

## ZERO, ŚŪNYA AND PŪRṂA: A COMPARATIVE ANALYSIS

ANIMISHA TEWARI

**ABSTRACT:** *Due to apparent duality in this world, one has to face a lot of difficulties while searching for the Truth. Our ego is the root cause for perception of duality and this in turn leads to suffering. This suffering can only be extinguished by attainment of the Truth, i.e, non-duality. However, in order to enable the finite intellect to comprehend the incomprehensible non-duality, this undifferentiated whole is sometimes denoted by nothingness (śūnya) or fullness (pūrṁa). Non-duality is usually understood by the numeral '1' which stands for unity or oneness. The main aim of this paper is to show that non-duality is best represented by the numeral '0', Mādhyamika śūnya (advaya) and Upaniṣadic pūrṁa (advaita). This paper also attempts to touch upon the hitherto untouched and profound implications of zero in view of its being a simultaneous indicator of nothingness as well as wholeness. It also humbly tries to tackle the perpetual mathematical problem of 'zero divided by zero' and in this regard tries to establish how the solution is consistent with the concept of non-duality. In a nutshell, this paper's endeavour is to approach the Ultimate Reality via threefold path: through the neutral means of mathematical zero "0" as a concept, negative means of Mādhyamika śūnya, and the positive means of Advaita Vedānta's pūrṁa.*

**Keywords:** *advaita, Buddhism, Mādhyamika, non-dualism, pūrṁa, śūnya, Vedānta, zero*

### 1. INTRODUCTION

The Ultimate aim of human life is to realize the Truth, which is real at any time and place.<sup>1</sup> In the search for the Truth about one's existence, one constantly grapples with the apparently dualistic notions of life & death, pain & pleasure, subject & object and so on. These dualistic notions have no scope of synthesis by the intellect. As a matter of fact, these dualistic approaches are the real cause of human suffering because "I" (the subject) is different from the other (the object of I's desire). Had this been all, there

---

TEWARI, ANIMISHA: Independent Scholar, India. Email: animishatewari20@gmail.com

---

<sup>1</sup> The term 'Truth' in this paper is used in the sense of underlying Ultimate Reality, in contrast to the "semantic" truth (i.e., the "semantic" relation between the linguistic truth bearer and the extra-linguistic truth maker) as conceived in people's pre-theoretic "way-things-are-capturing" understanding of truth.

would've been no problem, however, it is a natural tendency of the "I" ego, despite aforesaid apparent duality, to achieve completeness by trying to unite with the external objects, both psychological as well as material. Such a tendency of the illusory ego actually comes from our Consciousness or Awareness which is an undifferentiated whole. Now comes the real problem, i.e., representing this wholeness. This undifferentiated whole is sometimes denoted by nothingness (*śūnya*) or fullness (*pūrṇa*) according to various factors such as one's tradition, one's worldview, or stage of one's evolution. Our thoughts and languages arise from the intellect which itself divides the whole into fragments in order to ensure protection of the body-mind complex and to make sense of who we are and the world we live in. Although these fragments may seem to make our life easier, they actually complicate it due to unlimited constructions. It is like trying to see the world with muddy lenses or even trying to see the entire sky from a window. Thus, it becomes an obligation to deconstruct everything in order to realize the undifferentiated whole once again and free ourselves from suffering, or in other words, attain liberation.

The first section "Zero" deals with the mathematical numeral and concept of 0. It is a well-known fact that zero was discovered in India, but little is known about its history and reasons for being conceived. After all, when 'nothing' is so insignificant, why should anyone care about it at all. It is also the only neutral number and concept as it can neither be positive nor negative. Without zero it is practically impossible to think of how mathematics would've been today. Zero is also the major contributor to the formation of our Digital or Virtual Reality, which can be compared to Advaita's Dream & Phenomenal Reality. Due to these shared characteristics of the zero with the Ultimate, one can't help but wonder about the role of zero in and as the Absolute. Besides, zero also has profound mystical qualities as it is a simultaneous indicator of nothingness and wholeness. Moreover, zero literally means *śūnya* in Sanskrit/Hindi so in order to avoid confusion in this paper, 'zero' strictly stands for the mathematical zero while 'śūnya' stands for the Mādhyamika *śūnya*.

The next section is that of '*śūnya*' in Mādhyamika Buddhism. In the search for the Absolute, at least some of us will come across nothingness. Such an encounter involves the risk of falling into nihilism - rejection of existence. Though the idea is interesting, it doesn't fall short of criticism. Mādhyamika's *śūnya* which is frequently mistranslated as nihilism tries to show how it is beyond that. As a strict adherent of *pratīyasamutpada*, It criticizes both the extremes of Being and Non-Being. Thus, care should be taken before assuming that *śūnya* is establishing non-Being as the fundamental reality.

The third section talks about the Upaniṣadic '*pūrṇa*', a corollary to Mādhyamika *śūnya*. This *pūrṇa* is best explained through the teachings of Advaita Vedānta which is the epitome of Fullness. This too is misinterpreted as establishing the other extreme, i.e., of 'Being' as the fundamental reality. Śāṅkarācārya has very often been labeled as '*pracchanna-bauddha*' or Crypto-Buddhist due to uncanny resemblance between Advaita Vedānta and Mādhyamika Buddhism. This paper is an attempt to show that both these schools speak about the same Truth with different terminologies hence the bad blood among both traditions is unnecessary. After all, for the seeker of the Truth,

the main aim is to realize the Truth through whichever means is most suitable. Trying to uphold a particular tradition as the ultimate renders it as dogmatic, disabling one to attain Self-Realization.

The last section tackles the never-ending problem of zero divided by zero. It tries to establish how this problem is not only solvable but also consistent with the subject matter of this paper. Thus, the main aim of this paper is to approach the Ultimate Reality via three different insights because ultimately, they merge into a homogeneous whole which is both a beginning as well as an end. What is important is the journey undertaken to reach back at the starting point which is also described by the famous Zen proverb: “*Before enlightenment, chop wood, carry water. After enlightenment, chop wood, carry water*”.

## 2. ZERO

### 2.1 HISTORY OF ZERO: AN INDIAN PERSPECTIVE

Zero as a number is the building block of various branches of mathematics especially algebra, arithmetic, probability and calculus. In fact, all of our present mathematical and technological advancements can be attributed to zero. The digital world heavily relies on binary digits, a base 2 system consisting of numbers 0 and 1. The debate about the invention or discovery of zero as a number has remained inconclusive due to obliteration of historical records. However, the Indian contribution to zero as a number cannot be denied. It may be relevant to refer to the famous quote by the Indian mathematician B. Datta, “*The arithmetic of zero is entirely the Hindu contribution to the development of mathematical science. With no other early nations do we find any treatment of zero.*” (1).

The word ‘zero’ is a transliteration of Sanskrit ‘*śūnya*’, which literally means void, empty or nothing. According to the findings of Joseph (2008), ‘*śūnya*’ was translated into ‘*sifr*’ in Arabic by Al-Khwārizmī (c.780 A.D.), which too denoted nothing or empty. He was intrigued by the Indian numeral system consisting of ten digits one to nine, along with the tenth position occupied by the dot representation of zero, then known as *śūnya-bindu*. He was also instrumental in introducing the current Hindu-Arabic number system, a base-ten decimal system to Europe. Around 1200 A.D, Fibonacci (Leonardo of Pisa), an Italian Mathematician, further transliterated ‘*sifr*’ into ‘*zephirum*’ or ‘*zefiro*’ in his book *Liber Abaci*. As this number gained immense popularity for making tedious calculations simpler, it was banned by the religious authorities due to its threatening negative connotations as well as the impossibility of existence of non-existence. However, the people continued to use it for calculations in secrecy and thus came the English words ‘cipher’ (code), ‘nought’ (nothing) and ‘zero’ into being.

