1997

Dynamic motion of small objects within a velocity field

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DYNAMIC MOTION OF SMALL OBJECTS WITHIN A VELOCITY FIELD

A Thesis

Presented to

The Faculty of the Department of Mechanical and Aerospace Engineering

San José State University

In Partial Fulfillment of the Requirements for the Degree

Master of Science

By

Rufus Sizer Drury, Jr.

May 1997

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ABSTRACT

DYNAMIC MOTION OF SMALL OBJECTS WITHIN A VELOCITY FIELD

by Rufus Sizer Drury, Jr.

Sometimes loose objects such as fasteners, nuts, bolts, tools, broken components, or other loose debris are present in a reactor vessel. These objects can be carried by fluid flow to places in the system where they can adversely affect the performance of other components. It is therefore desirable to remove these objects whenever a planned power plant outage is scheduled (i.e., refueling), or sooner if necessary. The highest incidence of foreign object entry to the reactor vessel occurs during the fueling outage when the reactor vessel head is removed (e.g., workers dropping objects into the reactor vessel). Thus, it is useful to understand the dynamic motion of these potential loose objects within the various flow fields present in a large, modern nuclear reactor vessel.

A mathematical model, with three degrees of freedom (two translational: x , y , and one rotational: θ), based on the momentum principle with drag, lift, and moment coefficients, is developed to study the dynamic motion of small objects within a velocity field.

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NOMENCLATURE

F	Net force vector on object, lb_f .
m_o	Mass of object, lb_m .
V_{cg}	Velocity of the mass center, ft/s .
V_f	Velocity of the fluid, ft/s .
M	Net moment vector on object, $\text{lb}_f\text{-ft}$.
$\dot{H}_{cg}$	Rate of change of the angular momentum of the mass system about the mass center.
F_D	Drag force, lb_f .
F_L	Lift force, lb_f .
M_{cg}	Moment about the mass center, $\text{lb}_f\text{-ft}$.
F_B	Buoyant force, lb_f .
W	Weight of the object, lb_f .
Re	Reynolds Number.
L	Characteristic height of the tank or vessel.
L'	Characteristic length of the object used to define ω_{ref}
C_D, C_L, C_M	Drag, Lift, and Moment Coefficients.
α	Angle of attack
A_{ref}	Reference area of the object, ft^2 .
V_{rel}	Relative velocity (Relative Wind) ($V_f - V_{cm}$), ft/s .
ρ_f, ρ_o	Density of the fluid and the object, lb_m/ft^3 .
V	Volume of the object, ft^3 .
g	Acceleration due to gravity, 32.2 ft/s^2 .
g_o	Conversion constant (Newton's constant), $32.2 \text{ lb}_m \text{ ft} / \text{lb}_f \text{ s}^2$.
XYZ	Primary inertial system reference.
xyz	Reference axes attached to the body.
i, j, k	The unit vectors in the moving xyz system.

Subscripts

f	Property of fluid.
o	Property of object.

INTRODUCTION

There are instances when a foreign object enters or could enter an open reactor pressure vessel. The definition of a foreign object is a part that is either not normally found in the reactor or some reactor internal part that has broken free of its normal location in the vessel. If this foreign object is not found and removed, it then becomes a "lost part." The presence of the "lost part" could require an evaluation to determine if there is a safety concern.

Stopping or delaying operations during a refueling outage because of these small loose objects can have a significant impact on a critical path time, because every day a reactor is not generating electricity can cost a utility approximately a million dollars. It is useful to determine if a loose small object can be left where it is, or if it must be retrieved. The following considerations apply in this decision process:

- the likely migration path the loose objects would follow,
- the potential reduction of cooling water to a fuel bundle,
- the potential interference with control rod motion,
- the contribution to mechanical damage of other components, or
- the potential reaction with other components (chemical).

Sometimes loose objects such as fasteners, nuts, bolts, tools, broken components, or other loose debris are present in a reactor vessel. These objects

can be carried by coolant flow to places in the system where they can affect the performance of other components. It is therefore desirable to remove this material whenever a planned power plant outage is scheduled (i.e., refueling), or sooner if necessary. The highest incidence of foreign object entry to the reactor vessel occurs during the fueling outage when the reactor vessel head is removed (e.g., workers dropping objects into the reactor vessel). In order to determine the general neighborhood where a lost part resides, it is useful to predict its dynamic motion within the various flow fields present in a large, modern nuclear reactor vessel.

