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**Language *counts*: Early language mediates the relationship between
parent education and children's math ability**

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Research Highlights

- Findings from this longitudinal study show that early number word knowledge predicts later math achievement.
- The well-established relationship between parent education and math ability is mediated by early language skills.
- The relationship between ANS acuity and early math ability is mediated by children's number word knowledge.
- We propose a simple but comprehensive model of development with clear implications for practice and intervention.

Abstract

Children's early math skills have been hailed as a powerful predictor of academic success. Disparities in socioeconomic context, however, also have dramatic consequences on children's learning. It is therefore critical to investigate both of these distinct contributors in order to better understand the early foundations of children's academic outcomes. The current study tests an integrated model of children's developing math ability so as to 1) identify the specific skills and abilities most clearly linked to early math achievement and 2) measure the influence of children's socioeconomic context on each of these skills. We first evaluated the early vocabulary, number word knowledge (knower level), and Approximate Number System (ANS) acuity of a diverse group of preschoolers. Then, approximately one year later as they entered Kindergarten, we administered a test of early math achievement. We find that children's early language (general vocabulary and number word knowledge) fully mediates the relationship between parent education and math ability. Additionally, number word knowledge mediates the relationship between ANS acuity and early math. We argue that increased focus on number word knowledge, as well as general vocabulary, may help to minimize disparities in math ability as children enter kindergarten. We also highlight the role of parent education on children's learning and note that this may be an important locus for intervention.

Keywords: parent education, math ability, socioeconomic status, approximate number system (ANS), number words, language development

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Research across various fields and disciplines has recently converged to demonstrate that children's early math knowledge predicts later academic growth throughout school (e.g., Duncan et al., 2007; Geary, Hoard, Nugent, & Bailey, 2013; Jordan, Kaplan, Ramineni, & Locuniak, 2009). Importantly, however, children's socioeconomic status (SES) and family context contributes to math ability (LeFevre et al., 2009; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010) with differences in mathematical knowledge as a function of SES evident by kindergarten entry (Jordan & Levine, 2009; Melhuish et al., 2008). Despite the noted influence of socioeconomic context on math achievement, many studies on the cognitive underpinnings of early math learning report on middle- to upper-middle-class populations, or control for these factors without examining SES as a variable of interest. The current study aims to integrate socioeconomic context into current models of mathematical learning and cognition. Specifically, we test the hypothesis that children's early number word knowledge mediates the relationship between SES and later mathematical skills.

While various aspects of mathematical knowledge measured in kindergarten, such as counting procedures or numeral recognition, have been found to predict later math skills (Duncan et al, 2007; Göbel, Watson, Lervåg, & Hulme, 2014; Jordan, Kaplan, Ramineni, & Locuniak, 2009), it is clear that individual differences in mathematical cognition appear long before kindergarten entry (as early as three years old for symbolic number knowledge, Gunderson & Levine, 2011; and 6 months old in non-symbolic quantitative cognition, Starr, Libertus, & Brannon, 2013). Nevertheless, research has only recently begun to investigate the influence of preschoolers' math and number knowledge on later math outcomes (e.g., van Marle, Chu, Li, & Geary, 2014). This is despite the fact that there are good reasons to suspect that children's acquisition of number words (i.e., the words in the verbal count list, such as "one", "two", "three" and so on), in particular, will predict later math outcomes. For example, this achievement has been recognized as a crucial milestone in children's quantitative development (e.g., Fuson, 1988;

Gelman & Gallistel, 1978; Sarnecka, Goldman, & Slusser, 2014; Wynn, 1990) and children experiencing barriers in access to language (such as deaf children born to hearing parents who do not use sign language) show corresponding delays both in their acquisition of individual number words and in school-based math achievement (Kritzer, 2009).

The typical acquisition process begins around age two, as children learn to recite the count list, and extends through approximately the next two years, as children sequentially assign meaning to each number word over a series of distinct stages (“one”-, “two”-, “three”-, and “four”-knower levels). Eventually, around their 4th or 5th birthday, they come to realize that the last word in a count sequence represents the cardinal value of the set (i.e., the “cardinal principle”; Gelman & Gallistel, 1978) and can then productively use counting to establish sets of different sizes (Fuson, 1988; Sarnecka & Carey, 2008; Wynn, 1990).

Broadly speaking, children’s knower-levels correlate with general receptive vocabulary (Negen & Sarnecka, 2012). Therefore, number language may be regarded as a subset of general language acquisition with potentially unique ties to math ability and achievement. Exposure to number language facilitates children’s acquisition of number word meanings, and children’s knower-levels at 46 months can be predicted by the quality and quantity of number-specific language in the home (‘number talk’) years earlier (Gunderson & Levine, 2011; Levine et al., 2010). Critically, SES predicts parents’ number talk as well as children’s knower-levels (Levine et al., 2010; van Marle et al, 2014), and interventions supporting parents’ scaffolding of number language can facilitate children’s progress toward understanding cardinality (Berkowitz et al., 2015). While other variables, such as the quality of the input (Gunderson & Levine, 2011) and complexity of math-relevant activities at home (Saxe et al., 1987), also contribute to SES differences in math development, children’s language environment, in general, has been said to vary along an SES gradient (Hart & Risley, 2003; see also Sperry, Sperry & Miller (2018) and Golinkoff, Hoff, Rowe, Tamis-LeMonda, & Hirsh-Pasek, 2018). Given the established links between SES and general language acquisition (Hart & Risley, 2003; Hoff, 2003; Rowe, 2012); general vocabulary and number language (Negen & Sarnecka, 2012); and number knowledge and mathematics achievement

(Göbel et al., 2014), links between SES and mathematics achievement may be rooted in exposure to rich language generally, and to number language in particular.

In addition to language, children's mathematical development has been linked with the approximate number system (ANS), a non-verbal system for representing quantities. Evident in a range of species and newborn humans (Dehaene, 2011), the ANS supports mental representations of quantity that are limited in precision following Weber's Law (i.e., error increases in proportion to set size). ANS acuity correlates with performance on standardized measures of mathematics across the lifespan: In 6-month-old infants, the acuity of the ANS predicts symbolic mathematical knowledge at 3 years (Starr et al., 2013), and in 14-year-olds, ANS acuity correlates with performance on math assessments from kindergarten, even when controlling for other cognitive measures (Halberda et al., 2008).