Interestingly, the notion of zero has been in existence even before it was conceived as a number and operated upon. This notion led to the independent origin of positional notation in many cultures which helped in determining larger magnitude using place-values. The non-positional notation may be said to involve the Roman numerals. They

were non-positional because the system didn't determine the value of a number based on its position. For instance, V in XV or XLVII stands for 5 despite its position unlike the 2 denoting 200 in the value 205 or 20 in 28. Hence even a simple calculation such as  $V + VI = XI$  was a tedious feat. The earliest use of zero as placeholder is evident in the Babylonian system, a sexagesimal (60) base. Since there was no symbol for it, they used two slanted wedges to depict no magnitude, else 4 4 would've coalesced into 44. (Gironi 2012, 7). The Egyptians too used the hieroglyph 'nfr', meaning complete, to represent no magnitude. Meanwhile, Mayans used a mixed base of 20 and 18 where they only employed zero for counting and placeholder purposes and not calculations. They only required three symbols: a dot for one, bar for five and a snail shell for zero. (Joseph 2008)

India's zero, parallel to the Mayans, has had a multipurpose status of placeholder, concept, numeral etc. owing to the system of base 10. Due to only ten digits, it was possible to write any number as a combination of those digits. The earliest usage of zero as 'symbol' for chanda metrics is evident in *Chandaḥ-sūtra* of Pingala (200 B.C). The first use of *sūnya* as a proper dot symbol (*śūnya-bindu*) in calculations along with a full-fledged place-value notation appears in the Bakhshali manuscript on a piece of birch bark (c. 200 A.D). Āryabhata I (476 A.D) used alphabetical notation *kha* (aka *śūnya*, *ākāśa*, *pūrna*, *randhra*) to denote blank space in his work *Daśagītikā*; This was further clarified by Bhāskara I (c. 600 A.D) in his commentary wherein he states that *Kha* in 'khadvinavake' stands for *śūnya* which means the eighteen places marked by *śūnya*. (Datta & Singh 1962, 64-66). Zero was only used as positional notation till then to specify the place values in hundreds, tens, units place within a decimal base. Moreover, numbers were usually written in the same way as they were spoken or with names of colors instead of being denoted by symbols. The first symbol for zero as a circle is evident in the Gwalior inscription (876 A.D) consisting of a circle in the unit place of 270. (Gironi 2012, 8).

Datta (1927) observes that the first operation of addition and subtraction of a number with zero is found in Varāhamihira's *pañca-siddhāntikā* (505 A.D) which shows that it was accepted as a number. Moreover, it was only in Brahmagupta's *brāhmasphuṭasiddhānta* (628 A.D) that the property of zero w.r.t addition, subtraction and multiplication and division was propounded correctly except for the sole operation of division with zero as denominator. He states that the quotient of a number divided by zero is zero (*kha-cheda*, literally, 'having zero as the denominator') and that of zero divided by zero is also zero. (Datta 1927, 168-169). It was Bhāskarācārya (or Bhāskara II) who made some changes in this operation as well as introduced the concept of infinity in his *lilāvātī* and *bija-ganita* (1150 A.D). According to Bhāskara, for any finite number 'x' divided by zero, the quotient is infinity (*Kha-hara*). Therefore,  $x/0 = \text{infinity}$ . He also compares infinity with God and maintains that *kha-hara* ( $x/0$ ) remains as it is even on addition or subtraction by a finite quantity similar to how the process of creation or destruction has no effect on the infinite God. There is also some hint of infinitesimals in his work *Lilāvātī* since he states that the product of a number and zero is *kha-guna* which would mean that the division of *kha-guna* with zero is the number itself. (170, 174-176). Although deemed incorrect, these results do hold true in the case

of limiting function in calculus, but Bhāskara never explicitly used such terminology. In fact, this issue of division by zero still stands unsolvable and is considered *undefined* in case of a number divided by zero; *indeterminate* in case of zero divided by zero.

Despite such a rich history, zero was completely accepted as a full-fledged number in algebra only around the 16th or 17th century. Zero has paved a way for the advanced branch of mathematics called calculus which was developed independently by Newton and Leibniz. This further developed the ideas of the infinite and the infinitesimal. John Wallis gave the symbol for infinity '∞' in 1657 and shared the same view as Bhaskara that a number divided by zero is infinity. Meanwhile the concept of infinitesimal deals with extremely minute quantities very close to zero yet not zero. L'Hospital's Rule in limits does away with indeterminate forms like  $0/0$ ,  $\infty/\infty$  by using derivatives until these forms disappear in order to solve the equation. Yet these advancements have only made it possible to further solve equations which consist of these undefined and indeterminate 'forms' like: number/0,  $0/0$  and  $\infty/\infty$  etc., but do not provide any solution to these forms themselves. (Barukčić & Barukčić 2016). Another interesting aspect of calculus is that while the differential calculus tries to study the rate of change, analyzes the whole into infinitesimally small parts, reminding one of Buddhist philosophy; the integral calculus tries to assimilate the infinitesimally small parts once again to understand the original whole, thereby, reminding one of the philosophy of Advaita Vedānta.

## 2.2 CONCEPTION OF ZERO: PHILOSOPHICAL IMPLICATIONS

The conception of mathematical zero as a number owes much of its origin to the philosophical understanding of *śūnya* in Indian Philosophy. Although it may automatically make one think of Mādhyamika's *śūnya*, there is much more to it. Indian Logicians were already accustomed to the notions of *abhāva* (negation) and *anupalabdhi* (non-apprehension). The later Nyāya-Vaiśeṣika's gave an ontological status to *abhāva* by including it as the 7th *padārtha* which arose from the denial of other 6 *padārthas*. Buddhist logicians accept *anupalabdhi* (non-apprehension) as a characteristic mark of inference (*anumāna*). Absence for them is neither a real entity nor a separate means of knowledge (*pramāna*) because the absent Jar would have been perceivable if it were present on the floor. But since it is not, it can only be inferred. Meanwhile, Bhatta-Mīmāṃsākas and Advaita Vedānta accept *anupalabdhi* as a separate and valid means of knowledge (*pramāna*) for apprehending non-existence. They hold that absence of Jar on the floor is neither perceived nor inferred; rather it is known through *anupalabdhi* (non-apprehension) itself.

Furthermore, the concept of zero in the philosophical sense of void or nothingness has been programmed into our Being since the very Creation. The Nāsadīya Sūkta of Ṛgveda (RV 10.129), one of the oldest Creation Hymns, discusses about it in the following verses: “*nāsad āsīn no sad āsīt tadānīm*”, At that time, neither non-existence nor existence was there; “*tucchyenābhv apihitam yad āsīt*”, ‘That’ which came to be enveloped in nothingness. It is interesting to note that one of the ways in which ‘Amarakośa’, a Sanskrit lexicon by Amarasimha, defines *śūnya* or zero is as *tuccha*

(insignificant, trifling). (Datta 1927, 167). At the same time, zero was also referred to as *pūrṇa* (fullness) by some ancient scholars, the reasons for which shall be clear in third section. Therefore, mathematical zero was denoted by both *śūnya* and *pūrṇa*, as both emptiness and fullness respectively.

Apart from Indian philosophy, another ancient philosophical system which continues to influence the modern world is that of Greek philosophy. Ancient Greek philosophers tried to decipher nature in a rationalistic framework as they believed that the ability to reason was unique to humans. Due to their stringent adherence to reason and logic, they were not open to the idea of void or nothingness. Intellect cannot decipher nothingness because in saying something is nothing, it already is, thus, contradictory. Nor did they require zero in calculations as they were keener about geometry than arithmetic. However, some Greek philosophers like Anaximander, Parmenides, Plato and Aristotle did grapple with the idea of nothingness but dismissed it due to its lack of reference in the physical world along with their own fixations for an immutable Truth. (Grandy 2016). Parmenides argued against non-Being through the dictum “*ex nihilo nihil fit*” (Nothing comes out from Nothing). This implies that Being (Plenum) is all there is so even change is unreal because it necessarily presupposes a void, which is impossible. Aristotle postulated “*horror vacui*” (nature abhors vacuum) in his *Physics*. (Logan 1979). At the same time Boyer (1943) argues that Aristotle in *Physics* indirectly conceived of zero as a number and also dealt with its division operation stating its impossibility. However, this impossibility made motion in void a contradiction, thereby, leading to the rejection of the possibility of nothingness. The atomists - Leucippus and Democritus were the first to accept the presence of something like a void which made the movement of atoms possible, however they didn't go in-depth into it.

