on the *fixed-increment time advance* approach and against the backdrop of an AHS segment, the simulator models the behavior of vehicles from the moment they enter the segment, either as part of existing traffic on the AHS or as new-comers into the AHS through on-ramps, till they leave the AI-IS through off-ramps. We simulated a segment of a segregated AHS in which there is one automated lane and vehicles access the automated lane via the transition lane (dedicated to transition).

The platooning lane-flow rule has a number of variations. We simulated the platooning concept proposed by Hsu et al. [4] in detail. The inter-platoon spacings and intra-platoon spacings are parameters and can be determined based on safety considerations. Ranges for communication are also parameters and a maximum platoon size can be imposed. The free-agent lane-flow rule is simulated by setting the maximum platoon size to 1. The speed of vehicles is set as close to a target speed as safety spacing allows.

For a successful lane change with lane barriers, the lane-changing vehicle and the receiving gap in the neighboring lane have to align with the gate at the time of the maneuver. The simulator does not contain any logic to ensure such alignments. (Such alignment can be achieved with higher efficiency through coordination between the involved vehicles and platoons.) After the completion of the lane-change preparation, the simulator checks if there are lane barriers between the lane-change vehicle and the receiving gap and if not, whether a lane change can be completed safely. If a safe lane change cannot be completed, then the lane change vehicle will keep making requests until a lane change is completed. Note that each new request initiates a new set of preparation steps.

The simulator allows three basic maneuvers, upon which more complex maneuvers are built. A platoon can perform at most one basic maneuver at any time. The three maneuvers are:

- (M1) Join: This allows a platoon, possibly a single vehicle in an automated lane to join the platoon in front and become one platoon. This can occur only if the two parties are within communication range and the size of the new platoon does not exceed the maximum platoon size allowed.
- (M2) Split: This splits a platoon in an automated lane into two separate, autonomous platoons. The new platoon at the rear must decelerate to create a safe spacing from the front platoon.

(M3) Lane Change: This allows a single vehicle to change from its lane to an adjacent lane. A lane change can occur only if a sufficiently large gap is present in the destination lane. When such a gap is not present, the lane-change vehicle and the obstructing platoon together decide how to create a safe gap. The creation of a safe gap involves a full split maneuver. More precisely, the platoon will split into two with a gap of length larger than  $2 \times s_2$ . There are three cases:

Case 1 - The vehicle is alongside the front third of the platoon: the platoon decelerates to create a space in front of the platoon.

Case 2 - The vehicle is alongside the rear third of the platoon: the vehicle decelerates and uses the space behind the platoon.

Case 3 - The vehicle is alongside the middle third of the platoon: the platoon performs a (full) split to create a space.

To study the effect of lane changes on the performance of the AHS, a destination lane or an exit is randomly assigned to each vehicle, based upon a matrix of specified percentages for vehicle routing in the segment. An exiting vehicle initiates the preparation at a fixed location (or time) before the designated exit, which tends to create congestion at and before the location. This may be improved by evening out the location (or time) of preparation initiation.

The following statistics are generated to measure system performance:

Number and percentage of vehicles reaching each assigned automated lane and exit.

Average speed and standard deviation of speed

Traffic **flow** and density

Statistics on lane change times and distances

Distribution of platoon sizes

Difference between the average and the target speed.

For detailed descriptions of **SmartPath** and the major modifications, refer to [1,3].

## **(4) SIMULATION RESULTS**

### FOCUS

The primary focus is on the effect of (i) lane barriers (a major configuration option) and (ii) platooning (a major operating rule) on the capacity. Also studied is the effect of (iii) different longitudinal safety spacings, (iv) speed differences among different lanes, and (v) different exiting procedures. These five focal AHS design options led to the design of 6 sets of simulation experiments. A detailed description follows.

### 6 SETS OF SIMULATION CASES AND MOTIVATION

Against the backdrop of a common AHS segment, a simulation case is defined by a unique set of AHS physical configuration options, operating rules, and traffic levels. The simulation cases are organized into 6 sets. Table 4.1-1, to be explained in finer detail later, contains options and parameter values for each set. The only difference between Set 1 and Set 2 is the lane-flow rule and they use the platooning and free-agent rules respectively. They are selected for comparing the capacity impact of platooning. The only difference between Set 2 and Set 3 is in the target lane speeds. All lanes in Set 2 have a common target speed while the target speed for the transition lane and the manual lane in Set 3 is 1 meter/second lower than that of the automated lane. This difference is designed to study, by comparison to Set 2, the impact of the speed difference on capacity. Another set of cases, Set 4, is selected to further study the impact of speed difference on capacity. The difference in this case is 3 meters/second. Set 5, in contrast to Set 1, is designed to study the effect of the different exit procedures on the capacity. Finally, Set 6, in contrast to Set 1 again, is selected to understand the effect of a longer safety distance (60 meters as opposed to 40) on the capacity.

### PERFORMANCE METRICS

We use **SmartPath** to investigate the capacity of an AHS as a function of (i) configuration options, (ii) operating strategy and (iii) traffic demand. The capacity is measured by the three metrics (M1), (M2) and (M3) stated in the Introduction. Since the simulator keeps detailed information about vehicles' movement, we will augment (M1) with the success rate of exiting at the desired destination as

a direct measure of the ability of an AHS (mostly its operating strategy) to meet the stringent destination requirement.

## **(4.1) Experimental Design**

A simulation case is defined by a unique set of AHS design options and input parameters. A single run is performed for each case, i.e. there is no repetition with different random numbers for any case. In this subsection, we briefly describe (i) the options and parameters, (ii) how the traffic is generated, and (iii) how the performance measures are obtained.

### OPTIONS AND PARAMETERS

#### AHS Configuration

Figure 10 illustrates the configuration of the AHS segment used in the simulation. All simulation runs for cases with lane barriers use this exact configuration. Removal of the lane barriers gives the exact configuration for the cases without lane barriers.

#### 5 Focal Options and Parameters and 6 Sets of Simulation Cases

The 5 focal options and parameters can be found in Table 4.1-1 as column names.

Set	Max. Platoon Size	No Barriers/ Barriers	Safety Dist. (meters)	Lane Speed AL/TL+ML (m/sec)	Exit Proc.
1a	20	No div	40	27/27	B
1b	20	Gates	40	27/27	B
2a	1	Nodiv	5	27/27	B
2b	1	Gates	5	27/27	B
3a	1	No div	5	27/26	B
3b	1	Gates	5	27/26	B
4a	1	No div	5	27/24	B
5a	20	No div	40	27/27	A
5b	20	Gates	40	27/27	A
6a	20	No div	60	27/27	B
6b	20	Gates	60	27/27	B

Safety distance is the minimum distance, reserved for safety, between two traffic units. Under **platooning**, this distance is the minimum longitudinal separation between two platoons (inter-platoon spacing); in the case of the free-agent rule, it is the minimum separation between two vehicles †. Two inter-platoon spacings, 40 and 60 meters, are used. (An inter-platoon spacing of 50 meters is used in the examples illustrating the calculation of platoon-size distributions and the distributions for the lane-change completion times in Section 2.)

The target speed of the automated lane (AL) is always the highest among all three lanes and the transition lane (TL) and the manual lane (ML) are operated with a common target speed. AL, TL and ML refer to the automated, transition and manual lanes respectively.

Two exit procedures are used, A and B. The former was the only exit procedure available in the **SmartPath** before we added the latter as an enhancement. In “exit procedure A”, each vehicle that needs to exit begins a series of maneuvers at a fixed distance (500 meters) into the highway segment regardless of the desired exit. In “exit procedure B”, each vehicle that needs to exit begins the required preparation and maneuvers at a fixed distance (2000 meters) before the desired exit, i.e. the exit randomly assigned to the vehicle by the simulator. As noted in previous studies [8], the split maneuver

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† The 5-meter safety distance for the free-agent test cases was chosen only to match the high flow achievable by their platooning counterparts. This safety distance may actually be unsafe.



required for exiting vehicles is a significant source of congestion, especially when many exiting vehicles start the exit preparation at the same exact location.

Parameters with Common Values

Parameters whose values are common to all simulation runs are listed in Table 4.1-2. All except the last two (length of vehicle and intra-platoon spacing) are attributes of the physical configuration of the AHS segment.

Table 4.1-2		
Parameters Applicable to All Simulations		
Highway Length	10000	meters
Manual entrances	2000/6000	meters from start
Manual exits	4000/8000	meters from start
Gate lengths	100	meters
Distance between gates	1000	meters
Number of automated lanes	1	
Length of each vehicle	5	meters
Intra-platoon distance	1	meter
Simulation time	10-13	minutes
Number of vehicles generated	400-500	vehicle

The gate length was chosen to be large enough to accommodate a safe lane change. The distance between gates was chosen so that 10% of the highway segment length would accommodate gates.

Traffic Parameters

The parameters defining the traffic demand can be found in Tables 4.1-3 as column names.

Table 4.1-3				
Parameters used for generating balanced access/egress flow				
Flow Level	Desired Auto Flow (v./lanexhr.)	Actual Manual Flow (v./lanexhr.)	Auto Veh. Exit %	Manual Veh. Inter-arrival Time sec.
1	7000	1400	20	2.5
		700	10	5.0
		560	8	6.25
		420	6	8.57
		280	4	12.5
2	6000	1200	20	3.0
		600	10	6.0
		480	8	7.5
		360	6	10.0
		240	4	15.0
3	5000	1000	20	3.6
		500	10	7.2
		400	8	9.0
		300	6	12.0
		200	4	18.0
4	4000	800	20	4.5
		400	10	9.0
		320	8	11.25
		240	6	15.0
		160	4	22.5
5	3000	600	20	6.0
		300	10	12.0
		240	8	15.0
		180	6	20.0
		120	4	30.0
6	2000	400	20	9.0
		200	10	18.0
		160	8	22.5
		120	6	30.0
		80	4	45.0

Within each of the 6 sets, a number of identical cases, based on different demand levels (amount of input/exiting traffic) are simulated. Input traffic consists of the automated traffic entering the segment from upstream and the manual traffic entering from the manual entrances. To study the equilibrium, the amount of exiting traffic is set to the amount of manual traffic from entrances. Associated with each of the 6 flow levels for automated traffic are 5 flow levels from manual entrances. Table 4.1-3 shows different demand levels. Also shown are two derived quantities: (i) the percentage of automated vehicles from upstream that need to exit in the segment and (ii) the constant inter-arrival time of the manual vehicles at the entrances. (The constancy of the inter-arrival time was part of the SmartPath

design. We did not enhance it to accommodate random inter-arrival times. Rather, we modified Smart-Path so that the automated traffic entering the AHS segment from upstream can be organized in platoons and the inter-arrival times of platoons are random.) Note that the input flow in the automated lane will not be exactly equal to the desired flows given in Table 4.1-3 because the inter-platoon spacings and platoon sizes are randomly determined as traffic is generated.

TRAFFIC GENERATION

Automatic Traffic from Upstream

Free agents are generated as platoons of (maximum) size 1. The traffic flow on a lane is calculated according to:

$$Flow \text{ (# vehicles /lane -hour)} = \frac{3600 \times V \times N}{N \times (L + D) - D + INT}$$

where

$$INT = \text{mean inter-platoon distance} = D_{safe} + expm \times V,$$

and the parameters and their values are given below:

Parameter	Definition	Value	Unit
V	desired velocity	27	meters/second
N	mean platoon size	variable	
L	length of vehicle	5	meter
D	intra-platoon distance	1	meter
Dsafe	Safety Distance	40	meter
expm	mean of exponential inter-arrival time distribution	variable	Second

Note that the mean platoon size and the mean of the exponential inter-arrival time distribution are variable and used to produce the desired flow. Also note that, given a flow, each of these two variables is a function of the other. Table 4.1-4 tabulates a number of combinations of these two variables that produce the flows used in the simulation cases.

Table 4.1-4							
Desired input flow(vehicles/hour) versus average platoon size(N) and average inter-platoon distance(INT)							
Flow(veh/hr)	N						
7000	7	8	8	9	10	11	12
6000	5	6	6	7	8	8	9
5000	4	4	5	5	6	6	7
4000	3	3	4	4	4	5	5
3000	2	2	3	3	3	3	4
2000	1	1	2	2	2	2	2
INT(meters)	53	60	67	74	80	87	94

Table 4.1-5 gives the values for average inter-vehicle distance for the free-agent cases.

Table 4.1-5						
Flow versus Inter-vehicle spacing(free agent vehicles)						
Desired Input Flow	7000	6000	5000	4000	3000	2000
Average Inter-vehicle Distance(meters)	9	11	15	18	25	40

For any given values of these two variables, the automated traffic entering AHS is generated as follows:

The platoon size is assumed to have a truncated Poisson distribution (truncated at the maximum platoon size). The expected value of the distribution is set to the given mean platoon size. This distribution is different from the one obtained in Section 2, but it is a close approximation. The inter-platoon spacing is the sum of the (speed-dependent) minimum safety distance between two platoons and an exponentially distributed random variable with mean  $expm \times V$ . This selection of exponential distribution is consistent with the examples in Section 2.

When cases with the same input flow from upstream but different amounts of entering manual traffic (i.e. exiting traffic) are compared, the upstream flows generated are identical. More precisely, the sequences of platoon sizes and inter-platoon distances are identical because the same sequence of random numbers is used in generating the respective platoon sizes and inter-platoon distances. This is a standard variance-reduction technique.

Finally, the upstream automated traffic is regulated at the very beginning of the segment so that the in-flow is no larger than what the segment can safely accommodate. Therefore, the actual in-flow

may be lower than desired.

#### Entering Traffic From Manual Entrances

Vehicles entering the AHS from a manual entrance are generated at constant (deterministic) inter-arrival times. (See Table 4.1-3.)

#### Exiting Traffic

Exiting vehicles are randomly designated an exit upon generation according to the percentages given in Table 4.1-3 and the desired exits are evenly distributed between the two exits. Because of this random designation, the actual number of exiting vehicles in a simulation case is only approximately equal to the exit percentage multiplied by the total number of automated vehicles generated, i.e. the manual traffic from the entrances.

### MEASURING PERFORMANCE

#### Exit Success Rate

The exit success percentage is defined to be the number of automated vehicles that successfully exited the system divided by the number of vehicles that needed to exit. In any acceptable AHS operation, the exit success percentage should be close or equal to 100%.

A related measure is the number of unsuccessful lane changes, i.e. the number of lane change maneuvers that were started but were not completed before the end of the simulation. This includes the number of vehicles failing to exit and also the number of incoming manual vehicles that fail to reach the automated lane. In any acceptable AHS operation, this number should be equal or close to 0.

#### Traffic Flow in the Automated Lane

Flow is measured at 1 kilometer increments throughout the highway segment and the minimum flow for the segment, to be referred to as the “bottleneck flow”, is the minimum of the 10 measured flow values. This can be less than the desired input flow for reasons given earlier.

### Speed Distribution: Average and Standard Deviation

This is measured at 1 kilometer increments throughout the highway segment and the section with the “most congestion” has the lowest average velocity along with the highest standard deviation.

### REMARKS

A final note about the experimental design is that the length of simulation time is chosen so that all automated traffic would have enough time to travel through the complete length of the segment or to an assigned exit.

## **(4.2) Results**

This subsection summarizes the simulation results with some observations. The first 6 subsections, (4.2.1) through (4.2.6), are devoted to the 6 sets of simulation cases. Figures are provided to help understand the results. These results, in tabular format, are included in Appendix C. Section 4.2.7. summarizes the discussion.

Some test cases involve a large number of crashes. A suspected cause of the crashes is the coarseness of the fixed time increments <sup>1</sup>. After reducing the time increment from 0.1 second to 0.05 seconds, some of these test cases no longer suffer from the crash problem. These cases are marked with an asterisk (\*). Those test cases which are plagued by the crash problem even after the time increment reduction are marked with (\*\*). Since some performance data are invalid, the corresponding fields are left unfilled. If all the performance data for a test case are invalid, only the exit percentage (i.e. the % sign in the first column) is marked with a double asterisk (\*\*). In addition to these unfilled fields, a small amount of data is missing.

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<sup>1</sup> However, more serious possibilities have not been ruled out. The suspected relationship between the coarse time increments and the crashes will not be elaborated upon here, but is contained in [1].

## **(4.2.1) Set 1: Cases With Platoons**

### Exit Success Rate

For all input flows, low exit percentages lead to high exit success rates (see Figures 11.2 and 11.4). For high exit percentages ( $\geq 10\%$ ), exit success rate increases as the input flow decreases. This is true whether barriers are present or not. For cases with high flow and high exit percentage, the exit success rate is well below 100%.

### Longitudinal Flow

In the cases without barriers (Set 1a), there is high congestion near the beginning of the highway segment. The bottleneck flow is significantly less than the input flow for these cases. For example, the bottleneck flow is 5490 when the input flow is 7273 with an exit percentage of 20 (see Table C.1 and Figure 11.1). For the cases with the lowest input flows, the bottleneck flow is much closer to the input flow, i.e. there is less congestion. As expected, when the exit percentage is 0, the input flow is the same as the bottleneck flow.

With barriers (Set 1b), the high-demand cases result in much more congestion and more crashes near the beginning of the highway segment than the cases without barriers. For example, for an input flow of 6460 with exit percentages of 6-20 percent without barriers (Table C.3 and Figure 11.3), the bottleneck flows were 6435, 6202, 6460 and 5401 while their barriers counterparts led to excessive crashes. (see Figure 11.3). For the remaining cases, i.e. low-demand, the bottleneck flows and exit percentages are comparable.

### Speed Distribution

The deviation of average velocity from desired velocity and the standard deviation of velocity in the most congested section (1000 meters long) for some Set-1 cases are given below. (Desired velocity is 27 meters/sec.)

Input Flow veh/hour meters/sec	Specified Exit %	No barriers/ Gates	Velocity Deviation (desired - average)	Velocity Std Dev
7000	20	Gates	9-10	11-13
6000	20	Gates	1-2	5-6
7000	10	Gates	< 1	2-3
7000	20	No div	2-3	6-7
6000	20	No div	1-2	5-6
7000	10	No div	< 1	2-3
7000	6-8	No div	<.3	1-2
7000	4	No div	<.1	1
7000	0	No div	0	0

The cases with gates experience too many crashes and the data may not be valid.

### (4.2.2) Set 2: Cases with Free Agents Only

Generating high input flows of free agents requires (perhaps unrealistically) small inter-vehicle spacing. Recall that Table 4.1-5 contains the mean inter-vehicle distances as a function of the required flows. The safety distance must be even smaller than inter-vehicle distance.

#### Exit Success Rate

Without barriers, when the exit percentage is low (<10%), the exit success rate is high for input flows less than 5000 **veh/hr** (see Table C.6 and Figure 11.6). Also when the exit percentage is high ( $\geq 10\%$ ), the exit success rate increases as the input flow decreases. The presence of barriers (Set 2b) makes little difference in the ability to exit the vehicles (see and Table C.8 and Figure 11.8) except when a large number of vehicles need to exit. In these cases, as expected, more vehicles are able to exit when no barriers are present (see Table C.6 and Figure 11.6). For example, when the input flows are



equal to or larger than 5000 and the exit percentage is 20%, the exit success rates are 18%, 20% and 32% without barriers and 4%,10%, and 16% with barriers (see Table C.6/Figures 11.6 and Table C.8/Figure 11.8). The presence of barriers also makes little difference in the number of unsuccessful lane changes. Overall, the exit success rates for almost all cases in this set are very low.

#### Longitudinal Flow

In the cases without barriers (Set 2a) the bottleneck flow is significantly less than the input flow. For example, the bottleneck flow is 4325 when the input flow is 6424 with an exit percentage of 20 (see Table C.5 and Figure 11.5). For the cases with the lowest input flows, the bottleneck flow is closer to the input flow, but the flow still decreases as the exit percentage increases. As in Set 1, the input flow is the same as the bottleneck flow for the cases when the exit percentage is 0.

The presence of barriers (Set 2b) makes little difference in the ability to sustain the longitudinal flow (see Table C.7 and Figure 11.7) except when a large number of vehicles need to exit.

#### Speed Distribution

The velocities of the vehicles remain very nearly constant throughout the highway segment for all cases. The flow is much smoother than in the platooning cases of Set 1 because platoon splits are not necessary. The average velocity is close to the desired velocity of 27 meters/second and the standard deviation is at most .02 in a given 1 kilometer section.

