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# PLANNING FREEFORM EXTRUSION FABRICATION PROCESSES WITH CONSIDERATION OF HORIZONTAL STAIRCASE EFFECT

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## Abstract

An algorithm has been developed to estimate the “horizontal” staircase effect and a technique is proposed to reduce this type of geometrical error for freeform extrusion fabrication processes of 3D “solid” parts. The adaptive rastering technique, proposed in this paper, analyzes the geometry of each layer and changes the width of each line of the raster adaptively in order to reduce the staircase error and increase the productivity simultaneously. For each line, the maximum width that results in a staircase error smaller than a predefined threshold is determined for decreasing the fabrication time or increasing the dimensional accuracy, or both. To examine the efficacy of the proposed technique, examples are provided in which staircase errors and fabrication times are compared between uniform and adaptive rastering methods for each part. The results show a considerable improvement in accuracy and/or fabrication time for all parts studied when using the adaptive rastering technique.

## Introduction

One of the main challenges facing additive manufacturing processes is the geometrical errors caused by approximating complex shapes by lines of deposited material. In the additive manufacturing literature, this problem is commonly referred to as the staircase effect, which is typically considered only in the vertical direction. An obvious method to decrease this type of error is using finer lines. However, this will result in prolonged fabrication time. Approaches to reduce the horizontal staircase effect include printing outer contours, which follow the boundary of every layer, and machining the part after fabrication. These methods might be suitable for polymeric and metallic materials. However, for ceramics the former method may result in lower mechanical strength and the latter method is difficult and expensive. The reason that printing outer contours may considerably reduce the strength of the part is that it may introduce gaps between the outer contours and deposited lines of material in the inner regions and, since ceramics are very sensitive to voids and flaws, the part loses its strength.

In many freeform extrusion fabrication machines, the bottleneck in achieving higher productivity is the maximum attainable travel speed [1]. When the travel speed is set to its maximum value, productivity could be further increased by increasing the feed rate. However, at a constant travel speed, higher material feed rates result in larger lines and, thus, larger staircase errors, creating a compromise between productivity and accuracy. Another approach is setting the travel speed at its maximum value and adaptively changing the feed rate depending on changes in geometry of the part. In other words, when there is no abrupt change in the geometry, higher feed rates are used to decrease the fabrication time; however, lower feed rates are used to build steep slopes and delicate features with fine lines. This concept has been employed in “adaptive slicing” methods to reduce the “vertical” staircase effect. Figure 1 shows a simple example where a hemisphere is adaptively sliced to increase the dimensional accuracy. A brief review of adaptive slicing methods is provided in the following paragraphs.

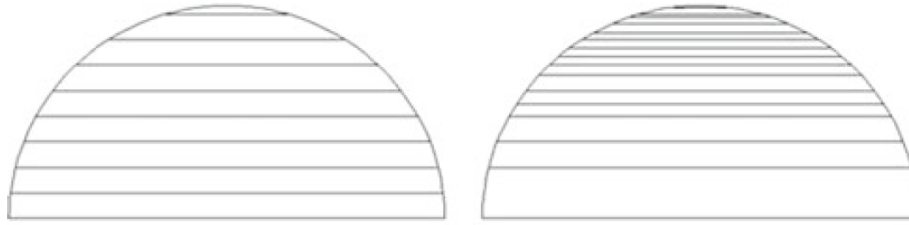


Figure 1. Uniform slicing (left) versus adaptive slicing (right) for a hemisphere.

Dolenc and Makela [2] introduced the concept of adaptive slicing. They used cusp height to calculate the part's dimensional error for each layer thickness. The user specifies a maximum allowable value for the cusp height, and the surface normal of the preceding intersection plane in the CAD file is compared with that value to determine the optimal layer thickness. Although many researchers still use the cusp height criterion (e.g. [3]), other methods have been proposed to calculate the error. Zhao and Laperriere [4] proposed an area deviation error criterion to obtain the appropriate layer thickness. Kumar and Choudhury [5] extended the error criterion to three-dimensional space and introduced a volume deviation criterion for direct adaptive slicing. Singhal et al. [6] used surface roughness to determine the optimal value for layer thickness between user-defined minimum and maximum values. Hayasi and Asiabanpour [7] projected all pairs of corresponding slices at the top and bottom of a layer onto the XY, XZ and YZ horizontal surfaces to detect any possible part geometry distortion. They also employed a bottom-up slicing approach where they start cutting at the minimum available thickness to avoid any large geometry deviation errors caused by sharply concave or convex corners.

Chen and Feng [8] considered the deviation between the final polished part and the CAD file boundary, and optimized the thickness as well as the position of each layer to minimize the number of layers for a given tolerance. Recently, the concept of adaptive slicing has been applied to additive manufacturing of Functionally Gradient Materials (FGM). For example, Su et al. [9] proposed a data format for modeling FGM objects and presented an adaptive slicing algorithm based on the finite element concept for FGM, which slices an FGM object into layers and then stores the data according to the proposed data format.

