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Error Correcting Codes for Cooperative Broadcasting

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Error Correcting Codes for Cooperative Broadcasting

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Outline

1. Motivation
2. Related work
3. Broadcast channels
4. Multilevel codes and the |u|u+v| construction
   - Time sharing |u|v| versus |u|u+v| construction
   - Decoding for |u|u+v| construction: Two stages versus single state
5. Demapping and decoding performance
   - BPSK and QPSK modulations
   - 4-PAM modulation: Hierarchical mapping
6. Conclusions and future work

1. Motivation

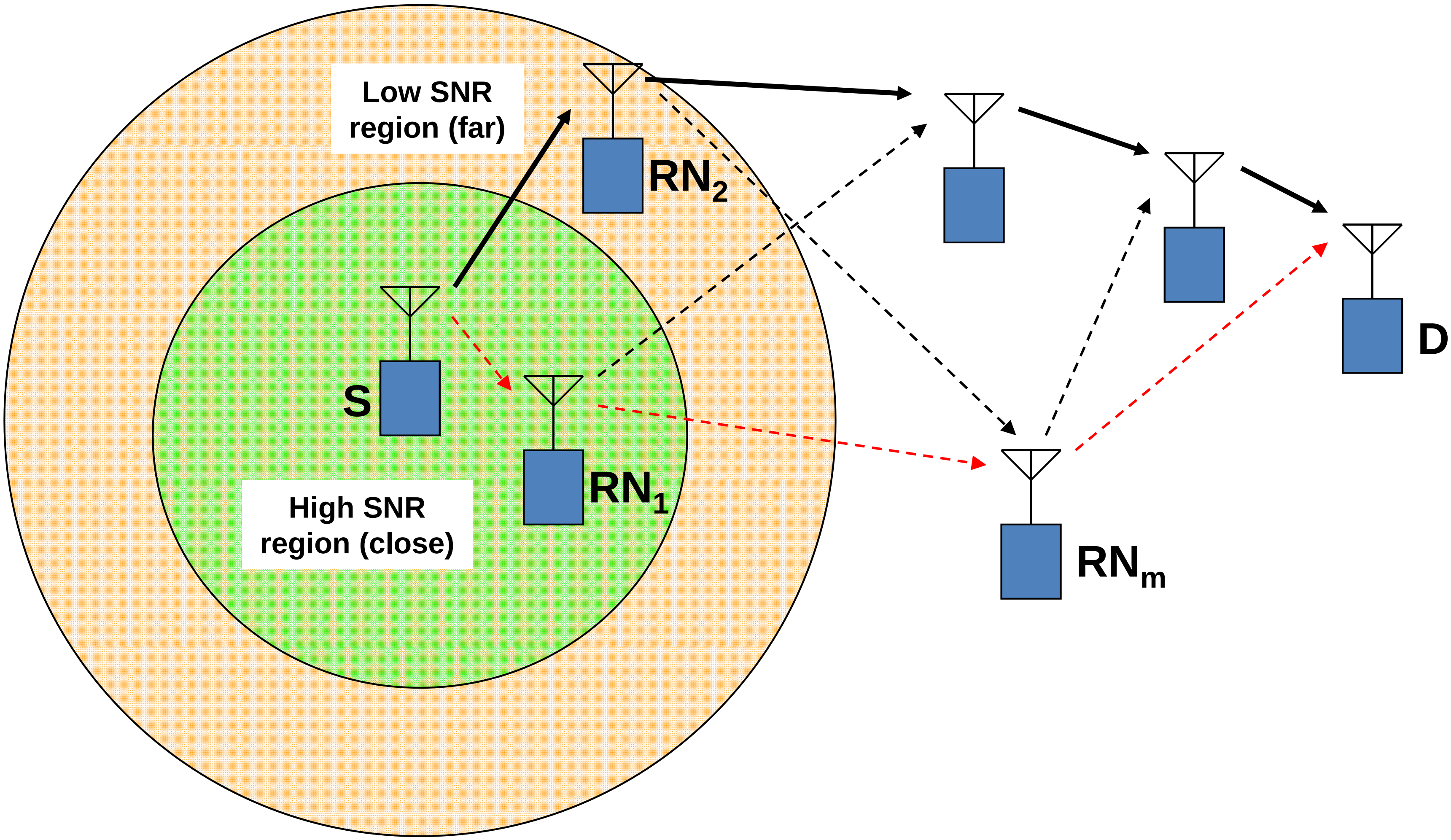
- Background on ***unequal error protection (UEP) codes***:

  Ph.D. thesis on multilevel error-correcting codes

- UEP codes are based on the idea of ***superposition coding*** proposed in Cover's paper (1972) on broadcast channels
- The nodes of a wireless network (cooperative or not) always broadcast information (i.e., every node, in principle, receives this information)
- ***Cooperative broadcasting*** paper (Bergmans and Cover ,1974)

  "Superposition coding always outperforms orthogonal (time-division or frequency-division) schemes"

Broadcasting in a Wireless Network



1. Relay RN_1 has higher SNR compared to RN_2 ⟶ $C_1 > C_2$

2. "Shortest path" (smallest number of hops) not always most reliable

2. Related work

- Cover (1972): “Broadcast Channels”
  - Cloud structure of capacity-achieving superposition codes
- Bergmans and Cover (1974): “Cooperative Broadcasting”
  - Superposition codes always outperform orthogonal assignment
- Laneman, Tse and Wornell (2004): “Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior”
  - Amplify-and-forward, decode-and-forward, adaptive relaying, incremental relaying
- Stefanov and Erkip (2004): “Cooperative Coding for Wireless Networks”
  - Propose superposition coding. Do not refer to Cover!!?
- Yi, Azimi, Kalyanaraman and Subramanian (2005): “Error Control Code Combining Techniques in Cluster-based Cooperative Wireless Networks”
  - Use Chase code-combining with hybrid ARQ

Related work (cont.)

- S. Katti, H. Rahul, W. Hu, D. Katabi, M. Medard, and J. Crowcroft,: XORs in The Air: Practical Wireless Network Coding," (2006)
  - Opportunistic coding
- Chen and Ahmed (2008): Throughput Enhancement in Cooperative Diversity Wireless Networks using Adaptive Modulation
  - Feedback CSI and adapt constellation accordingly
- Li, Ge, Tang and Xiong (2008): Cooperative Diversity Based on Alamouti Space-Time Code
  - Multiple-access stage of relay nodes achieved with Alamouti's scheme
- L. Xiao, T.E. Fuja, J. Kliewer and D.J. Costello (2009): Error Performance Analysis of Signal Superposition Coded Cooperative Diversity
  - Consider superposition coding performed at a single source, metrics
- H.J. Yang, Y. Choi and J. Chun (2010): Modified High-Order PAMs for Binary Coded Physical-Layer Network Coding
  - Constellation design for superposition (physical layer) network coding

3. Broadcast Channels

- Thomas Cover (1972) – Gaussian broadcast channel with one source broadcasting information to two users: User 1 has a larger signal-to-noise ratio (SNR) than User 2 ($N_1 < N_2$):

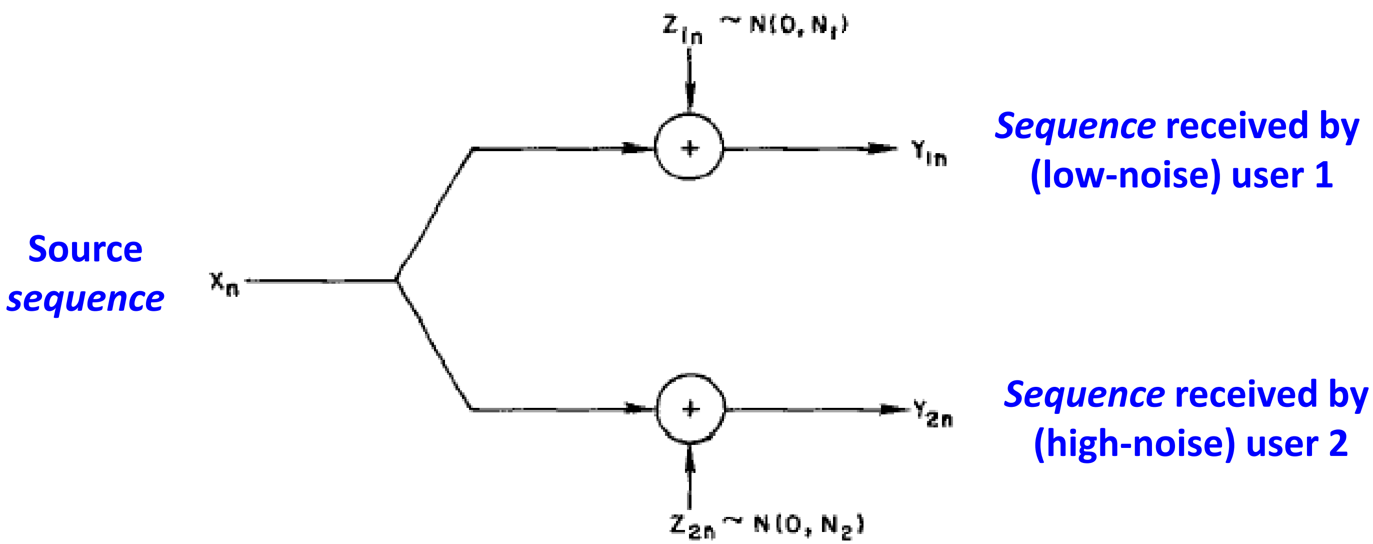


Fig. 16. Gaussian broadcast channel.

From: T. Cover, "Broadcast Channels," *IEEE Trans. Info. Theory*, vol. IT-18, no. 1, pp. 2-14, Jan. 1972

Coding for broadcast channels: Cloud concept

- Cover showed that a channel code achieving capacity has a ***cloud structure***, shown below for a binary symmetric broadcast channel:

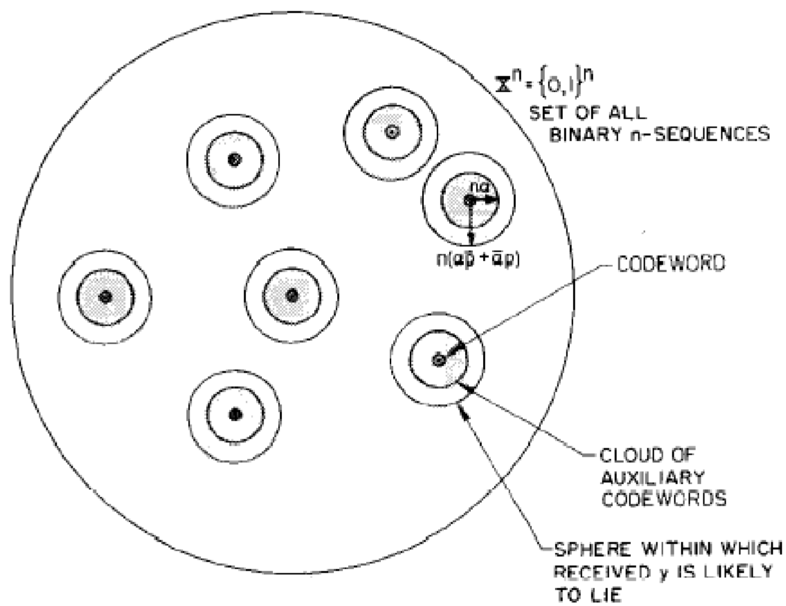
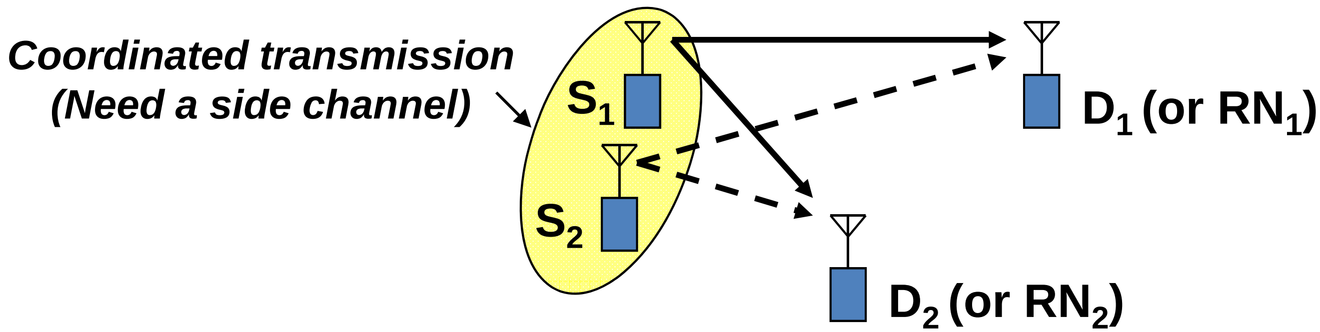


Fig. 4. Space of codewords for BSC.