### **(4.2.3) Set 3: Free Agents with Different Lane Velocities(27/26)**

#### Exit Success Rate

When the exit percentage is low(<10%), the exit success rate is low for all input flows except for the lowest flow of 2000 veh/hr with 4 or 6 % exit percentage (see Table C.10 and Figure 11.10). However, the large number of crashes renders the performance statistics useless for flows greater than 5000. The presence of barriers (Set 3b) makes little difference in the ability to exit the vehicles and in the number of unsuccessful lane changes (see Table C.12 and Figure 11.12). Overall, the exit success rates

are well below 100%.

#### Longitudinal Flow

As the cases in Set 2, the bottleneck flow is significantly less than the input flow. For example, in Set 3a the bottleneck flow is 2580 when the input flow is 4610 with an exit percentage of 20 (see Table C.9 and Figure 11.9). For the cases with the lowest input flows, the bottleneck flow is still significantly less than the input flow and the flow still decreases as the exit percentage increases. As usual, the input flow is the same as the bottleneck flow for the cases when the exit percentage is 0. However, the available data does not show a significant difference in the bottleneck flow between Sets 2 and 3. In other words, the speed difference may be too small to make a difference. The presence of barriers makes little difference in sustainable flow.

#### Speed Distribution

The velocities of the vehicles remain somewhat constant but decrease throughout the highway segment for most cases. In the case where the bottleneck flow is 2580, the most congested section has an average velocity of 26.14 meters/second with a standard deviation of 1.21 m/s.

### **(4.2.4) Set 4: Free Agents with Different Lane Velocities(27/24)**

#### Too Many Crashes; Too Little Meaningful Statistics

In no-barriers cases of this Set, too many crashes prevented any meaningful data for input flows greater than 5000. All cases with barriers experience too many crashes and hence do not produce any meaningful data. What follows describes the results for the remaining cases.

#### Exit Success Rate

There does not seem to be any statistically significant difference among this set and Sets 2 and 3.

### Longitudinal Flow

As in Set 3, the bottleneck flow is significantly less than the input flow. For example, the bottleneck flow is 1686 when the input flow is 4610 with an exit percentage of 20 (see Table C.13 and Figure 11.13). However, a major difference between this and Set 3 is that the bottleneck flows are significantly lower.

### Speed Distribution

As in Set 3, the velocities of the vehicles gradually decrease throughout the highway segment for most cases. In the case where the bottleneck flow is 1686, the most congested section has an average velocity of 24.4 meters/second with a standard deviation of 2.43 m/s. The performance in this category is also inferior to that of Set 2 and 3.

## **(4.2.5) Set 5: Platooning with “exit procedure A”**

All exiting vehicles and their platoons begin the exiting maneuvers (especially the split maneuver) at the same location (500 meters into the highway segment). For the cases with high input flows and high specified exit percentages, virtually all platoons must split to exit a vehicle. This results in congestion near the beginning of the highway segment.

### Exit Success Rate

It can be observed that, with the exit procedure A, the exit success rates for high-demand cases, unlike the bottleneck flow, are higher than their counterparts with exit procedure B. This is expected because exit procedure allows more time for exiting vehicles to change lanes and reach the desired exit. For the low-demand cases, the exit success rates are comparable also.

### Longitudinal Flow

With barriers, the bottleneck flows under high traffic demand are slightly smaller than those without. For the remaining cases, the bottleneck flow levels with or without barriers are comparable. Without barriers, the bottleneck flows are slightly lower than their counterparts with exit procedure B

(Set 1a). Since the bottleneck flows for Set-1 cases with barriers are unavailable, no clear comparisons with respect to their counterparts in this set can be made.

#### Speed Distribution

There is no observable difference in the speed and its variation with respect to the results for their counterparts with exit procedure B.

### **(4.2.6) Set 6: Platooning with Increased Safety Distance**

Cases in this set use a safety distance of 60 meters instead of 40. They also use a larger detection range of 80 meters as opposed to 60.

#### Longitudinal flow

Barriers, as expected, lead to lower bottleneck flows when compared to the bottleneck flows associated with a safety distance of 40 meters,

With high traffic demand, these cases have a lower bottleneck flow than the corresponding cases in Set 1 with safety distance at 40 meters. For example, when the input flow is  $> 7000$  and the exit percentage is 20 the bottleneck flow is 4622 in Set 6 (Table C.19 and Figure 11.19) and 5490 in Set 1 (Table C.1 and Figure 11.1). The bottleneck flows for cases with low system demand are comparable between the two sets. These are true regardless of the presence of barriers.

#### Exit Success Rate

When barriers are present fewer vehicles are able to exit. Since lane changes are only allowed when vehicles are further apart than the safety distance, cases with 60-meter safety distance result in more unsuccessful lane changes for high-demand cases.

#### Speed Distribution

There is no observable difference in the speed and its variation with respect to the results for their counterparts in Set 1.

## **(4.2.7) Summary of Simulation Results**

### Low Exit Success Percentage

In most test cases, the exit success rates are well below 100%. This poses a major challenge to designing AHS operating strategies. These rates are particularly low in cases where the flow rate and the exit percentage are both high. This clearly shows that high longitudinal flow hinders lateral flow and the lateral capacity can be increased at the expense of lower longitudinal capacity.

### Free-Agent Rule vs. Platooning

All free-agent cases tend to have a much larger percentage of incoming manual traffic failing to enter the automated lane. See Tables C.5 -C.14 in Appendix C. In many of the high-demand free-agent cases, more than half of the incoming manual traffic fails to reach the automated lane (see Table C.6). For example, in Set 2a (no barriers), when the input flow is 6424 and the specified exit percentage is 10, the percentage of incoming manual traffic that fails to reach the automated lane is  $50/65 = 77\%$ . Since a large amount of incoming manual traffic fails to reach the automated lane, they remain in the way of automated traffic that needs to exit. The reason for this is that the gaps between automated free agents are small. In the free-agent cases, the flow diminishes throughout the segment to the bottleneck flow at the end of the segment. In the platooning cases, the bottleneck flow occurs earlier in the segment due to platoon splits and the flow subsequently increases due to merging of platoons and incoming manual traffic. In the free-agent cases, high flows can be generated, but not as high as the highest possible flows under platooning.

The free-agent cases tend to have lower exit success percentages. Also, the number of unsuccessful lane changes is greater for the free agent cases than for the platooning cases because of the availability of larger gaps under platooning.

Under the free-agent rule, compared to cases with common lane speeds, lower speed in the transition and manual lanes (3 meters/second) results in lower bottleneck flow. This is true for all cases, especially for the cases with high traffic demand. Consider the free-agent cases where (i) there are no lane barriers, (ii) the input flow is 4610 and (iii) the exit percentages are 6-20 (see Table C.5). With

common lane speeds, the bottleneck flows are 2718, 2718, 2580, and 2752; they are 1824, 1479, 1445, and 1686 respectively when the common speed of the transition and manual lanes is 3 meters/second lower than the speed of the automated lane (see Table C.13). The flow is reduced because the vehicles entering the automated lane have a lower velocity. Also, this lower speed would slow down the existing traffic in the automated lane.

#### Effect of Lane Barriers

The presence of barriers results in lower bottleneck flows, i.e. there is more congestion. The presence of barriers is less significant with respect to exit success percentage than it is with respect to bottleneck flow. Lane change maneuvers take longer in the automated lane since the vehicle must change lanes at a gate, but vehicles manage to change lane successfully and exit. Many vehicles fail to exit in some cases; but, this is true with or without barriers.

#### Role of Safety Distance

When the safety distance is larger, lane changes are more restrictive and the bottleneck flows are lower. This has been demonstrated for platooning.

#### Effect of Different Lane Speeds

When the desired velocity in the automated lane is greater than the desired velocity in the other lanes, flows are lower and the velocities of vehicles reduce to a value lower than the desired velocity in the automated lane.

#### Effect of Exit Procedure

Exit procedure B tends to allow higher bottleneck flows. However, exit procedure A tends to produce higher exit success rates.

## (5) CONCLUSION

In this paper, we developed analytical models to study the vehicle/platoon and gap distributions on individual AHS lanes. Based on these models, we provide estimates for the time required for a complete lane change for several operating scenarios. We also modified **SmartPath**, an existing simulator, to study the effect of platooning and lane barriers on AHS capacity.

Among the analytical models developed is the model for predicting the platoon size distribution. Based on this model, we obtained the probability distribution under four representative traffic conditions (see figure 5). The average and the standard deviation of the platoon size distributions associated with a complete range of traffic conditions are also plotted (see Figure 6).

In most of the simulation test cases, the exit success percentage is well below 100%, which poses a major challenge to designing AHS operating strategies. Compared to the platooning lane-flow rule, the free-agent rule results in lower exit success percentages. The main reason is the lack of gaps sufficiently large for safe lane changes. A major challenge to improving the lateral capacity of AHS under the free-agent rule is to manage gaps more efficiently.

According to our simulation study, the presence of lane barriers results in lower longitudinal flow but makes little difference in exit success rate. This is because the lane changes are initiated well before the desired exits and, to accommodate the lane changes, the traffic has to slow down. According to our analytical study of a particular lane-change strategy, the average lane-change completion time increase with the flow in the destination lane and, at high flow levels, the increase is at a higher rate than the flow.

Note that neither the analytical models nor the simulation models studied in this paper adequately represent how a future AHS would be operated. More sophisticated operating strategies and models are required to optimize the capacity of an AHS. Long simulation time prevented the simulation of AHS traffic, for each test run, for a period long enough to reach traffic stability and also prevented a large size of test runs, for each test case, to reach indisputable statistical confidence.

Based on our study, we make the following remarks. Both the analytical models and the simulation results indicate a direct trade-off between the longitudinal and lateral capacities of an AHS.

Therefore, in predicting the maximum achievable flow of the AHS, the amount of lane changes assumed and the lateral capacity required to accommodate the lane changes must be explicitly considered. In particular, using the longitudinal flow, e.g. the number of vehicles per lane per hour, as the only measure for AHS capacity without any reference to the requirement for lateral flow is misleading. We suggest more study to accurately define the concept and measures of AHS capacity.

Most of the fundamental AHS concepts, e.g. shortening the longitudinal spacing between vehicles, have the potential of increasing only the longitudinal capacity of AHS. While the short spacing increases the longitudinal capacity, it may decrease the lateral capacity to such a degree that the lateral capacity becomes the bottleneck. Since exiting vehicles at the desired off-ramps without sufficient lateral capacity will lead to traffic slowdown, the longitudinal flow suffers as a result. Therefore, the issue of how to optimize the longitudinal flow subject to the requirement of lateral flow is an important issue to be resolved.



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## APPENDIX A: Geometric Gap Length Distribution

To illustrate the idea, calculate the joint probability distribution of the first two gap lengths,  $X_1$  and  $X_2$ :

$$p(X_1=i, X_2=j) = \frac{\binom{s-(i+j+2)}{v-2}}{\binom{s}{v}}$$

$$= \left(\frac{v}{s}\right) \left(\frac{v-1}{s-1}\right) \prod_{k=0}^{i+j-1} \left(\frac{s-v-k}{s-2-k}\right).$$

Since for each  $k=0,1,\dots,i+j-1$

$$\lim_{v/s=c, s \rightarrow \infty} \left(\frac{s-v-k}{s-2-k}\right) = 1-c,$$

by holding  $v/s=c$  and letting  $s$  go to infinity, we have

$$\lim_{v/s=c, s \rightarrow \infty} p(x_1=i, s_2=j) = c(1-c)^i \times c(1-c)^j.$$

Since the limiting distribution is of the product form, the two gap lengths are independent of each other. The common form of the two components shows that the two gap lengths are identically distributed with a Geometric Distribution with a success probability of  $v/s$ . We state the following theorem:

**Theorem A.1:** Suppose that the segment length  $s$  approaches infinity while  $v/s=c$ . Under Assumptions (AG1)-(AG3), the  $v+1$  gap lengths are independent and identically distributed according to the Geometric Distribution with a success probability of  $v/s$ .



## APPENDIX B: Exponential Gap Distribution

Instead of (AE4), we assume that the vehicle positions are a random sample of size  $v$  of a uniform random variable on the interval of  $[0, s - v \times (l+d)]$ .

Based on these assumptions, we can obtain the joint distribution of the  $v+1$  gap lengths. When  $v$  approaches infinity while  $v/s$  is kept constant, it can be shown that all the gap lengths are independent and identically distributed with an exponential distribution with a rate of  $v/(s - v \times (l+d))$ . To illustrate the idea, calculate the gap length distribution between the  $i$ -th and the  $(i+1)$ -th vehicle, from either end of the segment, as follows.

Denote the positions of the  $i$ -th and the  $i+1$ -th vehicles as  $Y_i$  and  $Y_{i+1}$  respectively. Also, denote  $v/(s - v \times (l+d))$  by  $b$ . Then, the joint p.d.f. (probability density function) of the  $Y_i$  and  $Y_{i+1}$  can be found to be:

$$g_{i,i+1}(Y_i=y_i, Y_{i+1}=y_{i+1}) = \frac{v!}{(i-1)!(v-i-1)!} \left(\frac{y_i}{b}\right)^{i-1} \left[1 - \frac{y_{i+1}}{b}\right]^{v-i-1} \left(\frac{1}{b}\right)^2, \text{ for } 0 < y_i < y_{i+1} < b.$$

To find the distribution of  $Y = Y_{i+1} - Y_i$ , first obtain the joint p.d.f,  $g_{Y,Z}^v(y, z)$ , of  $Y = y$  and  $Z \equiv Y_i \equiv z$ , an auxiliary random variable for derivation convenience. (Note that the superscript of  $g^v$  is included in indicate its dependence on  $v$ .) By change of variable,

$$g_{Y,Z}^v(y, z) = \frac{v!}{(i-1)!(v-i-1)!} \left(\frac{z}{b}\right)^{i-1} \left[1 - \frac{y+z}{b}\right]^{v-i-1} \left(\frac{1}{b}\right)^2, \text{ for } 0 < z < y+z < b.$$

Now, replacing  $b$  by  $v/c$ , where  $c \equiv v/(s - v \times (l+d))$  gives:

$$g_{Y,Z}^v(y, z) = \frac{c^{i+1}}{(i-1)!} \left[ \prod_{k=0}^i \frac{v-k}{v} \right] z^{i-1} \left[ 1 + \frac{-c(y+z)}{v} \right]^{v-i-1}, \text{ for } 0 < z < y+z < b.$$

Denote the marginal distribution of  $Y$  by  $g_Y^v(y)$ . Then,

$$g_Y^v(y) = \int_0^{(v/c)-y} g_{Y,Z}^v(y, z) dz.$$

Although both the integrand and the range of the integral depend on  $v$ , it can be shown that

$$\lim_{v \rightarrow \infty} g_v^y(y) = \int_0^{\infty} \left[ \lim_{v \rightarrow \infty} g_{v,z}^y(y,z) \right] dz .$$

But,

$$\lim_{v \rightarrow \infty} g_{v,z}^y(y,z) = \frac{c^{i+1}}{(i-1)!} z^{i-1} e^{-c(y+z)}, \text{ for } z > 0 \text{ and } y > 0.$$

Therefore,

$$\lim_{v \rightarrow \infty} g_v^y(y) = \int_0^{\infty} \frac{c^{i+1}}{(i-1)!} z^{i-1} e^{-c(y+z)} dz = \frac{c^{i+1}}{(i-1)!} e^{-cy} \int_0^{\infty} z^{i-1} e^{-cz} dz = ce^{-cy} .$$

Theorem B.1: Suppose that the segment length  $s$  approaches infinity while  $v/s$  remains constant.

Under assumption (1) -(3), the gap distributions are independent and identically distributed with an exponential distribution of rate  $c = v/(s-v \times (l+d))$ .

## **APPENDIX C: Simulation Output Data**

This appendix contains some of the simulation results in tabular form. For each specified exit percentage and each desired input flow, the following data are displayed:

Minimum(bottleneck) traffic flow in the segment. This occurs where the most lane change maneuvers occur, i.e. when vehicles need to exit **and/or** when vehicles need to enter the automated lane.

Number of automated vehicles generated

Number of manual vehicles generated

Number of attempted lane changes

Number of unsuccessful lane changes

Number of vehicles requesting an exit

Number of vehicles successfully exited

Exit success percentage

Listed below are the names of the tables along with the associated set of test cases(see Table 4.2-1).

Table	Set
c.1	1a
c.2	1a
c.3	1b
c.4	1b
c.5	2a
C.6	2a
c.7	2b
C.8	2b
c.9	3a
<b>C.10</b>	3a
c.11	3b
c.12	3b
c.13	4a
c.14	4a
c.15	5a
C.16	5a
c.17	5b
C.18	5b
c.19	6a
<b>C.20</b>	6a
c.21	6b
c.22	6b

Simulations which required a time increment (for updating vehicle state data) of  $T = .05 \text{ sec}$  instead of  $.1 \text{ sec}$  are denoted by an asterisk(\*). This was necessary to avoid crashes. Simulations which resulted in excessive crashes even with  $T = .05 \text{ sec}$  are denoted by a double asterisk(\*\*). Data is not shown for these cases due to lack of validity.



Set Ia - Bottleneck Flow(vehicles/hour) versus exit percentage						
Exit %	7000	6000	5000	4000	3000	2000
0	7273	6460	5329	3739	2949	1978
4	5514	4910	4918	3371	2686	1950
6	6992	6435*	4727	3562	2672	1808
8	7250	6202	4815	3439	2716	1851
10	6804	6460*	4830	3603	2759	1935
20	5490	5401*	5770*		2862	1978

Set IIa - Exiting Success Rate versus Exit Percentage and Desired Flow							
Exit%/Flow	Auto Veh	Manual Veh	Attempted Lane Chgs	Failed Lane Chgs	Exits Req	Exits Comp	Exit Rate
7000							
4	416	30	91	2	16	16	100.0
6	416	35	120	1	26	25	96.2
8	416	52	162	7	35	27	77.1
10*	416	65	204	13	42	37	88.1
20*	416	130	394	21	88	64	72.7
6000							
4	336	25	76	0	13	13	100.0
6	336	30	102	0	21	21	100.0
8	336	44	142	5	28	27	96.4
10	336	54	176	0	34	34	100.0
20	336	109	331	6	71	56	78.9
5000							
4	466	32	100	1	18	18	100.0
6	466	43	144	0	29	29	100.0
8	466	64	202	4	38	36	94.7
10	466	81	256	3	47	45	95.7
20	466	161	505	8	98	89	90.8
4000							
4	354	26	80	0	14	14	100.0
6	354	36	116	0	22	22	100.0
8	354	52	162	0	29	29	100.0
10	354	64	198	2	35	34	97.1
20**							
3000							
4	263	19	58	0	10	10	100.0
6	263	27	86	0	16	16	100.0
8	263	38	120	1	22	21	95.5
10	263	48	148	0	26	26	100.0
20	263	96	304	3	56	53	94.6
2000							
4	187	13	40	0	7	7	100.0
6	187	18	58	0	11	11	100.0
8	187	26	82	0	15	15	100.0
10	187	32	102	1	19	18	94.7
20	187	64	204	3	38	35	92.1

Set lb - Bottleneck Flow(vehicles/hour) versus exit percentage						
Exit %	7000	6000	5000	4000	3000	2000
0	7273	6460	5329	3739	2949	1978
4	5443	4858	4918	3330	2672	1973
6	6992	**	4639	3535	2643	1818
8	**	**	4771	3398	2701	1861
10	**	**	4757	3521	2759	1917
20	**	**	**	**	2613	1903

Set lb - Exiting: Success Rate versus Exit Percentage and Desired Flow							
Exit%/Flow	Auto Veh	Manual Veh	Attempted Lane Chgs	Failed Lane Chgs	Exits Req	Exits Comp	Exit Rate
7000							
4	416	30	91	2	16	16	100.0
6	416	35	120	0	26	21	80.8
8**							
10**							
20**							
6000							
4	336	25	76	0	13	11	84.6
6**							
8**							
10**							
20**							
5000							
4	466	32	100	1	18	16	88.9
6	466	43	144	0	29	28	96.6
8	466	64	203	3	38	33	86.8
10	466	81	255	7	47	33	70.2
20**							
4000							
4	354	26	80	0	14	14	100.0
6	354	36	116	0	22	22	100.0
8	354	52	162	0	29	27	93.1
10	354	64	198	2	35	31	88.6
20**							
3000							
4	263	19	58	0	10	10	100.0
6	263	27	86	1	16	15	93.8
8	263	38	120	1	22	20	90.9
10	263	48	148	0	26	25	96.2
20	263	96	304	2	56	47	83.9
2000							
4	189	13	40	0	7	7	100.0
6	189	18	60	0	12	12	100.0
8	189	26	82	0	15	15	100.0
10	189	32	102	1	19	18	94.7
20	189	64	206	3	39	34	87.2

