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Selfish Wavelength Assignment in Multifiber Optical Networks

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Selfish Wavelength Assignment in Multifiber Optical Networks

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Transparent all-optical networks

- Much more bandwidth than legacy copper wire
- No opto-electronic conversion
  - faster
  - cheaper
- Wavelength Division Multiplexing (WDM)
  - several “channels” per fiber

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  - several “channels” per fiber
- Multi-fiber setting
  - fault-tolerance
  - even more bandwidth

Non-cooperative model

- Large-scale networks: shortage of centralized control
  - provide incentives for users to work for the social good
- Social good: minimize fiber multiplicity
- Charge users according to the maximum fiber multiplicity incurred by their choice of frequency and/or routing

Non-cooperative model

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What will be the impact on social welfare if we allow users to act freely and selfishly?

Problem formulation

Def. PATH MULTICOLORING problems:

- input: graph $G(V, E)$, path set P, # colors w
- solution: a coloring $c : P \to W$, $W = \{\alpha_1, \ldots, \alpha_w\}$
- goals:
  - minimize the sum of maximum color multiplicities $\sum_{e \in E} \max_{\alpha \in W} \mu(e, \alpha)$ [NPZ01], or
  - minimize the maximum color multiplicity $\mu_{\max} \triangleq \max_{e \in E} \max_{\alpha \in W} \mu(e, \alpha)$ [AZ04]

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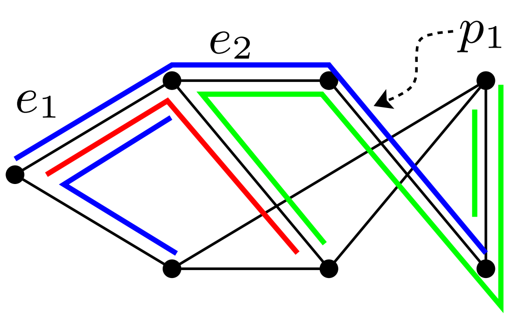
$$\mu_{\max} \triangleq \max_{e \in E} \max_{\alpha \in W} \mu(e, \alpha)$$



$$L(e_2) = 2 \qquad \mu_{e_1} = 2$$

$$L = 3 \qquad \mu_{\max} = 2$$

$$\mu(e_2, \text{green}) = 1 \qquad \mu(p_1, \text{blue}) = 2$$

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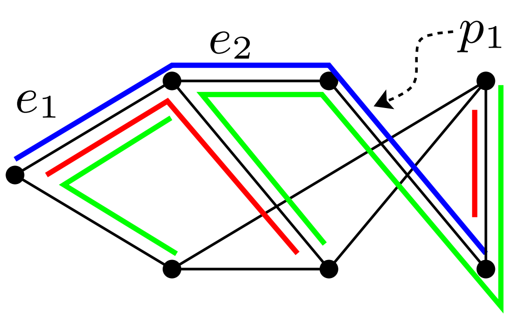

$$\mu_{\mathrm{OPT}} = 1$$
$$\mu_{\mathrm{OPT}} \geq \left\lceil \frac{L}{w} \right\rceil$$

Game-theoretic formulation

- Def. Given a graph G, path set P and w, define the game $\langle G, P, w\rangle$:
  - players: $p_1, \ldots, p_{|P|} \in P$
  - strategies: each p_i picks a color $c_i \in W$
  - strategy profile: a vector $\vec{c} = (c_1, \ldots, c_{|P|})$
  - disutility functions: for $p_i \in P$, $f_i(\vec{c}) = \mu(p_i, c_i)$
  - social cost:

$$\mathrm{sc}(\vec{c}) \triangleq \mu_{\max} = \max_{e \in E} \max_{\alpha \in W} \mu(e, \alpha)$$

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- **Def.** S-PMC: the class of all $\langle G, P, w \rangle$ games