ANALYSIS

SUBMERGED OBJECTS

The flow around a submerged object is generally more complicated than the flow in a straight pipe or channel, because it is two or three dimensional. Potential flow and boundary layer considerations are useful in understanding the details of the flow around any submerged object.

It may not be necessary to obtain fine details of the flow, but only the force on a body or object due to the flow of the fluid around it. For example, an airplane designer wants to know the “air forces,” namely “lift” and “drag,” of the plane so that she can select the “right” engine. By using experimental data for airfoil flows, we can predict lift and drag forces on submerged objects moving in a fluid.

Any object of any shape when immersed in a fluid stream will experience forces and moments from the flow. If the object has arbitrary shape and orientation, the flow will exert forces and moments about all three coordinate axes, as shown in Figure 1. Traditionally, one axis is selected parallel to the free stream and positive in the direction of flow. The force on the body along this axis is called drag, and the moment about that axis is called the rolling moment.

A second force, which is called the lift, is perpendicular to the drag. The moment about the lift axis is called yaw. The third force component is the side force, and

moment about this axis is the pitching moment. When the body has symmetry about the lift-drag axis and moves directly in to a stream, side force, yaw, and roll vanish, and the problem reduces to a two-dimensional case with lift and drag forces, and the pitching moment.

A final simplification often occurs when the object has two planes of symmetry. A wide variety of shapes such as cylinders, symmetric wings, and all bodies of revolution satisfy this requirement. If the free stream is parallel to the intersection of these two planes, called the principal chord line of the object, the object experiences drag only, with no lift, side force, or moments. This type of degenerate one-force drag data is what is most commonly reported in the literature, but if the free stream velocity is not parallel to the chord line, the object will have a non-symmetric orientation and all three forces and three moments can theoretically arise.

Figure 1. An object (washer) showing the orientation of aerodynamic forces and moments

LIFT, DRAG, AND MOMENT COEFFICIENTS

Borrowing aerodynamic terminology and nomenclature for airfoils, the free stream fluid velocity is the velocity of the fluid far upstream of the object. The direction of the free stream velocity is called the “relative wind.” The angle between the relative wind and the object is the angle of attack of the object. There is an aerodynamic force created by the pressure and shear stress distributions over the object surface. The resultant aerodynamic force can be resolved into two forces parallel and perpendicular to the free stream fluid velocity, corresponding to the drag D , and the lift L , respectively. Besides lift and drag, the surface pressure and shear distributions can also create a moment (M) which tends to rotate the object.

The actual magnitudes of lift, drag, and moment depend not only on the angle of attack, but also on velocity, density of the fluid, and geometry of the object. In principle, for a given object shape at a given angle of attack, one could find the variation of lift, drag, and moment by performing numerous wind-tunnel experiments wherein the critical parameters have individually varied. These results could then be correlated in the form of useful tables or graphs. Dimensional analysis has made it possible to obtain lift, drag, and moment coefficients, which are defined by

$$\text{Lift} = (\text{Dynamic Pressure}) \cdot (\text{Area}) \cdot (\text{Lift Coefficient})$$

$$\text{Drag} = (\text{Dynamic Pressure}) \cdot (\text{Area}) \cdot (\text{Drag Coefficient})$$

$$\text{Moment} = (\text{Dynamic Pressure}) \cdot (\text{Area}) \cdot (\text{Characteristic Length}) \cdot (\text{Moment Coefficient})$$

The development of the aerodynamic coefficients depends on the object shape and angle of attack. If the angle of attack were to vary, then the aerodynamic coefficients would also vary. Thus, for a given shape, the lift coefficient relation is given by:

$$C_L = f(\alpha, Re). \quad \text{Eq. 1}$$

Similar relationships hold for the drag and moment coefficients.

$$C_D = f(\alpha, Re). \quad \text{Eq. 2}$$

$$C_M = f(\alpha, Re). \quad \text{Eq. 3}$$

Thus, for an object of given shape, lift, drag, and moment coefficients can be stored in a data base.

