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Spatial and Temporal Factors in Estimating the Potential of Ride-sharing for Demand Reduction

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approach and not a quantitative analysis.

Gensch (1981) proposed a method to predict the switchable segment by using the basic cross-sectional survey data sets collected to calibrate a logit model. That multinomial logit model estimated the probability of choosing each of the alternatives (driving alone, carpooling, and public transit) by individuals. The data used in his analysis were collected from users of Santa Monica Freeway in Los Angeles, and he concluded that the empirical experiment tended to support this proposed approach. He concluded that “switchables are found often in homes where the number of licensed drivers is low, there is a male head of household, and the individual is a blue-collar worker (non-professional / managerial),” and emphasized that the purpose of his empirical analysis was “to provide some empirical support for two concepts. First, the difference in logit probabilities (i.e., the difference in the deterministic utility component of the probability value) is related to the individual’s propensity to try his second mode choice. .... In term of *actual behavior*, the empirical evidence supports the relationship for both bus and car-pool. The second concept is that there are demographic variables that can be related to groups with different propensities of switching modes.” The author suggested spending most of the promotional budget on those segments (or groups of commuters) that have been identified as most “switchable”.

2.6 Car-pool Promotion Programs and Concepts

This subsection reviews literature about programs aimed at promoting carpooling. The discussion is partitioned into six subjects:

- part-time carpooling

- flexible car-pool matching
- employer-based car-pool programs
- high-occupancy vehicle (HOV) Lanes
- preferential parking policies
- intelligent transportation system technologies for carpooling

2.6.1 Part-Time Carpooling

A common objection by solo drivers against carpooling is lack of flexibility. Due to this inflexibility, some researchers proposed strategies to remedy it. One approach is "part-time carpooling" (Glazer, Koval and Gerard, 1986). Their goal is two-person-two-days-per-week carpooling. Instead of inflexible every-day commitments, all participants were asked to commit to a two-person car-pool for only two days a week for three months. The authors concluded that "Because of the hard-to-please nature of the commuters in this target market, it appears that personalized matching attention is important to the success of a part-time carpooling promotional effort." Despite a high attrition rate (75 percent dropout in eight months), they still believed that "this demonstration project indicates that part-time carpooling is a promising technique for reaching beyond the commuter market traditionally served by conventional ridesharing programs."

2.6.2 Flexible Car-pool Matching

Another effort is "flexible car-pool matching" (Michael R. Ringrose, 1992). Flexible car-pool matching is a strategy for helping commuters to form car-pools with each other. The difference is that "the arrangements are made on a trip-by-trip basis, and thus do not require any long-term commitments." So this strategy combines the

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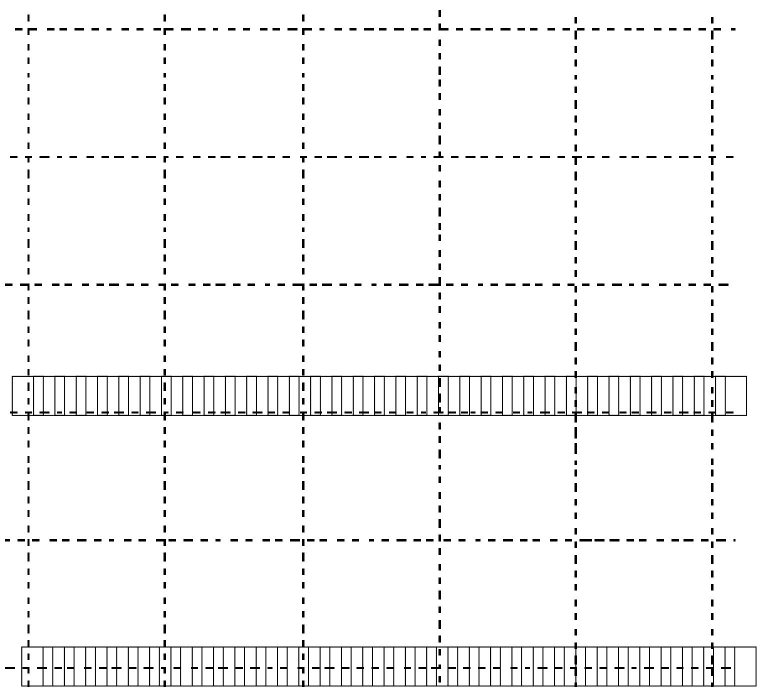

