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On Iterative Decoding of Two-Level Superposition Codes for Cooperative Broadcasting Based on QPSK and 4-PAM Constellations

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***Abstract*—This paper considers iterative decoding of two-level superposition codes used in cooperative broadcasting over wireless networks. The coding scheme consists of two low-density parity-check (LDPC) codes combined using Plotkin's $|u|u+v|$-construction and provides two levels of error protection. Coding for cooperative broadcasting differs from conventional single-source broadcast coding. Instead of binary addition of subcode codewords performed by a single-source encoder, in superposition coding two modulated subcode sequences, produced by two co-ordinated and independent sources, are combined at the antenna of the receiver. Expressions are derived for the bit metrics, or log-likelihood ratio values, used by an iterative decoder for two-level superposition coding schemes based on BPSK, QPSK and 4-PAM modulated sequences. It is shown that conventional equal-energy 4-PAM constellations do not work well with two-level superposition coding. A solution is proposed in which 4-PAM constellations of different levels of average energy are used.**

I. Introduction

Interest in cooperative wireless networks has grown considerably recently [1]-[7]. An important aspect of these types of networks is that communication between network nodes takes place in two stages: (1) broadcasting and (2) multiple access. On the other hand, Bergmans and Cover [8] showed that coordinated sources broadcasting information using superposition coding always outperform other schemes based on orthogonal channel assignments, such as time sharing or frequency division. The work reported here is based on the fundamental ideas put forward by Cover [9] on broadcast channels. Specifically, this paper considers superposition coding by two cooperating source nodes broadcasting information to two types of destination/relay nodes [8]. To provide two levels of error protection, here a coding scheme is considered that is based on the $|u|u+v|$-construction of Plotkin [10] with LDPC codes of the same length as components.

It is important to point out the difference between multilevel coding used in a single-source broadcasting system and superposition coding used in cooperative broadcasting. Contrary to the binary case, where addition is modulo two, the superposition of *modulated signal sequences* associated with $|u|u|$ and $|0|v|$ subcodes takes place at the receiver (via the antenna), with addition over the field of real numbers (or, depending on the signal constellation selected, over a multi-dimensional real-number field).

One way to obtain different levels of error protection is to partition a signal constellation into sets. The underlying signal labeling by bits induced by the partition needs to be selected in a way that a demapper at the receiver produces meaningful bit metrics — or likelihood ratio values — used by an iterative decoder. In this paper, however, we consider the case where conventional BPSK, QPSK and 4-PAM constellations are used by the broadcasters and examine the log-likelihood ratio (LLR) values needed for decoding with different bit mappings.

The coding schemes that are studied in this paper differ from those studied by Larsson and Vojcic [11] and later by Xiao et al. [12]. In the cooperative system studied in [11], a node in the wireless network transmits at the same time the data packet and the relay packet. Similarly, in [12], each node transmits the real-number addition (or *Euclidean superposition*) of the encoded and modulated local and relay packets. Other recent related work in this area is superposition mapping [13]-[15].

In contrast, in the wireless network considered here, the task of broadcasting information is shared by two cooperating node sources. Each source encodes and modulates data independently and sends it to the receiving nodes. The superposition of the transmitted signals occurs at the receiver.

II. Superposition coding with two levels of error protection

In two-level superposition codes for cooperative broadcasting there are two classes of destination/relay nodes: Close nodes and far nodes, as ranked by their spatial distance to a pair of coordinated source nodes. Those destination or relay nodes that are close to a source node are able to receive all of the transmitted information bits with a high degree of reliability, while nodes far from source nodes can only recover reliably a portion of the transmitted bits. Therefore, two levels of error protection are required. To provide two levels of error protection, in this paper the $|u|u+v|$ construction [10] will be used. Let C_1 and C_2 be two linear (n, k_1, d_1) and (n, k_2, d_2) codes, respectively. Then the linear code

$$C \triangleq \{|\bar{v}_1|\bar{v}_1 + \bar{v}_2| : \bar{v}_1 \in C_1, \bar{v}_2 \in C_2\},$$

