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An Economic Analysis of Factor Usage and Workplace Regulation: Comment

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I. Introduction

In a recent article in this Journal, Elder [1] examines the impact of workplace safety regulations upon a firm's input choices. His theoretical model, which treats accidents in the workplace as a by-product of production, shows that increases in safety are obtained at the expense of reduced output and altered factor usage. The results are empirically supported using state by state data for 1971 and 1976 where factor usage is regressed on factor returns and workers' compensation payments. When combined with work by Russell [2] and Viscusi [3] showing rather small benefits from safety regulation, Elder's work raises important questions about the social desirability of workplace regulation as implemented in the U.S.

We agree with Elder's general conclusions, but we find that the derivation of his theoretical results contain some errors. The purpose of this paper is to re-examine Elder's model and to illustrate how his theoretical model should have been developed.

II. Reexamination of the Model

Following Elder, we consider the case of a competitive firm that employs labor and capital to produce its output. The objective function of the profit maximizing firm can be defined by

$$\max \pi = pX(K, L) - CaL - rK - wL - bSL, \tag{1}$$

where π = profit, x = output, p = output price, K = capital, L = labor, a = accident rate, C = cost per accident, w = wage rate, r = capital return rate, b = price of safety, and S = safety expenditures. Accidents, $A = aL$, are modeled as a by-product of production, depending upon labor usage and the accident rate.[1] They are treated as a cost of production that can be reduced through safety expenditures. Hence, assume that: (i) $a = a(S)$, $a' = (da/dS) < 0$, $a'' = (d^2a/dS^2) > 0$; and (ii) $w = w(a)$, $w' = (dw/da) > 0$, and $w'' = (d^2w/da^2) = 0$. The assumption $w'' = 0$ is implicit in Elder's analysis. It can be relaxed so that $w'' > 0$ without affecting any of the qualitative results.[2] Intuitively, $w > 0$ seems likely since an increasing rate of accidents should increase workers' perceptions of risk.

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1. In his footnote 3, Elder mentions that this problem could also be modeled in a joint production framework, which is, "in fact, more intuitively appealing." Alternatively, the problem could be examined using a three-factor model where safety is the third factor of production. Elder's basic results would remain the same, but we feel this may be a more descriptive way of modeling the problem, especially since Elder already implicitly assumes the existence of safety as a factor of production.

2. Elder does not mention that $w'' = 0$. However, his equation (7) implies this important assumption. Even if $w'' < 0$, sufficient conditions are met if $a''(C + w') > -(a')^2 w''$.

A necessary consequence of the profit maximization problem is that the first order partial derivatives of (1) equal zero,

$$\partial \pi / \partial K = p X_K - r = 0. \tag{2}$$

$$\partial \pi / \partial L = p X_L - Ca - w(a) - bS = 0 \tag{3}$$

$$\partial \pi / \partial S = -a'CL - a'w'L - bL = -L[a'(C+w') + b] = 0. \tag{4}$$

Notice that our equation (4) differs from Elder's specification which reads as $\partial \pi / \partial S = -CLa' - L(w'/a') = 0$ [1, 317, eq. (4)].

Assuming that the production function, $X = X(K, L)$, is strictly concave implies that the second order derivatives satisfy $X_{KK} < 0$, $X_{LL} < 0$, and $(X_{KK}X_{LL} - X_{KL}^2) > 0$.[3] Sufficient conditions also require that the second order partial derivatives satisfy $\pi_{ii} < 0$, for $i = K, L, S$.[4] Then, the first order conditions can be solved for the optimal levels of factor usage, $K = K(C, r, b, p)$, $L = L(C, r, b, p)$ and $S = S(C, r, b, p)$.[5]

We next examine the effect of changes in worker's compensation payments, as approximated by C, on factor usage. The first order conditions are totally differentiated and Cramer's rule is used to solve for $\partial K / \partial C$, $\partial L / \partial C$ and $\partial S / \partial C$ as follows:

$$\partial K / \partial C = -a X_{KL} / [p(X_{KK}X_{LL} - X_{KL}^2)] \tag{5}$$

$$\partial L / \partial C = a X_{KK} / [p(X_{KK}X_{LL} - X_{KL}^2)] \tag{6}$$

$$\partial S / \partial C = -a' / [a''(C+w') + (a')^2 w''] \tag{7}$$

where $p(X_{KK}X_{LL} - X_{KL}^2) > 0$ and $[a''(C+w') + (a')^2 w''] > 0$ for profit maximization. Assuming that capital and labor are complementary, i.e., $X_{KL} > 0$, from (5), we obtain $\partial K / \partial C < 0$.[6] Furthermore, from (6) and (7), we obtain $\partial L / \partial C < 0$, $\partial S / \partial C > 0$. In other words, an increase in worker's compensation will increase safety investment and decrease the use of both capital and labor. Elder obtains this result by assuming that $w'' = 0$. We find the same result occurs in the more general case $w'' > 0$.