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IV. Formulation

Given a finite square-region-zone structure with common trip production and attraction characteristics for each zone and according to the discrete approach described in section (3.3), we use the following entropy optimization (gravity) model for trip generation and distribution.

$$\min \quad \sum_{i_1=-k}^{k} \sum_{i_2=-k}^{k} \sum_{j_1=-k}^{k} \sum_{j_2=-k}^{k} x_{(i_1,i_2)(j_1,j_2)} \ln x_{(i_1,i_2)(j_1,j_2)}$$

$$\text{s.t.} \quad \sum_{j_1=-k}^{k} \sum_{j_2=-k}^{k} x_{(i_1,i_2)(j_1,j_2)} = O_{(i_1,i_2)} \,,$$

$$\sum_{i_1=-k}^{k} \sum_{i_2=-k}^{k} x_{(i_1,i_2)(j_1,j_2)} = D_{(j_1,j_2)} \,,$$

$$\sum_{i_1=-k}^{k} \sum_{i_2=-k}^{k} \sum_{j_1=-k}^{k} \sum_{j_2=-k}^{k} c_{(i_1,i_2)(j_1,j_2)} x_{(i_1,i_2)(j_1,j_2)} = C \,,$$

$$x_{(i_1,i_2)(j_1,j_2)} \geq 0 \,,$$

where

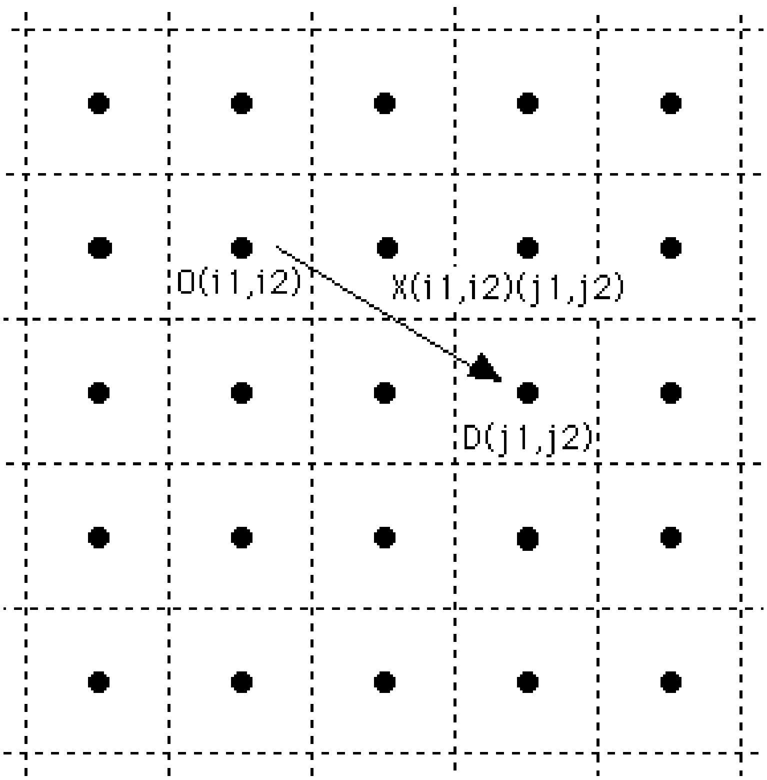
$x_{(i_1,i_2)(j_1,j_2)}$ is the number of trips from origin (i_1, i_2) to destination (j_1, j_2),

$O_{(i_1,i_2)}$ is the total production of origin (i_1, i_2),

$D_{(j_1,j_2)}$ is the total attraction of destination (j_1, j_2),

$c_{(i_1,i_2)(j_1,j_2)}$ is the travel distance between origin (i_1, i_2) to destination (j_1, j_2),

C is the total travel distance of all the commuters.