Although the vertical staircase effect has been considered in many papers and various approaches have been proposed to change the layer thickness adaptively, the horizontal staircase effect has rarely been taken into account. The reason lies in the fact that many additively manufactured parts are not solid (fully dense) and the voids and gaps between inner rasters and outer contours are insignificant. Moreover, the voids and gaps are not as critical for metals and polymers as they are for ceramics. In this paper, the horizontal staircase effect is considered and an algorithm is proposed to estimate this type of error. Furthermore, a technique is developed to reduce this error while increasing the productivity for freeform extrusion fabrication processes of 3D solid parts. Three examples are provided to illustrate the considerable improvement using the proposed method for various geometries.

### **Estimation and analysis of error**

As shown in Figure 2, each layer is composed of adjacent parallel lines. The heights of the lines (in the Z direction) are constant and equal to the layer thickness. Their width,  $w$ , is subject to change and can take different values in contiguous lines. The length of each line is limited by the stl file boundaries such that their midline intersects with the stl boundary at both ends (shown by blue points in Figure 2).

To estimate the staircase error for each line, the intersections of line boundaries and normals to the stl file boundaries are determined and the maximum of the lengths of these segments (between stl file and line) is defined as the cusp height and used as a measure of error (see Figure 2). In this figure, the green and red segments are perpendicular to the part boundaries in the stl file and intersect with line boundaries. Their maximum length is used as the measure of horizontal staircase error.

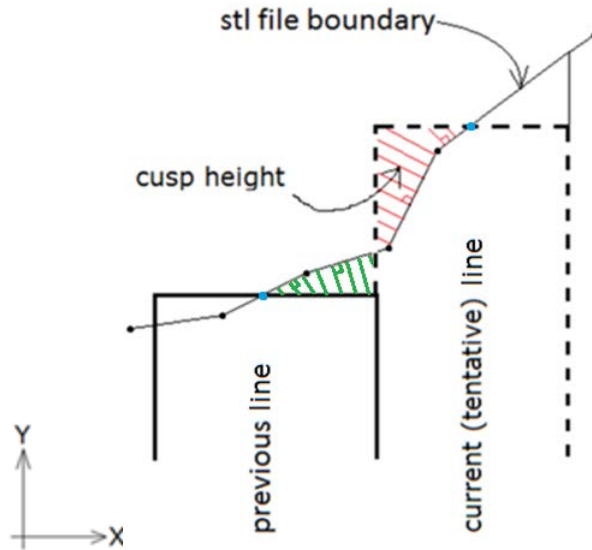


Figure 2. Stl file boundary and adjacent lines forming a layer. Cusp heights are a measure of error.

As shown in Figure 3, for a point at an arbitrary distance,  $s_i$ , from a segment end point  $(x_1, y_1)$ , the coordinates  $(x_i, y_i)$  are

$$\begin{cases} x_i = \frac{s_i}{l}(x_2 - x_1) + x_1 \\ y_i = \frac{s_i}{l}(y_2 - y_1) + y_1 \end{cases} \quad (1)$$

where  $l$  is the length of the segment. The cusp height,  $h_i$ , at this point is

$$h_i = l \frac{x_i - x_0}{y_2 - y_1} \quad (2)$$

Thus, for each segment, by calculating a sufficient number of cusp heights and choosing the maximum value, the error could be obtained. There are four error values to be determined at each step (two at one end as shown in Figure 2 in red and green colors, and two at the other end not shown in the figure) and the maximum of the four values is taken as the error for that line.

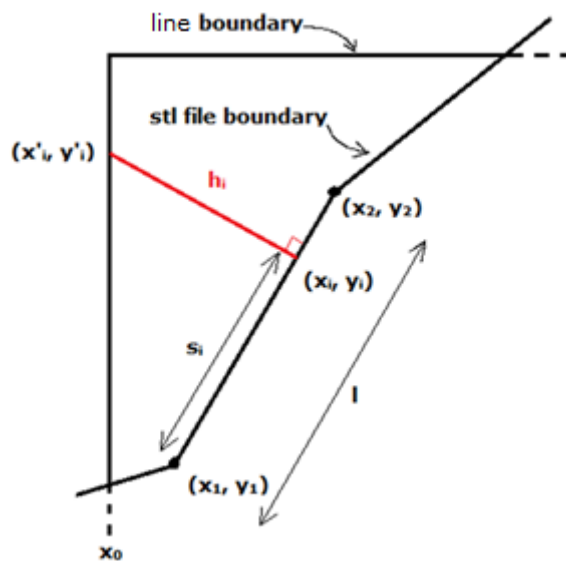


Figure 3. Calculation of cusp height.

Since each layer is composed of a finite number of lines, an error diagram for each layer could be plotted by calculating the error values for each line. A program is written in MATLAB which reads the geometry of the part from a CAD file in stl format, finds the intersections of the representative surfaces of the part with horizontal planes and forms the boundaries for each layer. After layer boundaries are obtained from the stl file, rastering is performed to fill in the build area with lines. Figure 4 shows the rastering and error diagrams for a layer of an object rastered with lines of 1.5 mm width. Each error corresponds to one line and each line is represented by a straight line passing through its center. The horizontal staircase effect manifests in regions with abrupt changes in the shape.

