1993

An overhead projector designed for side projection

Sujata V. Rao
San Jose State University

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An overhead projector designed for side projection

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San Jose State University, 1993

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**AN OVERHEAD PROJECTOR
DESIGNED FOR
SIDE PROJECTION**

A Thesis

Presented to

The Faculty of the Department of Physics

San Jose State University

In Partial Fulfillment

of the Requirements for the Degree of

Master of Science

By

Sujata V. Rao

May, 1993

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ABSTRACT

AN OVERHEAD PROJECTOR DESIGNED FOR SIDE PROJECTION

By

Sujata V. Rao

This thesis describes a new design that will replace the existing design for an overhead projector (OHP). The intent of this newly designed projector is to reduce distortion and improve focusing. With this new design one can place the OHP at the side of a classroom eliminating any obstruction caused in viewing .

All analyses and suggestions have been made on the basis of experimental observation. The new design was built using readily available optical elements. It was found that interpretation for distortion could not be made using the aberration curves of GENII-PLUS software. This was mainly due to the inability of the design software to handle tilted object and image planes.

To my father

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TABLE OF CONTENTS

CHAPTER 1	
Introduction	1
 CHAPTER 2	
Aberration Theory	
2.1 First order or Gaussian optics.....	3
2.1.1 Cardinal points, image position and size of an optical system.....	4
2.2 Aberration theory	7
2.2.1 The aberration polynomial	8
2.2.2 Spherical aberration.....	10
2.2.3 Coma	11
2.2.4 Astigmatism and field curvatures.....	13
2.2.5 Distortion.....	15
 CHAPTER 3	
Ray Tracing	
3.1 Ray tracing through the paraxial region.....	17
3.1.1 Paraxial ray tracing through several surfaces.....	18
3.2 Stops and apertures.....	21
3.3 Third order Seidel contributions	22
 CHAPTER 4	
Analysis of Existing Design	
4.1 Existing design	26
4.2 Scheimpflug condition	28
4.3 Using GenII-Plus software to verify the existing design	30
4.4 Verification of the radii of curvatures of the existing lens using a spherometer	31
4.5 Distortion correction	32

CHAPTER 5

The Improved Design

5.1 Experiment 1: Tilting the object plane with the existing design.....	34
5.2 Study of the effect of tilt of conjugate planes on distortion.....	34
5.3 Experiment 2: Tilting the conjugate planes and lenses to satisfy Scheimpflug condition.....	38
5.3.1 Description of the lenses used.....	38
5.3.2 Set-up description.....	39
5.3.3 Observation	40
5.3.3.1 Effect of tilt on the magnification produced by the field lens	41
5.3.3.2 Analysis for distortion produced by the entire system.....	43
5.3.3.3 Sample calculations.....	46

CHAPTER 6

Conclusion.....	51
------------------------	-----------

CHAPTER 7

Recommendations	53
------------------------------	-----------

BIBLIOGRAPHY	55
---------------------------	-----------

APPENDIX A

BASIC program that calculates the total contributions of the third order aberrations.....	56
--	----

APPENDIX B

Data of existing design from GenII Plus software	60
--	----

APPENDIX C

Proof of the Scheimpflug condition	61
--	----

CHAPTER 1

INTRODUCTION

In past years, the use of overhead projectors or vuegraphs in classrooms or lecture halls has proved to be very useful. This ubiquitous instrument is fairly simple in design. It consists of a light source imaged by a Fresnel condenser lens onto a projection lens. The projection lens images the object or transparency that is placed on the Fresnel lens onto the screen⁴⁻¹³. This instrument is generally placed in the front or center of a classroom. The speaker may stand beside the projector while giving his or her talk. There are two disadvantages as far as viewing is concerned.

Firstly, the speaker may obstruct the view of the audience and secondly, the instrument itself may cause obstruction in viewing. The second problem may be partially resolved by tilting the mirror present in a way so as to raise the image on the screen, thereby increasing visibility for the audience. But the drawback to this is that it produces a distorted or keystone image. This image shape has so far been accepted as inevitable by audiences. It is also seen that focusing is not very good. No projector available commercially has been designed to solve the problem to the best of our knowledge.

The purpose of this thesis is to eliminate all the above mentioned problems. We have been successful in attempting to produce this desired image. The chapters in this thesis include the following:

Chapter 2 covers the fundamentals of optical design. It also gives a detailed description of

the aberration theory.

In Chapter 3, a detailed description of ray tracing and aberration contribution are covered. This is required to test the performance of a newly designed system.

Chapter 4 covers a detailed description of the existing problems and an analysis of the present design of a projector.

Chapter 5 covers the description of the set up used in the new design.

Chapter 6 includes a summary that highlights the requirements of this newly built design to produce the image desired.

Chapter 7 covers recommendations for future research that may be pursued, the starting point of which is this thesis.

CHAPTER 2

ABERRATION THEORY

The use of optical systems has required that optical engineers design optical components to meet specific requirements. This task of designing an optical system is accomplished using a lens program. In the process of designing a lens system, the quality of the system is described by a number called the 'error function'. This function¹⁴ basically sums up all the defects. To obtain the best optical design meeting all the required needs, a designer would look for a design where this error function is reduced to its lowest possible level.

In this chapter, we shall briefly review the fundamentals of optics that are used for designing lenses. The following sections includes first order optics, paraxial optics, and aberration theory.

2.1 First order or Gaussian optics

First order or Gaussian optics is often referred to as the optics of a perfect imaging optical system. This applies to trigonometric expressions for which the ray heights and angles involved approach zero and are most valid for an infinitesimal region about the optical axis. This region is known as the 'Paraxial region'. These first order expressions provide a good reference for any aberrated or imperfect system. A well corrected system would closely match the first order expressions (and any deviation in the image size and location can be realized).

2.1.1 Cardinal points, image position and size of an optical system

Any optical system that is well corrected may be defined by its focal points, principal points and nodal points, all collectively called the 'Cardinal Points' (refer to Figure 2.1).

Figure 2.1: Diagrams indicating the focal points, principal points and nodal points of an optical system. 1 & 2 are the first and second principal points, 5 & 6 are the Nodal points and 3 & 4 are the first and second focal points.

The focal points are points at which rays that are parallel to the optical axis are brought to a point on the axis after passing through the lens system. If the incident ray on the system

and the emergent ray are extrapolated forward and backwards respectively so as to meet each other, the plane of intersection is defined as the 'Principal plane' and the point at which this plane intersects the optic axis is defined as the Principal point. Rays approaching from the left side of the system define the second focal and principal points and those from the right define the first focal and principal points. The effective focal length (efl) of a system is the distance from the principal to the focal points. The back focal length (bfl) is the distance from the vertex of the last surface of the system to the second focal point. The front focal length (ffl) is the distance from the front surface to the first focal point. The nodal points are axial points for which a ray directed towards the first nodal point appears to be coming from the second nodal point parallel to its original direction.

Once the cardinal points of an optical system are known, the location and size of the image formed can be determined. As a rule, the sign convention followed is as follows:

1. Heights above the optic axis are considered positive and the heights below are considered negative.
2. Distances to the left of a reference point (which could be a principal point) are negative.
3. Converging lens have positive focal lengths and diverging lens have negative focal lengths.

The image position and size may be determined as follows:

Figure 2.2: Diagram indicating the location, position and size of the image.

Let h and h' be the object and image heights; f , and f' be the focal lengths on the object and image sides; s , and s' correspond to the object and image distances; x , and x' are the object and image distances from the focal point respectively (refer to Figure 2.2).

From similar triangles,

$$\frac{h}{-h'} = \frac{-x}{f} \quad (2.1)$$

and

$$\frac{h}{-h'} = \frac{f'}{x'} \quad (2.2)$$

It can be shown that¹

$$s' = \frac{sf}{s + f} \quad (2.3)$$

where s' gives the position of the image. Using equations 2.1 and 2.2, the lateral or transverse magnification of an optical system is given by

$$m = \frac{h'}{h} = \frac{f}{x} = \frac{-x'}{f} \quad (2.4)$$

The longitudinal magnification along the optical axis for small thickness is given ¹

$$\overline{m} = m^2 \quad (2.5)$$

2.2 Aberration Theory

As mentioned in the previous section, a well corrected optical system closely follows paraxial optics. In this section the general behavior of lenses is considered. This would include rays that are not considered to be in the paraxial or first order region. All aberrations are measured by the amount by which rays miss the paraxial image point.