O(i1,i2)
X(i1,i2)(j1,j2)
D(j1,j2)

In fact, the formulation above is a special case of the following standard form of entropy optimization model:

Program P:

$$\min \quad \sum_{j=1}^{n} y_j \ln y_j$$

$$\text{s.t.} \quad \sum_{j=1}^{n} a_{ij} y_j = b_i, \qquad i = 1, 2, \ldots, m,$$

$$y_j \geq 0, \qquad j = 1, 2, \ldots, n.$$

We will solve this constrained entropy optimization problem by first obtaining its unconstrained dual, then solving the unconstrained dual and finally obtaining the optimal solution of this constrained problem through an effortless dual-to-primal conversion formula. To derive the dual, we utilized a simple inequality:

$$\ln z \leq z - 1, \qquad \text{for } z > 0. \tag{1}$$

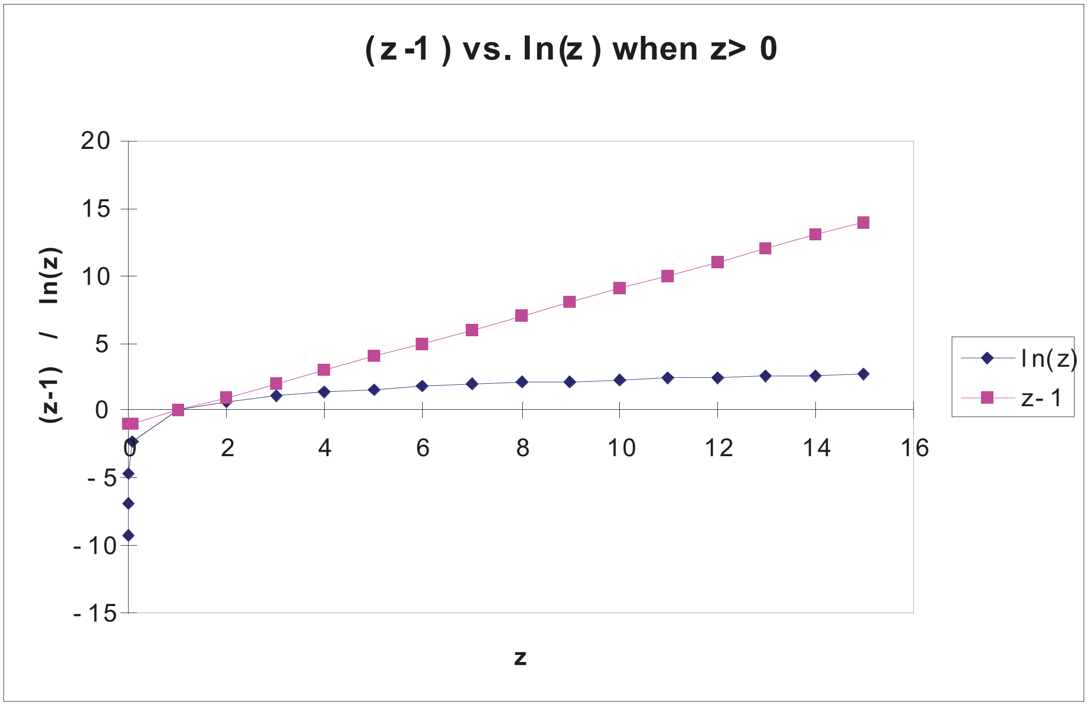


Fig. 3: the relationship between (z-1) and ln(z) when z > 0.

This inequality can be easily verified by the graph above, and notice that this inequality becomes an equality if and only if z = 1.