ANS acuity appears to be strongly related to number word knowledge, especially the acquisition of cardinality (Chu, Van Marle, & Geary, 2015; Shusterman, Slusser, Halberda, & Odic, 2016; Starr et al., 2013; van Marle et al., 2014). The directionality of this relationship, however, has not been established (see Shusterman et al., 2016) and there is no evidence for a relationship between ANS acuity and general language. The relationship between ANS acuity and SES is also unclear. Although literature on the ANS often emphasizes its status as an innate cognitive system (e.g., Dehaene, 2011; Starr et al., 2015), malleability of the ANS suggests that acuity might be affected by environmental factors (Tosto et al., 2014), and ANS acuity does appear to be sensitive to effects of formal education (Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Piazza, Pica, Izard, Spelke, & Dehaene, 2013).

Thus, two central accounts of early numeracy in the current literature focus on distinct aspects of development: the early-arising, potentially innate contributions of the ANS on the one hand, and the role of demographic variables and linguistic environment on the other. The aim of the current study, therefore, was to integrate these two approaches into a coherent framework so as to understand the relative contributions of environmental and individual factors on children's early mathematics knowledge. Based on the prior research, we sketched an initial theoretical model incorporating socioeconomic status, general vocabulary, number word knowledge, and approximate number representations (see Appendix A).

Our central hypothesis was that language mediates the relationship between SES and math achievement. Specifically, we hypothesized that (1) general vocabulary mediates the relationship between SES and knower-level, and (2) language (general vocabulary and/or number word knowledge) mediates the relationship between SES and math ability. Our second goal was to incorporate the ANS into this more comprehensive model of early numeracy. To this end, we first asked simply whether (3) ANS acuity varies with SES. Finally, given the hints that environmental variables affect the functioning of the ANS, and the known relationship between number language and ANS acuity measures, we wondered whether number language might help to explain the predictive relationship between ANS acuity and math outcomes. Therefore, we asked whether (4) number language mediates the predictive relationship between ANS acuity and math ability.

To develop and evaluate the plausibility of this multi-faceted model, we conducted a study with a diverse sample of preschoolers, administering two testing sessions one year apart, collecting data on family demographic information, receptive vocabulary, ANS acuity, knower-level, and early math ability.

Methods

Participants

Participants were recruited from preschool and daycare centers in San Jose, CA and Middletown, CT. One hundred and forty three children (72 females; $M_{\text{age}} = 4$ years, 3 months) completed Phase 1 ($n = 55$ in San Jose, CA and $n = 88$ in Middletown, CT). Approximately one year later ($M = 373$ days; $SD = 47$), we reached out to all of these participants and invited them to complete another testing session (Phase 2). The response rate was just over 60% and the 86 children who returned to complete Phase 2 were similarly distributed across location, age, and gender ($n = 35$ from San Jose, CA; 41 females; $M_{\text{age}} = 5$ years, 1 month). One additional child participated in Phase 2 but did not fully complete any of the tasks and was thus excluded from the analyses presented below. For both phases, children were tested in their respective classrooms or at a university lab and received a sticker or small prize for their participation (families who came into the lab additionally received a \$5 travel reimbursement).

Measures

In Phase 1 of this study, participants were tested on number word knowledge, ANS acuity, and receptive vocabulary, in this order. For the second phase, participants were tested on early math ability¹ and parents completed a questionnaire of family demographic information which included reports of average annual income, parent employment, and race/ethnicity.

Number Word Knowledge (Phase 1). Children completed the Give-N task (Wynn, 1990) to evaluate their understanding of individual number words and exact cardinality (as measured by their ability to count in order to produce exact set sizes; see Sarnecka, Goldman, & Slusser, 2014 for review). The task was administered according to a titration procedure such that children were assigned to a knower-level only if they produced consistent and reliable responses (see criteria outlined below) for any given number word (see Sarnecka & Carey, 2008). Knower-level is a metric used to operationalize number language that is both well-established and reliable (with high internal consistency, see Lee & Sarnecka, 2010; 2011, and test-retest stability, see Shusterman et al., 2016). This task also provides a nuanced picture of number word acquisition, dissociated from other mathematical skills like rote counting or numeral recognition (Wynn 1990).

Children were asked to put a certain number (1 to 8, in a fixed order) of fish in a bowl (“Can you put N fish in the swimming pond”). Incorrect responses to any given number, N , were followed by requests for $N-1$. After each response, children were prompted to “count and check to make sure it’s N ” and were allowed to fix errors. Children were assigned to the N -knower-level when they provided at least 2 out of 3 correct responses for that number (N), failed to do so at least 2 out of 3 requests for the next number ($N+1$), and avoided giving that number (N) on other trials. Children were assigned to the cardinal principle knower (CP-knower) level when they succeeded on all set sizes, thus demonstrating that they can use counting in order to determine the number of items in a set.

¹ A second visit during this phase included measures of ANS acuity and receptive vocabulary, not reported here. A subset of these children ($n = 48$) also completed a measure of executive functioning skills as part of another study.

ANS Acuity (Phase 1). Using an adaptation of the ‘Who Has More?’ task (Halberda et al., 2008), children compared sets of blue and yellow dots, presented on the left and right sides of a computer screen, respectively, for 2500 ms. Children were asked to indicate which side had more and received immediate feedback for all trials: a high-pitched beep for correct answers and a low-pitched beep for incorrect answers (accompanied by experimenter explanation as appropriate). Children completed six training trials and 60 test trials.

Each trial presented arrays of 4 to 15 dots on each side, with ratios between the two sets ranging from 1:2 to 6:7. Half the trials had more yellow dots, while half had more blue dots. The size of the individual dots varied within each array (each set included a range of smaller and larger dots) to ensure that the numerically larger set did not always have smaller dots. The numerically larger set had more total surface area (area-correlated) for half the trials and the numerically smaller set had more total surface area (area-anti-correlated) for the other half of the trials. Trial order was randomly assigned for each child.