Table C.5						
Set 2a - Bottleneck Flow(vehicles/hour) versus exit percentage						
Exit %	7000	6000	5000	4000	3000	2000
0	6424	5741	4610	4054	3076	1995
4			3097	2365	2097	1767
6	4579	4793	2718	2568	1986	1565
8	4389	4343	2718	2399	2097	1515
10	4452	4443	2580	2534	1958	1666
20	4325	4244	2752	2500	2014	1641

Table C.6							
Set 2a - Exiting Success Rate versus Exit Percentage and Desired Flow							
Exit%/Flow	Auto Veh	Manual Veh	Attempted Lane Chgs	Failed Lane Chgs	Exits Req I	Exits Comp-	Exit Rate
7000							
4	250	30	67	24	9	6	66.7
6	250	35	89	40	15	3	20.0
8	250	52	120	48	20	13	65.0
10	250	65	143	68	25	7	28.0
20	250	130	290	144	50	9	18.0
6000							
4	250	25	56	23	9	6	66.7
6	250	30	89	29	15	11	73.3
8	250	44	104	51	20	6	30.0
10	250	54	125	61	25	10	40.0
20	250	109	254	130	50	10	20.0
5000							
4	250	32	82	15	9	9	100.0
6	250	43	108	23	15	13	86.7
8	250	64	165	27	20	12	60.0
10	250	81	199	50	25	12	48.0
20	250	161	374	107	50	16	32.0
4000							
4	200	26	67	11	8	7	87.5
6	200	36	95	14	12	9	75.0
8	200	52	129	24	17	13	76.5
10	200	64	162	28	20	13	65.0
20	200	129	308	82	42	20	47.6
3000							
4	170	19	46	10	6	5	83.3
6	170	27	72	11	10	8	80.0
8	170	38	99	16	14	13	92.9
10	170	48	127	28	17	10	58.8
20	170	96	238	56	36	17	47.2
2000							
4	120	13	34	3	4	4	100.0
6	120	18	49	9	7	6	85.7
8	120	26	68	11	9	6	66.7
10	120	32	88	7	12	11	91.7
20	120	64	175	19	25	18	72.0

Set 2b - Bottleneck Flow(vehicles/hour) versus exit percentage						
Exit %	7000	6000	5000	4000	3000	2000
0	6424	5741	4610	4054	3076	1995
4			3097	2365	2097	1742
6	4579	4793	2718	2568	1986	1565
8	4389	4293	2718	2399	2097	1515
10	4452	4443	2580	2534	1930	1666
20	4389	4293	2752	2500	2042	1591

Set 2b - Exiting Success Rate versus Exit Percentage and Desired Flow							
Exit%/Flow	Auto Veh	Manual Veh	Attempted Lane Chgs	Failed Lane Chgs	Exits Req	Exits Comp	Exit Rate
7000							
4	250	30	66	24	9	6	66.7
6	250	35	89	40	15	3	20.0
8	250	52	118	48	20	11	55.0
10	250	65	141	69	25	5	20.0
20	250	130	285	146	50	2	4.0
6000							
4	250	25	56	23	9	6	66.7
6	250	30	89	29	15	10	66.7
8	250	44	105	50	20	6	30.0
10	250	54	124	62	25	10	40.0
20	250	109	256	131	50	5	10.0
5000							
4	250	32	82	15	9	9	100.0
6	250	43	108	23	15	13	86.7
8	250	64	165	37	20	11	55.0
10	250	81	199	50	25	12	48.0
20	250	161	370	112	50	8	16.0
4000							
4	200	26	67	11	8	7	87.5
6	200	36	95	14	12	9	75.0
8	200	52	129	24	17	13	76.5
10	200	64	162	28	20	11	55.0
20	200	129	307	83	42	15	35.7
3000							
4	170	19	46	10	6	5	83.3
6	170	27	72	11	10	7	70.0
8	170	38	99	16	14	13	92.9
10	170	48	127	28	17	9	52.9
20	170	96	238	57	36	10	27.8
2000							
4	120	13	34	3	4	4	100.0
6	120	18	49	9	7	6	85.7
8	120	26	68	11	9	6	66.7
10	120	32	87	9	12	10	83.3
20	120	64	174	21	25	14	56.0

Table C.9						
Set 3a - Bottleneck Flow(vehicles/hour) versus exit percentage						
Exit %	7000	6000	5000	4000	3000	2000
0			4610	4054	3076	1995
4			3131	2433	2153	1793
6			2752	2669	2014	1565
8			2787	2399	2153	1591
10			2649	2534	2069	1742
20			2580	2466	2097	1641

Table C.10							
Set 3a - Exiting Success Rate versus Exit Percentage and Desired Flow							
Exit%/Flow	Auto Veh	Manual Veh	Attempted Lane Chgs	Failed Lane Chgs	Exits Req	Exits Comp	Exit Rate
<b>5000</b>							
4	250	32	82	10	9	8	88.9
6	250	43	115	16	15	10	66.7
8	250	64	153	32	20	13	65.0
10	250	81	203	30	25	16	64.0
20							
<b>4000</b>							
4	200	26	66	8	8	6	75.0
6	200	36	95	9	12	9	75.0
8	200	52	136	24	17	11	64.7
10	200	64	158	22	20	15	75.0
20							
<b>3000</b>							
4	170	19	49	6	6	5	83.3
6	170	27	68	14	10	7	70.0
8	170	38	104	10	14	13	92.9
10	170	48	127	16	17	13	76.5
20	170	96	246	46	36	17	47.2
<b>2000</b>							
4	120	13	34	1	4	4	100.0
6	120	18	50	6	7	7	100.0
8	120	26	69	5	9	7	77.8
10	120	32	88	6	12	11	91.7
20	120	64	169	21	25	17	68.0

Table C. 11						
Set 3b - Bottleneck Flow(vehicles/hour) versus exit percentage						
Exit %	7000	6000	5000	4000	3000	2000
0			4610	4054	3076	1995
4			3200	2465	2153	1742
6			2752	2635	1986	1591
8			2821	2399	2181	1591
10			2684	2568	2125	1717
20			2821	2534	2209	1641

Table C.12							
Set 3b - Exiting Success Rate versus Exit Percentage and Desired Flow							
Exit%/Flow	Auto Veh	Manual Veh	Attempted Lane Chgs	Failed Lane Chgs	Exits Req	Exits Comp	Exit Rate
5000							
4	250	32	82	13	9	8	88.9
6	250	43	115	17	15	12	80.0
8	250	64	160	26	20	13	65.0
10	250	81	208	33	25	13	52.0
20	250	161	377	94	50	9	18.0
4000							
4	200	26	68	3	8	8	100.0
6	200	36	96	8	12	10	83.3
8	200	52	138	14	17	13	76.5
10	200	64	163	14	20	14	70.0
20	200	129	326	54	42	14	33.3
3000							
4	170	19	50	2	6	6	100.0
6	170	27	72	6	10	9	90.0
8	170	38	104	4	14	12	85.7
10	170	48	127	14	17	12	70.6
20	170	96	258	28	36	13	36.1
2000							
4	120	13	34	0	4	4	100.0
6	120	18	50	1	7	7	100.0
8	120	26	70	2	9	7	77.8
10	120	32	88	3	12	10	83.3
20	120	64	176	9	25	20	80.0

Set 4a - Bottleneck		Flow(vehicles/hour) versus exit percentage				
Exit %	7000	6000	5000	4000	3000	2000
0			4610"	4054*	3076	1995
4			3131"	1318*	1734	1742
6			1824"	1723*	1454	1439
8			1479*	1115*	1566	1010
10			1445*	1318*	1454	1565
20			1686*	1216*	1510	1237

Set 4a - Exiting Success Rate versus Exit Percentage and Desired Flow							
Exit%/Flow	Auto Veh	Manual Veh	Attempted Lane Chgs	Failed Lane Chgs	Exits Req	Exits Comp	Exit Rate
5000							
4*	250	32	82	7	9	9	100.0
6*	250	43	114	19	15	10	66.7
8*	250	64	162	31	20	11	55.0
10*	250	81	191	47	25	13	52.0
20*	250	161	369	102	50	17	34.0
4000							
4*	200	26	67	6	8	7	87.5
6*	200	36	96	11	12	10	83.3
8"	200	52	129	23	17	12	70.6
10*	200	64	165	24	20	12	60.0
20*	200	129	300	79	42	14	33.3
3000							
4	170	19	49	6	6	5	83.3
6	170	27	74	9	10	9	90.0
8	170	38	96	18	14	11	78.6
10	170	48	122	19	17	14	82.4
20	170	96	244	49	36	15	41.7
2000							
4	120	13	34	4	4	4	100.0
6	120	18	47	7	7	7	100.0
8	120	26	70	6	9	8	88.9
10	120	32	87	10	12	10	83.3
20	120	64	170	24	25	14	56.0

Set 5a - Bottleneck Flow(vehicles/hour) versus Exit Percentage						
Exit %	7000	6000	5000	4000	3000	2000
0	7273	6460	5329	3739	2949	1978
4	7039	6357	5315	3671	2949	1907
6	6452	4910	5227	3685	2818	1950
8	6727*	5966*	5013	3644	2920	1893
10	6687*	5866*	5212*	3685*	2905	1964
20	5255*	4910*	**	**	2759	1950

Set 5a - Exiting Success Rate versus Exit Percentage and Desired Flow							
Exit%/Flow	Auto Veh	Manual Veh	Attempted Lane Chgs	Failed Lane Chgs	Exits Req	Exits Comp	Exit Rate
7000							
4	416	30	88	6	16	14	87.5
6	416	35	120	0	26	25	96.2
8*	416	52			35	29	82.9
10*	416	65	201	8	42	35	83.3
20*	416	130	386	9	83	60	72.3
6000							
4	336	25	76	1	13	12	92.3
6	336	30	102	0	21	21	100.0
8*	336	44			28	28	100.0
10*	336	54	173	7	34	32	94.1
20*	336	109	323	22	71	54	76.1
5000							
4	466	32	100	1	18	17	94.4
6	466	43	142	4	29	27	93.1
8	470	64	13	5	39	32	82.1
10*	466	81	247	15	47	41	87.2
20**							
4000							
4	354	26	80	0	14	14	100.0
6	354	36	114	4	22	20	90.9
8	354	52	160	3	29	27	93.1
10*	354	64	193	12	35	27	77.1
20**							
3000							
4	263	19	58	0	10	10	100.0
6	263	27	86	1	16	14	87.5
8	263	38	119	3	22	20	90.9
10	263	48	146	4	26	24	92.3
20	263	96	290	25	56	45	80.4
2000							
4	187	13	40	0	7	7	100.0
6	187	18	58	0	11	11	100.0
8	187	26	80	4	15	12	80.0
10	187	32	101	2	19	18	94.7
20	187	64	203	6	38	32	84.2



Table C.17						
Set 5b - Bottleneck Flow(vehicles/hour) versus exit percentage						
Exit %	7000	6000	5000	4000	3000	2000
0	7273*	6460	5329	3739	2949	1978
4	7121*	6382	5315	3657	2935	1917
6	6816*	6082*	5039*	3671	2745	1959
8	6734*	5892*	5036*	3535	2935	1917
10	6006*	4212*	**	3562	2862	1959
20	**	**	**	3441*	2672	1804

Table C.18							
Set 5b - Exiting Success Rate versus Exit Percentage and Desired Flow							
Exit%/Flow	Auto Veh	Manual Veh	Attempted Lane Chgs	Failed Lane Chgs	Exits Req	Exits Comp	Exit Rate
7000							
4	416	30			16	16	100.0
6	416	35			26	25	96.2
8*	416	52	164	9	35	27	77.1
10*	416	65	207	6	42	37	88.1
20**							
6000							
4	336	25			13	13	100.0
6*	336	30			21	21	100.0
8**	336	44	144	1	28	27	96.4
10*	336	54	175	1	34	31	91.2
20**							
5000							
4	466	32			18	18	100.0
6*	466	43			29	27	93.1
8*	466	64	204	0	38	38	100.0
10**							
20**							
4000							
4	354	26	80	0	14	14	100.0
6	354	36	116	0	22	22	100.0
8	354	52	162	0	29	28	96.6
10	354	64	196	3	35	33	94.3
20*	386	129	27	15	82	80	97.6
3000							
4	263	19	58	0	10	10	100.0
6	263	27	86	0	16	15	93.8
8	263	38	120	0	22	22	100.0
10	263	48	148	0	26	26	100.0
20	263	96	302	3	56	55	98.2
2000							
4	189	13	40	0	7	7	100.0
6	189	18	60	0	12	12	100.0
8	189	26	82	0	15	15	100.0
10	189	32	102	0	19	19	100.0
20	189	64	204	2	39	38	97.4

Table C.19						
Set 6a - Bottleneck Flow(vehicles/hour)versus exit percentage						
Exit %	7000	6000	5000	4000	3000	2000
0	7156	6331	5329	3739	2949	1964
4	4997	4677	4845	3330	2657	1921
6	4786	5582	4492	3480	2657	1808
8	4669	4936	477 1	3385	2686	1822
10	**	**	4507*	3589	2730	1907
20	4622*	4574*	4125*	**	2832	

Table C.20							
Set 6a - Exit % Success Rate versus Exit Percentage and Desired Flow							
Exit%/Flow	Auto Veh	Manual Veh	Attempted Lane Chgs	Failed Lane Chgs	Exits Req	Exits Comp	Exit Rate
7000							
4	416	30	92	1	16	15	93.8
6	416	35	119	4	26	24	92.3
8	416	52	170	3	35	33	94.3
10**							
20	416	130	414	24	88	76	86.4
6000							
4	336	25	76	0	13	13	100.0
6	336	30	99	4	21	18	85.7
8*	336	44	142	3	28	27	96.4
10**							
20	336	109	326	42	71	53	74.6
5000							
4	466	32	100	1	18	18	100.0
6	466	43	144	0	29	29	100.0
8	470	64	204	1	38	38	100.0
10**							
20*	466	161	479	51	98	70	71.4
4000							
4	354	26	80	0	14	14	100.0
6	354	36	116	1	22	21	95.5
8	354	52	161	3	29	28	96.6
10*	354	64	198	2	35	34	97.1
20**							
3000							
4	263	19	58	0	10	10	100.0
6	263	27	86	0	16	16	100.0
8	263	38	120	2	22	20	90.9
10	263	48	148	2	26	25	96.2
20	263	96	298	12	56	47	83.9
2000							
4	187	13	40	1	7	6	85.7
6	187	18	58	1	11	10	90.9
8	187	26	82	0	15	15	100.0
10	187	32	102	0	19	19	100.0
20	187	64	204	3	38	35	92.1

Table C.21						
Set 6b - Bottleneck Flow(vehicles/hour) versus exit percentage						
Exit %	7000	6000	5000	4000	3000	2000
0	7156	6331	5329	3739	2949	1964
4	4997	4729	4830	3303	2657	1907
6	4716	4884	4346	3480	2643	1808
8	5514	**	**	3316	2643	1794
10	**	5608*	4478	3466	2716	1879
20	4058*	**	**	**	2686	1879

Table C.22							
Set 6b - Exiting Success Rate versus Exit Percentage and Desired Flow							
Exit%/Flow	Auto Veh	Manual Veh	Attempted Lane Chgs	Failed Lane Chgs	Exits Req	Exits Comp	Exit Rate
7000							
4*	416	30	92	0	16	15	93.8
6	416	35	118	5	26	22	84.6
8	416	52	164	8	35	25	71.4
10**							
20*	416	130	393	63	88	37	42.0
6000							
4	336	25	76	0	13	11	84.6
6	336	30	100	3	21	15	71.4
8**							
10*	336	54	168	11	34	19	55.9
20**							
5000							
4	466	32	100	1	18	18	100.0
6	470	43	144	0	29	27	93.1
8**							
10*	466	81	249	14	47	34	72.3
20**							
4000							
4	354	26	80	0	14	13	92.9
6	354	36	116	0	22	21	95.5
8	354	52	161	3	29	23	79.3
10	354	64	198	1	35	31	88.6
20**							
3000							
4	263	19	58	0	10	9	90.0
6	263	27	86	0	16	16	100.0
8	263	38	120	2	22	18	81.8
10	263	48	148	2	26	25	96.2
20	263	96	302	5	56	40	71.4
2000							
4	189	13	40	1	7	6	85.7
6	189	18	58	1	11	10	90.9
8	189	26	82	0	15	15	100.0
10	189	32	102	0	19	19	100.0
20	189	64	204	3	38	31	81.6



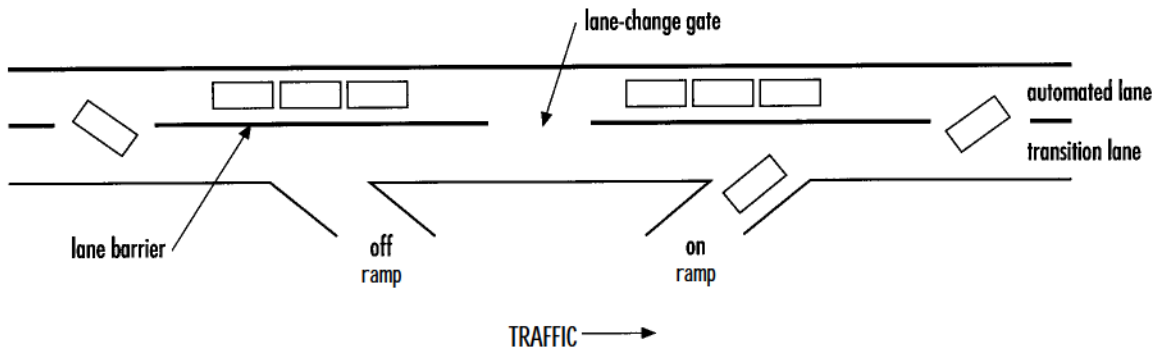
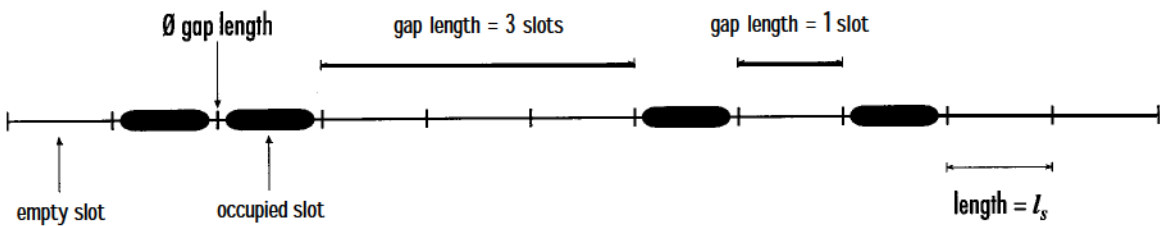


Figure 1: An AHS with one automated lane, one transition lane, and lane barriers under platooning.



total number of slots = 11  
total number of occupied slots = 4

Figure 2: Gaps in a slotted AHS.

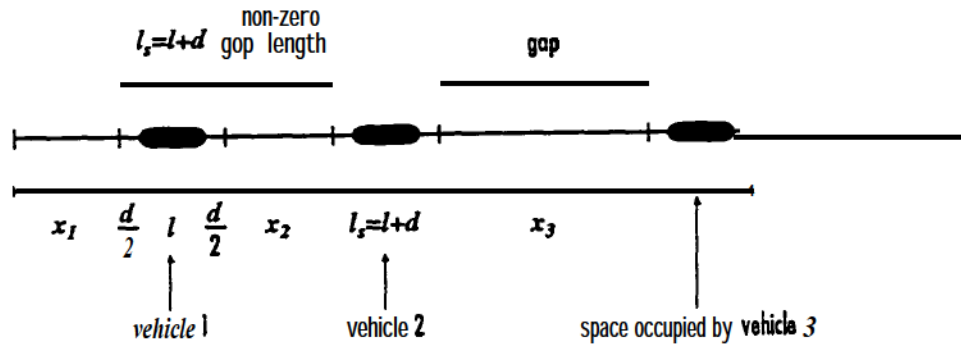
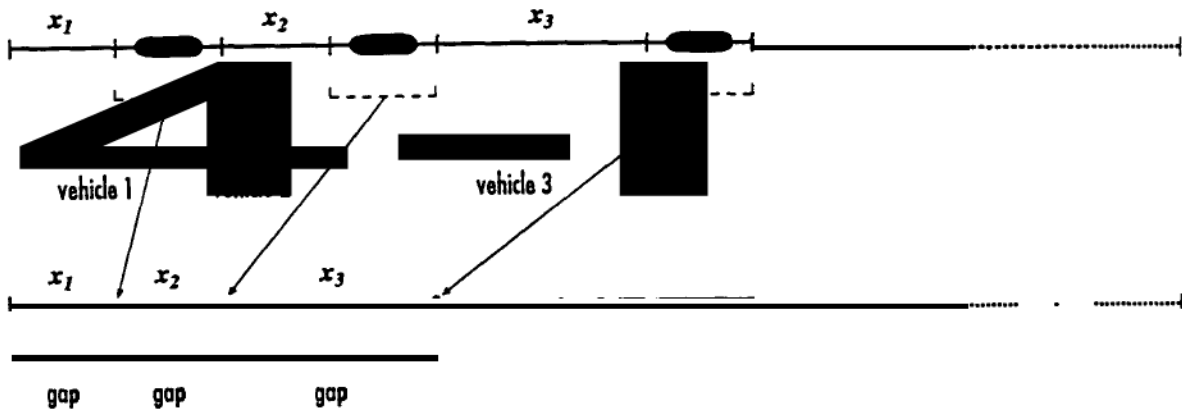


Figure 3.1: Continuous gap lengths (all gaps have non-zero lengths).



$$\text{reduced segment length} = s - v \times (l + d)$$

$$\text{number of vehicles (points)} = v$$

Figure 3.2: Continuous gap lengths, after removal of space occupied by vehicles.