Now, for any $w_i \in R$ (i = 1, ..., m), and $y_j > 0$ (j = 1, ..., n), we define:

$$z_j = \frac{\exp(\sum_{i=1}^{m} a_{ij} w_i - 1)}{y_j}, \qquad j = 1, ..., m.$$

Then, by using the simple inequality, i.e. Inequality (1), we have:

$$\ln z_j = (\sum_{i=1}^{m} a_{ij} w_i - 1) - \ln y_j \le z_j - 1 = \frac{\exp(\sum_{i=1}^{m} a_{ij} w_i - 1)}{y_j} - 1.$$

Consequently, for j = 1, …, n,

$$y_j\,(\overset{m}{\underset{i=1}{\bullet}}\, a_{ij}w_i) - \; y_j \ln y_j \;\bullet\; \exp(\overset{m}{\underset{i=1}{\bullet}}\, a_{ij}w_i \bullet 1)\ .$$

Summing up all n equations above gives

$$\overset{n}{\underset{j=1}{\bullet}}\; y_j\,(\overset{m}{\underset{i=1}{\bullet}}\, a_{ij}w_i) - \overset{n}{\underset{j=1}{\bullet}}\; \exp(\overset{m}{\underset{i=1}{\bullet}}\, a_{ij}w_i \bullet 1) \;\bullet\; \overset{n}{\underset{j=1}{\bullet}}\; y_j \ln y_j\ . \qquad (2)$$

If y_j's satisfy $\overset{n}{\underset{j=1}{\bullet}}\, a_{ij}y_j = b_i$, then

$$\overset{n}{\underset{j=1}{\bullet}} y_j(\overset{m}{\underset{i=1}{\bullet}}\, a_{ij}w_i) = \overset{m}{\underset{i=1}{\bullet}}(\overset{n}{\underset{j=1}{\bullet}}\, a_{ij}y_j)w_i = \overset{m}{\underset{i=1}{\bullet}}\, b_i w_i\ .$$

By substituting $\overset{n}{\underset{j=1}{\bullet}} y_j(\overset{m}{\underset{i=1}{\bullet}}\, a_{ij}w_i)$ with $\overset{m}{\underset{i=1}{\bullet}}\, b_i w_i$ in Inequality (2), we obtain

$$\overset{m}{\underset{i=1}{\bullet}}\, b_i w_i - \overset{n}{\underset{j=1}{\bullet}}\; \exp(\overset{m}{\underset{i=1}{\bullet}}\, a_{ij}w_i \bullet 1) \;\bullet\; \overset{n}{\underset{j=1}{\bullet}}\; y_j \ln y_j\ .$$

We are now ready to define the constrained dual program:

Max$\{\, d1(w) = \overset{m}{\underset{i=1}{\bullet}}\, b_i w_i - \overset{n}{\underset{j=1}{\bullet}}\; \exp(\overset{m}{\underset{i=1}{\bullet}}\, a_{ij}w_i \bullet 1)\}$.

It is easy to verify that $\underline{w}$ is an optimal solution to the dual program and $\underline{y}$ is an optimal solution to the entropy optimization problem, i.e., if $z_j \equiv \dfrac{\exp(\overset{m}{\underset{i=1}{\bullet}}\, a_{ij}w_i \bullet 1)}{y_j} = 1$. This is equivalent to:

$y_j = \exp(\overset{m}{\underset{i=1}{\bullet}}\, a_{ij}w_i \bullet 1)$, for all j.

Rewrite the dual program as:

Program D:

$$\min_{w \in R^m} \{d(w) \equiv \sum_{i=1}^{n} \exp(\sum_{i=1}^{m} a_{ij} w_i - 1) - \sum_{i=1}^{m} b_i w_i\}$$

Then, under some regularity conditions, Program D has an optimal solution $\underline{w}^*$.

Moreover, $\underline{y}^*$ defined by

$$y^*_j = \exp(\sum_{i=1}^{m} a_{ij} w_i^* - 1), \qquad j = 1, 2, ..., n,$$

is an optimal solution to Program P. Therefore, to solve the constrained Program P, one can instead solve the uncontrained Program D, which is much easier.

