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Size vs. Number: Assigning Number Words to Discrete and Continuous Quantities

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Symposium Title

Learning to count: New insights on the acquisition of symbolic numerical knowledge

Symposium Abstract

Humans have developed symbolic systems to represent and efficiently manipulate numerical information. Children gradually learn the rules and principles characterising the number system and how number words and numerals represent exact numerical quantities. The acquisition of symbolic number knowledge is a long and error-prone process that occupies children for several years, starting from the age of two until the first years of primary school, and it represents a crucial stepping stone for future mathematical achievement. In the symposium, we provide new insights on the acquisition of the symbolic numerical knowledge throughout development, including in how children initially learn number words, how they understand that they refer to number (as opposed to other dimensions of quantity), and how this achievement eventually leads to benefits in how number words relate to symbolic mathematics.

The first talk (Slusser) explores the developmental trajectory of children's understanding that number words refer to discrete numerical quantities, rather than continuous dimensions (e.g., "small", "a lot"). Children fully grasp the discrete nature of number words when they are 3-knowers, at an intermediate stage toward the mastering of the cardinality principle.

The second talk (Krajcsi) revalidates the widely used Give-a-Number task, in which children create numerical sets according to a required number (e.g., "give me four apples"). The current literature describes a sudden change in performance and strategy implementation when children extend their cardinal knowledge beyond four (i.e., cardinal-principle knowers). Contrary to this prevailing view, the proposed re-evaluation highlights the presence of a smooth, rather than sudden, change in performance as children slowly acquire the numerical meaning of number words beyond four, challenging traditional accounts of number words being learned through a sudden insight into general counting principles.

The third talk (Sella) describes how the mastering of different numerical concepts relate to the understanding of the exact numerical magnitude represented by number words and Arabic numbers. The mastering of the predecessor knowledge (i.e., removing one item from a set leads to the preceding number word in the counting list) and the knowledge of the spatial order of numbers relate to the performance in number comparison tasks beyond the acquisition of the cardinality principle and the later-greater principle.

The fourth talk (Odic) shows how the eventual mastery of number words leads to other benefits in children's numerical skills, most notably in children's ability to associate symbolic number words with their intuitive sense of number (the Approximate Number System; ANS), with non-numeric dimensions (e.g., estimating the length of a line), and as a source of error detection in symbolic mathematics.

The combination of the four talks provides an overview of the development of symbolic numerical knowledge from children who have just begun learning number words to those who have largely mastered it while giving new insights challenging current theoretical views.

Organiser
Francesco Sella
Centre for Mathematical Cognition
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Talk 1
Speaker: Emily Slusser
Position: Associate Professor and Department Chair
Institution: San Jose State University, US.
Authors: Slusser, Emily[1]; Cravalho, Patrick[1]
[1]San Jose State University

Size vs. Number: Assigning Number Words to Discrete and Continuous Quantities
Brief description of the talk: This talk explores when and how children come to understand that number words refer specifically to discrete numerosities (e.g., 1, 2, and 7), rather than various dimensions of continuous quantity (e.g., "small" in terms of surface area or "a lot" in terms of volume). Findings from a study of 2- to 4-year-old children indicate that number words are assigned exclusively to discrete numerosities at an intermediate stage of number word acquisition - after they have learned at least three number words ('three-knowers') - with evidence suggesting that executive functioning skills play an important role in this developmental trajectory.

Talk 2
Speaker: Attila Krajcsi
Position: Professor
Institution: ELTE Eötvös Loránd University, Hungary
Authors: Krajcsi, Attila[1]
[1] ELTE Eötvös Loránd University, Hungary

Revalidating the Give a Number task
Brief description of the talk: The Give a Number task is an essential tool to measure preschoolers' symbolic number knowledge. In a series of studies, reevaluating the original proposals by Wynn (1990, 1992), we found that contrary to the original findings there is no sudden increase in the task performance after understanding number 4. Additionally, strategy change between grabbing and counting can be observed, because only large numbers have to be counted; furthermore, the strategy does not change radically after understanding number 4. Based on these results, it might be reasonable to reevaluate the use of the task and former related findings of the literature.

Talk 3

Speaker: Francesco Sella
Position: Lecturer
Institution: Loughborough University, UK
Authors: Sella, Francesco[1]; Lucangeli, Daniela[2]; Cohen Kadosh, Roi[3]; Zorzi, Marco[4]
[1] Centre for Mathematical Cognition, Loughborough University, UK
[2] Department of Developmental Psychology, University of Padova, Italy
[3] Department of Experimental Psychology, University of Oxford, UK
[4] Department of General Psychology, University of Padova, Italy

Learning the number sequence: preschool children make sense of numbers

I will describe the acquisition of the cardinality principle, the mastering of successor and predecessor knowledge, the mapping between number words and external numerosities (ANS-to-word mapping), the spatial mapping of numbers and how all these abilities relate to the performance in number words and Arabic numbers comparison tasks. Our results revealed that, beyond the acquisition of the cardinality principle, the mastering of the predecessor knowledge and spatial order of digits relate to the understanding of the exact numerical magnitude represented by number words and Arabic digits.