While reliability reports for this measure are mixed (e.g., DeWind & Brannon, 2016; Norris & Castronovo, 2016), a recent analysis by Inglis and Gilmore (2014) finds that measures of general accuracy (e.g., percent correct) show the higher internal consistency and test-retest reliability (with coefficients greater than .65) as compared to calculated Weber fractions (w). Accordingly, for this study, we used the percent of correct responses (across ratios and trial types) for each child as the index of ANS acuity.

General Vocabulary (Phase 1). Children completed the Peabody Picture Vocabulary Test, 4th edition (PPVT-4) (Dunn & Dunn, 2007) as a measure receptive vocabulary. For each trial, the experimenter asked the child to indicate which of four pictures showed a specified noun or verb (e.g., “which picture shows *running*?”). The assessment concluded when the child reached eight errors within one set of 12 consecutive trials. Norm-referenced scores ($M = 100$, $SD = 15$) were generated for each child based on their raw score (defined as total number of items correct) and their age at the time of the test. The PPVT-4 is reported to have “excellent” internal consistency ($\alpha = .94$) and test-retest reliability (.93), with “good” alternative-form reliability ($r = .89$; Dunn & Dunn, 2007; Kline, 2000).

Math Ability (Phase 2). Upon returning one year later for Phase 2, children completed the Test of Early Mathematic Ability, 3rd Edition (TEMA-3) (Ginsburg & Baroody, 2003). This standardized assessment measures counting abilities, Arabic numeral recognition, number-comparisons, and calculation skills. Note that only two of the nearly 30 items normed for children 3 to 6 years old specifically measure an understanding of individual number words and cardinality (similar to the Give-N task described above, e.g., “how many stars did you count?” and “give me five tokens”). Other items ask children to engage in non-verbal comparisons (e.g., “which set has more?” or “make your set just like mine”), demonstrate their understanding of the count list (e.g., “what number comes after seven?”), identify or write Arabic numerals (e.g., “what number is this [7]?”), demonstrate an understanding of number conservation and constancy (e.g., “does the number of tokens change when they are arranged in a line?”) and engage in proportional reasoning or basic arithmetic (e.g., $_ + 3 = 7$).

Norm-referenced scores ($M = 100$, $SD = 15$) were generated for each child based on their raw score (defined as total number of items correct) and age at the time of the test. The TEMA-3 is reported to have “excellent” internal consistency (with coefficients greater than .92) and “good” test-retest and alternative form reliability (with coefficients greater than .80; Ginsburg & Baroody, 2003; Kline, 2000).

Parent Questionnaire. At Phase 2, parents were given a one-page questionnaire with questions about parents’ education² and income, as well as the child’s race/ethnicity. See Table 1 for a summary of responses.

² Parent education level was quantified as the average of both parents’ education. In single-parent homes ($n = 16$), the sole parent’s education level reported was used. Average level of parent education and highest level of parent education were highly correlated, $r = 0.932$, $p < .001$.

Table 1. Summary of participant demographics as reported on the Parent Questionnaire.

Parent Education	<i>n</i> (%)	Annual Income	<i>n</i> (%)	Race/Ethnicity	<i>n</i> (%)
< High School Diploma	1 (1%)	< \$30,000	8 (9%)	White/European	24 (28%)
High School Diploma	11 (13%)	\$30-60,000	9 (10%)	Latino/Hispanic	14 (16%)
Associate's Degree	27 (31%)	\$60-90,000	10 (12%)	African/African American	3 (3%)
Bachelor's Degree	21 (24%)	\$90-120,000	11 (13%)	Middle Eastern	4 (5%)
Master's Degree	21 (24%)	\$120-150,000	8 (9%)	Mixed Background	11 (13%)
Doctorate Degree	5 (6%)	> \$150,000	6 (7%)	Other	2 (2%)
Other/No Report	0 (0%)	No Report	34 (40%)	No Report	28 (33%)

Note that parent education does not systematically vary with race/ethnicity or income-to-needs ratios in our sample ($\chi^2(58) = 47.74, p = .565$ and $r_s = -.078, p = .639$ respectively). Of the families who returned questionnaires, nearly 75% ($n = 62$) identified as English only. In addition to English, 24% ($n = 20$) speak another language in the home. Few (2.5%, $n = 2$) reported that they speak two or more languages other than English in the home.

Data Analysis

The following analyses include only those participants who completed both Phase 1 and Phase 2 of the study ($n = 86$). We note that children who only participated in Phase 1 of the study were slightly younger than the children who participated in both phases ($M_{age} = 4.14$ years and $M_{age} = 4.36$ years respectively; $t(141) = 2.081, p = .039$), and tended to score higher on the assessment of general vocabulary ($M = 112$ versus $M = 100$ respectively; $t(138) = 4.488, p < .001$). However, these two groups did not systematically differ with regards to number word knowledge ($t(139) = 0.285, p = .776$) or ANS acuity ($t(133) = 1.804, p = .073$). Further analysis of missing data using Little’s MCAR test (Little, 1988) shows that missing data appear at random, $\chi^2(62) = 77.376, p = .090$. As such, listwise deletion was allowed such that only the 86 participants who completed both Phases 1 and 2 were included.

We first conducted a series of analyses to determine whether children’s performance on measures of number word knowledge, ANS acuity, general vocabulary, and early math ability is associated with

measures of SES. We then explored performance on measures of language (general vocabulary and number word knowledge) as independent and multiple mediators of the association between parent education³ and math ability. Finally, we investigate the association between ANS acuity and math ability, and test whether number word knowledge mediates the relation between the two. We report unstandardized regression coefficients (*b*) and standard errors (*SE*), as well as bootstrapped confidence intervals. Results are interpreted as statistically significant if the 95% confidence intervals do not include 0. Appropriate tests of statistical significance are included for reports of direct effects. Regression analyses employed listwise deletion of missing data. Final sample sizes for each of these analyses are reported below.

