1-31-2019

Counting and Basic Numerical Skills

Emily Slusser
San Jose State University, emily.slusser@sjsu.edu

Recommended Citation
Emily Slusser. "Counting and Basic Numerical Skills" *International Handbook of Mathematical Learning Difficulties: From the Laboratory to the Classroom* (2019): 521-542. https://doi.org/10.1007/978-3-319-97148-3_31

31

Counting and Basic Numerical Skills

Emily Slusser

Emily Slusser
San José State University
Child & Adolescent Development
emily.slusser@sjsu.edu
+1 408 924 3752

Abstract

The following chapter outlines a typical developmental trajectory of children's early number knowledge and counting skills. Using a series of anecdotal demonstrations of a young child's emergent knowledge as a guide, the chapter first outlines the conceptual and procedural building blocks for counting and basic numerical skills (Section 4.1 and 4.2), proceeds to an extended discussion of major conceptual achievements in counting (Section 4.3), and concludes with a review of our emerging understanding on how to best support and facilitate the development of these skills (Section 4.4). Throughout each of these sections, seminal studies are discussed to more clearly demonstrate the role of children's intuitive number sense in the construction of natural number concepts; specific challenges that children confront as they acquire the verbal count list (including several conceptual and linguistic obstacles that are often overlooked in early childhood curricula and assessments); and the effectiveness of low-cost, practical interventions that can be adopted by educators and parents to support and facilitate development.

Keywords: Counting, Cardinality, Number Words, Knower Levels, Approximate Number, Subitizing, Quantification, Acquisition

> You're enjoying a lovely day at the park with your 3-year-old nephew. A paddling of ducks waddles by and you start a conversation, *"Hey Charlie, look at the ducks! How many are there?"*
>
> A pretty straightforward question. Your nephew jumps at the opportunity to demonstrate his skills. Faithfully pointing to each duck, one-by-one, he responds, *"one…, two…, three…, four…, five!"*
>
> Ah, he's brilliant. You knew as much. Let's keep this conversation going. *"That's right!"* you say. *"So, how many ducks are there?"*
>
> He immediately responds, "*Eight!*"
>
> Right! Wait…what?

This narrative, having played out in countless situations, is likely familiar to any caretaker or educator. Indeed, the phenomenon is well documented: while most children appear to have learned to count by the time they are 2 or 2 ½ years old (Fuson 1988), most often, they are simply demonstrating their ability to reproduce a counting routine. Consequently, their behavior is often difficult to interpret – it is not, as we would be inclined to presume, a reliable indicator of their number knowledge. This is similar (and not unrelated) to that other pre-scholastic phenomenon of reciting the alphabetic without yet having developed an understanding of orthography or phonics.

In fact, even after a successful counting routine is achieved, children continue to face several underlying challenges on their way to acquiring early number concepts and basic counting skills. One of the core challenges follows from the fact that there is an important dissociation between conceptual and procedural knowledge of counting. In early phases of number acquisition, conceptual knowledge lags far behind that of procedural knowledge. Our nephew in the anecdote above has clearly learned some basic counting procedures (and recognizes that the question "*how many*" prompts these procedures) well before he will ultimately understand how this activity reveals the correct answer to this question. In fact, only over the next couple years will his incremental advances in both procedural and conceptual knowledge culminate in the ability to form and maintain precise representations of natural number (e.g., Carey 2010).

Number Sense

While ubiquitous in discussions of early education and mathematics, the term *number sense* is often used to refer to a variety of abilities and behaviors. Early childhood curricula and assessments often use the term to describe children's "fluidity and flexibility with numbers, the sense of what numbers mean and an ability to perform mental mathematics… and make comparison" (e.g., Gersten and Chard 1999). The following review, however, will adopt the term's primary definition, referring specifically to the evolutionarily primitive ability to represent non-symbolic quantity (Dantzig 1967; Dehaene 2011). This definition includes the ability to subitize (i.e., the ability to recognize the exact number of items in a small set without counting[1]; Kaufman et al. 1949), which manifests from our ability to represent and track individual items (e.g., Feigenson and Carey 2003). This definition of number sense also includes the ability to represent rough estimates of magnitude and number (e.g., Xu 2003).

Small Number Representations

> It's time for a snack. You offer your nephew two cookies but he immediately recognizes that you have given yourself three. He raises the alarm. "*How did he know?*" you think to yourself, "*didn't we just establish that he doesn't know how to count yet?*"

We can chalk this one up to the ability to represent and visually discriminate arrays of one, two, or three items, an ability available to even very young infants (Xu 2003). Consider the following experiment: 10- to 12-month old infants were presented with two adjacent buckets, one containing just 1 cracker and the other containing 2 crackers. When given the opportunity, the infants in this study

---

[1] The term 'subitize' also enjoys many definitions across early childhood curricula and assessment. The present chapter, however, will adopt and adhere to the definition provided above.

consistently chose (crawled to) the bucket with 2 crackers over the bucket with 1 (wouldn't you?) (Feigenson et al. 2004). Similarly, the infants chose the bucket with 3 crackers when the other had just 2 or 1. However, with choices of 4 *vs* 6, 3 *vs* 4, 2 *vs* 4, and even 1 *vs* 4 crackers, infants chose at random. Taken together, these results show that infants' preference for the greater number does not depend on the *relative* quantity, or the ratio of the two sets (infants consistently chose the bucket with 3 crackers to a bucket with 2 but seemed perfectly happy to go to either bucket when presented with a choice between 4 *vs* 6 crackers). Instead, their ability to make a meaningful choice is contingent upon *absolute* quantity (in this case the number of crackers), and their ability to represent these exact quantities is capped at 3 items. This limited (though impressive) ability has been demonstrated across a variety of experimental paradigms, each yielding similar results (e.g., Clearfield and Mix, 1999; Feigenson and Carey 2003; Starkey and Cooper 1980).

While greater number is generally correlated with greater continuous quantity (such as summed spatial extent or volume) in the natural world, these studies extensively control for continuous properties showing that these discriminations are based on number alone. Moreover, these representations are not limited to the visuo-spatial modality. Infants also assess exact quantities (up to 3) when presented with a series of temporal events and auditory sequences (e.g. puppet jumps and sounds; Wynn 1996).

This representational system then allows us to easily identify small, exact quantities immediately, accurately, and without counting (cf., Cordes et al. 2001). The signature limits of this system, however, remain relatively constant over the course of development (though older children and adults are often able to represent up to 5 or possibly 7 items in a set; Mandler and Shebo 1982; Trick and Pylyshyn, 1993) such that subitizing does not present a viable pathway to the representation of large, exact numbers like 27 or 308.

Approximate Number Representations

> So we've righted our mistake. Both of us now have three cookies. *Phew*. Wait… your astute (and somewhat righteous) nephew notices that yours has more chocolate chips! It seems there are a gazillion chocolate chips in each cookie, so we are well beyond subitizing. And, he's not counting… Enter the Approximate Number System.

The ability to represent large approximate quantities and detect differences between two large sets is supported by the Approximate Number System (ANS), a cognitive resource that is also available in early infancy (e.g., Lipton and Spelke 2003). Early access to this system is often demonstrated through the use of a habituation paradigm. For example, infants (as young as 6 months) are presented with a series of pictures, each with an array of 8 dots. Then, when presented with a picture with 16 dots, infants look longer at the novel array, showing that they discern the difference between sets of 8 and 16. While infants also respond to changes in overall spatial extent (e.g., summed area and/or contour length; Clearfield and Mix 1999), several studies that have controlled for alternative dimensions of quantity have shown that infants are able to make judgements on numerosity alone.

Judgements supported by the ANS, however, are imprecise and the threshold for a just noticeable difference follows Weber's Law, such that numerical discrimination is a function of the ratio between the two magnitudes under comparison, and not their absolute difference (e.g., Halberda and Feigenson 2008). Importantly, and unlike the small number representation system discussed above, ANS precision improves over the course of development (Halberda and Feigenson 2008; Odic et al. 2013). On average, 6-month-olds can reliably discriminate 1:2 ratios (such as was presented in the example above; Lipton and Spelke 2003), 9-month-olds can discriminate 2:3 ratios (Xu and Spelke 2000), 3-year-olds discriminate 3:4 ratios, 4-year-olds discriminate 4:5 ratios, and 5-year-olds discriminate 5:6 ratios (Odic et al. 2013); and adults can discriminate 10:11 ratios (Halberda and Feigenson 2008).

