

## A RUSSELLIAN ANALYSIS OF THE BUDDHIST *CATUṢKOṬI*

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**ABSTRACT:** *Names name, but there are no individuals who are named by names. This is the key to an elegant and ideologically parsimonious strategy for analyzing the Buddhist catuṣkoṭi. The strategy is ideologically parsimonious, because it appeals to no analytic resources beyond those of standard predicate logic. The strategy is elegant, because it is, in effect, an application of Bertrand Russell's theory of definite descriptions to Buddhist contexts. The strategy imposes some minor adjustments upon Russell's theory. Attention to familiar catuṣkoṭi from Vacchagotta and Nagarjuna as well as more obscure catuṣkoṭi from Khema, Zhi Yi, and Fa Zang motivates the adjustments. The result is a principled structural distinction between affirmative and negative catuṣkoṭi, as well as analyses for each that compare favorably to more recent efforts from Tillemans, Westerhoff, and Priest (among others).*

**Keywords:** *antirealism, catuskoti, definite description, empty name, Fa Zang, Nagarjuna, philosophy of language, reference failure, Russell, tetralemma*

### 1 INTRODUCTORY REMARKS

A *dilemma* presents an interlocutor with a choice between one of two mutually exclusive alternatives. A *catuṣkoṭi* (Chinese term 四句 *si-ju*, literally “four phrases”, colloquially “tetralemma”), similarly, presents an interlocutor with a choice between one of four mutually exclusive alternatives (Sanskrit term *koṭi*).<sup>1</sup> Consider, for example, an interlocutor asking, of a conversational partner, “Does the Buddha survive death?” The question seems to admit exactly one of two answers. The partner might answer, “Yes, the Buddha survives death.” The partner might answer, “No, the Buddha does not survive death.” Straightforward analysis of these answers using standard propositional logic entails that the interlocutor is presenting a dilemma. Let S stand for

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<sup>1</sup> Although I provide diacritical marks for the term *catuṣkoṭi*, I omit such marks for other Sanskrit and Pali terms in an effort to aid readability.

“The Buddha survives death”. Then the affirmative answer is  $S$ ; the negative answer,  $\neg S$ . Since  $S \vee \neg S$  and  $\neg(S \wedge \neg S)$  are theorems of standard propositional logic, it follows that  $S$  and  $\neg S$  are mutually exclusive alternatives.

The straightforward analysis of “yes or no” questions yields several puzzles for the exegesis of certain Buddhist discourses. Consider, for example, *Vacchagotta’s Discourse on Fire* (*Aggi-Vacchagotta Sutta* MN 72), which presents a conversation between the Buddha and a wandering ascetic named Vacchagotta. Vacchagotta raises several issues: whether the cosmos is eternal (in duration); whether the cosmos is finite (in size); whether the soul and the body are the same. Vacchagotta treats each issue as a dilemma. But the Buddha’s responses are unusual. He denies every alternative. “Is the cosmos eternal?” “No.” “Is the cosmos not eternal?” “No.” The discourse does not report Vacchagotta’s affective reaction to the Buddha’s responses, but bafflement would be appropriate. Straightforward analysis of the Buddha’s answers indicate that he is contradicting himself. Perhaps by virtue of noticing this result while holding the Buddha in great esteem, Vacchagotta alters his questioning strategy. On the issue of whether a *tathagata*—someone who has attained Buddhahood—survives death, Vacchagotta presents the Buddha with four alternatives rather than two: the *tathagata* survives death; the *tathagata* does not survive death; the *tathagata* both survives and does not survive death; the *tathagata* neither survives nor does not survive death. The Buddha denies each alternative. (See Table 1.)

Question	Answer
$S?$	$\neg S$
$\neg S?$	$\neg\neg S$
$S \wedge \neg S?$	$\neg(S \wedge \neg S)$
$\neg(S \vee \neg S)?$	$\neg\neg(S \vee \neg S)$

Table 1: Straightforward Analysis of Vacchagotta’s *Catuṣkoṭi*

The shift in Vacchagotta’s questioning strategy yields an exegetical puzzle. How does Vacchagotta’s third alternative (“both yes and no”) relate to the first two, and how does his fourth alternative (“neither yes nor no”) relate to the third? If the alternatives are part of a *catuṣkoṭi* rather than a dilemma, they should be mutually exclusive. An affirmative answer to one should not entail an affirmative answer to another. Standard propositional logic entails that affirming the third alternative is equivalent to affirming each of the first two, and that affirming the fourth alternative is equivalent to affirming the third. Moreover, if the third and fourth alternatives ask nothing beyond the content of the first two, why does Vacchagotta change his questioning strategy?

The Buddha’s answers yield another exegetical puzzle. How might the Buddha’s answers be jointly true? *Vacchagotta’s Discourse on Fire* depicts the Buddha as someone Vacchagotta rightly seeks out for wisdom. Regardless of whether Vacchagotta is presenting the Buddha with a dilemma or a *catuṣkoṭi*, the Buddha seems to provide contradictory answers. By denying  $S$ , the Buddha seems to be affirming  $\neg S$ . By denying  $\neg S$ , the Buddha seems to be affirming  $\neg\neg S$ . If the Buddha is affirming

both  $\neg S$  and  $\neg\neg S$ , standard propositional logic entails that he is contradicting himself. By denying  $S \wedge \neg S$ , the Buddha seems to contradict himself further, because this answer is inconsistent with the conjunction of his first two answers. Moreover, by denying  $\neg(S \vee \neg S)$ , the Buddha seems to be piling contradiction upon contradiction, because the answer is not only self-contradictory but also inconsistent with his other answers.

These puzzles about Vacchagotta's *catuṣkoṭi* ramify. For example, in Chapter 25 of his *Fundamentals of the Middle Way* (*Mulamadhyamakakarika*, hereafter MMK), Nagarjuna addresses the issue of whether the Buddha exists after death. He denies the four alternatives of a *catuṣkoṭi*.

It is not to be asserted that the Buddha exists beyond cessation, nor “does not exist” nor “both exists and does not exist,” nor “neither exists nor does not exist”—none of these is to be asserted (MMK 25.17; Siderits and Katsura 2013, 301).

This imitates *Vacchagotta's Discourse on Fire*, except that Nagarjuna proceeds to deny the four alternatives of a *catuṣkoṭi* about whether the Buddha exists prior to death (MMK 25.18). Likewise, in his *Great Cessation and Contemplation* (*Mo-He-Zhi-Guan* 摩訶止觀), Zhi Yi 智顓 (538-597), founder of the Chinese *Tian-Tai* 天台 tradition, considers whether thoughts arise. While acknowledging that we say, in a deluded way, that thoughts arise, he nonetheless denies that any thought arises from itself, from another, from both itself and another, or from neither itself nor another (Swanson 2018, 199-200). Zhi Yi offers a similar *catuṣkoṭi* in his *Profound Meaning of the Lotus Sutra* (*Miao-Fa-Lian-Hua-Jing-Xuan-Yi* 妙法蓮華經玄義), denying that *dharma*s arise from themselves, from others, from both [themselves and others], or without cause (*bu-zi, bu-ta, bu-gong, bu-wu-yin* 不自、不他、不共、不無因; T33.1716.699c20-c21).

## 2. APPROACHING THE CATUṢKOṬI

English-language efforts to resolve these exegetical puzzles typically search for logic-based solutions. The leading such efforts are analytic, ascribing hidden logical structure to each alternative of the *catuṣkoṭi*. Consider, for example, the analyses of Robinson (1957), Tillemans (1999, 199-200), and Westerhoff (2006). (See Table 2. I represent each analysis with quantifiers that are absent in the originals for the sake of facilitating structural comparisons. Priest adopts a similar approach (2010, 28).) Robinson motivates his analysis by speculative analogy: there are four alternatives in the *catuṣkoṭi*; there are four categorical forms in Aristotelian syllogistic logic; hence, perhaps, each affirmative alternative in the *catuṣkoṭi* corresponds to some categorical form or combination thereof. Tillemans, by contrast, is more principled: each alternative entails an existentially quantified claim; and Buddhists deny each alternative because they maintain that the domain of quantification is empty. Westerhoff blends the analyses of Robinson and Tillemans in an effort to preserve mutual exclusion among the four alternatives. He retains Robinson's analysis for the first two alternatives. He modifies Tillemans' analysis for the third by relativizing the predicate therein to distinct respects. Westerhoff also modifies the fourth alternative.

He replaces the outermost sentential negations with illocutionary denials, so that  $\sim S$  means “S is denied” rather than “S is false”. He stipulates, as well, that an illocutionary denial of an illocutionary denial is not an affirmation.

Question	Answer	Robinson’s Analysis	Tillemans’ Analysis	Westerhoff’s Analysis
S?	$\neg S$	$\neg \forall x Sx$	$\neg \exists x Sx$	$\neg \forall x Sx$
$\neg S$ ?	$\neg \neg S$	$\neg \forall x \neg Sx$	$\neg \exists x \neg Sx$	$\neg \forall x \neg Sx$
$S \wedge \neg S$ ?	$\neg (S \wedge \neg S)$	$\neg (\exists x Sx \wedge \exists x \neg Sx)$	$\neg \exists x (Sx \wedge \neg Sx)$	$\neg \exists x (S'x \wedge \neg S''x)$
$\neg (S \vee \neg S)$ ?	$\neg \neg (S \vee \neg S)$	$\neg (\neg \exists x Sx \wedge \neg \exists x \neg Sx)$	$\neg \exists x (\neg Sx \wedge \neg \neg Sx)$	$\sim \exists x (Sx \vee \neg Sx)$

Table 2: Analytic Interpretations of the *Catuṣkoṭi*

Are any of these analyses successful? Recent interpreters generally agree that a successful analysis should satisfy three adequacy conditions. The first is that ascriptions of quantificational structure should be well-motivated. There should be good reason for ascribing free variables to alternatives that seem to be variable-free, and there should be good reason for binding these variables with some specific combination and ordering of quantifiers. The second adequacy condition is that ascriptions of non-quantificational logical structure should be well-defined and well-motivated. There should be good reason for introducing extra structure, and its meaning should be well-understood. The third adequacy condition is that ascriptions of logical structure should present the four alternatives as mutually exclusive and exhaustive. Affirming one alternative should not require affirming another. Nor should any relevant alternative be neglected.