In a theoretical sense, the lift, drag, and moments on an object are developed from the detailed pressure and shear stress distributions on the surface of the object. The measurements and calculations of these distributions, especially for complex configurations, are not trivial undertakings. The physical complexity of the flow field around the object is inherently buried in the lift, drag, and moment coefficients. Thus, before the simple expressions of equations 1, 2, and 3 can be

used to calculate lift, drag, and moments for an object of arbitrary shape, the appropriate aerodynamic coefficients must be known.

EQUATION DEVELOPMENT

For a rigid body in planer motion (with three degrees of freedom: x , y , θ) about the center of mass, three independent scalar equations are required to describe the motion. the following conservation two equations hold. The first, Equation 4, is Newton's Second Law. The second, Equation 5, is a statement of the conservation of angular momentum relative to the object center of mass:

$$\mathbf{F} = m_0 \mathbf{V}_o \quad \text{Eq. 4}$$

$$\mathbf{M} = \dot{\mathbf{H}}_o \quad \text{Eq. 5}$$

where $\mathbf{H}$ is the angular momentum.

Lift, drag, weight, and buoyancy forces acting on the object, , and the moments caused by these forces on the object are defined in Equations 6 through 10, in which lift, drag, and moment coefficients have been incorporated.

$$\mathbf{F}_D = \frac{C_D}{2g_0} \rho r A_{ref} \mathbf{V}_{rel}^2 \quad \text{Eq. 6}$$

$$\mathbf{F}_L = \frac{C_L}{2g_0} \rho r A_{ref} \mathbf{V}_{rel}^2 \quad \text{Eq. 7}$$

$$\mathbf{M}_D = \frac{C_M}{2g_0} \rho_f A_{ref} \frac{C}{4} \mathbf{V}_{rel}^2 \quad \text{Eq. 8}$$

$$\mathbf{W} = \rho_0 \nabla_0 \frac{\mathbf{g}}{g_0} \quad \text{Eq. 9}$$

$$\mathbf{F}_B = \rho_f \nabla_0 \frac{\mathbf{g}}{g_0} \quad \text{Eq. 10}$$

The moment sum includes the moment of forces external to the object. Thus, the moment sum represents the sum of the moments about the center of mass of all external forces acting on the body. The moment equation and the two scalar components of the generalized Newton's second law of motion give Equations 11 through 13, which are the general equations of motion for a rigid body in plane motion.

$$\sum \mathbf{F}_x = m_0 \frac{dV_{o,x}}{dt} \quad \text{Eq. 11}$$

$$\sum \mathbf{F}_y = m_0 \frac{dV_{o,y}}{dt} \quad \text{Eq. 12}$$

$$\sum \mathbf{M}_o = I \frac{d^2\theta}{dt^2} \quad \text{Eq. 13}$$

If the external forces are substituted into Equations 11 and 12, the result is to yield:

$$\sum \mathbf{F}_x = \left[\frac{V_0 \rho_0}{g_0} \right] \frac{dV_{0,x}}{dt} = F_D \cos \beta - F_L \sin \beta \quad \text{Eq. 14}$$

$$\sum \mathbf{F}_y = \left[\frac{V_0 \rho_0}{g_0} \right] \frac{dV_{0,y}}{dt} = F_D \sin \beta - F_L \cos \beta + F_B - W \quad \text{Eq. 15}$$

When the definitions for $\mathbf{F}_D$, $\mathbf{F}_L$, $\mathbf{F}_B$, $\mathbf{W}$ and $\mathbf{M}$ are substituted into Equations 11 through 13, the following equations are obtained for plane motion of a rigid body with three degrees of freedom. Furthermore, to complete the set of governing equations, the differential relationships between distance and velocity and the differential relationships between angular position and angular velocity are included.