Results

Descriptive information about children's performance on each measure, and statistics reflecting associations with age and sex, are presented in Table 2. Age was significantly associated with performance across all measures ($ps < .05$), and is thus partialled out in subsequent analyses. There were no significant effects of sex ($ps > .05$), location (San Jose, CA vs. Middletown, CT; $ps > .05$), or setting (lab vs. preschool; $ps > .05$) on any measure. These variables were not factored into subsequent analyses.

³ To do this, we used the PROCESS macro in SPSS to estimate direct and indirect effects with bootstrapped standard errors (1000 bootstraps) (Hayes, 2013).

Table 2. Descriptive information reflecting children’s performance on each measure, as well as statistics reporting associations with age and sex.

Ability (Task)	Measure	n	Range	Mean (SD)	x Age	x Sex
Number Word Knowledge (Give-N)	6 levels (pre; one; two; three; four; CP)	85	n = 1; 9; 15; 9; 4; 47	n/a	$F(5,79)=5.70, p<.01$	$t(86)=1.88, p=.06^\dagger$
Approximate Number System (Who Has More?)	percent correct across all trials (0-100%)	82	31.7 - 98.3%	62.2% (15.0)	$r(81)=.57, p<.01$	$t(80)=1.14, p=.26$
General Vocabulary (PPVT)	age normed score (M=100, SD=15)	83	78 - 154	110.5 (16.2)	n/a	$t(81)=0.94, p=.35$
Math Ability (TEMA)	age normed score (M=100, SD=15)	58	77 - 132	104.6 (12.9)	n/a	$t(56)=0.09, p=.93$

Note: Correlations to age are not reported for norm-referenced measures (i.e., PPVT and TEMA).

[†] Females were more likely to have attained the CP-knower level status ($n = 30$), as compared to males ($n = 17$).

Correlations across all measures are reported in Table 3. Zero-order correlations were performed for age-standardized measures (i.e., math ability and general vocabulary), and partial correlations controlling for age at time of testing were performed for non-standardized measures (i.e., number work knowledge and ANS acuity). This analysis shows that parent education level was significantly associated with math achievement ($r(56) = 0.461, p < .001$), general vocabulary ($r(80) = 0.375, p < .001$), and number word knowledge ($r(82) = 0.336, p = .03$). Income-to-needs ratios, on the other hand, did not significantly correlate with any measure ($ps > .05$). Given that previous research has clearly identified meaningful and systematic associations between family income and math achievement (e.g., Jordan & Levine, 2009) and general vocabulary (e.g., Hart & Risley, 2003), we posit that our failure to detect or replicate this result is largely due to 1) a poor response rate on the annual family income question (nearly 40% chose not to respond to this question whereas all respondents indicated their level of education) and

2) relatively broad ranges income bins on the parent questionnaire that did not adequately capture the variability in our sample. One-way ANOVAs revealed no differences across reported race/ethnicity in age, general vocabulary, parent education, number word knowledge, or ANS acuity ($ps > .05$). The following analyses, therefore, use parent education as the primary index of socioeconomic status (we return to this issue in the Discussion section below).

Table 3. A complete correlation matrix including all measures of interest.

	1	2	3	4	5	6
1 Parent Education						
Parent Questionnaire	-	.258	.336*	.186	.375**	.461**
2 Income-to-Needs Ratio						
Parent Questionnaire		-	.107	.258	.197	.132
3 Number Knowledge						
Give-N; Phase 1			-	.414**	.390**	.459**
4 Approximate Number						
Who Has More? Phase 1				-	.451**	.428**
5 General Vocabulary						
PPVT; Phase 1					-	.583**
6 Math Ability						
TEMA; Phase 2						-

* $p < .05$

** $p < .01$

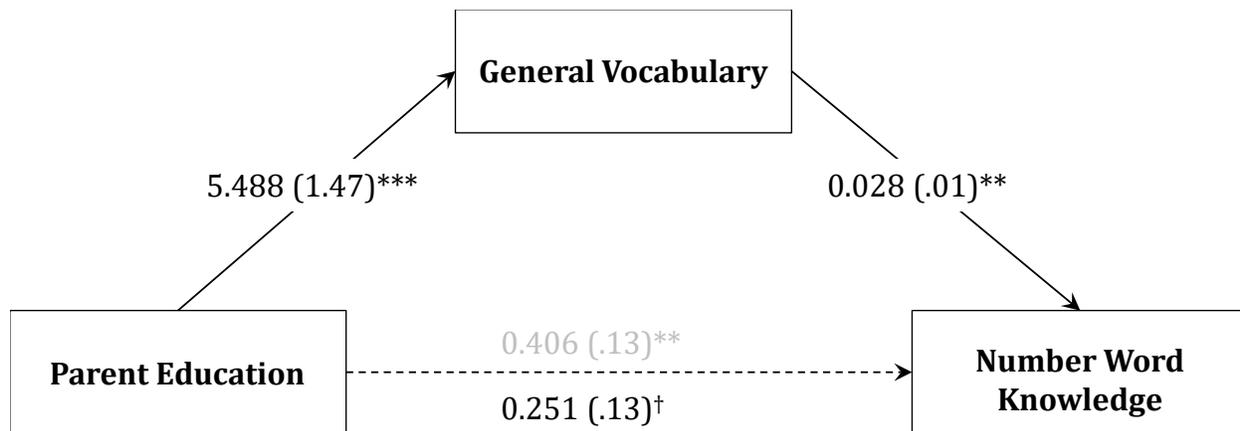
Note: Correlations with norm-referenced measures (i.e., PPVT and TEMA) are reported as zero-order Pearson correlations. All other correlations control for age at time of testing.

We first addressed the relationship between parent education and children’s number word knowledge, hypothesizing that the link between parent education and number word knowledge (knower-level) is mediated by general language abilities. In model 1 ($n = 83$), we regressed number word knowledge on parent education. Results show that parent education significantly predicts number word knowledge, $b = 0.406$, $SE = 0.13$, $p = .002$, 95% CI [0.155 – 0.658]. In model 2 ($n = 83$), we included general vocabulary as a mediator of the relation between parent education and number word knowledge. We found the direct effect of parent education on number word knowledge was attenuated, reaching a

marginal level of statistical significance, $b = 0.251$, $SE = 0.13$, $p = .057$, 95% CI [-0.007 – 0.510].