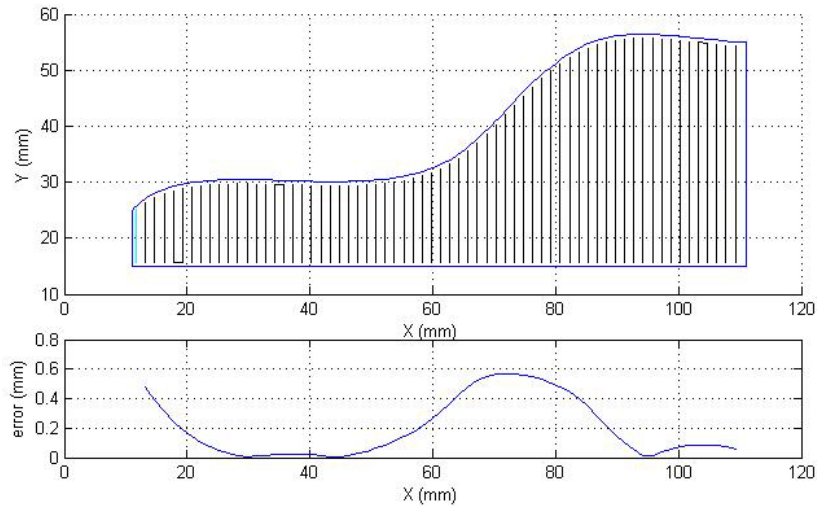


Figure 4. Rastering and error diagrams for a layer of an object rastered with 1.5 mm wide lines.

A smaller line width reduces the errors, but increases the fabrication time. As an example, the layer illustrated in Figure 4 is considered and it is assumed that the travel speed is 100 mm/s. For various line widths, the required fabrication time is plotted versus the maximum error value in Figure 5. The markers on the curve correspond to line widths of 0.2, 0.5, 1.0, 1.5, 2.0, 2.5 and 3.0 mm, from left to right, respectively (e.g. using a width of 1 mm results in a maximum error of 0.38 mm and 23.7 s of fabrication time). As the diagram clearly demonstrates, to achieve high accuracy, an unreasonably large amount of time is required. To overcome this problem, an adaptive rastering algorithm is proposed in the next section.

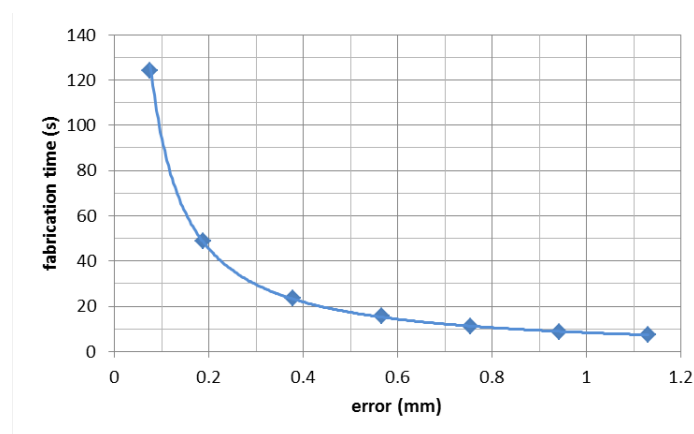


Figure 5. Fabrication time for each layer represented in Figure 4 as a function of maximum error.

### Adaptive rastering algorithm

In this section, adaptive rastering is proposed as an efficacious solution to reduce horizontal staircase errors or fabrication time, or both. This technique is based on using a small line width as needed and employing larger lines elsewhere. In other words, a small line width is used where there is an abrupt change in the geometry of the part and larger line widths for the rest of the part.

From a practical perspective, there are limitations on the minimum and maximum values of the line width and the number of different widths to be used. For a constant table speed (which is set at its maximum value), line width depends on the size of the extrusion nozzle as well as the feed rate. For a single nozzle and a certain table speed, width could only be adjusted in a certain range by changing the feed rate. Decreasing the feed rate below this critical value results in printing a discontinuous line. The upper bound of this range is limited by fluidity of the extrudate since it can only flow a certain distance,  $d$ , from the nozzle as shown in Figure 6. If the feed rate is further increased, the width remains constant and an increase in the height is observed. Furthermore, changes in the feed rate cannot follow a step reference, i.e., sudden jumps in the value of feed rate is not physically possible. This means that either the width of two consecutive lines has to be equal (so that no sudden change in feed rate is required), or the extrusion process has to be stopped between the two lines (so that the next line can be printed at desired width). Accordingly, to avoid an unreasonably large number of starts and stops, a limited number of line widths should be used. In this way, lines having the same width are printed together. As an example, Figure 7 shows a layer rastered with three line widths. Initially, all the lines with the smallest widths,  $w_1$ , are printed as shown in the left picture with cyan; next, lines of second width,  $w_2$ , are printed as shown in the middle picture with magenta; and finally, the widest lines,  $w_3$ , are printed as the right picture illustrates with green color. The rastering algorithm is explained in the next paragraph assuming three allowable widths.