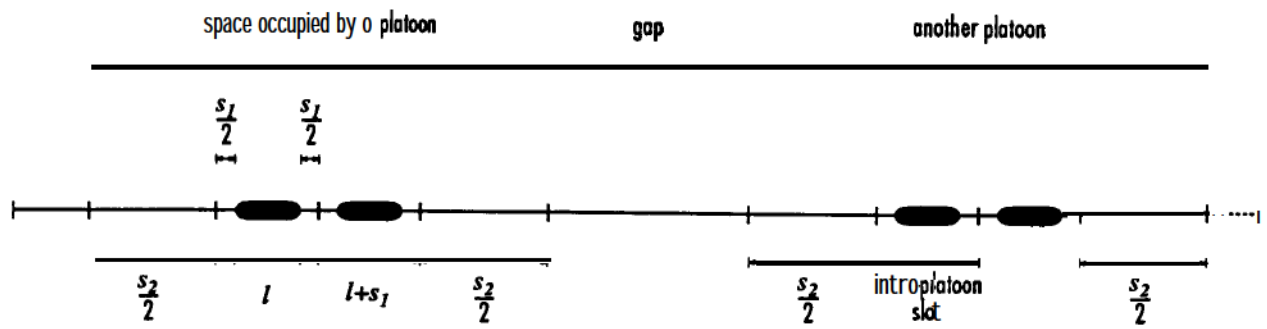


Figure 4: Platoon and gaps.

Figure 5: Platoon Size Probability Distributions

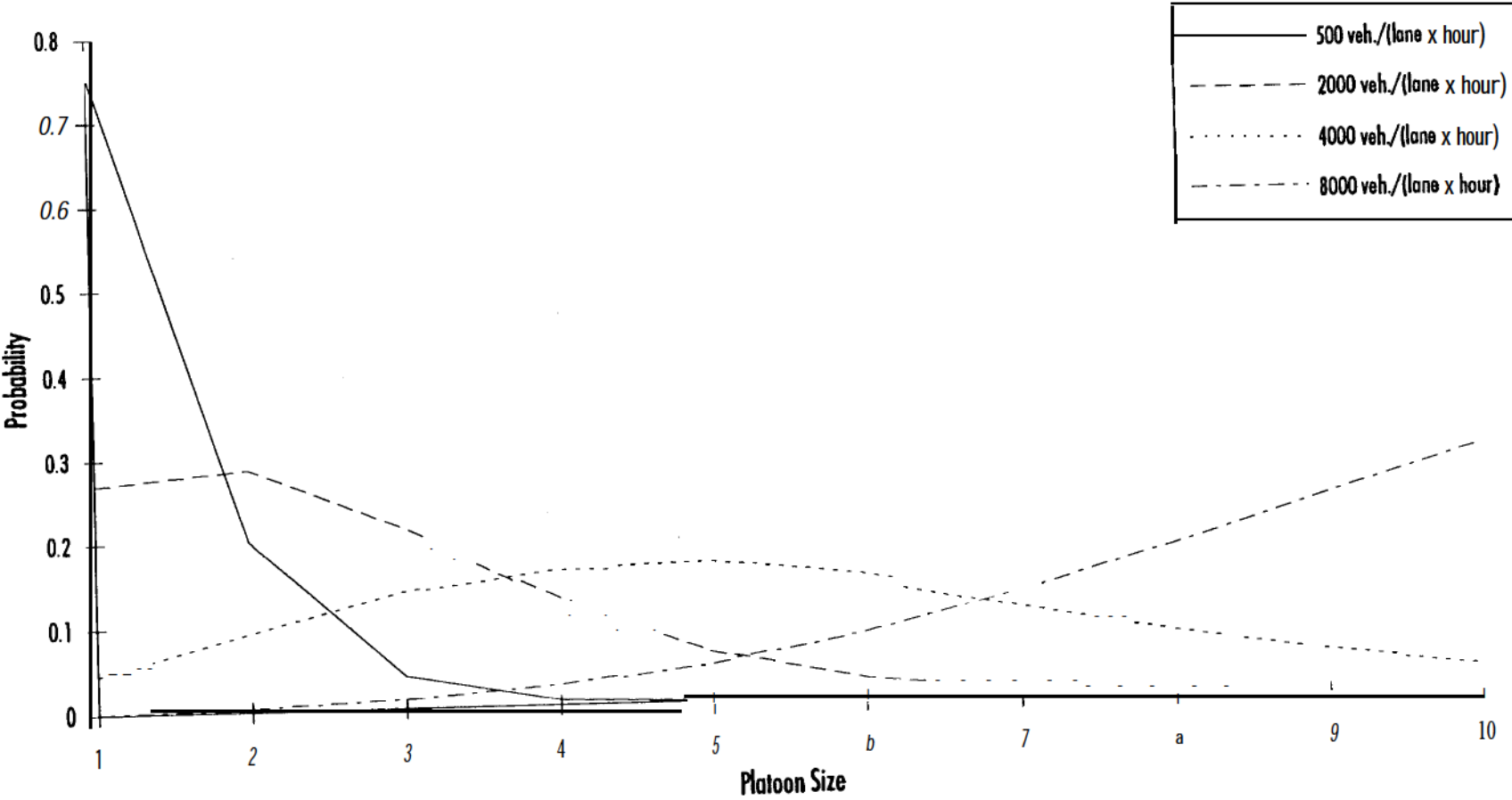
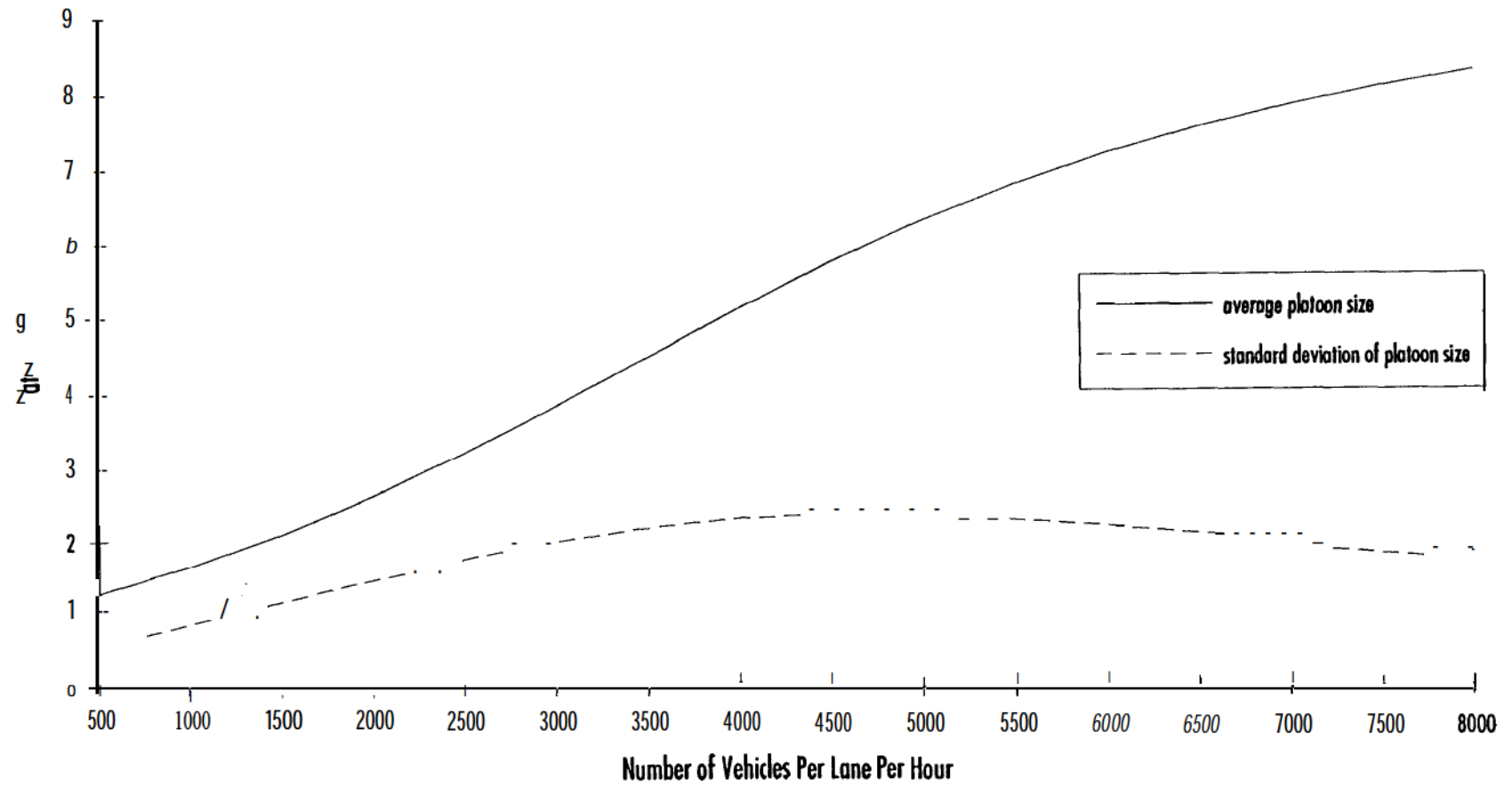




Figure 6: Average and Standard Deviation of Platoon Size Distributions



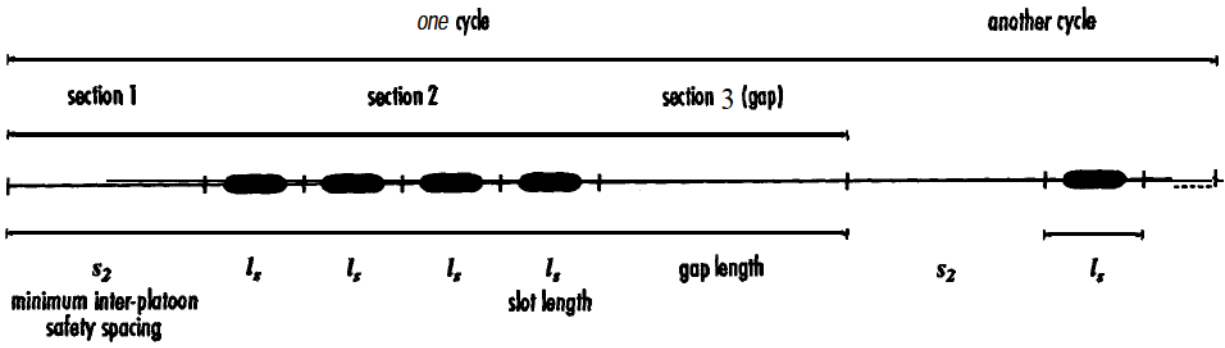


Figure 7: Platoon/gap cycle.

Figure 8: Probability Distributions for Lane-Change Completion Time

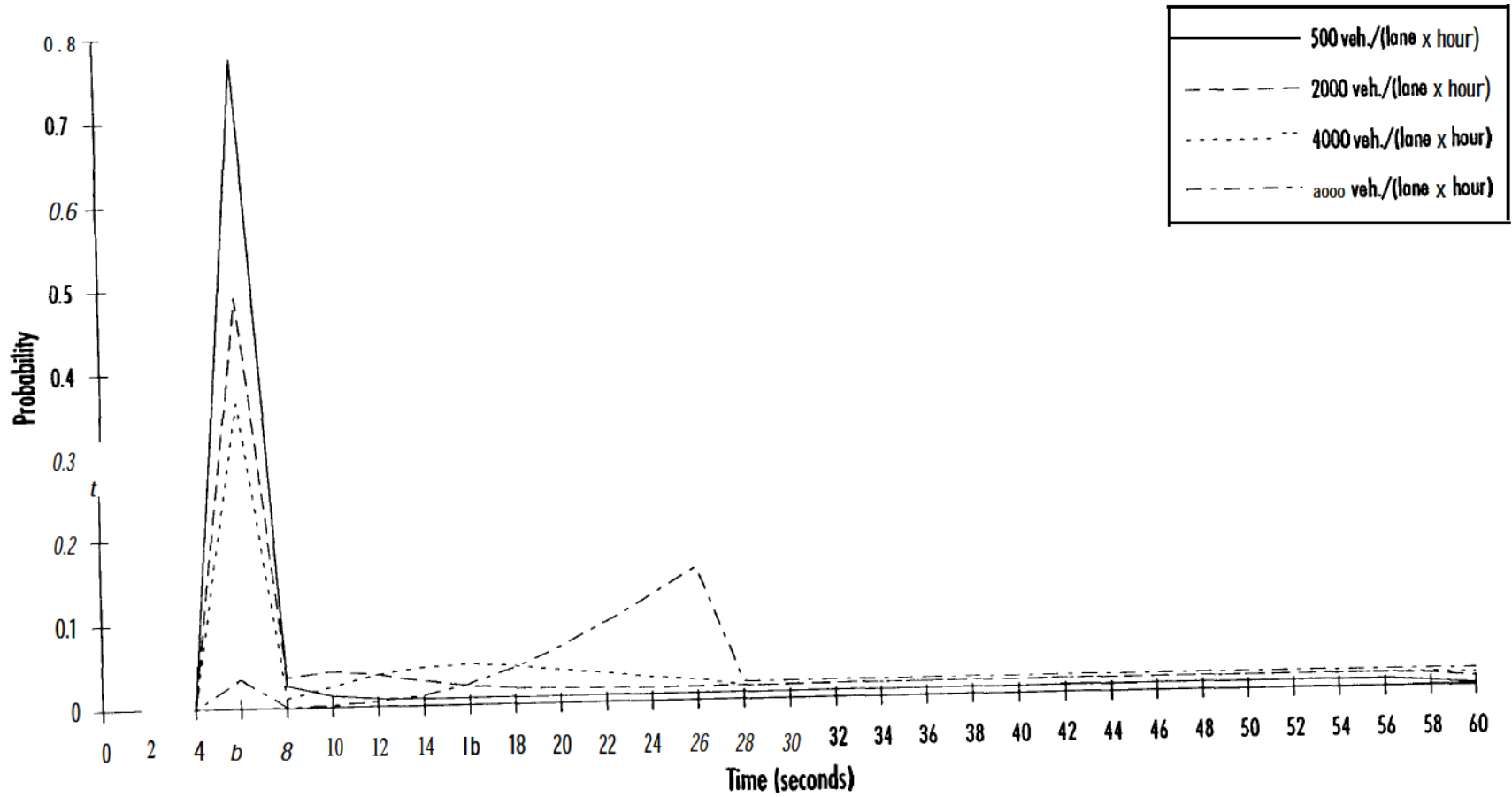
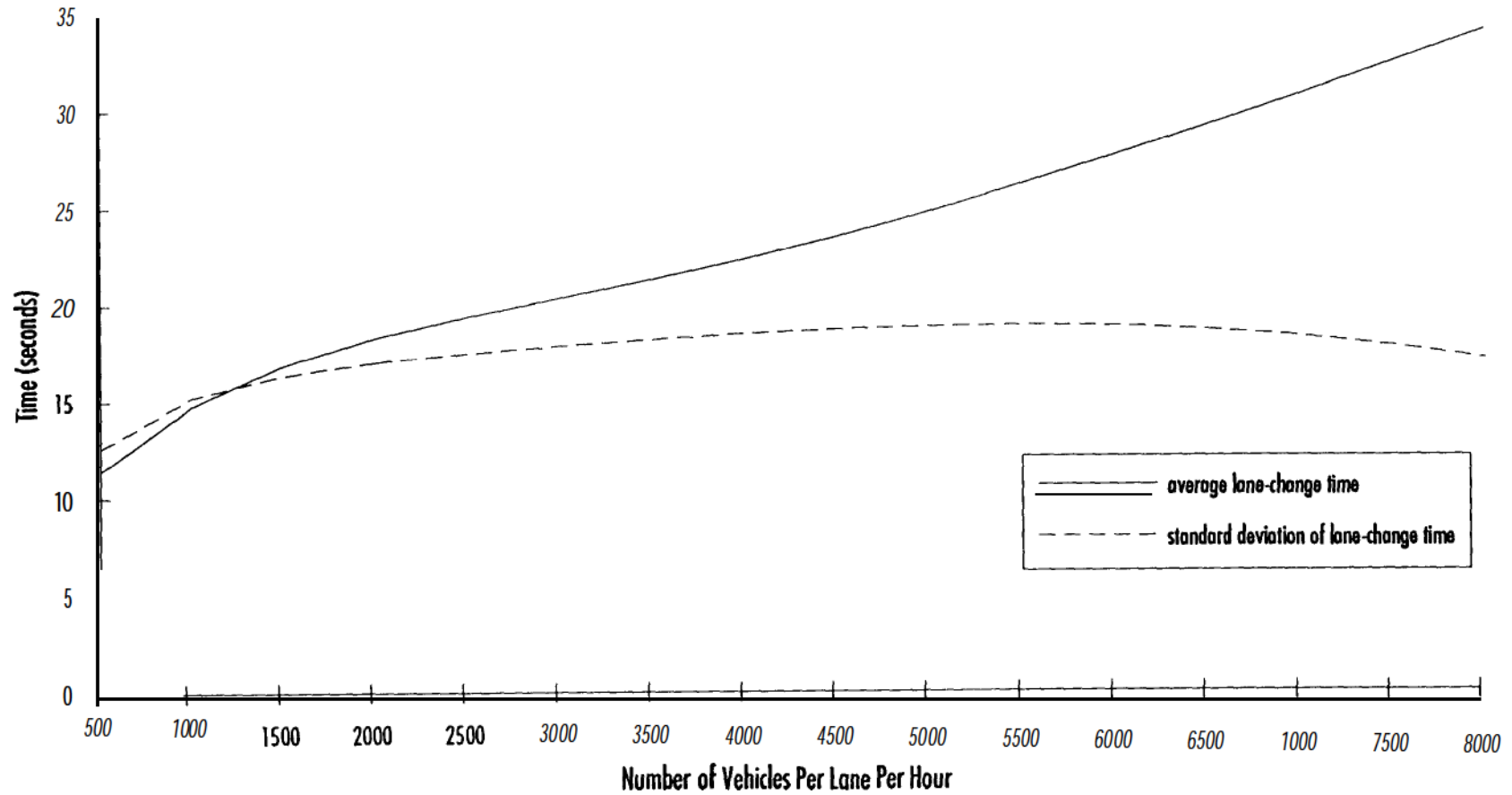
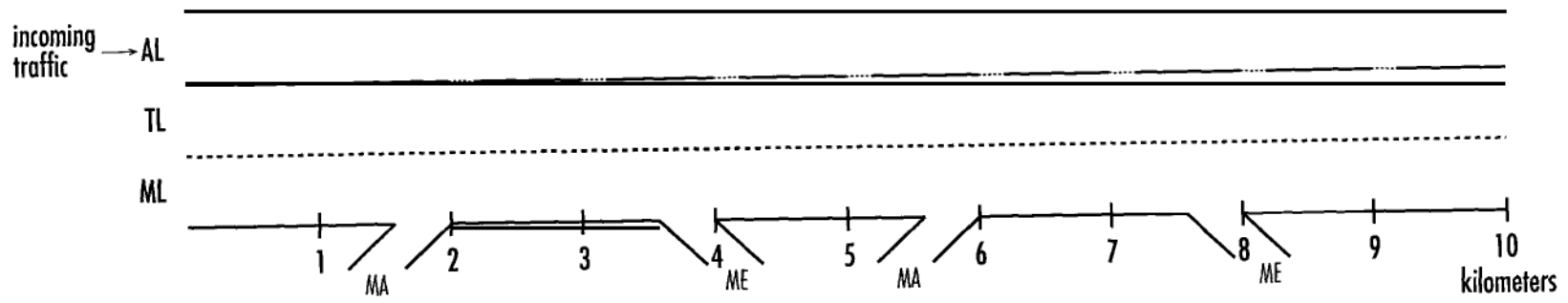


Figure 9: Average and Standard Deviation of Lane-Change Completion Times





- MA — manual entrance at 2 and 6 kms
- ME — manual exit at 4 and 8 kms
- ..... — 100 meter gates
- AL — Automated Lane
- TL — Transition Lane
- ML — Manual Lane

Figure 10: Configuration of AHS segment for simulation (barriers optional).