Importantly, general vocabulary remained a strong predictor of number word knowledge, $b = 0.028$, $SE = 0.009$, $p = .003$, 95% CI [0.010 – 0.046], see Figure 1. Further, the indirect effect of parent education on number word knowledge through general vocabulary was significant, $b = 0.155$, $SE = 0.065$, 95% CI [0.050 – 0.302], suggesting it is through general vocabulary that parent education affects number word knowledge. In total, the model predicted 41% of the variance in number word knowledge ($R^2 = 0.411$). This result aligns with the previously reported link between parental input and number word knowledge (Gunderson & Levine, 2011; Levine et al., 2010), but shows that children’s general language acquisition likely explains this relationship. Refer to Appendix B for the complete regression table with effect sizes.

Figure 1. Parent education significantly predicts number word knowledge, but the relation is mediated by general vocabulary.



- † $p = .06$
- * $p < .05$
- ** $p < .01$
- *** $p < .001$

Note: Solid lines show predicted relationships. Values indicate the regression coefficient, b , and standard error, in parentheses. Values in gray represent the direct path before mediation.

Next, we explored the relationship between parent education and early math outcomes. For this analysis, we tested general vocabulary and number word knowledge (knower-level) as individual mediators of the relation between parent education and math ability, and both as multiple mediators. In

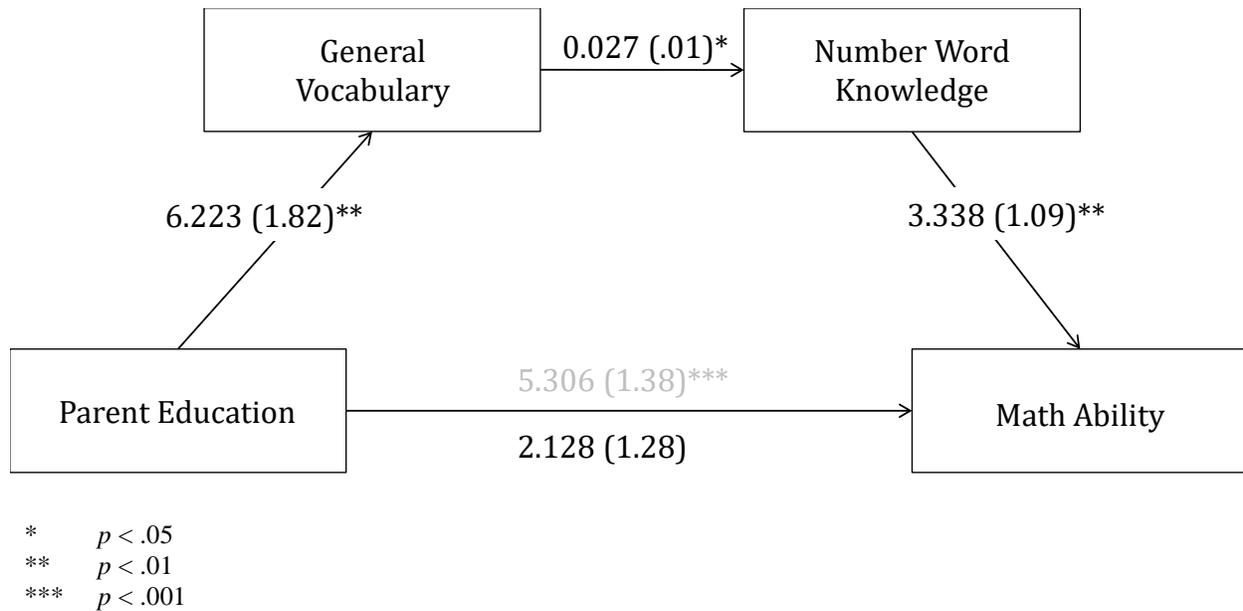
model 1 ($n = 58$), we regressed math ability on parent education. Indeed, parent education was a significant predictor of children's early math ability, $b = 5.306$, $SE = 1.385$, $p < .001$, 95% CI [2.528 – 8.085].

In model 2 ($n = 58$), we included number word knowledge as a mediator of the relation between parent education and math ability, and found the direct effect of parent education on math ability decreased, but remained statistically significant, $b = 3.585$, $SE = 1.278$, $p = .007$, 95% CI [1.021 – 6.148]. The indirect effect of parent education on math ability through number word knowledge was also significant, $b = 1.667$, $SE = 0.807$, 95% CI [0.511 – 3.952], suggesting that a portion of the variance that parent education contributes to math ability is through number word knowledge.

In model 3 ($n = 58$), we tested general vocabulary as the sole mediator of the relation between parent education and math ability. Here again, we found a modest decrease in the magnitude of the relationship between parent education and children's math, but not a complete mediation, $b = 2.921$, $SE = 1.350$, $p = .035$, 95% CI [0.212 – 5.630]. The indirect effect of parent education on math ability, through general vocabulary, did reach statistical significance, $b = 2.385$, $SE = 0.887$, 95% CI [0.943 – 4.598]. These results suggest that while parent education remains a reliable predictor of children's developing math skills, the noted link between parent education and early language may, at least in part, serve to explain this relationship.

Finally, in model 4 ($n = 58$), we included both general vocabulary and number word knowledge as multiple mediators of the relation between parent education and math ability. Here, we found that the direct effect of parent education on math ability disappeared, $b = 2.128$, $SE = 1.277$, $p = 0.102$, 95% CI [-0.437 – 4.693], see Figure 2. The indirect effect of parent education on math ability through both general vocabulary and number word knowledge was statistically significant and together fully explained the variance contributed by parent education ($b = 1.816$, $SE = 0.761$, 95% CI [0.644 – 3.691]; $b = 1.363$, $SE = 0.767$, 95% CI [0.312 – 3.315], respectively). In total, the variables in Model 4 explained 50% of the variance in math ability ($R^2 = .501$). Refer to Appendix C for the complete regression table with effect sizes.

