1-1998

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Recommended Citation

Robert H. Morelos-Zaragoza, Tadao Kasami, Shu Lin, and Hideki Imai. "On Block-Coded Modulation Using Unequal Error Protection Codes Over Rayleigh-Fading Channels" *Faculty Publications* (1998): 1-4. https://doi.org/10.1109/26.655394

On Block-Coded Modulation Using Unequal Error Protection Codes Over Rayleigh-Fading Channels

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***Abstract*— This letter considers block-coded 8-phase-shift-keying (PSK) modulations for the unequal error protection (UEP) of information transmitted over Rayleigh-fading channels. Both conventional linear block codes and linear UEP (LUEP) codes are combined with a naturally labeled 8-PSK signal set, using the multilevel construction of Imai and Hirakawa [1]. Computer simulation results are presented showing that, over Rayleigh-fading channels, it is possible to improve the coding gain for the most significant bits with the use of binary LUEP codes as constituent codes, in comparison with using conventional binary linear codes alone.**

I. Introduction

In recent years, coded modulation schemes that offer nonuniform or uequal error protection of information have received considerable attention. Application examples of these schemes are satellite and terrestrial broadcasting of digital high-definition television signals, as well as transmission of coded speech and image. A single code that offers different levels of error protection is called an unequal error protection (UEP) code. Binary linear UEP (LUEP) codes were first studied by Masnik and Wolf [2]. Previous work on combining LUEP codes and phase-shift-keying (PSK) modulation for fading channels is reported in [3] and [4]. Hagenauer *et al.* [3] proposed rate-compatible punctured convolutional codes combined with differential quadrature-phase-shift-keying (DQPSK) modulation to provide UEP by means of their inherent variable rate structure. In [4], Gray labeling of a quadrature-phase-shift-keying (QPSK) signal set was used to map binary LUEP codes of even length onto block modulation codes with UEP capabilities over Gaussian channels. Seshadri and Sundberg [5] studied the UEP capabilities of the Imai–Hirakawa multilevel construction over fading channels with short block codes and nonuniform Gray mapped 8-PSK signal sets.

In this paper, binary LUEP block codes are combined with an 8-PSK signal constellation to obtain block modulation codes for *unequal error protection* over Rayleigh-fading channels. The goal is to obtain coded signal sequences, associated with the most significant message bits, separated by a distance[1] greater than the minimum distance of the modulation code. As a result, the most significant (or more error sensitive) message bits are expected to have a bit-error rate (BER) lower than the average minimum BER guaranteed by the minimum distance of the code. Good LUEP codes should be chosen in such a way that: 1) their minimum distance is at least equal to that of the conventional linear codes they replace and 2) their rates satisfy a bandwidth efficiency constraint (e.g., at least 2 b/symbol, for coded 8-PSK modulation). For given multiple error protection requirements, a good LUEP code has a rate higher than that of the combination of linear codes, one for each requirement.

Analytical bounds on the error performance *at each level* of a block-coded modulation (BCM) scheme over Rayleigh-fading channels are difficult to derive, specially at practical BER's. Therefore, the error performance is demonstrated by Monte Carlo computer simulations of selected examples.

II. LUEP Codes

In this section, the basic concepts of LUEP are briefly overviewed. For more details, interested readers are referred to [2]. Let C be an (n,k) binary linear block code. As usual, a vector $\overline{\boldsymbol{m}}$ from $\{0,1\}^k$ is called a *message*, and an element $\overline{\boldsymbol{c}}(\overline{\boldsymbol{m}}) \in \{0,1\}^n$ from C is called a *codeword*. Let the *message space* $\{0,1\}^k$ be expressed as the direct product of two disjoint *message subspaces* $\{0,1\}^{k_i}, i=1,2$ such that $\{0,1\}^k = \{0,1\}^{k_1} \times \{0,1\}^{k_2}$. In other words, a message can be written as $\overline{\boldsymbol{m}} = (\overline{\boldsymbol{m}}_1, \overline{\boldsymbol{m}}_2), \boldsymbol{m}_i \in \{0,1\}^{k_i}, i=1,2$. Vector $\overline{\boldsymbol{m}}_i$ is called the ith *message part* $i=1,2$. The *separation vector* of C is defined as the vector $\overline{\boldsymbol{s}} = (\boldsymbol{s}_1, \boldsymbol{s}_2)$ where