With the aid of general theory, we know that the solution to our trip distribution problem has the following form:

$$x_{(i_1,i_2)(j_1,j_2)} = \exp(u_{(i_1,i_2)} + v_{(j_1,j_2)} + c_{(i_1,i_2)(j_1,j_2)} * w_c - 1),$$

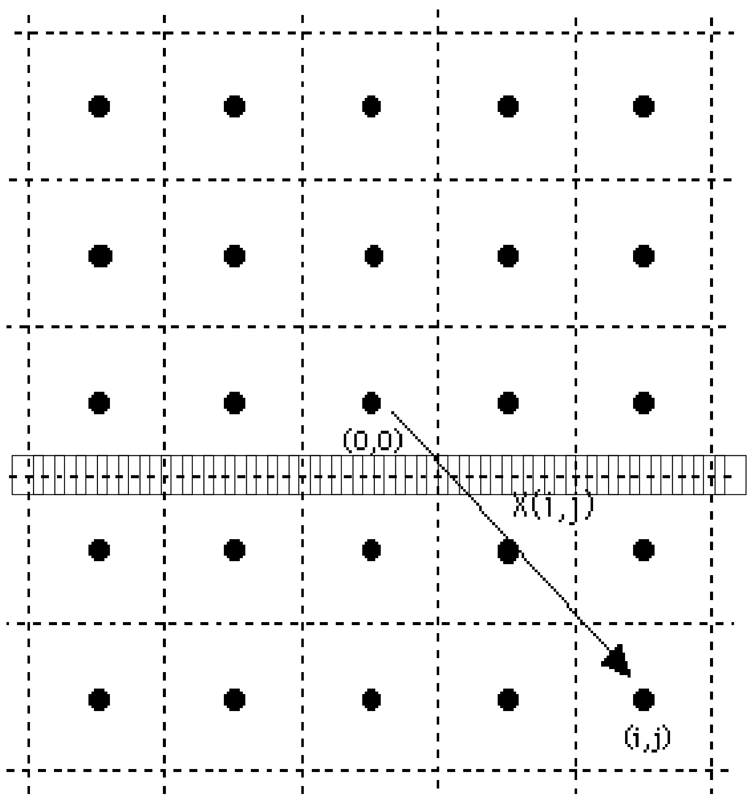
where

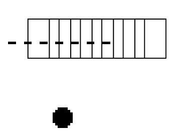
$u_{(i_1,i_2)}$ is the dual variable associated with the constrain corresponding trip production,

$v_{(j_1,j_2)}$ is the dual variable associated with the constrain corresponding trip constraint,

w_c is the dual variable associated with the constraint on total travel distance.

The conjecture is that the trip number between any two zones, when the size of the square region goes to infinity, will converge to a certain number. However, proving the conjecture using the above model seems quite complicated. Therefore, we simplify


(0,0)
X(i,j)
(i,j)



Also, the average commute distance for the workers in a particular zone should be identical to the average commute distance for all the workers. We also add a constraint on "transportation cost" based on the assumption on average commuting distance. Now our problem formulation becomes

$$\min \quad \sum_{i=-k}^{k} \sum_{j=-k}^{k} x_{(i,j)} \ln x_{(i,j)}$$

$$\text{s.t.} \quad \sum_{i=-k}^{k} \sum_{j=-k}^{k} x_{(i,j)} = O_{(0,0)} \,,$$

$$\sum_{i=-k}^{k} \sum_{j=-k}^{k} c_{(i,j)} x_{(i,j)} = d * O_{(0,0)} \,,$$

where

$x_{(i,j)}$ is the number of trip from origin (0,0) to destination (i,j),

$c_{(i,j)}$ is the travel time between origin (0,0) and destination (i,j),

$O_{(0,0)}$ is the trip production of origin (0,0),

d denotes average commuting distance.

With the aid of the general theory discussed earlier, we obtain:

$$x_{(i,j)} = \frac{\exp^{w_c c_{(i,j)}}}{\sum_{all(i,j)} \exp^{w_c c_{(i,j)}}} * O_{(0,0)} \,,$$

$$d = \frac{\sum_{all(i,j)} c_{(i,j)} x_{(i,j)}}{\sum_{all(i,j)} x_{(i,j)}} \,,$$

where

$x_{(i,j)}$ is the number of trip from origin (0,0) to destination (i,j),

$c_{(i,j)}$ is the travel distance between origin (0,0) and destination (i,j),

$O_{(0,0)}$ [illegible]

w_c [illegible]