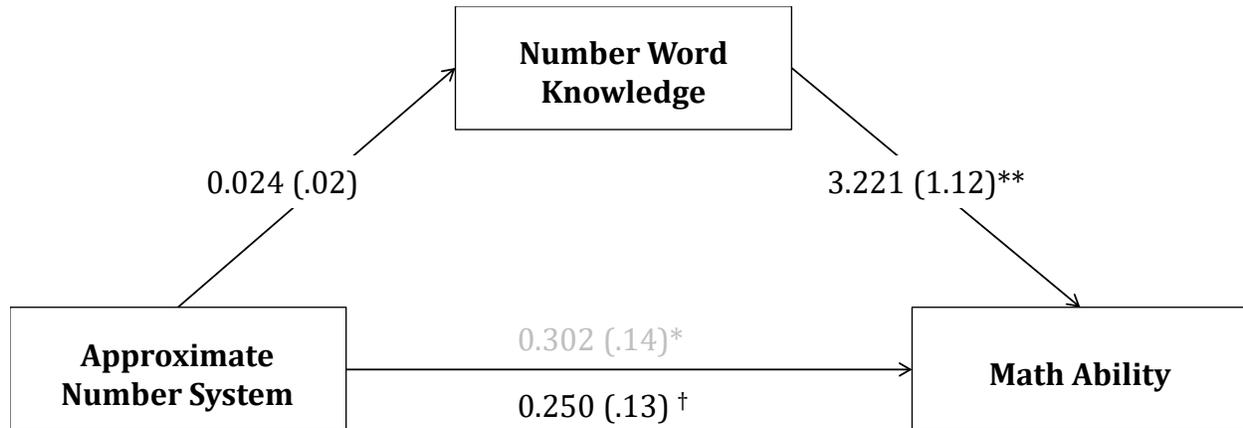
Figure 2. Parent education significantly predicts math ability, but the relation is fully mediated by general vocabulary and number word knowledge.



Note: Solid lines show predicted relationships. Values indicate the regression coefficient and standard error, $b (SE)$. Values in gray represent the direct path before mediation.

Lastly, we tested whether ANS acuity, which theoretically represents a non-linguistic system supporting mathematical reasoning, contributes independently to variance in math ability. Consistent with prior investigations of the relation between ANS acuity and math ability, results from model 1 ($n = 62$) suggested ANS acuity predicted math ability a year later ($b = 0.302, SE = 0.137, p = .021, 95\% CI [0.052 - 0.605]$) over and above age at time of Phase 1 assessment, time between Phases 1 and 2, and general vocabulary. Then, in model 2 ($n = 61$), we tested number word knowledge as a potential mediator of ANS acuity and math ability. When number word knowledge was included in the model, we found the direct effect of ANS acuity on math ability was attenuated and reduced to marginal significance, $b = 0.250, SE = 0.133, p = 0.064, 95\% CI [-0.015 - 0.516]$, see Figure 3. Further, there was a marginally significant indirect effect of ANS acuity on math ability through knower level, $b = .078, SE = .065, 95\% CI[-.043 - .220]$. In total, the variables in this final model explained nearly 50% of the variance in math ability ($R^2 = .474$). Refer to Appendix D for the complete regression table with effect sizes.

Figure 3. ANS acuity significantly predicts math ability, but the relation is mediated by number word knowledge.

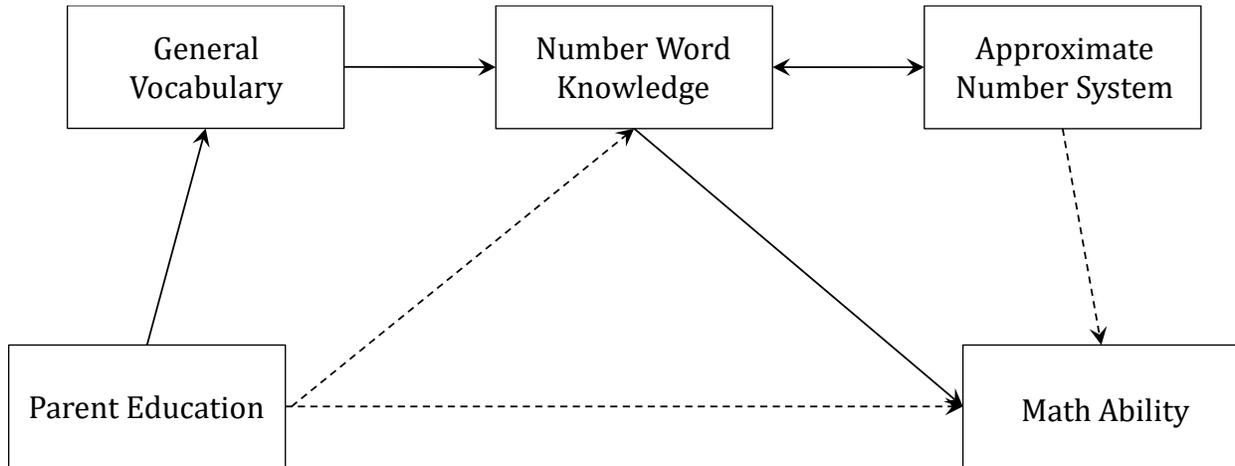


† $p = .06$
 * $p < .05$
 ** $p < .01$

Note: Solid lines show predicted relationships. Values indicate the regression coefficient and standard error, b (SE). Values in gray represent the direct path before mediation.

Summary of Results

The current study explores possible predictors of children’s emerging math skills. Results replicate previously reported links between parent education and math ability (e.g., Levine et al, 2010). However, we find that this link is fully mediated by children’s early language skills. These findings suggest that general vocabulary serves as a foundation for children’s emerging number word knowledge, which then predicts later math ability. While previous reports have identified children’s ANS acuity as a reliable predictor of later math achievement (Halberda et al., 2008), the current findings suggest that ANS acuity may be more directly linked to children’s number word knowledge, which then influences performance on measures of math ability. See Figure 4.

Figure 4. A full model of factors proposed to explain variance in children's early math ability.

Note: Solid lines show significant relationships ($p < .05$). Dashed lines show non-significant or mediated relationships.