$$\boldsymbol{s}_i \triangleq \min\{\mathrm{wt}(\overline{\boldsymbol{c}}(\overline{\boldsymbol{m}})) \colon \overline{\boldsymbol{m}}_i \neq \overline{0}, \qquad \overline{\boldsymbol{m}}_i \in \{0,1\}^{k_i}\}$$

for $i=1,2$ where $\mathrm{wt}(\overline{\boldsymbol{x}})$ denotes the Hamming weight (or number of nonzero entries) of a vector $\overline{\boldsymbol{x}}$. Without loss of generality, it is assumed that code C has both components of its separation vector distinct and arranged in decreasing order $\boldsymbol{s}_1 > \boldsymbol{s}_2$ such that C is an (n,k) binary linear block code of minimum distance $d_{\min} = \boldsymbol{s}_2$. The first message part $\overline{\boldsymbol{m}}_1$ will

Paper approved by S. B. Wicker, the Editor for Coding Theory and Techniques of the IEEE Communications Society. Manuscript received May 22, 1996; revised February 20, 1997. This work was supported in part by NASA under Grant NAG 5-931, by the National Science Foundation under Grant NCR-9415374, by the Japanese Society for the Promotion of Science (JSPS) under Fellowship 93157, and by the Ministry of Education of Japan under Grant (C) 06650416. This paper was presented in part at the Sixth IEEE International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC'95), Toronto, Canada, September 1995.

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Publisher Item Identifier S 0090-6778(98)01054-X.

[1] For a Rayleigh-fading channel, by *distance* it is meant both *symbol and product distances*.

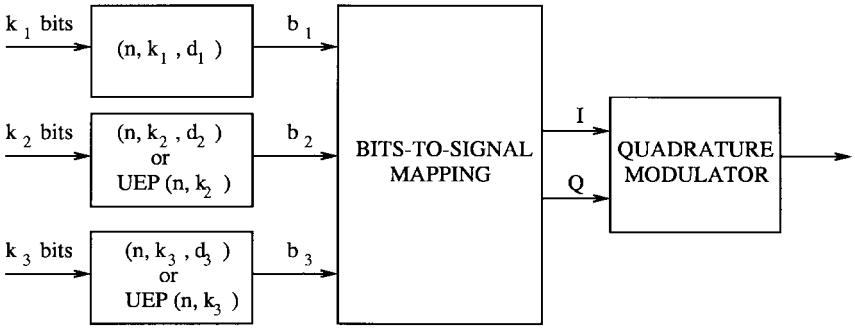

Fig. 1. An encoder of block 8-PSK modulation codes for UEP.

be referred to as the *most significant bits* (MSB) and the second message part $\overline{\boldsymbol{m}}_2$ the *least significant bits* (LSB).

Code C is said to be an (n,k) binary *two-level LUEP code*, denoted UEP(n,k) with separation vector $\overline{\boldsymbol{s}} = (\boldsymbol{s}_1, \boldsymbol{s}_2)$ for the message space $\{0,1\}^{k_1} \times \{0,1\}^{k_2}$. In terms of levels of error correction, it can be shown [2] that k_i information bits can be successfully decoded in the presence of up to $\lfloor (\boldsymbol{s}_i - 1)/2 \rfloor$ random errors $i = 1,2$ where $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.

III. Block-Coded 8-PSK Modulation for UEP

Let S represent a uniform unit-energy 8-PSK signal set. In this section, natural labeling (i.e., standard mapping by set partitioning) of set S is considered. That is, a label $\ell_i = b_1 + 2b_2 + 4b_3$ represents the signal point $e^{j\ell_i \pi/4}$ for $0 \le i < 8$ where $j = \sqrt{-1}$ and $b_i \in \{0,1\}, 1 \le i \le 3$. In three-level block-coded 8-PSK modulation [1], codewords of three binary linear codes C_i of length n, dimension k_i, and minimum distance d_i are used to select n trios of label bits b_i for $1 \le i \le 3$. The set of resulting length-n sequences of 8-PSK signals is said to be a *three-level block 8-PSK modulation code* $\boldsymbol{\Lambda}$ of length n and bandwidth efficiency $\eta = (k_1 + k_2 + k_3)/n$ (b/symbol).