$$\frac{dV_{o,x}}{dt} = \left(\frac{1}{2}\right) \left(\frac{\rho_f}{\rho_o}\right) \left(\frac{A_{ref}}{V}\right) V_{ref}^2 \{C_D \cos \beta - C_L \sin \beta\} \quad \text{Eq. 16}$$

$$\frac{dV_{o,y}}{dt} = \left(\frac{1}{2}\right) \left(\frac{\rho_f}{\rho_o}\right) \left(\frac{A_{ref}}{V}\right) V_{ref}^2 \{C_D \sin \beta - C_L \cos \beta\} + \left(\frac{g}{\rho_o}\right) (\rho_f - \rho_o) \quad \text{Eq. 17}$$

$$\frac{d\omega_o}{dt} = \left(\frac{C_M}{2g_o I_z}\right) \rho_f A_{ref} \left(\frac{C}{4}\right) V_{ref}^2 \quad \text{Eq. 18}$$

$$\frac{dx_o}{dt} = V_{o,x} \quad \text{Eq. 19}$$

$$\frac{dy_o}{dt} = V_{o,y} \quad \text{Eq. 20}$$

$$\frac{d\theta_o}{dt} = \omega_o \quad \text{Eq. 21}$$

Equations 16 through 21 describe the dynamic motion of an object with two translational degrees of freedom and one angular degree of freedom (i.e., x , y , and θ), in a velocity flow field.

Next, the governing equations are cast into non-dimensional form. Normalized variables are listed in Table 1. The resulting non-dimensional model coefficients yield model laws for preserving dominant phenomena. This type of information could be used in designing a scale model test.

Table 1. Dimensional and Non-dimensional Variables

Variable	Description
L	Characteristic height of tank or vessel, ft.
V_{ref}	Characteristic velocity, ft. per second.
Δt	Characteristic time, $[L \div V_{ref}]$
ω_{ref}	Characteristic angular velocity, $[V_{ref} = L_d \omega_{ref}]$
$t^* = \frac{t}{\Delta t}$	Non-dimensional time.
$x^* = \frac{x}{L}$	Non-dimensional distance: x-axis.
$y^* = \frac{y}{L}$	Non-dimensional distance: y-axis.

REFERENCE VELOCITY

The first step in the normalization process is to define a meaningful reference velocity. When first released in a stationary fluid, the object will accelerate downward because the forces acting on it are out-of-balance. This acceleration will result in an increase in the velocity of the object. After a while the drag force will increase to the point where the forces acting on the object are in balance. When that point is reached the object will attain a constant or terminal velocity. This terminal velocity is a useful reference since it represents the maximum velocity attainable for the object. Thus, for constant velocity of the object,

$$\sum F = \text{mass} \times \text{acceleration} = 0 \quad \text{Eq. 22}$$

which can be rewritten as

$$\frac{dV}{dt} = \frac{-F_D - F_B + W}{m_o} \quad \text{Eq. 23}$$

Thus, at constant velocity, and using the defined sign convention the drag force is

$$F_D = F_B - W. \quad \text{Eq. 24}$$

Now, substituting the earlier definitions for W , F_D , and F_B , the force balance Equation can be written as

$$\frac{\bar{C}_D A_{ref} \rho_f}{2g_0} (V_o - V_f) \left| (V_o - V_f) \right| = \rho_o \nabla_o \frac{g}{g_0} - \rho_f \nabla_o \frac{g}{g_0}, \quad \text{Eq. 25}$$

which after simplifying becomes:

$$(V_o - V_f) \left| (V_o - V_f) \right| = \frac{2g_0}{\bar{C}_D A_{ref} \rho_f} \left(\rho_o \nabla_o \frac{g}{g_0} - \rho_f \nabla_o \frac{g}{g_0} \right) \quad \text{Eq. 26}$$

or

$$V^2 = (V_o - V_f) \left| (V_o - V_f) \right| = \frac{2g \nabla_o}{\bar{C}_D A_{ref} \rho_f} (\rho_o - \rho_f) \quad \text{Eq. 27}$$

From the relationship shown in Equation (27), it can be seen that, the square of the terminal velocity is inversely proportional to the average drag coefficient and is proportional the difference between the object and fluid densities. The relationship also is directly influenced by the volume to surface area ratio.