Discussion

This longitudinal study, which includes a large sample of children from diverse backgrounds, highlights the role of language in emerging numeracy and represents a step toward meaningfully integrating socioeconomic status (SES) into models of early mathematical development. Current models emphasize the contributions of early-arising cognitive factors, especially approximate number system (ANS) acuity, and demographic variables, like SES, to children's numeracy, but rarely attempt to integrate the two. We argue that more comprehensive theories of early mathematics are needed to develop a better understanding of the multiple influences on academic achievement and maximally effective interventions.

Our results show that parent education and children's approximate number system (ANS) acuity exhibit only indirect effects on math ability through a common factor: children's number word knowledge. We find that children's early language acquisition (which is influenced by SES; Hart & Risley, 2003; Hoff, 2003; Rowe, 2012) correlates with knowledge of number word meanings (see also Negen & Sarnecka, 2012), which then predicts mathematical achievement one year later. Knowledge of

number words also mediates the noted relationship between ANS acuity and subsequent math achievement (see also Chu et al., 2015; Starr et al., 2016).

Our study joins others in demonstrating a clear relationship between preschool number knowledge and subsequent math achievement (Göbel et al., 2014; Chu et al., 2015; van Marle et al., 2014). Like van Marle et al. (2014), we use a nuanced measure of children's number word learning (Give-N), and we show that the relationship to math achievement holds even when accounting for general language and parent education. Thus the knower-level framework has high predictive validity for an array of mathematical skills above and beyond an understanding of the verbal count list and cardinality (e.g., conservation, basic arithmetic and calculation, part-whole concepts, and numerical fluency). The number word knowledge task used here (Give-N) can thus be considered a practical metric for early mathematical development.

Notably, and in contrast to previous studies, we explore SES as a variable of interest in mediation analyses, rather than controlling for it as a background variable. While many potential mechanisms have been proposed to explain the relationship between SES and mathematical outcomes (Jordan & Levine, 2009; Noble, Norman, & Farah, 2005), SES itself is a complex construct comprising various factors. Of the three indices in most composite measures of socioeconomic status (family income, parent occupation, and parent education), our results align with previous reports in emphasizing parent education as a particularly strong influence on children's academic achievement (e.g., Cheadle, 2008). We propose, based on our results, that parent education assumes a central role because it sets the pace of general language development, which in turn influences number word knowledge. Previous research showed that parents' number talk at young ages uniquely predicts children's number word knowledge at 46 months, over and above parents' language in general (Gunderson & Levine, 2011). We suspect that both factors – specific input about number words and concepts, and children's global language development – buttress children's acquisition of number words, with consequences for later math ability.

Regarding the relationship between ANS acuity and math achievement, our findings offer several points: First, like Chu et al. (2015), we find that the predictive contribution of ANS acuity to math ability

is mediated by number word knowledge. However, while Chu et al. (2015) take this as an indication that ANS representations are ‘foundational’ for learning number words, we suggest that the directionality may go the other way. While it is possible that individual differences in ANS drive differences in number word knowledge, as Chu et al. (2015) and others suggest, increasing evidence points toward the opposite direction: that learning number words facilitates ANS acuity (see, Shusterman et al., 2016; Lyons, Bugden, Zheng, De Jesus, & Ansari, 2018), a possibility that is consistent with the data here and in Chu et al. (2015). Number word learning would then emerge as a mediator for the correlation between ANS and math ability if it causally contributes to both variables. Given that mediation analyses do not provide evidence about causality, we suggest that these data merit careful interpretation.

These findings also align with Jordan et al (2009) as they highlight early numeracy as a predictor of later math achievement. However, our findings point to children’s understanding of individual number words and their ability to use of counting procedures to determine exact cardinality as a key predictor of later math achievement, rather than children’s understanding of route counting procedures, non-verbal number comparison (as shown by demoted role of ANS acuity in our model), or verbal arithmetic (which we posit extends from or builds upon early number word knowledge).

Nevertheless, we note that our analyses are correlational and, as is always the case with these lines of research, there are plausible alternatives. For instance, children’s number language might simply be a proxy for general parental input, or scaffolding, which would then independently predict both children’s general vocabulary and their math knowledge. Previous research makes clear that not all input is equivalent. For example, Gunderson and Levine (2011) highlight the fact that different aspects of parents’ linguistic input have distinct consequences for children’s math learning, and only number talk by parents predicts children’s number word knowledge. We did not evaluate parent input directly in this study, so we cannot rule out this possibility. Furthermore, while we find that parent education is not related to ANS acuity in preschoolers, it is possible that environmental influences on the ANS emerge later in development, when representations become integrated with symbolic representations of quantity. Moreover, recent reports suggest a more complex story, with executive functioning (EF) skills interacting

with SES in the relationship between symbolic math and ANS acuity (Fuhs & McNeil, 2013; Hassinger-Das, Jordan, Glutting, Irwin, & Dyson, 2014; Keller & Libertus, 2015). Further research is therefore necessary to confirm that children's *number word knowledge*, in particular, is a key predictor of math ability and to fully understand the interplay between SES and cognitive abilities such math and EF.

Even with the clear need for more research, these findings carry implications for early education and intervention. The present study suggests that an increased focus on number word knowledge, as well as general vocabulary, may help to minimize disparities in math ability as children enter kindergarten. Intervention to support children's number sense through training ANS acuity (e.g., Wang, Odic, Halberda, & Feigenson, 2016) may remain justified, but interventions that focus on number word meanings, either in symbolic contexts, or in the mapping of number words to non-symbolic representations, are likely to gain more traction toward long term math achievement.

Finally, our findings align with others in highlighting the role of socioeconomic status—particularly parent education—as a key influence on linguistic and conceptual development in the domain of mathematics (Chu et al., 2015; Jordan & Levine, 2009), pointing to parents' educational attainment as a locus for intervention for children's academic prospects. Public policy that supports this goal, such as well-implemented dual-generation or wrap-around programs that educate and support parents and children simultaneously (Shonkoff & Fisher, 2013), could be useful levers to intervene on socioeconomic status and its consequences for important school outcomes like math achievement.