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is a linear $(2n, k_1 + k_1, \min\{2d_1, d_2\})$ code with generator and parity-check matrices

$$G = \begin{pmatrix} G_1 & G_1 \\ 0 & G_2 \end{pmatrix}, \quad H = \begin{pmatrix} H_1 & 0 \\ H_2 & H_2 \end{pmatrix},$$

respectively. If the condition $2d_1 < d_2$ is satisfied [19], then code C is said to be a linear unequal-error-protection (LUEP) code with separation vector $\bar{s} = (d_2, 2d_1)$ for the message space $M = \{0,1\}^{k_2} \times \{0,1\}^{k_1}$. This LUEP code is designed for a degraded binary broadcast channel. It is also interesting to note that with component LDPC codes, the parity-check (PC) submatrix associated with the k_2 most significant bits (MSB) is more dense than the PC submatrix associated with the k_1 least significant bits (LSB). As a result, iterative decoding provides two levels of error protection.

A. Over-the-air mixing

In over-the-air mixing [8] or superposition, the subcode codewords $|\bar{v}_1|\bar{v}_1|$ and $|\bar{0}|\bar{v}_2|$ are modulated and sent *separately* by each of the two source nodes. This is shown schematically in Fig. 1.

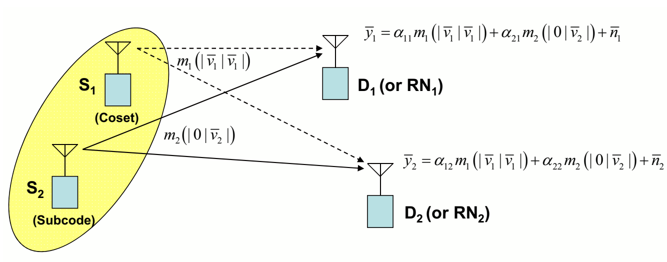

Fig. 1. A two-source cooperative broadcasting system for two users.

The transmitted signal sequences are combined by the receive antennas at the destination (or relay) nodes. Let $m_1(|\bar{v}_1|\bar{v}_1|)$ and $m_2(|\bar{0}|\bar{v}_2|)$ denote the mappings of bits to signal sequences drawn from two (generally two-dimensional) signal sets, $\mathcal{M}_1$ and $\mathcal{M}_2$, each associated to a source node. Assuming an AWGN channel, the received signal sequence is given by

$$\bar{y} = m_1(|\bar{v}_1|\bar{v}_1|) + m_2(|\bar{0}|\bar{v}_2|) + \bar{n},$$

where addition (per dimension) is over the field of real numbers, $m_i(\cdot) \in \mathcal{M}_i$, $i = 1,2$, and $\bar{n}$ is a (generally two-dimensional) Gaussian random vector with i.i.d. components of zero-mean and variance $N_0/2$.

In a more general case, over a block-fading channel as illustrated in Fig. 1, the signal sequence received by destination (or relay) node j is given by

$$\bar{y} = \alpha_{1j} m_1(|\bar{v}_1|\bar{v}_1|) + \alpha_{2j} m_2(|\bar{0}|\bar{v}_2|) + \bar{n},$$

where α_{ij} denotes the fading amplitude (in general a vector) from source node i to destination (or relay) node j, and $i = 1,2$. The block fading assumption means that the same fading amplitude affects all the symbols in a transmitted signal sequence. To achieve two levels of error protection for broadcasting, over either AWGN or flat Rayleigh fading channels, two sets of signal constellation points $\mathcal{M}_1$ and $\mathcal{M}_2$ are selected so as to obtain two different values of *minimum Euclidean distance* (MED),