Figure 11.1 Bottleneck Flow as a Function of Specified Exit Percentage: Set Ia

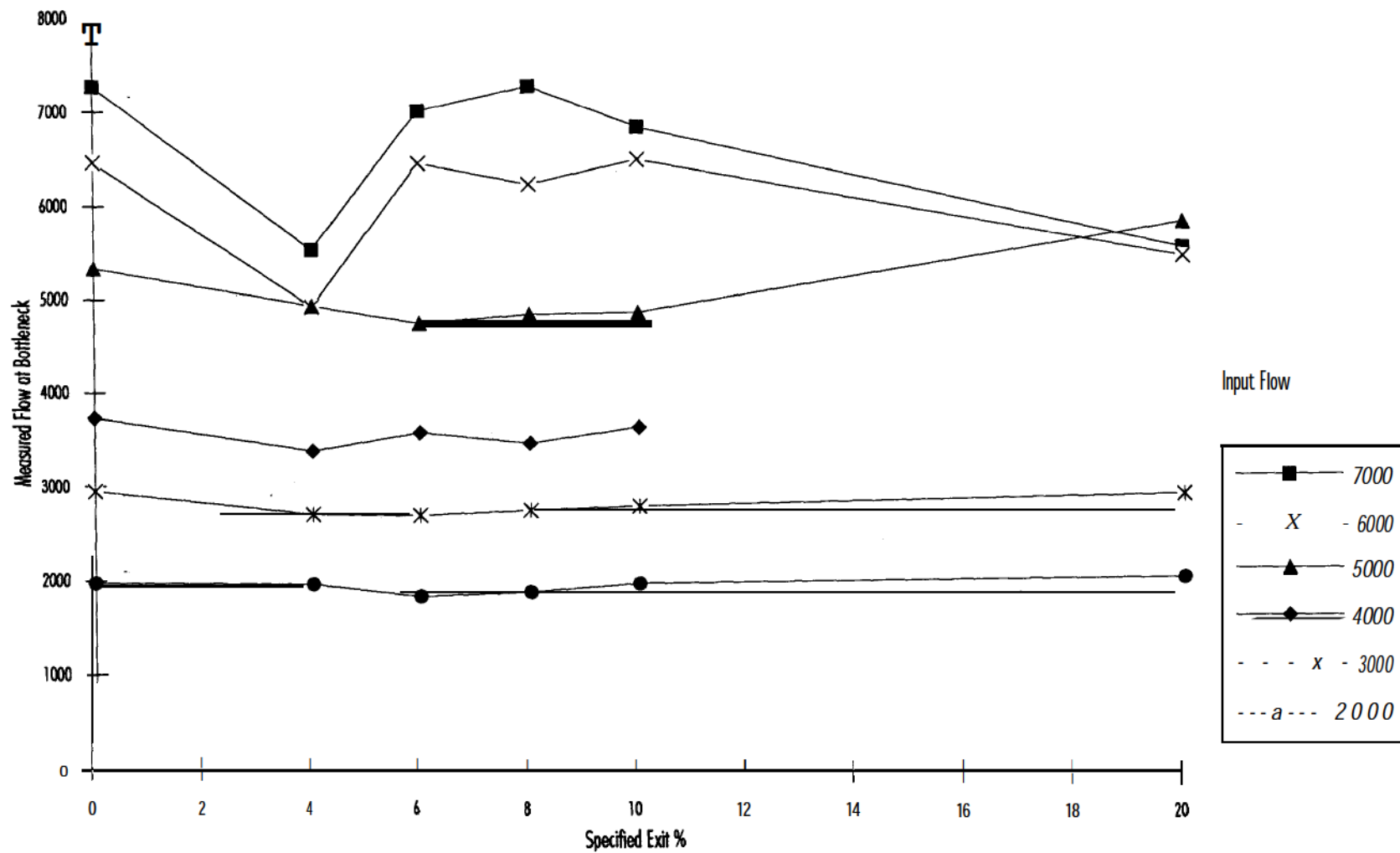


Figure 11.2 Exiting Success Rate as a Function of Specified Exit Percentage: Set Ia

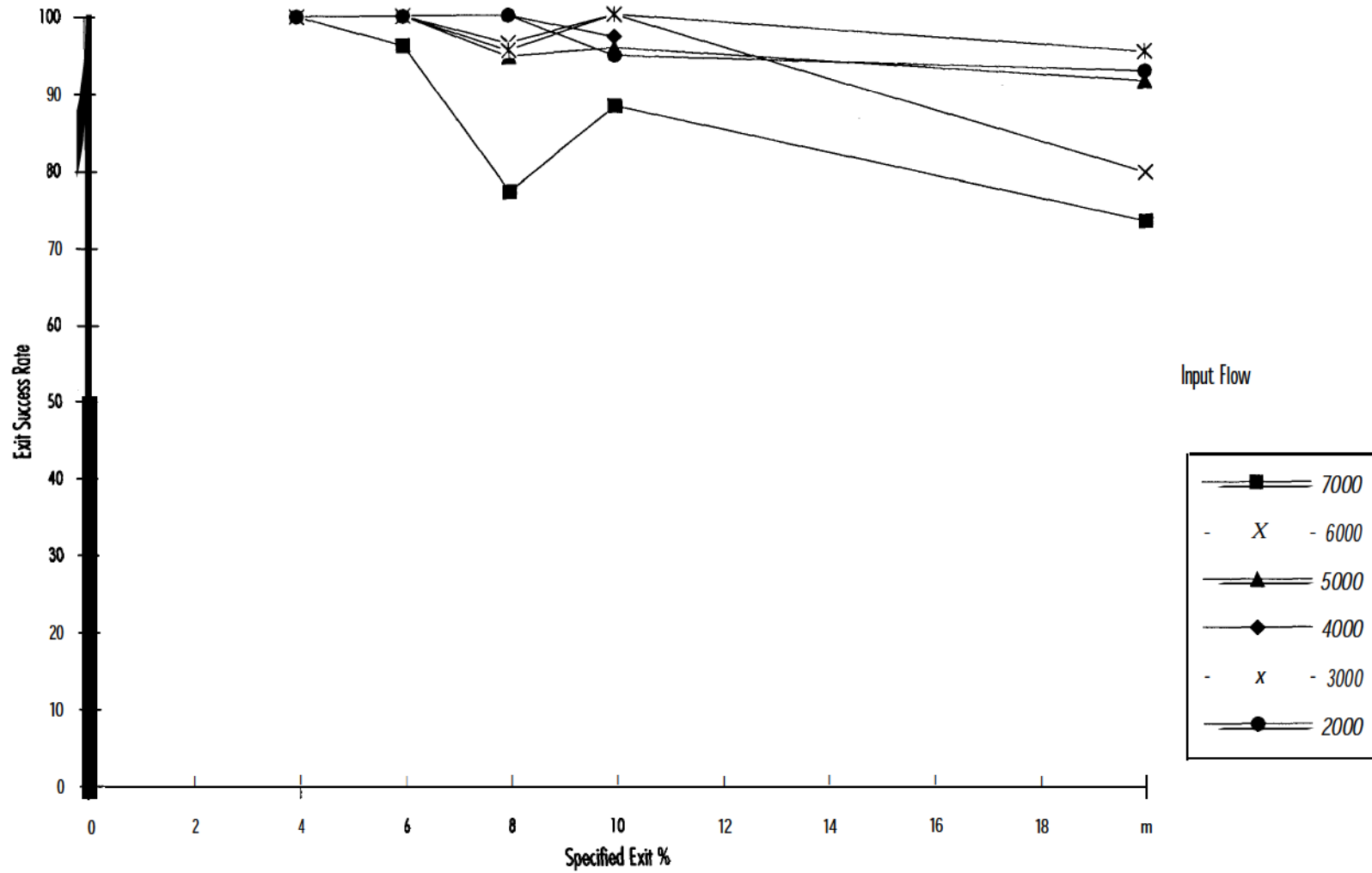


Figure 11.3 Bottleneck Flow as a Function of Specified Exit Percentage: Set 1 b

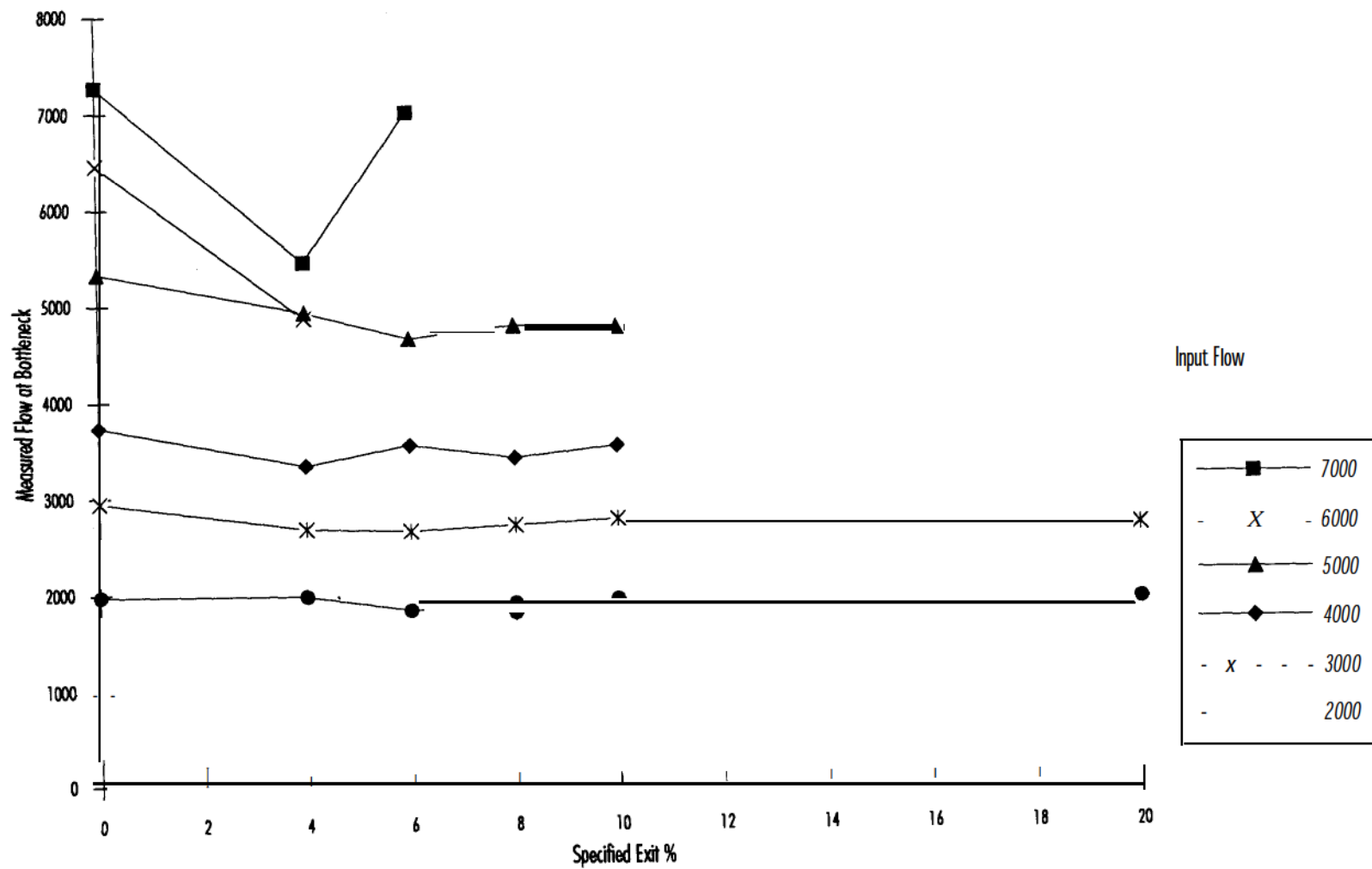




Figure 11.4 Exiting Success Rate as a Function of Specified Exit Percentage: Set 1b

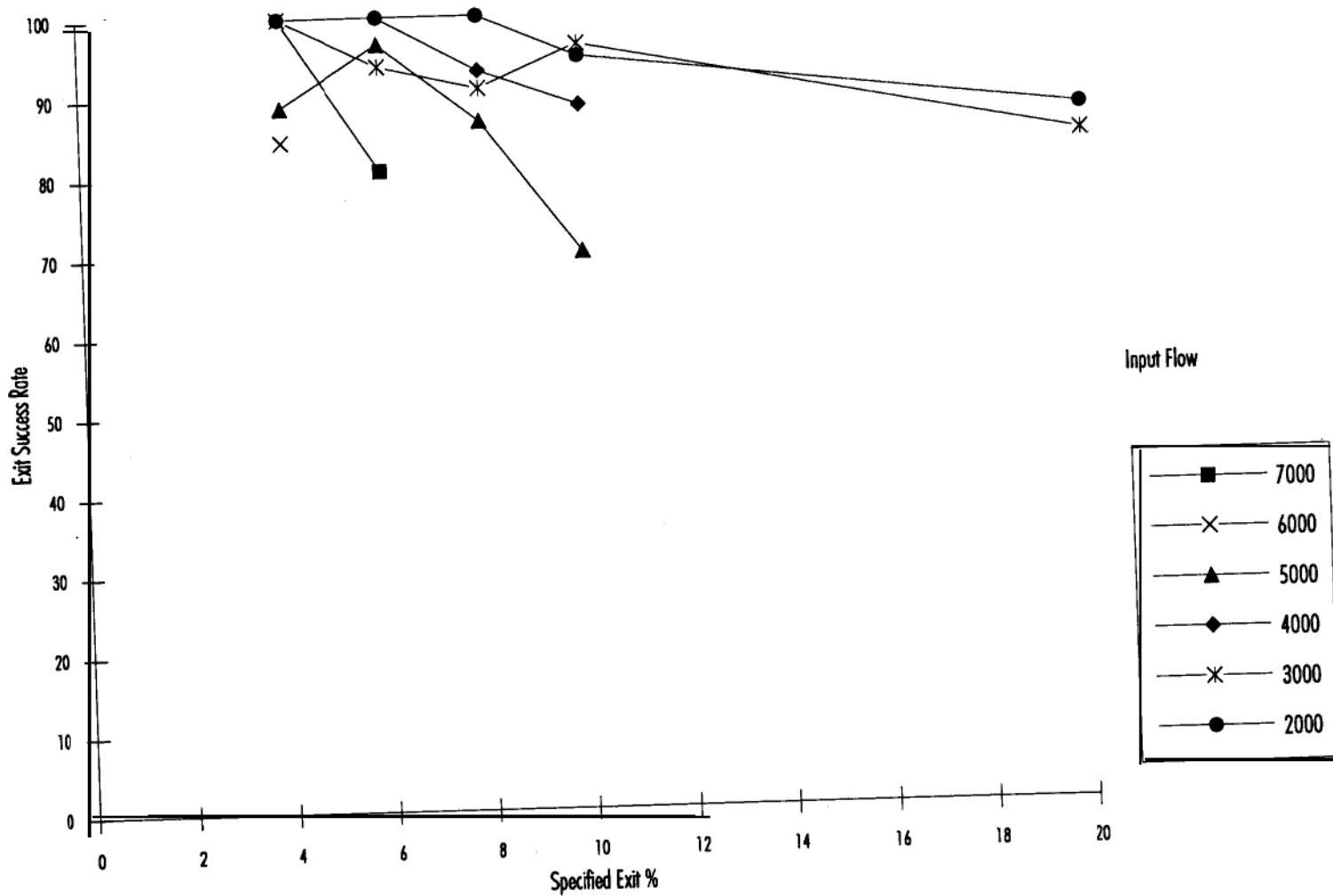


Figure 11.5 Bottleneck Flow as a Function of Specified Exit Percentage: Set 2a

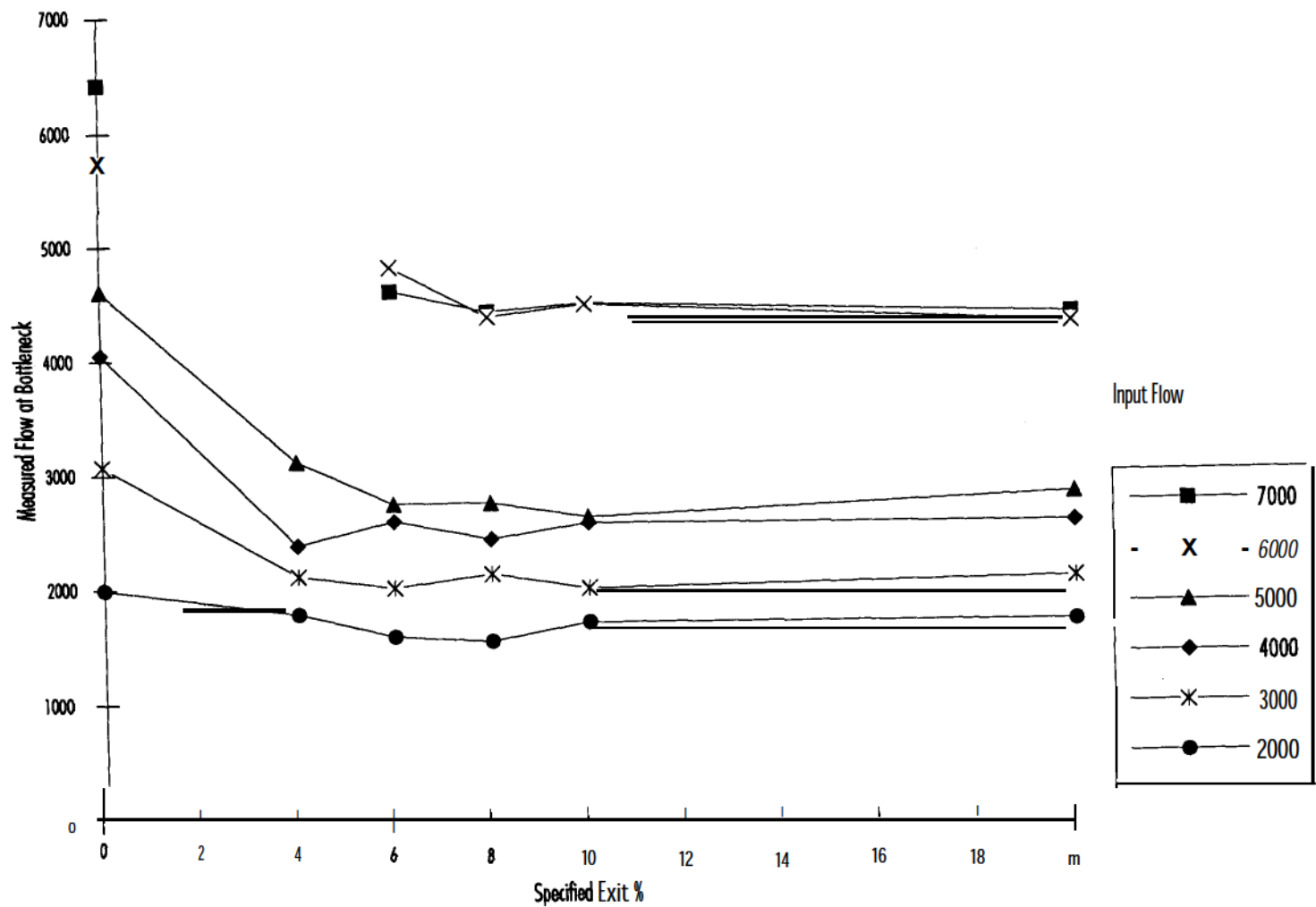


Figure 11.6 Exiting Success Rate as a Function of Specified Exit Percentage: Set 2a