The German Mathematician, Ludwig Von Seidel, studied and classified aberrations now called the Seidel aberrations. This classification and understanding has greatly simplified the work²⁻³ of determining the aberrations of a lens system. For monochromatic light, there are five Seidel aberrations namely, spherical, coma, astigmatism, curvature of field and distortion. An additional aberration results from the wavelength dependence of the imaging properties of an optical system. This is known as the chromatic aberration.

2.2.1 The aberration polynomial

Assume an optical system that is symmetric about the optic axis (refer to Figure 2.3). Let an object point along the y axis be at a distance h from the optic axis. The polar coordinates s , and θ define the position of the ray passing through the aperture of the optical system. Since it is impractical to derive an exact expression for the image coordinates y' and z' as a function of h , s and θ , a general form is considered assuming an axially symmetrical system. For instance, a ray passing through an axial point in object space must pass through the same axial point in the image space after passing through the same annular zone (or same value of s) in the aperture.

Figure 2.3: A ray of height h in the object plane passes through the aperture of the optical system defined by the polar coordinates (s, θ) and intersects the image surface at $y'z'$.

Using this axially symmetric system, y' is defined as

$$y' = \begin{aligned} & A_1 s \cos \theta + A_2 h \\ & + B_1 s^3 \cos \theta + B_2 s^2 h (2 + \cos 2\theta) + (3B_3 + B_4) s h^2 \cos \theta + B_5 h^3 \\ & + C_1 s^5 \cos \theta + (C_2 + C_3 \cos 2\theta) s^4 h + (C_4 + C_6 \cos 2\theta) s^3 h^2 \cos \theta \\ & + (C_7 + C_8 \cos 2\theta) s^2 h^3 + C_{10} s h^4 \cos \theta + C_{12} h^5 + D_1 s^7 \cos \theta + \dots \end{aligned} \quad (2.6)$$

and z' is defined as

$$z' = \begin{aligned} & A_1 s \sin \theta \\ & + B_1 s^3 \sin \theta + B_2 s^2 h \sin 2\theta + (B_3 + B_4) s h^2 \sin \theta \\ & + C_1 s^5 \sin \theta + C_3 s^4 h \sin 2\theta + (C_5 + C_6 \cos 2\theta) s^3 h^2 \sin \theta \\ & + C_9 s^2 h^3 \sin 2\theta + C_{11} s h^4 \sin \theta + D_1 s^7 \sin \theta + \dots \end{aligned} \quad (2.7)$$

A_n , B_n , C_n and D_n are constants. The exponents of s and h are unity for the A terms.

These are referred to as the first order terms. The exponents of the B terms add up to three, these are called the third order terms. The exponents of C add up to five, these are known as the fifth order terms and so on. In any axially symmetric system only the odd terms are present.

The A_1 term is a measure of the distance from the paraxial focus to the image plane. In equation 2.6, A_2 is the magnification given by h'/h . The remaining terms, apart from the first order terms, are called 'transverse aberrations'.

The third order terms or the B terms are referred to as Seidel or primary aberrations. The B_1 term is spherical aberration, B_2 is coma, B_3 is astigmatism, B_4 is petzval, and B_5 is distortion.

The C terms are called the fifth order or secondary aberrations, where C_1 is the fifth order spherical aberration, C_2 and C_3 are linear coma, C_4 , C_5 and C_6 are oblique spherical aberration, C_7 , C_8 and C_9 are elliptical coma, C_{10} and C_{11} are Petzval and astigmatism and C_{12} is distortion. The D terms correspond to the seventh order aberrations. A brief description of each of the Seidel aberrations are given in the following sections.

2.2.2 Spherical aberration

Spherical aberration arises from the $B_1 s^3 \cos \theta$ term. This is the only aberration that does not depend on h . This indicates that it exists for axial object and image points. It is the only on-axis aberration. The variation of the focus with aperture defines spherical aberration. It is found that as the aperture size of the lens increases, the ray intercept separations BI and AI increases (refer to Figure 2.4). The distance BI is referred to as the 'longitudinal spherical aberration' and AI is referred to as the 'transverse spherical aberration'. Both these separations depend on the object distance.

Let L' and l' be the respective ray intercepts for the trigonometric and paraxial ray from the same axial object point. It is to be noted that in the case of trigonometric rays, the angles made by the rays with respect to the optical axis is not as small as that of the paraxial rays. The longitudinal spherical aberration of the image point LA is given by

$$LA = L' - l' \quad (2.8)$$

The transverse spherical aberration is given by

$$TA = - LA \tan U'_R \quad (2.9)$$

where U'_R is the angle the ray makes with the axis. From the sign convention spherical aberration with negative sign is called undercorrected spherical aberration and spherical aberration with positive sign is called overcorrected spherical aberration. The former is associated with converging elements and the latter with diverging elements.

Figure 2.4: Illustration of a lens having undercorrected spherical aberration. Rays farther from the axis are brought to a focus closer to the lens.

2.2.3 Coma

Coma is an off axis aberration that arises due to the variation of magnification with aperture. This aberration increases rapidly with aperture. The rays farther from the axis intersect the image plane at the same point whereas the chief ray intersects the image plane at a different point. If the magnification for the outer rays through a lens is greater

than that of the central ray, the coma is said to be positive; otherwise it is negative. The distance from P' to the intersection of the edge rays is called the 'Tangential Coma' (refer to Figure 2.5).

Figure 2.5: Illustration of a lens having coma. The marginal rays focus at a different height than the central ray.

Coma is given by

$$\text{Coma}_T = H_{a'} - H_{p'} \quad (2.10)$$

Coma varies with the shape of the lens element¹ and the position of the apertures that obstruct the rays forming an image. This off axis aberration varies directly with its distance from the optic axis. It is to be noted that for symmetrical systems, coma, lateral color and distortion are zero.

2.2.4 Astigmatism and field curvatures

Before defining astigmatism, the definition of tangential or meridional rays, sagittal and skew rays should be understood. Meridional or tangential rays are rays that are coplanar with the optical axis. The plane in which both ray and axis lie is called the meridional plane. Rays that do not lie on the meridional plane are defined as skew rays. The skew rays that lie in a plane passing through the principal ray and perpendicular to the meridional plane are defined as sagittal rays, the plane is referred to as sagittal planes. Astigmatism occurs when the tangential and sagittal images do not coincide (refer to Figure 2.6). For a poorly made lens where astigmatism exists, a point source is not imaged as a point but forms two separate lines. Astigmatism increases as the ray moves away from the axis. It depends on the shape of the lens as well as the aperture. In the case of a single lens (where the aperture is at the surface of the lens) astigmatism is equal to the square of image height divided by the focal length, *i.e.*, $\text{astigmatism} = h'^2/f$

Figure 2.6: Illustration of a lens having astigmatism.

Field curvature: This is the longitudinal distance of the focal surfaces from the ideal flat surfaces. The basic field curvature present in any optical system is defined as the Petzval curvature. Petzval curvature depends on the refractive index of the lens and the radii of curvature.

$$\text{Petzval curvature} = \left(\frac{h'}{2}\right)^2 \frac{1}{nf} \quad (2.11)$$

where n and f are the refractive index and focal length of the element. In the absence of astigmatism, the tangential and sagittal surfaces coincide with the Petzval surface. But in the presence of primary astigmatism, the tangential image surface lies three times as far from the Petzval curvature as the sagittal surface. If the tangential surface is to the left of the sagittal, and these two surfaces are to the left of the Petzval surface, the astigmatism is said to be undercorrected or negative (refer to Figure 2.7).

Figure 2.7 Illustration of the sagittal, tangential and Petzval surfaces.

2.2.5 Distortion

A non-uniform lateral magnification of a system over its entire field gives rise to distortion. It may also be defined as the amount of displacement of an image from the paraxial image point. There are two types of distortion. The first is known as the 'barrel distortion' which arises due to decrease in magnification towards the edge of a field. The second is known as 'pincushion distortion' which is due to the increase in magnification towards the edge of the field. The position of the stop influences the distortion. If a stop is placed in front of the lens, the distortion produced is of barrel type. For a stop placed behind the lens, the distortion produced is pin cushion. It is found that for a symmetric doublet with a stop placed at the center, distortion is negligible. Thus using a combination of lenses, the distortion effects produced may be cancelled. It should be noted that the shape of the distorted image produced by an overhead projector differs from the two standard forms.