$$\begin{aligned} s_1 &= \min_{\bar{v}_1' \neq \bar{v}_1} \{D\left(m_1(|\bar{v}_1|\bar{v}_1|) + m_2(|\bar{0}|\bar{v}_2|),\right. \\ &\qquad \left. m_1(|\bar{v}_1'|\bar{v}_1'|) + m_2(|\bar{0}|\bar{v}_2|)\right)\}, \\ s_2 &= \min_{\bar{v}_2' \neq \bar{v}_2} \{D\left(m_1(|\bar{v}_1|\bar{v}_1|) + m_2(|\bar{0}|\bar{v}_2|),\right. \\ &\qquad \left. m_1(|\bar{v}_1|\bar{v}_1|) + m_2(|\bar{0}|\bar{v}_2'|)\right)\}, \end{aligned}$$

where $D(\bar{x}_1, \bar{x}_2)$ denotes the Euclidean distance between two vectors $\bar{x}_1, \bar{x}_2$, and $s_2 > s_1$. The $|u|u+v|$ construction gives two values of minimum Hamming (symbol) distances and two values of minimum product distances. This means that with transmission over flat Rayleigh fading channels, two levels of diversity order are obtained. In the case of QPSK modulation with Gray labeling, the MED values become proportional to the square roots of the minimum Hamming distances of the constituent codes C_1 and C_2. That is, $s_i = \sqrt{2d_i}$, $i = 1,2$. Binary LUEP codes can be applied to obtain good superposition codes with different levels of error protection.

B. Metrics for two-level cooperative broadcasting using BPSK and QPSK modulations

The analysis of two-level superposition coding for two cooperative broadcasting sources is not the same as multilevel coding with two levels of error protection for one broadcasting source. In this section, this difference is clarified by considering the bit metrics for BPSK modulation in each case. Since a symmetric QPSK signal set can be interpreted as a Cartesian product BPSK×BPSK signal set, the results presented in this section apply also to superposition coding based on QPSK modulation.

In two-source cooperative broadcasting with BPSK modulation and the bit mapping $0 \mapsto \sqrt{E_b}$ and $1 \mapsto -\sqrt{E_b}$, the possible received values r in the absence of noise are shown in Table I. For QPSK modulation, as expected, the received values form the Cartesian product of two sets of three possible values per dimension. The received values (r_1, r_2), normalized with respect to $\sqrt{E_b}$, for two-level superposition coding with QPSK modulation are shown in Fig. 2. Also shown in the figure are the bit labels $(v_{11} \oplus v_{21}, v_{12} \oplus v_{22})$. Since $r_i = 0$ when the coded bits $v_{1i} \oplus v_{2i} = 1$, and $r_i \neq 0$ whenever $v_{1i} \oplus v_{2i} = 0$, $i = 1,2$, the iterative decoding process will work properly with bit metrics per dimension computed in the same manner as for BPSK modulation.

The a-posteriori log-likelihood ratio values, or LLR values, are used as metrics in iterative decoding at the receiver. For binary modulation, it is well know that the LLR value of an information bit u given the received value r is

$$L_u(r) = \log \left[\frac{\Pr\{u=1|r\}}{\Pr\{u=0|r\}} \right].$$

For single-source broadcasting over an AWGN channel, using a max-log approximation, the LLR values are the same

TABLE I
RECEIVED VALUES WITH BPSK MODULATION

Source 1 v_1	Source 2 v_2	Conventional $m(v_1 \oplus v_2)$	Cooperative $m(v_1) + m(v_2)$
0	0	$\sqrt{E_b}$	$2\sqrt{E_b}$
0	1	$-\sqrt{E_b}$	0
1	1	$\sqrt{E_b}$	$-2\sqrt{E_b}$
1	0	$-\sqrt{E_b}$	0

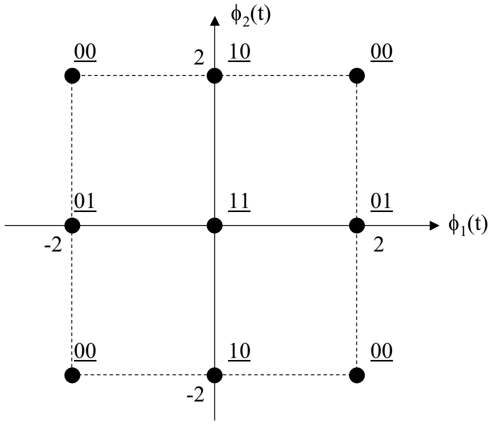

Fig. 2. The noiseless received constellation for two-level superposition coding with QPSK modulation.