Talk 4

Speaker: Darko Odic
Position: Assistant Professor
Institution: University of British Columbia, Canada
Authors: Odic, Darko[1]; Dramkin, Denitza[1]; Wong, Harris[1]
[1]University of British Columbia, Canada

Developing the interface between number word and the intuitive number sense: challenges and benefits

Young children have access to two distinct representations of number: an intuitive, but imprecise, perceptual number sense (ANS), and the slowly developing symbolic number words. In this talk, I show that the development of this interface is initially slow and protracted but, once children master the ability to translate between number words and the ANS, this gives them two key advantages for other numerical skills: (1) children can immediately form a novel number word interface with non-numeric dimensions, such as estimating area; (2) children can use their intuitive number sense as a source of error detection in formal mathematics.

Together, the talks argue that: a) Children's understanding of the discrete nature of number words only emerges at an intermediate stage toward mastering the cardinality principle; b) widely used tasks and their underlying models should be carefully reconsidered to obtain a more detailed understanding of the developmental stages children go through when learning number words; c) Learning the directional property of the counting list and the spatial order of numbers can scaffold the understanding of symbolic numerical magnitude; d) The ability to translate between number words and the ANS gives children crucial advantages for other numerical skills.

Size vs. Number

Assigning Number Words to Discrete and Continuous Quantities

Emily Slusser and Patrick Cravalho

San José State University

MCLS Conference

August 7, 2020

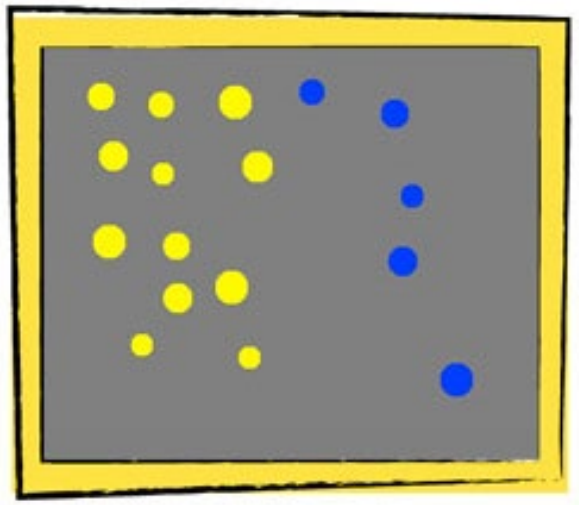
Overview

- **Background**
  - Early Representations of Number
  - Number Knower-Levels
  - Quantity vs. Numerosity
- **Methods & Results**
  - Give-N (Knower-Level)
  - Elephant Crocodile (Executive Functioning)
  - Transform Sets (Quantity v. Number)
- **Conclusions and Future Directions**