Usually, distortion is defined by the difference in ray height between a trigonometric ray and the paraxial ray height at the image plane, *i. e.*,

$$\text{Distortion} = H'_p - h'$$

where H'_p and h' are the trigonometric and paraxial ray heights. The percent distortion is given by

$$\text{Percent distortion} = \frac{H'_p - h'}{h'} 100 \quad (2.12)$$

In this thesis, distortion is redefined in terms of the magnification produced for paraxial rays, the definition of which is:

$$\text{Percent distortion} = \frac{m' - m}{m} 100 \quad (2.13)$$

m' is the magnification produced for an off-axis paraxial ray that includes distortion and m is the magnification produced along the axis with no distortion. Equation 2.13 is obtained from equation 2.12 by using

$$H_p' = \frac{HP'}{P}$$

$$h' = \frac{Hs'}{s}$$

where H is the object distance, P' and P are the object and image distances for off-axis rays, and s' and s are the object and image distances along the axis. The total contribution produced by a lens system is found by multiplying the contribution produced due to each lens element. For large image and object distances, the tilt of a lens does not produce any significant effect on distortion. But for small distances, distortion is altered. The effect of distortion with the tilt of the conjugate planes is studied in Section 5.3 of Chapter 5.

CHAPTER 3

RAY TRACING

To study the performance of a designed optical system, a bundle of rays is traced through the system starting from the object plane, passing through the system, and finally meeting the image plane. The position of the rays at each intermediate surface of the system and at its final plane may be calculated. Based on these values, the overall performance of the system is enhanced to meet the specific needs of the designed system. Further, the contribution of each of the aberrations considered in the previous chapter may be estimated.

This chapter first describes ray tracing through the paraxial region. The transfer equations through several surfaces are described. Finally, the third order aberration contributions are discussed.

3.1 Ray tracing through the paraxial region

The paraxial relations of an optical system determines the general performance of the system. The simplicity of these equations enables one to make calculations easily. Often it is found that paraxial relations can be extended for large angles which make them useful for estimating the quality of images formed and the required diameters of the optical elements.

These paraxial equations thus serve as good references for aberration determination that may be done on the basis of how far a trigonometrically computed ray departs from its ideal location.

3.1.1 Paraxial ray tracing through several surfaces

Consider a paraxial ray incident on a surface at a height y from the axis. Let l and l' be the distances before and after refraction. u and u' are the angles made before and after refraction with the optical axis. Let N and N' be the refractive indices of the medium before and after refraction. Using the paraxial approximation, $u = -y/l$ and $u' = -y/l'$ (refer to Figure 3.1).

Figure 3.1: Illustration of the relationship of y , l , u , l' , and u' for paraxial rays.

Substituting for l and l' in

$$\frac{N'}{l'} - \frac{N}{l} = \frac{N' - N}{R} \quad (3.1)$$

and multiplying by y , it can be shown that

$$N' u' = Nu - y(N' - N) c \quad (3.2)$$

where c is the curvature of the surface that can be expressed as the reciprocal of its radius, *i. e.*, $c=1/R$. This angle u' becomes the input angle for the next surface. To find the ray height at the second surface, we need to derive a set of transfer equations. Let y_1 and y_2 be the heights of the ray at surfaces 1 and 2 respectively. Let t be the distance separating the two surfaces (refer to Figure 3.2).

Figure 3.2: Illustration of the transfer equation of a paraxial ray from surface #1 to surface #2.

From figure 3.2 it can be seen that

$$y_2 = y_1 + tu'_1 \quad (3.3)$$

A positive sign is used in equation 3.2 since the angle at the second surface is positive.

We can also write

$$y_2 = y_1 + \frac{t N_1' u_1}{N_1'} \quad (3.4)$$

Since the slope of the incident ray on surface #2 is the same as that of surface #1, the second transfer equation may be obtained from the equations

$$u_2 = u_1'$$

or

$$N_2 u_2 = N_1' u_1'$$

The ray height at the second surface may be found using equation 3.4. Moving from surface to surface, the refraction at the surface and the ray heights at each of the surfaces are determined. The paraxial image may be determined for a point intersecting the optic axis. At this point, the ray height is zero. The location of the image plane from the last surface is given by $l' = -y_k/u_k'$. Here y_k and u_k' correspond to the ray height and the angle at the last surface. The effective focal length (efl) and the back focal length (bfl) of a system is determined by tracing a ray parallel to the axis for which the initial slope is zero. The efl is given by the relation $efl = -y_1/u_k'$ where y_1 is the ray height at the first surface and u_k' is the final angle. The bfl = $-y_k/u_k'$.

3.2 Stops and apertures

Apertures or stops in an optical system (in paraxial approximation) define the amount of energy that can pass through a system. The amount of energy that can pass through the system from an axial point on the object is defined by the 'aperture stop', the size of which determines the illumination at the image plane. An aperture in an optical system may also limit the imaging for off axis rays from the object plane. The aperture that limits the size of the object or image is called as the 'field stop'. For complex optical arrangements, the entrance and exit pupils of the system are the images of the aperture stop in object and image space respectively. In all systems, the principal ray (or chief ray) passes through the center of the aperture stop, and is directed towards the centers of the entrance and exit pupils. The amount of radiation entering or exiting the system is determined by the size and location of the pupils. The images of the field stop in object and image space are called the entrance and exit windows respectively.

The relationship between the semi-apertures and the aberrations discussed in the previous chapters are given in Table 3.1. This table is true for rays close to the axis and is not valid for rays with significant heights. To illustrate the use of this table, let us assume that for a given lens the aberrations are known. If the aperture diameter is increased or decreased the aberrations produced by this new aperture may be determined from the table. These new aberration values can then be compared to the existing values. Thus, the table serves as a quick and useful check for aberrations while designing an optical system.

<u>Aberration</u>	<u>versus Aperture</u>
Spherical (longitudinal)	y^2
Spherical (transverse)	y^3
Coma	y^2
Petzval curvature (longitudinal)	--
Petzval curvature (transverse)	y
Astigmatism and Field curvature (transverse)	y
Distortion	--

Table 3.1: Illustrates the relationship between semi-apertures and the aberrations.

3.3 Third order Seidel contributions

The transverse aberration of a general ray with respect to a reference ray can be expressed in terms of powers of image height and system aperture. The first order aberrations are defects of focus or image size which vary linearly with aperture. The third order terms correspond to primary aberrations. Here, the term s^3 correspond to spherical aberration indicating that it is independent of height h . The term s^2h corresponds to coma, sh^2 corresponds to astigmatism and field curvature, and h^3 to distortion. The total contribution due to these terms are the third order aberrations. The terms contributing to the fifth order contribution will be s^5 , s^4h , s^3h^2 , s^2h^3 , sh^4 and h^5 . The contribution due to these aberrations decreases rapidly as the powers increase. It is found that the performance of a system may be based on paraxial and third order contributions, as the higher orders change very slowly with the design parameters. The third order contributions can be obtained from an axial and principal ray. Let y , u , i be the ray

height, exit angle and incident angle of the paraxial ray and y_p , u_p , i_p be that for the principal ray. The optical invariant¹ can then be defined as $I = y_p N u - y N u_p = h N'_k u'_k$. h , the final image height, can be written as $h = I / N'_k u'_k$ where N'_k and u'_k are the final index of refraction of the medium and the final angle after refraction, respectively. Using the optical invariant, the ray heights and angles at the object and image planes can be determined without tracing the rays through the intermediate surfaces. For each surface of the system, the following equations may be defined:

$$i = cy + u \quad (3.5)$$

where i is the paraxial angle of incidence

For the principal ray

$$i_p = cy_p + u_p \quad (3.6)$$

we define

$$B = \frac{N(N'-N)}{2N'I} y(u'+i) \quad (3.7)$$

and

$$B_p = \frac{N(N'-N)}{2N'I} y_p(u'_p+i_p) \quad (3.8)$$

The transverse third order spherical aberration (TSC) contribution is then given by

$$TSC = B i^2 h \quad (3.9)$$

This is independent of i_p .