Notably, individual differences in ANS acuity within these age groups are associated with math achievement. In fact, several studies have shown that individuals with more precise ANS acuity perform better on tests of formal mathematics (Libertus et al. 2011; Libertus et al. 2012; Lyons and Beilock 2011). In one study, performance on the Test of Early Math Ability (TEMA-3; Ginsburg and Baroody 2003) could be predicted from ANS acuity measured at 6 months (Libertus et al. 2011). In another, numerical acuity measured in 14-year-olds correlated with their performance on standardized math tests as far back as kindergarten (Halberda et al. 2008). Furthermore, there is evidence to suggest that ANS acuity is malleable, and may be influenced by environmental factors (Tosto et al. 2014) and formal instruction (Halberda et al. 2012; Piazza et al. 2013).

Summary

Together, these two systems are considered core cognitive resources that serve as a foundation for the construction of natural-number concepts (Carey 2010). Each is clearly necessary for the development of counting and basic number skills, however, neither is sufficient. The following sections will review how children’s developing understanding of the verbal count list (e.g., individual number words such as *one*, *two*, and *three*) ultimately allows for the construction of natural number concepts (i.e., the ability to represent exactly 27 or 308).

Number Language

As discussed above, the ability to represent small, exact numbers and large, approximate numerosity is available in early infancy, but mapping these representations to symbolic representations of number (e.g., number words) is no small feat. Whereas children as young as two years old have little difficulty mapping approximate quantifiers (such as *more* and *a lot*) to representations of quantity (Dale and Fenson 1996), children can spend upwards of two years sequentially assigning meaning to individual number words and figuring out how the verbal count list works.

While a long and protracted process, the acquisition of number language is a crucial milestone in children's quantitative development (Fuson 1988; Gelman and Gallistel 1978; Wynn 1990; 1992). As the following section will discuss, the language system itself is largely responsible for the ability to represent large exact number. In fact, children who experience significant language barriers, such as may happen when born deaf to hearing parents, show delays not only in their acquisition of individual number words but also in later math achievement (Kritzer 2009). Moreover, individuals who grow into adulthood without learning to count proficiently demonstrate poorer performance on tasks assessing representations of exact number and cardinality (Frank et al. 2008; Spaepen et al. 2011).

Knower Levels

> "*The kid's really put one over on me,*" you think. When it comes to cookies, he clearly knows what he's talking about (*three* cookies is more than *two*, and don't even think about saving the cookie with more chocolate chips for yourself!). But you're not entirely satisfied so you decide to put it to the test…
>
> You give him the whole bag of cookies, but ask him if you can have just *one*. He happily obliges. One cookie, no problem. "*Can you give me two cookies?*" you ask. Sure, he hands you two. One last time for good measure – this time you ask for *three* cookies. "*Sure!*" he says as he hands over as many as he can grab. Not *three*, not *two*, but an entire handful!

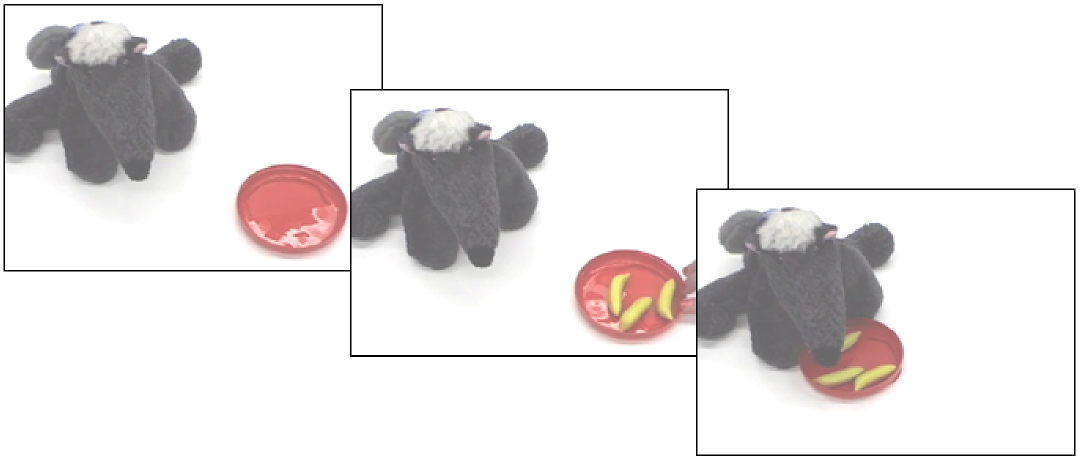
While seemingly inconsistent and unpredictable, it turns out that our nephew's response is not unusual for a 3-year-old. In fact, it often takes two or more years to learn even a subset of number words, during which time children work out the cardinal meanings of each number word one at a time and in order (Le Corre et al. 2006; Sarnecka and Lee 2009; Wynn 1990, 1992). Interestingly, as they go through this process, children appear to traverse a predictable series of knowledge states, or "knower" levels (see Sarnecka et al. 2014 for review).

This incremental progression shows up on assessments such as the Give-N (or Give-a-Number) task in which children are asked to create sets in response to specific prompts (e.g., "Can you give *three* bananas to the puppet?”) (Wynn 1992; see Figure 1). In such tasks 2- to 4-year-olds, who can generally recite the count list up to 10 or so without error, are often unable to give the correct number of items when asked for those same numbers in the Give-N task. In response to a Give-N trial asking for *six* bananas, for example, these children may simply grab a handful of items without counting, even when prompted to count or check their response (e.g., “Can you count and make sure you gave the puppet *six* bananas?” or “Can you fix it so that the puppet gets *six* bananas?”) (e.g., Le Corre et al. 2006).

At the earliest knower level (often referred to as the “pre- knower” level; e.g., Slusser et al. 2013b), children’s responses to any given prompt are generally unrelated to the number of items requested. These children may give just one item, or even a handful of items, regardless of the specific prompt. At the next level, children reliably give 1 item when asked for *one*, but give 2 or more items when asked for any other number. Note that their responses seem to be simple guesses, not counting or estimation errors (Sarnecka and Lee 2009), and these children appear to understand that number words that they do know are not used to refer to sets of any other size (i.e., they will not offer 1 item when asked for any number other than *one*; Wynn 1990, 1992). The one-knower level is followed by the “two-knower” level, then the “three-knower” level, and sometimes the “four-knower” level. At each *N*-knower level children demonstrate predictable and accurate performance up to *but not* beyond *N*. Eventually, around the time they reach the three- or four-knower level (often 2 years after they first entered the one-knower level), children realize that the final number word in their count sequence refers to the cardinal value of the set they are enumerating. At this point they may be said to have induced the “cardinality principle” (Gelman and Gallistel 1978), and can henceforth employ counting procedures felicitously to create any set size within their count list (Sarnecka and Carey 2008; Wynn 1990; cf. Davidson et al. 2012). It has been argued that, as children progress through these individual knower levels, they are incrementally assigning each of the first three or four number words to their representations of small, exact sets (see Section 4.1 above; Carey 2010). Numbers exceeding the set size limit of 3 or 4 items must then be represented through counting. For this reason, we don’t typically see children who would be characterized as “five-”, “six-”, or “seven-knowers” (cf. Wagner and Johnson 2011).

Figure 31.1. The Give-a-Number task can be used to assess children’s number-knower levels (e.g., Wynn 1992). For this task, children are typically asked to create set sizes of 1 to 6 items. Children are given the opportunity to check and fix their responses after each trial.

“Can you give the puppet *three* bananas?”



““Is that *three*? Can you check to make sure that’s *three*?”

The one- through four-knower levels are found not only for speakers of English, but also for speakers of Japanese (Sarnecka et al. 2007), Mandarin Chinese (Li et al. 2003), and Russian (Sarnecka et al. 2007). Furthermore, bilingual children who have memorized the counting lists in both of their languages before learning the exact meanings of these words in either language show the same or similar knower-levels in both languages (Goldman et al. 2014).

There is, however, notable variability across children with different learning backgrounds and experiences. For example, while children from relatively high socioeconomic backgrounds typically reach an understanding of cardinality sometime between 3 to 4 years old (see Sarnecka and Lee 2009), children from less privileged backgrounds often do not reach this level of understanding until well after their fourth birthday (e.g., Dowker 2008; Jordan and Levine 2009).

While the cardinality induction is often recognized as a major conceptual achievement, we will put this aside for now (but revisit it in Section 4.3 below). The following sections will instead explore what subset-knowers (a term used to describe children at the one-, two-, three-, and four-knower levels; Le Corre et al. 2006) know and have yet to learn about number.

Discrete Quantification

One piece of knowledge that is integral to understanding natural-number concepts is the idea that number is a property of sets and that sets are comprised of discrete individuals. Indeed, a conceptual dissociation between continuous substances (such as water and sand) and discrete objects (such as blocks and coins) is available in infancy (Hespos et al. 2009), and as children acquire language they reflect this distinction through their appropriate use of linguistic morphology (i.e., the English singular/plural marking) to denote the difference between mass and count nouns (e.g., Barner et al. 2007).