Having determined a reasonable reference velocity, a reference time can be expressed as the time required for an object moving at reference velocity V_{ref} to pass through a distance L , or $t_{ref} = \frac{L}{V_{ref}}$. Table 2 gives non-dimensional

variables and derivatives for eventual use in the governing equations.

Table 2. Non-dimensional Operators

$$V_x^* = \frac{V_x}{V_{ref}}$$

$$V_y^* = \frac{V_y}{V_{ref}}$$

$$\frac{d}{dt} = \frac{d}{dt^*} \frac{dt^*}{dt} = \frac{V_{ref}}{L} \frac{d}{dt^*}$$

$$\frac{dx^*}{dt^*} = V_x^*$$

$$\frac{dy^*}{dt^*} = V_y^*$$

The $\frac{dV_x^*}{dt^*}$ term can be expressed as:

$$\frac{dV_x^*}{dt^*} = \left(\frac{1}{2}\right) \left(\frac{\rho_f}{\rho_o}\right) \left(\frac{LA_{ref}}{V_o}\right) (V_{rel}^*)^2 \{c_D \cos \beta - c_L \sin \beta\} \quad \text{Eq. 28}$$

The $\frac{dV_y^*}{dt^*}$ term also includes the weight and buoyant terms and is given by:

$$\frac{dV_y^*}{dt^*} = \left(\frac{1}{2}\right) \left(\frac{\rho_f}{\rho_o}\right) \left(\frac{LA_{ref}}{V_o}\right) (V_{rel}^*)^2 \{c_D \sin \beta - c_L \cos \beta\} + \left(\frac{L}{V_{ref}^2}\right) \left(\frac{g}{\rho_o}\right) (\rho_f - \rho_o)$$

Eq. 29

The reference velocity V_{ref} is substituted into the second term on the right-hand-side of equation 29, giving

$$\left(\frac{L}{V_{ref}^2}\right) \left(\frac{g}{\rho_o}\right) (\rho_f - \rho_o) = \frac{Lc_D A_{ref} \rho_f}{2g V_o (\rho_o - \rho_f)} \left(\frac{g}{\rho_o}\right) (\rho_f - \rho_o) = -\left(\frac{\bar{c}_D}{2}\right) \left(\frac{LA_{ref}}{V_o}\right) \left(\frac{\rho_f}{\rho_o}\right)$$

In this form, the equation 29 for $\frac{dV_y^*}{dt^*}$ can now be written as

$$\frac{dV_y^*}{dt^*} = \left(\frac{1}{2}\right) \left(\frac{\rho_f}{\rho_o}\right) \left(\frac{LA_{ref}}{V_o}\right) (V_{rel}^*)^2 \{c_D \sin \beta - c_L \cos \beta\} - \left(\frac{\bar{c}_D}{2}\right) \left(\frac{LA_{ref}}{V_o}\right) \left(\frac{\rho_f}{\rho_o}\right)$$

Eq. 30

Furthermore, the remaining equations of motion are written as follows:

$$\frac{d\omega^*}{dt^*} = \left(\frac{c_M}{2l_z} \right) \rho_f L \left(\frac{c}{4} \right) L' A_{ref} (V_{rel}^*)^2 \quad \text{Eq. 31}$$

$$\frac{dx^*}{dt^*} = V_x^* \quad \text{Eq. 32}$$

$$\frac{dy^*}{dt^*} = V_y^* \quad \text{Eq. 33}$$

$$\frac{d\theta^*}{dt^*} = \omega^* \quad \text{Eq. 34}$$

Thus, equations 28 and 30 through 34 represent a seventh order system of ordinary differential equations that describe the dynamic motion of an object under the constraints of planar motion (i.e., three degrees of freedom) incorporating the aerodynamic coefficients for lift, drag, and moment.

NUMERICAL SOLUTION OF SYSTEM OF EQUATIONS

The solution of equations 28 and 30 through 34 is accomplished numerically using a Runge-Kutta method. The Runge-Kutta subroutine, which uses Gills method, for N simulations differential equations was taken from Appendix C of Reference 6. The main program and other subroutines, which are written in the FORTRAN language, are listed in the Appendix. An outline of the steps taken to solve the specific set of equations is shown in Table 3.