as in conventional BPSK modulation over an AWGN channel and given by

$$L_u^{(1)}(r) = \frac{4\sqrt{E_b}}{N_0}\, r,$$

where N_0 is the one-sided power of the AWGN process. For two-source cooperative broadcasting over an AWGN channel, the LLR value is given by

$$L_c(r) = \log\left[\frac{\Pr\{u_1 \oplus u_2 = 0|r\}}{\Pr\{u_1 \oplus u_2 = 1|r\}}\right].$$

A max-log approximation of the LLR value, shown in Fig. 3, for BPSK modulation (and for each bit in QPSK modulation) is given by

$$L_c(r) \approx -\frac{8\sqrt{E_b}}{N_0}\, (1 - |r|) - \log(2).$$

It follows that decoding of superposition codes is quite different from that of multilevel codes. Also, when assuming that transmission occurs over a flat Rayleigh fading channel (not block fading), the approximated LLR value for two-level superposition coding becomes

$$L_c(r) \approx \frac{2\sqrt{E_b}}{N_0}\, \left(2\alpha_1\alpha_2 + |r(\alpha_1 - \alpha_2)| - |r(\alpha_1 + \alpha_2)|\right),$$

where α_1 and α_2 denote the gains of the channels from source nodes S_1 and S_2, respectively, to the destination node.

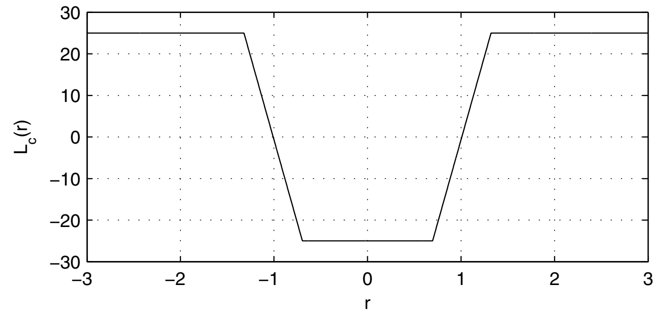

Fig. 3. LLR metrics for BPSK. $E_b = 1$ and $E_b/N_0 = 10$ dB.

III. EXAMPLES OF TWO-LEVEL SUPERPOSITION CODES

In this section, the performance is examined of various two-level superposition codes based on the $|u|u+v|$ construction. The channel models used here assume ideal synchronization at the receiver and a demapper with complete knowledge of N_0, the one-sided AWGN power spectral density. In the case of Rayleigh fading, knowledge of the channel gains is also assumed. The simulation results reported below were obtained using iterative belief-propagation decoding with a maximum of 50 iterations. The decoding process finishes earlier whenever a codeword is found. For each value of bit energy-to-noise ratio, the transmission of 10,000 sequences was simulated.

A. Superposition codes with BPSK and QPSK modulations

The performances of a superposition coding scheme based on binary LDPC codes of length 96 using BPSK and QPSK modulations over AWGN and Rayleigh fading channels are presented in Figures 4 and 5, respectively. The scheme is based on two constituent codes: C_1, a regular $(96, 50)$ LDPC code with degree distribution $(3, 6)$; and C_2, a regular $(96, 49)$ LDPC code with degree distribution $(4, 8)$. The data description files (i.e., the adjacency lists of the parity-check matrices) of these codes were obtained from David MacKay's web page [21].