Nash Equilibria

- Def. A strategy profile is a Nash Equilibrium (NE) if no player can reduce her disutility by changing strategy unilaterally:

$$\forall p_i \in P, \forall c_i' \in W : \; f_i(\vec{c}; c_i) \leq f_i(\vec{c}; c_i')$$

- Def. ε-approximate Nash Equilibrium: no player can reduce her disutility by more than a factor of $1 - \varepsilon$
- Def. We denote the social cost of the worst-case NE by $\hat{\mu}$:

$$\hat{\mu} = \max_{\vec{c} \text{ is NE}} \mathrm{sc}(\vec{c})$$

Efficiency of Nash Equilibria

- Def. The price of anarchy (PoA) of an S-PMC game:

$$\text{PoA} = \frac{\hat{\mu}}{\mu_{\text{OPT}}}$$

- Def. The price of stability (PoS) of an S-PMC game:

$$\text{PoS} = \frac{\min_{\vec{c} \text{ is NE}} \text{sc}(\vec{c})}{\mu_{\text{OPT}}}$$

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- Rate of convergence to some NE?
  - by repeatedly changing some player's strategy to improve her disutility (Nash dynamics)

Results in this work

- Any Nash dynamics converges in at most $4^{|P|}$ steps
- Efficient computation of NE:
  - optimal NE for S-PMC(ROOTED-TREE)
  - $\frac{1}{2}$-approximate NE for S-PMC(STAR)
- Upper and lower bounds for the PoA:
  - # colors
  - minimum length of any path that contributes to the cost of some worst-case NE
  - matching lower bounds for graphs with $\Delta \geq 3$
  - constant for a large subclass of S-PMC(RING)

Related work

- Price of anarchy [KP99], price of stability [ADK$^+$04]
- Congestion games [MS96, Ros73]
  - player cost: SUM of delays of selected resources
  - large body of work
- Bottleneck network games
  - player cost: MAX of delays along her path
  - players pick among several possible routings [BM06]
  - latency functions on edges [BO06]

Convergence to NE

Thm. Any Nash dynamics converges in at most $4^{|P|}$ steps

- consider the vector

$$(d_L(\vec{c}), d_{L-1}(\vec{c}), \ldots, d_1(\vec{c}))$$

- lexicographic-order argument (attributed to Mehlhorn in [FKK$^+$02])
- $\mathrm{PoS} = 1$

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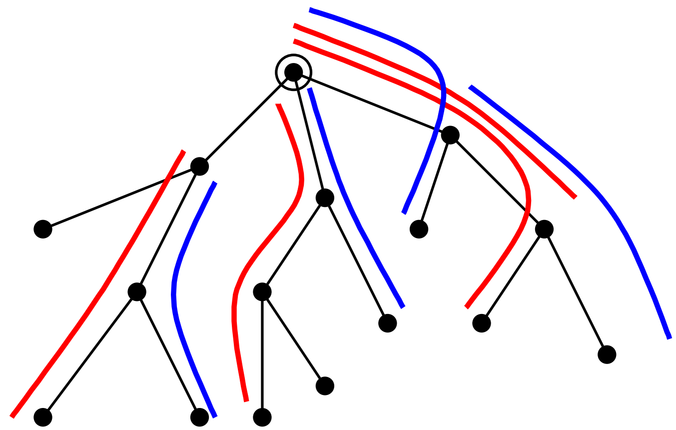
- how many such vectors?