The sagittal third order coma contribution (cc) is given by

$$CC = B_{ip} h \quad (3.10)$$

The tangential third order coma contribution is given by 3CC

The transverse third order astigmatism contribution (TAC) is given by

$$TAC = B_{ip}^2 h \quad (3.11)$$

The transverse third order Petzval contribution TPC is

$$TPC = - \frac{(N - N') c l h}{2 N N'} \quad (3.12)$$

The third order distortion DC is given by

$$DC = h \left[B_{ip} i_p + \frac{1}{2} (u_p'^2 - u_p^2) \right] \quad (3.13)$$

The paraxial transverse axial chromatic aberration contribution TAChC is given by

$$TAChC = \frac{-y_i}{N'_k u'_k} \left(\Delta N - \frac{N}{N'} \Delta N' \right) \quad (3.14)$$

The paraxial lateral chromatic aberration contribution TchC is

$$\text{TchC} = \frac{-y_p^i}{N'_k u'_k} (\Delta N - \frac{N}{N'} \Delta N') \quad (3.15)$$

where ΔN is the dispersion in the medium which is equal to the difference in index of refraction for short and long wavelength. $\Delta N = N_F - N_C$ or $\Delta N = \frac{(N-1)}{V}$. The longitudinal contributions can be obtained by dividing each contribution by the final angle u'_k .

$$\text{SC} = \frac{-\text{TSC}}{u'_k} \quad (3.16)$$

$$\text{AC} = \frac{-\text{TAC}}{u'_k} \quad (3.17)$$

$$\text{PC} = \frac{-\text{TPC}}{u'_k} \quad (3.18)$$

$$\text{LchC} = \frac{-\text{TAchC}}{u'_k} \quad (3.19)$$

The Seidel coefficients are obtained by multiplying the transverse contributions by $(-2N'_k u'_k)$. The total contributions of the third order aberrations may be found by adding the contributions of each surface. Therefore, ΣTSC , ΣCC , $3\Sigma\text{CC}$, ΣTAC , ΣTPC , ΣDC , ΣTAchC and ΣTchC are the final contributions of the third order for each aberration (refer to Appendix A).

CHAPTER 4

ANALYSIS OF EXISTING DESIGN

An overhead projector is normally placed in the center or front of a classroom. The image that is projected may not be seen completely by the audience, as both the speaker and the projector may obstruct their view. This problem may be partially solved by projecting the image upwards. However, the ideal choice would be to place the projector to the side of a classroom.

This chapter discusses and verifies the existing design of an overhead projector and the image produced by it. It also suggests the criteria to be met to eliminate the existing defects of the image.

4.1 Existing Design

The existing design of a projector⁸⁻¹² is fairly simple. Some projectors are designed with single meniscus lenses, others make use of two meniscus lenses normal to each other. The basic set-up of a projector consists of a source of light whose rays are directed towards a condenser or Fresnel lens. In earlier designs, a concave mirror was used in place of a Fresnel lens. The light source (tungsten iodine lamp) of the projector is imaged by the condenser lens onto a projection lens. Thus, the condensing system used, concentrates the light as much as possible into the system apertures. The object or transparency may be placed on the Fresnel lens. Light from the illuminated object then

passes through the lens system and the final image is imaged by the lens system onto a screen (refer to Figure 4.1).

Figure 4.1: Existing Design. A- Source, F- Fresnel Lens, O- Object, L- Meniscus Lens , S- Screen.

A mirror is placed above the projection lens at an angle of 45° . This is used to fold the path of the beam so as to project the image onto the screen. The whole lens arrangement is moveable on a support which enables the user to improve focusing at the image plane. With this projector placed in the center of the classroom, the projected image on the screen may be viewed (refer to Figure 4.2). The image formed is distorted, and an appreciable amount of defocusing exists. This defocusing may be due to the violation of the Scheimpflug condition discussed in the next section.

Figure 4.2: Illustration of the image produced by the overhead projector.

4.2 Scheimpflug condition

To produce good focusing with object and the image planes that are not perpendicular to the optical axis, the Scheimpflug condition¹ requires that they meet and intersect each other at the plane of the lens (refer to Figure 4.3). For the proof of this condition refer to Appendix C. If this condition is satisfied, there is no defocusing. In an existing projector, the object and the image planes are intended to lie parallel to each other.

Figure 4.3: The Scheimpflug condition. F- Fresnel lens, O- object, L- lens , S- screen.

If this projector is placed to the side of a classroom, the image formed will appear distorted (refer to Figure 4.4).

Figure 4.4: Image of the overhead projector when placed to the side of the classroom.

This distortion is very annoying because the lines slope upwards in the upper half of the image and downward in the lower half of the image, and focusing is not very sharp for the same reason mentioned earlier, namely, the violation of the Scheimpflug condition. We therefore require a new projector design that can produce a distortion-free image and improve on the focusing when placed to the side of the classroom. Such a system does not exist to the best of our knowledge. This new system designed in this thesis should be of reasonable cost as well.

4.3 Using GenII-Plus software to verify the existing design

The existing design consisting of a simple meniscus lens was evaluated using GenII-Plus. In the set-up the convex side of the meniscus lens faces the object, that is, it is towards the shorter conjugate and the concave side faces the longer conjugate or image side. The object and image distances for the best focusing were measured, and it was found that focusing was best when the object distances were between 30 to 34 cm for an image distance of 230 cm. Usually, while entering the paraxial data in GenII Plus we enter the data for the longer conjugate first, and then the shorter conjugate. This means that the object distance is treated as the image distance and vice versa. In order to optimize the program specific bundle of rays needs to be specified (refer to Appendix B). Each specific ray meets a specific point on the lens surface. For instance, the 0 0 ray is the on-axis ray, and the 1 1 ray indicates that the ray starts from the negative side of 1 of the object plane and crosses the axes to the positive side of the lens, meeting it at the edge. The 0.8 0.8 ray starts from a height of 0.8 cm at the object plane meeting the lens surface at a height of 0.8 cm from the optic axis and so on. With each of these rays acting as central rays a cone of radius 0.15 cm is chosen. This radius was chosen through rough

estimation. Once this data is entered, keeping the two surfaces as variables, the design is optimized. The design program sets the radii of curvatures for the lens surfaces to meet the required input data. The radii that were obtained are -9.5 cm towards the shorter conjugate and -25.7 cm towards the longer conjugate.

4.4 Verification of the radii of curvatures of the existing lens using a spherometer

A spherometer is a convenient instrument¹⁵ that can be used to measure the radii of curvature of a lens. A description of the way this instrument works is as follows:

Before making measurements with the spherometer, the zero curvature reading is made by keeping the spherometer on a plane surface with its micrometer screw touching the surface. The spherometer is then placed on the spherical surface and the reading is taken when the micrometer screw touches the surface. Let d be the difference in the micrometer readings between the curved and flat surfaces. If L is the distance between the centers of the legs of the tripod then

$$R = \left(\frac{d}{2} + \frac{L^2}{6d} - r \right) \quad (4.1)$$

where r is the radius of the spheres at the ends of the spherometer legs. If the ends are in point form, then $r = 0$.

Result:

Surface# 1 facing longer conjugate

Flat surface $d_1 = 0.24 \text{ mm}$

Curved surface $d_2 = 1.40 \text{ mm}$

$$d = d_1 - d_2 = 1.16 \text{ mm}$$

$$R = 260 \text{ mm or } 26 \text{ cm.}$$

Surface# 2 facing shorter conjugate

Flat surface $d_1 = 0.24 \text{ mm}$

Curved surface $d_2 = 3.50 \text{ mm}$

$$d = d_1 - d_2 = 3.26 \text{ mm}$$

$$R = 96.15 \text{ mm or } 9.6 \text{ cm.}$$

The results from Sections 4.3 and 4.4 are compared. It is found that the two results are in close correspondence, thus verifying the existing design.

4.5 Distortion correction

As mentioned in Section 4.1, a well focused, distortion free image is desired (refer to Figure 4.5). To produce such an image one makes use of the Scheimpflug condition, keeping the projector to the side of the classroom.

Figure 4.5: Illustration of an ideal undistorted image that is required.

CHAPTER 5

THE IMPROVED DESIGN

The intent of improving the existing design is to produce a distortion free image for sideways projection. There were two approaches used to achieve this goal which are discussed in the following sections. All conclusions drawn are based on experimental observations. The lenses used in these experiments can be readily purchased and are the ones available in our laboratory. By using these available lenses the cost of the improved design may be reduced.