Early Representations of Number

Small-Exact

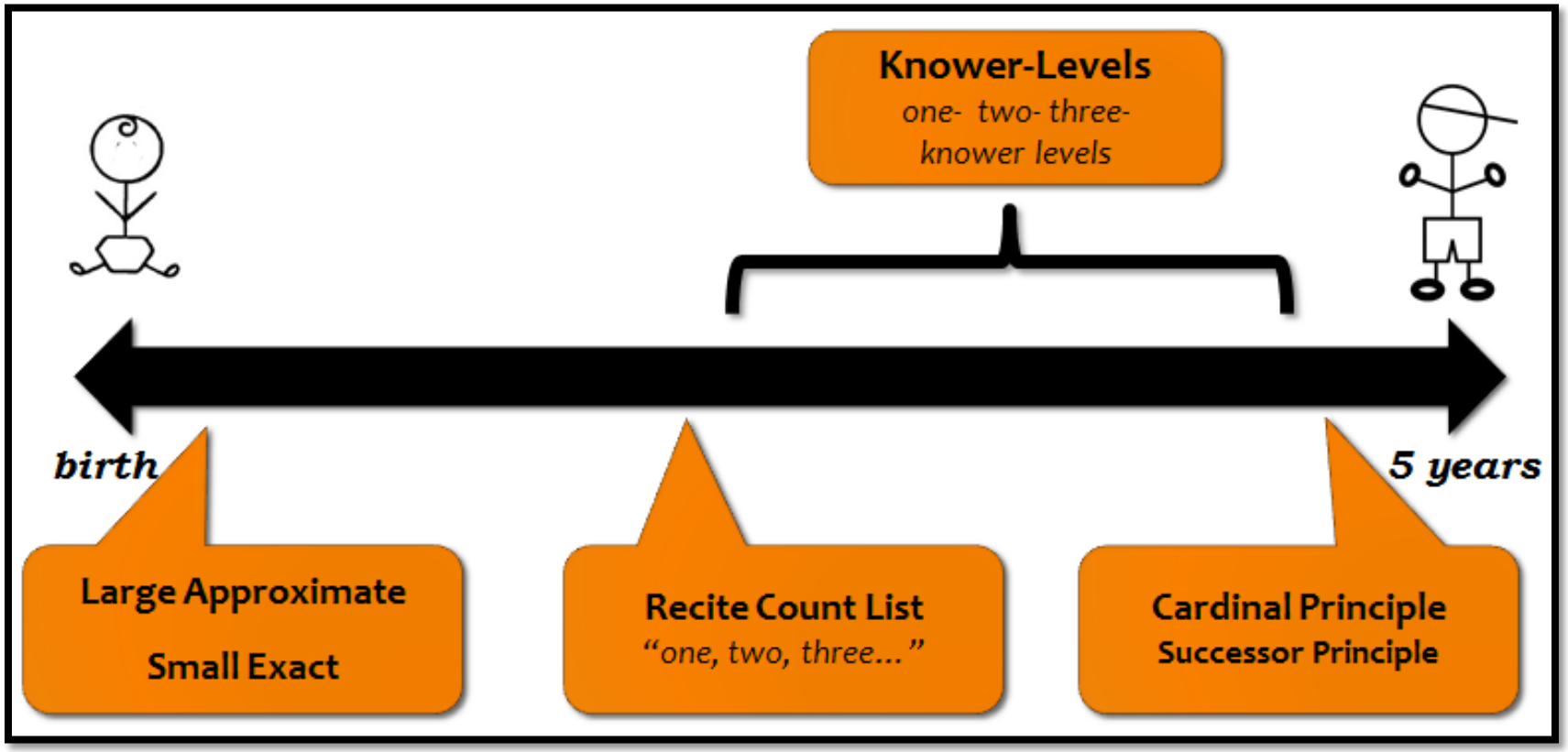
e.g. Feigenson & Carey, 2003



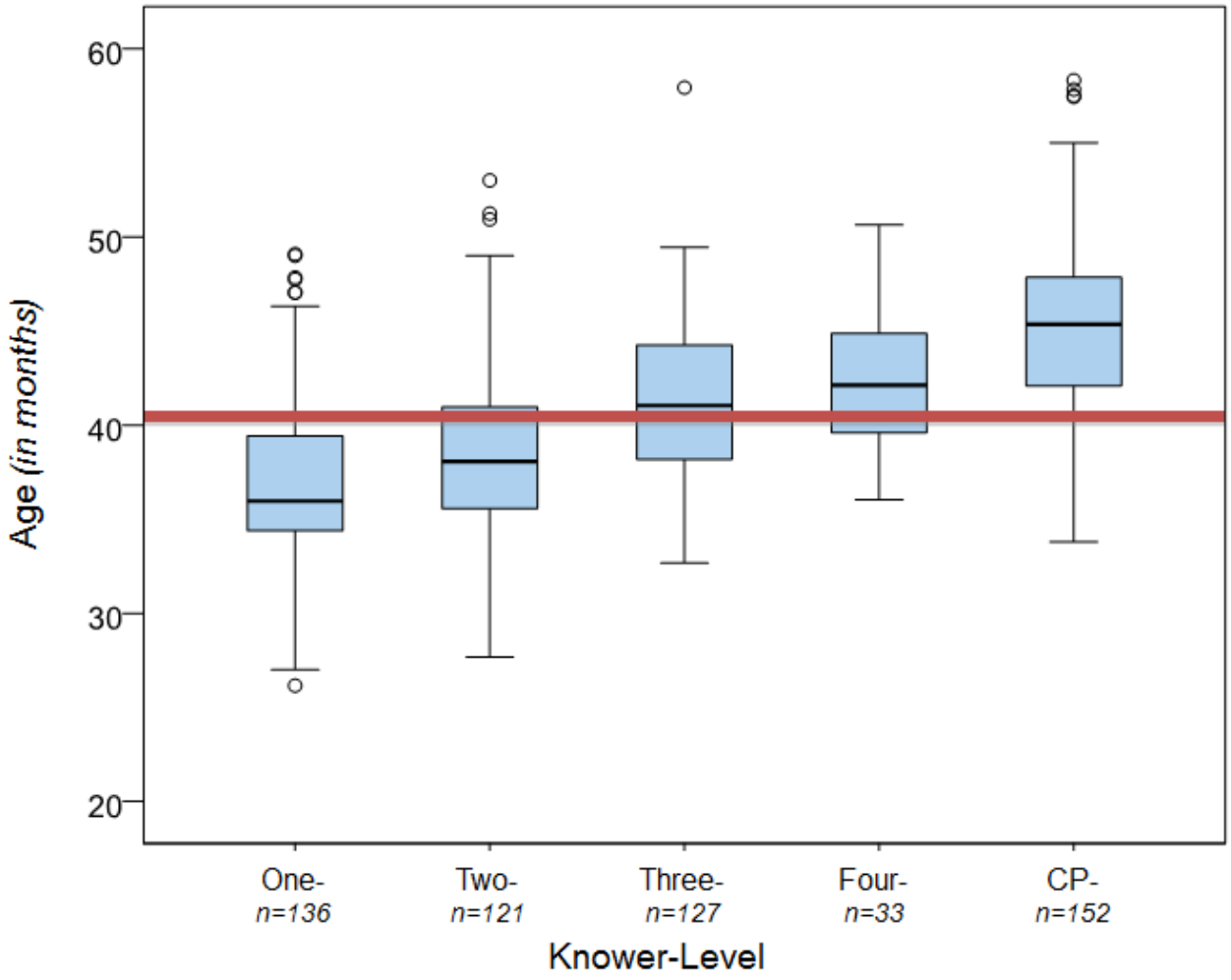
Large-Approximate

e.g. Halberda et al. 2013

Large, Exact Representations

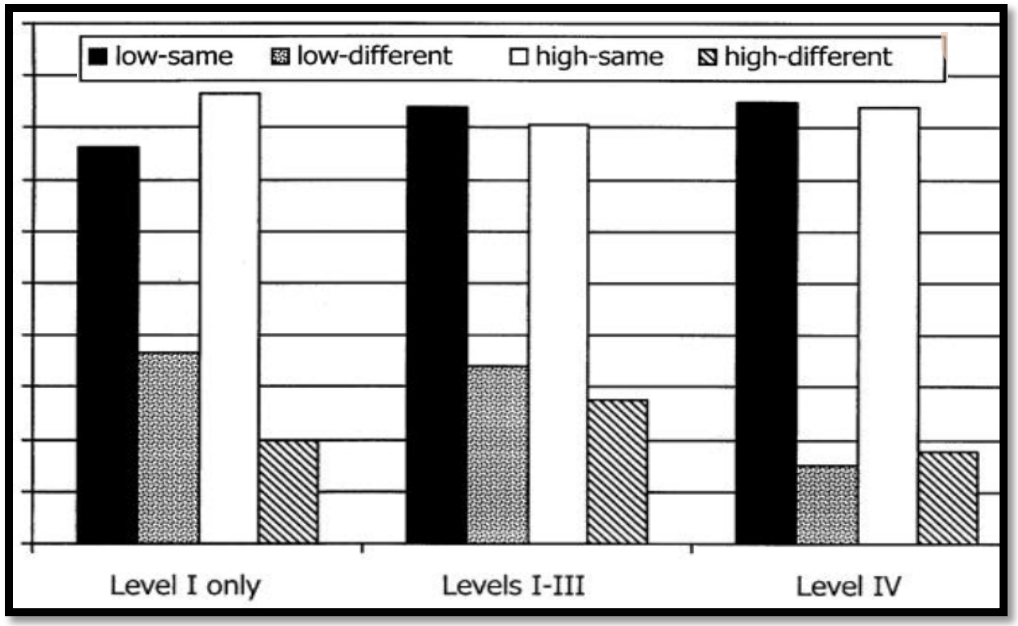


Knower Levels



Sarnecka, Goldman, & Slusser, 2014

Sarnecka & Gelman *(2004)*