The leading analytic efforts for understanding *catuṣkoṭi* seem to violate at least one of these conditions. (Here I follow Priest 2010, 28-30.) On Robinson’s analysis, the fourth alternative entails the second. Robinson also provides no good reason for the specific quantificational structure he ascribes. Tillemans’ analysis likewise violates the third adequacy condition. His third and fourth alternatives are equivalent; and the third entails each of the first two. Westerhoff’s analysis seems to violate the second. The predicate relativity in the third alternative seem to be *ad hoc*, and the illocutionary denial he ascribes to the fourth alternative is ill-defined.

The inadequacies of analytic efforts to understand the *catuṣkoṭi* lend credence to more revisionary efforts. These efforts interpret the four alternatives using atypical constructions. Consider, for example, the analyses of Priest (2010) and Kreutz (2019). (See Table 3.) Priest introduces angle brackets that act as name-forming operators, so that  $T\langle S \rangle$  means “S is true” and  $F\langle S \rangle$  means “S is not true”. He then analyzes each alternative of the *catuṣkoṭi* as positing one of four distinct truth-values for the same claim: only true, only false, both true and false, neither true nor false. Kreutz combines Priest’s innovations with Westerhoff’s appeal to illocutionary denial, interpreting all of the negations in Priest’s analysis as *denials of truth* or *denials of falsity* (as appropriate).

Question	Answer	Priest's Analysis	Kreutz's Analysis
S?	$\neg S$	$\neg(\mathbf{T}\langle S \rangle \wedge \neg \mathbf{F}\langle S \rangle)$	$\sim(\mathbf{T}\langle S \rangle \wedge \sim \mathbf{F}\langle S \rangle)$
$\neg S$ ?	$\neg\neg S$	$\neg(\neg \mathbf{T}\langle S \rangle \wedge \mathbf{F}\langle S \rangle)$	$\sim(\sim \mathbf{T}\langle S \rangle \wedge \mathbf{F}\langle S \rangle)$
$S \wedge \neg S$ ?	$\neg(S \wedge \neg S)$	$\neg(\mathbf{T}\langle S \rangle \wedge \mathbf{F}\langle S \rangle)$	$\sim(\mathbf{T}\langle S \rangle \wedge \mathbf{F}\langle S \rangle)$
$\neg(S \vee \neg S)$ ?	$\neg\neg(S \vee \neg S)$	$\neg(\neg \mathbf{T}\langle S \rangle \wedge \neg \mathbf{F}\langle S \rangle)$	$\sim(\sim \mathbf{T}\langle S \rangle \wedge \sim \mathbf{F}\langle S \rangle)$

Table 3: Revisionary Analyses of the *Catuṣkoṭi*

I have no decisive objection to the revisionary efforts by Priest and Kreutz. They are laden with more ideology than analytic efforts. Ideology concerns concepts that resist definition in terms of other concepts (see Cowling 2013, 3892-3893). Revisionary efforts have additional ideology in the sense that, in addition to appealing to concepts from standard predicate logic, they also appeal to concepts that resist definition in terms of concepts from standard predicate logic. Priest's analysis introduces a name forming operator,  $E\langle S \rangle$ , meaning "S is empty". He defines the operator informally, so that a claim is empty whenever it is neither true but not false, false but not true, both true and false, and neither true nor false (Priest 2010, 37). However, for the sake differentiating being empty from being neither true nor false, he also stipulates that a claim is neither true nor false when neither it nor its negation is true and it is not empty (Priest 2010, 38). The result is that  $E\langle S \rangle$  is not definable using only resources available from standard predicate logic.<sup>2</sup> Kreutz's analysis introduces a logical operator for illocutionary negation to represent the speech act of denial (2019, 84-85). Because speech acts are not truth-functionally definable, Kreutz's analysis also requires ideology that is not available with standard predicate logic.

The inadequacies of extant analytic efforts indicate that some sort of ideological escalation is, perhaps, necessary. I aim to demonstrate that this is not so. There is an elegant analysis of the *catuṣkoṭi* available that satisfies all three conditions of adequacy. The analysis takes seriously Tillemans' contention that, in denying each alternative of a *catuṣkoṭi*, Buddhists mean to affirm that no individual satisfies the relevant predicate. The analysis I offer improves upon Tillemans' by making explicit why the denials are apt and how the *catuṣkoṭi* arises for those who misunderstand what their Buddhist interlocutors mean to affirm. It invokes no resources beyond those available in standard predicate logic and thereby qualifies as less ideological than revisionary efforts.<sup>3</sup> For reasons that should become apparent, I shall refer to the analysis as Russellian.

<sup>2</sup> Whether Priest's analysis requires further ideology is difficult to determine. Cotnoir argues that Priest's analysis requires two primitive truth predicates, one for conventional truth and one for ultimate truth, in order to accommodate various patterns of conditional reasoning (Cotnoir 2015). Kreutz offers two responses on behalf of Priest. He argues, first, that adding a set of inferences that are usually, but not always, valid suffices (Kreutz 2019, 83). He argues, second, that proper attention to Priest's emptiness operator removes the motivation for Cotnoir's objection (Kreutz 2019, 83-84). If Cotnoir's objection or Kreutz's first response thereto is correct, Priest's analysis requires a second extra piece of ideology beyond standard predicate logic, in the form of an extra truth predicate or an extra sort of validity.

<sup>3</sup> Tillemans motivates his analysis by restricting the scope of the *catuṣkoṭi* to the domain of ultimate truths in contrast to truths of convention. The distinction between kinds of truth remains poorly defined

### 3. MOTIVATING A RUSSELLIAN ANALYSIS

*Catuṣkoṭi* from the Buddhist tradition present two exegetical challenges. The first is to understand how the four alternatives relate to each other such that affirming one does not require affirming another. The second is to understand how one might simultaneously deny all four alternatives as incorrect. Analyses that purport to resolve these challenges are subject to three adequacy conditions: any reinterpretation of the alternatives that invokes hidden quantificational structure should be well-motivated; any reinterpretation that invokes other logical ideology should be well-motivated, and the ideology should be well-defined; and any reinterpretation of the alternatives should present them as mutually exclusive. The analysis I offer aims to satisfy all three conditions. The analysis is, in effect, a synthesis of Buddhist antirealism with an interpretation of names as predicates. It begins from the observation that when each alternative of a *catuṣkoṭi* is false, the source of falsity is a failure of reference.

When a claim exhibits reference failure, it contains a name that designates an individual and yet the named individual does not exist. Paradigmatic examples of reference failure include “Phlogiston is an ignitable matter present in flammable substances”, “Vulcan is a planet between Mercury and the sun”, and “Persephone preserves seeds from dangerous weather”. I maintain that Buddhists deny claims such as “The Buddha survives death”, not because they affirm that the Buddha fails to survive death, but because they affirm that the Buddha’s name fails to designate an existing individual. If Sb translates “The Buddha survives death”, Buddhists deny Sb and so affirm  $\neg$ Sb, because they affirm there is no one who is named the Buddha. They also deny  $\neg$ Sb, and so affirm  $\neg\neg$ Sb, for the same reason. Or so I maintain, subject to suitable analysis of the affirmations and denials. Call this assumption *Reference Failure*.

#### 3.1 BUDDHIST ANTIREALISM

There is good reason, internal to the Buddhist tradition, for endorsing *Reference Failure*. Consider, first, *Khema’s Discourse* (*Khema Sutta* SN44.1), which reports a brief dialogue between the eminent nun Khema and a king named Pasenadi (6th c. BCE). The king presents Khema with a *catuṣkoṭi* about whether the *tathagata* survives death. It is identical to the *catuṣkoṭi* that Vacchagotta presents to the Buddha. Khema declares that the Buddha denies each alternative. But unlike Vacchagotta, who changes the subject, the king asks Khema to explain the Buddha’s reason. Khema obliges with a metaphor. She asks the king whether he employs someone to count the drops of water in the great ocean (*mahasamudda*). The king answers that he does not. Khema asks him to explain. He answers that the ocean is “deep, boundless, hard to fathom” (*gambhira*,

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(see McDaniel 2019, Brenner 2020). For the sake of respecting the second constraint on a successful analysis of the *catuṣkoṭi*, according to which ascriptions of non-quantificational logical structure should be well-defined and well-motivated, I do not invoke the distinction between kinds of truth.

*appameyya, duppariyogaha*). (The intended implication is that efforts to count the ocean's drops of water are doomed to fail, because the number of drops is innumerable, immeasurable, inexhaustible.) Khema then completes the metaphor: the *tathagata*, too, is deep, boundless, hard to fathom. The *tathagata*'s profundity ensures that all efforts to describe the *tathagata* fail. The efforts fail, according to Khema, not because language cannot match the *tathagata*'s infinite grandeur, but because the *tathagata* is "released from conceptions that pertain to materiality" (*rupa-sankhaya-vimutta*). *Reference Failure* offers a natural interpretation for Khema's explanation. Because there is no one who is named the *tathagata*, there is no one named the *tathagata* who encounters death. Since there is no one named the *tathagata*, the *tathagata* is released from being counted as either surviving or failing to survive death. Just as efforts to count the number of drops in the ocean—as one million, one billion, one trillion, and so on—are doomed to fail, so too are efforts to count the *tathagata* as surviving death, failing to survive death, both surviving and failing to survive, or neither surviving nor failing to survive.