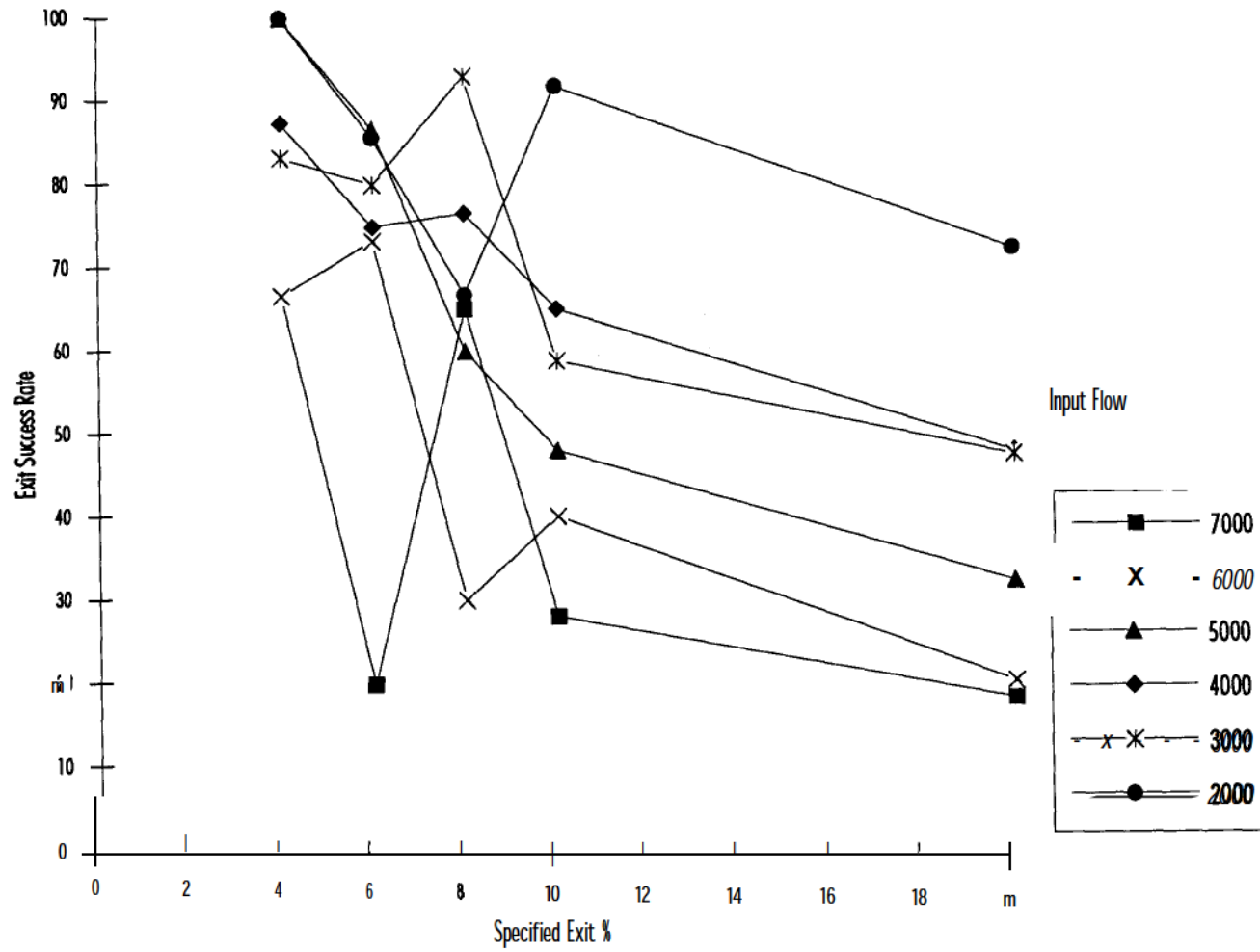


Figure 11.7 Bottleneck Flow as a Function of Specified Exit Percentage: Set 2b

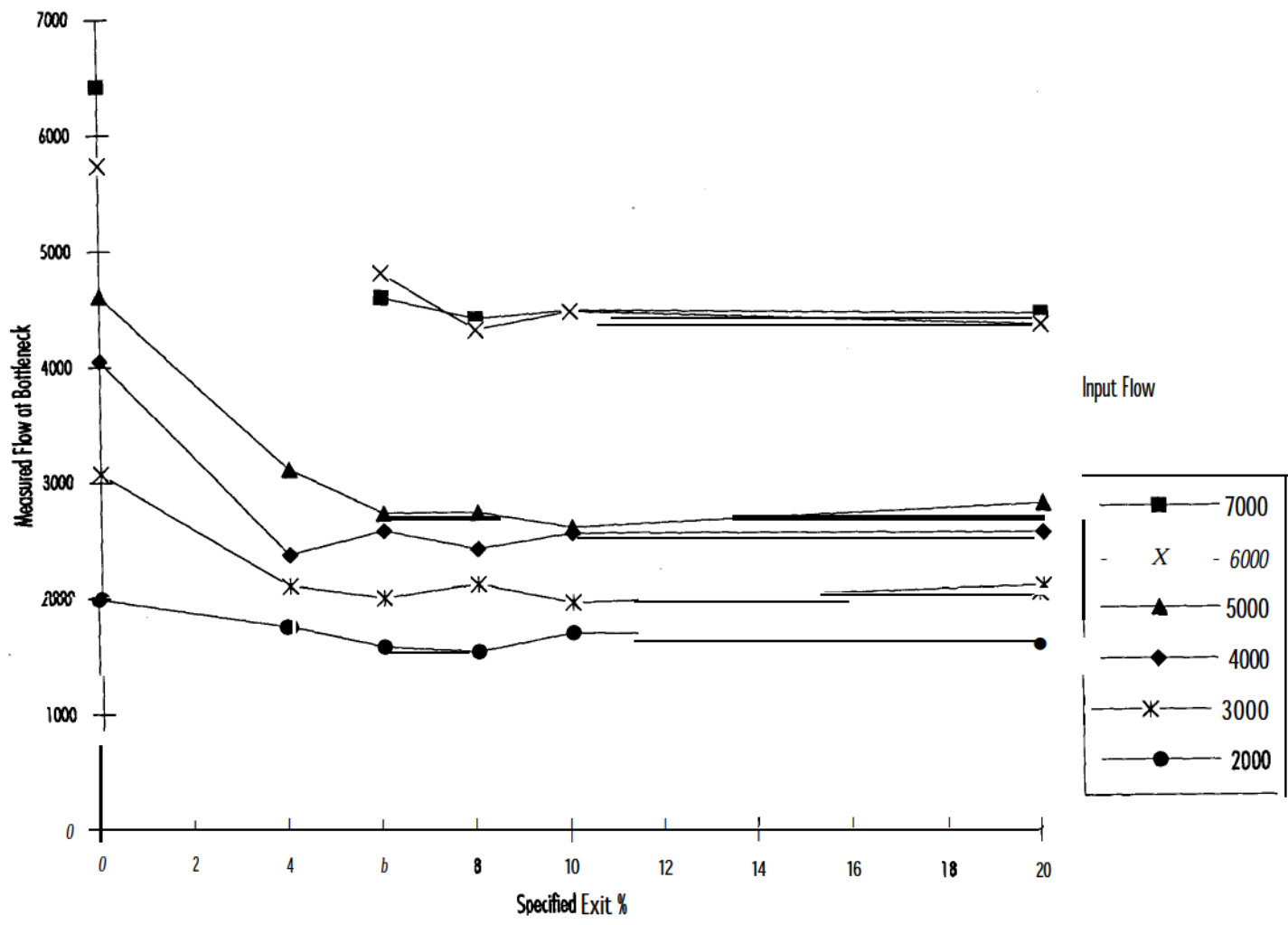


Figure 11.8 Exiting Success Rate as a Function of Specified Exit Percentage: Set 2b

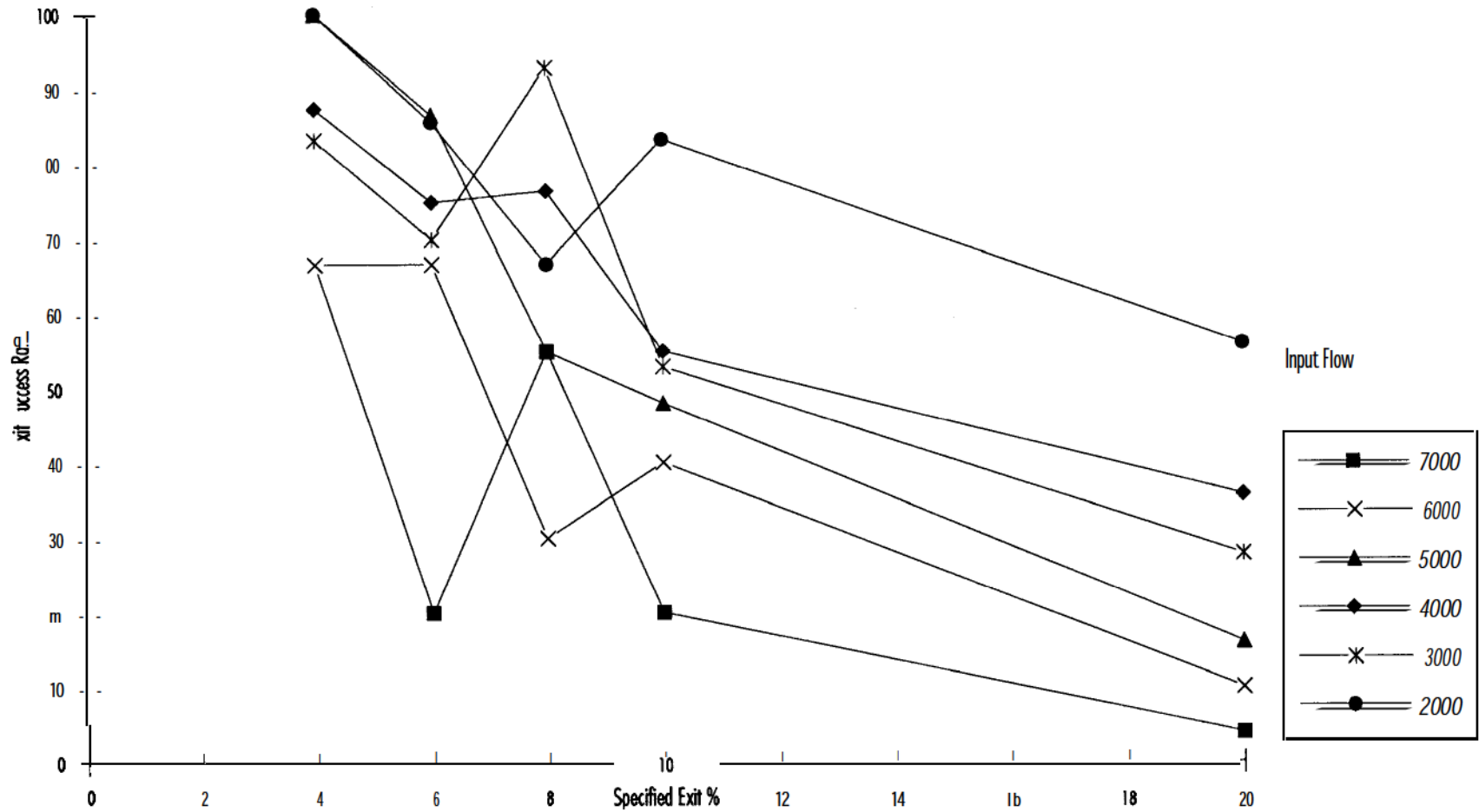


Figure 11.9 Bottleneck Flow as a Function of Specified Exit Percentage: Set 3a

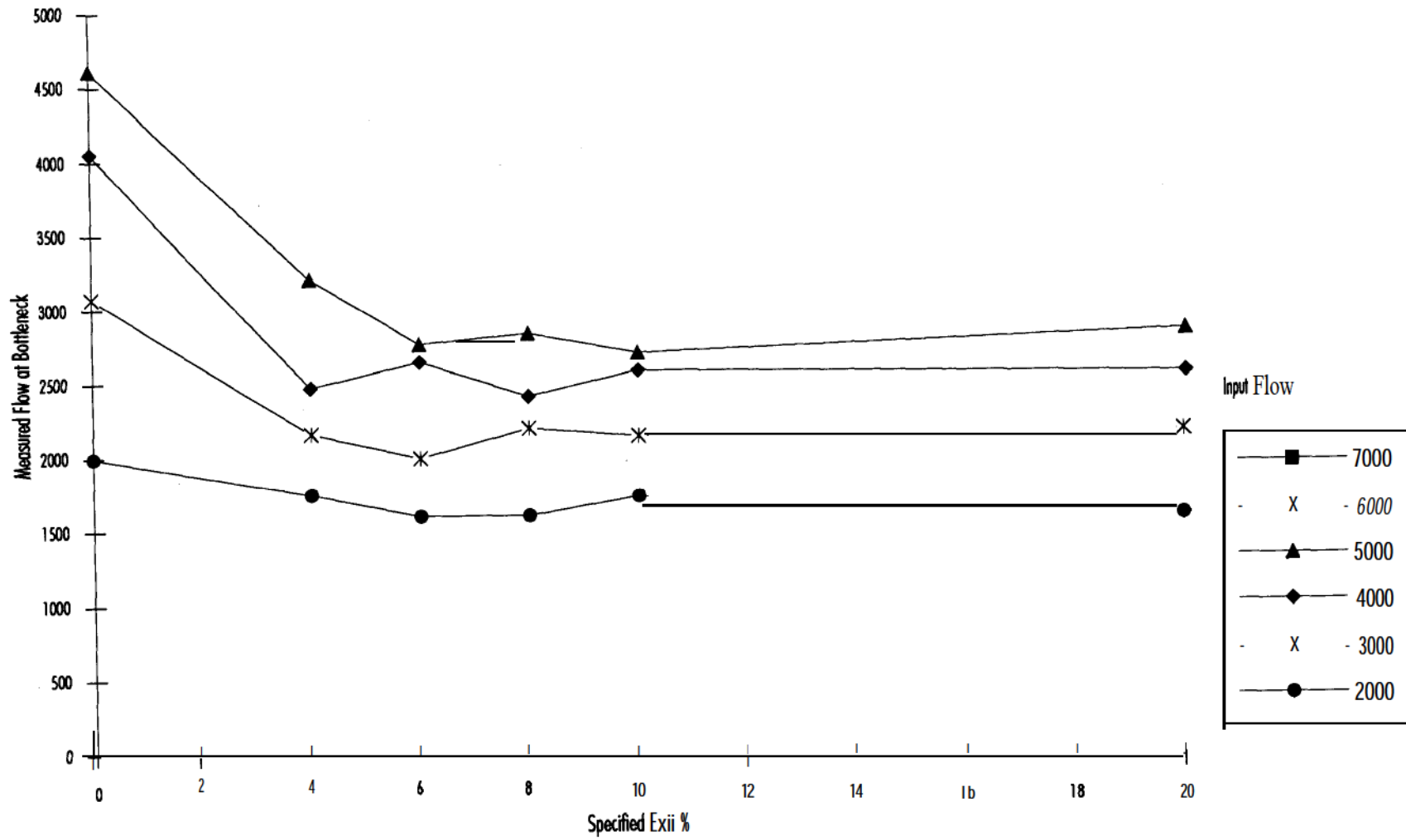


Figure 11.10 Exiting Success Rate as a Function of Specified Exit Percentage: Set 3a

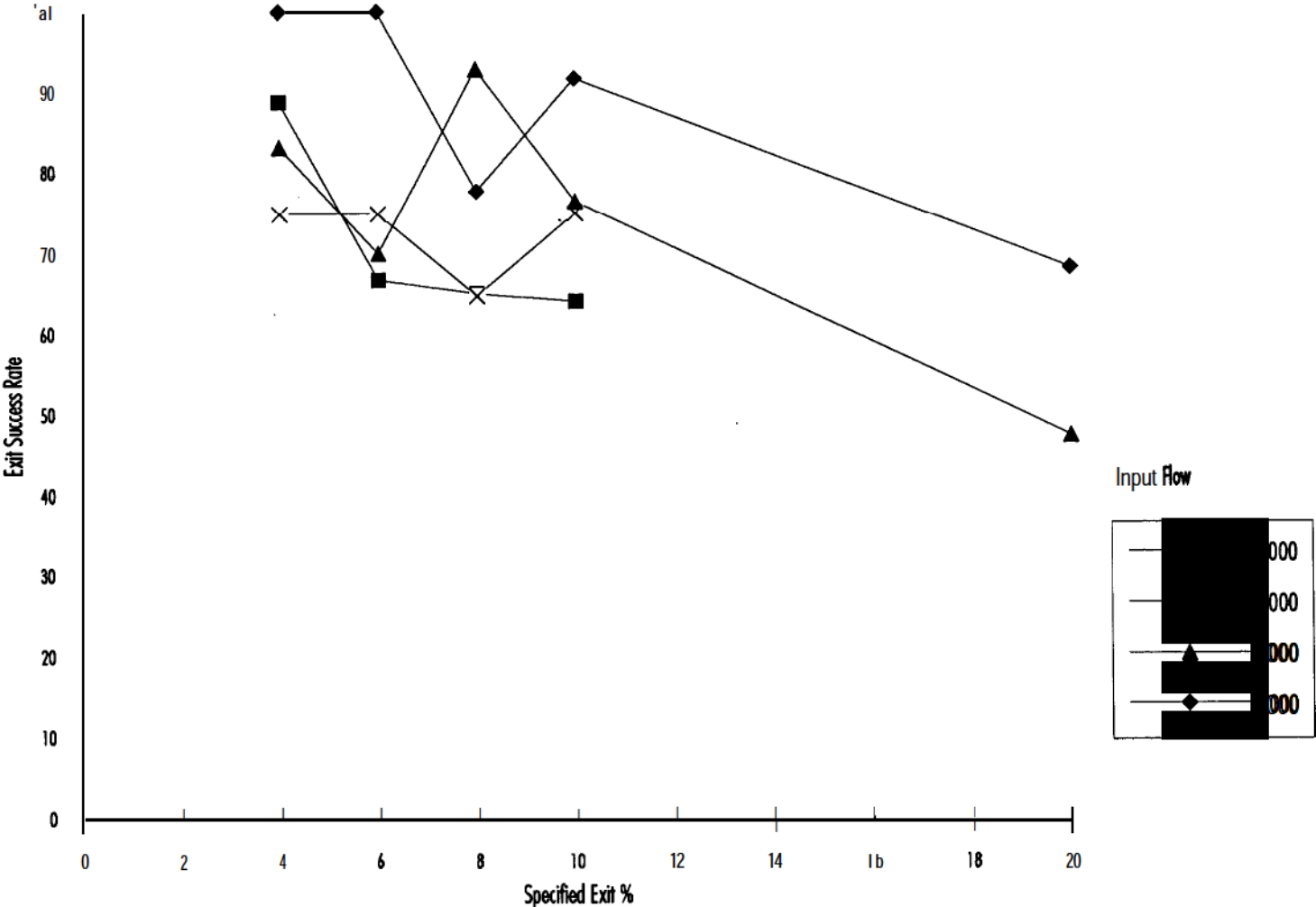


Figure 11.11 Bottleneck Flow as a Function of Specified Exit Percentage: Set 3b

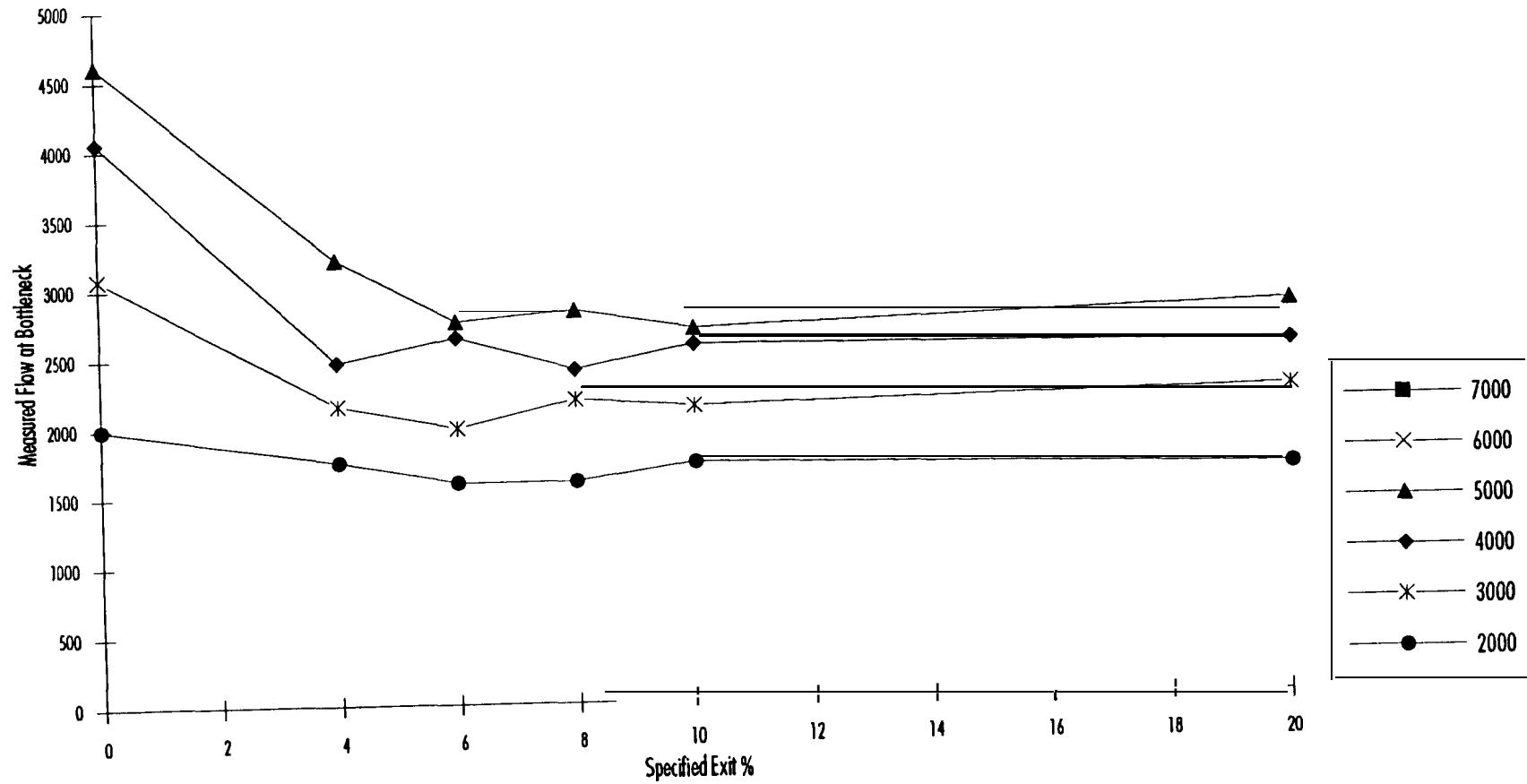




Figure 11.12 Exiting Success Rate as a Function of Specified Exit Percentage: Set 3b