$$\binom{|P| + L - 1}{|P|} \leq 2^{|P|+L-1} < 4^{|P|}$$

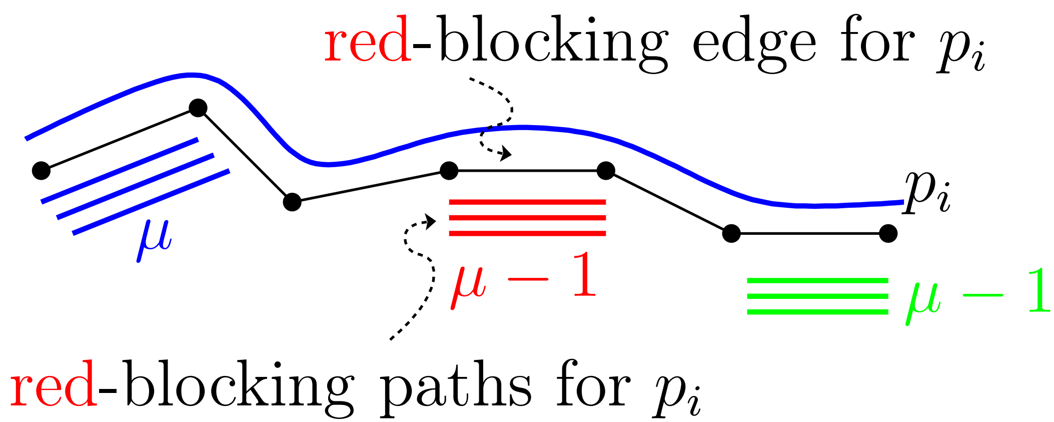
Efficient computation of optimal NE



- $\langle G, P, w \rangle$ is in S-PMC(ROOTED-TREE) if $\exists r$ s.t. each path in P lies entirely on some simple path from r to a leaf
- consider edges in BFS order: color paths with min-multiplicity color in the partial solution

A structural property of NE

- If $\vec{c}$ is a NE, then for any $p_i \in P$ and for any $\alpha \in W$ there is an $e \in p_i$ s.t. $\mu(e, \alpha) \geq f_i(\vec{c}) - 1$



An upper bound on the PoA

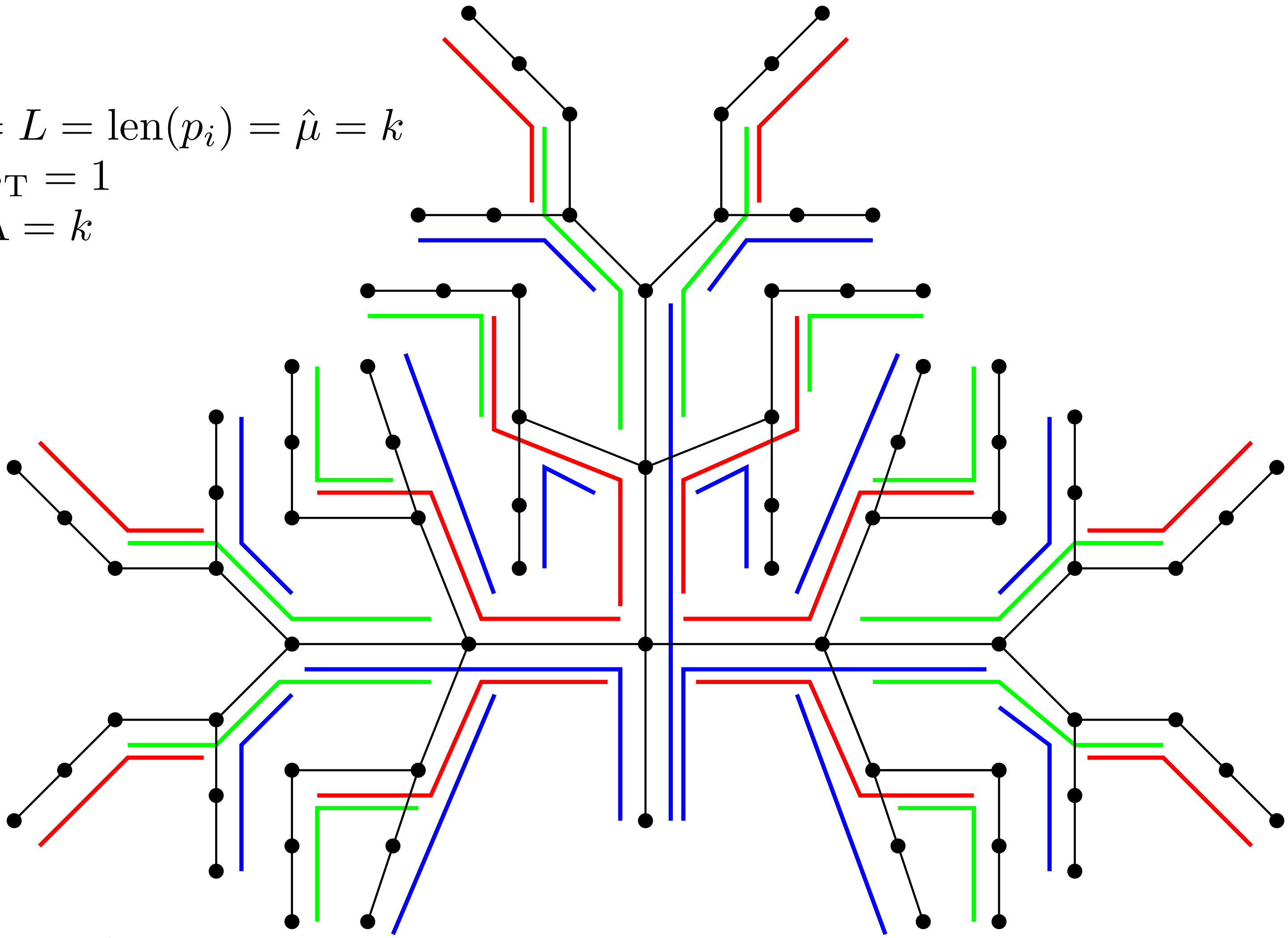
Thm. If $\mathrm{sc}(\vec{c}) = f_i(\vec{c}) = \hat{\mu}$ then $\mathrm{PoA} \leq \mathrm{len}(p_i)$