Fig. 1 shows the block diagram of an encoder structure of three-level block 8-PSK modulation codes for UEP. The selection of codes at each level is done as follows. A conventional (n, k_1, d_1) linear block code C_1 is used in the first encoding level (to select label bit b_1) to ensure that the *minimum symbol distance*

$$\delta_H \triangleq \min\{d_1, d_2, d_3\}$$

will occur at the second or third encoding levels, so that the *minimum product distance*

$$\Delta_p^2 \triangleq (\delta_\ell)^{d_\ell}, \ell = \min\{i\colon \delta_H = d_i\}$$

will be greater than or equal to 2, where $\delta_1 = 0.586$, $\delta_2 = 2$, and $\delta_3 = 4$; and δ_i denotes the minimum intraset squared Euclidean distance at level i [6], [8]. Let UEP(n,k) denote a binary LUEP code of length n and dimension k. In addition, a binary (n, k_2, d_2) linear code or a UEP(n,k_2) code C_2 is used in the second encoding level. The third encoding level employs a binary (n, k_3, d_3) linear code or a UEP(n,k_3) code.

A. A Length-8 Three-Level Block 8-PSK Modulation Code for UEP

Let C_1, C_2, and C_3 be (8, 4, 4), (8, 7, 2), and (8, 7, 2) linear codes, respectively. The Imai–Hirakawa three-level construction results in a block modulation code $\boldsymbol{\Lambda_1}$ of length 8, $\eta = 2.25$ b/symbol, minimum symbol distance $\delta_H = 2$, and minimum product distance $\Delta_p^2 = 4$. Computer simulations of this code were reported in [5].

Now let C_3 be a binary optimal LUEP code, UEP(8,5), from [7] with separation vector $\overline{\boldsymbol{s}} = (3,2)$ for the message space $\{0,1\}^4 \times \{0,1\}$. The corresponding block 8-PSK modulation code will be denoted $\boldsymbol{\Lambda_2}$. A generator matrix for C_3 is given by

$$G_3 = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

and it can be readily verified that the minimum Hamming weight of C_3 is 2. Any two codewords of C_3, in correspondence to information vectors whose first four information bits differ (equivalently, any linear combination of the top four rows of G_3), are at a Hamming distance of at least 3.

Modulation code $\boldsymbol{\Lambda_2}$ has length 8, bandwidth efficiency $\eta = 2$ b/symbol, minimum symbol distance $\delta_H = 2$, and minimum product distance $\Delta_p^2 = 4$. Note that $\boldsymbol{\Lambda_2}$ has the same distance parameters as $\boldsymbol{\Lambda_1}$. Moreover, 25% of the information bits (the four MSB encoded by the UEP(8,5) code) have corresponding symbol and product distances equal to 3 and 64, respectively. That is, a subset of the signal sequences encoded by $\boldsymbol{\Lambda_2}$, those corresponding to the MSB encoded by the LUEP code, have larger symbol and product distances than those of the conventional BCM $\boldsymbol{\Lambda_1}$. Consequently, a better performance will be achieved for the MSB. Also note that, since the other distance parameters remain unchanged with respect to the conventional BCM, the average performance is improved.

Three-level code $\boldsymbol{\Lambda_2}$ is compared with a three-level code for UEP, that uses conventional linear codes, of about the same average error performance: Time-sharing of (7, 4, 3) and (2, 1, 2) linear codes, which produces a UEP(9,5) code, denoted $|(7,4,3)|(2,1,2)|$, is used as C_3. C_1 and C_2 are (9, 4, 4) and (9, 8, 2) linear codes, respectively. These component codes produce a block modulation code $\boldsymbol{\Lambda_3}$ of length 9 and $\eta = 1.89$