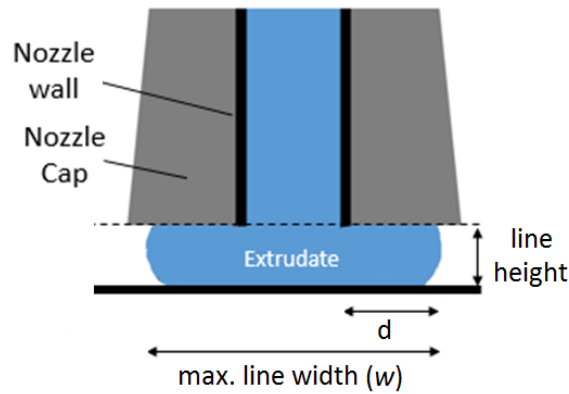


Figure 6. Dimensions of the extrudate.

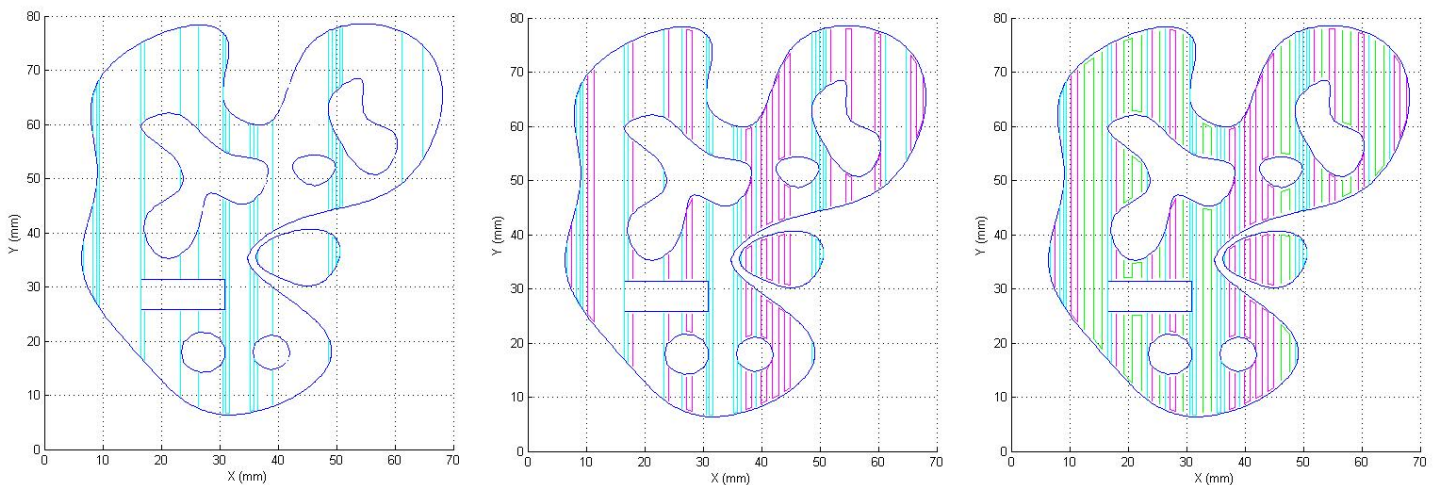


Figure 7. Rastering sequence of a layer with three line widths.

Rastering starts from the leftmost line in the layer where the smallest width is chosen. To determine the width of the second line, the smallest line width ( $w_1$ ) is examined initially and the corresponding error is calculated (the intersections of its midline with stl file boundary are obtained and the region between the midline of the second line and the midline of the first line is considered as illustrated in Figure 2 and cusp heights in this region are calculated). The error is compared against the maximum allowable value predefined by the user. If the error is below the tolerance, the second line width ( $w_2$ ) is tested. If the error for  $w_2$  is above

the threshold,  $w_1$  will be used to print that line. Otherwise, the largest line ( $w_3$ ) is tested and, if the error is above the tolerance,  $w_2$  will be used. Otherwise,  $w_3$  will be chosen. This process is repeated for all lines until the entire layer is covered.

Figure 8 shows the flowchart of this algorithm assuming three line widths are allowed. Although three widths are assumed in the flowchart, any number of line widths can be assumed using the same algorithm. To implement this method, a program is developed in MATLAB which reads the geometry of the CAD file in stl format, slices the part into layers, determines the sequence and width of each line for all the lines to print each layer, and generates a g-code for a freeform extrusion fabrication machine to print the part.