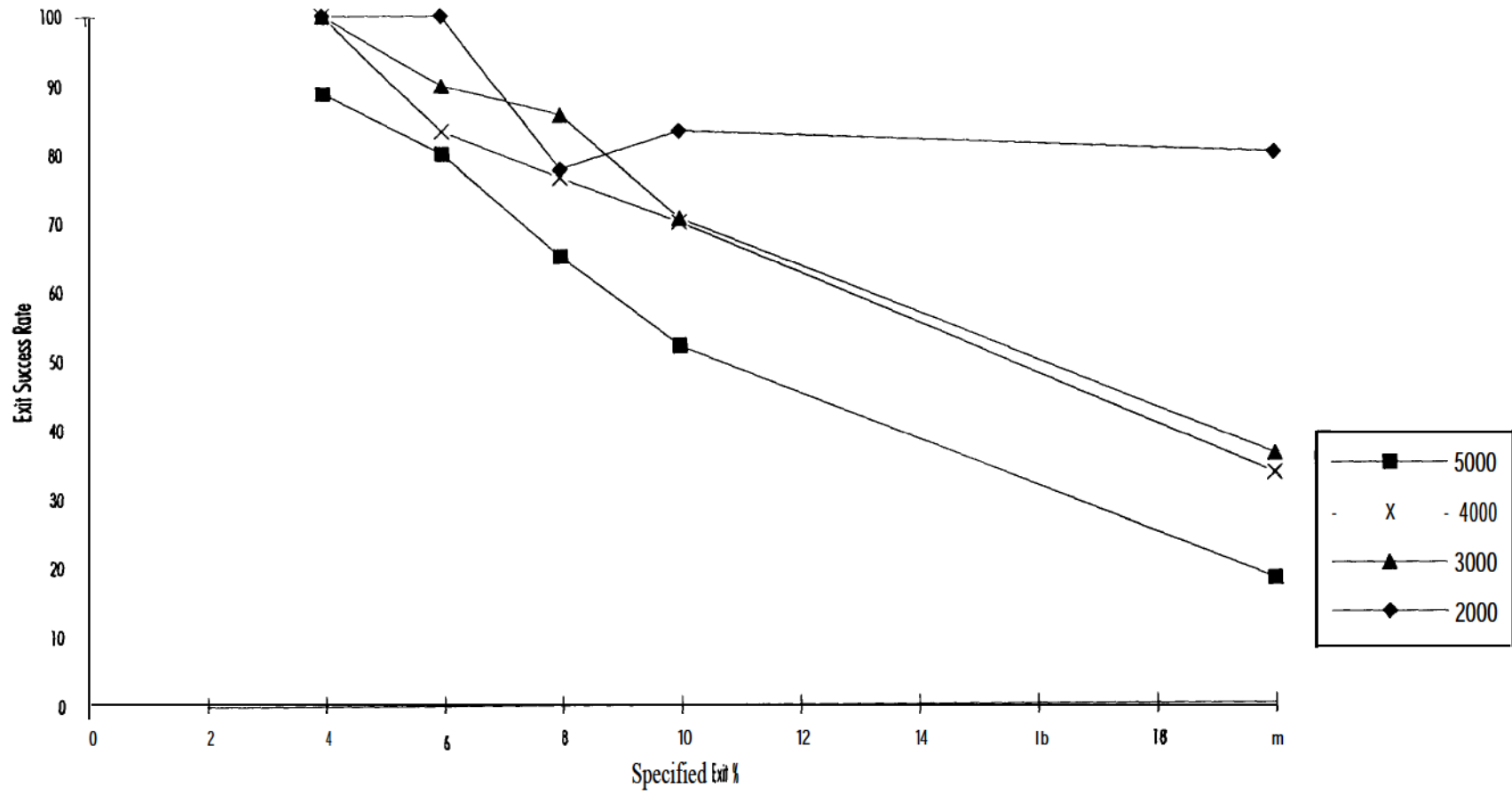


Figure 11.13 Bottleneck Flow as a Function of Specified Exit Percentage: Set 4a

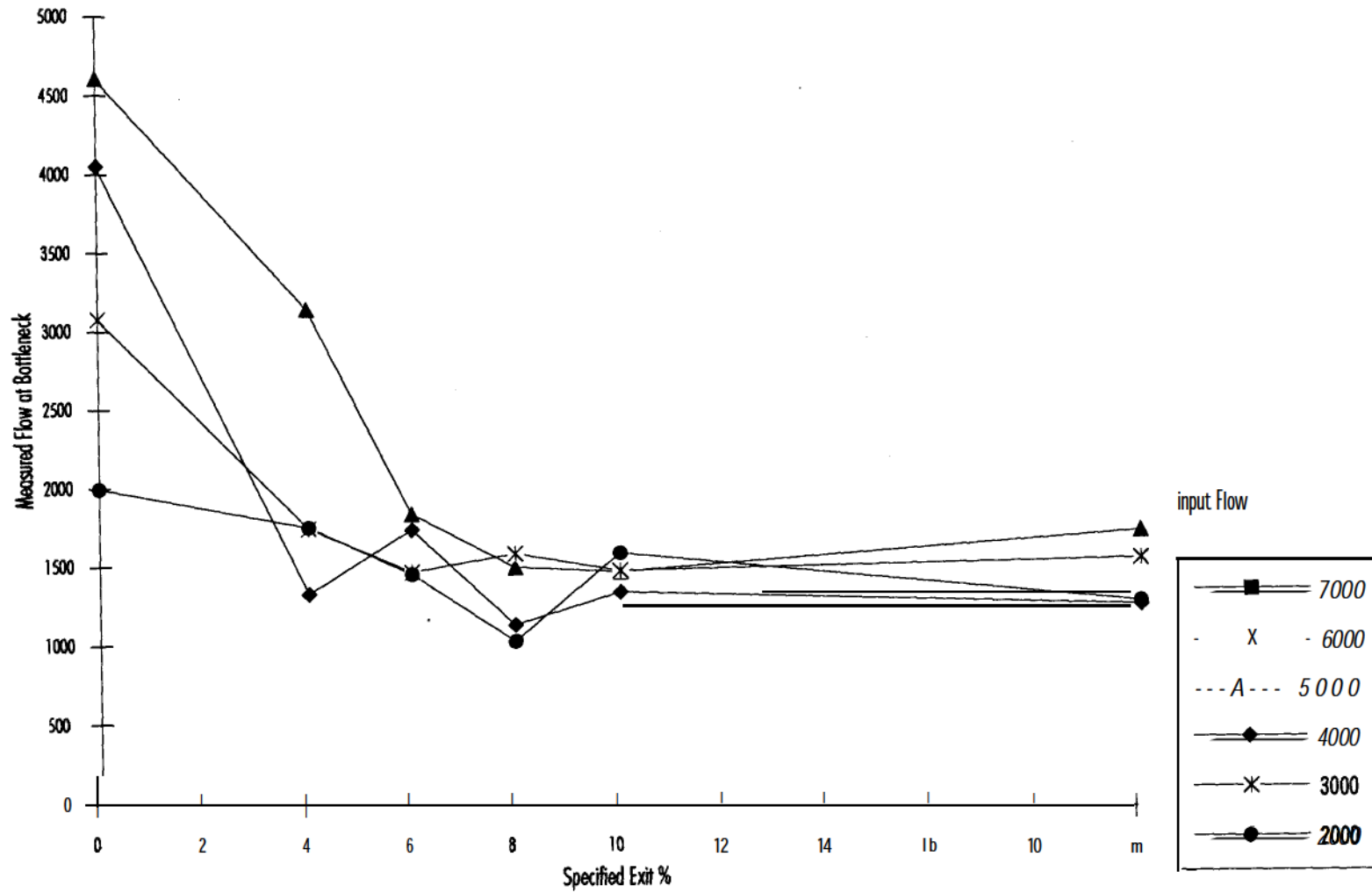


Figure 11.14 Exiting Success Rate as a Function of Specified Exit Percentage: Set 4a

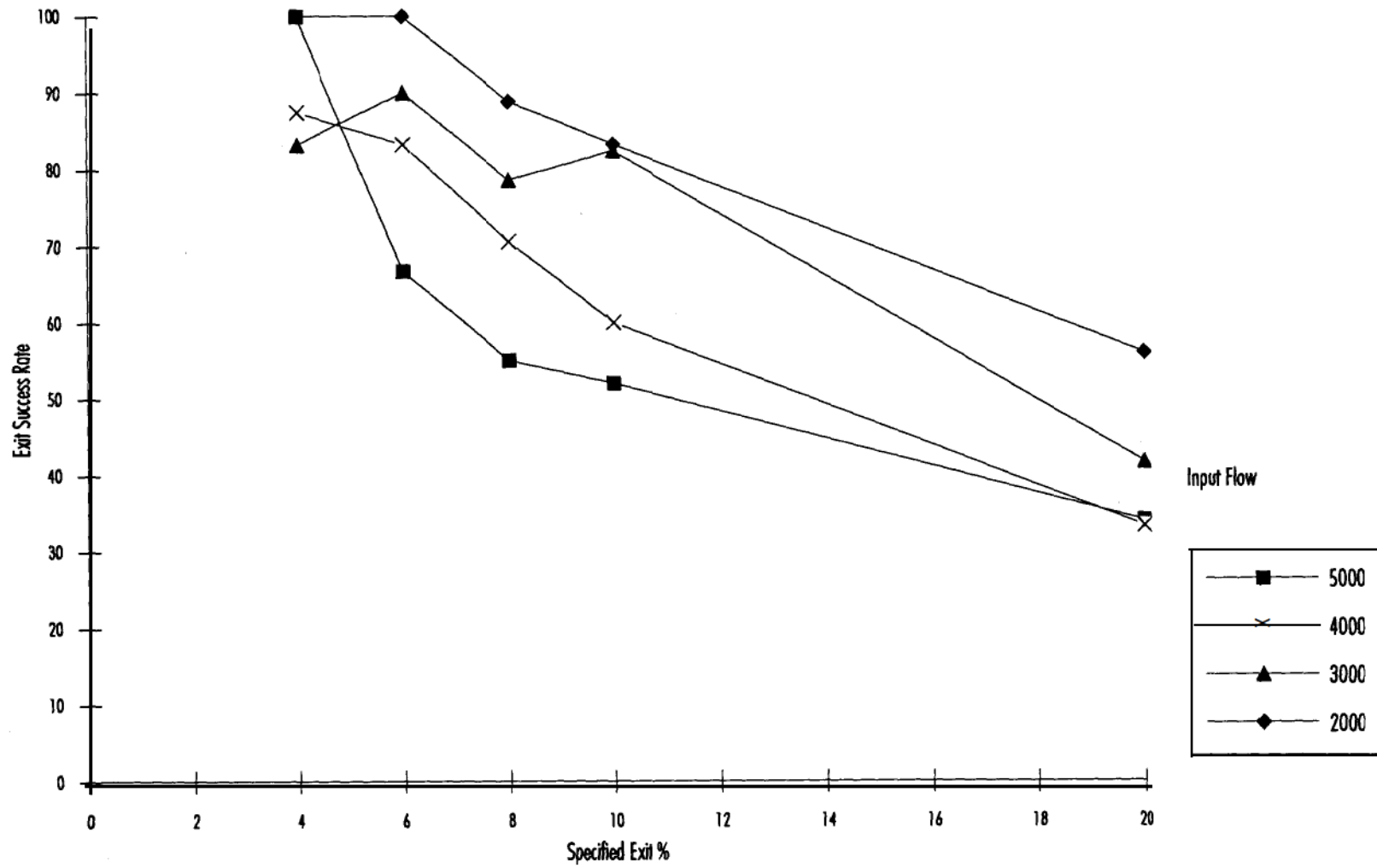


Figure 11.15 Bottleneck Flow as a Function of Specified Exit Percentage: Set 5a

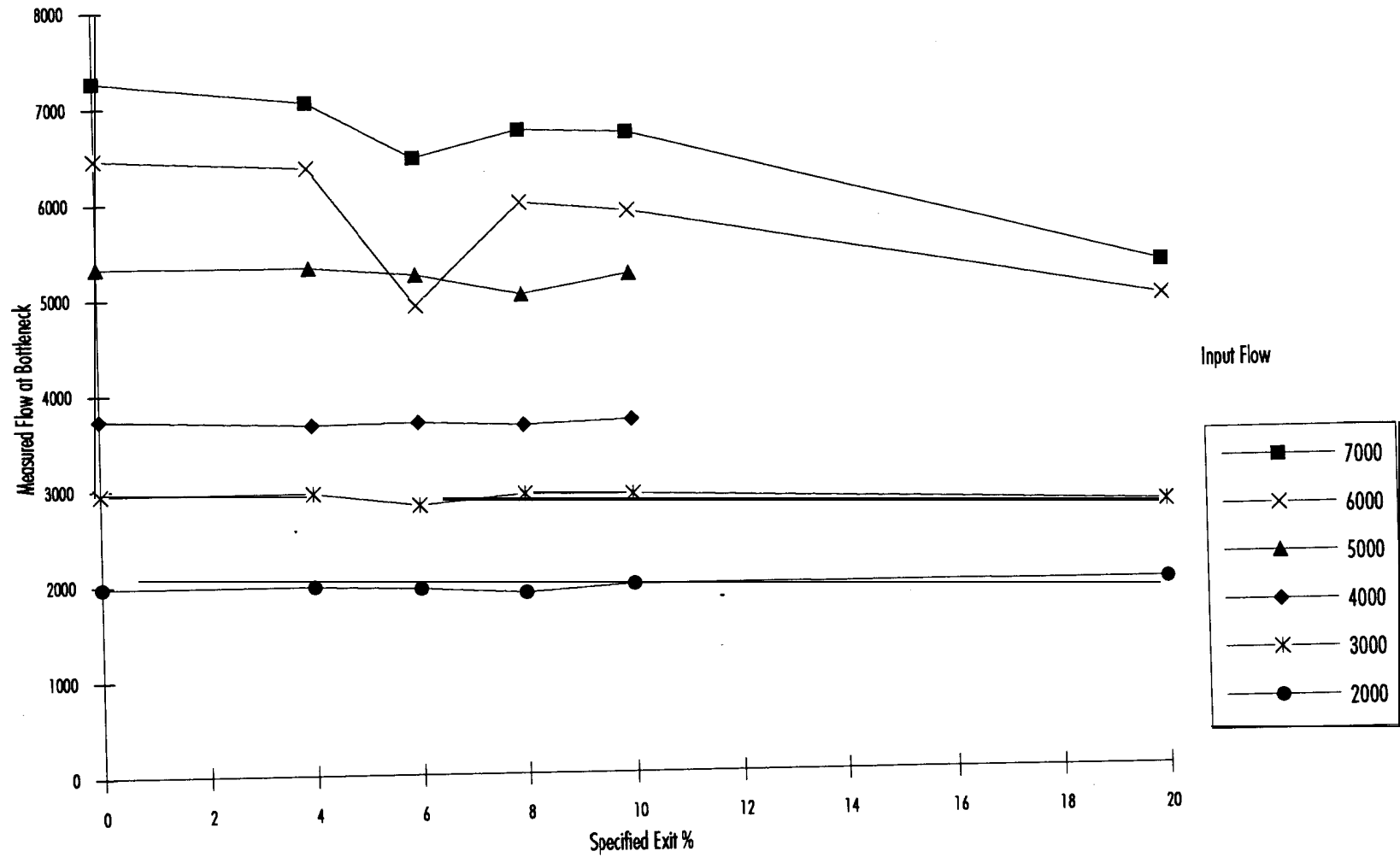


Figure 11.16 Exiting Success Rate as a Function of Specified Exit Percentage: Set 5a