From: T. Cover, "Broadcast Channels," ***IEEE Trans. Info. Theory*****, vol. IT-18, no. 1, pp. 2-14, Jan. 1972**

- A cloud is a set of codewords (or sequences) that is selected with the information bits (most important or MSB) to be transmitted to the high-noise user

Cooperative Broadcasting (1974)



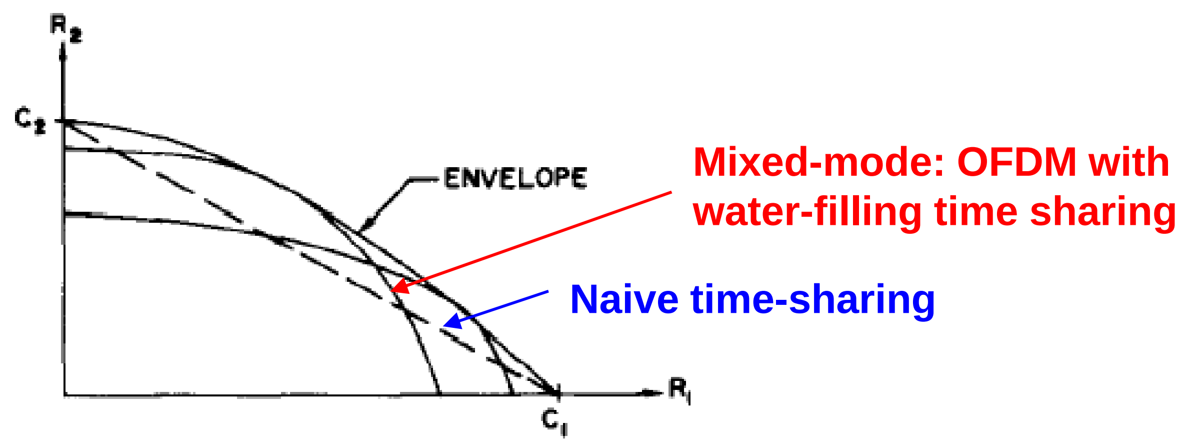


Fig. 3. Rates achievable by frequency division and variable power time sharing.

From: P. Bergmans and T. Cover, "Cooperative Broadcasting," *IEEE Trans. Info. Theory*, vol. IT-20, no. 3, pp. 317-324, May 1974.

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4. Multilevel LUEP codes and |u|u+v| construction

- *An LUEP code has subcode partition-chain* $C \supset C_1 \supset \cdots \supset C_L$ with

$$G = \begin{pmatrix} G_1 \\ G_2' \\ \vdots \\ G_L \end{pmatrix}, \quad G_2 = \begin{pmatrix} G_2' \\ G_3' \\ \vdots \\ G_L \end{pmatrix}, \quad \cdots, \quad G_L,$$

  and $d_1 > d_2 > \cdots > d_L$.

- Practical *two-level LUEP codes* can been constructed based on *block, convolutional or LDPC codes* and Plotkin's (or |u|u+v|) *construction*:

$$G = \begin{pmatrix} 0 & G_1 \\ G_2' & G_2' \end{pmatrix},$$

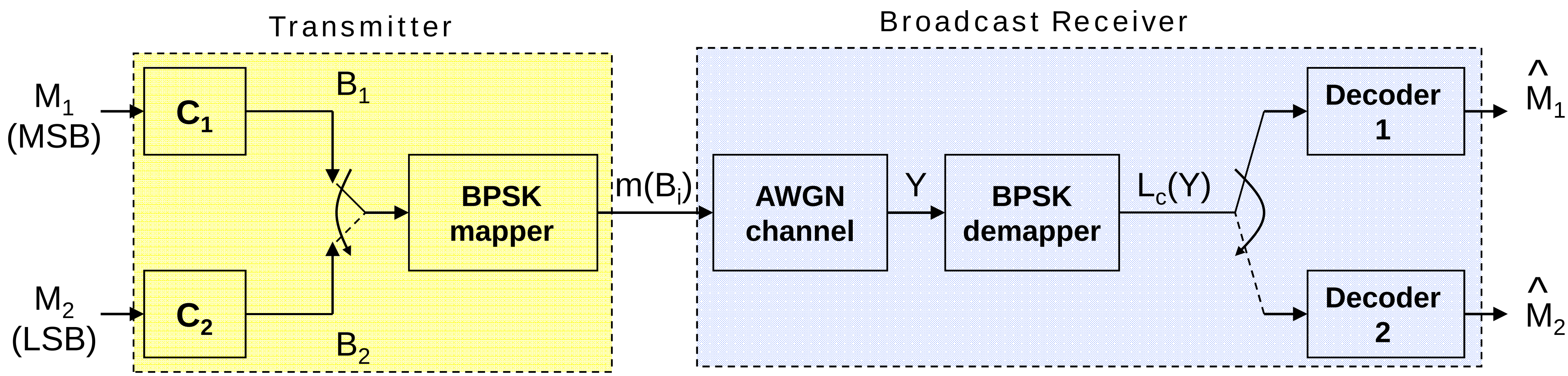
  ← Cloud centers (coset leaders)

  ← Cloud (subcode) codewords

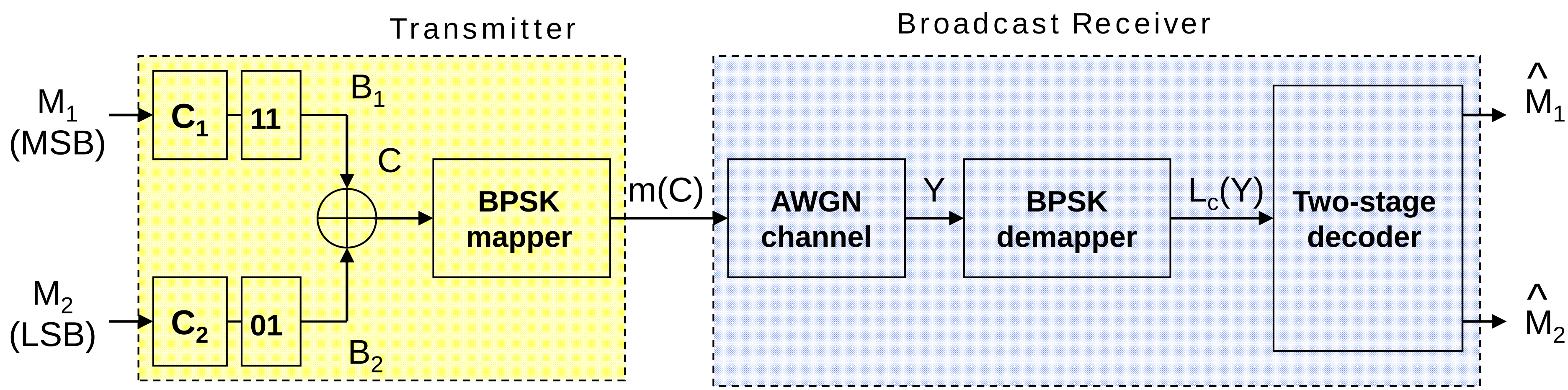
  where $d_2 = 2d_2' < d_1$.

Time-sharing (|u|v|) versus Plotkin (|u|u+v|)

- Time-sharing:

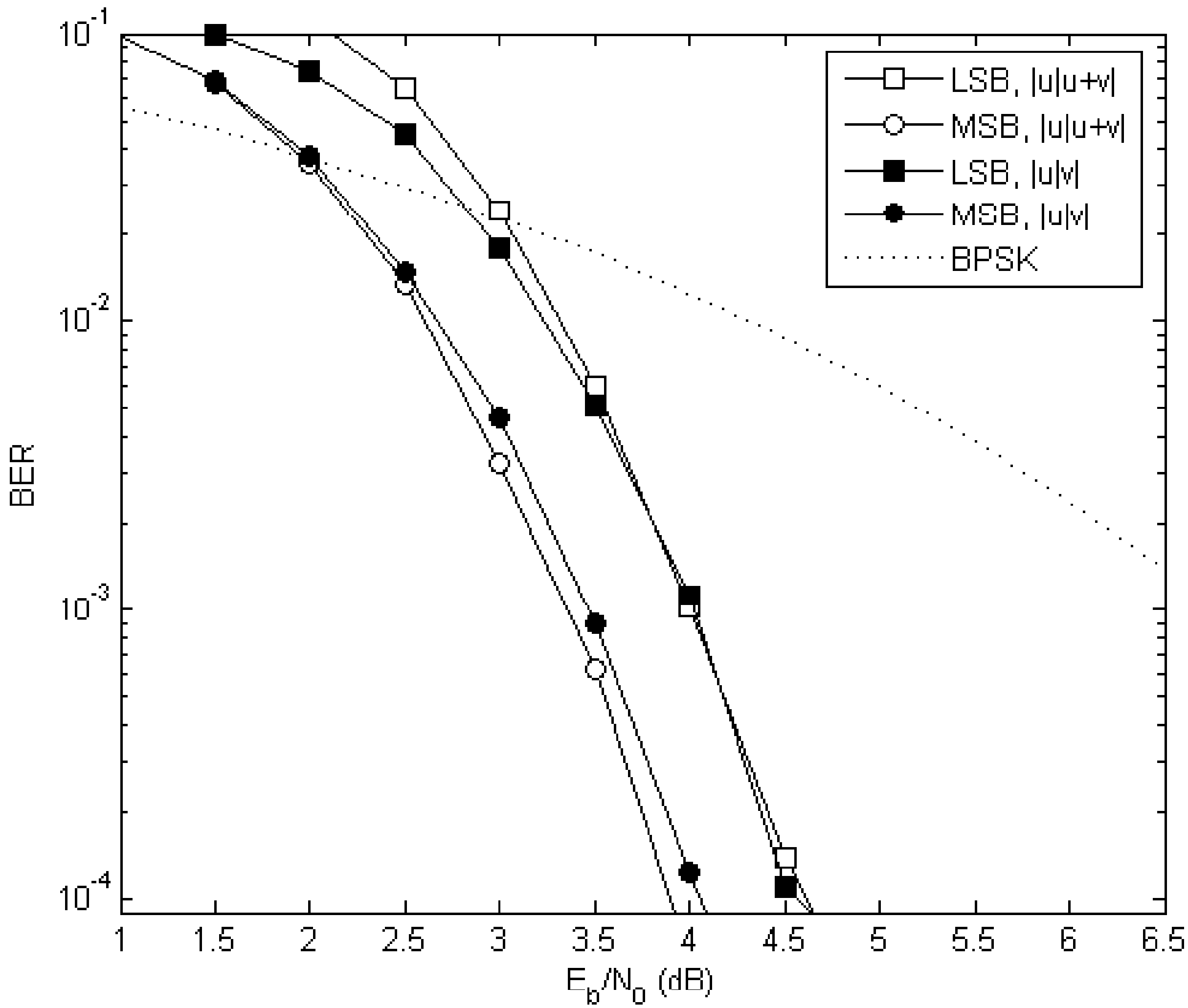


- Plotkin:



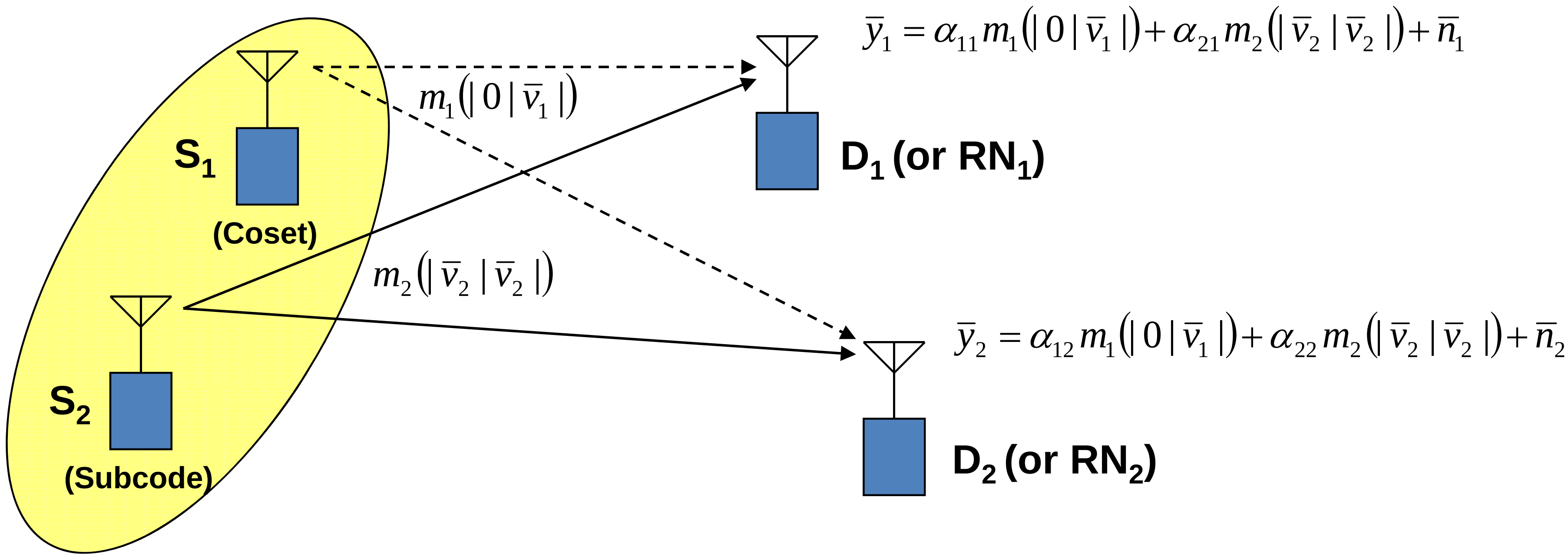
Performance of |u|v| vs. |u|u+v|: short LDPC codes

C_1, C_2: LDPC (96,50) codes of degrees (3,6) and (4,8)



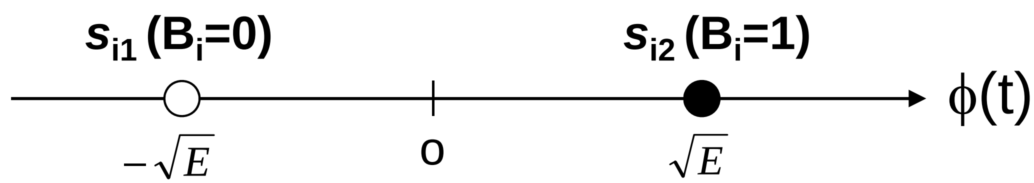
A Plotkin |u|u+v| coding scheme

- Multilevel codes ***always improve the throughput*** over any orthogonal (time- or frequency-division) approach
- Follow Bergmans' and Cover's idea: Design an ***"over-the-air" |u|u+v| (Plotkin) coding scheme***:

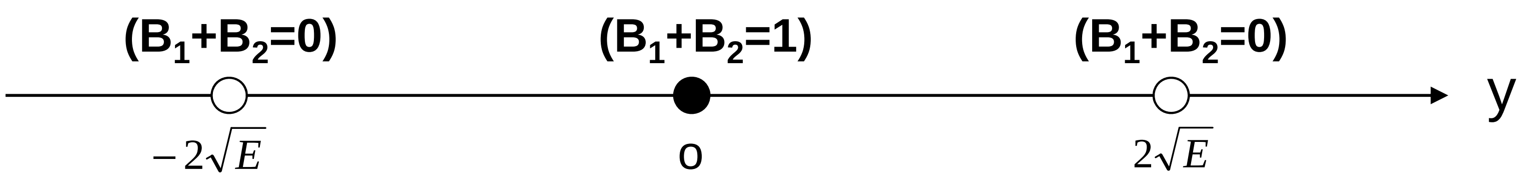


Two sources with BPSK mapping

- ***BPSK Mapping*** m_i from a bit to a signal set M_i , *i=1,2. Assume* $\alpha_{i1}=\alpha_{i2}$
- $M_1=\{s_{11},s_{12}\}$ and $M_2=\{s_{21},s_{22}\}$:

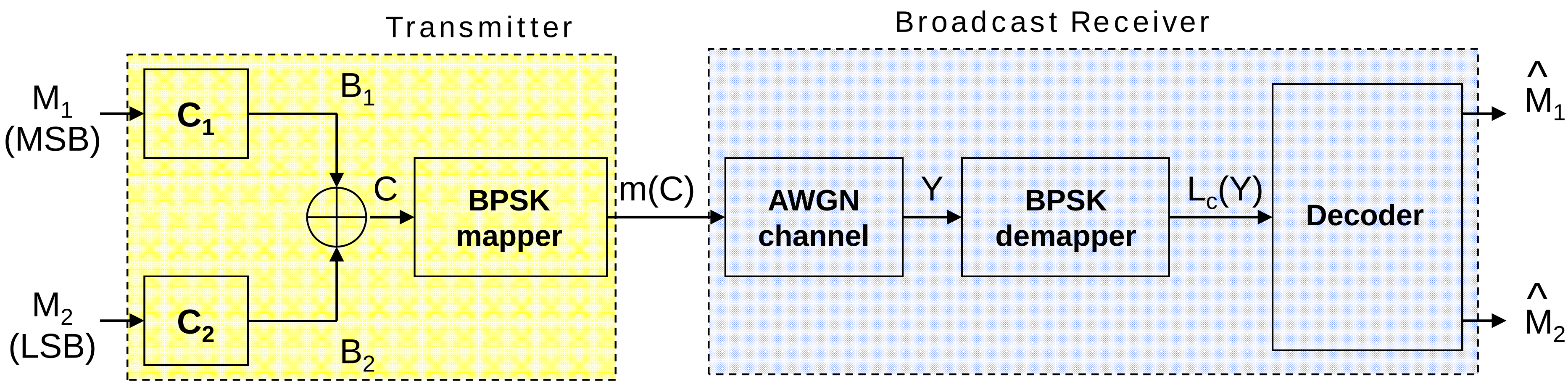


- At the receiver, the ***direct-sum*** M_1+M_2 is equal to a <u>ternary</u> signal set:

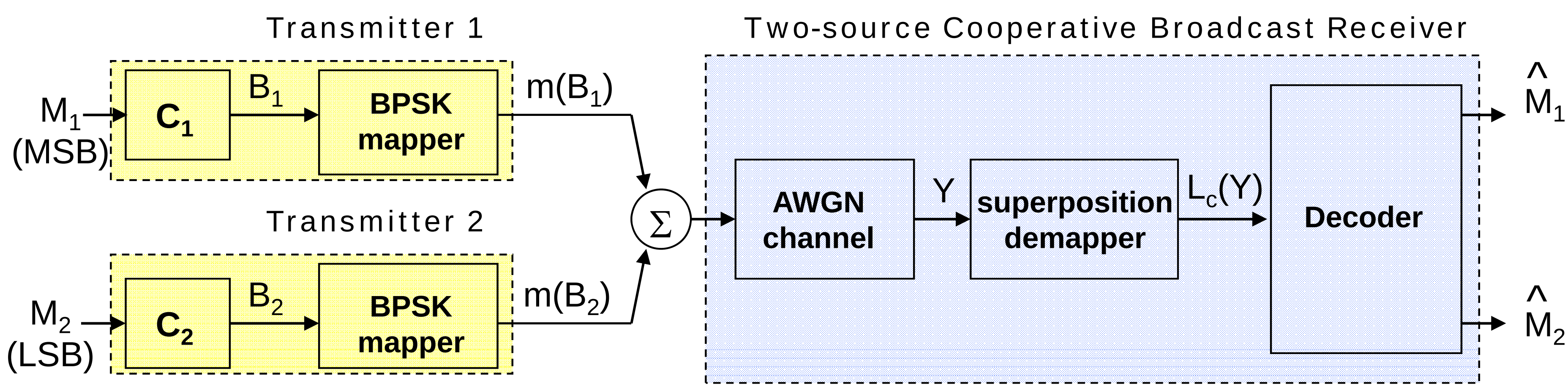


Single-source and cooperative broadcasting

- Single source:

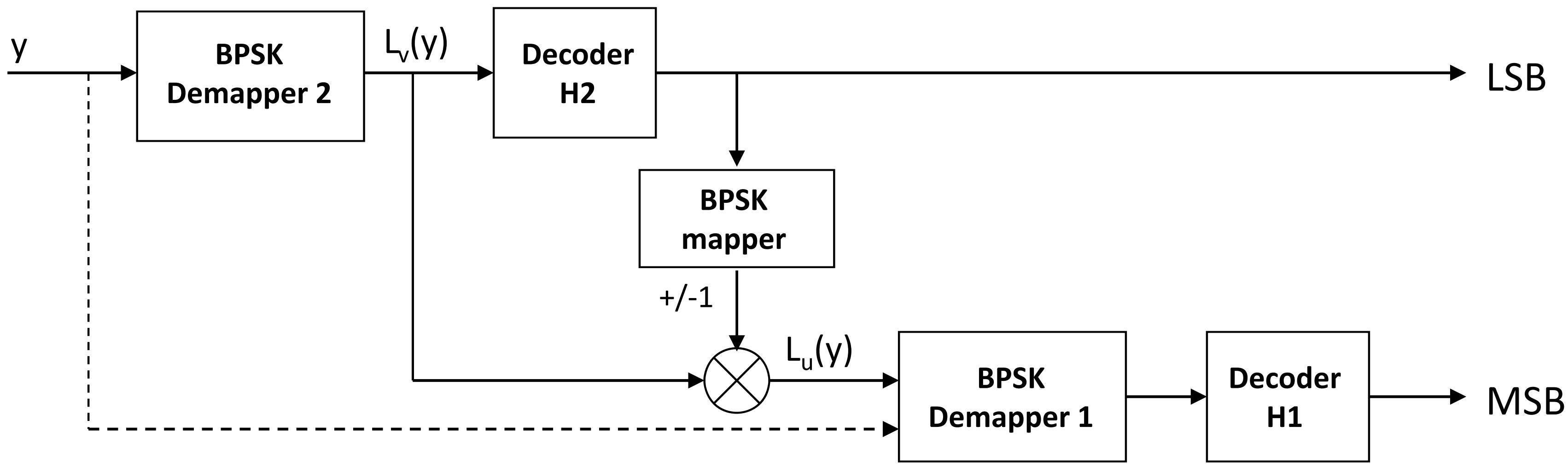


- Cooperative (two sources):

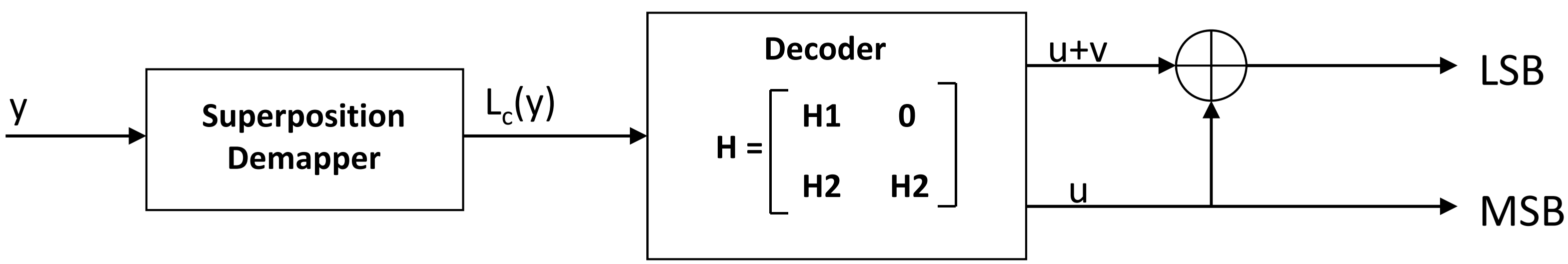


Decoding for |u|u+v| construction

Two stages (Kumar-Milenkovic, 2006)

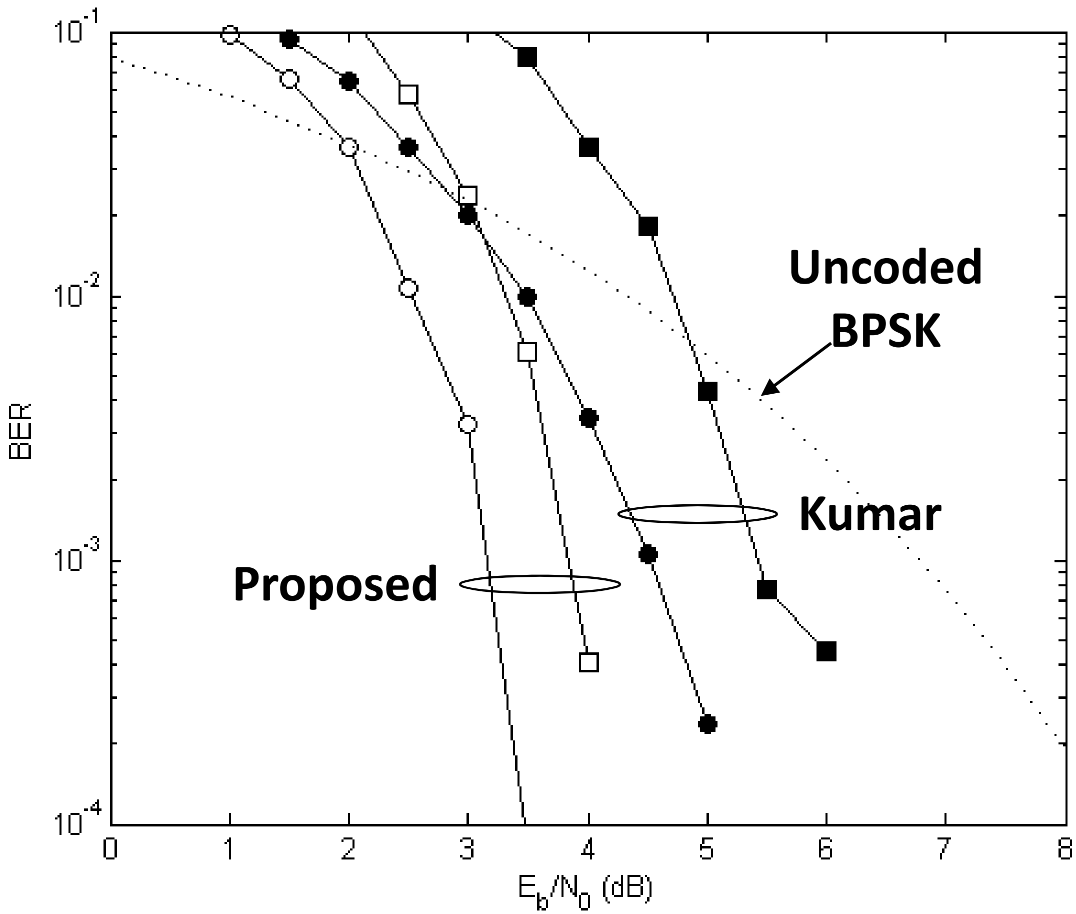


Single stage (SJSU, 2009)