B. Superposition codes with subset BPSK mapping

In this scheme, a QPSK signal set is received as the Cartesian product of two BPSK signal subsets: An in-phase BPSK subset and a quadrature BPSK subset, denoted as BPSK-I and BPSK-Q, respectively. This is known in the literature as quadrature multiplexing. The first source node S_1 uses a mapping based on the BPSK-I subset with encoding based on C_1. The second source node S_2 in turn uses a mapping that is based on the BPSK-Q subset with encoding based on C_2. Due to the orthogonality between the BPSK-I and BPSK-Q subsets, iterative decoding proceeds in parallel, with two identical demmapers for BPSK-I and BPSK-Q used to make an independent estimate of the MSB and LSB message bits, respectively.

The performance of two-level superposition coding of length 204 with subset BPSK mapping is compared to BPSK modulation in Fig. 6. In this scheme, codes C_1 and C_2 are

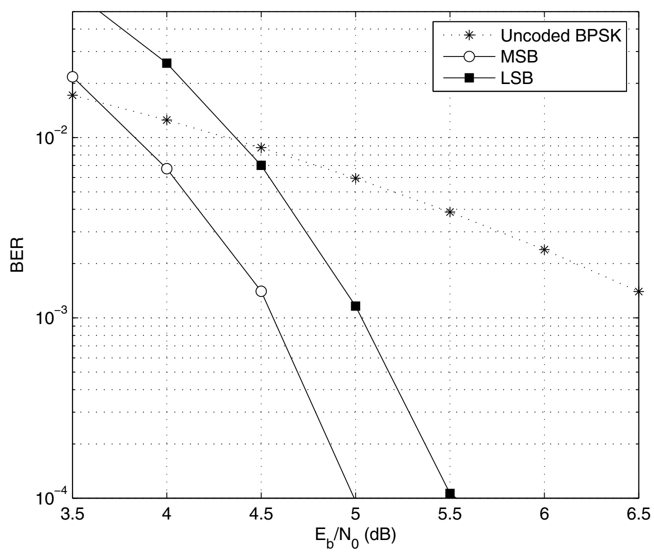

Fig. 4. Performance of a two-level superposition code with BPSK modulation and LDPC codes of length 96. AWGN channel.

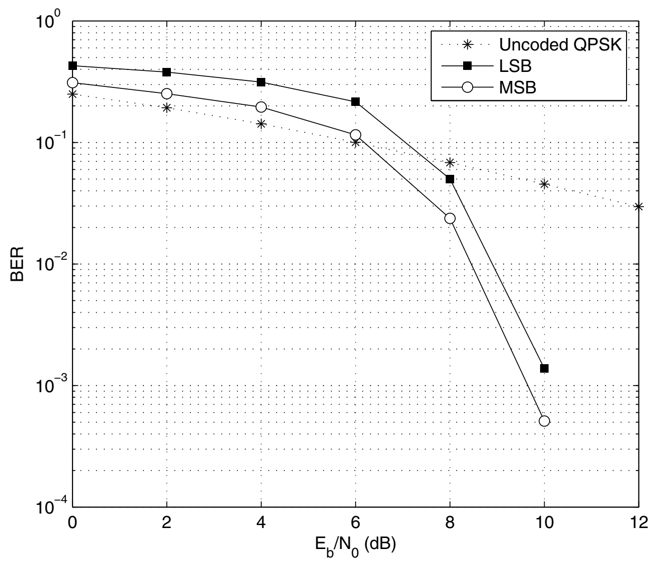

Fig. 5. Performance of a two-level superposition code with QPSK modulation and LDPC codes of length 96. Rayleigh fading channel.

regular $(204, 102)$ LDPC codes with degree distributions $(3, 6)$ and $(5, 10)$, respectively [21]. The simulation results reveal that superposition coding with mapping based on orthogonal BPSK subsets can outperform mapping based on a conventional BPSK signal set.