To determine whether children with an incomplete understanding of number words (i.e., subset-knowers) understand that number words, in general, are used to refer only to sets of discrete individuals, we invited a group of subset knowers (2 to 4 years old) to complete the Blocks and Water task (Slusser et al. 2013b, see Figure 2). For this task, children watched as an experimenter placed five objects (e.g., blocks) in one cup and five scoops of a continuous substance (e.g., water) in another cup. Four trials asked children about a number word outside the range of numbers know by any subset-knower (e.g., “Which cup has five?”) and another four trials asked about a quantifier (e.g., “Which cup has more?”) [2]. For half of the trials, the cup with discrete objects was full; for the other half, the cup with the continuous substance was full. Results showed that, while children correctly chose the full cup when asked which cup has “more”, they had to have reached the three-knower level before reliably choosing the cup with discrete objects as an example of “five”. A series of follow up experiments seem to indicate that one- and two-knowers have an emergent but tenuous understanding of this constraint but are, in general, as likely to extend the word “five” to continuous substances as to sets of discrete objects.

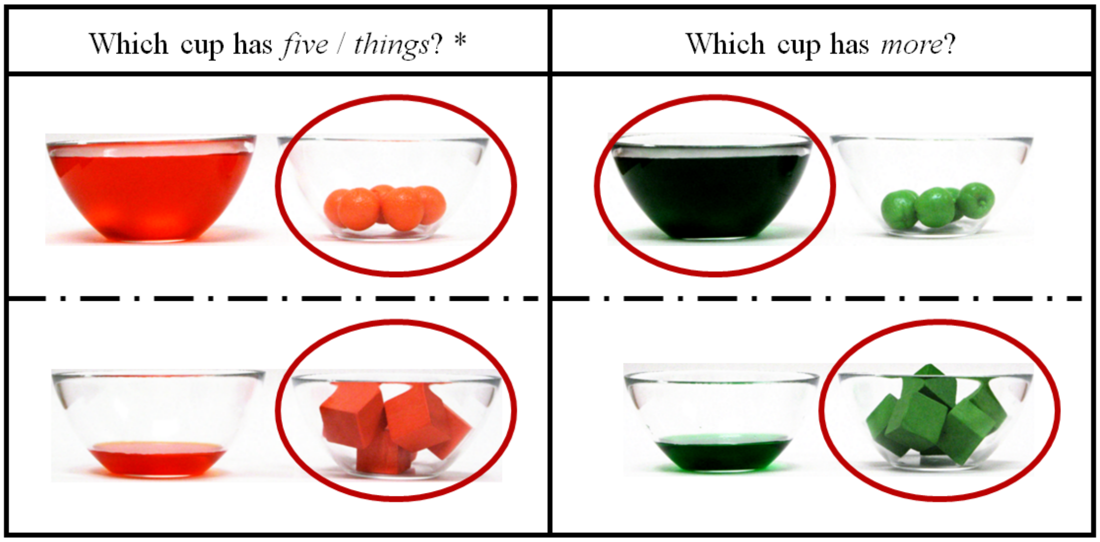
---

[2] Note that approximate quantifiers such as “more” and “a lot” can take a wide range of referents, with few constraints, while number words refer only to collections of discrete individuals.

Thus, it seems that children come to understand that number words are used for discrete quantification only after learning the precise meanings of at least a subset of number words. It is possible then that children use their understanding of the number words “one” and “two” to draw inferential connection between number words and discrete objects. Alternatively, children may use the linguistic context that generally occurs in natural speech to form this connection (Bloom and Wynn 1997). This argument arises from the observation that number words reference nouns morphologically coded according to their conceptual category (i.e. count *vs* mass) – that is to say, count nouns take the plural marking, “-s”, whereas mass nouns do not. After first confirming that number words are in fact most often accompanied by an adjacent count noun and plural marking (e.g., “Look, five *ducks*!”) in both child and child-directed speech (Slusser 2010), we tested whether children use this information to establish that number words reference count nouns, and consequently collections of discrete objects.

The 2- to 4-year-old children in this study completed the Blocks and Water task above, but in this iteration each test question was presented within a syntactically “rich” linguistic context (Slusser 2010; see Figure 2). For example, children were asked, “Which cup has five *things*?” rather than “Which cup has five?” Results show that English-speaking children connect number words to discrete quantification before learning the specific meaning of any number words *so long as* the number word is paired with an adjacent count noun and plural marking. Similarly, Mandarin-speaking children demonstrate similar learning trajectories when presented with a number word in isolation and when accompanied by the noun classifier 個 (pronounced “*ge*”).

Figure 31.2. The Blocks and Water task was used to determine whether and when children understand that number words reference discrete sets (Slusser et al. 2013b) and whether linguistic context (in the form of a count noun + plural marking in English or the general noun classifier, 個 [*ge*], in Mandarin) facilitates this understanding (Slusser 2010). (Figure adapted from Slusser et al. 2013b)



* Prompt differed according to the experiment and trial type.

Note: The cup with continuous substance is full for half of the trials. Red circles indicate the correct response.

Overall, this series of experiments shows that children use their emerging understanding of number words as well as linguistic cues that occur in natural speech to connect number words to discrete quantification. Moreover, these data constrain future hypotheses on how children learn number words: The fact that this process may involve generalization from certain exemplars and surrounding language provides evidence that number word knowledge is not entirely built upon a priori principles.

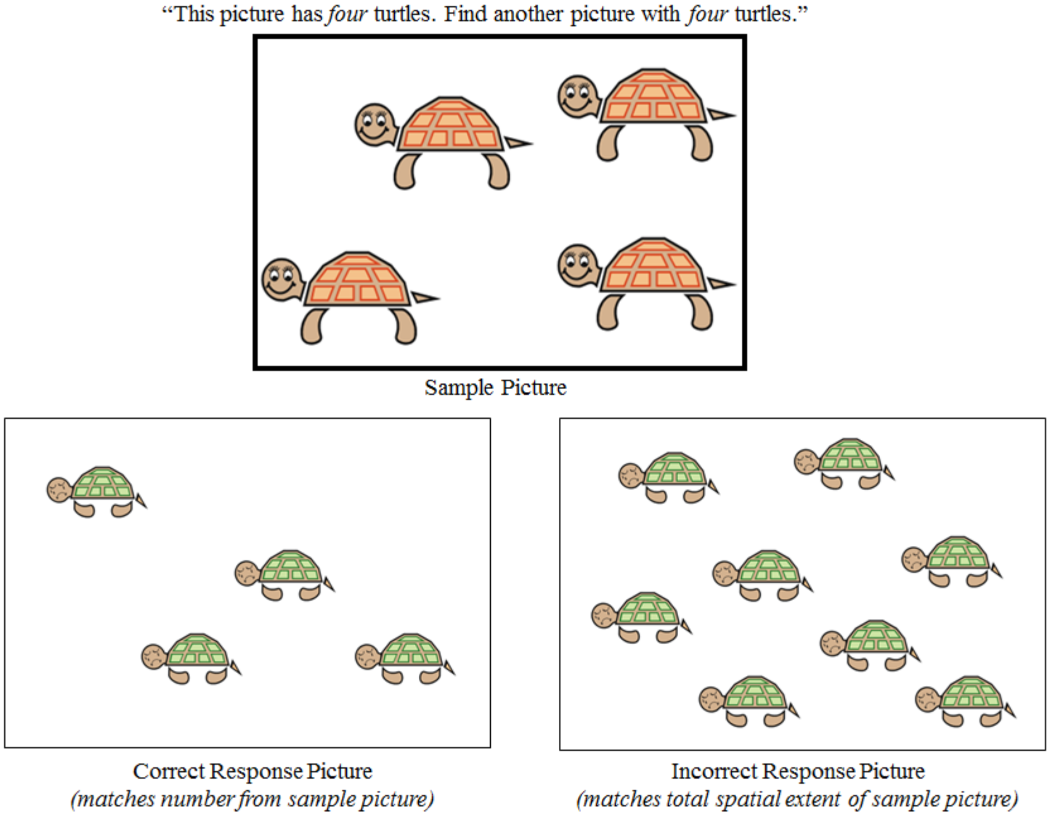
Numerosity

Connecting number words to discrete quantification, is only one step in acquiring an understanding of natural numbers. Children must also understand that number words denote numerosity (and not, for example, some other characteristic of set, such as total volume or spatial extent). Setting out to address this question, Sarnecka and Gelman (2004) invited 2 to 5 year old subset- and CP-knowers to complete the Transform Sets task. For this task, the experimenter placed a certain number of objects in a box while saying (e.g.), “I’m putting *six* buttons in this box.” The experimenter then performed some action with the box (either shaking it, turning it around, adding one object, or removing one object). The children were then asked (e.g.), “Now how many buttons are in the box? *Five* or *six*?” Results show that subset-knowers (and CP-knowers) do indeed understand that the number word should change when an item has been added or removed from the box (and that the number word does not change when a non-numerical transformation takes place, such as when the experimenter simply shakes the box). It seems that, while they still do not understand the precise meanings of the number words *five* and *six* (as illustrated through their performance on the Give-N task) subset-knowers do understand something about these number words – that they denote some aspect of quantity.