Aristotle also laid down the axioms of The Law of Excluded Middle and The Law of Non-Contradiction which form two out of three fundamental laws of classical logic. The law of noncontradiction states that a proposition cannot be simultaneously true and false,  $\sim (p \wedge \sim p)$  whereas the law of excluded middle states that a proposition can either be true or false with no middle solution,  $(p \vee \sim p)$ . The law of excluded middle outrightly rejects the existence of zero which can be said to be a middle ground between the negative and positive integers since zero by itself is neither positive nor negative and has no properties of its own. These laws also depict the inherent dualistic notions prevalent in that period. This can be contrasted by the Indian Logicians acceptance of fourfold alternatives (*catuskoti*) wherein, a proposition, apart from being True, False, can be simultaneously both true and false as well as neither true nor false. The last alternative has also been used in *Nāsadīya Sūkta* as already mentioned above. This will be taken up in the next section in detail.

Finally, as Logan (1979) argues, it must be the intuitive insight of Indian Philosophy over the logical attitude of the western philosophy that led to the discovery of zero. (27). Thus, this further alludes to the fact that deep rooted philosophical thinking influenced the formulation of zero as a number in India, even if indirectly!

This understanding of mathematical zero leads us towards philosophical *śūnya* which gives us more insight into the underlying Ultimate Reality.

### 3. ŚŪNYA

#### 3.1 BUDDHISM

In order to understand *śūnya* it is important to get acquainted with the basic teachings of Buddhism that was founded by Gautam Buddha, born Siddhārtha Gautam (c. 600 BC). He observed that life is gripped with suffering and set on a journey to find a way to get rid of it. Although Buddha himself never wrote anything, the main teachings (*saddharma*) ascribed to him are the Four noble truths and the doctrine of dependent origination. the four noble truths (*ārya satya*) are: *duḥkha* (there is suffering), *duḥkha samudaya* (there is a cause of suffering), *duḥkha nirodha* (there is a cessation of suffering), *duḥkha nirodha gāminī pratipat* (there is a path leading to the cessation of suffering). These four truths briefly describe the totality of the nature of human existence which is suffering itself as well as a way to eliminate it.

The doctrine of Dependent Origination or *Pratītyasamutpāda* is the Buddhist Theory of Causation which is a middle way (*madhyamā-pratipat*) between the two extremist views of causation. It ensues the second and the third noble truth. The extreme views comprise of eternalism (*śāśvatavāda*) which maintains that cause and effect are identical, giving rise to a permanent entity; and nihilism (*ucchēdavāda*) which holds that cause and effect are different, reducing everything to nothingness. Meanwhile, the middle path holds that one cause is dependent on another cause for its production which then manifests as effects. so, there is only a psychophysical entity (*pañcaskandha*) against the notion of an eternal soul. this gives rise to the no-soul doctrine (*nairātmyavāda* or *anātmavāda*) as well as the doctrine of momentariness (*kṣaṇikavāda*). *Pratītyasamutpāda* is the twelve spoked wheel of causation which explains *samsāra*, the cycle of birth and death, as the cause of suffering and *Nirvāṇa* as the cessation of suffering. Everything has a cause, eliminating which will eliminate the effects, thereby eradicating suffering altogether. The twelve spokes (*nidāna*) are: *avidyā* (Ignorance), *saṃskāra* (Karmic impressions), *viññāna* (Consciousness), *nāma-rūpa* (Name and form), *ṣaḍāyatana* (Six sense organs including mind), *sparśa* (Contact), *vedanā* (Sense experience), *trṣṇā* (Thirst/desire), *upādāna* (Clinging to desire), *bhava* (Will to be born), *jāti* (Re-birth), *jarā-maraṇa* (old age and death). Each link is the cause of the succeeding link which traps one in *samsāra* due to the root cause of *avidyā* (ignorance). This chain can be broken by eradicating the Ignorance which is only possible by realizing the mutual interdependence of everything.

Various interpretations of these teachings have led to different *-yāna*'s (vehicles) in Buddhism, the main ones being the Hīnayāna (The Little Vehicle), Mahāyāna (The Great Vehicle). The Hīnayāna consists of the Sautrāntika and the Sarvāstivāda (Vaibhāṣika) schools, while the Mahāyāna consists of the Vijñānavāda (Yogācāra) and Mādhyamika (Śūnyavāda) school of Buddhism.

### 3.2 NĀGĀRJUNA'S ŚŪNYA

The basic tenets of the Mādhyamika School were already present in the *Prajñapāramitā sūtra* of Māhayāna, which were systematically propounded by Nāgārjuna (200 A.D) in his fundamental text *Mūlamadhyamakakārikā* (The Root verses of the Middle Way). It roughly consists of 450 verses and is divided into 27 chapters. Nāgārjuna is also known as the second Buddha by the followers of Mādhyamika school due to his interpretation being the closest to Gautam Buddha's own teachings who himself sought the middle way. Although not all Buddhists (especially those of Hīnyāna tradition) agree with this claim. Mādhyamika school further flourished through the works of his disciple Āryadeva and commentators Candrakīrti, Buddhapālita and Bhāviveka.

*Śūnya*, as mentioned above, literally means void, nothing, emptiness. However, the proper translation in this context is usually held by modern interpreters to be 'de-void'. It may come across as a surprise to the skeptics that *śūnya* doesn't mean nihilism but that this emptiness directs one to a non-dual reality (*Advaya*). Sometimes *śūnya* is synonymously used with *śūnyatā* (emptiness), but the addition of *ta-pratyaya* may make it seem as an abstract concept which enables one to mistranslate it as a distinct view (*śūnyata-dṛṣṭi*) and further into nihilism. *Śūnya* refutes all other views (*dṛṣṭi*) but it itself cannot be refuted as it is neither a philosophy nor a doctrine (*dṛṣṭi-śūnyata*). This is why it is slightly inappropriate to call it *śūnya-vāda* as well. What then is *śūnya*? Well, *śūnya* is *śūnya* of *śūnya*, i.e, an emptiness of emptiness. This makes it impossible to conceive *śūnya* as an ontological or substantial entity. This is also in stark contrast to Parmenides dictum "*ex nihilo nihil fit*" mentioned above, which states nothing comes out of nothing. When nothing is considered synonymous with *śūnya* in this context, it shows that emptiness can very well come out of emptiness without changing anything. Interestingly, while refuting the *dharmas*, five skandhas, four noble truths, dependent origination and even *Nirvāṇa*; Buddha and Nāgārjuna claim that these are all ultimately *śūnya*.

#### 3.2.1 THE TWO TRUTHS

*dve satye samupāśritya buddhānāṃ dharmadeśanā /  
lokasaṃvṛtisatyam ca satyam ca paramārthataḥ //  
ye 'nayo na vijānanti vibhāgam satyayor dvayoḥ /  
te tattvam na vijānanti gambhīram buddha-śāsane //*<sup>2</sup>

Truth has two aspects instead of four: loka-saṃvṛti-satya (conventional) and paramārtha-satya (Absolute). (Sharma, 1996, 47). The purpose of this so-called distinction is not to downplay the empirical reality in the face of some Absolute Reality. Saṃvṛti, which means covering, is still a conventional truth because without it one

<sup>2</sup> MMK, XXIV, 8-9, See Kalupahana (1986) for sanskrit verses



cannot reach the Absolute. It is like a journey which takes us to our destination and is equally important and real in a way. However, one shouldn't take this empirical reality to be the only truth, nor is it supposed to be transcended per se as there is no division of higher or lower realm in the Mādhyamika philosophy. The empirical can only be negated from the ultimate standpoint so one needs to get to the ultimate in order to deny the phenomenal reality. This is why *śūnya* shouldn't be misunderstood as nihilism. Nāgārjuna also maintains that one needs to understand this distinction of the two truths in order to understand Buddha's Saddharma.