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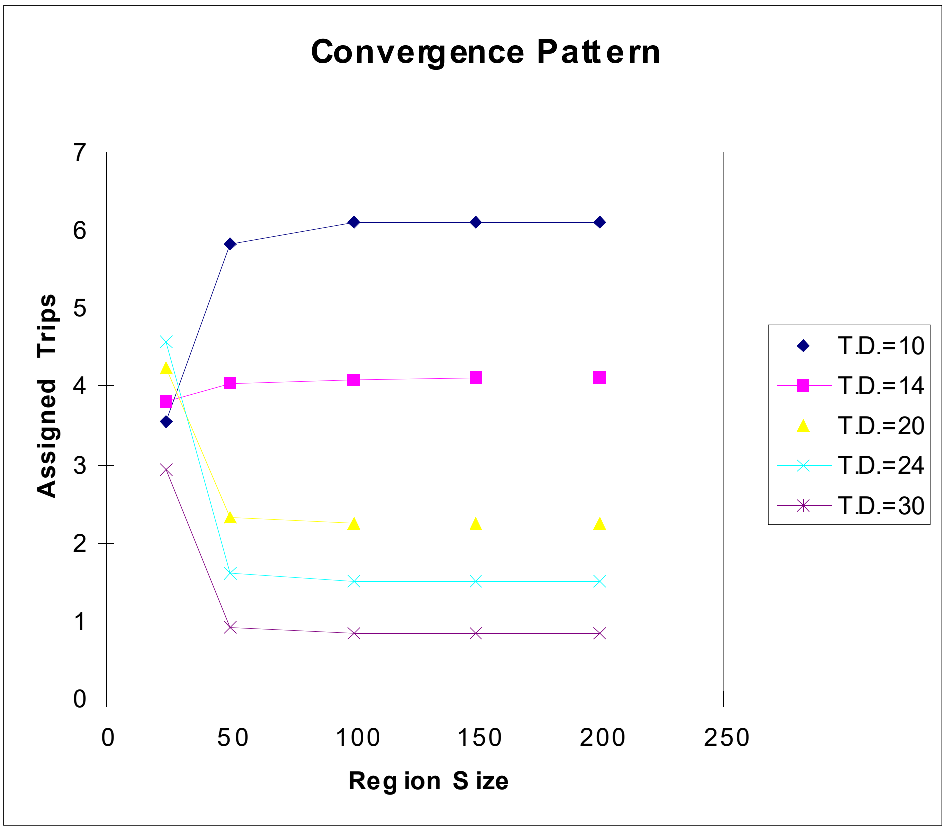
Convergence Pattern
Assigned Trips
Region Size
0
1
2
3
4
5
6
7
0
50
100
150
200
250
T.D.=10
T.D.=14
T.D.=20
T.D.=24
T.D.=30