Note the use of the term *quantity*, not *numerosity*. Upon careful inspection, we see that the Transform-Sets task does not disambiguate number or numerosity from the broader dimension of quantity. Remember, children’s intuitive number sense supports representations of both numerosity and continuous spatial extent (see Section 4.1 above). In the Transform-Sets task described above, the number of items in the box changed, but so did other dimensions of quantity (i.e., area, volume, weight). While subset-knowers clearly associate number words with quantity, it is not entirely clear whether they understand that number words refer specifically to numerosity.

To address this specific confound, we developed a Match-to-Sample task with careful controls and manipulations of continuous spatial extent (either summed area or contour length, depending on the trial) so as to pit dimensions of quantity directly against numerosity (Slusser and Sarnecka 2011; see Figure 3). For this task, children were presented with a sample picture as the experimenter said (e.g.), “This picture has *four* turtles.” The experimenter then presented two additional pictures and said (e.g.), “Find another picture with *four* turtles.” One picture had the same number of items as the sample but different overall spatial extent (e.g., 4 small turtles). The other had a different number of items, but the same overall spatial extent (e.g., 8 small turtles). Results showed that while CP-knowers understand that two sets of the same numerosity should be labeled with the same number word, subset-knowers are as likely to extend that number word (e.g., *four*) to other dimensions of continuous quantity (by, in this case, selecting a picture of 8 small turtles).

Figure 31.3. A Match-to-Sample task was used to determine whether children understand that number words denote numerosity, rather than some other dimension of quantity (e.g., summed spatial extent) (Slusser and Sarnecka 2011). (Figure adapted from Slusser & Sarnecka, 2011)



"This picture has *four* turtles. Find another picture with *four* turtles."

Sample Picture

Correct Response Picture
(matches number from sample picture)

Incorrect Response Picture
(matches total spatial extent of sample picture)

Note. On this particular trial, there is no possible match on the characteristics of the individuals comprising the set (e.g., the color or mood of the turtles).

Summary

Taken together, these findings reveal that subset-knowers' understanding of numbers matures immensely as they acquire the meanings of individual number words. In addition to enriching our understanding of how children's understanding develops over time, these studies highlight a series of additional conceptual and linguistic challenges that are often overlooked in the development of early childhood curricula and assessments.

Counting Principles

The previous section discusses how children learn each of the number words in their count list one-by-one and in order. The process appears to take upwards of 2 years, and as they do this they learn some of the fundamental properties of number (i.e., number words refer only to discrete sets and are used to denote numerosity, not continuous quantity). Whereas the counting routine, in and of itself, does not appear to be integral to this process, children are certainly gaining experience and learning about counting procedures over this period of time.

As Gelman and Gallistel (1978) pointed out in their seminal work on *Young Children's Understanding of Numbers*, in order to count productively, children (and adults) must at the very least (1) recite the count list in the same sequence every time (e.g., *one, two, three, four* and not *one, four, three, two*); (2) count each object in a set without skipping or double-counting, (3) understand that they can count the objects in any order (e.g., counting from left-to-right yields the same answer as when counting from right-to-left), and (4) understand that the last number word recited in the counting routine indicates the total number of items in the set. While the first three rules seem to unfold with experience and practice, the following sections will focus on the final counting principle in this list – the cardinality principle.

Cardinality Principle

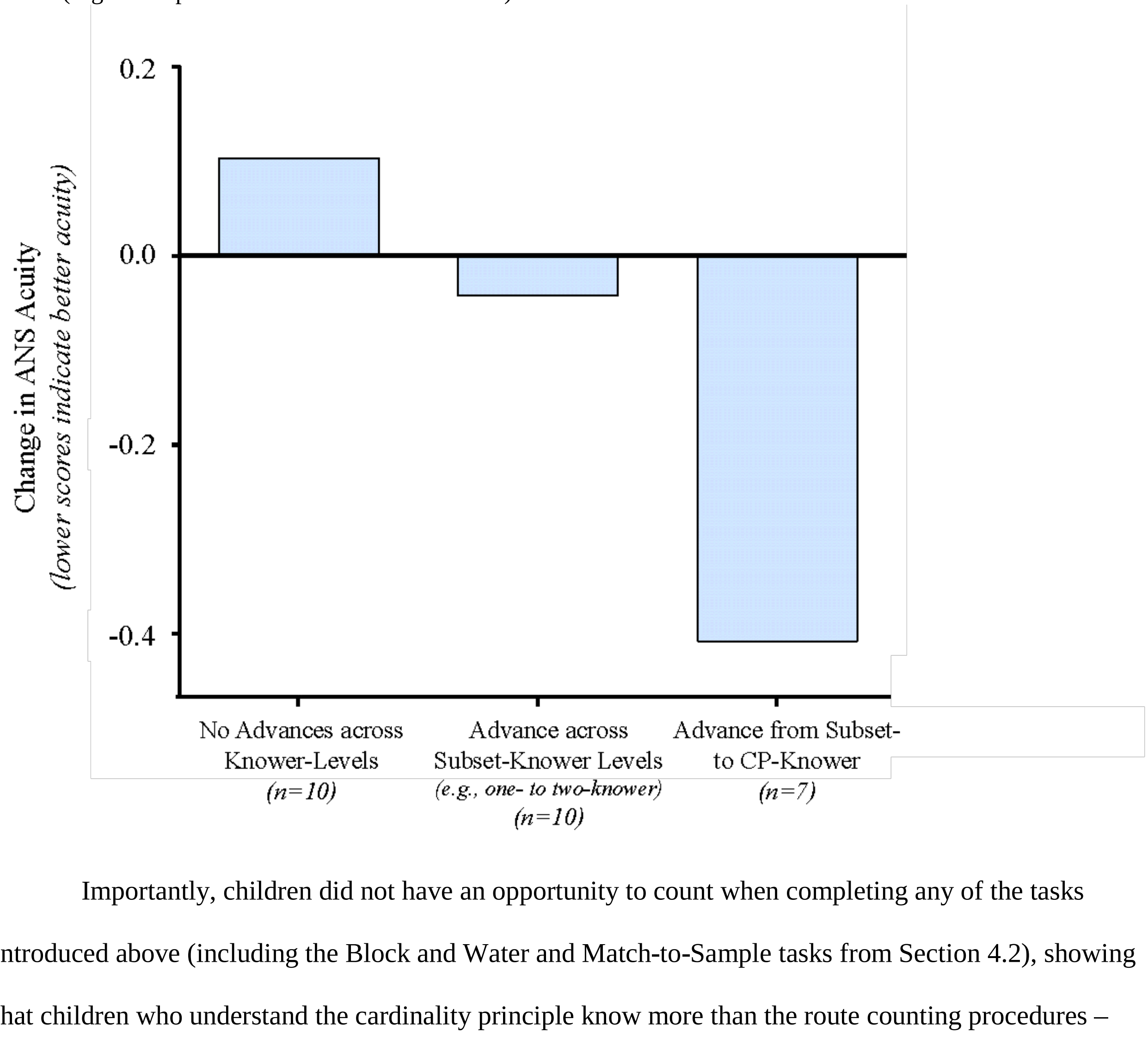
> After your little experiment with the cookies, you think back to your conversation about the ducks in the park. Your nephew *did* recite the count list in order; he *did* count each duck in one-to-one correspondence, and he didn't seem too concerned with the order or arrangement of the ducks. But wait… there's just one thing missing. He did *not* seem to understand that the last word in his count list should indicate the total number of ducks. Well, jeez, that seems simple enough…

When considered a part of Gelman and Gallistel's (1978) list of counting principles, the cardinality principle (or 'last word rule') simply stipulates that the last number word in a count sequence represents the cardinal value of that set. In reality, however, it seems children's understanding of this specific procedure is contingent upon a crucial conceptual induction – often referred to as the cardinality principle induction (Carey 2010). As mentioned previously (Section 4.2), prior to this induction, children progress through a series of intermediate knowledge states (knower levels), during which time they do not seem to understand how counting is used to generate or identify specific set sizes (e.g., Le Corre et al 2006). Importantly, children who understand the cardinality principle (i.e., CP-knowers) perform differently from subset-knowers on a variety of tasks assessing early number knowledge. Some of these tasks explicitly involve counting. For example, on the Give-N task, CP-knowers use counting to generate specific set sizes and can fix their answers when they make mistakes. While subset-knowers often engage in counting behaviors (extensively abiding by the counting principles outlined above), they fail to use counting to generate specific set sizes. Some tasks, however, do not explicitly involve counting. Examples of these include the Blocks and Water and Match-to-Sample tasks discussed above, which reveal that subset-knowers do not yet understand the fundamental properties of number words (i.e., that they are used for discrete quantification and denote exact numerosities).