From an empirical perspective of *avidyā* (ignorance), *śūnya* is *pratītyasamutpāda* itself, that is to say, it is akin to Relativity. Nāgārjuna accepts the middle path but feels that it too has to be given up. Objects are devoid of self-existence, i.e., *svabhāva-śūnya* or *niḥsvabhāva*. For instance, a 'Book' does not have any self-existence (*svabhāva*) of its own; it is dependent upon an author, minimum page numbers, the content, the type of information, binding, target audience and so on. The essence 'book' does not have any *svabhāva* and is, therefore, *śūnya*. If the book were *aśūnya* then it would have existed eternally and independently of the above-mentioned factors. If the book were *śūnya* in the nihilistic sense, then it would have never been able to come into existence nor been readable.

Moreover, there is another lesser-known aspect of *śūnya*;

*aparapratyayaṃ śāntaṃ prapañcair aprapañcitam /  
nirvikalpam anānārtham etat tattvasya lakṣaṇam //*<sup>3</sup>

From an ultimate perspective of *prajñā-parāmitā* (direct intuition), *śūnya* is non-dual Reality or Truth (*advaya tattva*), tranquil (*śāntam*), devoid of all plurality and suffering (*prapañca-śūnya*), inexpressible (*anirvacanīya*), indeterminate (*nirvikalpam*), infinite (*anānārtham*), beyond four categories of thought (*catuṣkoṭi-vinirmukta*) and *nirvāṇa* itself.

Thus, the conventional Truth is *svabhāva-śūnya* (devoid of self-existence) while the Ultimate Truth is *prapañca-śūnya* (devoid of plurality). (Sharma 1996, 42). This paper is mainly concerned with this latter perspective, i.e., *prapañca-śūnya*.

### 3.2.2 PRAPAÑCA-ŚŪNYA

The opening verses of *Mūlamadhyamakakārikā* states “*yaḥ pratītyasamutpādaṃ prapañcopaśamaṃ śivam.*”<sup>4</sup> *Pratītyasamutpāda* is referred to as “*prapañcopaśamaṃ*” and “*śivam*”, i.e., non-dual (devoid of plurality) and blissful (*śivam*). *Śūnya* is *Pratītyasamutpāda* from an empirical point of view. Therefore, *śūnya* is non-dual and blissful.

<sup>3</sup> MMK, XVIII, 9

<sup>4</sup> MMK, Dedicatory Verse

That *śūnya* is non-dual shall be clear from Nāgārjuna's use of the Mādhyamika's destructive as well as constructive dialectic. In destructive dialectic,<sup>5</sup> he employs *catuskoṭi* or tetra-lemma (four-cornered alternatives) according to which only four views are possible in any category of thought. *Catuskoṭi* refutes everything by bringing out the inherent absurdity in views through the method of *reductio ad absurdum* (*prasaṅga*). The four alternatives are:

1. It Is (*sat*),
2. It Is Not (*asat*),
3. It Both Is and Is not (*ubhaya*)
4. It Neither Is nor Is not (*anubhaya*)

The first two alternatives of affirmation and negation are primary as they correspond to the problem of identity and difference respectively. Meanwhile, the other two alternatives are their derivatives with the third being a conjunctive affirmation and fourth being a disjunctive denial of identity and difference. (Sharma, 1996, p. 23). Buddha himself used this method in devising *Pratītyasamutpāda* as a middle way where the two extremes consisting of eternalism and nihilism correspond to Being and Non-Being, respectively. However, in doing so He supposedly put forth his own doctrine of universal flux (*dharma*) which according to Hīnayāna tradition ought not be defied. Nāgārjuna pointed out that *dharma* is misinterpreted by them as universal, but it is, in fact, only a means to understand *śūnya* and hence unreal (*dharma-nairātmya*).

Through the destructive use of dialectic, Nāgārjuna negates all four alternatives to bring forth *śūnya* underlying within. Thus, saying that *śūnya* is either absolutism or nihilism fails the very purpose of the dialectic. Moreover, two extremes are to be reconciled through non-dualism. Raju (1954) compares how the negation of the four alternatives applies to the mathematical zero as well. That "0" is positive is false, that it is not positive (i.e., negative) is false; it is both positive and negative is false; and lastly, it is neither positive nor negative is also false. In this way "0" is similar to Nāgārjuna's *śūnya*. Since the fourfold negation applies to everything, the resulting plurality of the phenomenal world is *śūnya*. *Prapañca-śūnya* is then beyond these four alternatives of thought (*catuskoṭi-vinirmukta*).

How is 'prapañca' to be interpreted?

*karmakleśakṣayān mokṣaḥ karmakleśā vikalpataḥ/  
te prapañcāt prapañcas tu śūnyatāyāṃ nirudhyate//*<sup>6</sup>

*Prapañca* is generally accepted by Mādhyamika scholars as the conceptual or ideological proliferation which leads to defilements of thoughts (*vikalpa*) in addition to inherent contradictions in our own viewpoints. These defilements lead to suffering and

<sup>5</sup> See Sharma 1987, 89 for destructive dialectics

<sup>6</sup> MMK, XVIII, 5

drive us away from the Truth. In this regard, *prapañca* is broadly accepted to be either an “activity” of conceptualisation, the “objects” of conceptualisation or the ‘instruments’ as concepts themselves. (Saito 2019). Moreover, Kalupahana (1986) translates *prapañca* as ‘obsession’ with objects supposedly having *svabhāva* which ceases in *śūnyata*. All in all, this interpretation of *prapañca* is a cognitive interpretation undertaken to do away with metaphysical realism. Matilal (1987) adds an anti-realism element to Mādhyamika as he thinks that it doesn’t deny the external reality but our conceptions of a metaphysical reality.

However, according to Sharma (1996), *prapañca* when taken as ‘empirical existence of plurality’ rather than just a ‘conceptual proliferation’ brings into light the underlying non-dualism (*advaya*). This can be seen from the following verse:

*prapañcayanti ye buddham prapañcātītām avyayaṃ /*  
*te prapañcahatāḥ sarve na paśyanti tathāgatam //*  
*tathāgato yat svabhāvas tat svabhāvam idaṃ jagat /*  
*tathāgato niḥsvabhāvo niḥsvabhāvam idaṃ jagat //*<sup>7</sup>

The Buddha is beyond plurality of existence (*prapañca-śūnya*) and can only be grasped by one who has transcended plurality and realized non-duality. That is to say, Buddha cannot be grasped by those who are trapped in this pluralistic empirical world and dualistic notions of “I”, “Mine” etc., while seeing Buddha as an “Other” Enlightened Being, distinct from themselves. Buddha, who is the *Tathāgata* (Suchness) can only be revealed through the lens of non-dualism (*advaya*). Empirically, whatever is the *svabhāva* of the *Tathāgata* is also the *svabhāva* of “this world” (*idaṃ jagat*) but since the *Tathāgata* is devoid of self-existence (*svabhāva-śūnya*), so is this world. As already pointed above, *svabhāva-śūnya* is the empirical perspective of *śūnya*. However, this understanding is still an intellectual understanding as Reality is above the categories of intellect. It is our intellect followed by thoughts and speech which lead to subject-object duality. This duality further leads to plurality reducing everything into having an independent existence, and this eventually leads to suffering as already discussed above. Destructive dialectics satisfies the intellect by employing logic and reasoning to attack the phenomenal. This is not the end as the intellect too needs to be transformed into intuition (*prajñā*) in order to realize the Ultimate non-dual Reality. The intellectual understanding paves the way for direct intuition of non-dual awareness to dawn upon us which is devoid of all pluralities (*prapañca-śūnya*).