Table 3. Solution Steps

Step 1	PROBLEM DEFINITION
Step 2	READ INPUT
Step 3	DETERMINE OBJECT VELOCITY IN THE REFERENCE FRAME X,Y (V_x & V_y) DETERMINE THE VELOCITY UNIT VECTOR
Step 4	DETERMINE OBJECT ORIENTATION IN THE X,Y REFERENCE FRAME; THE ANGLE θ (i.e., THE ANGLE BETWEEN THE "FRONT" OF THE OBJECT AND THE X AXIS). DETERMINE UNIT VECTOR IN THE DIRECTION (FRONT OF THE OBJECT)
Step 5	DETERMINE FLUID VELOCITY IN THE X,Y REFERENCE FRAME GIVEN OBJECT LOCATION LOOK-UP VELOCITY DETERMINE UNIT VECTOR DIRECTION ANGLE
Step 6	DETERMINE RELATIVE VELOCITY (USE THE VECTOR COMPONENT RELATIONSHIP)
Step 7	DETERMINE ANGLE OF ATTACK
Step 8	DETERMINE OBJECT REYNOLDS NUMBER (IN TERMS OF ABSOLUTE VELOCITY)
Step 9	LOOK-UP CORRESPONDING VALUES OF C_D , C_L , AND C_M (GIVEN THE REYNOLDS NUMBER AND ANGLE OF ATTACK)
Step 10	TRANSLATE THE LIFT AND DRAG FORCE FROM OBJECT COORDINATES TO THE REFERENCE FRAME COORDINATES (USE COORDINATE TRANSFORMATION TO TRANSLATE THE LIFT AND DRAG FORCES IN TO THE REFERENCE FRAME X,Y)
Step 11	INTEGRATE THE EQUATION SET
Step 12	WRITE SELECTED OUTPUT AT THE CURRENT TIME STEP
Step 13	REPEAT STEPS

RESULTS AND DISCUSSION

TEST SOLUTIONS

A necessary step in the problem solving process involves testing the solution (i.e., the system of equations developed). Two simple test cases involving spheres free-falling in a tank of water were used to check the solution process outlined in Table 3 of the previous section. The first test case is for a one inch steel sphere and the second test case is a 8.5 inch bowling ball. These sample test cases were taken from References 7 and 8.

TEST CASE #1: A 1" STEEL SPHERE

The solution for the terminal velocity as described in Reference 7 uses a graphical approach. The drag coefficient used for the sphere is taken from a graph by Lapple and Shepherd (Reference 14). The input used and the output of the computer simulation for this test case are shown in the Appendix. A comparison of the simulation and the reference case is shown in Table 4.

Although the solutions do not exactly match, the agreement is good because the "given" solution is based on reading a graph and as such is approximate. The computer simulation uses an empirical relationship for the drag coefficient of the sphere (equation 3-265 of Reference 6):

$$C_D \approx \frac{24}{Re} + \frac{6}{1 + \sqrt{Re}} + 0.4 \quad ; \quad 0 \leq Re \leq 2 \times 10^5 \quad \text{Eq. 35}$$

The accuracy of equation 35 is about ± 10 percent until the data deviate at the drag crisis, where the boundary layer on the sphere surface changes from laminar to turbulent. The drag crisis occurs slightly earlier for rough surfaces or with free stream turbulence (Reference 6). However, with a Reynolds number of about 6×10^4 , the transition is sufficiently removed such that the empirical relationship is applicable.

Figure 2 shows the dimensionless velocity as well as the dimensionless distance fallen for the 1" steel sphere. Figure 3 shows the dimensionless velocity but this time compared with the drag coefficient. More output, graphical and tabular, for this simulation (i.e., a free falling 1" steel sphere) is contained in the Appendix.

TEST CASE #2: AN 8.5" BOWLING BALL

The solution for the terminal velocity as described in Reference 8 uses a graphical approach. The drag coefficient used for the sphere is taken from a graph adapted from Prandtl (Reference 15). The input used and the output of the computer simulation for this test case is shown in the Appendix. A comparison of the simulation and the reference case is shown in Table 5.