Another notable difference between subset- and CP-knowers is that only CP-knowers understand that any set with *N* items can be put into one-to-one correspondence with any other set labeled with the same number word (*N*) – an idea referred to as 'equinumerosity' (Muldoon, Lewis and Freeman 2009; Sarnecka and Wright 2013). Like many of the skills outlined above, children's understanding of equinumerosity seems to align closely with their induction of the cardinality principle. For example, if one child were to have a handful of grapes for a snack and the other was offered the same (both snacks are recognized to be "just the same" through one-to-one correspondence) then each snack should also be labeled with the same number word. Results on a task that evaluated children's understanding of this concept show that only CP-knowers know that sets that are "just the same" are labeled with the same number word (and if the sets are not the same then a different number word should be used) (Sarnecka and Wright 2013).

Furthermore, there is emerging evidence to suggest that children tap into ANS representations as they learn how counting represents number (Carey et al. 2017; Chu et al. 2015; Shusterman et al. 2016; van Marle et al. 2014). One such study tracked 2-to 4-year-old's understanding of individual number words and counting procedures (through the Give-N task) as well as their ANS acuity over a 6-month period (Shusterman et al. 2016). Results show that children's acquisition of the cardinality principle is tightly linked to marked improvement in ANS acuity and that there is little evidence to suggest that ANS representations underlie advancements across subset-knower levels (e.g., moving from the one-knower to two-knower level) (see Figure 4). These findings provide further evidence for the notion that the cardinality principle is not just a counting rule – it is essential to the creation and representation of natural number concepts.

Figure 31.4. A 6-month longitudinal study evaluating children's developing number knowledge, counting skills, and ANS acuity shows that the acquisition of the cardinality principle is tightly linked to notable increases in ANS acuity (Shusterman et al. 2016). Note that ANS acuity is not clearly linked to advances across number-knower levels. (Figure adapted from Shusterman et al. 2016)



Importantly, children did not have an opportunity to count when completing any of the tasks introduced above (including the Block and Water and Match-to-Sample tasks from Section 4.2), showing that children who understand the cardinality principle know more than the route counting procedures –

they have developed deeper insight about numbers and number words. Thus the promotion from subset- to CP-knower seems to be far more profound than it initially appears.

Successor Function

With the cardinality principle comes an understanding of the successor function, which reflects another fundamental property of number – with each additional item in a set, we advance one step (i.e., word) along the verbal count list. In conjunction with the cardinality principle, an understanding of the successor function allows children to represent the cardinal meanings of every word in their count list (Sarnecka et al. 2014).

To explore children's understanding of the successor function, Sarnecka and Carey (2008) showed a group of 2 to 4 year old children a box with 5 items inside. Similar to the Transform Sets task described above, experimenters explained (e.g.), "There are *five* apples in this box," and then added an item to the box. In this task, however, the experimenter asked (e.g.), "Now how many are in the box? *Six* or *seven*?" As with the tasks reviewed above, only the CP-knowers seemed to understand that adding 1 item to a set moves the total count one step (word) forward along the count list (and adding 2 items moves the count two steps forward).

Together, children's understanding of the cardinality principle and successor function are often considered to be "the final piece of the puzzle" (Sarnecka et al. 2014) – the last thing that children must figure out in order to use counting to construct natural number concepts.

Summary

While your 3 year old nephew at the beginning of this chapter has clearly memorized several words in the verbal count list and has acquired at least some of Gelman and Gallistel's (1978) counting principles, it seems this routine serves no meaningful purpose other than offering the expected response to the question "how many?". Gradually, however, over the next several months or years, he will come to realize that counting is used to determine the exact number of items in a set, and that cardinality changes with each additional item.

Facilitating the Acquisition of Exact Number Concepts

Sections 4.2 and 4.3 above outline several challenges that children inevitably face as they develop counting and basic numerical skills, while presenting the argument that children must confront and conquer these challenges in order to construct and represent exact number concepts. Moreover, recent research has identified these achievements as central to children's eventual success in school (Aunio and Niemivirta 2010; Bartelet et al. 2014; Duncan et al. 2007; Göbel et al. 2014), with the unfortunate caveat that children who start school without these fundamental number concepts are at a serious disadvantage, both in the short- and long-term (Dowker 2008; Jordan et al., 2009).

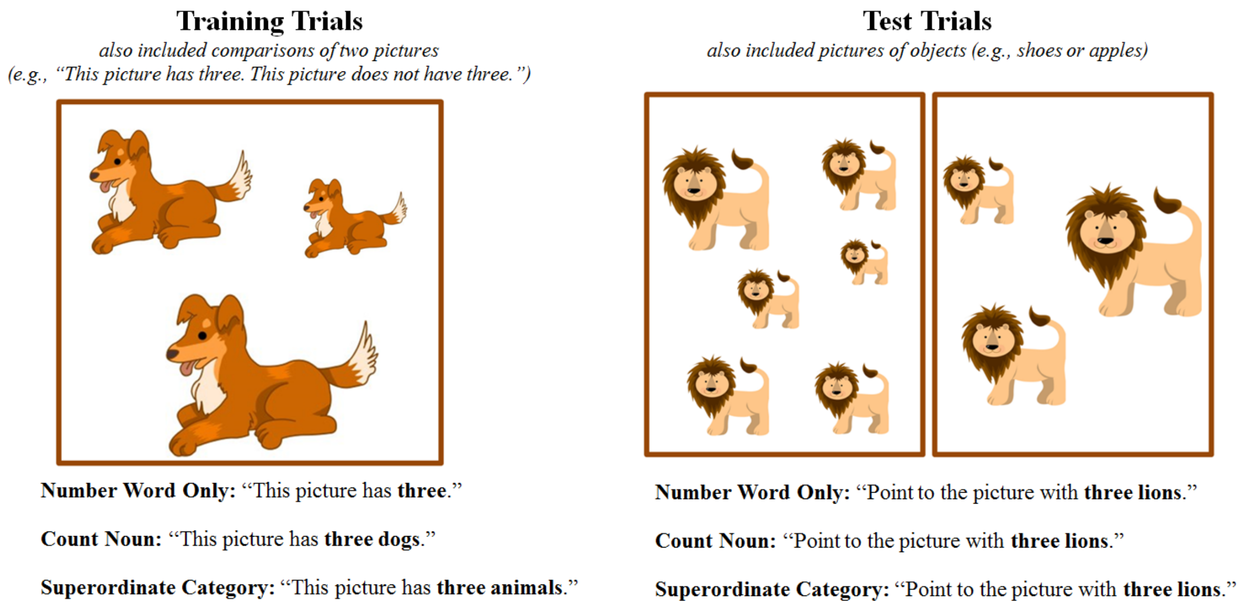
> Even though you realize that your simple 'judgement calls' on who has more chocolate chips will have to be supported with clear empirical evidence from here on out, you nevertheless decide to help your nephew out (that's what family's for, right?). Lucky for you, researchers' evaluations of both small- and broad-scale interventions have culminated in a collection of best practices that can be easily implemented even in informal settings.

Facilitating the Acquisition of Individual Number Words

In addition to the four counting principles outlined in Section 4.3 above, Gelman and Gallistel (1978) noted that children must also understand abstraction – the idea that number is an inherent property of any set of discrete items and that a set of 10 apples, for example, shares something in common with a set of 10 oranges (who said that we can't compare apples and oranges?). Unfortunately (though interestingly) many researchers who have attempted to teach children the meaning of a new number word (e.g., teach a two-knower the exact meaning of the word *three*) find limited success. Whereas these children may come to recognize that the new number word can be used to label a set of (e.g.) three marbles, they often do not understand that the word *three* can be applied or generalized to other sets of 3 (e.g., 3 blocks, 3 buttons, 3 meals) (Carey et al. 2017; Huang et al. 2010; Mix et al. 2002).