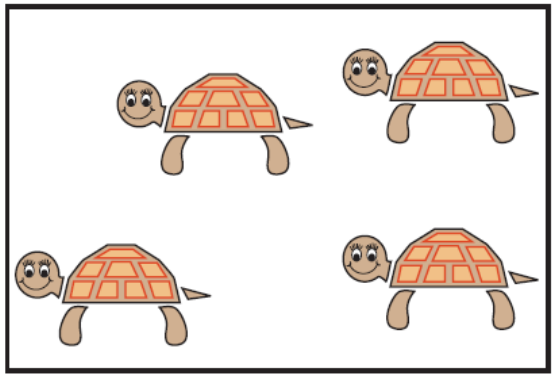


Take Aways…

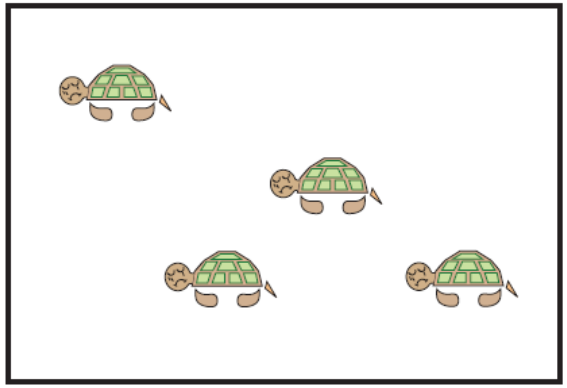
The number word does ***not*** change when shaken or rotated, but ***does*** change when an item is added or removed.

Slusser & Sarnecka *(2011)*

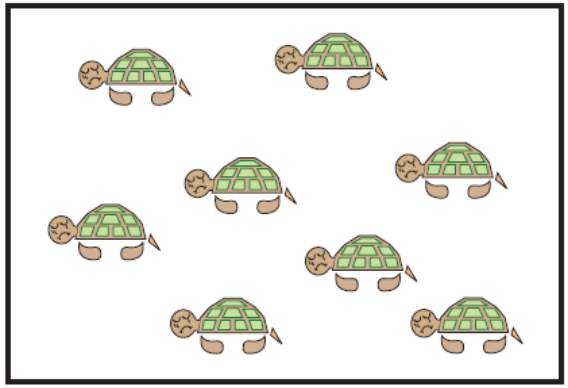
"This Picture has *four* turtles.Find another picture with *four* turtles.