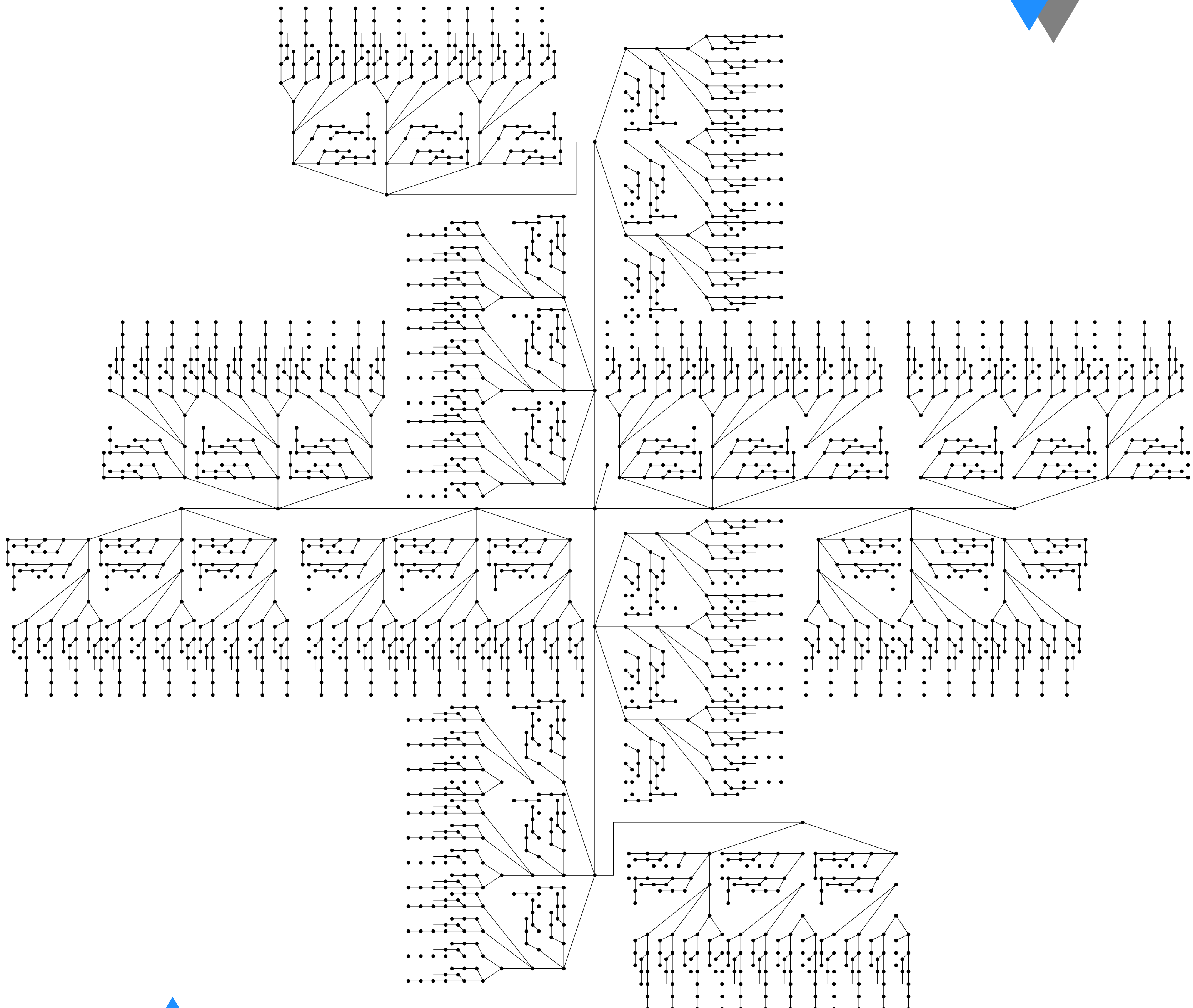
Proof.