To explore this phenomenon further, we introduced a group of two-knowers to the word *three* (Slusser et al. 2017) through one of three training conditions. Children randomly assigned to the Number Word Only condition were presented with several pictures of 3 animals and were told, “This picture has *three*.” Children in the Count Noun condition were presented with this same series of pictures but were told, (e.g.) “This picture has *three dogs*”. And children in the Superordinate Category condition were told, “This picture has *three animals*.” Following training trials with corrective feedback, two-knowers in the Count Noun and Superordinate category conditions failed to extend the new number word (*three*) to sets of new animals (e.g., lions) or objects (e.g., shoes), while children in the Number Word Only condition succeeded. These findings suggest that the specificity of the linguistic context in which a number word is introduced influences children’s ability to generalize newly acquired number words. Thus, while a rich linguistic context seems to facilitate children’s understanding of number word semantics (see Section 4.2.1.1), when introducing a specific number word, adults and educators should provide varied input and avoid coupling a number word with a specific noun or category label unnecessarily.

Figure 31.5. Examples of training and test trials: To evaluate the role of linguistic context in children’s acquisition of individual number words, we designed 3 training conditions. Children who were trained with the Number Word Only were more likely to generalize the newly acquired number word to new sets than children assigned to the Count Noun or Superordinate Category conditions.



Number Word Only: "This picture has **three**."

Count Noun: "This picture has **three dogs**."

Superordinate Category: "This picture has **three animals**."

Number Word Only: "Point to the picture with **three lions**."

Count Noun: "Point to the picture with **three lions**."

Superordinate Category: "Point to the picture with **three lions**."

Facilitating the Acquisition of the Cardinality Principle

Efforts to teach children the cardinality principle over a short period of time have also been met with mixed success (e.g, Mix et al. 2012). Nevertheless, it seems there is growing evidence that adults can effectively scaffold children's understanding of the cardinality principle by presenting the counting routine in close temporal contiguity with an appropriate label of cardinality. Most recently, Paliwal and Baroody (2017) found that modeling a counting procedure that emphasizes the total number of items in a set facilitates children's understanding of the cardinality principle. For this study, 3- to 5-year-olds were randomly assigned to one of three training groups. Children practiced counting 1 to 6 items with an experimenter several times over a 6-week period. Upon post-test (which included a measure similar to the Give-N task described above), children who practiced counting using a procedure that emphasized the total number of items in a set (e.g., "One, two, three. *Three*. There are *three* elephants!") outperformed children who simply counted the items (e.g., one, two, three) without repeating or emphasizing the cardinal value of the set.

Notably, however, adults often do not approach counting activities in this way (Mix et al. 2012). While they may count or provide a cardinal label, they do not often do both. This coupled with the observation that number talk, in general, is relatively rare in everyday interactions (Levine et al. 2010) suggests that many children are not, on a daily basis, exposed to input that facilitates this understanding.

Broad Scale Intervention

Following participation in “broad-scale” mathematics intervention programs (meaning that they include a multitude of both classroom- and home-based activities), children from low and middle-socioeconomic backgrounds have consistently demonstrated improved performance on composite mathematical assessments (e.g., Arnold et al. 2002; Starkey et al 2004). Not only do children's math scores improve, but other numerically related skills, such as measurement and problem solving, also improve.

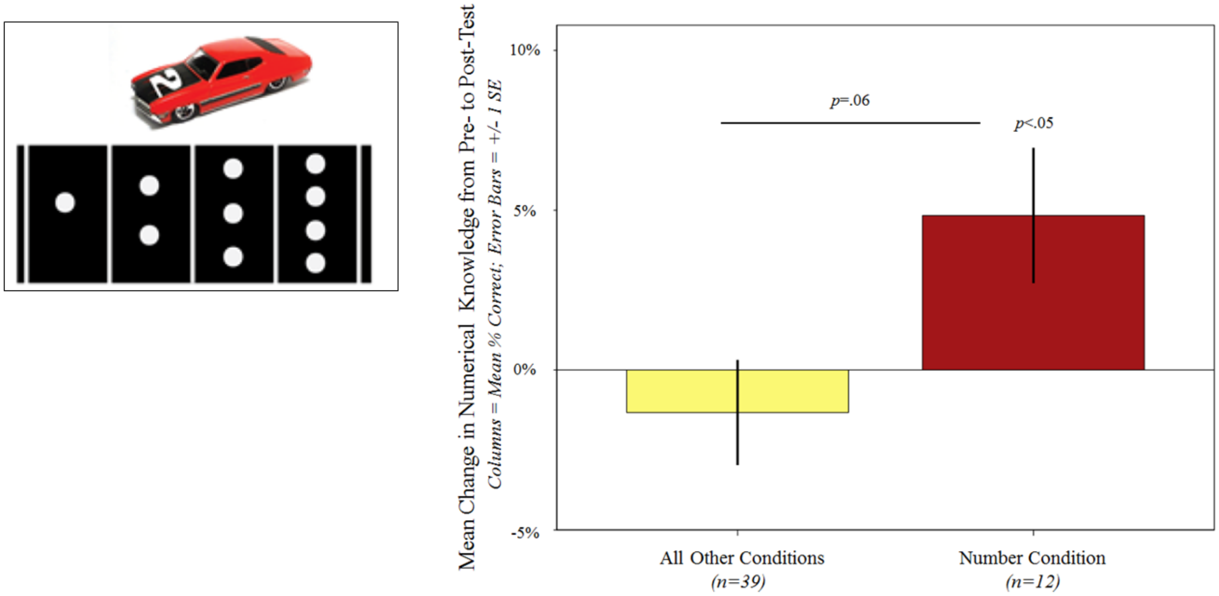
One notable demonstration of these noted benefits follows Greenes et al.’s (2004) evaluation of their Big Math for Little Kids program. This curriculum, designed to increase mathematical competency among 4 to 5 year old children, includes a series of engaging number-based games that encourage and facilitate critical thinking related to number. The studies presented in the following two sections, however, suggest that meaningful experience and intervention need not take the form of established curriculum. Instead, it seems that parents and educators can facilitate children’s counting and basic numerical skills by simply offering or creating numerically based games and toys, and by incorporating ‘number talk’ into daily conversations.

Numerically Based Toys

Over the last several years researchers have begun to study the direct cognitive benefits associated with children’s play with numerically based toys. One study linked cognitive benefits to play with numbered board games in preschoolers from low-income backgrounds (Siegler and Ramani 2008). Children (ages 4 to 5) completed 4 sessions of play using a board game with squares labeled 1-10. Even though they initially struggled with math related tasks as compared to their more affluent peers at pre-test, these children consistently demonstrated improvements at post-test, suggesting that numerically based play can have profound effects on mathematical cognition.

More recently, in a study funded by the toy manufacturing giant Mattel©, 3- and 4-year-old children were randomly assigned to one of four conditions, each with a specific toy predicted to support development within a particular cognitive domain (Slusser et al. 2013a). Children in the Number Condition were given a set of ten small race cars (think Hot Wheels™) and a parking garage. Each car was labeled with a numeral from 1 to 10 and the parking garage included a series of parking spaces, each with an array of 1 to 10 dots. After a 1-month period (during which time children were encouraged to play with the toy but received no other specific instruction from the researchers) children's counting and basic numerical skills increased dramatically, significantly more than children assigned to any other condition[3] (see Figure 6). Thus, simply playing with numbered toys appears to promote improvement in numerical understanding.

Figure 31.6. Children's independent play with numerically based toys (left) over a 1-month period promotes their numerical understanding (right) (Slusser et al. 2013a).



---

[3] Children in the other conditions received either a set of ethnically diverse dolls, dress up clothes, or wooden blocks.