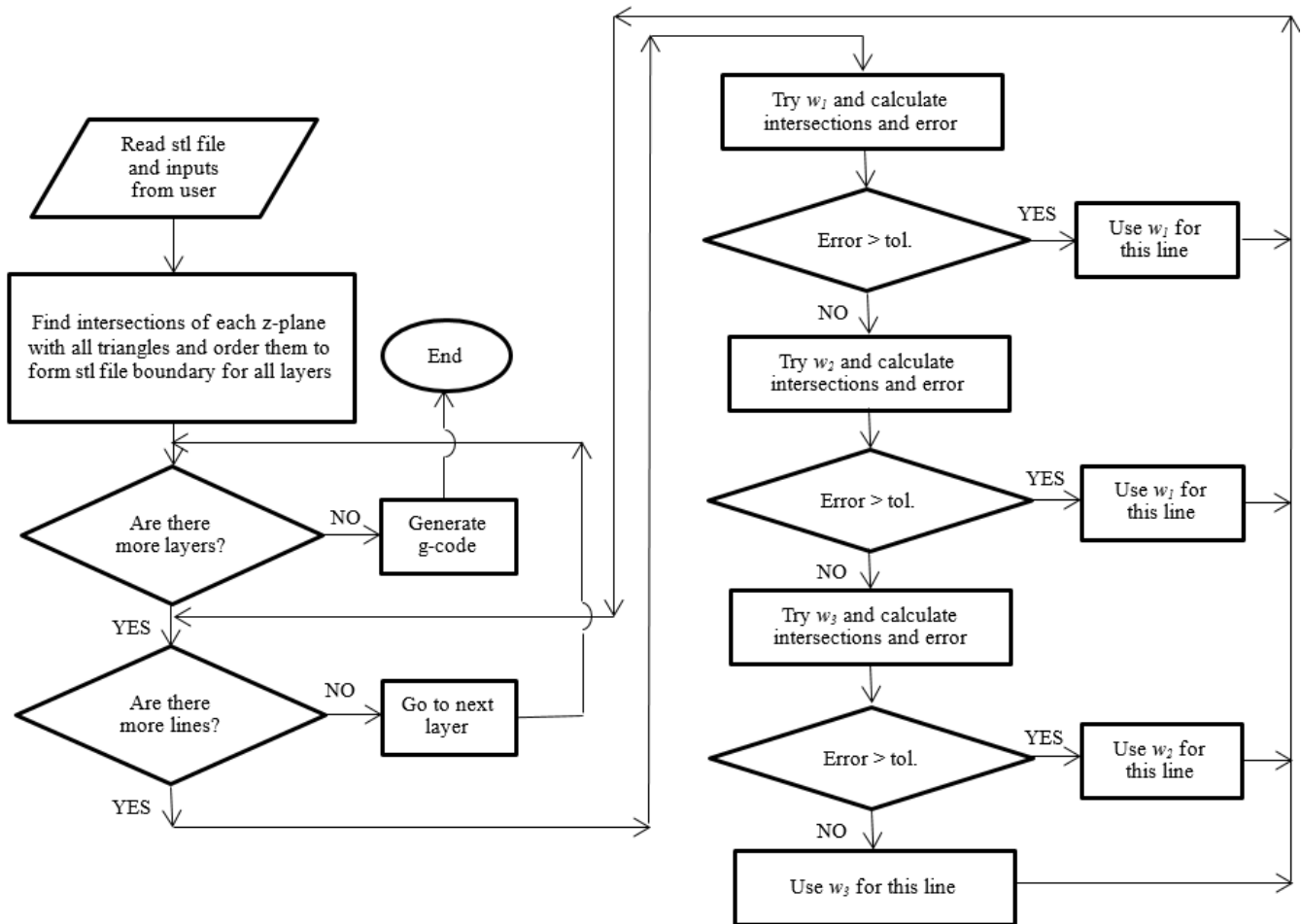


Figure 8. Flowchart of adaptive rastering algorithm.

Three different approaches could be made for adaptive rastering of a part depending on the choices for line widths. The highest attainable accuracy mainly depends on the choice of the smallest line width while other widths are obtained by trial and error to reduce the fabrication time as much as possible. As an example, for the layer shown in Figure 4, values of 0.8, 1, and 3 mm for  $w_1$ ,  $w_2$ , and  $w_3$ , results in a maximum error of 0.118 mm and a fabrication time of 44.5 s. The adaptively rastered layer along with the error diagram is illustrated in Figure 9 where cyan, magenta and green colors correspond to line widths of 0.8, 1.0 and 3 mm, respectively.