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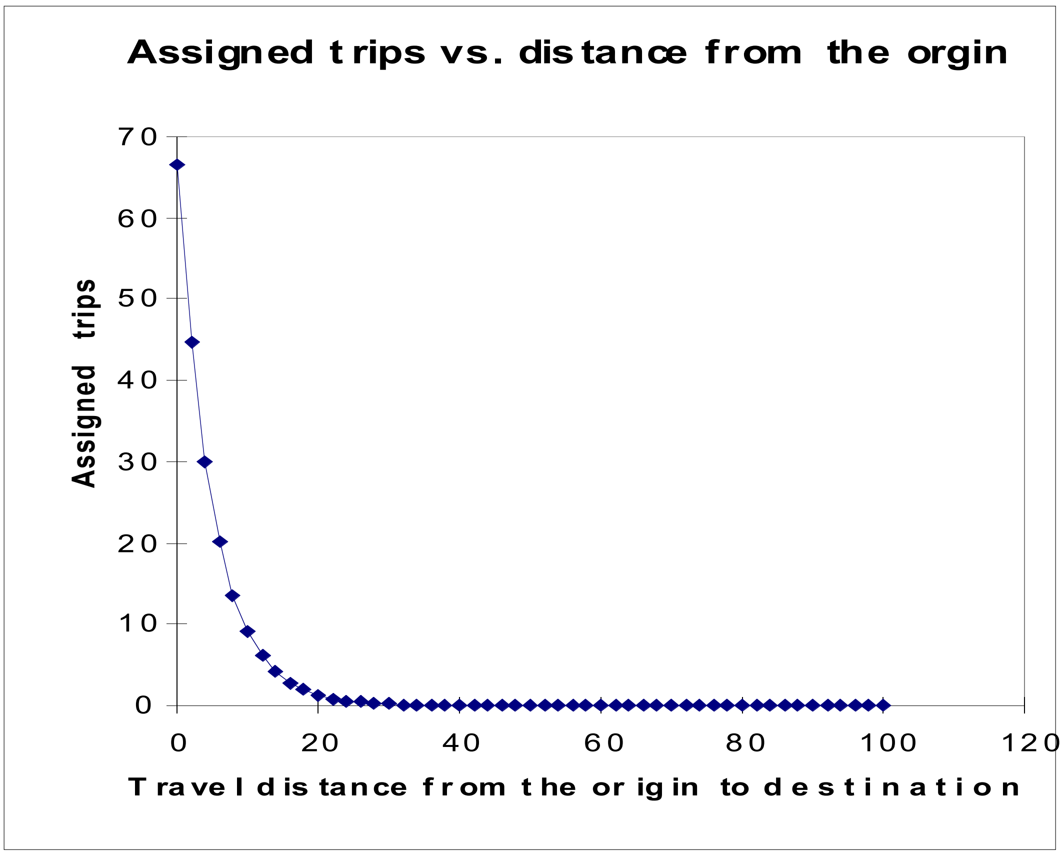
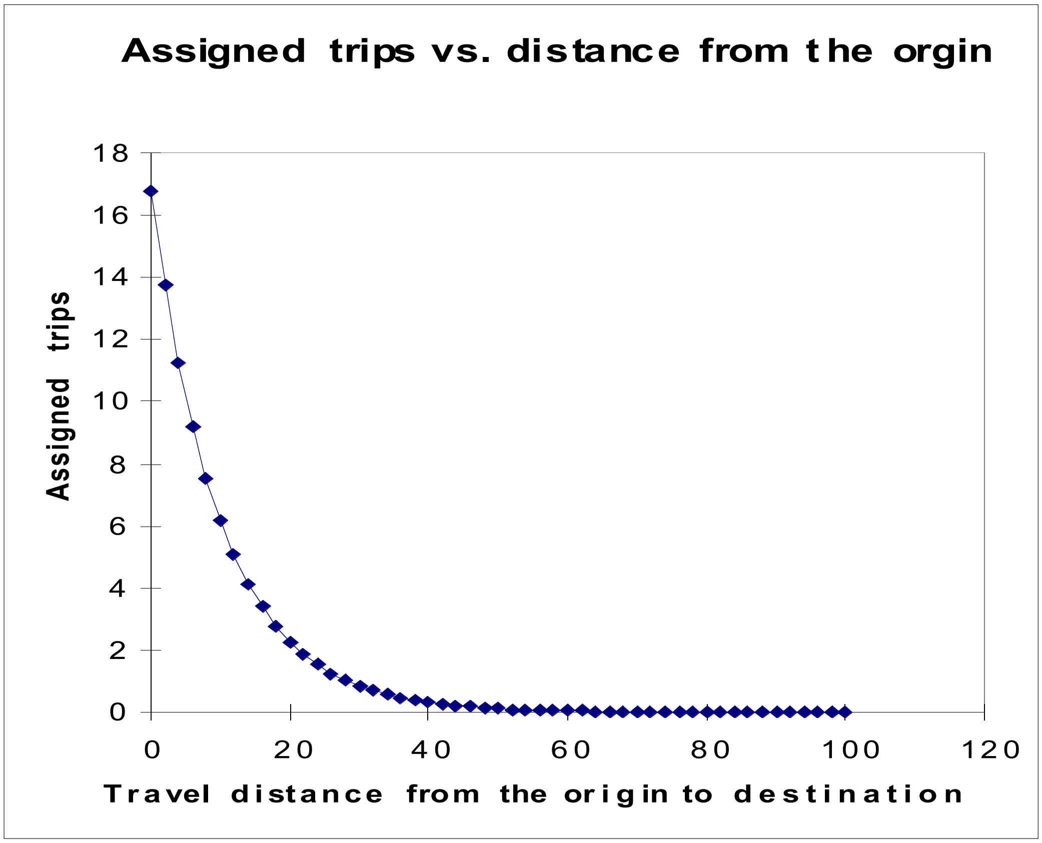
Density = 581 Jobs/Sq. Mi. (2324 Jobs/grid)