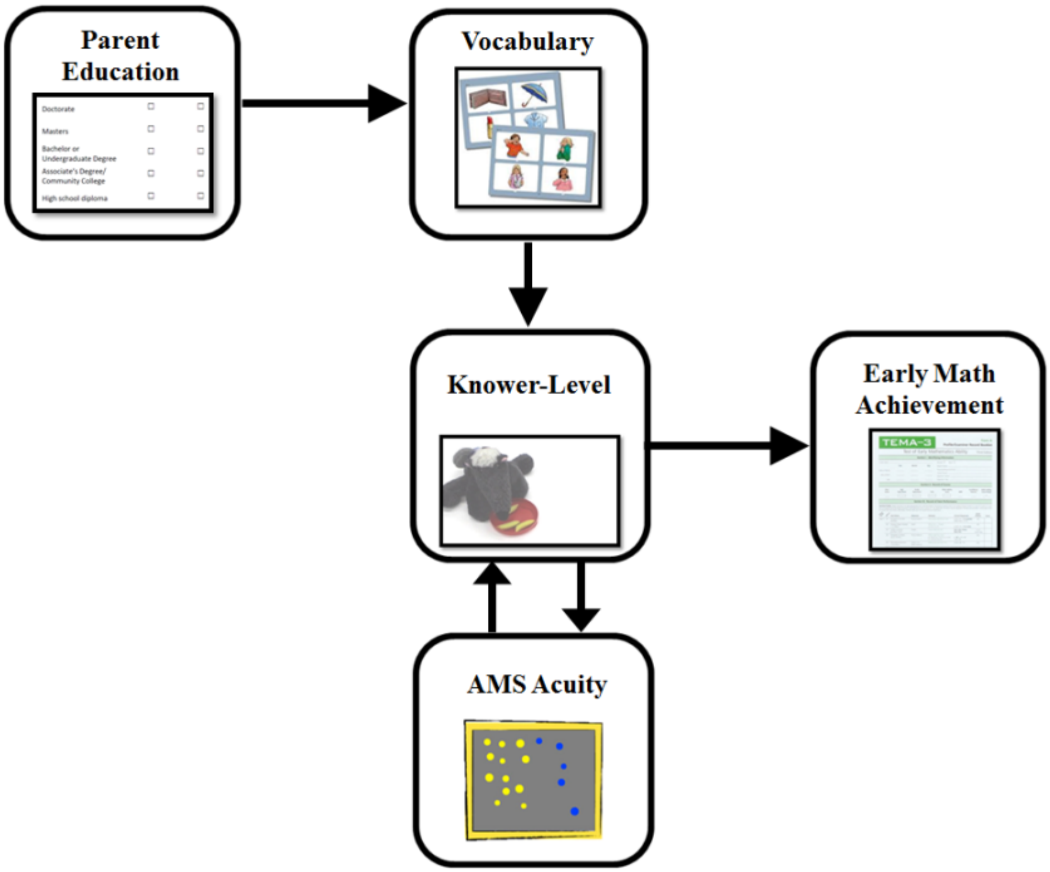
Number Language

Even without the use of games or toys, recent research has shown that exposure to number language facilitates children's acquisition of number word meanings. In fact, children's knower levels can be predicted by the quality and quantity of number-specific language in the home (Gunderson and Levine 2011; Levine et al. 2010) and interventions that help parents engage in meaningful number talk can facilitate children's progress toward understanding cardinality (Berkowitz et al. 2015).

This important link between number knowledge and early language exposure is further demonstrated through a recent study that evaluates and models the influence of parent education, general vocabulary, ANS acuity, and number word knowledge on children's early math achievement (Slusser et al. under revision). For this study, we first evaluated the receptive vocabulary, number-knower level, and ANS acuity of a diverse group of 3- to 5-year-old preschoolers. We then administered the TEMA-3 approximately one year later, as they entered Kindergarten. We found that children's early language (general vocabulary and number word knowledge) fully mediates the relationship between parent education and math ability. Additionally, number word knowledge mediates the noted relationship between ANS acuity and early math (see Figure 7).

Even with a clear need for additional research, these findings carry implications for early education and intervention. For example, while proposals for early intervention to support children's developing number sense (ANS acuity; e.g., Wang et al. 2016) remain justified, these findings suggest that an increased focus on number language and general vocabulary may help to minimize disparities in math ability as children enter kindergarten.

Figure 31.7. A diagram that illustrates the relationship of parent education and early math. Results from a 1-year longitudinal study following preschoolers through to kindergarten show that early language skills are linked to number word knowledge and these factors fully mediate the relationship between parent education and math ability (Slusser et al. under revision).



Summary

In sum, a sampling of research across various disciplines (including early education and instruction, child development, psychology, and cognitive science) shows that children's intuitive number sense, understanding of individual number words, as well as their procedural and conceptual counting knowledge serve as the key building blocks for future math ability. While idiosyncrasies in each result in predictable developmental outcomes, researchers have identified a series of effective, low-cost, and practical interventions that can be easily adopted by parents and practitioners alike.

References

Arnold, D. H., Fisher, P. H., Doctoroff, G. L., Dobbs, J. (2002). Accelerating math development in Head Start classrooms. *Journal of Educational Psychology, 94*, 762-770.

Aunio, P., & Niemivirta, M. (2010). Predicting children's mathematical performance in grade one by early numeracy. *Learning and Individual Differences, 20,* 427-435.

Barner, D., Thalwitz, D., Wood, J., & Carey, S. (2007). On the relation between the acquisition of singular-plural morpho-syntax and the conceptual distinction between one and more than one. *Developmental Science, 10,* 365-373.

Bartelet, D., Vaessen, A., Blomert, L., & Ansari, D. (2014). What basic number processing measures in kindergarten explain unique variability in first-grade arithmetic proficiency? *Journal of Experimental Child Psychology, 117,* 12-28.

Berkowitz, T., Schaeffer, M. W., Maloney, E. A., Peterson, L., Gregor, C., Levine, S. C., & Beilock, S. L. (2015). Math at home adds up to achievement in school. *Science, 350,* 196-198.

Bloom, P. & Wynn, K. (1997). Linguistic cues in the acquisition of number words. *Journal of Child Language, 24,* 511-533.

Carey, S. (2010). *The Origin of Concepts*. New York: Oxford University Press.

Carey, S., Shusterman, A., Haward, P., & Distefano, R. (2017). Do analog number representations underlie the meanings of young children's verbal numbers? *Cognition, 168*, 243-255.

Chu, F., van Marle, K., & Geary, D. (2015). Early numerical foundations of young children’s mathematical development. *Journal of Experimental Child Psychology, 132,* 205-212.

Clearfield, M. W. and Mix, K. S. (1999). Number versus contour length in infants' discrimination of small visual sets. *Psychological Science, 10,* 408-411.

Cordes, S., Gelman, R., Gallistel, C. R., & Whalen, J. (2001). Variability signatures distinguish verbal from nonverbal counting for both large and small numbers. *Psychonomic Bulletin & Review, 8*, 698-707.

Dale, P. S. & Fenson, L. (1996). Lexical development norms for young children. *Behavior Research Methods, Instruments, & Computers, 28,* 125-127.

Dantzig, T. (1967). *Number: The Language of Science*. New York: Free Press.

Davidson, K., Eng, K., & Barner, D. (2012). Does learning to count involve a semantic induction? *Cognition, 123*: 162-173.

Dehaene, S. (2011). *The Number Sense: How the Mind Creates Mathematics*. Oxford, England: Oxford University Press.

Dowker, A. (2008). Individual differences in numerical abilities in preschoolers. *Developmental Science, 11,* 650-654.

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., . . . Brooks-Gunn, J. (2007). School readiness and later achievement. *Developmental Psychology, 43*, 1428.

Feigenson, L. & Carey, S. (2003). Tracking individuals via object files: Evidence from infants' manual search. *Developmental Science 6,* 568-584.

Feigenson, L., Dehaene, S., & Spelke, E. S. (2004). Core systems of number. *Trends in Cognitive Sciences, 8,* 307-314.

Frank, M. C., Everett, D. L., Fedorenko, E., & Gibson, E. (2008). Number as a cognitive technology: Evidence from Pirahã language and cognition. *Cognition, 108,* 819-824.

Fuson, K. C. (1988). *Children's Counting and Concepts of Number*. New York: Springer Science & Business Media.

Gelman, R., & Gallistel, C. (1978). *Young Children's Understanding of Numbers.* Cambridge, MA: Harvard University Press.

Gersten, R., & Chard, D. (1999). Number sense: Rethinking arithmetic instruction for students with mathematical disabilities. *The Journal of Special Education, 33*, 18-28.

Ginsburg, H., & Baroody, A. (2003). *Test of Early Math Ability* (3rd ed.). Austin, TX: Pro-Ed.

Göbel, S. M., Watson, S. E., Lervåg, A., & Hulme, C. (2014). Children's arithmetic development: It is number knowledge, not the approximate number sense, that counts. *Psychological Science,* 1-10.

Goldman, M.C., Negen, J. & Sarnecka, B.W. (2014). Are bilingual children better at ignoring perceptually misleading information? A novel test. *Developmental Science, 17,* 956-64.

Greenes, C., Ginsburg, H. P., & Balfanz, R. (2004). Big math for little kids. *Early Childhood Research Quarterly, 19,* 159–166.

Gunderson, E. A., & Levine, S. C. (2011). Some types of parent number talk count more than others: relations between parents’ input and children’s cardinal-number knowledge. *Developmental Science, 14,* 1021-1032.

Halberda, J. & Feigenson, L. (2008) Developmental change in the acuity of the Number Sense: The approximate number system in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental Psychology, 44,* 1457–1465.