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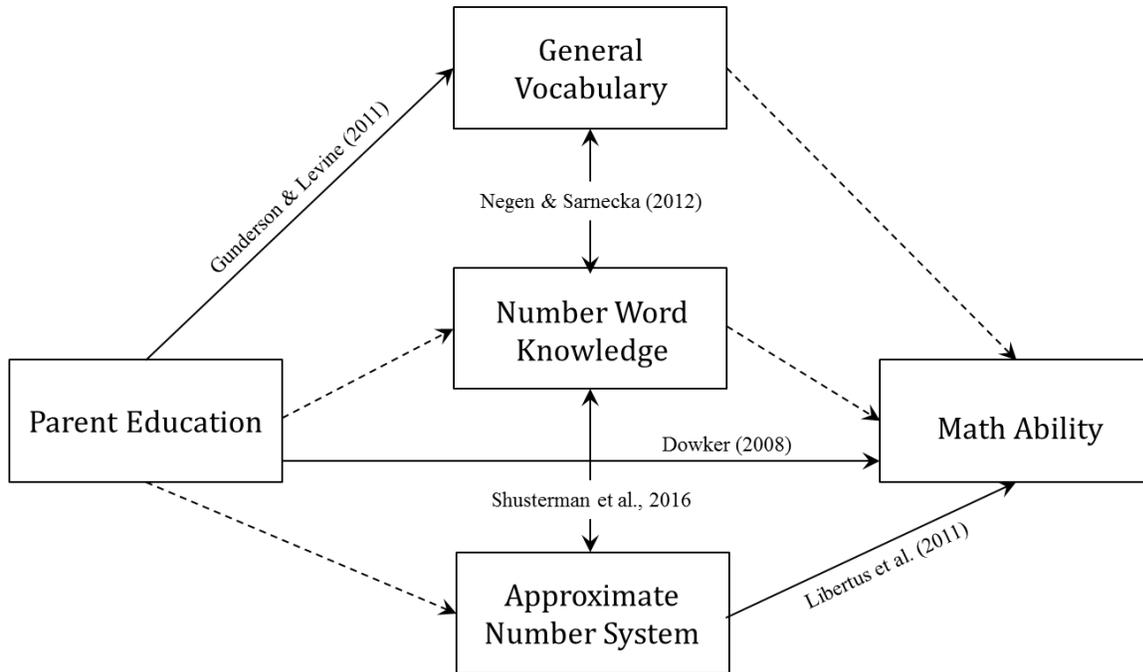
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Appendix A: An initial theoretical model for the influence of parent education on early math ability based on relevant literature to date.



Appendix B: Regression models evaluating influences on number word knowledge. Results show that general vocabulary mediates the relation between parent education and number word knowledge.

Model Variables	Model 1				Model 2			
	<i>b</i>	SE	β	<i>p</i>	b	SE	β	<i>p</i>
Intercept	-3.504	1.133		0.003	-5.796	1.307		<.001
Age (<i>covariate</i>)	1.358	0.226	0.552	<.001	1.293	0.216	0.526	<.001
Parent Education	0.406	0.126	0.296	0.002	0.251	0.130	0.183	0.057
General Vocabulary					0.028	0.009	0.290	0.003
<i>R</i> ²	0.340				0.411			

Notes: *b*, unstandardized regression coefficient; SE, standard error; β , standardized regression coefficient; *p*, significance (p-value); *R*², proportion variance explained.

Appendix C: Regression models evaluating influences on early math achievement. Results show that number word knowledge and general vocabulary mediate the relation between parent education and early math achievement.

Model 1					Model 2			
Model Variables	<i>b</i>	SE	β	<i>p</i>	<i>b</i>	SE	β	<i>p</i>
Intercept	84.646	16.306		<.001	100.234	14.517		<.001
Age (<i>covariate</i>)	0.916	2.504	0.036	0.716	-5.668	2.688	-0.278	0.040
Time between Sessions (<i>covariate</i>)	-0.288	1.013	-0.35	0.778	-0.806	0.888	-0.012	0.091
Parent Education	5.306	1.385	0.469	<.001	3.585	1.278	0.311	0.007
Number Word					4.315	1.064	0.564	<.001
R^2	0.219				0.401			
Model 3					Model 4			
Model Variables	<i>b</i>	SE	β	<i>p</i>	<i>b</i>	SE	β	<i>p</i>
Intercept	49.898	16.663		0.004	71.063	16.905		<.001
Age (<i>covariate</i>)	-0.935	2.246	-0.045	0.679	-5.686	2.590	-0.276	0.033
Time between Sessions (<i>covariate</i>)	0.397	0.906	0.048	0.663	0.376	0.840	0.046	0.656
Parent Education	2.921	1.350	0.260	0.035	2.128	1.278	0.189	0.102
Number Word					3.338	1.093	0.423	0.003
General Vocabulary	0.383	0.094	0.492	<.001	0.292	0.092	0.374	0.003
R^2	0.409				0.501			

Notes: *b*, unstandardized regression coefficient; SE, standard error; β , standardized regression coefficient; *p*, significance (p-value); R^2 , proportion variance explained.

Appendix D: Regression models evaluating influences on early math achievement. Results show that number word knowledge mediates the relation between ANS acuity and early math ability.

Model Variables	Model 1				Model 2			
	<i>b</i>	SE	β	<i>p</i>	<i>b</i>	SE	β	<i>p</i>
Intercept	47.122	15.954		0.004	69.942	16.722		<.001
Age (<i>covariate</i>)	-4.397	3.016	-0.213	0.093	-8.744	3.139	-0.42	0.007
Time between Sessions (<i>covariate</i>)	1.407	0.928	0.164	0.135	1.178	0.877	0.138	0.185
General Vocabulary (<i>covariate</i>)	0.356	0.091	0.458	<.001	0.259	0.091	0.334	0.064
Approximate Number	0.302	0.137	0.348	0.021	0.250	0.133	0.289	0.006
Number Word					3.221	1.119	0.411	0.006
<i>R</i> ²	0.395				0.474			

Notes: *b*, unstandardized regression coefficient; SE, standard error; β , standardized regression coefficient; *p*, significance (p-value); *R*², proportion variance explained.