	Avg. Commuting Distance							
T.D.	**10**	**12**	**14**	**16**	**18**	**20**	**22**	**24**
0	58.64	40.88	30.09	23.06	18.24	14.78	12.22	10.27
2	39.36	29.31	22.62	17.97	14.60	12.10	10.19	8.69
4	26.42	21.02	17.01	14.00	11.70	9.91	8.49	7.36
6	17.74	15.07	12.78	10.90	9.37	8.11	7.08	6.23
8	11.90	10.80	9.61	8.49	7.50	6.64	5.90	5.27
10	7.99	7.75	7.22	6.62	6.01	5.44	4.92	4.46
12	5.36	5.55	5.43	5.15	4.81	4.45	4.11	3.78
14	3.60	3.98	4.08	4.01	3.85	3.65	3.42	3.20
16	2.42	2.86	3.07	3.13	3.09	2.99	2.85	2.71
18	1.62	2.05	2.31	2.44	2.47	2.44	2.38	2.29
20	1.09	1.47	1.73	1.90	1.98	2.00	1.98	1.94
22	0.73	1.05	1.30	1.48	1.58	1.64	1.65	1.64
24	0.49	0.75	0.98	1.15	1.27	1.34	1.38	1.39
26	0.33	0.54	0.74	0.90	1.02	1.10	1.15	1.18
28	0.22	0.39	0.55	0.70	0.81	0.90	0.96	1.00
30	0.15	0.28	0.42	0.54	0.65	0.74	0.80	0.84

Density = 660 Jobs/Sq. Mi. (2640 Jobs/grid)

	Avg. Commuting Distance							
T.D.	**10**	**12**	**14**	**16**	**18**	**20**	**22**	**24**
0	66.6	46.4	34.2	26.2	20.7	16.8	13.9	11.7
2	44.7	33.3	25.7	20.4	16.6	13.7	11.6	9.87
4	30	23.9	19.3	15.9	13.3	11.3	9.65	8.36
6	20.1	17.1	14.5	12.4	10.6	9.22	8.04	7.08
8	13.5	12.3	10.9	9.65	8.52	7.55	6.71	5.99
10	9.08	8.8	8.21	7.51	6.82	6.18	5.59	5.07
12	6.09	6.31	6.17	5.85	5.47	5.06	4.66	4.29
14	4.09	4.52	4.64	4.56	4.38	4.14	3.89	3.63
16	2.75	3.24	3.49	3.55	3.51	3.39	3.24	3.08
18	1.84	2.33	2.62	2.77	2.81	2.78	2.7	2.6
20	1.24	1.67	1.97	2.16	2.25	2.27	2.25	2.2
22	0.83	1.2	1.48	1.68	1.8	1.86	1.88	1.87
24	0.56	0.86	1.11	1.31	1.44	1.52	1.57	1.58
26	0.37	0.61	0.84	1.02	1.15	1.25	1.31	1.34
28	0.25	0.44	0.63	0.79	0.92	1.02	1.09	1.13
30	0.17	0.32	0.47	0.62	0.74	0.84	0.91	0.96


Assigned trips vs. distance from the orgin
Assigned trips
70
60
50
40
30
20
10
0
0
20
40
60
80
100
120
Travel distance from the origin to destination


Assigned trips vs. distance from the orgin
Assigned trips
18
16
14
12
10
8
6
4
2
0
0
20
40
60
80
100
120
Travel distance from the origin to destination

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