5.1 Experiment 1: Tilting the object plane with the existing design:

The first approach to producing a distortion free image was to tilt the object or transparency alone, retaining the existing design intact. This produced an undistorted image but the focusing was not good. The focusing was found to be non-uniform on the screen. The position of the lens system was changed to improve the focusing, but this had very little effect. Therefore, a different design was needed to produce both a distortion free image as well as good focusing. In the next section, the effect of tilt of the conjugate planes on distortion is studied.

5.2 Study of the effect of tilt of conjugate planes on distortion

As indicated in Section 4.2, the object and image planes need to be tilted to satisfy the Scheimpflug condition. In this section, the effect of tilt of conjugate planes on distortion

is studied for general case and this is then applied to the elements used in the design. Let s and s' be the object and image distances. Let the object and image planes be tilted by the angles θ_1 and θ_2 , respectively. Let ϕ be the field angle (refer to Figure 5.1).

Figure 5.1: Illustration of the effect of tilt of conjugate planes on distortion.

Considering the image side,

$$y = x \tan \phi \quad (5.1)$$

and

$$y - 0 = \tan(90^\circ - \theta_2) (x - s')$$

or

$$y = \cot \theta_2 (x - s') \quad (5.2)$$

From equations 5.1 and 5.2

$$x \tan \phi = \cot \theta_2 (x - s') \quad (5.3)$$

$$x = - \frac{s' \cot \theta_2}{\tan \phi - \cot \theta_2} \quad (5.4)$$

$$y = \frac{s' \cot \theta_2 \tan \phi}{\cot \theta_2 - \tan \phi} \quad (5.5)$$

Therefore,

$$P' = \sqrt{x^2 + y^2} \quad (5.6)$$

Using equations 5.4 and 5.5,

$$P' = \frac{s' \cot \theta_2 \sec \phi}{[\cot \theta_2 - \tan \phi]} \quad (5.7)$$

Considering the object side,

$$y = x \tan \phi \quad (5.8)$$

and

$$y - 0 = \tan(90^\circ + \theta_1) (x - s)$$

or

$$y = -\cot\theta_1(x-s) \quad (5.9)$$

$$x = -\frac{\cot\theta_1(x-s)}{\tan\phi} \quad (5.10)$$

$$P = \sqrt{x^2 + y^2} \quad (5.11)$$

Using 5.9 and 5.10 in 5.11, we get

$$P = \frac{s \cot\theta_1 \sec\phi}{(\cot\theta_1 + \tan\phi)} \quad (5.12)$$

The magnification m is given by

$$m' = -\frac{P'}{P}$$

Using equations 5.7 and 5.12,

$$\frac{m'}{m} = \frac{\cot\theta_2(\cot\theta_1 + \tan\phi)}{\cot\theta_1(\cot\theta_2 - \tan\phi)} \quad (5.13)$$

Equation 5.13 indicates that the field angle and the amount of tilt on the conjugate planes influences distortion.

Two cases leading to no distortion are as follows:

Case (i) For no tilt on the conjugate planes, θ_1 and θ_2 are zero. $\tan\phi$ can be neglected in both numerator and denominator of equation 5.13 giving $m'/m = 1$ or $m' = m$. Hence distortion is zero.

Case (ii) For a zero field angle, ϕ is zero, which leads to $m'/m = 1$ or $m' = m$. Thus distortion is zero again.

5.3 Experiment 2: Tilting the conjugate planes and the lenses to satisfy Scheimpflug condition:

Since the first method (refer to Section 5.1) to produce a distortion free image was unsuccessful in producing a sharp image, a new design was considered. In this case three lenses are used to satisfy specific needs (refer to Figure 5.2).

5.3.1 Description of the lenses used

Source and Fresnel Lens: Bell-Howell Projector

120V 60Hz 400W

Lamp - 360W Max

Lens A RICOH 1:1.4 50 mm

Lens B NIKON 1:2.8 60 mm

Lens F Biconvex lens of focal length 7.5 cm

Figure 5.2: Top view of the new design.

5.3.2 Set-up description

The basic arrangement consists of a source-Fresnel lens combination, and the existing projector box are used for this. Lenses A and B are camera lenses (with perhaps six or more elements) having very small focal lengths and fairly large diameters for collecting sufficient light. The whole lens arrangement is placed on an optical bench, each lens capable of being moved independently. The entire set up makes use of the Scheimpflug condition (refer to Section 4.2). Focusing problems are eliminated making use of this condition.

The object or transparency is placed on the Fresnel lens, and the light from the Fresnel lens condensed to a spot of minimum size. The location of this minimum spot size is of

importance as it reduces off axis aberrations. Lens A is placed at the plane of the minimum spot size. At this plane, the condenser lens images the filament. An image of the object is produced at the image plane of lens A. Lens B is used to project the image produced by lens A onto the screen. This lens is a camera lens (consists of six or more elements) having short focal length.

5.3.3 Observation

To satisfy the Scheimpflug condition, it is found that the tilt on lens B should be greater than on lens A. Lens A is placed at the minimum spot size, which is important since if this were shifted to other positions, the final image would be out of focus and the illumination on the screen would be non-uniform. It was found that not all information from the object was transferred to the screen. This was due to the limited aperture size of lens B. Ray heights exceeding the aperture size were chopped, thus permitting a limited set of rays to pass through. Also, it was found that distortion was reduced but not completely eliminated. There was a residual distortion of about 13% that visually was not easily detected, but focusing produced by this system was not uniformly sharp.

To eliminate the above mentioned problems, a field lens is placed at a position for which the focusing is the best. The purpose of this lens is to increase the field of view by converging the rays incident on the surface of lens B. It is found that any residual distortion can be partially corrected by tilting the field lens from an angle θ_2 to α . This is because the field lens produces a varying magnification that contributes to the compensation of distortion. Further, it was seen that the introduction of the field lens produced a barrel type of distortion which could not be removed. The tilt of the field lens

skewed the barrel shape of the image, thus making the image appear a little curved towards one edge. An analysis of the tilt effect is discussed in the next section.

5.3.3.1 Effect of tilt on the magnification produced by the field lens

Let θ_2 be the tilt on the field lens at the image plane of lens A. This image plane is the object plane for the field lens. Let s_f be the distance through which the field lens is shifted to obtain the best focus. This distance will appear to be the object distance along the axis. Let α be the angle through which the field lens is rotated to compensate for residual distortion (refer to Figure 5.3).

Figure 5.3: Illustration of the effect of the tilt on the magnification produced by the field lens.

From figure 5.3, it is seen that the object distance varies for off axis points. For the lower half (with respect to the optical axis) of the field lens, the object distance s_2 is given by

$s_2 = s_f + \alpha r_f$ where r_f is the radius of the field lens. For the upper half of the lens, the object distance s_1 is given by $s_1 = s_f - \alpha r_f$. The corresponding image distances for the upper and lower half are determined from the general expression $s' = sf/s+f$. For the upper half (with respect to the optical axis) of the lens, the image distance is given by

$$s'_u = \frac{f s_1}{(s_1 - f)} \quad (5.14)$$

and for the lower half of the field lens the image distance is

$$s'_l = \frac{f s_2}{(s_2 - f)} \quad (5.15)$$

The magnification for the upper half of the lens is determined using equation 5.14 and is given by

$$m'_{uF} = -\frac{s'_u}{s_1} = \frac{f}{(f - s_1)} \quad (5.16)$$

Using the expression for the central ray, the focal length f can be expressed in terms of s'_f and s_f . s'_f being the image distance along the optical axis.

$$f = \frac{s'_f (s_f - f)}{s_f} \quad (5.17)$$

Using equation 5.17 in 5.16

$$m'_{uF} = m_F \left[\frac{f - s_f}{f - s_1} \right] \quad (5.18)$$

where m_F is the magnification produced along the axis. Similarly for the lower part of the field lens the magnification is given by

$$m'_{lF} = m_F \left[\frac{f - s_f}{f - s_2} \right] \quad (5.19)$$

It is to be noted that s_f , s_1 , and s_2 in equations 5.18 and 5.19 are negative. From equations 5.18 and 5.19, it is seen that magnification depends on the tilt of the field lens.