If safety regulation is introduced (as a binding constraint), the regulated level of safety investment (S_R) exceeds the optimal level of safety investment (S^*). Compliance with regulations is assumed so that

$$S = S_R. \tag{8}$$

The objective function of the regulated firm becomes

$$\max \pi = p X(K, L) - CaL - rK - wL - bS_R L \tag{9}$$

and the first order conditions are

$$\partial \pi / \partial K = p X_K - r = 0 \tag{10}$$

$$\partial \pi / \partial L = p X_L - Ca - w(a) - bS_R = 0. \tag{11}$$

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3. Elder assumes that capital and labor are complementary factors, so that $X_{KL} > 0$.

4. The second order sufficient conditions are $\pi_{KK} = p X_{KK}$, $\pi_{KL} = p X_{KL}$, $\pi_{KS} = \pi_{SK} = 0$, $\pi_{LK} = \pi_{KL}$, $\pi_{LL} = p X_{LL}$, $\pi_{LS} = -[a'(C+w') + b]$, $\pi_{SL} = \pi_{LS}$, and $\pi_{SS} = -L[a''(C+w') + (a')^2 w'']$.

5. Note that optimal levels of K, L and S are not a function of w. w is already endogenous, $w = w(a(S))$.

6. There is an error in Elder's equation (5). It should be read as our equation (5). The initial minus was omitted in Elder's paper.

These first order conditions differ from (2) – (4) in that S_R replaces S, which is no longer a choice variable.

Since the production function is strictly concave, the second order sufficient conditions are met. Hence, equations (10) and (11) can be solved for $K = K(S_R, C, r, p, b)$, $L = L(S_R, C, r, p, b)$.

Totally differentiating (10) and (11) and using Cramer's rule gives

$$\partial K / \partial S_R = (1/H) p V X_{KL} \tag{12}$$

$$\partial L / \partial S_R = - (1/H) p V X_{KK} \tag{13}$$

where $H = p^2 (X_{KK} X_{LL} - X_{KL}^2) > 0$ and $V = -(Ca' + w'a' + b)$.[7] Since $H > 0$, $p > 0$, $X_{KL} > 0$, and $X_{KK} < 0$, the sign of $\partial K / \partial S_R$ and $\partial L / \partial S_R$ is determined by the sign of V.[8]

According to Elder,

> It can be shown that $V > 0$ by considering equation (4), [i.e., $\partial \pi / \partial S = - CL(da/ds) - L[(dw/da)/(da/ds)] - bL = 0$], which determines S^*. When safety is unregulated, (4) holds and $V = 0$ (since dividing both sides of (4) by L yields V). When safety is regulated ($S_R > S^*$), (4) does not hold and $V = 0$. The latter implies that $da/dS_R > da/dS^*$ and $b < - C(da/dS_R) - (dw/da)(da/dS_R)$. Thus $V > 0$ [1, 319, footnote 10; bracket is added].

If $V > 0$, equations (12) and (13) imply that $\partial K / \partial S_R$ and $\partial L / \partial S_R > 0$, contradicting Elder's basic behavioral proposition and empirical findings. However, using the model that we have developed, it can be shown that $V < 0$.

From equation (4) in the unregulated case, we note that $V = \pi_S / L = (Ca' + w'a' + b)$, so that $V = 0$ at the optimal level of safety, S^*. Under binding regulation $S_R > S^*$ and $a(S^*) > a(S_R)$, which means there are less accidents under regulation. Since $a'(S) < 0$ and $a''(S) > 0$, $a'(S_R) > a'(S^*)$. Both $a'(S_R)$ and $a'(S^*)$ are negative numbers, so that in terms of magnitude, $a'(S^*) > a'(S_R)$. This implies that accident reduction from the last dollar spent on safety is smaller in the regulated case. Similarly, $w' > 0$ and $w'' > 0$ imply that $w'(a(S_R)) > w'(a(S^*))$, or that reductions in the wage rate from each additional dollar spent on safety are smaller in the regulated case.

Under safety regulation,

$$V = V_R = - \{Ca'(S_R) + [w'(a(S_R))][a'(S_R)] + b\},$$

and in absence of regulation

$$V = V^* = - \{Ca'(S^*) + [w'(a(S^*))][a'(S^*)] + b\} = 0.$$

Assuming b is constant,

$$V_R - V^* = C[a'(S^*) - a'(S_R)] + [w'(a(S^*))][a'(S^*)] - [w'(a(S_R))][a'(S_R)]$$

is negative since $a'(S_R) > a'(S^*)$ and $w'(a(S_R)) > w'(a(S^*))$. Hence, $V_R < 0$, and the firm reduces output and factor employment when faced with workplace safety regulation.

7. Note that $V = \pi_{SL} = \pi_{LS}$, as obtained from the second order conditions. Equivalently, $V = \pi_S / L$.
8. In Elder's equation (12), the minus sign should be omitted. On the other hand, a minus sign should be added to equation (13).

III. Conclusion

We have illustrated how Elder's theoretical model of the effects of safety regulation should have developed. In Elder's paper, the intuitively correct results were obtained through a series of offsetting errors. Our respecification simply strengthens Elder's theoretical and empirical results. We believe that alternative specifications of the model (such as, considering joint production, or explicitly including safety as a factor input in production) would produce qualitatively similar results. In the empirical section, Elder has employed CES cost functions instead of a more flexible functional form. Relaxing this assumption could affect measured values of the elasticity of substitution of K and L between periods, however, the conclusion that safety affects factor choices and reduces output would remain unaltered.

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