Consider, next, *The Questions of King Milinda (Milinda Panha)*, an extended dialogue in which a king named Milinda learns about Buddhism from a visiting monk named Nagasena. Milinda respectfully asks Nagasena to identify himself by declaring his name and how he is known. Nagasena answers that his name is Nagasena and that, although his name is a designation (*pannatti*) for a person, his name does not refer to a person (*puggalo*). The subsequent dialectical exchange is rich, and it rewards careful study. Disregarding subtle details, Nagasena's answer indicates that he affirms three metaphysical principles. The first is that each person is one rather than many. The second is that each person is composite rather than atomic. The third is that no composite is one.<sup>4</sup> These principles jointly entail that there are no persons.

The absence of persons entails, as a corollary, that insofar as Nagasena's name designates a person, it fails to refer. Let the miniscule *n* be a name for the person Nagasena. This name either succeeds or fails in referring to a person. Suppose, for *reductio ad absurdum*, that Nagasena's name, *n*, refers to a person. Then there is a person who is Nagasena. The first principle entails that this person is one; the second and third, that the person is not one. Hence, there is nothing that is a person and named Nagasena. Yet the same supposition, that Nagasena's name refers to a person, also

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<sup>4</sup> There is good reason, within the Buddhist tradition, for affirming each of these principles. Here I only gesture toward relevant justifications. Regarding the first, those lacking *nirvana* conceptualize themselves as one, and they take this conceptualization to be a truth of reality. So persons, *real* persons, are one rather than many. Regarding the second, atomic persons—persons lacking proper parts—would be immune to *dukkha* by virtue of existing independently of others. Because persons are not immune to *dukkha*, persons are not atomic; and because anything not atomic is composite, persons are composite. Regarding the third, a composite is one only if it is identical to either one of its proper parts, more than one (in relation or not, exhaustively or not), or something other than its proper parts. Because every composite has at least two disjoint proper parts, no composite is identical to one of its proper parts. Because no individual is both one and many, no composite is identical to more than one of its proper parts. Were a composite identical to something separate from its proper parts, it would be possible to encounter the composite without encountering any of its proper parts. Because such encounters are impossible, it follows that no composite is one.

entails Nagasena is a person. Since everything is self-identical, it follows that there is something named Nagasena. This completes the *reductio*. So Nagasena's name does not refer to a person. Because the name, *n*, is a name for a person, it follows that Nagasena's name fails to refer.

Subsequent Buddhist tradition extends Nagasena's conclusion from composites to (atomic) *dharmas*. For example, in his MMK, Nagarjuna endorses two relevant principles. The first is that every *dharma* has an intrinsic nature (*svabhava*); the second, that nothing has intrinsic nature. These principles entail that any name that designates a *dharma* fails to refer. This argument, together with Nagasena's argument for *Reference Failure* regarding composite individuals, entails *Universal Reference Failure*: all names fail to refer. In *The Dispeller of Disputes (Vigrahavyāvartanī)*, Nagarjuna explicitly endorses *Universal Reference Failure*.

[W]e do not assert a referring name. This is because a name, too, due to the absence of substance [*svabhava*] in things, is insubstantial and therefore empty. Because of its emptiness it is non-referring (Westerhoff 2010, 37; see also Westerhoff 2019, 782-785).

The *25,000 Line Discourse on the Perfection of Wisdom (Pancavimsatisahasrika-prajnaparamita-sutra)* also endorses *Universal Reference Failure*, comparing names to the traces left in the sky by flying birds, and warning against tendencies to cling to names themselves (*namasanketa-prajnapati*), names for composites (*avavada-prajnapati*), and names for partless atoms (*dharma-prajnapati*) (see Wang 2001, 118).

### 3.2 CHALLENGES FOR REFERENCE FAILURE

Regardless of its interpretive aptness, *Reference Failure* seems to be an ill-advised assumption. Standard predicate logic satisfies *Bivalence*, the principle according to which, for every claim, either it or its negation is true. If  $P_n$  translates "Nagasena is a person", *Bivalence* entails that either  $P_n$  is true or  $\neg P_n$  is true. The first disjunct entails  $\exists x(P_x \wedge x=n)$ ; the second,  $\exists x(\neg P_x \wedge x=n)$ . Constructive dilemma yields, as a corollary, that Nagasena's name, abbreviated as the miniscule *n*, designates an existing individual. *Reference Failure* entails, to the contrary, that Nagasena's name designates no one. It follows that Nagasena's name refers and fails to refer. But standard predicate logic also validates, as a theorem, *Non-Contradiction*. This is the principle according to which, for any predicate  $\Phi_x$ ,  $\neg \exists x(\Phi_x \wedge \neg \Phi_x)$ . So there seems to be a choice: endorse *Reference Failure* and jettison some part of standard predicate logic; or retain standard predicate logic and reject *Reference Failure*.<sup>5</sup>

*Reference Failure* also seems to be an ill-formed assumption. The most natural articulation for the assumption is this: some names fail to refer to existing individuals. But this is unsatisfying. Consider a name that does not refer to any existing individual.

<sup>5</sup> For reasons to suppose that Buddhists typically endorse *Non-Contradiction*, see Robinson 1957, 295-296. For reasons to suppose that Buddhists typically endorse *Bivalence*, see Ruegg 1997, 47-52. (Ruegg discusses *Bivalence* under the label *excluded middle*.) Nagarjuna endorses instances of both principles. For *Non-Contradiction*, see MMK 7.30 and 8.7; for *Bivalence*, see MMK 5.6.

Because all names designate individuals, this name designates some individual. Because the name fails to refer, the designated individual is not an existing individual. There are no non-existing individuals.<sup>6</sup> But every individual either exists or not. So there is not, after all, a name that fails to refer. Every name, it seems, refers to some existing individual. Similar considerations tell against articulating *Reference Failure* as the principle that some names do not refer at all. Whatever its proper articulation, *Reference Failure* seems to entail that when a name fails to refer, it designates an individual that is (numerically) distinct from everything. Because everything is, of necessity, (numerically) identical to itself, it seems to follow that *Reference Failure* never obtains.

### 3.3 NAMES AS PREDICATES

These challenges for articulating and making coherent *Reference Failure* vanish if names function as predicates rather than individual constants. If names function as constants, translating “Nagasena is a person” into the language of standard predicate logic proceeds as follows: designate an individual constant,  $n$ , as naming Nagasena; assign a predicate,  $Px$ , to stand for “ $x$  is a person”; then translate “Nagasena is a person” as  $Pn$ . If, by contrast, names function as predicates, translating “Nagasena is a person” into the language of standard predicate logic proceeds as follows: assign a predicate,  $Nx$ , to stand for “ $x$  is the one named Nagasena”; assign  $Px$  to stand for “ $x$  is a person”; then translate “Nagasena is a person” as  $\exists x(Nx \wedge Px)$ .

*Reference Failure* contravenes the necessity of self-identity if names function as constants, but not if names function as predicates. Consider, for example, the claim “Nagasena does not exist”. If names function as constants, this translates as  $\neg \exists x(x=n)$  and so violates the necessary truth  $n=n$ . If names function as predicates, the same claim translates as  $\neg \exists xNx$ . This translation is perfectly consistent with the necessity of self-identity. It also reveals that the miniscule  $n$  is not a name for Nagasena. This result, which generalizes for all names, is exactly correct. Insofar as all names refer to existing individuals and Nagasena does not exist, no name refers to Nagasena.<sup>7</sup>

Similarly, *Reference Failure* conflicts with *Non-Contradiction* and *Bivalence* if names function as constants, but not if names function as predicates. Consider, again,

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<sup>6</sup> Suppose, for *reductio ad absurdum*, that there is a non-existing individual—the planet Vulcan, perhaps, residing between Mercury and the sun. Because Vulcan does not exist, there is no individual that is (numerically) identical to Vulcan. But everything is self-identical. So there is an individual that is identical to Vulcan, namely, Vulcan. This completes the *reductio*.

<sup>7</sup> An insistence upon treating names as predicates (or descriptions), and never as individual constants, is the main innovation for the analysis to follow. It complicates the procedure for translating from natural language into the language of standard predicate logic, but the complication is necessary for preserving standard predicate logic. Beginning with Leonard 1956, some take the complication to warrant abandoning standard logic for a system of free logic. I ignore this option, for the sake of minimizing ideological innovation. For efforts toward an interpretation of Dharmakīrti’s *apoha* semantics using a dual-domain free logic, see Schweizer 2015. Yao (2009) and McGeever (2018), by contrast, offer further motivation for treating names as predicates in Buddhist contexts.

the claim “Nagasena is a person”. If names function as constants, *Bivalence* entails that either  $P_n$  is true or  $\neg P_n$  is true. Constructive dilemma yields, as a corollary,  $\exists x(x=n)$ . *Reference Failure* entails, to the contrary,  $\neg \exists x(x=n)$ . *Non-Contradiction* thereby forces a choice: deny  $\exists x(x=n)$  and reject *Bivalence*, or else deny  $\neg \exists x(x=n)$  and reject *Reference Failure*. If names function as predicates, there is no such forced choice. *Bivalence* entails that either  $\exists x(Nx \wedge Px)$  is true or  $\neg \exists x(Nx \wedge Px)$  is true. *Reference Failure* entails  $\neg \exists xNx$ . No contradiction follows.