It may, thus, be said that *svabhāva* corresponds to Subject (Identity) while *prapañca* corresponds to the Object (difference). Addition of *śūnya* in front of these terms implies transcendence of subject-object duality into non-dualism. T. Murti (1955) also observes that the main role of Mādhyamika dialectic is to purify one’s intuition by removing impurities so that the non-dual *prajñā* may dawn upon us. When this *prajñā* can see the Truth in its totality, for instance seeing the flower, the mountains etc., via pure awareness, that is the moment we attain the Absolute Truth. This is the

<sup>7</sup> MMK, XXII, 15-16

constructive aspect of the Mādhyamika dialectic.<sup>8</sup> He also says that *advaya* is the epistemological approach towards non-dualism since its main aim is purification of intellect more than anything.

Śūnya understood in the sense of prapañca-śūnya directs us to a lesser-known interpretation of the ‘word’ śūnya, i.e. pūrṇa (fullness). According to S. Rinpoche (2005), to understand śūnya and pūrṇa one needs to recognise what and how much is to be negated in order to deconstruct the intellect. If everything is negated, one is bound to fall into nihilism and if nothing is negated then one automatically falls into eternalism. The amount to be negated then corresponds to the middle path which is best depicted by both śūnya and pūrṇa. Śūnya is pūrṇa and pūrṇa is śūnya and the numeral zero represents as well as means both.

#### 4. PŪRṆA

##### 4.1 ADVAITA VEDĀNTA

The Upaniṣadic philosophy advocates the notion of *pūrṇa* to establish the Ultimate Reality. *Pūrṇa*, which literally means full, whole, complete, perfect etc, is usually taken as an antithesis of *śūnya*. These two terms are in such stark contradiction to the intellect that synthesizing them is beyond the question. It is our intellect which discriminates *pūrṇa* and *śūnya* as two opposites, otherwise these two are the two aspects of the same coin. The view that *śūnya* is the beginning while *pūrṇa* is the end goal is correct only if we assume linear progression of events. However, Indian philosophy has always accepted a cyclic progression of events which constitutes neither any beginning nor an end. This is best represented by the symbol of mathematical zero “0”. Since there is no beginning nor an end but only this present moment which holds the power to liberation, *pūrṇa* and *śūnya* coalesce into one other turning an intellectual impossibility into an Intuitional Reality.

The teachings of the *Upaniṣads* along with the *Brahma Sūtras* and the *Bhagavadgīta* (*Prasthānatrayī*) constitute the essence of the Indian philosophical school of *Vedānta*. *Vedānta* means the goal or end of the Vedas. It summarizes the creamy portion of the Vedas as well as the *Upaniṣads*. *Brahman*, the Ultimate Reality, is the central theme of the *Upaniṣads*. Various understanding of *Brahman* and its relation to the world has led to different schools of *Vedānta* like Advaita Vedānta, *Viśiṣṭādvaita*, *Dvaita*, *Dvaitādvaita* and so on. This paper shall focus on the interpretation of *Ādi Śaṅkarācārya* (c. 700 A.D.) whose commentary upon the Brahmasutra and systematization of the Upaniṣadic philosophy constitutes the school of Advaita Vedānta. Advaita Vedānta is “logically the most consistent and spiritually the most advanced philosophy of India”. (Sharma, 1996, 96). Advaita means non-dual, That which is “not-two”. It is most commonly translated as ‘monism’ due to its propagation of an inherent unity in diversity. However, this is a mistranslation because as a matter of fact the category of monism automatically presupposes the possibility of

<sup>8</sup> See Sharma 1987, 97 for constructive dialectic

another. The *Upaniṣads* frequently emphasize upon Brahman as “one without a second”. The intellect analyzes the whole into parts for the sake of simplicity but in doing so it considers the parts as the ultimate. Thus, when *advaita* talks about the unity or oneness of all pluralities, it is not agreeable to the intellect which subsequently levies criticisms. This paper posits that non-dualism is best represented by “0” instead of “1”.

*Advaita Vedānta* distinguishes three levels of *satta* or reality. The *pratibhāsika-satta* or illusory reality operates on the level of dream which is considered ‘real’ as long as we are asleep. Yet they are non-real, i.e., neither real nor unreal. It consists of dream or illusory objects which are mostly subjective to the experiencer. For instance, mirage. The falsity of this reality vanishes after waking up. The *vyavaharika-satta* operates on the level of the spatiotemporal phenomenal world. It consists of sense experiences and world-objects which have a more objective nature since they are shared by multiple experiencers. For instance, River Ganges. The phenomenal world relies on the trinity of the knowledge, knower and known. There is a subject which is the experiencer, an object which is experienced, and the resulting experience forms the knowledge. The plurality of existence in the *vyavaharika-satta* is brought forth by *Māyā* or *Avidyā* (Ignorance), which is a power of Brahman Itself. The falsity of this phenomenal world too disperses when its nature of superimposition is realized just like that of waves superimposed on the ocean. And lastly, the *paramarthika-satta* or the Absolute Reality operates on the level of consciousness. Here, the subject-object dualities, pluralities merge into pure non-dual awareness. Brahman is the knowledge, knower and known, all at once. “*Brahma-Satyam Jagan-Mithyā Jivo Brahmaiva Nāparaḥ*” states that Brahman is the only Truth, the world is an illusion, and the Individual self is no different from the Self or Brahman but identical.

By now one may question what is this Brahman? This question is redundant because Brahman cannot be trapped into the limits of definitions. It is to be intuited in Its wholeness (*pūrṇatva*). Yājñavalkya, in his dialogue with Gārgī, discusses the views of the knowers of Brahman. who employ “*neti-neti*” or “not this, not this” to eliminate all the descriptions associated with It. Brahman is never seen, heard, felt etc., but is the Witness-Consciousness. It is neither a dream-object nor a sense-object, rather It is the witness of all the objects. It is the seer and hearer Itself. Brahman is the Self within all of us, It is the highest of all Truths. (*Bṛhadāraṇyaka Upaniṣad*, 3.8.11).

Śaṅkarācārya expounds the same idea in an elegant composition of six verses called the ‘*Nirvāṇaṣatkam*’. After eliminating all possible descriptions using “*neti-neti*” in the first five verses, he lays down the immediate spiritual experience (*Prajñā*) of Brahman or Self in the following last verse:

*ahaṃ nirvikalpo nirākārarūpo /  
vibhutvācca sarvatra sarvendriyāṇām //  
na cāsaṅgataṃ naiva muktirna meyaḥ /  
cidānandarūpaḥ śivo'haṃ śivo'haṃ //6//*

The Brahman or Self is non-dual, unconditioned, Immutable, devoid of all attributes and forms. Being omnipresent It pervades all the senses and is just the self-

luminous Witness. It is detached from all worldly desires including the desire for Liberation. The Self is never in bondage so there is nothing to be freed of either. The only bondage is the superimposition of the individual self (*jīva*) on the Transcendental Self which considers itself as a separate, real entity. Brahman is the only Reality, rest everything is Its appearances (*vivartavāda*). These appearances dissolve when the veil of ignorance is removed. Brahman is the *Sacchidānanda* or Existence-Consciousness-Bliss. It is *śivam* or non-dual blissful awareness. It is pertinent to note that Nāgārjuna in his dedicatory verse of *Mādhyamika-kārika* also uses the adjective ‘*śivam*’ for *prāṭhyasamutpada* which is *śūnya* from empirical point of view.