5.3.3.2 Analysis for distortion produced by the entire system

Due to the tilt of the field lens and object and image planes, the distortion produced for the upper and lower half (with respect to the optical axis) of the entire new system need to be considered separately. The variation in the off axis object and image distances causes a variation in the distortion produced (refer to Figure 5.4).

Figure 5.4: Illustration of varying image and object distances for off axis rays considered above and below the axis.

From figure 5.4, m_u' and m_l' are defined as

$$m_u' = -\frac{P_u'}{P_l'} \quad (5.20)$$

where m_u' remains the same as the general expression considered in section 5.2. Hence equation 5.13 defines equation 5.20. For a ray meeting the upper half of the image plane of lens A

$$m_{uA}' = \frac{m_A \cot\theta_2 (\cot\theta_1 + \tan\phi_A)}{\cot\theta_1 (\cot\theta_2 - \tan\phi_A)} \quad (5.21)$$

where $m_A = -S_A'/S_A$. Considering $m_l' = -P_l'/P_u'$, it is found that P_l' will remain the same as P_l except that S_A in equation 5.12 is replaced with S_A' and θ_1 by θ_2 . The sign convention used in the analysis is such that the sign associated with ϕ is of no importance. Using equation 5.12, P_l' for lens A becomes

$$P_l' = \frac{S_A' \cot\theta_2 \sec\phi_A}{(\cot\theta_2 + \tan\phi_A)} \quad (5.22)$$

P_u will remain the same as P_u' except that S_A' is replaced with S_A and θ_2 by θ_1 . Using equation 5.7, P_u for lens A is

$$P_u = \frac{S_A \cot\theta_1 \sec\phi_A}{[\cot\theta_1 - \tan\phi_A]} \quad (5.23)$$

Using equations 5.22 and 5.23 in m_l' , we get

$$m'_{IA} = \frac{m_A \cot\theta_2 (\cot\theta_1 - \tan\phi_A)}{\cot\theta_1 (\cot\theta_2 + \tan\phi_A)} \quad (5.24)$$

The same analogy can be applied to lens B. With the preceding equations, an analysis for distortion produced by the entire system is considered. The following analysis applies to the distortion produced for a ray starting from the object plane below the optical axis. For this ray, m'_{uA} is considered as it crosses the plane of lens A reaching the upper half of the image plane of lens A. This ray further proceeds towards lens B crossing its plane to reach the final image plane at a point below the optical axis. Therefore, m'_{IB} needs to be considered for this ray. A discussion of the effect produced by lenses A, B and the field lens follows.

Lens A: The magnification m'_{uA} produced at the upper end of the image plane for this lens is greater than that produced at the lower half, *i. e.*, $m'_{uA} > m'_{IA}$. The reason for this being the varying image and object distances considered for both m'_{IA} and m'_{uA} . This indicates that distortion is introduced by this lens at the final image plane.

Lens B: The rays from the upper half of the image plane of lens A reaches the lower half of the screen. Hence m'_{IB} is considered for this lens. m'_{IB} has a reducing effect on the magnification produced as compared to the upper half of the screen. The reason for which is due to the variation in image and object distances. Lens B partially compensates for any distortion introduced by lens A, as verified by observation. It was found that almost all distortion was removed by these two lens systems. However, the focusing was non-uniform. The residual distortion of 13% seemed to exist indicating that lens B does not fully compensate the distortion introduced by lens A by the required amount. A further correction on this residual distortion and focusing is considered using a field lens.

Field Lens: The field lens is moved to a position where the focusing is the best. This position appears before the image plane of lens A. The image plane of lens A acts as an object plane for the field lens. This object distance is negative, thereby forming a real image closer to the lens. Due to the tilt of the lenses, the magnification produced by the field lens varies across the field lens (refer to Section 5.3.3.1). This variation in magnification, though small, appears to partially compensate for any residual distortion produced by the combination of lenses A and B.

In the next section, the magnifications produced by each element are calculated. The distortion produced by the entire system is also estimated.

5.3.3.3 Sample calculations

This section gives a numerical set of calculations in support of the observations mentioned in Section 5.3.3.2. Distances were measured experimentally with a meter stick and measured with respect to the principal planes of the lenses. For lenses A and B, the principal planes lie very close to each other. Hence, the average distance between the two planes was considered. All angles were measured using the triangulation method using a laser beam. Using the initial tilt angle of lens A and the distances measured, the tilts on the rest of the lenses and the image plane were calculated. There was a close correspondence between these values that were calculated and the actual values measured in the experimental set up. For simplicity, the estimated values are considered.

Figure 5.5: Illustration of the entire set up.

OQ is the distance from the on-axis point in the object plane to the point of intersection with the other planes.

ϕ_A is the field angle for lens A.

θ_2 is the tilt of the image plane of lens A.

(x,y) are the coordinates for a point considered at the upper half of the image plane.

ϕ_B is the field angle for lens B.

θ_4 is the tilt of the image plane of lens B.

S_A' , S_A , S_B' and S_B are the distances measured with respect to the principal planes of the respective lenses. The measured values are:

S_A	45 cm
S_A'	5 cm
S_B	6.5 cm
S_B'	318.75 cm

The tilt β_1 on lens A = 11.25°

The tilt θ_1 on the object plane = 0°

The object height $h = 26$ cm. The parameters calculated using the preceding values are given in Table 5.1.

Parameters	Formula used	Value obtained
OQ	$S_A \tan(90^\circ - \beta_1)$	226.23 cm
φ_A	$\arctan \left[\frac{\frac{h}{2}}{S_A} \right]$	16.11°
θ_2	$90^\circ - \arctan \left[\frac{OQ}{S_A + S'_A} \right]$	12.46°
(x, y)	$x = \frac{S'_A \cot \theta_2}{\cot \theta_2 - \tan \varphi_A}$	5.34 cm
	$x - S'_A$	0.34 cm
	$y = x \tan \varphi_A$	1.54 cm
	$S_B - (x - S'_A)$	6.16 cm
φ_B	$\arctan \left[\frac{y}{S_B - (x - S'_A)} \right]$	14.05°
θ_4	$\frac{OQ}{S_A + S'_A + S_B + S'_B}$	32.24°

Table 5.1: Illustration of parameters calculated.

Using the preceding values, the off-axis magnification produced by lens A for a ray meeting the upper half of the image plane of lens A is determined using equation 5.21

$$m'_{uA} = 1.068 m_A \quad (5.25)$$

The off axis magnification produced by lens B for a ray meeting the lower half of the image plane of lens B is found using equation 5.24, where m_A is replaced by m_B , θ_2 by θ_4 , θ_1 by θ_3 , ϕ_A by ϕ_B . It is to be noted that θ_2 of lens A is the same as θ_3 for lens B.

$$m'_{lB} = 0.815 m_B \quad (5.26)$$

Using 5.25 and 5.26 the calculated percent distortion produced in the absence of the field lens is given by

$$\text{Percent distortion} = \frac{m'_{uA}m'_{lB} - m_A m_B}{m_A m_B} 100 = 13\%$$

Field Lens Calculations: Referring to Section 5.3.3.1, the measured distance $s_f = 1$ cm.

The measured radius of the field lens $r_f = 2.5$ cm. The focal length f of the field lens is 7.5 cm.

$$s_1 = s_f - \alpha r_f = 0.76 \text{ cm}$$

$$s_2 = s_f + \alpha r_f = 1.23 \text{ cm}$$

where $\alpha = 5.39^\circ$. Using equations 5.18 and 5.19, the magnification produced at the upper and lower halves of the field lens are calculated.

The results obtained are

$$\text{and } m'_{uF} = 1.03 m_F \quad (5.27)$$

$$m'_{lF} = 0.97 m_F \quad (5.28)$$

From equations 5.27 and 5.28, it can be seen that the field lens produces a 6% difference in magnification between the lower and upper halves of the lens. This varying magnification is used to partially correct any residual distortion produced by the lens A and B combination. The overall percent distortion is found using:

$$\text{Percent distortion} = \frac{m'_{uA} m'_{lB} m'_{uF} - m_A m_B m_F}{m_A m_B m_F} 100$$

where m_A , m_B and m_F are the magnifications produced along the optical axis.

Substituting equations 5.25, 5.26 and 5.28 in 5.29, the percent distortion = 10%. Ideally, the desired percent distortion should have been zero. The cause of this additional 10% is discussed in Chapter 6.