- all w colors are blocked along p_i
- some edge of p_i must block at least $\left\lceil \frac{w}{\mathrm{len}(p_i)} \right\rceil$ colors
- max load is $L \geq 1 + \left\lceil \frac{w}{\mathrm{len}(p_i)} \right\rceil (\hat{\mu} - 1)$
- $\mu_{\mathrm{OPT}} \geq \left\lceil \frac{L}{w} \right\rceil$
- $\mathrm{PoA} = \frac{\hat{\mu}}{\mu_{\mathrm{OPT}}} \leq \frac{\hat{\mu}}{\left\lceil \frac{1+\left\lceil \frac{w}{\mathrm{len}(p_i)} \right\rceil (\hat{\mu}-1)}{w} \right\rceil} \leq \mathrm{len}(p_i)$

A matching lower bound

$w = L = \text{len}(p_i) = \hat{\mu} = k$
$\mu_{\text{OPT}} = 1$
$\text{PoA} = k$





What about graphs with degree 2?

Lem. In a NE of an S-PMC(RING) game, $\forall$ edge e and $\forall \alpha_i$ there is an arc s.t.:

- $\forall \alpha_j$ the arc contains an α_j-blocking edge for some path that uses edge e and is colored with α_i, and
- $\forall e'$ in the arc, $\mu(e', \alpha_i) \geq \left\lceil \frac{\mu(e,\alpha_i)}{2} \right\rceil$

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Repeated application of the Lemma in a worst-case NE proves $L \geq \frac{\hat{\mu} w}{4}$, therefore

$$\mu_{\text{OPT}} \geq \frac{L}{w} \geq \frac{\hat{\mu}}{4} \Rightarrow \text{PoA} \leq 4$$

Further work

- Tackle convergence
- Tighten the PoA analysis for rings
- Selfish routing and wavelength assignment
- Pricing mechanisms

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... Thank you!

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