Sample picture

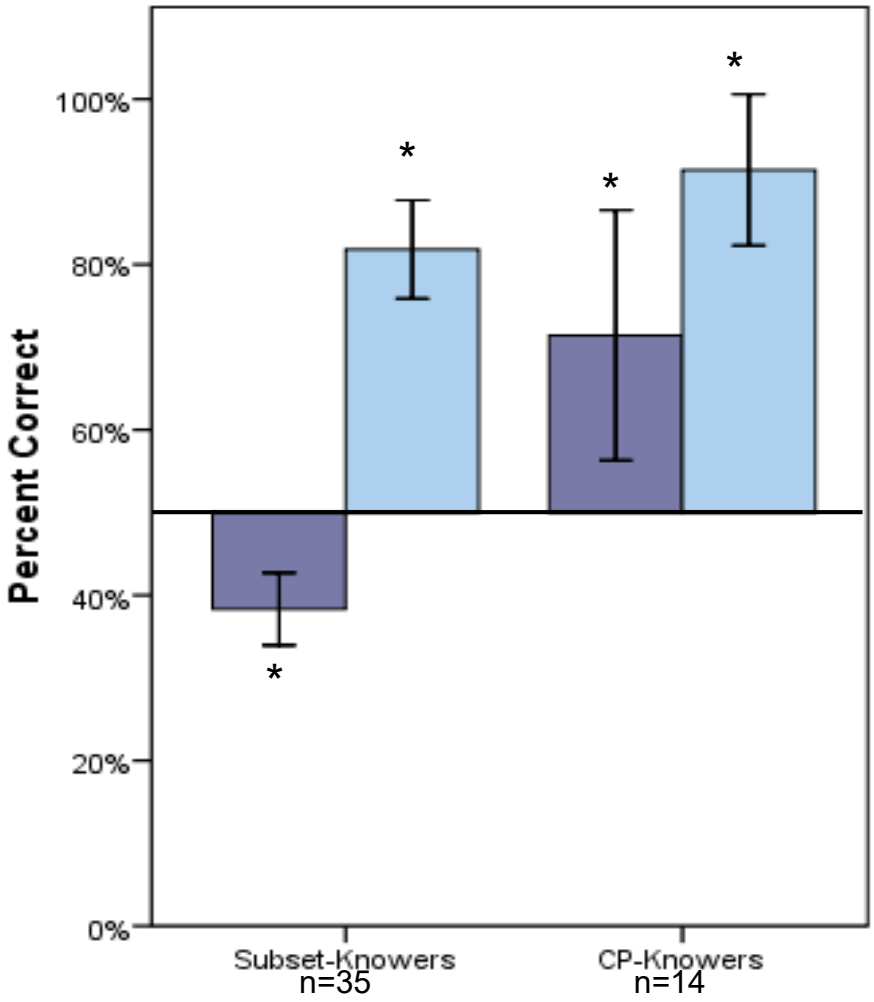


Correct response picture
(matches sample picture on number)



Incorrect response picture
(matches sample picture on summed contour length)

Slusser & Sarnecka *(2011)*



Take Aways…

Match when prompted with **color** or **mood** (“*find another picture with* ***green*** *turtles*”).

Do ***not*** match on **numerosity** when prompted with a **number word** until CP level.

Size vs. Number

- When and how do children come to understand that **number words refer to discrete numerosity**?
- Do children think the number word changes as the individual items **grow or shrink** in size?
- Influence of **executive functioning** (domain-general) and **language and counting** (domain-specific) skills?

2- to 4-year-olds (n=45; 25 males; M_{age}= 3;10)

Give-N *(Wynn 1990)*

Can you put two fish in the pond?

Is that two fish?

Determine how many number words a child knows (**Knower-Level**) and whether they understand the **Cardinality Principle.**

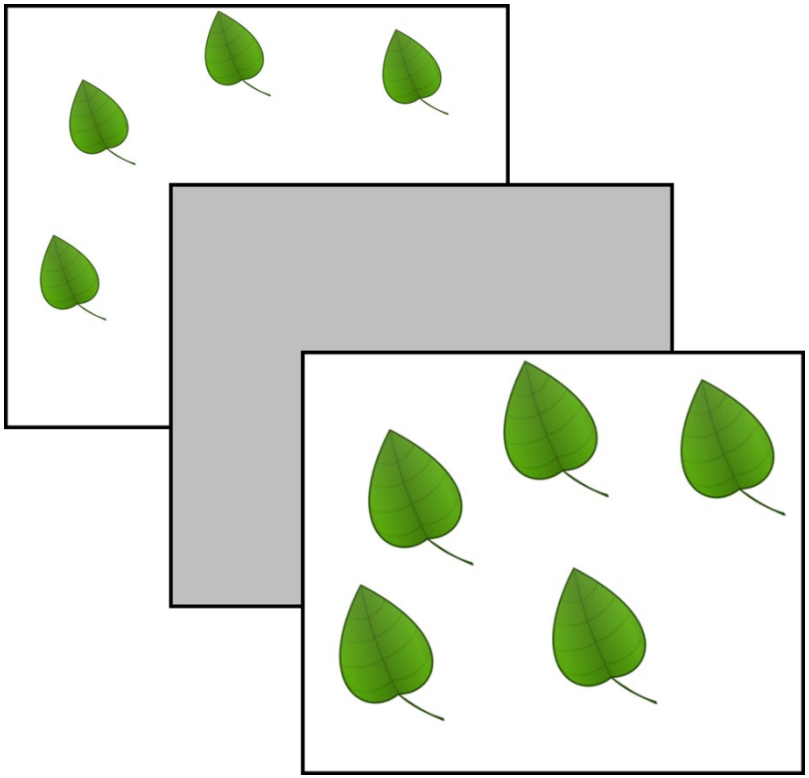
Elephant-Crocodile *(Leonard et al., 2014)*

When the elephant presses a button, press the same button that he presses.

When the crocodile presses a button, press the other button, the button that he didn't press.

Measure children's **executive functioning** skills (impulse control and cognitive flexibility).