### 3.4 THE RUSSELLIAN CONNECTION

The challenges that beset *Reference Failure*, and that admit to resolution if names function as predicates, resemble the challenges that motivated Bertrand Russell to develop his theory of definite descriptions. Russell (1905) examines the claim, “The present king of France is bald”. This is a description, because it describes someone or something as being a certain way (or satisfying a predicate). The description is definite, because the name for the someone or something so described contains the definite article. The background assumption is that *there is no present king of France*, because France at present is a democracy rather than a monarchy (and democracies lack kings). The challenge, as Russell understands it, is to analyze the claim’s logical form in a way that exhibits why it is false while preserving both *Bivalence* and *Non-Contradiction*. His solution involves analyzing the claim as an implicit threefold conjunction. One conjunct serves as an existence condition, declaring that at least one individual satisfies the predicate for being the present king of France. A second conjunct serves as a uniqueness condition, declaring that at most one individual satisfies the predicate for being the present king of France. A third, finally, serves as a maximality condition, declaring that any individual satisfying the “is the present king of France” predicate also satisfies the predicate for being bald. If  $Kx$  stands for “ $x$  is the present king of France” and “ $Bx$ ” stands for “ $x$  is bald”, Russell’s analysis translates “The present king of France is bald” as:

$$\exists xKx \wedge \forall x\forall y(Kx \wedge Ky \rightarrow x=y) \wedge \exists x(Kx \rightarrow Bx).$$

Because there is no present king of France, the first conjunct of this translation is false. So the analysis explains why “The present king of France is bald” is false. It does so, moreover, without violating *Bivalence* or *Non-Contradiction*. Russell’s theory of definite descriptions maintains that definite descriptions are properly analyzed as threefold conjunctions having the same logical form as this paradigm case.

Russell’s theory has limitations. It fares poorly as an analysis for definite mass descriptions and definite plural descriptions. (Here I follow Sharvy 1980, but see also Gendler Szabó 2005.) For example, “The Buddha drank the tea in this room” is a definite description that contains a mass predicate (‘is tea in this room’). According to Russell’s theory, exactly one individual satisfies this predicate. But, pre-theoretically, if the Buddha is drinking in a room with other monks, the predicate applies to the tea

in their cups too. Similarly, “The Buddha addressed the monks in this room” is a definite description that contains a plural predicate (‘are monks in this room’). According to Russell’s theory, exactly one person satisfies this predicate. But pre-theoretically, it applies to each of them. Russell’s theory also fails to apply to proper names (in English), which contain no article.

Objections pertaining to the scope of Russell’s theory admit to solution through creative reinterpretation of naming conventions. For example, Graff Fara interprets proper names as denuded definite descriptions—definite descriptions with an unpronounced definite article (Graff Fara 2015, 70). So interpreted, the name ‘Nagasena’ is semantically equivalent to ‘(the) Nagasena’ or ‘the one who is named Nagasena’. Objections pertaining to the strength of Russell’s theory admit to solution by modifying one or more of his preferred conjuncts. For example, Sharvy (1980) demonstrates that a slight adjustment to the uniqueness condition suffices for extending the theory to mass predicates and plural predicates. For Russell’s paradigm case, Sharvy prefers the following analysis (with  $y \leq x$  standing for the relation “y is a proper part of x or  $y=x$ ”):

$$\exists x Kx \wedge \forall x \forall y (Kx \wedge Ky \rightarrow x \leq y) \wedge \exists x (Kx \rightarrow Bx).$$

Although Sharvy’s analysis is not Russell’s, it is Russellian for three reasons. First, it treats names as predicates. Second, it posits that an individual satisfies such a predicate only if the individual named by the predicate exists. Third, it analyzes definite descriptions as bearers of scope, so that negations thereof are ambiguous between negating the description as a whole and negating some specific part, and so that negating the description as a whole does not entail negating one specific part rather than another.

Evans provides notation that helps to represent the core components of a Russellian analysis. He introduces a binary quantifier  $I_x$  (read: “the x”) to represent “The  $\Phi$  is  $\Psi$ ” as  $I_x[\Phi x; \Psi x]$  (read: “the x that is the one named  $\Phi$  is  $\Psi$ ”) (Evans 1982, 58). This binary quantifier admits the following partial definition:

$$\text{If } I_x[\Phi x; \Psi x], \text{ then } \exists x(\Phi x \wedge \Psi x).$$

The binary quantifier is not part of standard predicate logic. But nothing it adds to standard predicate logic—if, indeed, it adds anything at all—is necessary for the Russellian analyses to follow. The partial definition suffices to reveal how “The  $\Phi$  is *not*  $\Psi$ ” is ambiguous between

$$\neg I_x[\Phi x; \Psi x], \text{ which entails only } \neg \exists x(\Phi x \wedge \Psi x); \text{ and}$$

$$I_x[\Phi x; \neg \Psi x], \text{ which entails } \exists x(\Phi x \wedge \neg \Psi x).$$

It suffices to reveal, as well, that  $\neg Ix[\Phi x; \Psi x]$  does not entail  $Ix[\Phi x; \neg\Psi x]$ . This has the virtue of allowing  $Ix[\Phi x; \Psi x]$  to be false regardless of whether  $\exists x\Psi x$  is true. The partial definition for the binary quantifier  $Ix$  also has virtues beyond its suitability for representing the core components of a Russellian analysis. It provides a complete definition for indefinite descriptions. This guarantees that Russellian analyses are possible for languages—such as Sanskrit, Latin, and Chinese—that lack articles.<sup>8</sup> The definition also allows applications of Russellian analysis to disregard concerns about Russell’s uniqueness condition. I turn now to just such an application.

#### 4. A RUSSELLIAN ANALYSIS OF THE *CATUṢKOṬĪ*

Buddhist antirealism seems to entail that some names, although they name, are such that no individuals are the ones who are named. Treating names as predicates rather than individual constants helps to make sense of this entailment, and a Russellian analysis of name-containing descriptions indicates that, when names are treated as predicates, such descriptions contain hidden quantificational structure. This suggests that a Russellian analysis of the alternatives of Buddhist *catuṣkoṭi* might resolve the exegetical challenges associated with those *catuṣkoṭi*. I shall proceed, accordingly, to demonstrate that Russellian analysis does just this, and that it does so in ways that satisfy the three adequacy conditions for successful analyses. I demonstrate, as well, that the resulting analysis compares favorably to other analytic interpretations of the *catuṣkoṭi*.

##### 4.1 PRELIMINARY ANALYSIS

Consider, once more, *Vacchagotta’s Discourse on Fire* and the issue of whether the *tathagata* survives death. Let  $Tx$  stand for “ $x$  is the one named *tathagata*”. Let  $Sx$  stand for “ $x$  survives death”. Russellian analysis of “The *tathagata* survives death” is straightforward:  $Ix[Tx; Sx]$ . Analysis of “The *tathagata* does not survive death” yields two alternatives:  $\neg Ix[Tx; Sx]$  or  $Ix[Tx; \neg Sx]$ . *Reference Failure* selects the second of these as appropriate. According to *Reference Failure*, the Buddha denies that the *tathagata* does not survive death because he denies that there is an individual named the *tathagata*. Since  $\neg Ix[Tx; Sx]$  is true if there is no individual named the *tathagata*, the Buddha, in denying that the *tathagata* does not survive death, must be denying

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<sup>8</sup> Consider, for example, Gongsun Long (公孫龍)’s so-called White Horse Paradox: *bai-ma-fei-ma* 白馬非馬. Translating with minimal interpretation, this is the claim “white horse not horse”. There is a Russellian analysis that interprets the claim as follows: let  $Wx$  stand for “ $x$  is white”,  $Hx$  stand for “ $x$  is a horse”, and  $Ox$  stand for “ $x$  is the one who is named as a white horse”; then analyze “white horse not horse” as  $\neg Ix[Ox; Hx]$ . This entails  $\neg\exists x(Ox \wedge Hx)$ . Given the reasonable background assumption that  $\forall x(Ox \leftrightarrow (Wx \wedge Hx))$ —every white horse is white and a horse, and whatever is white and a horse is a white horse—it follows that  $\neg\exists x(Ox)$ . This Russellian analysis of the White Horse Paradox suggests, therefore, that Gongsun Long means to declare that ‘white horse’ (*bai-ma* 白馬), the name, fails to refer. But, regardless of whether the analysis is apt, it demonstrates the applicability of Russellian analysis to an article-free language.

$Ix[Tx; \neg Sx]$ . Similar reasoning selects similarly scoped claims for Vacchagotta's third and fourth alternatives. A Russellian analysis of Vacchagotta's *catuṣkoṭi*, together with *Reference Failure*, thereby yields a preliminary interpretation of the four alternatives.<sup>9</sup> (See Table 4.)

The <i>tathagata</i> survives death.	$Ix[Tx; Sx]$
The <i>tathagata</i> does not survive death.	$Ix[Tx; \neg Sx]$
The <i>tathagata</i> both survives death and does not survive death.	$Ix[Tx; (Sx \wedge \neg Sx)]$
The <i>tathagata</i> neither survives death nor does not survive death.	$Ix[Tx; \neg(Sx \vee \neg Sx)]$

Table 4: Preliminary Analysis of Vacchagotta's *Catuṣkoṭi*

This preliminary interpretation satisfies two of three adequacy conditions for a successful analysis of Vacchagotta's *catuṣkoṭi*. Ascriptions of quantificational structure are well-motivated. Free variables are necessary to accommodate the analysis of names as predicates, and the quantifiers binding these variables are appropriate for Russellian reasons. Ascriptions of non-quantificational logical structure are also well-motivated. The only such structure is the placement of the negation operator for the final three alternatives, and *Reference Failure* justifies placing the operator within the scope of Evans' binary quantifier.