CHAPTER 6

CONCLUSION

The primary concern of this thesis is the distortion present in the projected image of a vuegraph. This distortion which differs from the two standard forms, the barrel shape and the pin cushion, was practically eliminated in the new set-up .

All analysis were based on experimental observation. Available lenses in the lab were used in designing and assembling a new projector. This new design consists of a source and condenser lens, two camera lenses producing canceling effects in distortion, and a field lens placed intermediate in position to the two lenses to produce the best focusing. The need for camera lenses is to maintain small focal distances and large apertures in order to keep the lens system compact. To produce sharp focusing, the Scheimpflug condition has to be satisfied. This condition requires the object and image planes that are not perpendicular to the optical axis, meet and intersect each other at the plane of the lens. It was found that the tilt on the second camera lens had to be greater than that of the first to satisfy this condition. The field lens is used to increase the field of view by converging the rays incident on the second camera lens. Further, the tilt of the field lens is used to remove any residual distortion. This is partially accomplished with the varying magnification of 6% that is produced by the field lens. The overall distortion for the entire system is about 10%; ideally a 0% distortion is desired. The barrel shape of the image is due to the inherent distortion present in the single-element field lens and is skewed due to the tilt of the lens. This distortion is, however, not very obvious to the eye; in fact, very few people notice it. This new arrangement is placed at the side of the classroom, thereby removing any

obstruction caused in viewing by the speaker and the projector itself.

In conclusion, the results obtained with this new set-up met the goal of producing a well focused and virtually a distortion free image, when placed at the side of the classroom.

CHAPTER 7

RECOMMENDATIONS

The main emphasis in this thesis is to obtain a well focused distortion free image. The analytical calculations show that the image produced by the entire system should be practically distortion free. However, a small amount of distortion does exist for reasons which are described below.

The measurement of the tilt of the image plane of lens A is critical. Experimentally, it was difficult to measure this angle. A small change in the tilt did not produce any visual change in the image produced. A more accurate method of determining this tilt is to place a rotating ground glass at the object plane. A microscopic objective is then placed near the ground glass to visually obtain the best focus. This method of locating the exact tilt was restrictive to us, due to limited equipment available.

The second reason is due to the mounts and the optical bench used. The optical mounts used were incapable of measuring very small angles, and a well calibrated optical bench was not available. Mounts with both translational and rotational capabilities are highly recommended to produce more accurate readings.

Although the field lens consisted of a single lens, it produced a distortion much greater than that of a camera lens consisting of six or more elements. One of the options to eliminate this problem is to use another high quality multi-element camera lens at the image plane, but the cost of such a system would definitely increase.

Chromatic aberration was not studied in detail in this thesis. The lens systems used in the set-up are normally corrected for chromatic aberration, but experimentally, it was found that there was residual chromatic aberration present. A detailed study of chromatic aberration and the implementation of its results in a hardware set up would certainly be an important contribution in the development of a side projection overhead projector. This may be taken as a new project, the starting point of which is this thesis.

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APPENDIX A

This program calculates the total contributions of the third order aberrations, the entrance and exit pupils. The data that needs to be specified initially includes: the ray height and the angle at the first surface, the number of surfaces, curvatures and clear apertures(CA) comprising the optical system. To determine which of the elements of the system limits the diameter of the axial cone of rays, the ratio of the clear aperture to the axial ray height at each surface is calculated. The smallest ratio indicates the aperture stop. The entrance pupil is then determined by tracing the ray imaged by the left part (with respect to the aperture stop) of the system.

```

5 INPUT "# of surfaces"; K
10 FOR X=1 TO K
13 INPUT "Curvatures";C(X)
15 NEXT X
20 INPUT "Y(1)";Y(1)
21 INPUT "U(1)";U(1)
28 FOR X=1 TO K-1
30 INPUT "thicknesses";T(X)
31 NEXT X
32 FOR X=1 TO K+1
34 INPUT "refractive indices";N(X)
40 NEXT X
41 FOR X=1 TO K
42 INPUT "clear aperture";CA(X)
43 NEXT X
55 LET P=1
56 FOR Q=1 TO K
60 U(P+1)=N(P)*U(P)/N(P+1)-C(P)*Y(P)*(N(P+1)-N(P))/N(P+1)
65 Y(Q+1)=Y(Q)+T(Q)*U(Q+1)
66 P=P+1
70 NEXT Q
71 FOR X=1 TO K
72 PRINT "Y("X")=";Y(X)
73 NEXT X
74 FOR X=1 TO K+1
75 PRINT "U("X")=";U(X)
76 NEXT X
77 DIM AT(50)
78 FOR X=1 TO K
79 AT(X)=CA(X)/Y(X)
80 NEXT X
81 REM to find the smallest value of ca/y
82 LET E=AT(1)
83 LET F=1
84 FOR X=2 TO K
85 M=AT(X)
86 IF M>E THEN 90
87 IF M<E THEN 88
88 E=AT(X)
89 F=X
90 NEXT X
91 PRINT "the aperture stop is at surface "F"ca/y=";E
92 BF=-Y(K)/U(K+1)
93 PRINT BF
94 DIM RI(50),CU(50),TH(50)
95 LET M=1

```

```

96 FOR X=F TO 1 STEP -1
97 RI(M)=N(X) GOTO 98
98 CU(M)=C(X) GOTO 99
99 IF M=F THEN GOTO 102
100 TH(M)=T(X-1) GOTO 101
101 M=M+1
102 NEXT X
109 BF=-Y(K)/U(K+1)
110 PRINT BF
111 LET Y1=.001
112 U1=Y1/TH(1)
113 IF RI(1)<>1 THEN GOTO 114
114 FOR X=1 TO F GOTO 115
115 CU(X)=CU(X+1)
116 NEXT X
117 LET Y1=.001
118 U1=Y1/TH(1)
120 LET P=1
121 FOR Q=1 TO F-1
122 U(P+1)=(RI(P)*U(P)/RI(P+1))-(CU(P)*Y(P)*(RI(P+1)-RI(P))/RI(P+1))
123 Y(Q+1)=Y(Q)+TH(Q+1)*U(Q+1)
124 P=P+1
126 NEXT Q
127 FOR X=1 TO F-1
128 PRINT "Y("X")=";Y(X)
129 NEXT X
130 FOR X=1 TO F
131 PRINT "U("X")=";U(X)
132 NEXT X
135 D=Y(F-1)/-U(F)
136 PRINT "The entr pupil is at a dist" D"from sur with curv= ";C(1)
200 REM principal ray calculations
210 REM Y(1)=DISTANCE OF ENTRANCE PUPIL FROM FIRST SURFACE *HALF FIELD ANGLE
220 REM Y(1) =24.905*.25=6.22625
230 DIM YP(50),UP(50)
240 INPUT "YP(1)=";YP(1)
250 INPUT "UP(1)=";UP(1)
260 LET R=1
270 FOR S=1 TO K
280 UP(R+1)=N(R)*UP(R)/N(R+1)-C(R)*YP(R)*(N(R+1)-N(R))/N(R+1)
290 YP(S+1)=YP(S)+T(S)*UP(S+1)
300 R=R+1
310 NEXT S
320 REM h=final image height
330 H=YP(6)+UP(7)*(79.33)
340 FOR A=1 TO K
350 PRINT "UP("A")="; UP(A), "YP("A")="; YP(A)
355 NEXT A
356 PRINT "UP("K+1")=";UP(K+1)
360 PRINT "H="; H
370 INV=YP(1)*N(1)*U(1)-Y(1)*N(1)*UP(1)
380 PRINT "Inv="; INV
390 REM to verify image height by from invariant I; hp is the same as h
400 HP=INV/(N(K+1)*U(K+1))
410 PRINT "HP="; HP
419 DIM DN(50)
420 DIM I(50),IP(50),B(50), BP(50),TSC(50),CC(50),TAC(50),TPC(50)
421 DIM DC(50),TACHC(50),TCHC(50),SC(50),AC(50), PC(50),LCHC(50)
422 FOR X=1 TO K+1
423 INPUT "v-values";V(X)