Transform Sets *(Sarnecka & Gelman '04)*

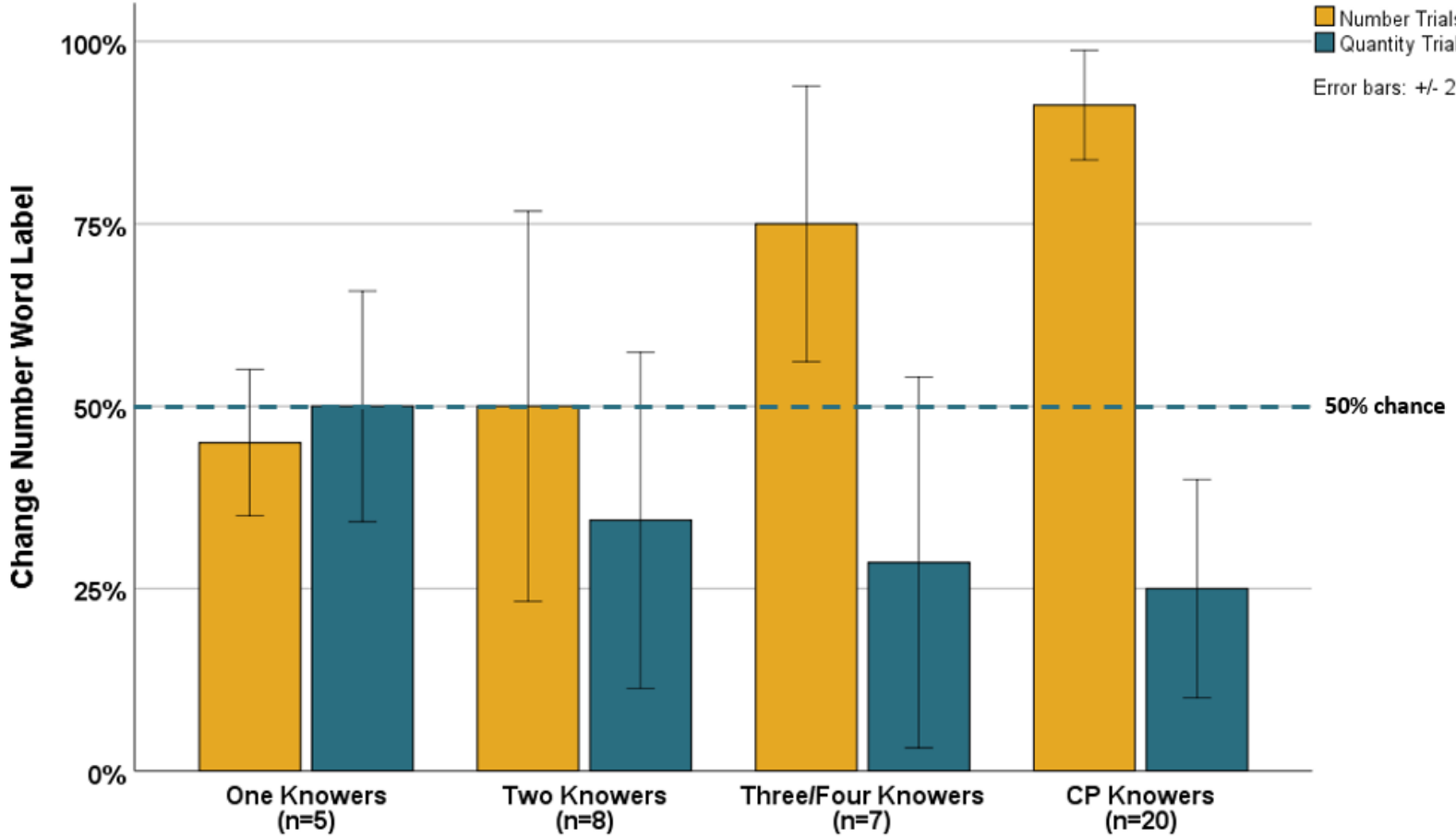


I'm going to put six leaves in the box.

Now, are there five or six leaves in the box.