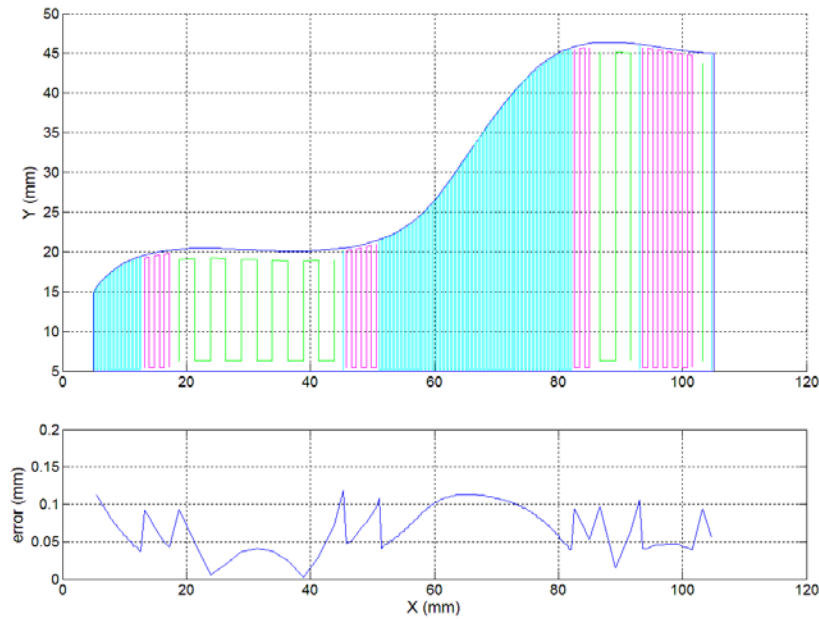


Figure 9. A layer adaptively rastered with 0.8, 1 and 3 mm line width shown in cyan, magenta and green colors, respectively. Each point in the error diagram corresponds to one line.

The above mentioned procedure is repeated for various sets of line widths, and fabrication time is plotted versus error in Figure 10 for each combination of line widths. The uniform rastering results represented in Figure 5 is also plotted in this figure for comparison. For a point, A, on the blue curve, one approach could be choosing a set of line widths for adaptive rastering which reduces the fabrication time without an increase in error (point B on the red curve). Another approach is choosing line widths corresponding to point C where there is a considerable decrease in error while fabrication time remains constant. Third approach is moving from A to D which reduces both error and time.

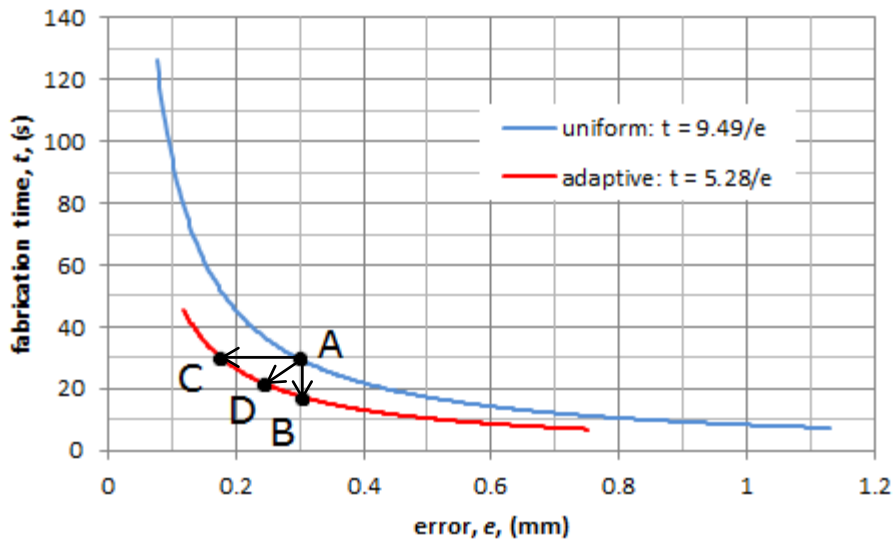


Figure 10. Uniform and adaptive rastering for layer represented in Figure 4.

## Case studies

### Part #1

As an illustrative example, a cylinder is considered and three approaches explained above are examined to adaptively raster each layer of the cylinder. In the first adaptive rastering approach only the fabrication time is decreased; in the second approach errors are reduced, whereas the third approach reduces both errors and



fabrication time simultaneously. The results are compared to uniform rastering with a line width of 1 mm which results in a maximum error of 0.485 mm and a fabrication time of 27.58 s for each layer (assuming 100 mm/s for the travel speed). As shown in Figure 11 (left), the error values decrease linearly toward the middle of the part. Figure 11 (right) shows an adaptively rastered layer using the second approach. Cyan, magenta and green colors correspond to line widths of 0.6, 1.0 and 1.6 mm, respectively. As the error diagram below the picture indicates, the maximum error is 0.325 mm. The results of the three approaches are represented in Table 1 and are compared against the uniform rastering. If the errors are not to be reduced, there is 34.5% decrease in fabrication time and, if the productivity is not to be improved, errors decrease by 33.0%. There could also be a 14.0% and 20.6% simultaneous reduction in error and time, respectively, when using the third approach.