IV. Superposition coding based on 4-PAM constellations

There are two not equivalent bit mappings for a 4-PAM constellation: Gray and natural (Ungerboeck [22]). These are shown in Table II, where E_s denotes the average symbol

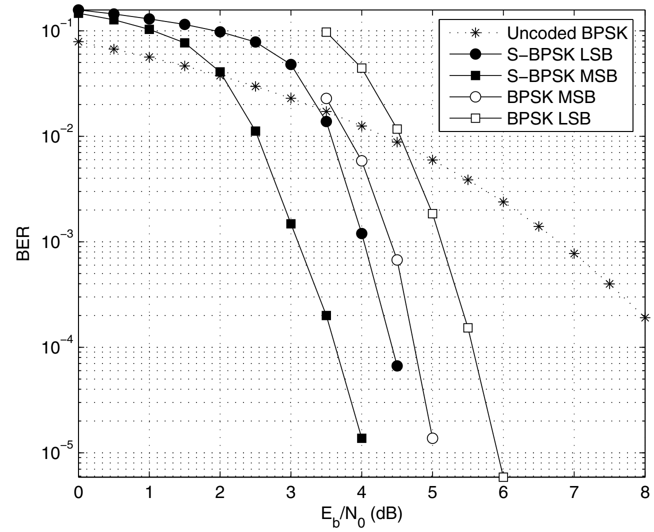

Fig. 6. Comparison of two-level superposition coding with BPSK modulation and with subset BPSK mapping. Component LDPC codes of length 204.

TABLE II
Two mappings of a 4-PAM constellation

LSB	MSB	Gray	Natural
v_1	v_2	$m(v_1, v_2)$	$m(v_1, v_2)$
0	0	$-3\sqrt{E_s/10}$	$-3\sqrt{E_s/10}$
0	1	$-\sqrt{E_s/10}$	$-\sqrt{E_s/10}$
1	1	$+\sqrt{E_s/10}$	$+3\sqrt{E_s/10}$
1	0	$+3\sqrt{E_s/10}$	$+\sqrt{E_s/10}$

energy. The bit metrics $L_{c_i}(r)$ for these two mappings (with $E_s/N_0 = 10$ dB), for $i = 1$ (LSB) and $i = 2$ (MSB) are shown in Figs. 7 and 8, respectively. In the figures, the received value is normalized with respect to $\sqrt{E_s/10}$

Consider a Gray mapped 4-PAM constellation. If the received channel value is close to $r = \pm 2\sqrt{E_s/10}$ then the LLR value is close to zero. This is shown in Fig. 7 and occurs because the associated two bits do not all agree for this received channel value. Without noise, three different combinations of pairs of 4-PAM values $m(v_1, v_2)$ result in the same received magnitude $|r| = 2\sqrt{E_s/10}$. Consequently, the average probability of bit error of superposition coding exhibits an error floor of approximate value $P_b = \frac{3}{16}$.

A partial improvement is obtained with a natural labeling of the 4-PAM constellation points. The metrics in Fig. 8 show that only the LSB is affected by the dichotomy that occurs when receiving a value close to $r = \pm 2\sqrt{E_s/10}$. The error floor is reduced to approximately $P_b = \frac{3}{32}$ which still renders the 4-PAM superposition coding scheme impractical.

To solve the error floor problem that arises in two-level superposition coding with the use of two equal-energy 4-PAM constellations, it is proposed to use two Gray-mapped 4-PAM constellations of unequal energies as follows.