Despite these virtues, the preliminary interpretation does not yield four alternatives that are mutually exclusive.  $Ix[Tx; (Sx \wedge \neg Sx)]$  entails both  $Ix[Tx; Sx]$  and  $Ix[Tx; \neg Sx]$ , and  $Ix[Tx; (Sx \wedge \neg Sx)]$  is equivalent to  $Ix[Tx; \neg(Sx \vee \neg Sx)]$ . Attending to the dialectical structure of Vacchagotta's *catuṣkoṭi* motivates a modified interpretation that avoids this problem. Each question-answer pair in Vacchagotta's conversation with the Buddha involves a spoken component and an unspoken component. The spoken component is the public question-answer pair. The unspoken component, by contrast, is private and includes the Buddha's reason for his answer and the inference Vacchagotta makes from the Buddha's spoken answer. The Buddha's reason, I maintain, is always the failure of the *tathagata*'s name to refer. I maintain, further, that the *next* question Vacchagotta asks indicates Vacchagotta's unspoken inference. Doing so helps to explain why Vacchagotta continues the conversation in the specific way he does. Doing so also portrays Vacchagotta as one who incorrectly and unwittingly presupposes that the *tathagata*'s name designates an existing individual.

<sup>9</sup> Strawson (1950) famously objects to Russell's view that reference failure renders definite descriptions false. Strawson argues that it is more intuitive to treat such claims as neither true nor false. Shaw offers a compromise: Strawson's view is appropriate when the interlocutors know there is a failure of reference, and Russell's view is appropriate otherwise (Shaw 1988, 164-167). Insofar as Vacchagotta incorrectly presumes referential success, Shaw's compromise removes Strawsonian-style objections to applying Russell's view to Vacchagotta's *catuṣkoṭi*. Regardless of whether Shaw's compromise is appropriate, I maintain that Strawson's objections to the truth of Russell's view are not objections to applying Russell's view to *catuṣkoṭi*. There is no good reason to suppose that the intuitions to which Strawson appeals are intuitions Buddhists would endorse.

## 4.2 DIALECTICAL STRUCTURE

The dialectical structure of Vacchagotta's first exchange with the Buddha is relatively clear. The Buddha denies that the *tathagata* survives death, and Vacchagotta interprets this answer as meaning that some existing individual, the *tathagata*, does not survive death. The structure of the second exchange is more obscure. The Buddha denies that the *tathagata* does not survive death. There are two ways for Vacchagotta to interpret this answer while presupposing, incorrectly, that the *tathagata's* name refers. He can interpret the Buddha as meaning to affirm that some existing individual, the *tathagata*, survives death. Or he can interpret the Buddha as meaning to affirm, instead, that *Non-Contradiction* does not hold of the *tathagata*. If *Non-Contradiction* holds for the (existing) *tathagata*, the Buddha speaks falsely when answering Vacchagotta's questions. Insofar as Vacchagotta's esteem for the Buddha inclines him to interpret the Buddha as speaking truthfully, Vacchagotta likely infers, from the Buddha's answer, that the Buddha means to affirm that *Non-Contradiction* does not hold of the *tathagata*.

In the third exchange, the Buddha denies that the *tathagata* both survives and does not survive death. Vacchagotta's inferences from the first two exchanges should incline him to infer, in the third exchange, that *Non-Contradiction* is not the Buddha's concern. Insofar as Vacchagotta presupposes, incorrectly, that the *tathagata's* name refers, and insofar as he inclines toward interpreting the Buddha as speaking truthfully, this leaves Vacchagotta with one option: infer that, in answering the third question, the Buddha means to affirm that Vacchagotta's first two alternatives are neither true nor false. If this is the Buddha's meaning, the Buddha is rejecting *Bivalence*. So when Vacchagotta asks the Buddha, in the fourth exchange, whether he affirms that the *tathagata* neither survives nor does not survive death, he likely means to ask whether the Buddha rejects *Bivalence*. Since Vacchagotta changes the topic of conversation, the inference Vacchagotta draws from the Buddha's final answer is unknown.

Because Vacchagotta's conversation with the Buddha contains unspoken dialectical structure, it is helpful to make the structure explicit. In doing so, I maintain that Vacchagotta considers *Non-Contradiction* and *Bivalence* as candidates for rejection in the conversation.<sup>10</sup> I assume that, in the third alternative, Vacchagotta is foregrounding *Non-Contradiction* as a target for rejection and disregarding the status of *Bivalence*. I assume, as well, that in the fourth alternative he is foregrounding *Bivalence* as a target for rejection and disregarding the status of *Non-Contradiction*. I also maintain that the Buddha, in declining to affirm an alternative, directs his denial to the spoken content of the alternative. Then, if NC abbreviates *Non-Contradiction* and BV abbreviates *Bivalence*, Vacchagotta's exchanges with the Buddha have the structure indicated by Table 5.

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<sup>10</sup> Neither of *Bivalence* nor *Non-Contradiction* entails the other. Intuitionistic logic satisfies *Non-Contradiction* but not *Bivalence*. Belnap's four-valued logic satisfies *Bivalence* but not *Non-Contradiction* (see Belnap 1977).

<b>Vacchagotta's First Alternative</b>		
Vacchagotta's Question	Do you affirm that the <i>tathagata</i> survives death?	$Ix[Tx; Sx] \wedge NC \wedge BV?$
Buddha's Answer	I do not.	$\neg Ix[Tx; Sx] \wedge NC \wedge BV$
Buddha's Reason	There is no individual named the <i>tathagata</i> .	$\neg \exists xTx$
Vacchagotta's Inference	He affirms that the <i>tathagata</i> does not survive death.	$Ix[Tx; \neg Sx] \wedge NC \wedge BV$
<b>Vacchagotta's Second Alternative</b>		
Vacchagotta's Question	Do you affirm that the <i>tathagata</i> does not survive death?	$Ix[Tx; \neg Sx] \wedge NC \wedge BV?$
Buddha's Answer	I do not.	$\neg Ix[Tx; \neg Sx] \wedge NC \wedge BV$
Buddha's Reason	There is no individual named the <i>tathagata</i> .	$\neg \exists xTx$
Vacchagotta's Inference	He affirms that the <i>tathagata</i> survives and does not survive death.	$Ix[Tx; (Sx \wedge \neg Sx)] \wedge \neg NC \wedge (BV \vee \neg BV)$
<b>Vacchagotta's Third Alternative</b>		
Vacchagotta's Question	Do you affirm that the <i>tathagata</i> survives and does not survive death?	$Ix[Tx; (Sx \wedge \neg Sx)] \wedge \neg NC \wedge (BV \vee \neg BV)?$
Buddha's Answer	I do not.	$\neg Ix[Tx; (Sx \wedge \neg Sx)] \wedge \neg NC \wedge (BV \vee \neg BV)$
Buddha's Reason	There is no individual named the <i>tathagata</i> .	$\neg \exists xTx$
Vacchagotta's Inference	He affirms that the <i>tathagata</i> neither survives nor does not survive death.	$Ix[Tx; \neg(Sx \vee \neg Sx)] \wedge (NC \vee \neg NC) \wedge \neg BV$
<b>Vacchagotta's Fourth Alternative</b>		
Vacchagotta's Question	Do you affirm that the <i>tathagata</i> neither survives nor does not survive death?	$Ix[Tx; \neg(Sx \vee \neg Sx)] \wedge (NC \vee \neg NC) \wedge \neg BV?$
Buddha's Answer	I do not.	$\neg Ix[Tx; \neg(Sx \vee \neg Sx)] \wedge (NC \vee \neg NC) \wedge \neg BV$
Buddha's Reason	There is no individual named the <i>tathagata</i> .	$\neg \exists xTx$
Vacchagotta's Inference	Unknown.	Unknown.

Table 5: The Dialectical Structure of Vacchagotta's *Catuskoṭi*

### 4.3 MODIFIED ANALYSIS

Attending to the dialectical structure of Vacchagotta's conversation with the Buddha indicates that each of Vacchagotta's alternatives contains an unspoken contention about *Non-Contradiction* and *Bivalence*. Revising the preliminary Russellian analysis to make these declarations explicit yields a modified analysis that satisfies the three adequacy conditions for a successful analysis of Vacchagotta's *catuṣkoṭi*. (See Table 6.)

The <i>tathagata</i> survives death.	$Ix[Tx; Sx] \wedge NC \wedge BV$
The <i>tathagata</i> does not survive death.	$Ix[Tx; \neg Sx] \wedge NC \wedge BV$
The <i>tathagata</i> both survives death and does not survive death.	$Ix[Tx; (Sx \wedge \neg Sx)] \wedge \neg NC \wedge (BV \vee \neg BV)$
The <i>tathagata</i> neither survives death nor does not survive death.	$Ix[Tx; \neg(Sx \vee \neg Sx)] \wedge (NC \vee \neg NC) \wedge \neg BV$

Table 6: Modified Analysis of Vacchagotta's *Catuṣkoṭi*

Vacchagotta's esteem for the Buddha and his mistaken presupposition motivate ascribing implicit declarations about logical principles. Moreover, the four alternatives in the modified analysis are mutually exclusive with respect to the following topic: Does the *tathagata* survive death or not? Consider, as evidence, the table in Table 7.