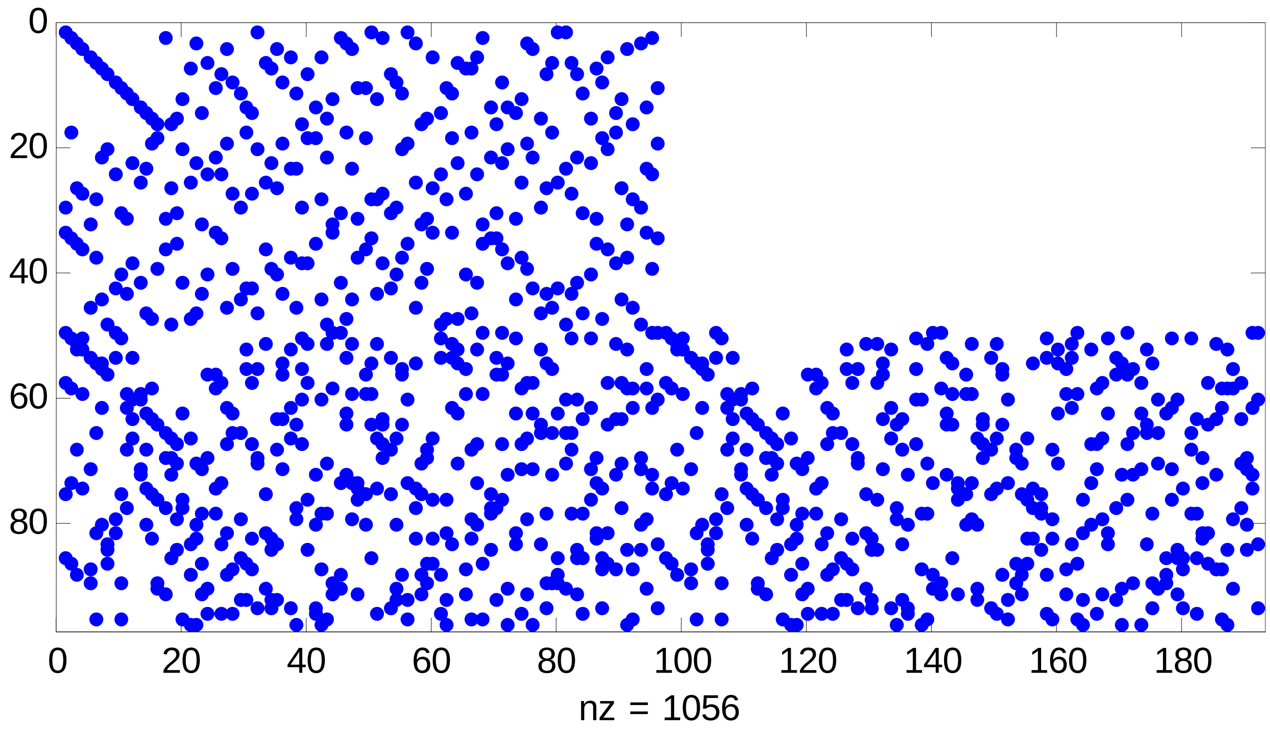


|u|u+v| decoding: Simulation results

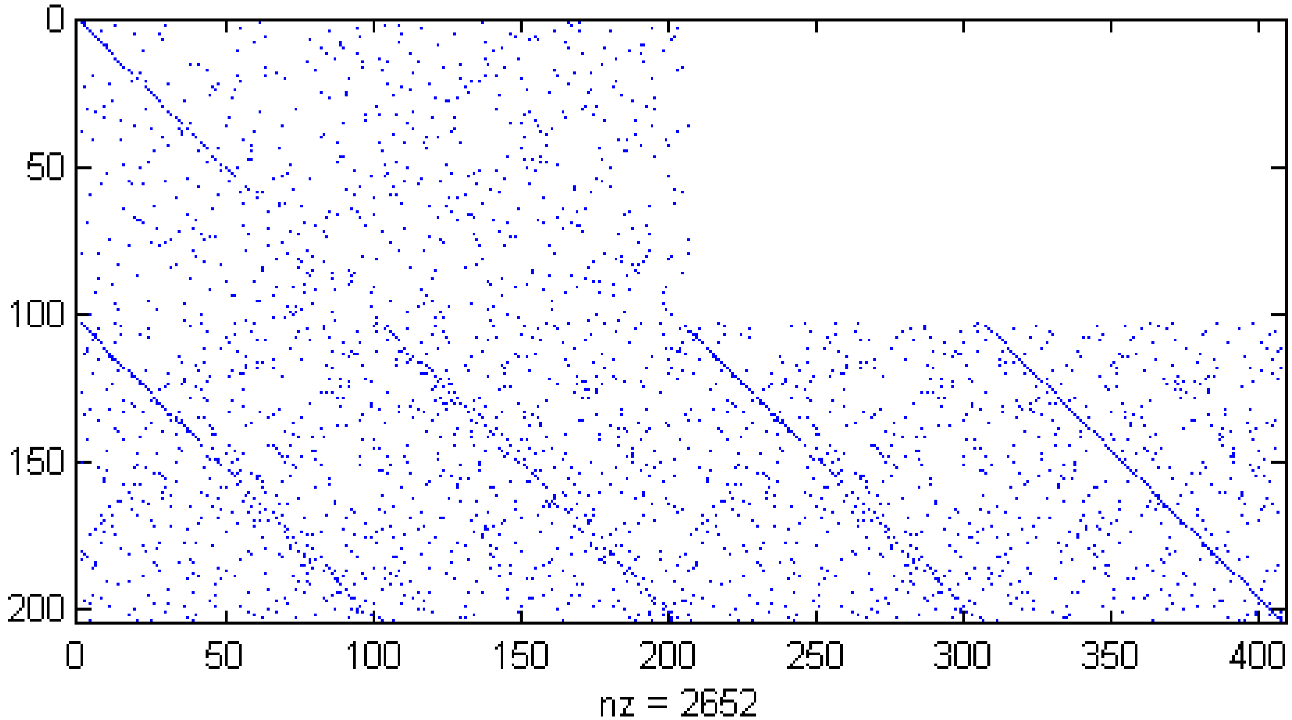
C_1: Regular (96,50) LDPC code with degrees (3,6); C_2: Regular (96,49) LDPC codes with degrees (4,8)



H matrix used in |u|u+v| construction with LDPC codes of length 96



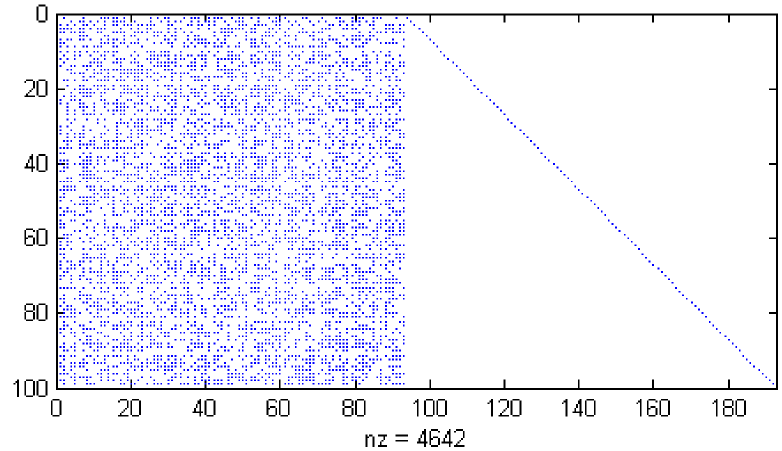
H matrix used in |u|u+v| construction with LDPC codes of length 204



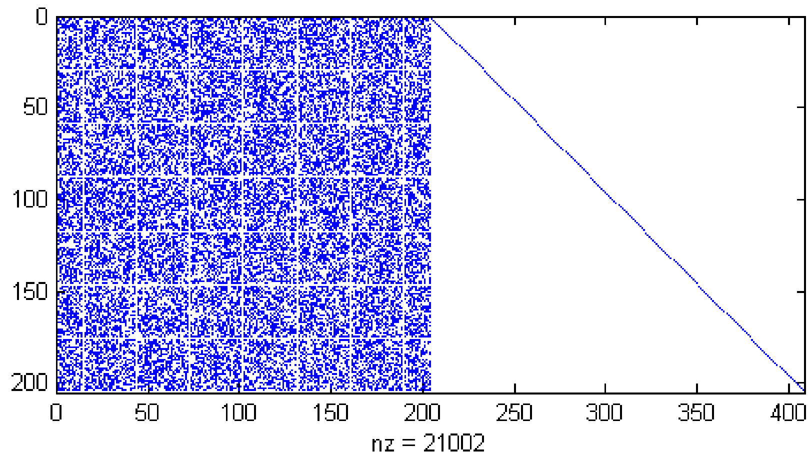
Systematic encoding

- Use Gaussian elimination to produce systematic generator matrices and a permutation:

Length 96:



Length 204:



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Single-source versus cooperative: BPSK Metrics

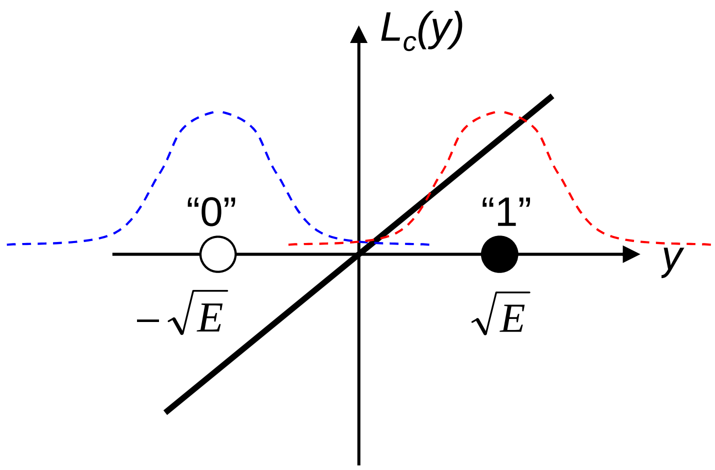
Single source

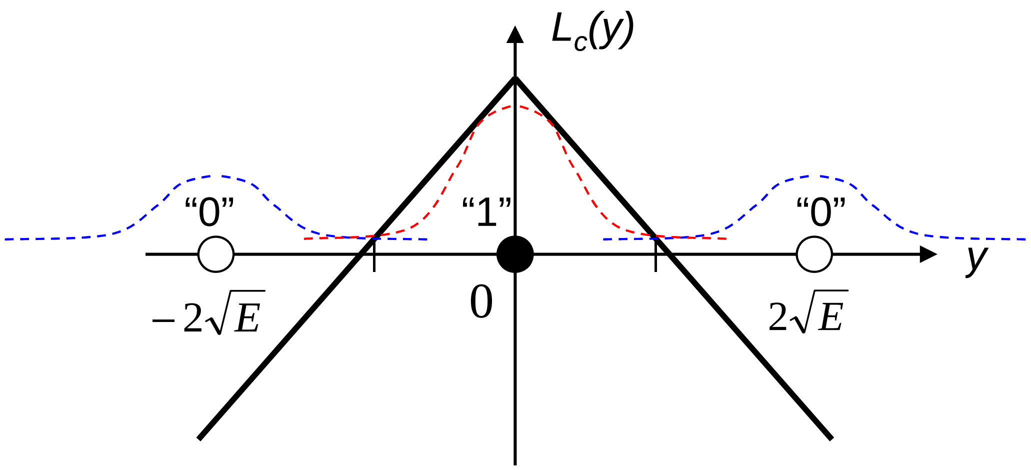
B_1	B_2	$C=B_1+B_2$	y
0	0	0	-1
0	1	1	+1
1	0	1	+1
1	1	0	-1

Cooperative (dual source)

B_1	B_2	y
0	0	-2
0	1	0
1	0	0
1	1	-2

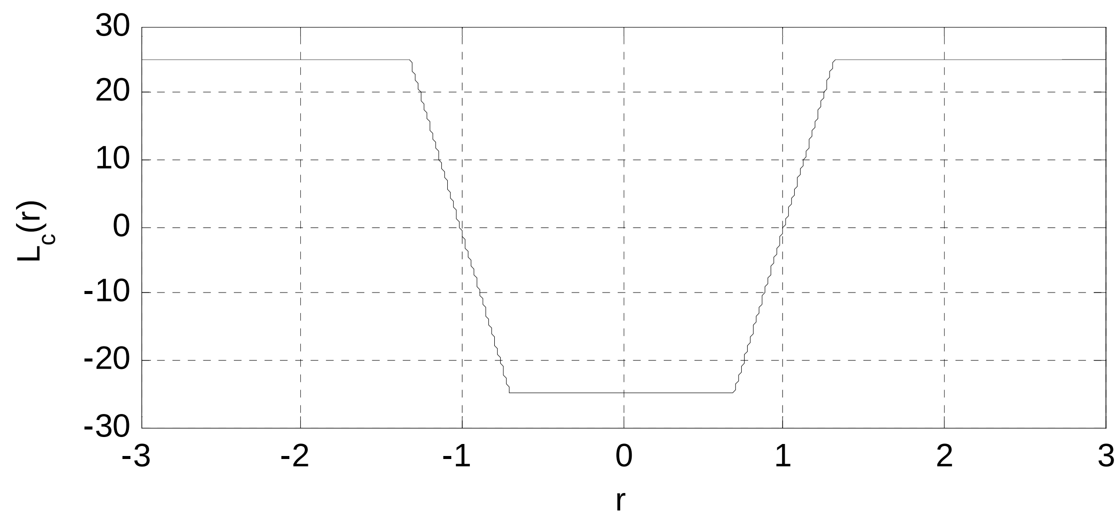
Log-likelihood ratio (LLR) metric: $$L_c(y) = \frac{\Pr\{c=1|y\}}{\Pr\{c=0|y\}}$$