The Absolute or the Transcendental Reality doesn’t imply an “other-worldliness” which is away from this phenomenal reality, accessible to the fated few. This Absolute Reality is very much here in the ‘Now’ accessible ‘to’ and ‘as’ each one of us. “*Sarvam Khalu Idam Brahma*” (*Chāndogya Upaniṣad*, 3.14.1) that is, “All this is, verily, Brahman”. In fact, the Absolute Reality is immediately within our reach as compared to the other two. Its only subject matter is that of Self-Knowledge. The teachings of *Advaita Vedānta* can best be summarized by the four *Mahāvākya* of the Upanishads which also represent the four Vedas each.<sup>9</sup> They are:

1. “*Prajñānam Brahma*” (*Aitareya Upaniṣad* 3.3, Rgveda) Consciousness or Pure Awareness of the present moment is Brahman.
2. “*Ayam Ātma Brahma*” (*Mandukya Upaniṣad* 1.2, Atharvaveda) This very Self which is also called the Atman is Brahman.
3. “*Ahaṃ Brahmasmi*” (*Bṛhadāraṇyaka Upaniṣad* 1.4.10, Yajurveda), “I am Brahman or the non-dual consciousness” emphasizes the inherent divinity in man.
4. “*Tat Tvam Asi*” (*Chāndogya Upaniṣad* 6.8.7, Sāmaveda), That thou art lays down an inherent unity with otherness. Since I and You are that non-dual consciousness, we all are non-dual consciousness.

All these *mahāvākya*’s are essentially saying the same thing in different ways, that is, All is non-dual, non-dual is All. This non-dual consciousness has no senses to perceive, no mind to feel, nor an intellect to think. However, when the *antahkaraṇa* (inner instruments) are inundated by the Consciousness, the perceptions and feelings arise which further enable the *jīva* (individual self) act accordingly.

The philosophy of *Advaita Vedānta* can be best understood by two analogies. The first analogy is that of element water. Let the water in the ocean be Brahman. The water in the rivers be the *Ātman* and the individual rivers having name & form be the individual self or *jīva*. Now these rivers are bound to merge into the ocean sooner or later. The water in the river identifies itself through its name and form thereby strengthening its I-Consciousness throughout. But where is the individual river? In addition, the river is constantly flowing so what exactly constitutes the river. After a series of speculations and eliminations on the question “Who am I?”, the veil of ignorance disappears, and the river realizes its true nature of being the water. This

<sup>9</sup> See Vedanta New York (2017) *Mahāvākya* by Swami Sarvapriyananda

realization reveals the underlying ātman. Now as its journey progresses and it merges with other streams and rivers, its I-consciousness loses hold over it and it starts flowing in a more blissful manner. As we have already seen above, this *Ātman* is Brahman, so it has attained liberation and is flowing as a *jīvanmukta* (liberated while alive). It is still alive as it is fulfilling its *prarabdha-karma* while showing the way to those trapped in the *vyavaharika-satta*. Once it merges with the ocean, it reaches the final stage of *videhamukti* (liberation after death). All that is there is a non-dual consciousness.

The second analogy is that of electricity. Let electricity be equivalent to Brahman, the electrical energy required to power an appliance/device be *ātman* and the electric appliance itself be the *jīva*. Since electricity is produced mainly from solar, wind or hydro energies, it is closer to Consciousness and human nature in a way. Each appliance has different functions and requires different amounts of that same electrical energy in terms of amperes and voltage to operate effectively, e.g., cooling, heating etc. The energy consumption is also proportional to the wattage of the electrical appliance, i.e., a heater requires more energy than a light bulb. Now the electrical energy required to run the appliances/devices may initially associate itself with the appliance, forgetting who It really is. After a series of reflection and introspection on “Who am I?”, it starts realizing its true nature. Also here the appliance cannot change itself, that is to say ‘evolve’ itself with this newly attained Self-Realization, hence, it will short-circuit if the current drawn out is more than it can handle.

Somewhat similar case occurs with human beings. When the body is not prepared to receive greater inflow of Consciousness, the mind short-circuits and leads to various psychopathological disorders. However, the body and mind can evolve through practices such as meditation, yoga the threefold discipline of: *Sravana* (Studying), *Manana* (Reflection), *Nididhyāsana* (Meditation on mahāvākya). This journey consists in the evolution of mind to Supermind.<sup>10</sup> So when we say one’s consciousness is evolving or expanding, it doesn’t mean that there is a change in the quantity of consciousness; rather there is a change in how much consciousness our body-mind can draw out or accommodate. Thus, the body-mind complex is an equally important instrument that leads us to Self-Realization since the Truth cannot be revealed just like that. This is also the main principle of Sri Aurobindo’s Integral (*pūrṇa*) Yoga.

## 4.2 UPANIṢADIC PŪRṆA

*Pūrṇa*, being consistent with non-dualistic philosophy, refers to *Nirguṇa* Brahman as It is already complete in Itself. This is evident in the *Aṣṭāvakraḡītā*, another text of *Advaita Vedānta*, which discusses the nature of the Self, one’s true nature, as *pūrṇa*.

*ātmā sāksī vibhuh pūrṇa eko muktaścidakriyah /  
asaṅgo niḡsprhah śānto bhramātsamsāravāniva //*<sup>11</sup>

<sup>10</sup> Supermind is akin to *Turiya* (The Fourth), the fourth and the highest stage of deep sleep in which the Absolute manifests itself and is the pure witness consciousness. See *Mandukya Upanishad*.

<sup>11</sup> *Aṣṭāvakraḡītā*, 1.12, See *Vivekananda Samiti*, IIT Kanpur. (2017) *Swami Sarvpriyananda on Ultimate Truth* (14:53)

The Self/*Ātman* or Brahman is Full, all-pervading, self-illuminating, tranquil, actionless and desireless Witness consciousness or *sākṣī*. The Witness consciousness is the one who experiences the world through the *Antaḥkaraṇa* (inner instrument consisting of ego, intellect, mind, memory) without itself being involved in it. All actions are performed without identifying with a ‘doer’ (*kartabhava*). The agency or doership over actions is an illusion of the *samsāra*. This witness consciousness, the *pūrṇa* is actually the witness of the *śūnya* and vice-versa. There is really no difference between *śūnya* and *pūrṇa*.

The earliest occurrence of *pūrṇa* can be traced back to the Atharvaveda in a hymn which mainly deals with ritualistic and philosophical speculations about the Supreme Reality.

*pūrṇātpūrṇāmūdyati pūrṇām pūrṇéna sicyate /  
utó tádyá vidyāma yátas tát pariṣicyáte//*<sup>12</sup>

From the full arises the full which in turn is also sustained by the Full. The source, that is, the Full is but that Supreme Being (Brahman).

This verse seems to have had a major influence on the invocation verse of both *Íśavāsaya* and *Bṛhadāraṇyaka Upaniṣad* mentioned below. This single mantra contains the essence of both these Upanishads. Here, *pūrṇa* is an allusion to Brahman thereby describing the nature of the Ultimate Reality.

*om pūrṇamadaḥ pūrṇamidaṃ pūrṇātpūrṇamudacyate /  
pūrṇasya pūrṇamādāya pūrṇamevāvaśiṣyate //*<sup>13</sup>

Om, THAT is the whole (*pūrṇa*), THIS is also the whole (*pūrṇa*). Here, *adaḥ* or ‘That’ means something remote which cannot be perceived through the senses but intuition, i.e., the Ultimate Reality; Whereas *idaṃ* or ‘This’ refers to something perceptible by the senses and intellect, i.e., the phenomenal reality. Both are whole in themselves but are not different from each other. From that wholeness of Ultimate Reality emerges this wholeness of Phenomenal Reality but as an appearance. They don’t have any causal relation. Thus, taking away wholeness from wholeness, i.e., taking away the phenomenal reality from the Ultimate Reality (or vice-versa), wholeness (*pūrṇa*) still remains. For a moment when we take away the whole phenomenal reality all there is left is *śūnya* or nothing. This is one way to interpret this. Advaita maintains that Existence is the only ultimate as it is real in all three dimensions of time; so, all that is left is Existence in its wholeness (*pūrṇa*) without names and forms. Thus, everything is *pūrṇa*. When the phenomenal world arises out of or dissolves into the Ultimate, there is no change in the quantity per se. This verse employs higher mathematics which is accessible to our intuition (Osho 1980). When something

<sup>12</sup> Atharvaveda, x.8.29, See Sharma 2005.