$Ix[Tx; Sx]?$	$Ix[Tx; \neg Sx]?$	Alternative
yes	no	Option 1, with NC and BV
no	yes	Option 2, with NC and BV
yes	yes	Option 3, with $\neg NC$ and $(BV \vee \neg BV)$
no	no	Option 4, with $\neg BV$ and $(NC \vee \neg NC)$

Table 7: Mutual Exclusion in Vacchagotta's *Catuṣkoṭi*

The table neglects some alternatives, such as affirming that the *tathagata* survives death while rejecting *Non-Contradiction* and *Bivalence*. These neglected alternatives are rhetorically extravagant. Their extravagance is good reason to suppose that Vacchagotta and the Buddha consider them to be irrelevant to their conversation. The table also neglects the alternative of referential failure. There is good rhetorical reason for this neglect: the conversation portrays Vacchagotta as an interlocutor who unwittingly endorses referential success. Hence, although the alternatives are not *logically* exhaustive, they are *rhetorically* exhaustive. They are exhaustive in all the ways that matter for Vacchagotta's exchange with the Buddha.

## 4.4 THE LESSON OF THE CATUṢKOṬĪ

The modified Russellian analysis of Vacchagotta's *catuṣkoṭī* presumes that Vacchagotta, in proceeding through his alternatives, treats *Non-Contradiction* and *Bivalence* as principles the Buddha might reject. However, it does not presume that *the Buddha* treats these as candidates for rejection. The analysis also presumes that names function as predicates and that, in denying each alternative, the Buddha denies that any existing individual satisfies such predicates. It does not, however, presume that *the Buddha* treats names as predicates. The result is an interpretation of the Buddha's answers that compares favorably to its predecessors. (See Table 8.)

Robinson's Analysis	Tillemans' Analysis	Westerhoff's Analysis	Russellian Analysis
$\neg\forall xSx$	$\neg\exists xSx$	$\neg\forall xSx$	$\neg\exists x[Tx; Sx]$
$\neg\forall x\neg Sx$	$\neg\exists x\neg Sx$	$\neg\forall x\neg Sx$	$\neg\exists x[Tx; \neg Sx]$
$\neg(\exists xSx \wedge \exists x\neg Sx)$	$\neg\exists x(Sx \wedge \neg Sx)$	$\neg\exists x(S'x \wedge \neg S''x)$	$\neg\exists x[Tx; (Sx \wedge \neg Sx)]$
$\neg(\neg\exists xSx \wedge \neg\exists x\neg Sx)$	$\neg\exists x(\neg Sx \wedge \neg\neg Sx)$	$\sim\exists x(Sx \vee \neg Sx)$	$\neg\exists x[Tx; \neg(Sx \vee \neg Sx)]$

Table 8: Comparing the Russellian Analysis to Predecessors

The Russellian analysis most closely resembles Tillemans', with the exception of treating each answer as correct by virtue of reference failure rather than by virtue of some individual failing to satisfy the ascribed predicate. It resembles Westerhoff's as well, because it permits all four negations to be jointly and simultaneously true. Westerhoff achieves this result by introducing illocutionary denials and relativized predicates. The Russellian analysis achieves it, by contrast, by treating names as predicates. If the modified Russellian analysis is correct, the lesson of Vacchagotta's *catuṣkoṭī* is not that one or more standard logical principles fail. Nor, in contrast to Priest 2010, is the lesson that claims about reality have some atypical truth status. If the modified Russellian analysis is correct, the lesson of Vacchagotta's *catuṣkoṭī* is that, because there is no one named the *tathagata*, there are no truths about the *tathagata*.

This lesson coheres with the Buddha's subsequent conversation with Vacchagotta. The Buddha offers a fire analogy. When there is a fire burning, it is correct to say that the fire is burning, that the fire depends upon its fuel, that the fire travels this way rather than that. But when the fire ceases, it is incorrect to say that the fire travels this way or that way. So, too, the Buddha declares, the *tathagata*, when "released from conceptions that pertain to materiality", neither survives death nor fails to survive, is neither reborn nor not reborn, is neither liberated from nor ensnared within *samsara*. Moreover, the Buddha continues, because the *tathagata* is, like the great ocean, "deep, boundless, hard to fathom", there is no one who is named the *tathagata*.

Because Vacchagotta's *catuṣkoṭī* is a paradigm case, the lesson also generalizes. In the face of *Reference Failure*, conceptualizations fail to limn the structure of reality. There is no one who is named the *tathagata*. So the *tathagata* is neither empty nor non-

empty (see MMK 22.11). There is no one who is named the Buddha. So the Buddha neither survives nor fails to survive death. There is no individual that is named the cosmos. So the cosmos is neither finite nor infinite. There is no individual that is named the soul. So the soul is neither the same as the body nor distinct from the body. There is no individual that is named a *dharma*. So *dharmas* neither arise from themselves nor arise from another. There is no individual that is named dependent upon another. So the one who depends is neither identical to nor distinct from the one depended upon (see MMK 18.10).

## 5. FURTHER CHALLENGES AND INSIGHTS

Russellian analysis of Vacchagotta's *catuṣkoṭi* indicates that the alternatives therein contain hidden quantificational structure and ideological content. The analysis readily generalizes to other *catuṣkoṭi*. Yet Russellian analysis seems to fail for *catuṣkoṭi* about existence and for *catuṣkoṭi* that merit affirmation of their alternatives. I address these challenges sequentially. Modifying the Russellian strategy for analyzing *catuṣkoṭi* about existence requires careful attention to the logical structure of singular existentials. For purposes of motivating and explaining these modifications, I introduce an example that has yet to receive sustained attention in recent literature on *catuṣkoṭi*. The example is apt, because it appears in the context of analyzing the misunderstandings that tend to befall those who endeavor to understand *catuṣkoṭi*. Attending to advice about how to forestall these misunderstandings motivates a strategy for extending the Russellian strategy to affirmative *catuṣkoṭi*.

### 5.1 EXISTENTIAL *CATUṢKOṬI*

*Catuṣkoṭi* about existence are exceedingly common in the discourses of Chinese Buddhists. Fa Zang 法藏 (643-712), the third patriarch of the *Hua-Yan* 華嚴 tradition, provides a paradigmatic and instructive example. The example appears in the first section of the final chapter of his *Treatise on the Five Teachings* (*Hua-Yan-Wu-Jiao-Zhang* 華嚴五教章). Fa Zang argues that every *dharma* has three distinct but inseparable natures. He realizes that some might understand him to be claiming that his names for these natures refer to existing individuals. But the natures are empty, and so the names fail to refer. For the sake of guarding against attachment to the existence of the natures, he develops a series of *catuṣkoṭi* (T45.1866.499c5-500a16).

Consider Fa Zang's first *catuṣkoṭi*. The issue is whether the real nature (*zhen ru* 真如) of the *dharmas*, *tathata*, exists (T45.1866.499c5-c11).<sup>11</sup> Fa Zang denies that *tathata*

<sup>11</sup> Whether the distinction between ultimate truth and conventional truth is relevant to understanding Fa Zang's *catuṣkoṭi* is an open and difficult question. His theory of the three natures develops a similar theory from the Yogacara tradition (see Liu 1982). Yogacarins tend to associate real natures with ultimate truths (D'Amato 2005, 201-202). Fa Zang also provides *catuṣkoṭi* for two other natures, one of which—the discriminated nature (*suo-zhi* 所執)—the Yogacarins tend to associate with conventional truth. That Fa Zang uses *catuṣkoṭi* to discuss both kinds of truth provides some reason to suppose that

exists, because *tathata* responds to conditions (*sui-yuan* 隨緣) despite being constant (*bu-bian* 不變). He denies that *tathata* does not exist, because *tathata* is constant despite responding to conditions. He denies that *tathata* both exists and does not exist, because *tathata* lacks a dual nature (*er-xing* 二性). He denies that *tathata* neither exists nor does not exist, because *tathata*, like the great ocean, is endowed with profundity (*ju-de* 具德).<sup>12</sup>

When names function as individual constants, the standard practice among contemporary logicians is to translate singular existentials as existentially quantified claims of numerical identity—for example, translating “*Tathata* exists” as  $\exists x(x=h)$ , where the miniscule *h* is a name for *tathata*. This practice coheres poorly with *Reference Failure*. When names function as predicates, the standard practice is to translate singular existentials as existential quantifications over designating predicates—for example,  $\exists xHx$  where *Hx* stands for “*x* is the one named *tathata*”. This practice coheres well with Russellian analyses of *catuṣkoṭi*, which treat names as predicates. Yet the practice causes trouble for Russellian analyses of *catuṣkoṭi* about existence.

Consider, for example, a modified Russellian analysis for Fa Zang’s examination of whether *tathata* exists, following standard conventions for translating singular existentials. (See Table 9.)

Alternative	Russellian Analysis (Standard)	Fa Zang’s Answer
<i>Tathata</i> exists.	$\exists xHx \wedge NC \wedge EM$	$\neg\exists xHx$
<i>Tathata</i> does not exist.	$\neg\exists xHx \wedge NC \wedge EM$	$\neg\neg\exists xHx$
<i>Tathata</i> both exists and does exist.	$\exists x(Hx \wedge \neg Hx) \wedge \neg NC \wedge (EM \vee \neg EM)$	$\neg\exists x(Hx \wedge \neg Hx)$
<i>Tathata</i> neither exists nor does not exist.	$\exists x\neg(Hx \vee \neg Hx) \wedge (NC \vee \neg NC) \wedge \neg EM$	$\neg\exists x\neg(Hx \vee \neg Hx)$

Table 9: Standard Russellian Analysis for an Existential *Catuṣkoṭi*

The problem with the analysis is that the interpretation of Fa Zang’s answer for the second alternative is inconsistent with the presumed reason for that answer. Fa Zang *denies* that *tathata* does not exist. But this contradicts *Reference Failure*, which *affirms* that *tathata* does not exist.