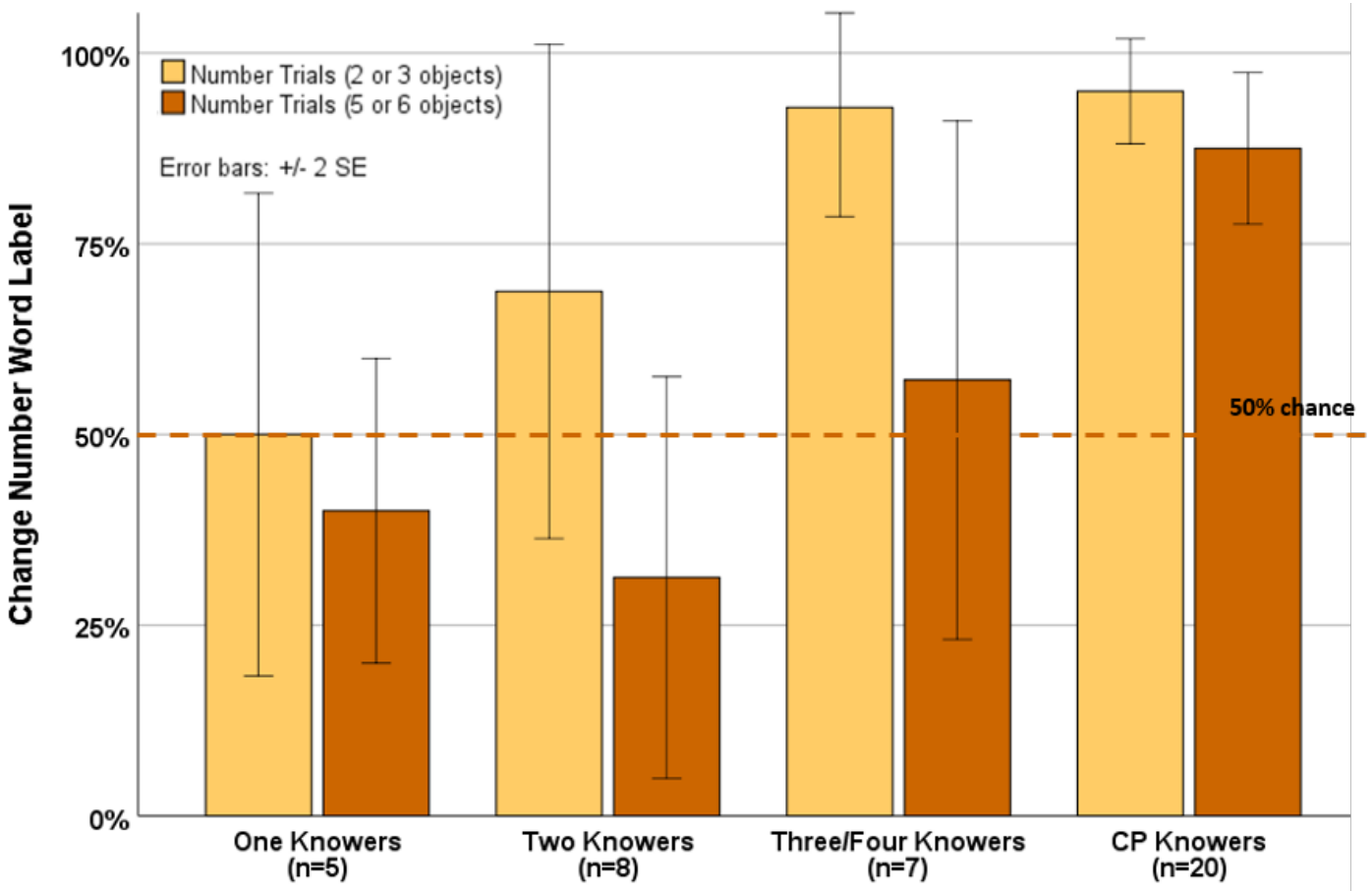
Assess children's understanding of number words –
do number words refer to (discrete) numerosity?

Transform Sets x Knower Level



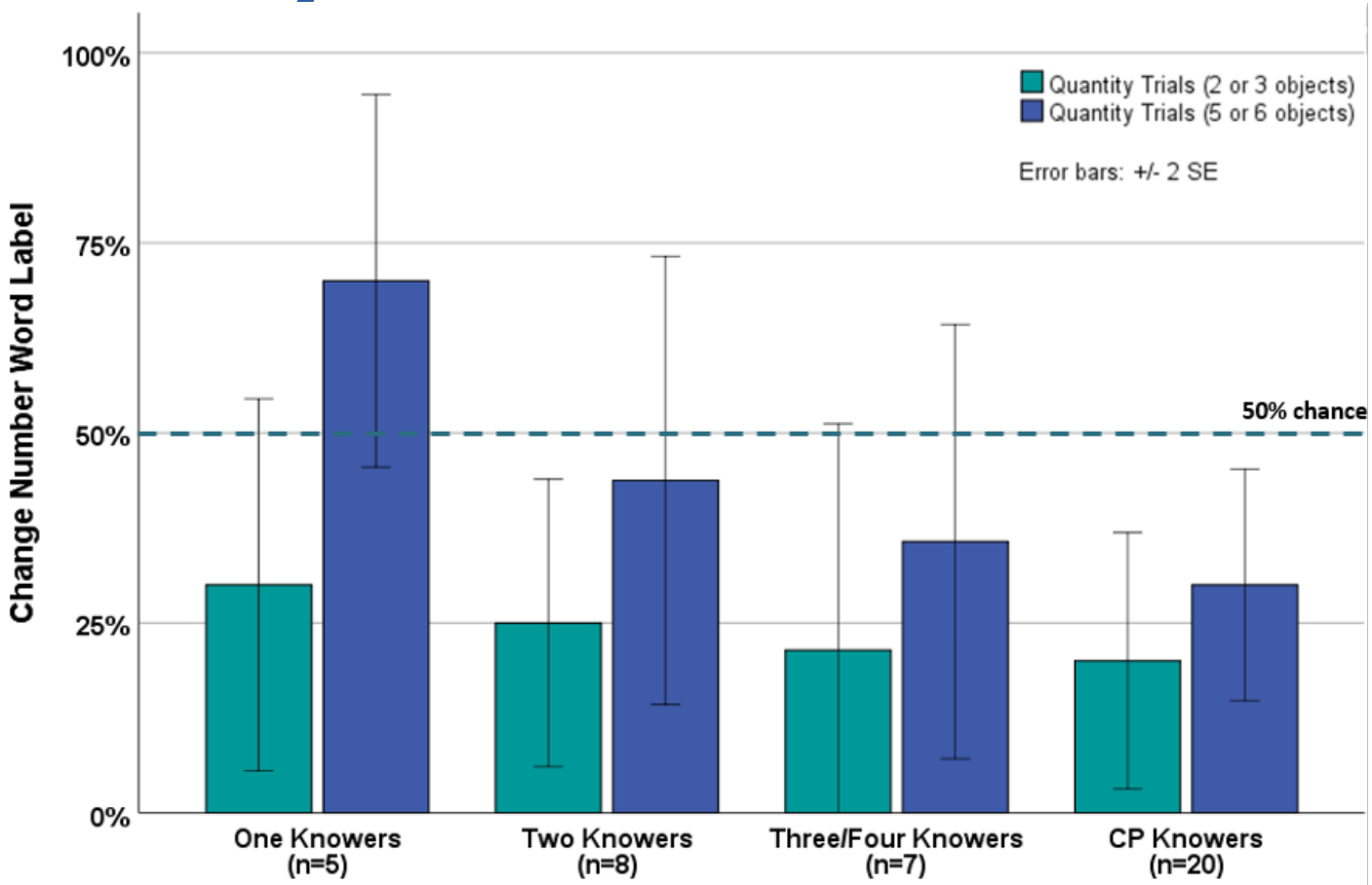
Three/Four and CP Knowers understand that **number labels** change *if, only if,* an item is **added** or **subtracted** from a set.