<sup>13</sup> *Íśavāsaya*, *Bṛhadāraṇyaka Upaniṣad*, *The Peace Chant*



is taken away from something, it is obvious that the remainder is less than the original quantity. But here no matter which operation is performed, the net result is *pūrṇa*. This is comparable to the law of conservation of energy wherein energy can neither be created nor be destroyed but can be converted from one form to another. So, it is constant regardless of where and how it manifests.

*Pūrṇa* is popularly translated into infinity, for it is plausible to conceive the whole as an infinite. This paper asserts that infinity does not stand for Brahman but rather the power or infinite manifestations of *māyā* (and *maya* is the power of Brahman itself). Infinity and zero are the two aspects of the same Underlying Reality. Infinity is a huge number that doesn't end or rather we don't know if it will end. Just because something is endless doesn't imply it is complete. It just means more constructions which take us far away from the Truth. Similarly, through elementary mathematics we see that adding infinity to infinity does give infinity; taking away infinity from infinity, infinity may not remain. It is rather indeterminate since infinity is not a fixed number but only a concept. Furthermore, there can be different sizes of various infinite sets. Thus, infinity subtracted from infinity will not necessarily yield infinity but most likely a finite number, depending on the circumstances. Meanwhile, 0 has already been given the status of a numeral etc., along with a concept. It is further said that 'Brahm' means expansion. Modern science also claims that the universe is expanding. Even if we perform any operation on zero for an infinite number of times, the net result will always be a zero. Advaita also says that creation, preservation, and destruction of this universe is simply a play (*līlā*). This can most suitably be represented by the numeral 0 instead of infinity, as there is no purpose or meaning in 0 yet it gives value to anything it is attached to. Thus, Brahman or *pūrṇa* or even *śūnya* is most suitably represented by zero.

Over and above that, on substituting *pūrṇa* with *śūnya* or even 0 in this verse, the meaning remains unchanged, viz,

*om śūnyamadaḥ śūnyamidam śūnyāśūnyamudacyate /  
śūnyasya śūnyamādāya śūnyamevāśiṣyate //*<sup>14</sup>

THAT is the nothing (*śūnya*), THIS is also the nothing (*śūnya*). With respect to Mādhyamika's two truths, 'That' is the *Paramārtha* while 'This' is *Samvṛti*. From that nothingness of *paramārtha* emerges this nothingness of *samvṛti*. Both are *śūnya* from different perspectives. Adding nothing to nothing gives nothing; Taking nothing from nothing, nothing (*śūnya*) still remains. This is exactly what the statement *śūnya* is *śūnya* of *śūnya*, or emptiness is empty of emptiness also means.

Similarly in the case of mathematical zero, That is 0, This is 0. On either addition or subtraction of zero from zero, zero remains.

Thus, Brahman or non-dual consciousness can be referred to by either 0, *śūnya* or *pūrṇa*.

<sup>14</sup> This verse is obtained by simply substituting *śūnya* with *pūrṇa* in the invocation verse, *pūrṇamadaḥ pūrṇamidam...etc.*



$$a = b \quad (1)$$

Multiply 'a' on both sides,

$$a^2 = ab \quad (2)$$

Subtract 'b<sup>2</sup>' from both sides

$$a^2 - b^2 = ab - b^2 \quad (3)$$

$$(a + b)(a - b) = b(a - b) \quad (4)$$

Divide by (a - b) from both sides

$$a + b = b \quad (5)$$

Since a = b,

$$2b = b \quad (6)$$

Therefore, 2 = 1

Kindly note that in the fourth step where we divide by (a - b) from both sides, the operations performed henceforth are *not valid in mathematics* since (a - b) is 0 (since the initial assumption was a = b). Therefore, from a mathematical point of view the proof can no longer proceed since division by 0 is an undefined/indeterminate act (discussed above in the three cases of division). As such, the conclusion that 1 = 2 is invalid in mathematics. However, for the sake of establishing the philosophical parallels of zero, *śūnya* and *pūrna* presented in this paper, we hypothetically consider with respect to the invocation verse that division by zero is indeed possible thereby proving 1=2. Now if we further subtract 1 from both sides [1 - 1 = 2 - 1] we get 0 = 1; and if we add 1 to both sides [1 + 1 = 2 + 1] we have 2 = 3. Similarly, by extension we will have 3 = 4 = 5 = 6 and so on. Thus, for any number x, we can say that x = 0.<sup>15</sup>

This shows that all numbers are equal and hence the resulting quotient will be zero without resulting in indeterminacy. I shall call this resulting quotient, i.e., 0, a 'philosophical 0' which incorporates the profound implications of Higher Truth of non-duality within it.<sup>16</sup>

Therefore, we have a new philosophical solution to the old problem of division of zero by zero altogether, that is;

*zero divided by zero = non-dual consciousness*

From a mathematical point of view, the above solution may come across as absurd due to two reasons - division by zero is not possible due to its contradictory nature and here since we assume that a = b, dividing by (a - b) takes the form of 0/0; and secondly, each number obviously has different magnitude so equating them (0 = 1 = 2....) is an impossibility. The mathematicians have withdrawn from this absurdity by rendering it undefined/indeterminate and feel there's no need to revisit it. Even from a philosophical

---

<sup>15</sup> Using the proof of 1 = 2, I have proved that 0 = 1 and 1 = 2 = 3 = 4.....etc. So, this means that all numbers are equivalent to 0. Now when we try to solve 0/0, regardless of the number we take as the quotient, the result will always be 0 (since any number multiplied by zero gives 0). And since all numbers are equivalent to 0, the result of 0/0 will be 0.

<sup>16</sup> Just as there are apparent distinctions in the empirical world which dissolve on realization of the non-dual reality, the same happens in the case of numbers after realizing the profound power of 0.

point of view, it is indeed an indisputable fact even with regards to the scope of this paper that the numbers (a, b, c...) have different “apparent” magnitudes in the phenomenal reality. However, it is not contradictory to say that these numbers are indeed equal in Absolute Reality as everything is non-dual.<sup>17</sup> The fact that all beings are different only in name, form and function in this empirical reality, but only non-duality prevails in the Absolute Reality - has already been established by Mādhyamika Buddhism as well as Advaita Vedānta in the above sections. This conclusion is further consistent with the statement mentioned in the beginning of the paper that the Truth, i.e., non-duality, is real at any time and place. Moreover, all numbers are relative to the absolute, i.e. to ‘0’ in this case and get their meaning from this relativity. Further, zero is also the only numeral that can change the value of any numeral depending upon where it is placed, a concept essentially given by the positional notation system discussed earlier. This is why the knower of non-dual Reality is able to transcend this apparent absurdity without getting tangled in contradictions as they apprehend the totality of the universe using *prajñā* (intuition). It is therefore established that zero can be substituted in the place of *pūrṇa* in the invocation verse.

## 6. CONCLUSION

This paper takes three different approaches which lead to non-duality. All existence (and non-existence) derives its meaning from the undifferentiated whole, i.e. 0/*sūnya/pūrṇa*. The Absolute is equivalent to 0 or *sūnya* or *pūrṇa* which are themselves not any entity per se. Instead of saying non-duality is “one undivided by a second”, it is best represented by ‘zero divided by zero’. Mathematics, like philosophy, may have the potential to delve into the Absolute Realm if one is aware of the subtleties occurring in the discipline. It is also important to note that Truth cannot and should not be numbered. In saying that the Truth is One, we already presuppose the other, i.e. a non-Truth, and this just leads to haphazard intellectualization. It is best represented by facets of zero: *sūnya & pūrṇa*.