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the distinction among kinds of truths might be irrelevant to analyzing at least some *catuṣkoṭi*. I thank an anonymous reviewer for raising this issue.

<sup>12</sup> In declaring that *tathata* lacks a dual nature, Fa Zang likely means that *tathata* does not satisfy contradictory predicates. In declaring that *tathata* is endowed with profundity, Fa Zang likely means that *tathata* is simultaneously chief (*zhu* 主) and attendant (*ban* 伴), determining the characteristics of *dharma*s even as those others determine its characteristics. As chief, *tathata* is akin to the pole star, a constant standard by which sailors at sea orient themselves. As attendant, *tathata* is akin to a sailor at sea, responsive to changing winds and tumultuous waves.

The solution to this problem is to depart from standard practice by treating existence as a predicate. Nakhnikian and Salmon (1982) demonstrate that such a departure is harmless (see also Evans 1982, 345-348). Let  $Ex$  stand for “ $x$  exists”, and define this predicate explicitly and completely as follows:

$$Ex =_{df} (x=x).$$

Russellian analyses of singular existentials thereby yield familiar binary structures. For example, the Russellian analysis of “*Tathata* exists” yields  $Ix[Hx; Ex]$ . This approach supports a variant Russellian analysis for Fa Zang’s *catuṣkoṭi* about whether *tathata* exists. (See Table 10.)

Alternative	Russellian Analysis (Variant)	Fa Zang’s Answer
<i>Tathata</i> exists.	$Ix[Hx; Ex] \wedge NC \wedge EM$	$\neg Ix[Hx; Ex]$
<i>Tathata</i> does not exist.	$Ix[Hx; \neg Ex] \wedge NC \wedge EM$	$\neg Ix[Hx; \neg Ex]$
<i>Tathata</i> both exists and does exist.	$Ix[Hx; (Ex \wedge \neg Ex)] \wedge \neg NC \wedge (EM \vee \neg EM)$	$\neg Ix[Hx; (Ex \wedge \neg Ex)]$
<i>Tathata</i> neither exists nor does not exist.	$Ix[Hx; \neg(Ex \vee \neg Ex)] \wedge (NC \vee \neg NC) \wedge \neg EM$	$\neg Ix[Hx; \neg(Ex \vee \neg Ex)]$

Table 10: Variant Russellian Analysis for Existential *Catuṣkoṭi*

This variant analysis, unlike its standard cousin, makes Fa Zang’s denying that *tathata* does not exist consistent with his affirming that no individual is designated by the name *tathata*. Indeed, whereas the standard analysis presents Fa Zang’s answer to the second alternative as inconsistent with his reason, the variant analysis presents Fa Zang’s answer as overdetermined by available reasons. One reason for declining to affirm  $Ix[Hx; \neg Ex]$  is reference failure. Another is descriptive failure. The second alternative,  $Ix[Hx; \neg Ex]$ , entails  $\exists x \neg Ex$ . The definition for the existence predicate yields, as a corollary, that some existing individual is non-self-identical:  $\exists x(x \neq x)$ . This is impossible. So the second alternative is doubly incorrect, because *tathata*’s name fails to refer and everything exists ( $\neg \exists x Hx$  and  $\forall x Ex$ ).

## 5.2 EASY TRUTHS AND SUBTLE ERRORS

The variant translation of singular existents supports a Russellian analysis of Fa Zang’s *catuṣkoṭi* about whether *tathata* exists. The analysis supports hermeneutical insight into some puzzling claims Fa Zang makes about his reasoning. These insights extend to an atypical kind of *catuṣkoṭi* wherein the correct answers affirm all four alternatives. The insights are important, because they justify analyzing “affirmative” *catuṣkoṭi* differently than more typical “negative” *catuṣkoṭi*, and because they explain why the alternatives for affirmative *catuṣkoṭi* should not be mutually exclusive.

Fa Zang examines two ways of misunderstanding his *catuṣkoṭi* about whether *tathata* exists. The first, which he labels the error of eternalism, interprets his arguments as demonstrating that *tathata* exists. Fa Zang's discussion is dense, but the error seems to rely upon some attractive reasoning. Suppose *tathata* does not exist because *tathata* is constant and responds to conditions. Then *tathata* is constant. So, it seems, there is some existing individual that is constant, and this individual is identical to *tathata*. It follows that *tathata* exists. Fa Zang categorizes this result as eternalist, presumably because it entails that *tathata*'s name refers to an existing individual. Fa Zang categorizes the result as an error, because he maintains that *tathata*'s name fails to refer.

The way to avoid the error of eternalism is to deny that *tathata*'s name refers to an existing individual. But this risks a second error that Fa Zang labels annihilationism. Suppose *tathata*'s name fails to refer. Then any existing individual identical to *tathata* is inconstant. Because nothing is non-self-identical, it follows that *tathata* is inconstant. Hence, there is some existing individual who is identical to *tathata* but inconstant. It follows that *tathata* exists. Fa Zang categorizes this result as an error, presumably because it serves as a *reductio ad absurdum* for the claim that *tathata*'s name fails to refer.

Although Fa Zang is clear in declaring that eternalism and annihilationism are errors, he is less clear when explaining why the reasoning in their support fails (T45.1866.500b5-b6). He warns those prone to such reasoning to know that *tathata* being constant does not differ from *tathata* being inconstant (*zhen-ru-bu-yi-chang-zhi-wu-chang* 真如不異常之無常). Russellian analysis vindicates Fa Zang's advice. If there is no existing individual named *tathata*, predicating constancy of *tathata* does not differ from predicating inconstancy of *tathata*, because both predications yield falsehoods. Fa Zang's advice, unfortunately, does not explain how the reasoning for eternalism and annihilationism goes awry. Here, again, formal analysis is helpful.

The argument for eternalism begins from Fa Zang's declaration that *tathata* is constant and responds to conditions. Let  $Cx$  stand for "x is constant" and  $Rx$  stand for "x is responsive to conditions". Let the miniscule  $t$  be an individual constant that names *tathata*. Then the supporting inferences seem to proceed as follows: from Fa Zang's declaration, infer  $\exists x((Cx \wedge Rx) \wedge x=t)$ ; after simplifying to  $\exists x(x=t)$ , conclude that *tathata* exists. The argument for annihilationism begins from Fa Zang's presumption that *tathata*'s name fails to refer. The supporting inferences seem to proceed as follows: from Fa Zang's presumption, infer  $\neg \exists x(x=t)$ ; by addition, infer  $\neg \exists x(x=t \wedge Cx)$ ; because  $\neg t \neq t$ , simplify to  $\neg \exists x Cx$ ; then instantiate to  $\neg Ct$  and generalize to  $\exists x(\neg Cx \wedge x=t)$ ; after simplifying to  $\exists x(x=t)$ , conclude that *tathata* exists.

The arguments for eternalism and annihilationism seem to be valid. The errors are difficult to discern. But the argument for annihilationism errs almost immediately by positing that some individual, witnessed by the miniscule  $t$ , is the referent of *tathata*'s name. This posit vindicates the reasoning from Fa Zang's presumption (that *tathata*'s name fails to refer) to  $\neg \exists x(x=t)$ . If, however, *tathata*'s name functions as a predicate (as Russellian analysis advises), the proper formalization of Fa Zang's presumption is  $\neg \exists x Hx$ . So formalized, Fa Zang's presumption does not support the subsequent

reasoning for annihilationism: if no individual is the one named *tathata*, *tathata* is not a self-identical individual. (Nor, for that matter, is *tathata* a non-self-identical individual.) Unfortunately, treating *tathata*'s name as a predicate seems to vindicate the argument for eternalism. From Fa Zang's declaration (that *tathata* is constant and responsive to conditions), infer  $\exists x(Hx \wedge (Cx \wedge Rx))$ ; simplify to  $\exists xHx$ ; then conclude that *tathata* exists.

The error in the reasoning for eternalism is extremely subtle. The error in the argument for annihilationism provides a clue. Properly formalized, Fa Zang's presumption is  $\neg \exists xHx$ . This entails both  $\forall x(Hx \rightarrow Cx)$  and  $\forall x(Hx \rightarrow \neg Cx)$ : whatever is *tathata* is constant, and whatever is *tathata* is inconstant. These are the truths embedded in Fa Zang advice that *tathata* being constant does not differ from *tathata* being inconstant. More significantly, however,  $\forall x(Hx \rightarrow Cx)$  is a plausible candidate for what Fa Zang means in declaring that *tathata* is constant. The argument for eternalism thereby fails, because  $\forall x(Hx \rightarrow Cx)$  does not entail  $\exists xHx$ .

### 5.3 AFFIRMATIVE *CATUṢKOṬI*

Many *catuṣkoṭi* in the Buddhist tradition aim to diminish attachment by fostering insight into the ontological innocence of conceptual designation (Skt. *prajñapti*). The correct answer to each alternative in these *catuṣkoṭi* is denial, because each alternative suffers from referential failure. There are, however, some *catuṣkoṭi* for which the correct answer to each alternative is affirmation. In his *Great Cessation and Contemplation*, Zhi Yi offers a sophisticated example, affirming that everything arises and perishes, neither arises nor perishes, both arises and perishes and neither arises nor perishes, and neither both arises and perishes nor neither arises nor perishes (see Swanson 2018, 178-180). But Nagarjuna provides the paradigm case:

All is real, or all is unreal, all is both real and unreal, all is neither unreal nor real; this is the graded teaching of the Buddha (MMK 18.8; trans. Katsura and Siderits 2013, 200).