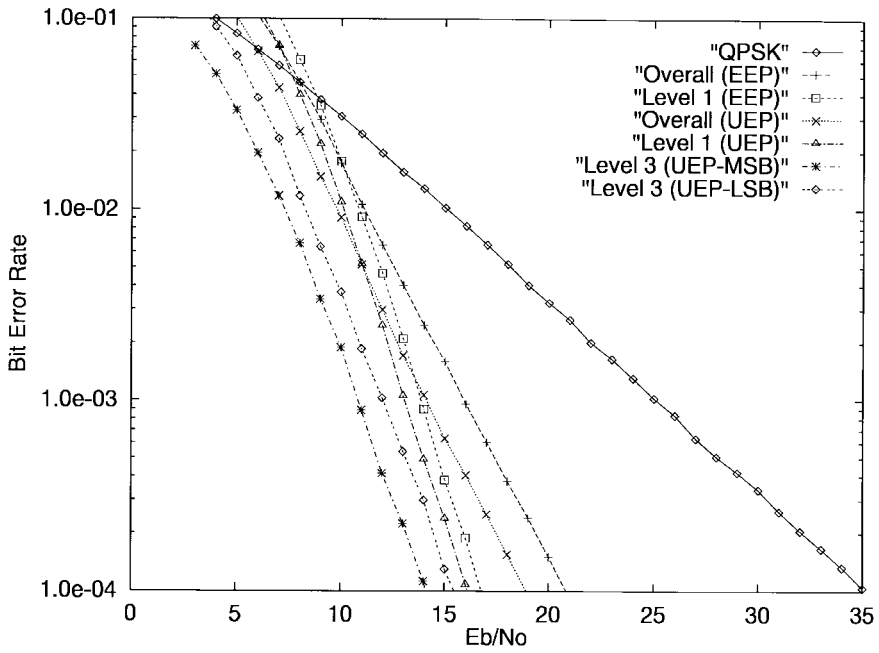

Fig. 2. Simulation results for $\boldsymbol{\Lambda_1}$ equal error protection (EEP) and $\boldsymbol{\Lambda_2}$ (UEP).

b/s/Hz, with the same minimum symbol and product distances as $\boldsymbol{\Lambda_2}$, but reduced bandwidth efficiency.

Computer simulation results for $\boldsymbol{\Lambda_1}$ and $\boldsymbol{\Lambda_2}$ are shown in Fig. 2. The results were obtained using a naturally labeled uniform 8-PSK signal set, and single-stage maximum likelihood soft-decision decoding using squared Euclidean distance as metric and the Viterbi algorithm. The channel model used in the simulations is as follows. The received complex symbols are $r_i = \rho_i s_i + n_i, 1 \le i \le n$, where ρ_i are independently identically distributed (i.i.d.) Rayleigh random numbers with $E\{\rho_i^2\} = 1$, and n_i is a two-dimensional Gaussian noise sample with power spectral density N_0. Note that this model implicitly assumes perfect interleaving and coherent detection.

The increase in coding gain for the MSB is impressive. At a BER of 10^{-3}, the coding gain in the third stage is at least 13 dB for $\boldsymbol{\Lambda_2}$, much more than the about 8.5 dB for $\boldsymbol{\Lambda_1}$, with respect to uncoded QPSK modulation. The largest coding gain for $\boldsymbol{\Lambda_1}$, at the BER of 10^{-3}, occurs in the four bits encoded by the first stage. This coding gain is about 11.5 dB compared to 14 dB in the third encoding stage (the four MSB encoded by the UEP(8,5) code) for $\boldsymbol{\Lambda_2}$. In addition, the average coding gain of $\boldsymbol{\Lambda_2}$ at the BER of 10^{-3} is about 2 dB larger than for the conventional BCM.

B. A Length-32 Three-Level Block 8-PSK Modulation Code for UEP

Let C_1, C_2, and C_3 be (32, 16, 8), (32, 26, 4), and (32, 26, 4) Reed–Muller (RM) codes, respectively. The three-level construction yields a block 8-PSK modulation code $\boldsymbol{\Lambda_4}$ of length 32, $\eta = 2.125$ bits/symbol, minimum symbol distance $\delta_H = 4$, and minimum product distance $\Delta_p^2 = 16$. This code was presented in [6] and shown to achieve high performance over a Rayleigh-fading channel.