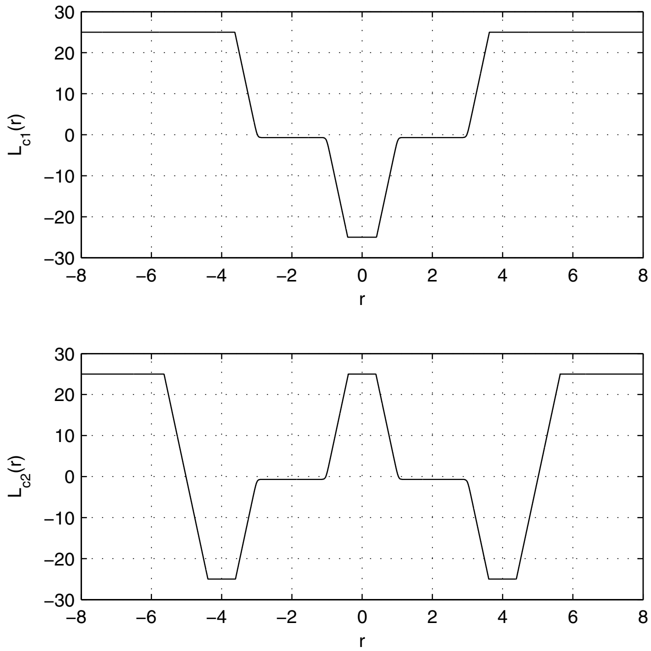

Fig. 7. 4-PAM LLR metrics with Gray mapping. $E_s/N_0 = 10$ dB.

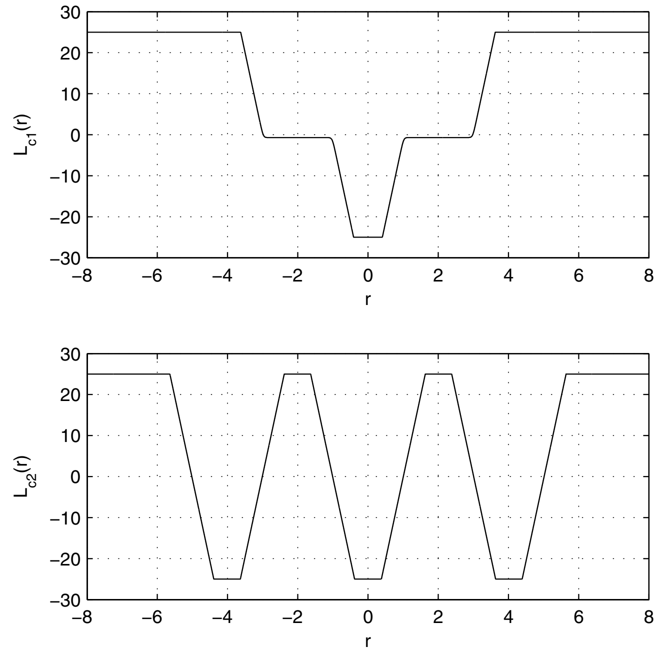

Fig. 8. 4-PAM LLR metrics with natural mapping. $E_s/N_0 = 10$ dB.

One 4-PAM constellation is set to four times the average energy of the other constellation. Those coded bits mapped into this high-energy constellation will be more protected compared to the other bits. At the receiver, a superimposed 16-PAM constellation is observed, for which the bit metrics computed by the demapper allow successful estimation of the message bits with an iterative decoding algorithm. This is similar to the construction of a hierarchical 16-PAM constellation for single-source broadcasting. The metrics are shown in Fig. 9, for $E_s/N_0 = 10$ dB.

The error performance of this scheme obtained via computer simulations is shown in Fig. 10 with two LDPC codes: C_1 a regular $(96, 50)$ LDPC code with degree distribution $(3, 6)$, and C_2 a regular $(96, 49)$ LDPC code with degree distribution $(4, 8)$. The results show that two levels of error protection can be achieved with 4-PAM modulation as long as the energies of the two constellations are different. Care must be taken in designing each constellation to ensure that correct bit metrics will be produced and iterative decoding will work.

V. Final remarks

A construction technique of superposition codes for cooperative broadcasting has been introduced. Two modulated subcode sequences are transmitted independently by two cooperating source nodes. The antenna at a destination/relay node performs "over-the-air mixing" of these sequences. A modulation scheme based on orthogonal BPSK subsets was introduced and shown to outperform superposition coding based on conventional BPSK modulation.

Expressions were derived for the bit metrics used in iterative decoding of two-level superposition codes based on BPSK, QPSK and 4-PAM constellations. It has been shown that conventional equal-energy 4-PAM constellations do not work well with two-level superposition coding and LDPC codes with iterative decoding. The bit metrics of two 4-PAM constellations with Gray and natural labeling were examined. The performance of superposition coding based on equal-energy 4-PAM constellations exhibits an error floor that renders this scheme impractical.