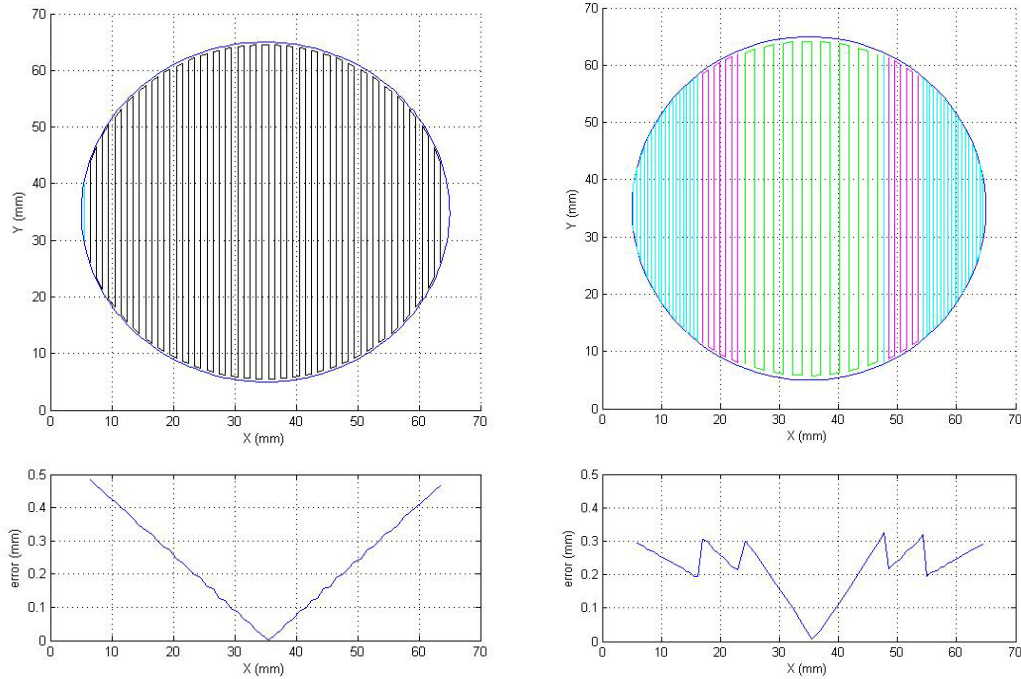


Figure 11. Uniform rastering of a layer of a cylinder (left) and adaptive rastering (right).

Table 1. Comparison between uniform and adaptive rastering for a layer of a cylinder.

Rastering	Line width(s) (mm)	Maximum error (mm)	Reduction (%)	Fabrication time (s)	Reduction (%)
Uniform	1.0	0.485	-	27.58	-
Adaptive #1	1.0, 1.3, 2.0	0.485	<b>0</b>	18.06	<b>34.5</b>
Adaptive #2	0.6, 1.0, 1.6	0.325	<b>33.0</b>	27.58	<b>0</b>
Adaptive #3	0.8, 1.0, 1.6	0.417	<b>14.0</b>	21.90	<b>20.6</b>

## Part #2

In this example, the part shown in Figure 4 is considered. First, assuming uniform rastering, the maximum allowable error is determined to the extent that the manufacturer of the part attains maximum profit. Next, it is demonstrated how employing the adaptive rastering technique enables the manufacturer to produce cheaper parts with higher quality, and gain more profit.

The manufacturer’s objective is to maximize the “amount of profit he acquires per unit time” which is

$$P = (p - c) \times n \quad (3)$$

where  $P$  is profit per unit time (\$/h),  $p$  is price a customer is willing to pay per part,  $c$  is cost of fabricating each part and  $n$  is number of parts produced per unit time.

Assuming the price a customer pays linearly decreases with error,  $e$ , ( $p = a - be$ ) and cost linearly depends on time,  $T$ , plus raw material,  $d$ , ( $c = cT + d$ ), the profit (\$/h) will be

$$P = ((a - be) - (cT + d)) \times 60/T \quad (4)$$

It is assumed that a perfect part ( $e \approx 0$ ) could be sold at \$100 and an inferior part ( $e = 1$  mm) at \$30 (so,  $a = \$100$ ,  $b = 70$  \$/mm), cost of using the 3D printer is 1 \$/min ( $c = 1$  \$/min) and raw material is \$20 ( $d = \$20$ ), and the part is made of 60 layers.

To write the profit in terms of only error, the relationship between required fabrication time,  $T$ , and error,  $e$ , should be substituted in equation (4). Using Figure 10, the required time for manufacturing each part for both uniform and adaptive rastering is approximated by a homographic function of the form

$$T(\text{min}) = \frac{f}{e \text{ (mm)}} \quad (5)$$

where  $f = 9.49$  for uniform rastering and  $f = 5.28$  for adaptive rastering. By substituting equation (5) into equation (4)

$$P \left( \frac{\$}{h} \right) = [(a - be) - \left( \frac{cf}{e} + d \right)] \times \frac{60e}{f} \quad (6)$$

and taking the derivative with respect to  $e$ , the optimum value of  $e$  for which the manufacturer will gain maximum profit could be easily found ( $e = \frac{a-d}{2b} = 0.571$  mm). This results in a price of \$60, a cost of \$36.6, a productivity of 3.61 parts per hour, and a profit of 84.5 \$/h for uniform rastering, and a price of \$60, a cost of \$29.2, a productivity of 6.49 parts per hour, and a profit of 199.7 \$/h for adaptive rastering. Thus, the manufacturer gains 2.36 times more profit by employing adaptive rastering technique.

### Part #3

The third part is very similar to one of the examples chosen by Panhalkar et al. [10] for adaptive slicing and is illustrated in Figure 12 (left). Similar to Part #1, three approaches are examined to adaptively raster each layer. In the first adaptive rastering approach, only the fabrication time is decreased; in the second approach errors are reduced, whereas the third approach reduces both errors and fabrication time simultaneously. The results are compared to uniform rastering with a line width of 0.5 mm and layer thickness of 1 mm which results in a maximum error of 0.249 mm and a fabrication time of 230 min for the whole part (assuming 100 mm/s for travel speed).