Now let C_1 and C_2 be as above and let C_3 be a UEP (32,22) code with separation vector $\overline{\boldsymbol{s}} = (6, 4)$ for the message space $\{0,1\}^7 \times \{0,1\}^{15}$. This UEP(32,22) code is obtained from an extended (16, 7, 6) Bose–Chaudhuri–Hocquenghen (BCH) code and a (16, 15, 2) RM code using the $|u|u+v|$ construction [7]. The result is a block 8-PSK modulation code $\boldsymbol{\Lambda_5}$ of length 32, $\eta = 2$ bits/sec/Hz, minimum symbol distance $\delta_H = 4$, and minimum product distance $\Delta_p^2 = 16$, that provides coded signal sequences in correspondence to 10.94% of the information (the seven MSB encoded by C_3) with symbol and product distances of 6 and 4096, respectively. This is to say that enhanced UEP capabilities and *better average error performance* with respect to $\boldsymbol{\Lambda_4}$ are achieved, with the *same bandwidth efficiency* as uncoded QPSK modulation. Indeed, the computer simulation results reported in [10] confirm the superiority of $\boldsymbol{\Lambda_5}$ over $\boldsymbol{\Lambda_4}$.

$\boldsymbol{\Lambda_5}$ is now compared with BCM for UEP using conventional linear block codes. To obtain the same error protection capabilities as the UEP code C_3 used by $\boldsymbol{\Lambda_4}$, the time-sharing of (16, 7, 6) and (21, 15, 4) linear codes, which results in a UEP(37,22) code, denoted $|(16,7,6)|(21,15,4)|$, may be used. We note that these codes are selected such that the error protection capabilities and message space are the same as those of UEP(32,22). Let C_1 be a (37, 22, 8) linear code, from the table of best linear codes [11]. Let C_2 be a (37, 30, 4) linear code (a shortened Hamming code), and let C_3 be the aforementioned $|(16,7,6)|(21,15,4)|$ code. By the three-level construction, a modulation code $\boldsymbol{\Lambda_6}$ of length 37 and rate $\eta = 2$ b/s/Hz is obtained, i.e., with the same bandwidth efficiency and error protection capabilities as $\boldsymbol{\Lambda_5}$. However, $\boldsymbol{\Lambda_6}$ has larger decoding complexity and a longer delay (a block length of 37) than $\boldsymbol{\Lambda_5}$.

IV. Concluding Remarks

The use of binary LUEP codes as component codes in three-level coded 8-PSK modulation has the potential to achieve

TABLE I
BLOCK 8-PSK MODULATION CODES OF LENGTHS 8 AND 32

Modulation Code	Encoding Level i	Component Code C_i	Rate R	Information bits	Information %	Symbol $\delta_{H,i}$	Product $\Delta^2_{p,i}$
	1	$(8,4,4)$		4	22.22	4	0.118
$\mathbf{\Lambda_1}$	2	$(8,7,2)$	2.25	7	38.89	2	4
(EEP)	3	$(8,7,2)$		7	38.89	2	16
	1	$(8,4,4)$		4	25.00	4	0.118
$\mathbf{\Lambda_2}$	2	$(8,7,2)$	2.00	7	43.75	2	4
(UEP)	3	UEP$(8,5)$		4	25.00	3	64
				1	6.25	2	16
	1	$(9,4,4)$		4	23.53	4	0.118
$\mathbf{\Lambda_3}$	2	$(9,8,2)$	1.89	8	47.06	2	4
(T.S.)	3	$\|(7,4,3)\|(2,1,2)\|$		4	23.53	3	64
				1	5.88	2	16
	1	$(32,16,8)$		16	23.53	8	0.014
$\mathbf{\Lambda_4}$	2	$(32,26,4)$	2.12	26	38.23	4	16
(EEP)	3	$(32,26,4)$		26	38.23	4	256
	1	$(32,16,8)$		16	25.00	8	0.014
$\mathbf{\Lambda_5}$	2	$(32,26,4)$	2.00	26	40.62	4	16
(UEP)	3	UEP$(32,22)$		7	10.94	6	4096
				15	23.44	4	256
	1	$(37,22,8)$		22	29.73	8	0.014
$\mathbf{\Lambda_6}$	2	$(37,30,4)$	2.00	30	40.54	4	16
(T.S.)	3	$\|(16,7,6)\|(21,15,4)\|$		7	9.46	6	4096
				15	20.27	4	256

enhanced UEP capabilities and increased error performance, both on the average and for the MSB, compared to using binary linear block codes. As a reference, in Table I the examples of three-level 8-PSK modulation codes presented in Sections III-A and III-B are summarized. It should be noted that BCM schemes using linear block codes do have UEP capabilities, because of the different Hamming distances of the component codes at each level. However, the examples and computer simulations presented in this work suggest that the performance can be improved by using UEP codes in BCM over Rayleigh-fading channels.

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