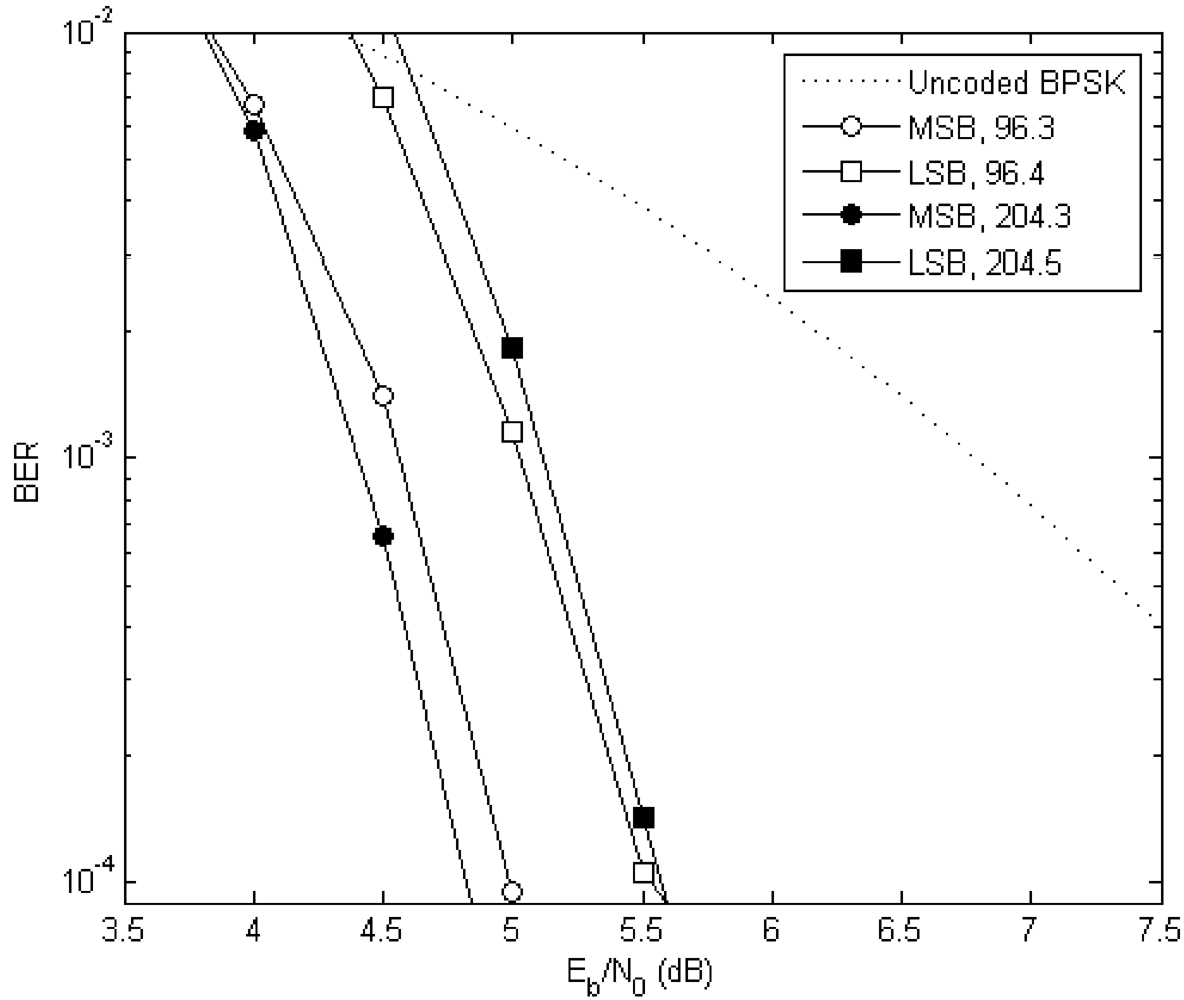


LLR metrics for cooperative broadcasting and BPSK modulation (E/N_0=10 dB)

B	m(B)
0	-1
1	1



Cooperative |u|u+v|: Lengths 96 and 204 AWGN channel



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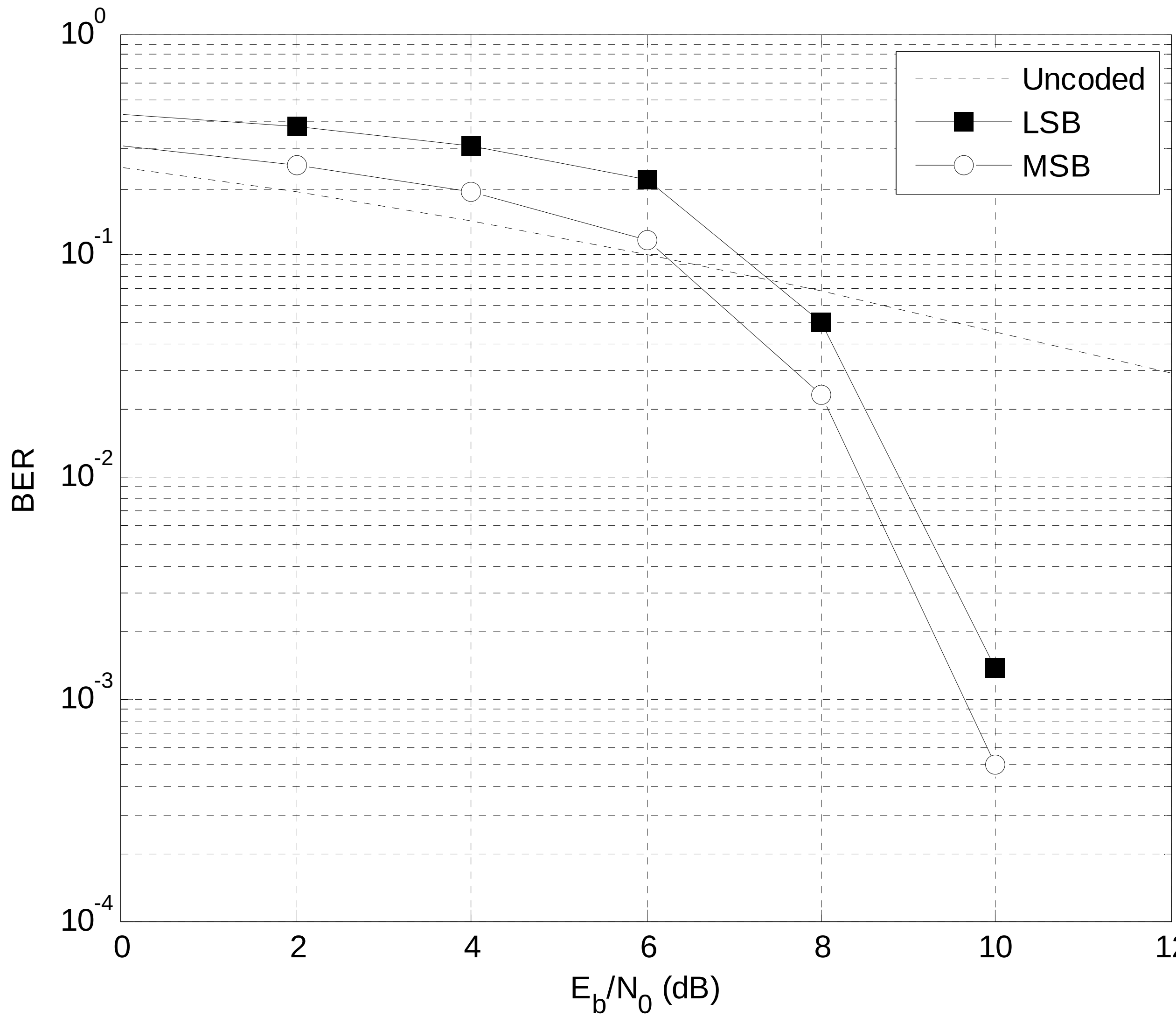
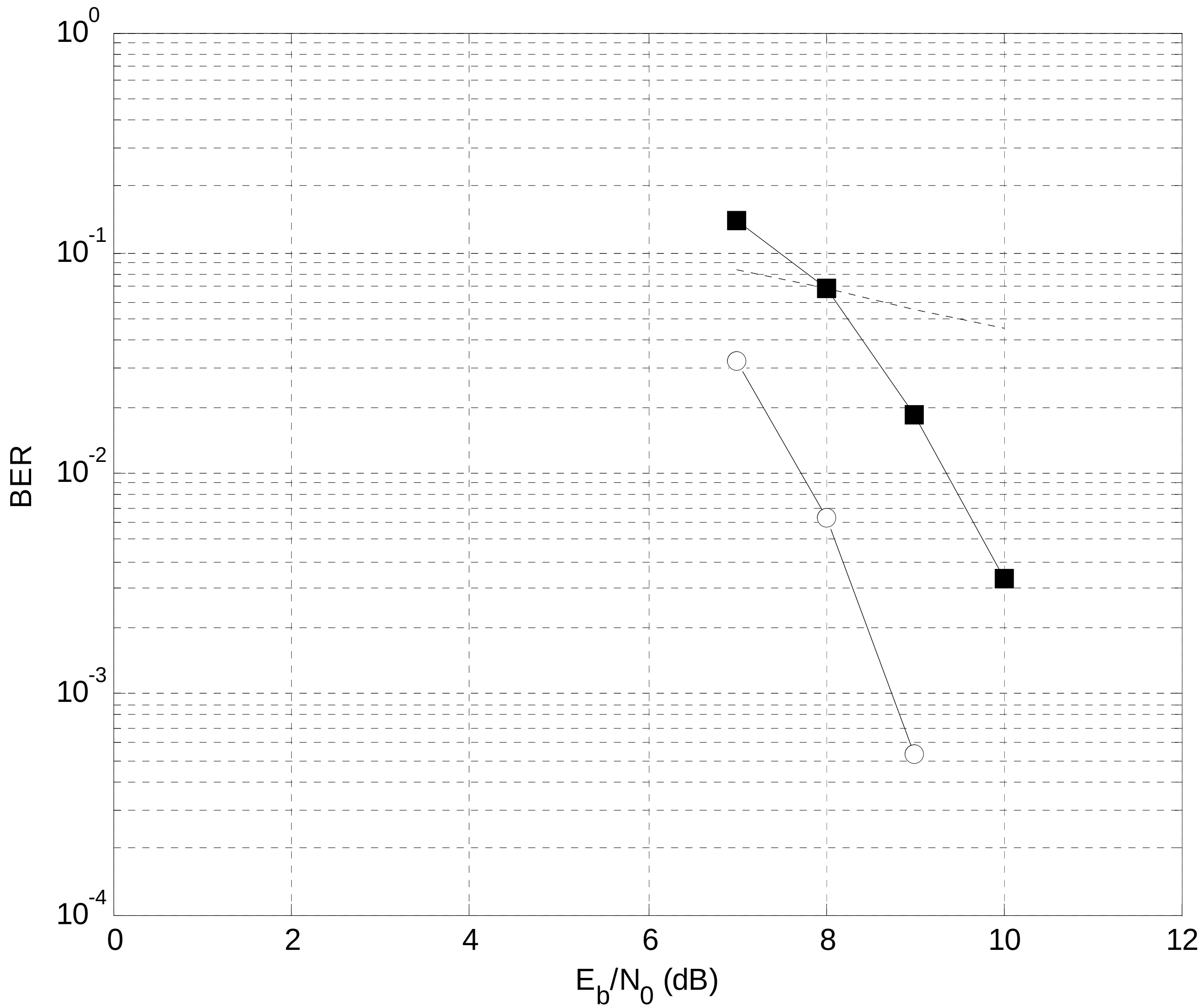
Cooperative |u|u+v|: Length 96
Flat Rayleigh fading channel



Cooperative |u|u+v|: Length 204
Flat Rayleigh fading channel



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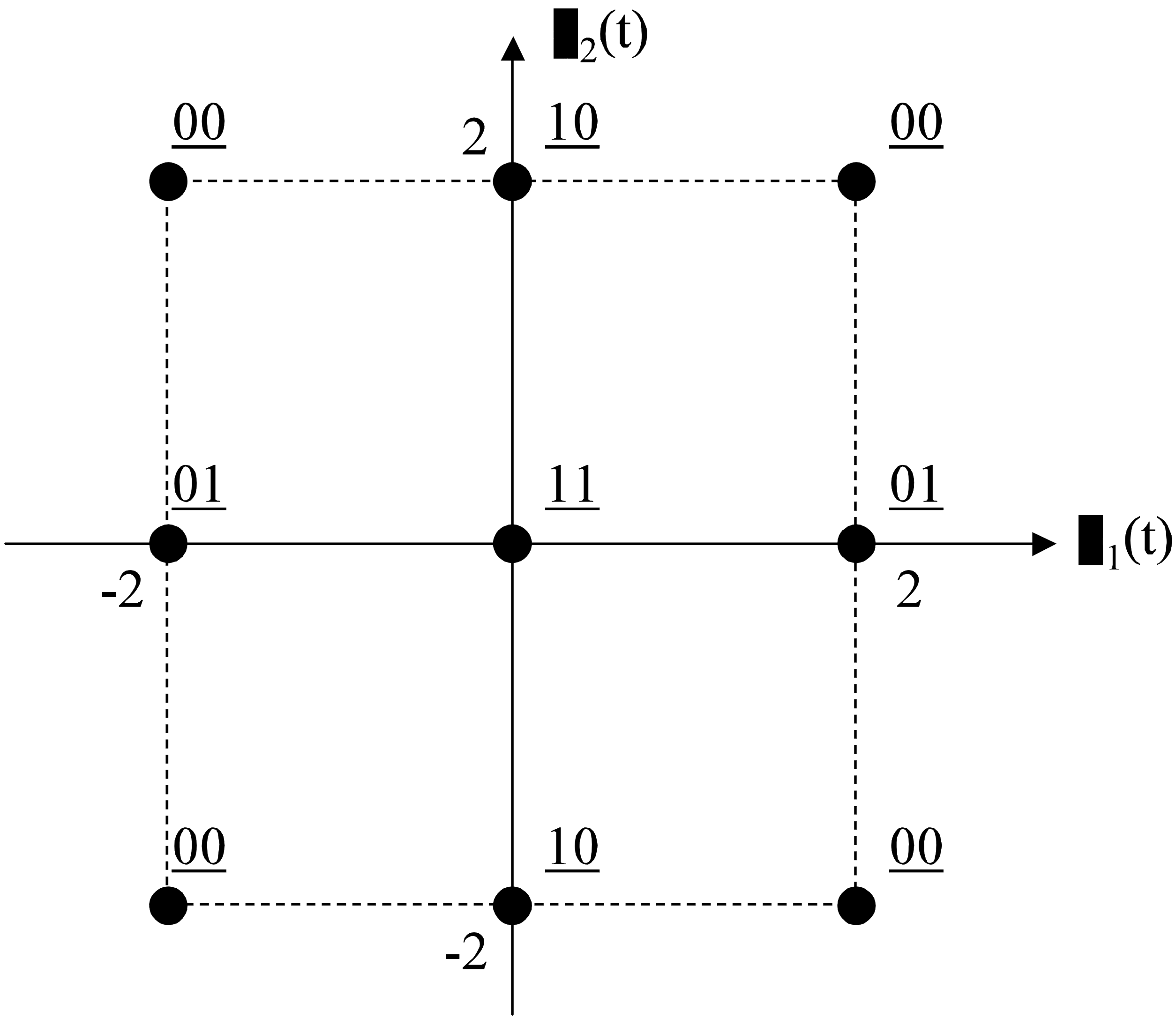
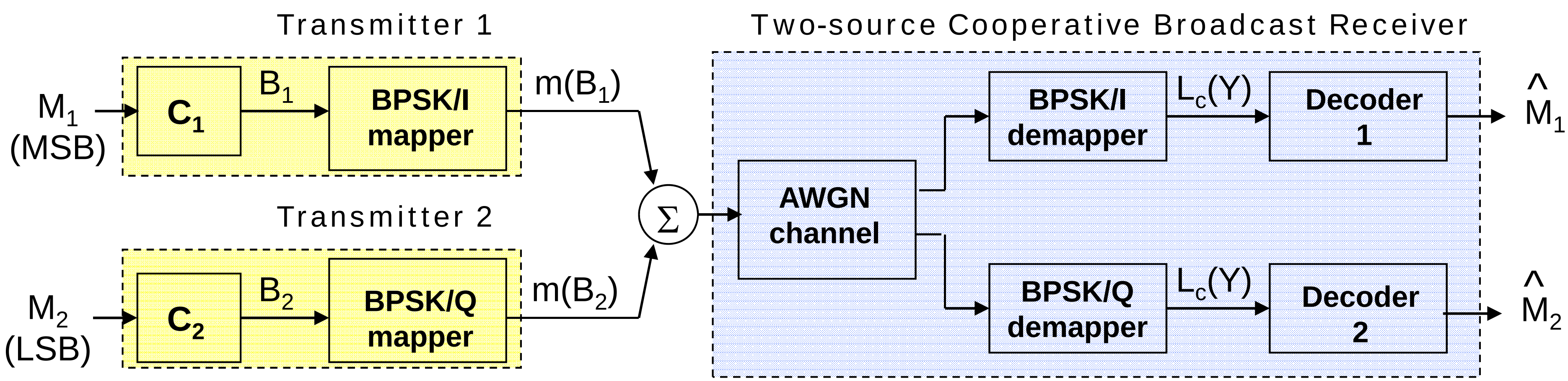
Results are applicable to QPSK modulation