Figure 12 (middle and right) shows two adaptively rastered layers using the first approach. Cyan, magenta and green colors correspond to line widths of 0.5, 0.7 and 1.2 mm respectively. The results of the three approaches are represented in Table 2 and are compared against the uniform rastering. If the errors are not to be reduced, there is 40.9% decrease in fabrication time, and if the productivity is not to be improved, errors decrease by 37.7%. There could also be a 14.0% and 25.6% simultaneous reduction in error and time, respectively when using the third approach.

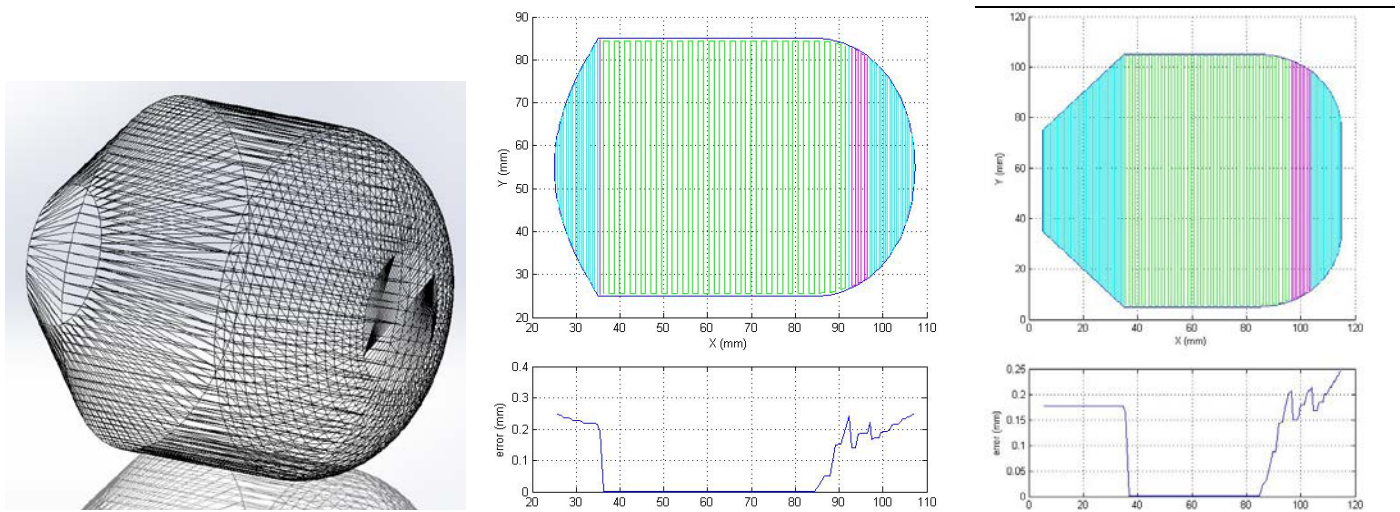


Figure 12. Stl representation of Part #3 (left) and adaptive rastering of two sample layers (middle and right).

Table 2. Comparison between uniform and adaptive rastering for a layer of a cylinder.

Rastering	Line width(s) (mm)	Maximum error (mm)	Reduction (%)	Fabrication time (min)	Reduction (%)
Uniform	0.5	0.249	-	230	-
Adaptive #1	0.5, 0.7, 1.2	0.249	<b>0</b>	136	<b>40.9</b>
Adaptive #2	0.3, 0.5, 0.8	0.155	<b>37.7</b>	230	<b>0</b>
Adaptive #3	0.4, 0.6, 1	0.214	<b>14.0</b>	171	<b>25.6</b>

## Summary and Conclusions

An algorithm was developed to estimate the “horizontal” staircase effect and a technique was proposed to reduce this type of geometrical error and/or to increase the productivity for freeform extrusion fabrication processes. To estimate the error for each line, the intersections of line boundaries and lines perpendicular to stl file boundaries were determined and the lengths of these segments (between stl file and line) were taken as the error. The “adaptive rastering” technique was then developed to change the width of every line of the rasters adaptively in order to reduce the error and/or increase the productivity simultaneously.

Three representative parts were studied to examine the efficacy of the proposed technique. In the first case study, a cylinder was chosen for which the adaptive rastering resulted in 20.6% and 14% reduction in fabrication time and error, respectively. In the second example, an optimization problem was considered to maximize the amount of profit in a small manufacturing unit. The maximum attainable profit per hour was 2.36 times higher when adaptive rastering was employed. In the last case study, a more realistic part previously studied in adaptive slicing was adaptively rastered using different approaches. When the errors were not to be reduced, a 40.9% reduction was observed in fabrication time; when the productivity was held constant, errors decreased by 37.7%. Thus, it could be concluded that the adaptive rastering technique proposed in this paper can considerably improve the dimensional accuracy and/or fabrication time.

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