Silence is the best medium that can approach the Truth since the latter cannot be verbalized to the extent that Truth has to be “realized” intuitively rather than being “understood” intellectually. But for that realization to dawn intuitively, one needs to transcend the intellect and this is precisely what various philosophical schools and disciplines give in their own ways & means of approaching as well as referring to that Truth. It must be understood in this context that the moment we try to capture the flow of a river, we render it senseless and dead. Therefore, the better approach is to appreciate the flow by becoming the flow rather than modifying or interfering with the flow. This can only be achieved by a series of reflection and contemplation on one’s Self using own experience as guiding light. In this background, one can comprehend

---

<sup>17</sup> This conclusion is a pure philosophical solution to the problem of division by zero and in no means tries to undermine the ability of mathematics. Philosophy is probably the only discipline which has the power to speculate as well as inquire into the Absolute to some extent. Hence, this paper is moving beyond the mathematical considerations and establishing this point for purely reflective/introspective purposes.

why division by zero, Buddha's silence on the *avyākṛta* (14 metaphysical questions), and the definition of Brahman are all indescribable and undefined. The mind and intellect dysfunctions at such junctures and we conveniently render the indescribable as being senseless, notwithstanding the fact that there is tremendous sense involved therein.

Lastly, the analogy of the *Nyagrodha* (banyan) tree in the *Chāndogya Upaniṣad* (6.12)<sup>18</sup> finds its relevance here. In order to show how the world, full of names & forms, emerges out of an undifferentiated whole, Sage Uddālaka asks his son Śvetaketu to bring a fruit of Nyagrodha tree and break it until it is no longer breakable. After breaking the tiny seed, Śvetaketu perceives nothing, i.e., *śūnya*. Yet this *śūnya* is *pūrṇa* since a full grown Nyagrodha tree emerges out of that very nothingness which is brimming with fullness. As mentioned earlier, the invocation verse of *Iśa* and *Bṛhadāraṇyaka Upaniṣads* state that when the phenomenal reality is taken away, all that remains is *pūrṇa* without any names and forms. Similarly, Nāgārjuna states that *śūnya* is the negation of *catuṣkoṭi*, the 4 categories of intellect - is, is not, both is and is not, neither is nor is not; it is *advaya*, "not-two". Likewise, the application of Advaitin's *neti-neti* to all descriptions of Brahman shows that Brahman is above all categorization of the intellect as well; it is *advaita*, "not-two". The mathematical zero too cannot be grasped by the intellect and it obviously is "not-two". Thus, non-duality is established as the merging point of zero, *śūnya* and *pūrṇa*.

#### ACKNOWLEDGEMENTS

I wish to thank various people who have made this research possible. This article has been extensively built upon an essay that I had written for my Master's essay course. I would like to thank my father for constantly encouraging and helping me by re-reading the article numerous times and providing helpful suggestions despite his non-philosophical academic background. I am also grateful to the anonymous reviewers of the journal *Comparative Philosophy* for taking out their valuable time to offer constructive criticism and insightful feedback to improve the quality of this article. I would like to extend my appreciation to my family and friends who supported me throughout this endeavor. Lastly, I must also candidly admit that the recent transcendence of my Labrador 'Goldie' into the non-dual consciousness made me introspect upon numerous issues regarding life & death, and the same has somehow an indelible imprint in this article.

#### REFERECNCES

---

<sup>18</sup> See, *Chāndogya-Upaniṣad with Śaṅkara-bhāṣya*, (2001), chapter 6, section 12.

- Barukčić, J.P., & Barukčić, I. (2016), “Anti Aristotle—The Division of Zero by Zero”, *Journal of Applied Mathematics and Physics* 4: 749-761. <<http://dx.doi.org/10.4236/jamp.2016.44085>>
- Boyer, C. B. (1943), “An Early Reference to Division by Zero”, *The American Mathematical Monthly* 50.8: 487–491. <<https://doi.org/10.2307/2304187>>
- Boyer, C. B. (1944), “Zero: The Symbol, the Concept, the Number”, *National Mathematics Magazine* 18.8: 323–330. <<https://doi.org/10.2307/3030083>>
- Chāndogya-Upaniṣad with Śaṅkara-bhāṣya*, (2001), Chapter 6, in Som Raj Gupta (trans.), *The Word Speaks to the Faustian Man, Vol. 4* (Delhi: Motilal Banarsidass).
- Datta, B. (1927), “Early history of the Arithmetic of Zero and Infinity in India”, *Bulletin of the Calcutta Mathematical Society* 18: 165-176.
- Datta, B., & Singh, A. N. (1962), *History of Hindu Mathematics, A Source Book Parts I and II* (Bombay: Asia Publishing House).
- Gironi, F. (2012), “Śūnyatā and the Zeroing of Being: A reworking of empty concepts”, *Journal of Indian Philosophy and Religion* 15: 1-42.
- Gopinath, Kaviraj (2005), “On Pūrṇa” (H. N. Chakravarti, trans), in B. Bäumer & J. R. Dupuche (eds.), *Void and Fullness in the Buddhist, Hindu, and Christian Traditions: Sunya-Pūrṇa-Pleroma* (D.K. Printworld), 239-242.
- Grandy, D. (2016), “Sunyata in the West”, *Comparative Philosophy* 7.1: 35-58. <[https://doi.org/10.31979/2151-6014\(2016\).070106](https://doi.org/10.31979/2151-6014(2016).070106)>
- Joseph, G. (2008), “A Brief History of Zero”, *Tārīkh-e-‘Elm: Iranian Journal for the History of Science* 6: 37-48.
- Kalupahana, D. J. (1986), *Mūlamādhyamakakārikā of Nagarjuna: The Philosophy of the Middle Way* (1st ed.) (Motilal Banarsidass Publishers).
- Knifong, J. D., & Burton, G. M. (1980), “Intuitive Definitions for Division with Zero”, *The Mathematics Teacher* 73.3: 179–186. <<http://www.jstor.org/stable/27961941>>
- Logan, R. K. (1979), “The Mystery of the Discovery of Zero”, *A Review of General Semantics* 36.1: 16–28. <<http://www.jstor.org/stable/42575888>>
- Motilal, B. K. (1987), “Mādhyamika”, *Annals of the Bhandarkar Oriental Research Institute* 68.1/4: 215–224. <<http://www.jstor.org/stable/41693320>>
- Murti, T. R. (1955), *The Central Philosophy of Buddhism: A Study of the Mādhyamika System* (London: George Allen and Unwin).
- Osho, R. (1980), *I Am That: Talks on The Isha Upanishad*, English Discourse Series, 2-18.
- Pappu, S. (2019), “Śūnyā and Pūrṇa”, in S. R. Bhatt (ed.), *Quantum Reality and Theory of Śūnyā* (Springer), 39-45. <<https://doi.org/10.1007/978-981-13-1957-0>>
- Raju, P. T. (1954), “The Principle of Four-Cornered Negation in Indian Philosophy”, *The Review of Metaphysics* 7.4: 694–713. <<http://www.jstor.org/stable/20123413>>
- Rinpoche, S. (2005), “Śūnyā and Pūrṇa”, in B. Bäumer & J. R. Dupuche (eds.), *Void and Fullness in the Buddhist, Hindu, and Christian Traditions: Sunya-Pūrṇa-Pleroma* (D.K. Printworld), 19-31.
- Saito, A. (2019), “Prapañca in the Mūlamādhyamakakārikā”, *Bulletin of the International Institute for Buddhist Studies* 2: 1-9. <[https://researchmap.jp/read0012109/published\\_papers/29362895/attachment\\_file.pdf](https://researchmap.jp/read0012109/published_papers/29362895/attachment_file.pdf)>

- Sharma, C. (1987), *A Critical Survey of Indian Philosophy* (15th ed.) (Motilal Banarsidass).
- Sharma, C. (1996), *The Advaita Tradition in Indian Philosophy: A Study of Advaita in Buddhism, Vedānta and Kāshmīra Shaivism* (Motilal Banarsidass).
- Sharma, D. B. S. (2005), “Concept of Fullness or Pūrṇatva in Indian Philosophy”, in B. Bäumer & J. R. Dupuche (eds.), *Void and Fullness in the Buddhist, Hindu, and Christian Traditions: Sunya-Pūrṇa-Pleroma* (D.K. Printworld), 33-48.