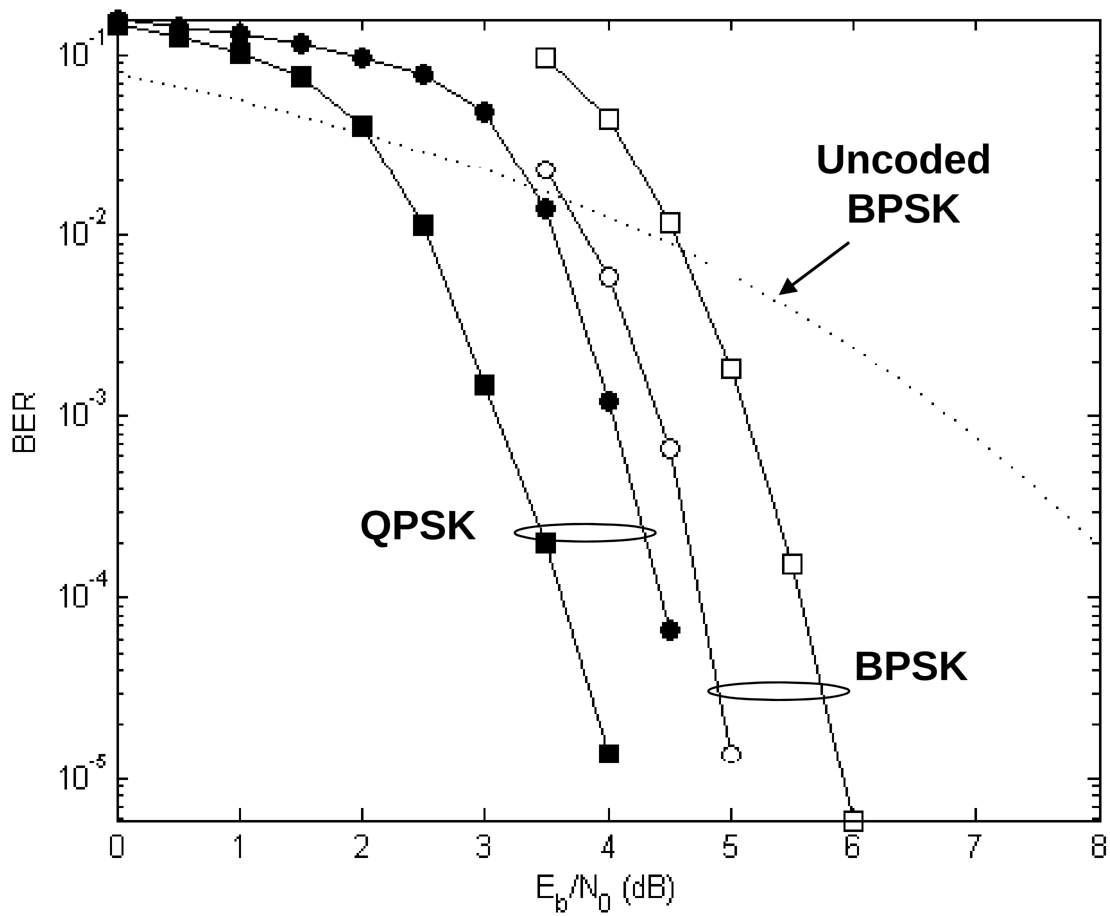
QPSK = BPSK x BPSK: Subset mapping

- Two basis functions: $\phi_1(t)$, $\phi_2(t)$.
- Basic idea: Quadrature multiplexing
- Source *i*: BPSK mapping with $\phi_i(t), i=1,2$
- Receiver processes two BPSK sequences in parallel branches



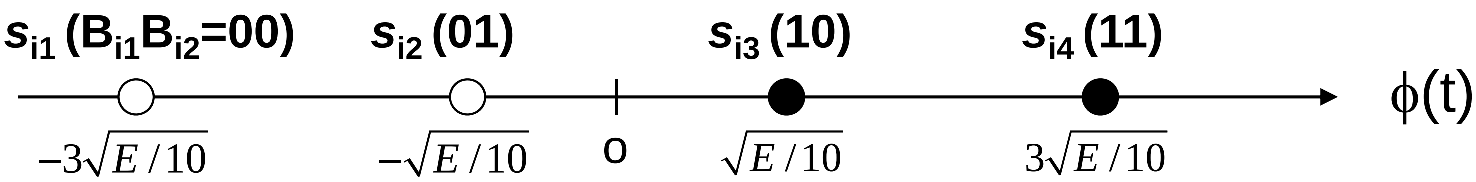
QPSK subset vs Cooperative |u|u+v| BPSK

C_1, C_2: LDPC (204,102) codes of node degrees (3,6) and (5,10)

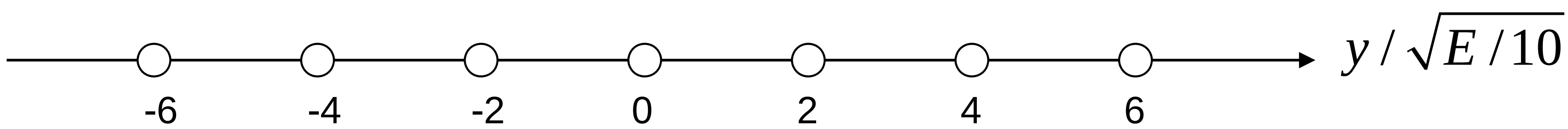


Cooperative |u|u+v| with 4-PAM and natural mapping

- ***4-PAM Mapping*** m_i from two bits to a signal set M_i , *i=1,2. Assume* $\alpha_{i1}=\alpha_{i2}$
- $M_1=\{s_{11},s_{12},s_{13},s_{14}\}$ and $M_2=\{s_{21},s_{22},s_{23},s_{24}\}$:



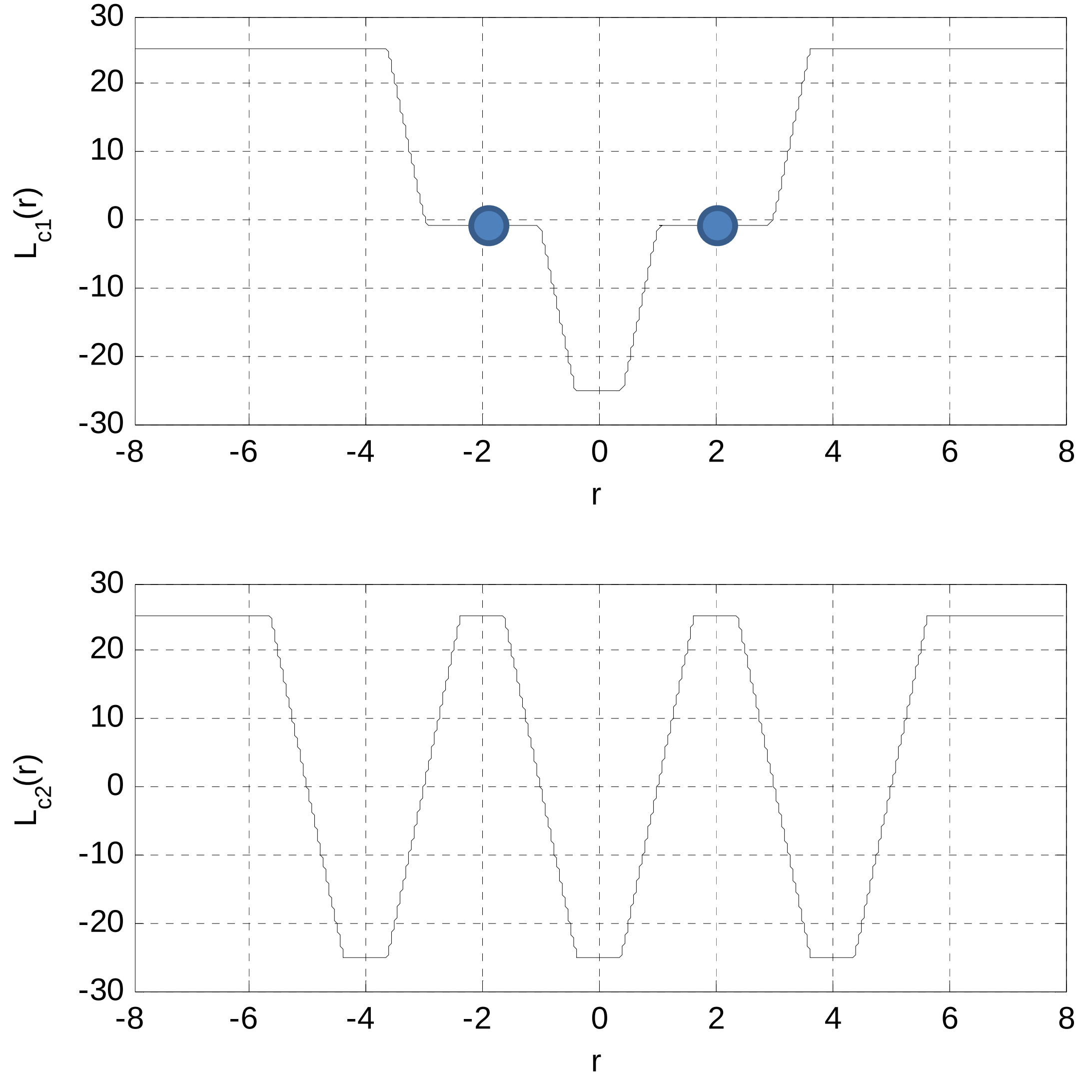
- At the receiver, the ***direct-sum*** M_1+M_2 is equal to a 7-ary signal set:



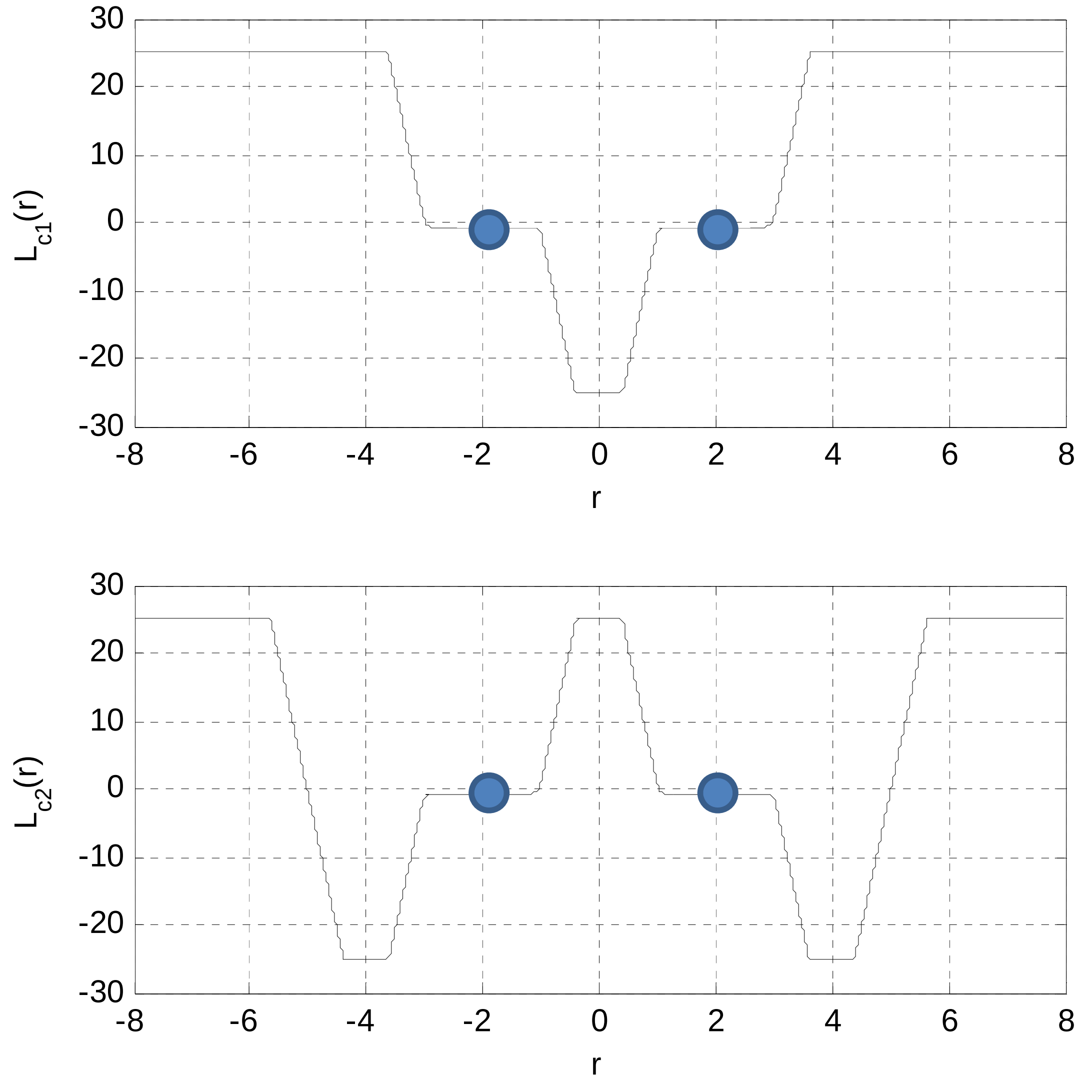
$B_{11}+B_{12}=$	0	0	1,0,1	1	1,0,1	0	0
$B_{21}+B_{22}=$	0	1	0,0,0	1	0,0,0	1	0