```

```

425 REM V is chosen to be = 0 for for air for convenience
429 NEXT X
430 FOR S=1 TO K
435 IF V(S)=0 THEN GOTO 439
438 DN(S)=(N(S)-1)/V(S): GOTO 443
439 DN(S)=0
443 NEXT S
444 FOR S=1 TO K
445 I(S)=C(S)*Y(S)+U(S)
450 IP(S)=C(S)*YP(S)+UP(S)
460 B(S)=(N(S)*(N(1+S)-N(S))*Y(S)*(U(1+S)+I(S)))/(2*N(1+S)*INV)
470 BP(S)=(N(S)*(N(1+S)-N(S))*YP(S)*(UP(1+S)+IP(S)))/(2*N(1+S)*INV)
480 PRINT "b(";"S"=";B(S),"bp(";"S"=";BP(S)
490 TSC(S)=B(S)*I(S)*I(S)*HP
500 CC(S)=B(S)*I(S)*IP(S)*HP
510 TAC(S)=B(S)*IP(S)*IP(S)*HP
520 TPC(S)=-((N(S)-N(S+1))*C(S)*INV*HP)/(2*N(S)*N(S+1))
530 DC(S)=HP*(BP(S)*I(S)*IP(S)+.5*(UP(S+1)*UP(S+1)-UP(S)*UP(S)))
560 TACHC(S)=-Y(S)*I(S)*(DN(S)-(N(S)*DN(S+1))/N(S+1))/(N(K+1)*U(K+1))
570 TCHC(S)=-Y(S)*IP(S)*(DN(S)-(N(S)*DN(S+1))/N(S+1))/(N(K+1)*U(K+1))
580 SC(S)=-TSC(S)/U(K+1)
590 AC(S)=-TAC(S)/U(K+1)
600 PC(S)=-TPC(S)/U(K+1)
610 LCHC(S)=-TACHC(S)/U(K+1)
620 NEXT S
630 FOR X=1 TO K
640 PRINT "tsc(";"X"=";TSC(X),"cc(";"X"=";CC(X),"tac(";"X"=";TAC(X)
650 PRINT "tpc(";"X"=";TPC(X),"dc(";"X"=";DC(X),"dn(";"X"=";DN(X)
660 PRINT "dn(";"X+1"=";DN(X+1),"tachc(";"X"=";TACHC(X),"tchc(";"X"=";TCHC(X)
670 PRINT "sc(";"X"=";SC(X),"ac(";"X"=";AC(X),"pc(";"X"=";PC(X)
680 PRINT "lchc(";"X"=";LCHC(X)
690 NEXT X
700 PRINT "
910 FOR X=1 TO K
920 SUM= SUM+TSC(X)
930 NEXT X
940 PRINT "SUM[TSC]= ";SUM
950 FOR X=1 TO K
960 MUM= MUM+SC(X)
970 NEXT X
980 PRINT "SUM[SC]= ";MUM
990 FOR X=1 TO K
1000 GUM= GUM+CC(X)
1005 NEXT X
1010 PRINT "SUM[CC]= ";GUM
1020 FOR X=1 TO K
1030 LUM=LUM+TAC(X)
1035 NEXT X
1040 PRINT "SUM[TAC]= ";LUM
1050 FOR X=1 TO K
1060 TUM=TUM+AC(X)
1070 NEXT X
1080 PRINT "SUM[AC]= ";TUM
1090 FOR X=1 TO K
1100 RUM=RUM+TPC(X)
1110 NEXT X
1120 PRINT "SUM[TPC]= ";RUM
1130 FOR X=1 TO K
1140 FUM=FUM+PC(X)
1145 NEXT X
1150 PRINT "SUM[PC]= ";FUM
1170 FOR X=1 TO K

```

```
1180 QUM=QUM+DC(X)
1190 NEXT X
1200 PRINT "SUM[DC]= ";QUM
1210 FOR X=1 TO K
1220 PUM=PUM+TACHC(X)
1230 NEXT X
1240 PRINT "SUM[TACHC]= ";PUM
1280 FOR X=1 TO K
1290 DUM=DUM+LCHC(X)
1300 NEXT X
1310 PRINT "SUM[LCHC]= ";DUM
1320 FOR X=1 TO K
1330 YUM=YUM+TCHC(X)
1340 NEXT X
1350 PRINT "SUM[TCHC]= ";YUM
Ok
```

APPENDIX B

Verification of the existing design using Genesee software¹⁴.

```

File      Edit      GENII+      View
TITLE projector using a meniscus with tolerance 0 for trig y      ZOOM 1 OF 1
WAVLN 0.5876 OBJHT -93      ENPHT 3      F/R 5.65 HFOV 22.02 EFL 29.68 1
      GENII-PLUS (R) v1.3.0 - (C) 1990-1991 Genesee Optics Software, Inc.

Surfaces
0  Sph  0      230      air      1      0
#1 Sph  0      0      air      1      -3.45  0
2  Sph  v-25.648 0.9      glass  1.5      -3.368  0
3  Sph  v-9.5111 0      air      1      -3.511  0
4  Sph  0      34      air      1      -3.673  0
5  Sph  0      0      air      1      16      0

Extras
Rays
1  ray      0      0      0      0
2  ray      0      0.15  0      0
3  ray      0      -0.15 0      0
4  ray      1      1      0      0
5  ray      1      0.85  0      0
6  ray      0.8      0.8      0      0
7  ray      0.8      0.95  0      0
8  ray      0.8      0.65  0      0
9  ray      0.6      0.6      0      0
10 ray      0.6      0.75  0      0
11 ray      0.6      0.45  0      0
12 ray      0.4      0.4      0      0
13 ray      0.4      0.55  0      0
14 ray      0.4      0.25  0      0
15 ray      0.2      0.2      0      0
16 ray      0.2      0.35  0      0
17 ray      0.2      0.05  0      0

Targets
RayNo  ColorNo  Target  Tolerance SurfNo
1 1 trig y 1 1 0 0 0
2 2 trig dy 2 1 0 0.01 0
3 3 trig dy 3 1 0 0.01 0
4 4 trig y 4 1 94.5 0 0
5 5 trig dy 5 1 0 0.01 0
6 6 trig y 6 1 75.6 0 0
7 7 trig dy 7 1 0 0.01 0
8 8 trig dy 8 1 0 0.01 0
9 9 trig y 9 1 56.7 0 0
10 10 trig dy 10 1 0 0.01 0
11 11 trig dy 11 1 0 0.01 0
12 12 trig y 12 1 37.8 0 0
13 13 trig dy 13 1 0 0.01 0
14 14 trig dy 14 1 0 0.01 0
15 15 trig y 15 1 18.9 0 0
16 16 trig dy 16 1 0 0.01 0
17 17 trig dy 17 1 0 0.01 0

End Data

```


APPENDIX C

Proof of Scheimpflug condition based on the expressions obtained in Section 5.2 of Chapter 5.

Statement: Scheimpflug condition requires that the object and image planes that are not perpendicular to the optical axis, meet and intersect each other at the plane of the lens. To prove this statement, it is required that

From the figure,

$$s' \tan(90^\circ - \theta_2) = s \tan(90^\circ - \theta_1)$$

where θ_1 and θ_2 are the tilts on the object and image planes.

or

$$s' \cot(\theta_2) = s \cot(\theta_1)$$

Proof:

$$\frac{1}{s'} + \frac{1}{s} = \frac{1}{P'} + \frac{1}{P} = \frac{1}{f}$$

where s' , s are the on-axis image and object distances and P' and P are the off-axis image and object distances respectively. Using equations 5.7 and 5.12, (for P' and P) of Section 5.2, it can be shown that

$$\frac{1}{s'} + \frac{1}{s} = \frac{\cot\theta_1 + \tan\phi}{s \cot\theta_1 \sec\phi} + \frac{\cot\theta_2 - \tan\phi}{s' \cot\theta_2 \sec\phi}$$

For small field angles ($\phi \approx 16^\circ$) $\sec\phi \approx 1$. Simplifying, we get

$$\frac{s + s'}{s s'} = \frac{s + s'}{s s'} + \frac{\tan\phi (s' \cot\theta_2 - s \cot\theta_1)}{s s' \cot\theta_1 \cot\theta_2}$$

This implies that

$$\frac{\tan\phi (s' \cot\theta_2 - s \cot\theta_1)}{s s' \cot\theta_1 \cot\theta_2} = 0$$

or

$$s' \cot\theta_2 - s \cot\theta_1 = 0$$

Hence, the tilted object and image planes meet and intersect at the plane of the lens.