Number Trials x Knower Level



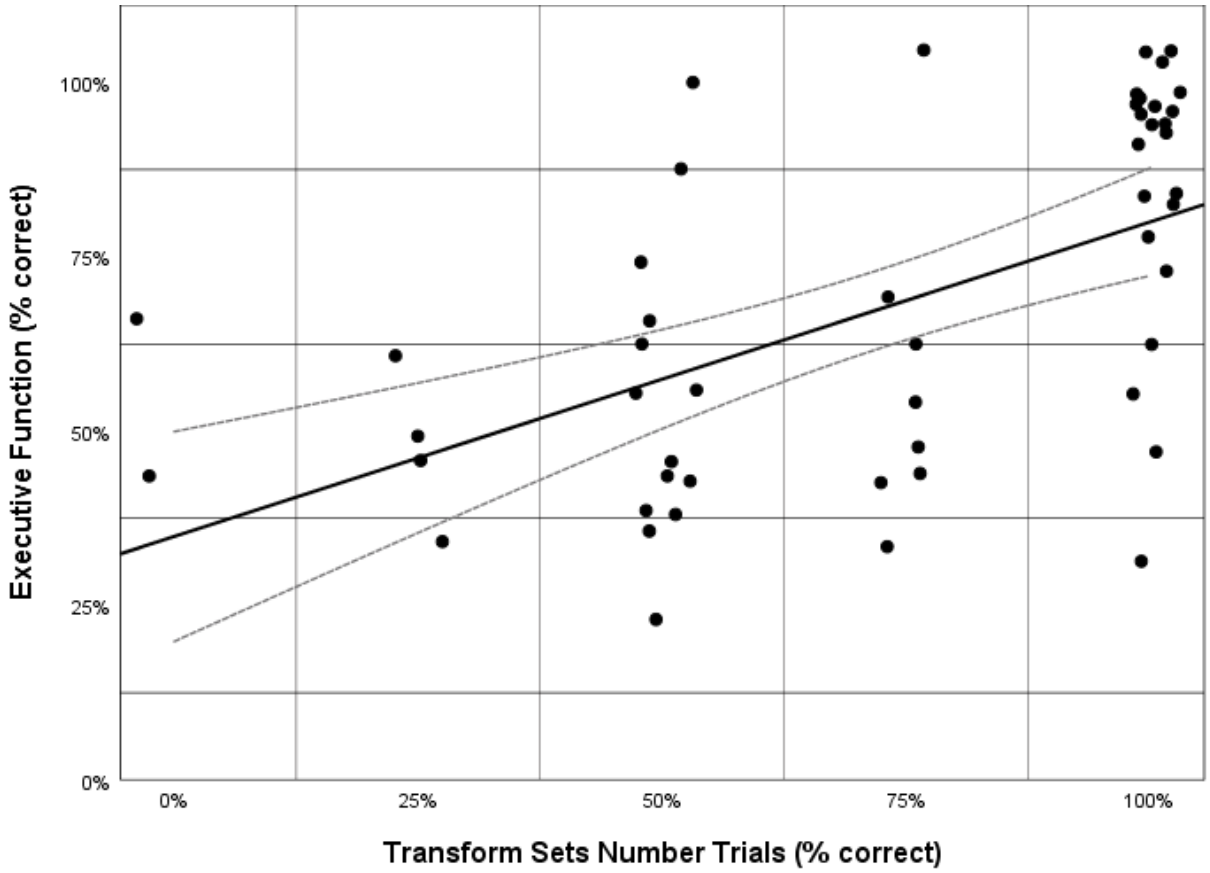
This understanding is **first applied to *known* numbers** ("two" and "three" for Three-Knowers).

Quantity Trials x Knower Level



One Knowers may change number word labels in response to changes in continuous quantity (*grow/shrink*).

Number Trials x Executive Function



EF correlates with performance on number trials.

EF does ***not*** predict performance on quantity trials.

Conclusions

Children draw on AMS representations as they construct number concepts.

AMS supports representation of discrete and continuous quantities.

∴ Children must figure out whether number words label numerosity or continuous quantity (often conflated).

- Children appear to link number words to numerosity as they learn/induce the Cardinality Principle.
- **Executive functioning** influences children's developing understanding of number, but...?

Thank You!

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