← ***No dichotomy for bit B2***

LLR metrics for cooperative broadcasting and 4-PAM modulation with natural mapping ($E/N_0=10$)



Metrics for cooperative broadcasting and 4-PAM modulation with Gray mapping (E/N_0=10)



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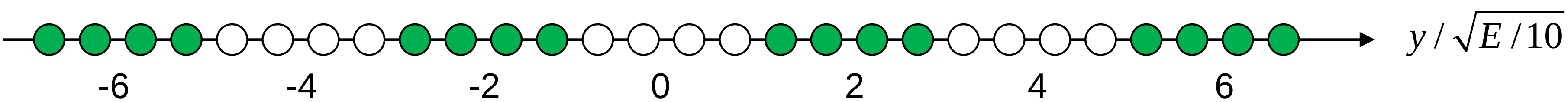
Cooperative |u|u+v| with 4-PAM and hierarchical mapping

- ***4-PAM Mapping*** m_i from two bits to a signal set M_i , *i=1,2. Assume* $\alpha_{i1}=\alpha_{i2}$
- Again, two sets: $M_1=\{s_{11},s_{12},s_{13},s_{14}\}$ and $M_2=\{s_{21},s_{22},s_{23},s_{24}\}$ with ***two power levels***
- At the receiver, the ***direct-sum*** M_1+M_2 is equal to a 28-ary signal set:

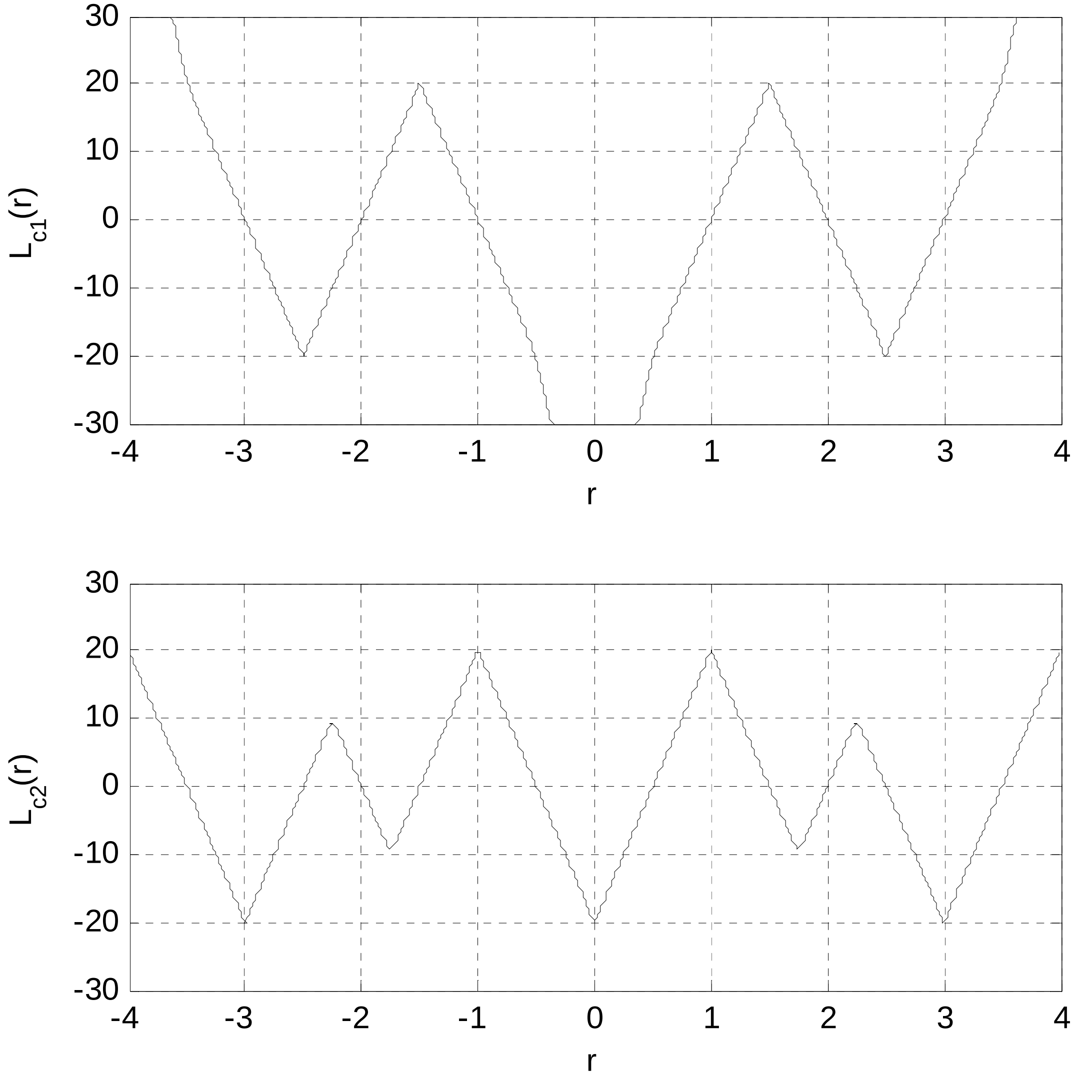


This idea was proposed by L. Xiao, T.E. Fuja, J. Kliewer and D.J. Costello (2009)

Note: In their scheme, superposition takes place at the transmitter …

- All dichotomies (i.e., metric $L_c=0$) are removed, in exchange for decreased error performance
- ***Results are applicable to 16-QAM modulation***

Metrics for cooperative broadcasting with 4-PAM modulation and <u>hierarchical mapping</u>



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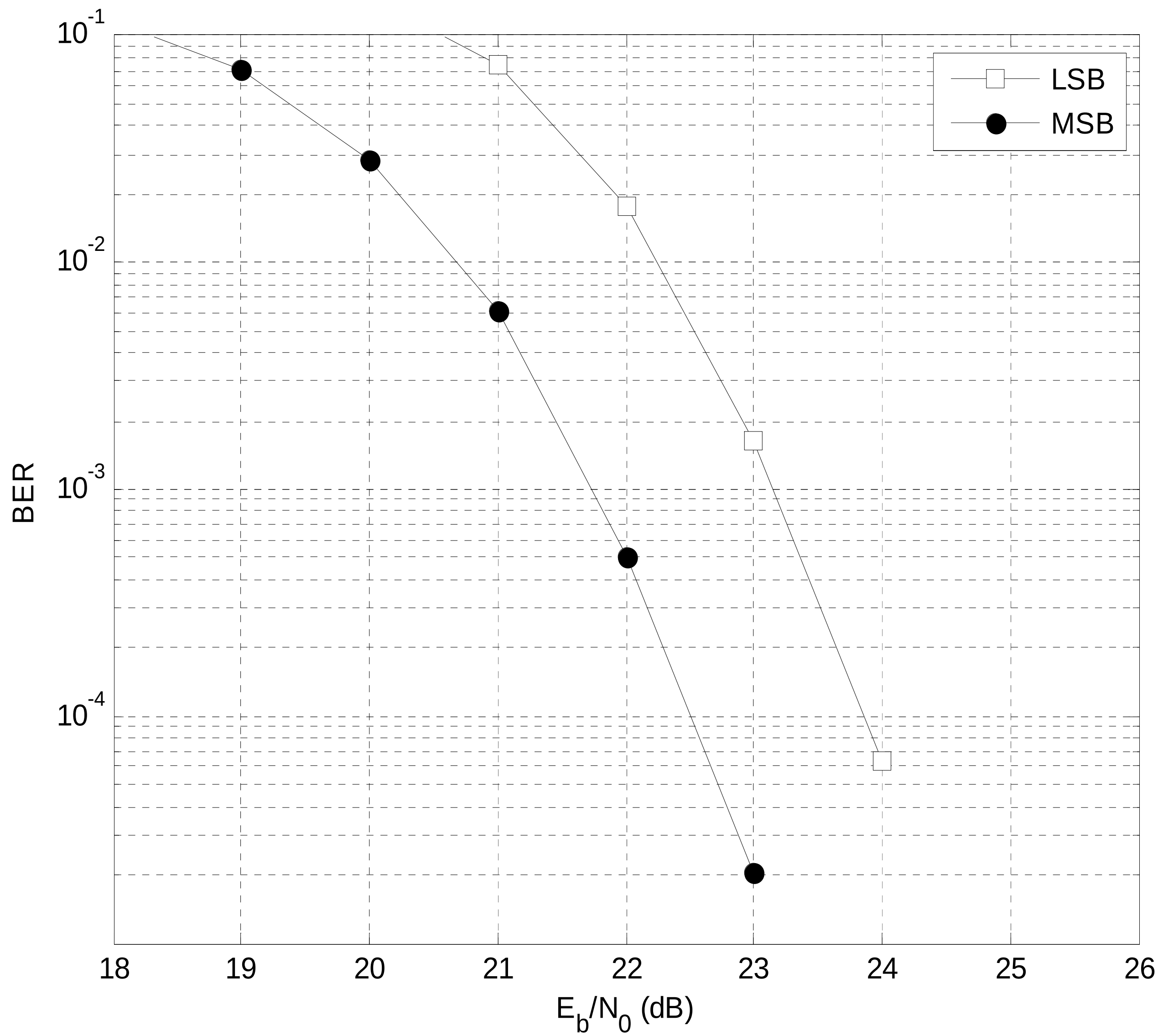
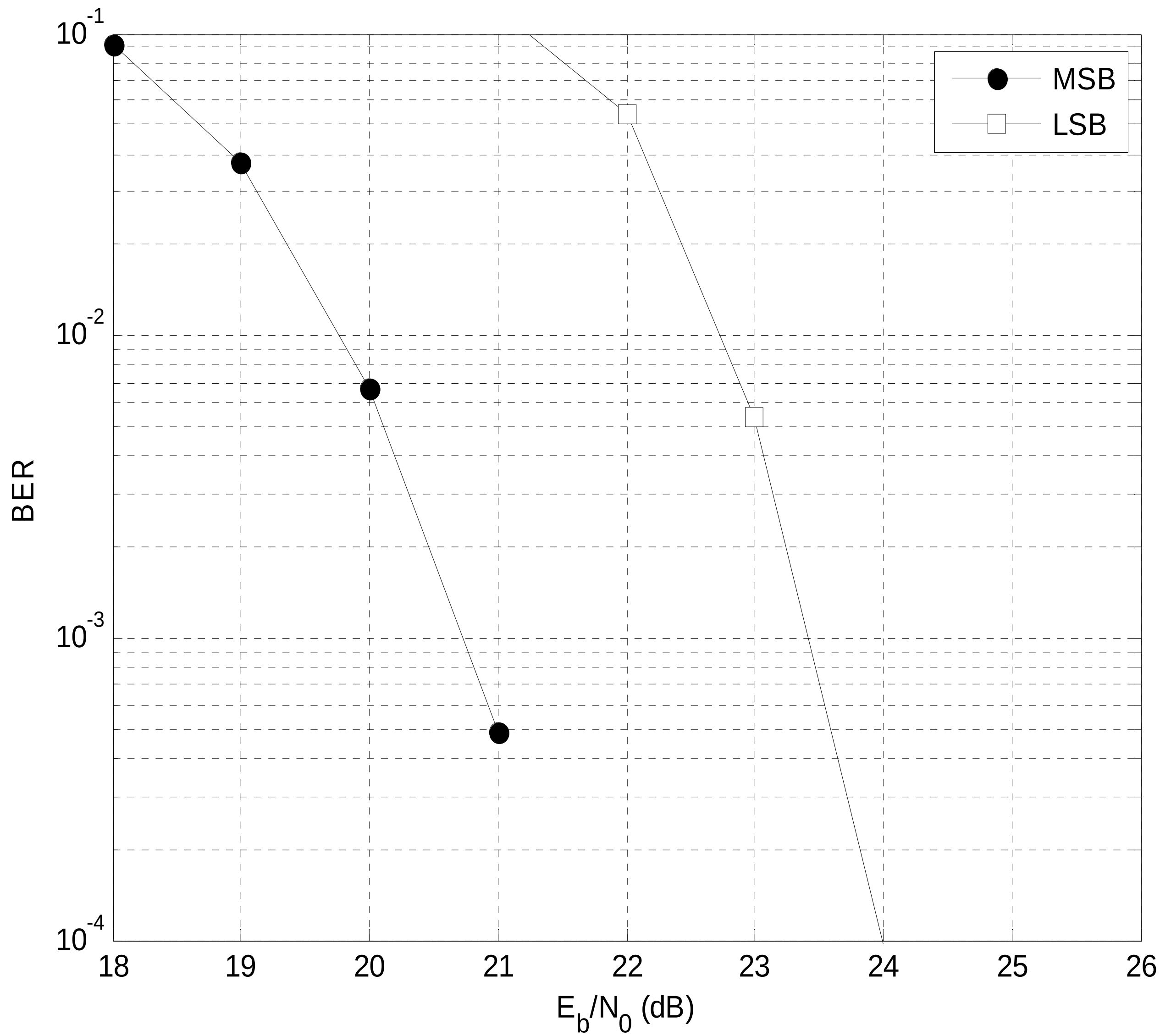
Performance of 4-PAM with hierarchical mapping. Length 96 LDPC codes