Halberda, J., Ly, R., Wilmer, J., Naiman, D., & Germine, L. (2012). Number sense across the lifespan as revealed by massive internet-based sample. *Proceedings of the National Academy of Sciences, 109,* 11116-11120.

Halberda, J., Mazzocco, M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature, 455*, 665-668.

Hespos, S. J., Ferry, A., & Rips, L. (2009). Five-month-old infants have different expectations for solids and substances. *Psychological Science, 20*, 603-611.

Huang, Y.T., Spelke, E., & Snedeker, J. (2010). When is four far more than three? Children's generalization of newly acquired number words. *Psychological Science, 21,* 600-606.

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology, 45,* 850.

Jordan, N. C., & Levine, S. C. (2009). Socioeconomic variation, number competence, and mathematics learning difficulties in young children. *Developmental Disabilities Research Reviews, 15*, 60-68.

Kaufman, E. L., Lord, M. W., Reese, T. W., & Volkmann, J. (1949). The discrimination of visual number. *The American Journal of Psychology, 62,* 498-525.

Kritzer, K. L. (2009). Barely started and already left behind: A descriptive analysis of the mathematics ability demonstrated by young deaf children. *Journal of Deaf Studies and Deaf Education,* 1-13.

Le Corre, M., Van de Walle, G., Brannon, E. M., & Carey, S. (2006). Re-visiting the competence/performance debate in the acquisition of the counting principles. *Cognitive Psychology, 52,* 130-169.

Levine, S. C., Suriyakham, L. W., Rowe, M. L., Huttenlocher, J., & Gunderson, E. A. (2010). What counts in the development of young children's number knowledge? *Developmental Psychology, 46,* 1309-1319.

Li, P., Le Corre, M., Shui, R., Jia, G., & Carey, S. (2003). Effects of plural syntax on number word learning: A cross-linguistic study. *Proceedings of the 28th Boston University Conference on Language Development.* Somerville, MA: Cascadilla Press.

Libertus, M., Feigenson, L., & Halberda, J. (2011). Preschool acuity of the approximate number system correlates with school math ability. *Developmental Science, 14,* 1292-1300.

Libertus, M., Odic, D., & Halberda, J. (2012). Intuitive sense of number correlates with math scores on college-entrance examination. *Acta Psychologica, 141,* 373–379.

Lipton, J. S. & Spelke, E. S. (2003). Origins of number sense: Large number discrimination in human infants. *Psychological Science, 15,* 396-401.

Lyons, I.M., & Beilock, S.L. (2011). Numerical ordering ability mediates the relation between number-sense and arithmetic competence. *Cognition, 121*, 256–261.

Mandler, G., & Shebo, B. J. (1982). Subitizing: An analysis of its component processes. *Journal of Experimental Psychology: General, 111*, 1-22.

Mix, K., Huttenlocher, J., & Levine, S. (2002). *Quantitative Development in Infancy and Early Childhood.* New York: Oxford University Press.

Mix, K., Sandhofer, C., Moore, J., & Russell, C. (2012). Acquisition of the cardinal word principle: The role of input. *Early Childhood Research Quarterly, 27,* 274-283.

Muldoon, K., Lewis, C. and Freeman, N. (2009). Why set-comparison is vital in early number learning. *Trends in Cognitive Sciences, 13,* 203-208.

Odic, D., Libertus, M., Feigenson, L., & Halberda, J. (2013). Developmental change in the acuity of approximate number and area representations. *Developmental Psychology, 49,* 1103.

Paliwal, V. & Baroody, A. (2017). How best to teach the cardinality principle? Paper presented at the American Education Research Association, San Antonio, TX.

Piazza, M., Pica, P., Izard, V., Spelke, E., & Dehaene, S. (2013). Education enhances the acuity of the nonverbal Approximate Number System. *Psychological Science, 24,* 1037-1043.

Sarnecka, B. W., & Carey, S. (2008). How counting represents number: What children must learn and when they learn it. *Cognition, 108,* 662-674.

Sarnecka, B. W. & Gelman, S. A. (2004). Six does not just mean a lot: Preschoolers see number words as specific. *Cognition, 92,* 329-35.

Sarnecka, B., Goldman, M., & Slusser, E. (2014). How Counting Forms the Basis for Children's First Representations of the Natural Numbers. In R. Cohen Kadosh & A. Dowker (Eds.). *The Oxford Handbook of Numerical Cognition* (pp. 291-309). Oxford, UK: Oxford University Press.

Sarnecka, B. W., Kamenskaya, V., Yamana, Y., Ogura, T., & Yudovina, Y. (2007). From grammatical number to exact numbers: Early meanings of 'one', 'two', and 'three' in English, Russian, and Japanese. *Cognitive Psychology, 55*, 136-168.

Sarnecka, B. W. & Lee, M. D. (2009). Levels of number knowledge in early childhood. *Journal of Experimental Child Psychology, 103*, 325-337.

Sarnecka, B. W. & Wright, C. E. (2013). The exact-numbers idea: Children's understanding of cardinality and equinumerosity. *Cognitive Science, 37,* 1493-1506.

Shusterman, A., Slusser, E., Halberda, J., & Odic, D. (2016). Acquisition of the Cardinal Principle coincides with improvement in Approximate Number System acuity in preschoolers. *PLoS ONE, 11,* 1-22.

Siegler, R. S., & Ramani, G. B. (2008). Playing linear numerical board games promotes low-income children's numerical development. *Developmental Science, 11*, 655-661.

Slusser, E. (2010). *The development of number concepts: Discrete quantification and numerosity*. (Doctoral dissertation). University of California, Irvine, CA.

Slusser, E., Chase, E., Berkowitz, T., George, E., Swee, M., Cho, D., Barth, H., & Shusterman, A. (2013a). *The power of play: Promoting preschoolers' social and numerical development through independent play with toys*. Poster presented at the Biennial Meeting for the Society for Research in Child Development, Seattle, WA.

Slusser, E., Ditta, A., & Sarnecka, B. (2013b). Connecting numbers to discrete quantification: a step in the child's construction of integer concepts. *Cognition, 129,* 31-41.

Slusser, E., Ribner, A., & Shusterman, A. (*under revision*). Language counts: Early language mediates the relationship between parent education and children's math ability.

Slusser, E. & Sarnecka, B. (2011). A picture of eight turtles: the child's understanding of cardinality and numerosity. *Journal of Experimental Child Psychology, 110*, 38-51.

Slusser, E., Stoop., T., Lo., & Shusterman, A. (2017). *Children's use of newly acquired number words in novel contexts*. Poster presented at the Biennial Meeting for the Society for Research in Child Development, Austin, TX.

Spaepen, E., Coppola, M., Spelke, E., Carey, S., & Goldin-Meadow, S. (2011). Number without a language model. Proceedings of the National Academy of Sciences, 108, 3163-3168.

Starkey, P. & Cooper, R.G. (1980). Perception of numbers by human infants. *Science*, *28,* 1033-1035.

Starkey, P., Klein, A., & Wakeley, A. (2004). Enhancing young children's mathematical knowledge through a pre-kindergarten mathematics intervention. *Early Childhood Research Quarterly, 19*, 99-120.

Tosto, M. G., Petrill, S. A., Halberda, J., Trzaskowski, M., Tikhomirova, T. N., Bogdanova, O. Y. … Kovas, Y. (2014). Why do we differ in number sense? Evidence from a genetically sensitive investigation. *Intelligence, 43,* 35-46.

Trick, L. & Pylyshyn, Z. (1993). Why are small and large numbers enumerated differently? A limited-capacity preattentive stage in vision. *Psychological Review, 101*, 80–102.

van Marle, K., Chu, F. W., Li, Y., & Geary, D. C. (2014). Acuity of the approximate number system and preschoolers' quantitative development. *Developmental Science, 17,* 492-505.

Wagner, J. B. & Johnson, S. C. (2011). An association between understanding cardinality and analog magnitude representations in preschoolers. *Cognition, 119*, 10–22.

Wang, H., Odic, D., Halberda, J., & Feigenson, L. (2016). Changing the precision of preschoolers' approximate number system representations changes their symbolic math performance. *Journal of Experimental Child Psychology, 147,* 82-99.

Wynn, K. (1990). Children's understanding of counting. *Cognition, 36,* 155-193.

Wynn, K. (1992). Children's acquisition of the number words and the counting system. *Cognitive Psychology, 24,* 220-251.

Wynn, K. (1996). Infants' individuation and enumeration of actions. *Psychological Science, 7,* 165-169.

Xu, F. (2003). Numerosity discrimination in infants: Evidence for two systems of representations. *Cognition, 89*, B15-B25.

Xu, F. & Spelke, E. S. (2000). Large number discrimination in 6-month old infants. *Cognition, 74*, B1-B11.