A solution was proposed that uses two unequal-energy 4-PAM constellations, similar to [11]. The important difference is that superposition takes place at the receiver. This scheme is equivalent to labeling of a hierarchical 16-PAM constellation. Naturally, synchronization is required so that the modulated sequences can be properly aligned and added. As an illustration, the performance of a two-level superposition code was examined. This is based on two different length-96 LDPC codes and two 4-PAM constellations of different levels of average energy.

Finally, it is notable that the results obtained for 4-PAM modulation are also applicable to two-level superposition coding using two unequal-energy 16-QAM constellations. The computation of bit metrics for each of the in-phase and quadrature components of the received 16-QAM modulated sequences proceeds in the same way as in the 4-PAM modulation scheme presented in this paper. Future work includes further extensions to other constellations, the examination of the error performance with longer LDPC codes and design rules for given requirements of proportion of MSB and desired coding

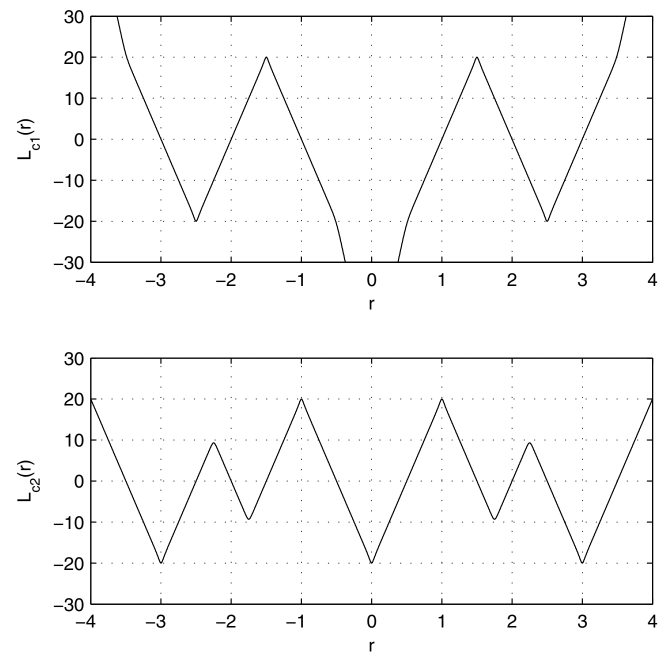

Fig. 9. 4-PAM LLR metrics with hierarchical mapping. $E_s/N_0 = 10$ dB.

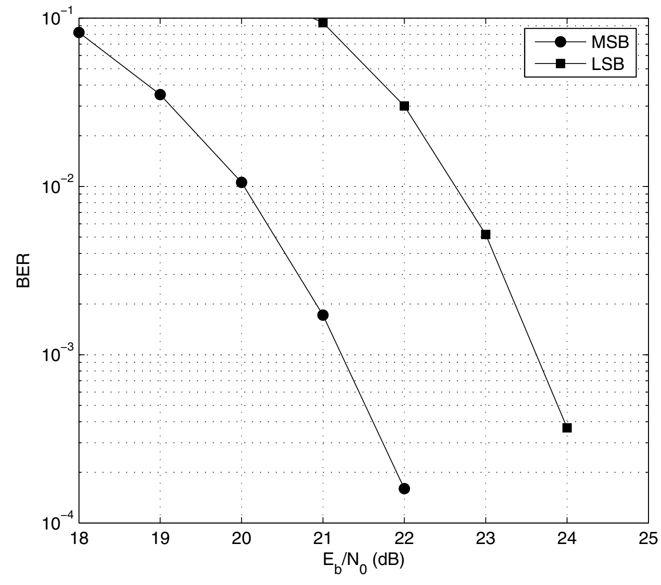

Fig. 10. Performance of two-level superposition coding using a hierarchical 4-PAM constellation and LDPC codes of length 96.

gains and diversity levels. Also of interest is the study of synchronization techniques for cooperative broadcasting with over-the-air mixing.

Acknowledgment

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