Performance of 4-PAM with hierarchical mapping. Length 204 LDPC codes



Conclusions and future work

- Proposed a coding scheme for two-user cooperative broadcasting ("over-the-air mixing"), based on Plotkin's |u|u+v| construction using BPSK, QPSK, 4-PAM and 16-QAM modulations

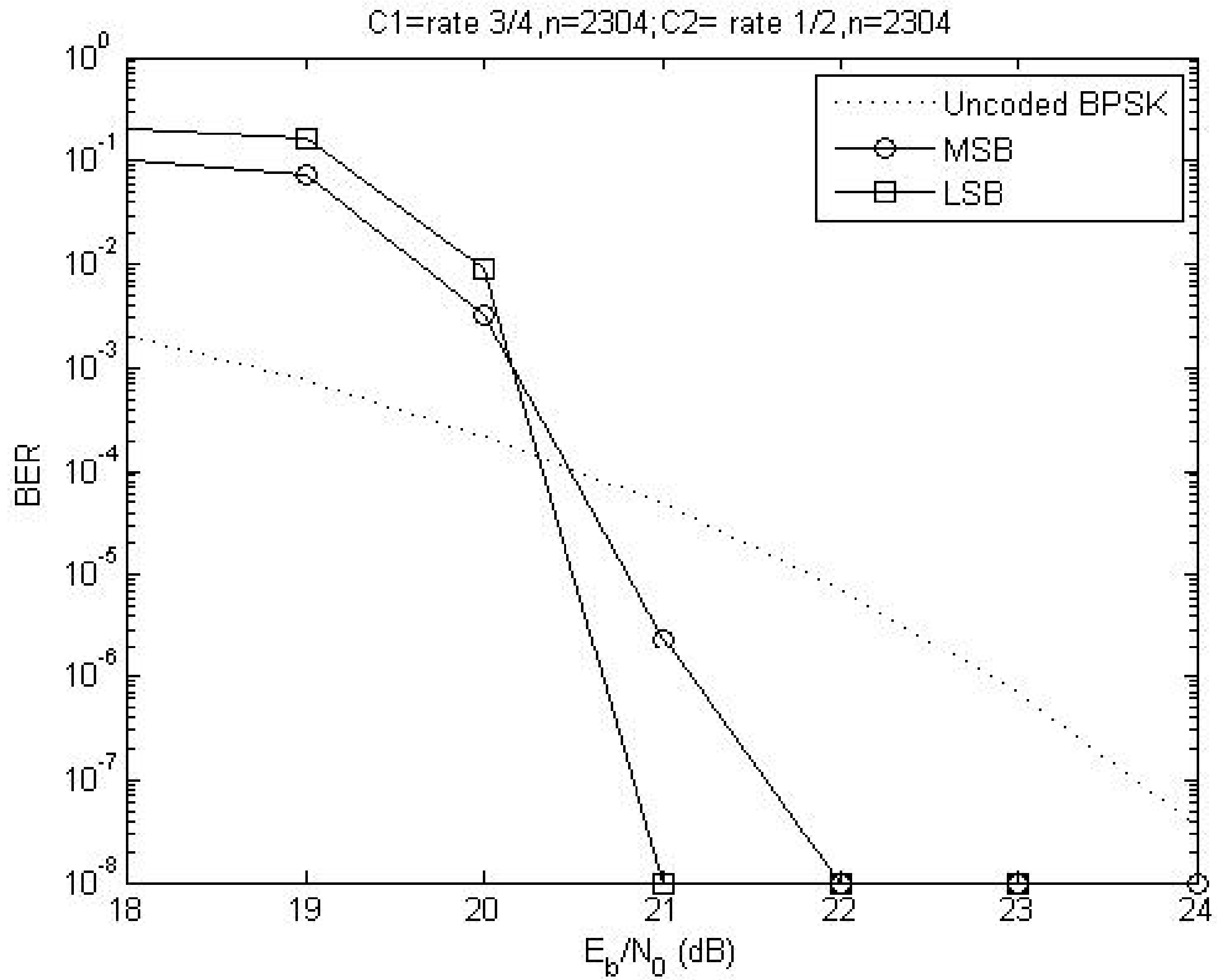
- ***Cooperative broadcasting = Network coding over physical layer***

  MILCOM 2010 presentation, comment from Matthew C. Valenti: (His paper was on "Receiver Design for Noncoherent *Digital Network Coding*")

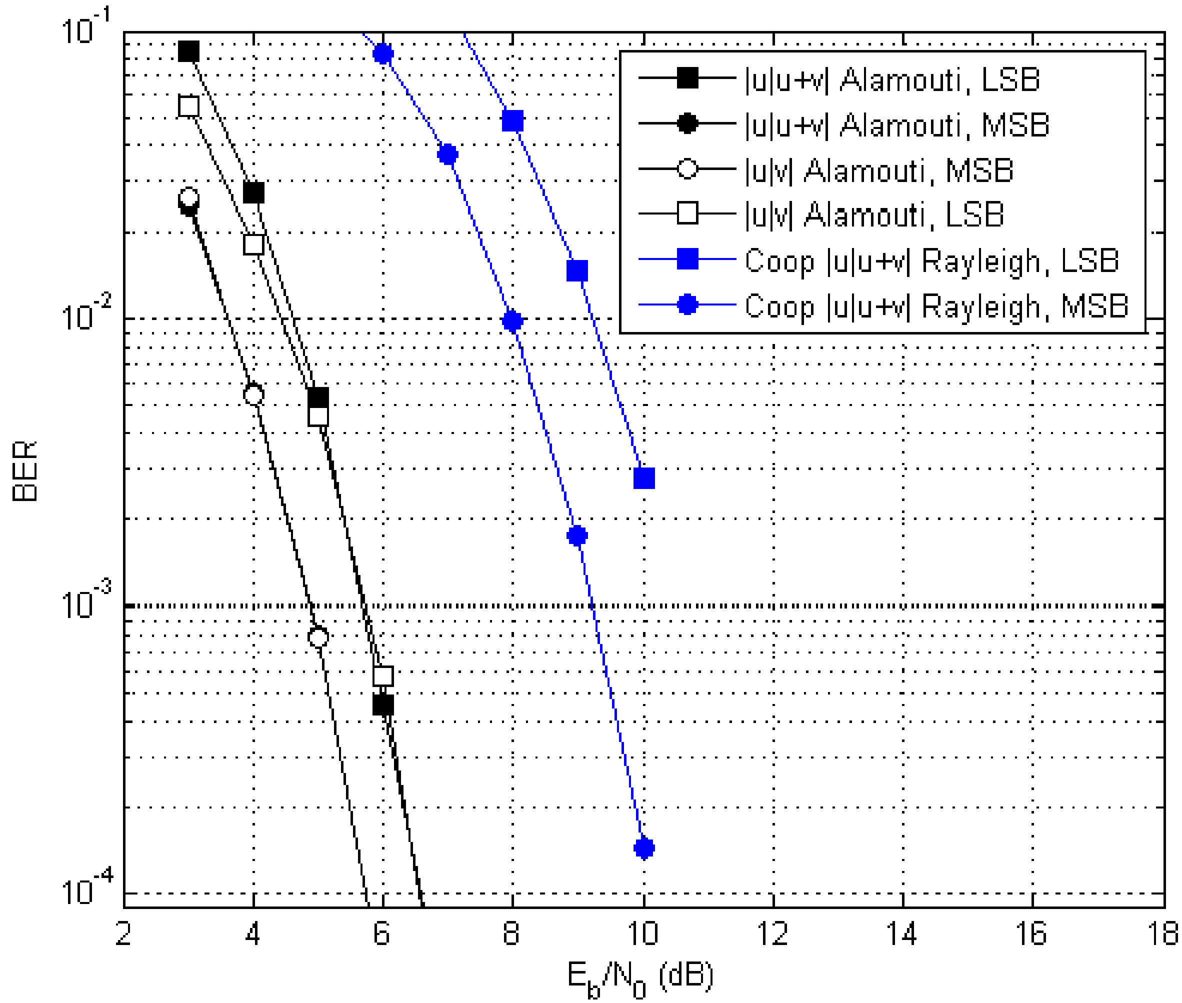
- Future directions
  - Performance with ***longer LDPC codes*** (such as those used in WiMax)
  - Design rules based on LDPC code parameters (minimum distance, node distributions) versus proportion of MSB and performance
  - Use ***a software radio*** platform to study
    - Synchronization techniques for over-the-air mixing
    - Effect of channel estimation errors
    - Error performance over realistic (frequency-selective) wireless channels
    - Noncoherent modulation: Differential encoding, FSK (as in Valenti's paper)

Supporting slides

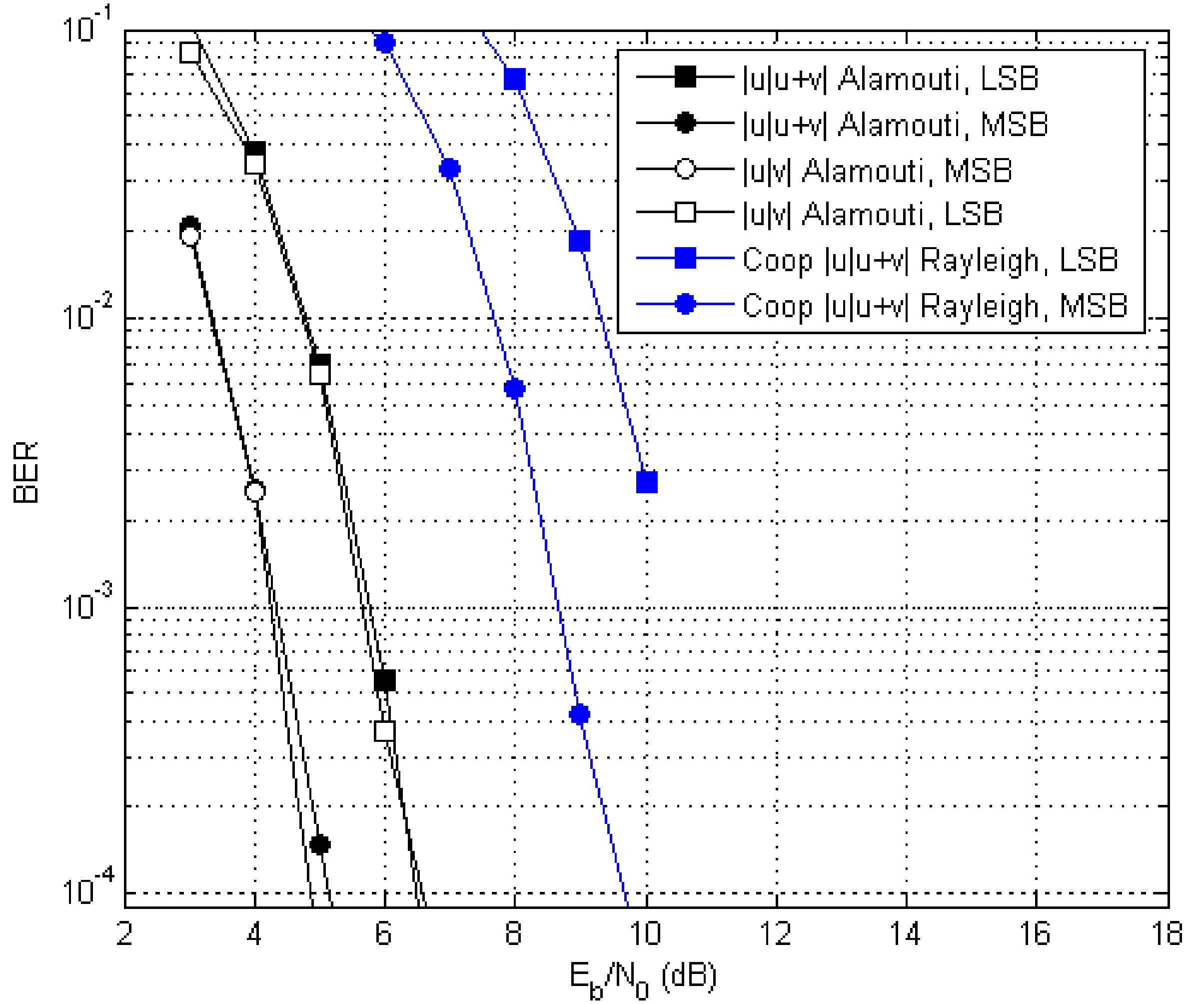
Performance of WiMax codes



Combining cooperative |u|u+v| with Alamouti: Length 96 codes



Combining cooperative |u|u+v| with Alamouti: Length 204 codes



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