The scope of this *catuṣkoṭi* is everything (Skt. *sarva*), and by this Nagarjuna seems to mean whatever his Buddhist peers and predecessors are inclined to affirm as existing: *ayatanas*, *skandhas*, *dhatus*, *dharmas*, and so on. In categorizing the *catuṣkoṭi* as “graded teaching” (Skt. *anusasana*), Nagarjuna signals that each alternative is more sophisticated than its predecessor and, therefore, that more sophisticated teachings are appropriate for more sophisticated audiences. In categorizing the *catuṣkoṭi* as the Buddha's teaching, Nagarjuna signals that all four alternatives merit affirmation—and so all Buddhist traditions, including those he criticizes as incorrect, convey some truth.

Consider, then, a specific *skandha*. Let it be materiality (*rupa*). Nagarjuna's contention is that materiality is real, unreal, both real and unreal, and neither real nor unreal. Let  $Mx$  stand for “x is the one named materiality”, and let  $Rx$  stand for “x is real”. Then, disregarding the status of *Non-Contradiction* and *Bivalence*, the Russellian analysis for the materiality-instance of Nagarjuna's affirmative *catuṣkoṭi* seems to be mimic the Russellian analysis for Vacchagotta's negative *catuṣkoṭi*. (See Table 11.)

Materiality is real.	$I_x[Mx; Rx]$
Materiality is unreal.	$I_x[Mx; \neg Rx]$
Materiality is both real and unreal.	$I_x[Mx; (Rx \wedge \neg Rx)]$
Materiality is neither real nor unreal.	$I_x[Mx; \neg(Rx \vee \neg Rx)]$

Table 11: Naive Analysis of an Affirmative *Catuṣkoṭi*

The affirmative *catuṣkoṭi*, so interpreted, is disastrous. The analysis presents Nagarjuna as affirming a contradiction by virtue of affirming the first two alternatives. It presents him as affirming two further contradictions, as well, by virtue of affirming the third and fourth alternatives. Fortunately, the problem here is not the *catuṣkoṭi* or Nagarjuna's affirmations therein. The problem is the naive analysis of the alternatives. Nagarjuna's central teaching is *Universal Reference Failure*. According to his predecessors in the Abhidharma tradition, only names for composites fail to refer. But for Nagarjuna, all names fail to refer, because names always only designate empty individuals and empty individuals do not exist. The naive analysis of the materiality *catuṣkoṭi* thereby yields four false alternatives. Each is false, because the analysis incorrectly presumes that materiality's name designates something that exists.

There is a more sophisticated Russellian analysis for the materiality-instance of Nagarjuna's affirmative *catuṣkoṭi*. This alternative analysis begins from the assumption that materiality fails to refer:  $\neg \exists x Mx$ . Nagarjuna affirms this assumption at MMK 18.9. The assumption entails both  $\forall x (Mx \rightarrow Rx)$  and  $\forall x (Mx \rightarrow \neg Rx)$ : whatever is materiality is real, and whatever is materiality is unreal. These are, respectively, the first two alternatives for Nagarjuna's *catuṣkoṭi*. Whoever affirms *Reference Failure* for materiality should affirm both alternatives. Nagarjuna's assumption ( $\neg \exists x Mx$ ) also entails  $\forall x (Mx \rightarrow (Rx \wedge \neg Rx))$ , which in turn entails  $\forall x (Mx \rightarrow \neg (Rx \vee \neg Rx))$ : whatever is materiality is both real and unreal, and whatever is materiality is neither real nor unreal. These are, respectively, the final two alternatives for Nagarjuna's *catuṣkoṭi*. Whoever affirms *Reference Failure* for materiality should affirm them both. (See Table 12.)

Materiality is real.	$\forall x (Mx \rightarrow Rx)$
Materiality is unreal.	$\forall x (Mx \rightarrow \neg Rx)$
Materiality is both real and unreal.	$\forall x (Mx \rightarrow (Rx \wedge \neg Rx))$
Materiality is neither real nor unreal.	$\forall x (Mx \rightarrow \neg (Rx \vee \neg Rx))$

Table 12: Sophisticated Analysis of an Affirmative *Catuṣkoṭi*

The sophisticated analysis ascribes to each alternative hidden logical structure. These ascriptions, quantificational and otherwise, are well-motivated. They follow naturally from the assumption that the name for materiality fails to refer, and they honor Fa Zang's advice for avoiding misunderstandings. The analysis does not, however, present the alternatives as mutually exclusive. If the analysis is correct, affirming the

third requires affirming each of the first two; and affirming the third is tantamount to affirming the fourth. The analysis fails, therefore, to satisfy the aforementioned three adequacy conditions for successful analyses. I maintain, however, that this does not count against the aptness of the analysis. The requirement of mutual exclusivity among the alternatives, while appropriate for negative *catuskoṭi*, is inappropriate for affirmative ones.

That a successful analysis of Buddhist *catuskoṭi* ought to present the four alternatives as mutually exclusive is a requirement more often asserted than justified. Kreutz offers a concise argument in its favor.

A model for the *catuskoṭi* has to maintain the *mutual exclusivity* and *exhaustive nature* of the kotis – call this the exclusivity/exhaustivity constraint. The reason is twofold: (1) we want to be charitable to the logical abilities to the authors who were using the *catuskoṭi*, and (2) is the way the (negative) *catuskoṭi* is commonly employed as an argument, supposed to undermine all possible ways a predicate can be attributed to something needs the four kotis to exhaust the logical space, which they can only do if they are mutually exclusive. This argument, which is a kind of *reductio* argument to reveal the deficiency of the concept in question is called a *prasanga* argument (Kreutz 2019, 68).

The reasons Kreutz adduces for mutual exclusivity are fair and plausible for negative *catuskoṭi*. Negative *catuskoṭi* typically appear in rhetorical contexts directed toward disabusing an interlocutor of the presumption that certain names designate existing individuals, and the dialectical structure implicit in these contexts seems to structure the sequential presentation of alternatives in a way that presumes their mutual exclusivity. But Kreutz's reasons are less fair and plausible for affirmative *catuskoṭi*. Affirmative *catuskoṭi* appear in rhetorical contexts directed toward instructing those who realize that names fail to refer, and the dialectical structure implicit in these contexts seems to structure the sequential presentation of alternatives as aiming to emphasize their mutual inclusivity. That is, whereas the goal of negative *catuskoṭi* is to guide the interlocutor toward realizing that many distinct alternatives are false because of referential failure, the goal of positive *catuskoṭi* is to guide the interlocutor toward realizing that, because of referential failure, distinctions among alternatives are merely apparent. So it is a virtue, rather than a vice, of the sophisticated analysis that it presents the four alternatives as mutually inclusive.

A further virtue of the sophisticated analysis is the insight it reveals regarding the soteriological significance of the four alternatives in affirmative *catuskoṭi*. The sophisticated analysis for the materiality-instance of Nagarjuna's affirmative *catuskoṭi* is applicable to anything within the scope of the Buddha's graded teaching. Everything named is real, unreal, both, and neither, because all names fail to refer. Affirmative *catuskoṭi* qualify as the Buddha's teaching, because the Buddha teaches that names fail to refer. The teaching qualifies as graded, with different alternatives appropriate for different audiences, not because some audiences lack the logical facility to derive the truth of some alternatives, and not because some alternatives are more correct than

others, but because each alternative presents different risks for error.<sup>13</sup> The third alternative risks the confusion of supposing that *Non-Contradiction* fails; the fourth, of supposing that *Bivalence* fails. Ruegg notes the soteriological danger of each:

if there really existed such a dialectical synthesis [failure of *Non-Contradiction*] or third value [failure of *Bivalence*], there would be something on which conceptual thinking could base itself and cling, and the entire purpose of the Madhyamaka method could then no longer be achieved (Ruegg 1997, 47).

The second alternative risks the confusion of supposing that reference fails because nothing exists, which in turn risks attachment to non-existence. The first, finally, risks the confusion of supposing that names designate existing individuals, and this risks attachment to existence.

## 6. CONCLUDING REMARKS

Russellian analyses of Buddhist *catuṣkoṭi* yield several insights. The analyses clarify the role and meaning of *Reference Failure* in securing uniform truth valuations for all four alternatives. They highlight dialectical connections among adjacent alternatives. They motivate different interpretations for affirmative and negative *catuṣkoṭi*, and they justify differences in the conditions for evaluating analyses of each kind of *catuṣkoṭi*. They reveal, as well, some unexpected risks associated with deciding whether to treat names as predicates or individual constants, and with deciding how to interpret singular existentials. Russellian analyses achieve these insights, moreover, with minimal ideology. They are well-motivated in their ascriptions of logical structure to all four alternatives, and they do not rely upon non-standard devices such as paraconsistent logic or illocutionary denial.<sup>14</sup>

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<sup>13</sup> Here I develop Katsura and Siderits' suggestion that "perhaps the hierarchy [among the alternatives of the *catuṣkoṭi* at MMK 18.8] is not based on increasing accuracy but on increasing usefulness for achieving our goal (in this case, the cessation of suffering)" (Katsura and Siderits 2013, 202). For more on the significance of language for Madhyamakan soteriology, see Tsai 2014.

<sup>14</sup> I do not claim to have resolved all exegetical issues concerning Buddhist *catuṣkoṭi*. For example, I discuss neither the extent to which Russellian analyses facilitate insight into the distinction between ultimate truth and conventional truth, nor the ways in which *Universal Reference Failure* might relate to the practice of skillful means (*upaya*). These substantial topics remain for future work. For some indication of the issues at stake, see Garfield 2010 as well as Matsunaga and Matsunaga 1974, respectively.

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