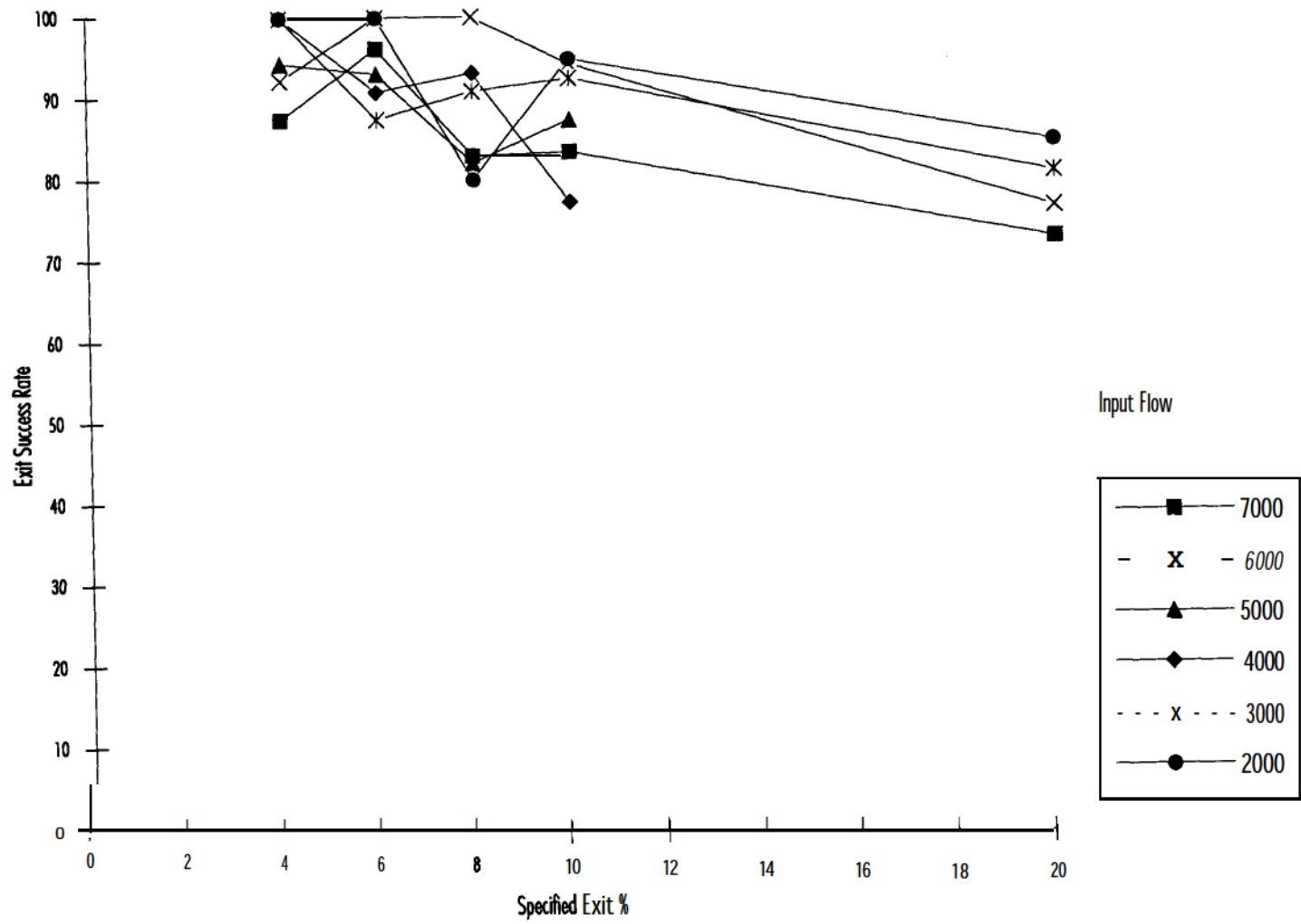


Figure 11.17 Bottleneck Flow as a Function of Specified Exit Percentage: Set 5b

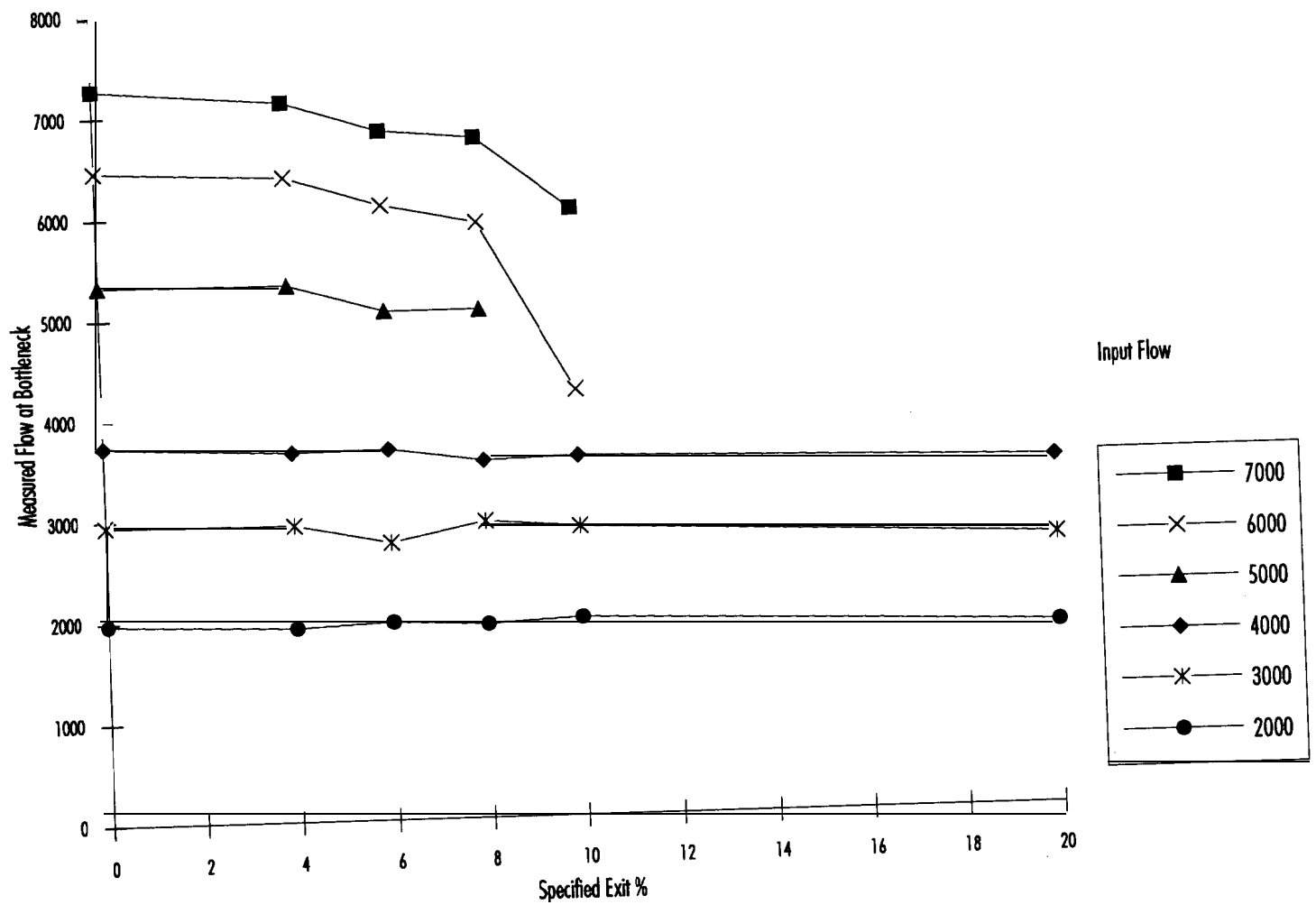


Figure 11.18 Exiting Success Rate as a Function of Specified Exit Percentage: Set 5b

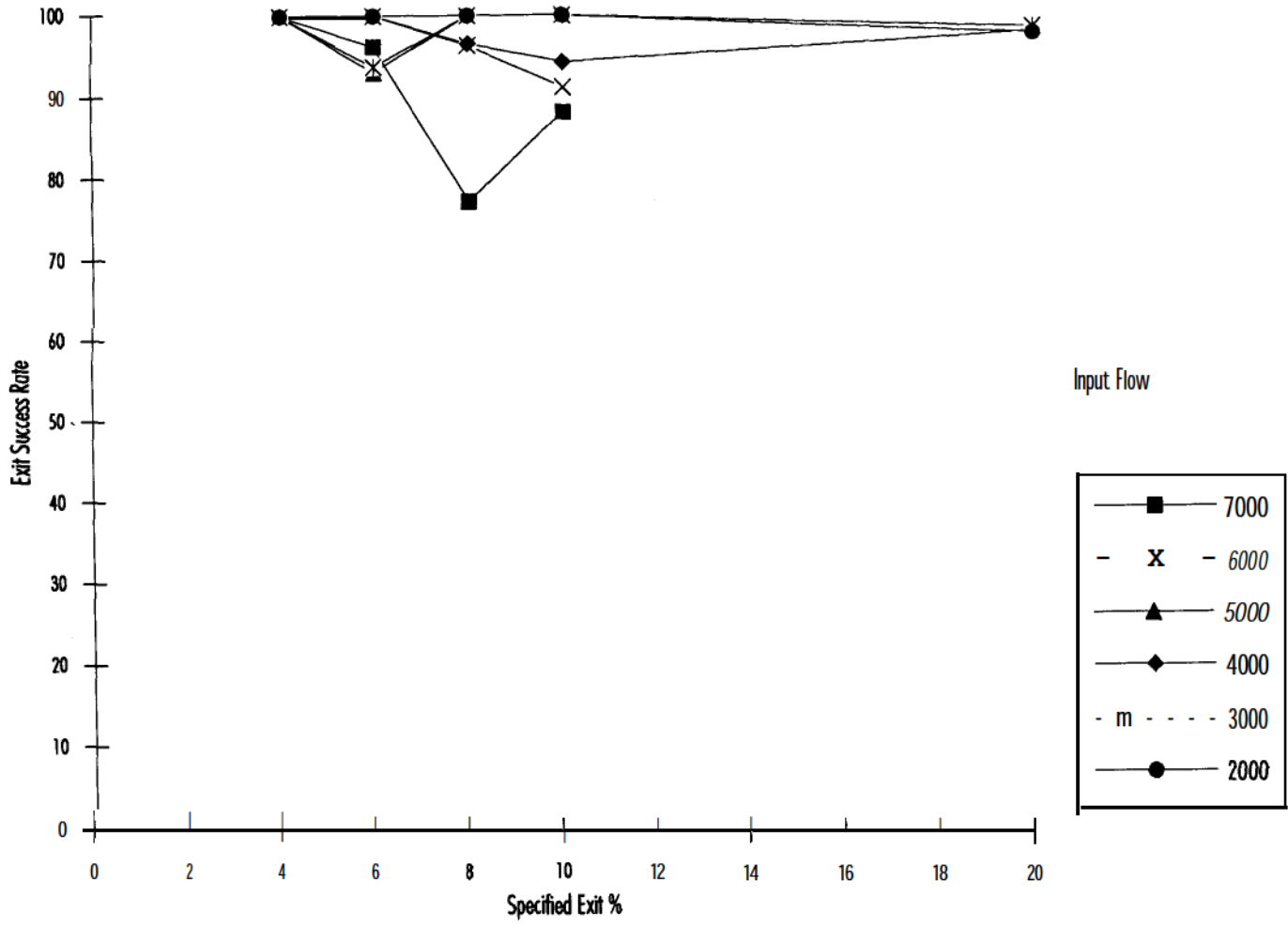


Figure 11.19 Bottleneck Flow as a Function of Specified Exit Percentage: Set 6a

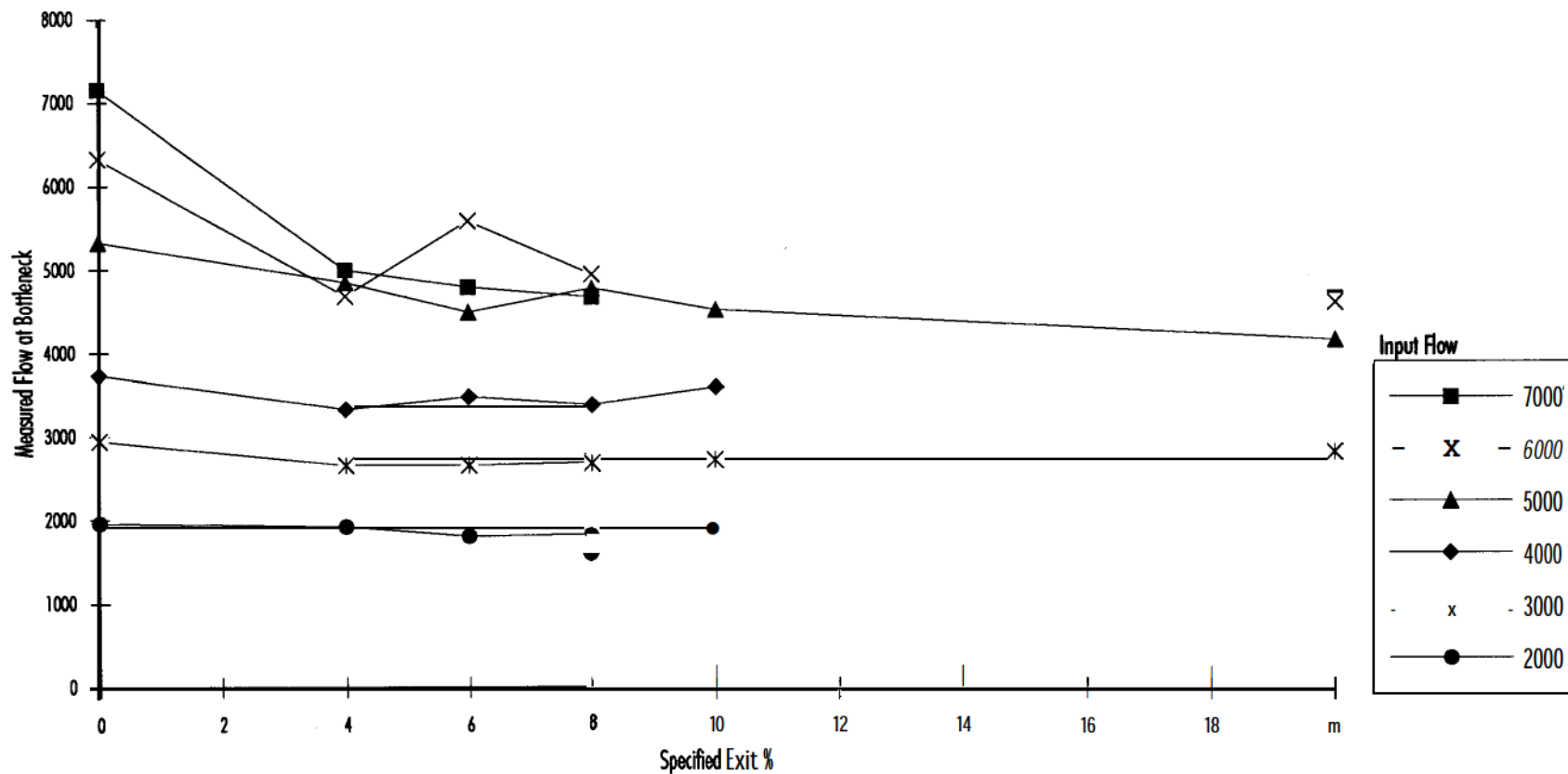




Figure 11.20 Exiting Success Rate as a Function of Specified Exit Percentage: Set 6a

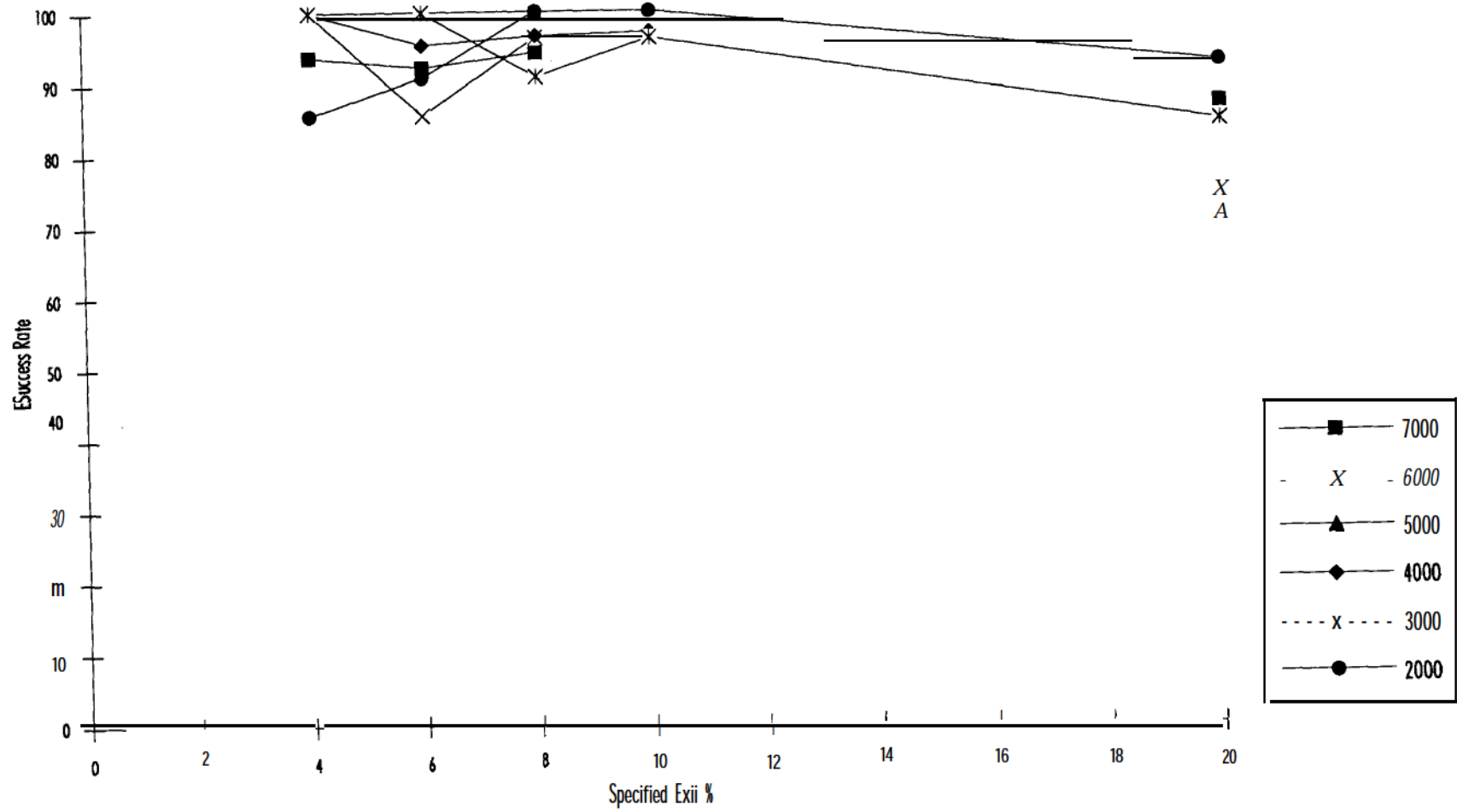


Figure 11.21 Bottleneck Flow as a Function of Specified Exit Percentage: Set 6b

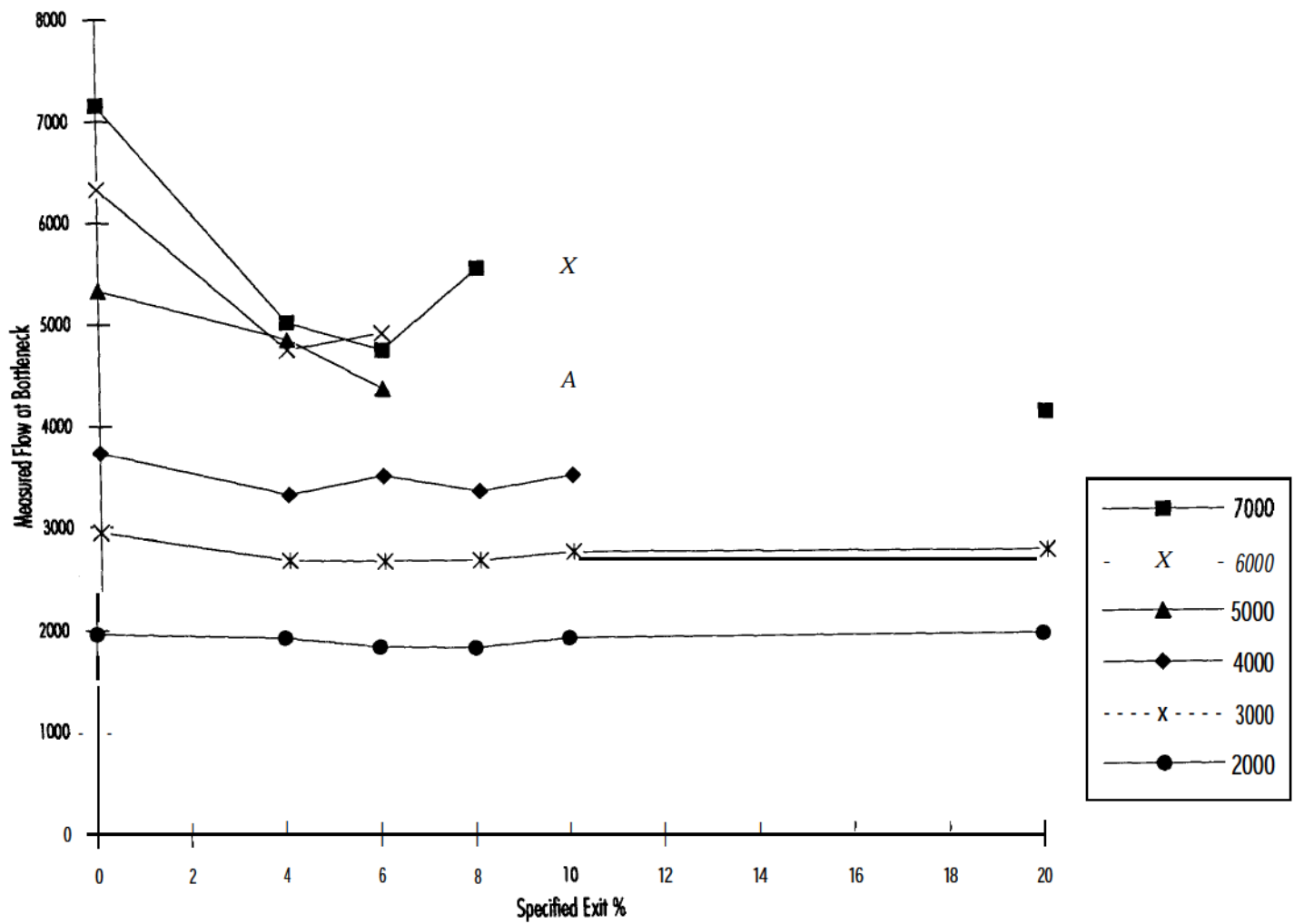


Figure 11.22 Exiting Success Rate as a Function of Specified Exit Percentage: Set 6b