Figure 4 shows the position of the bowling ball assuming an initial velocity condition. More output, graphical and tabular, for this simulation (i.e., the free falling bowling ball) is contained in the Appendix.

COMPLEX SHAPES

As stated earlier, the lift, drag, and moments on an object of arbitrary shape are developed from the detailed pressure and shear stress distributions on the surface of the object. The measurements and calculations of these distributions, especially for complex configurations, are not trivial undertakings. All the physical complexity of the flow field around the object is inherently buried in the lift, drag, and moment coefficients. Thus, before these simple equations can be used to calculate lift, drag, and moments for an arbitrary shape object, the appropriate aerodynamic coefficients must be known in advance.

Of the three coefficients, the most difficult to determine is the moment coefficient. Although this information is available for wing sections

(Reference 5), the data is limited in its usefulness because of the limited range of the angle of attack (i.e.. most designers are not interested in the post-stall behavior of wings at large angles of attack, $\alpha \geq 30^\circ$). However, from the perspective of calculating the dynamic motion and trajectory, this information is needed for an accurate solution.

PAST OBSERVATIONS OF FALLING OBJECTS

Without the complete aerodynamic coefficient set for an object, the solution of the trajectory may be inaccurate. However, we can learn useful information from past experiments that may give insight to the type of data needed. A number of investigators from different fields have studied the free-falling, tumbling, behavior of irregularly shaped objects (References 9,10,11,12, and 13). These investigations focus on the determination of a relationship between a drag coefficient and the Reynolds number associated with the object. Some of these investigations (e.g., References 10 and 11) included a modification of the drag term (i.e., a shape factor correction) to account for irregularities of the object. However, most of these studies are concerned with the settling rate or terminal velocity of the object.

Some aspects of flow about a disk and other bluff bodies are already known (Reference 12). However, this data tends to involve objects in flow fields with

very low Reynolds numbers such an example is when the inertial forces of the fluid can be neglected and the flow is steady (Stokes flow). This information is not useful for the solution of the trajectory of small objects in a reactor vessel flow field because the fluid velocities (Reynolds numbers) are not in the range of interest.

Willmarth et. al. (Reference 12) studied the stability of freely falling disks. The dynamic behavior of the disk was initially stable at low Reynolds numbers (i.e., below 100). The disk held a horizontal orientation throughout its free-fall in a still fluid. Near a Reynolds number of approximately 100, damped pitching oscillations were sometimes observed. Beyond a Reynolds number of 100 the motion of the disk became unstable in that it began to pitch. As the Reynolds number increased so did the pitching motion. Small pitching oscillations about a diameter with little transitional motion of the center of mass increased in amplitude until appreciable transitional motion of the center of mass was observed. This pitching motion or side to side motion occurred in a vertical plane normal to the disk diameter. As the Reynolds number was increased, the pitching motion accompanied by large-amplitude translation increased. At some point the motion was observed to shift from pitching and translation to tumbling.

As observed in Reference 12:

“When the disk is released it may complete one or two oscillations of increasing pitching amplitude and then completely overturn. Once the disk has overturned it falls quite rapidly in an apparently random manner. The disks usually hit the container sides. The ultimate location on the bottom of a large container could never be predicted.”

Another interesting motion pattern was observed in Reference 10 and 12. A definite spiral motion was observed about a vertical axis parallel to the direction of falling. With the disk, the irregularity of the motion prevented any quantitative measurements (Reference 12).

Table 4. Comparison of results for Case #1: Small sphere (1" diameter)

	Terminal Velocity (ft. per second)	Reynolds Number	Final C_D
Test Case	7.0	54,000	0.5
Simulation	7.6	58,712	0.43

Table 5. Comparison of results for Case #1: Large sphere (8.5" diameter bowling ball)

	Terminal Velocity (ft. per second)	Reynolds Number	Final C_D
Test Case ¹	5.2	-	-
Simulation	5.3	224,232.	0.41

¹ The values of C_D and Re are not specifically identified for this example.

Figure 2. Steel Sphere (1" diameter) falling in still water: vertical velocity, Vy^* and vertical distance, y^* .

Figure 3. Steel Sphere (1" diameter) falling in still water: vertical velocity, V_{y^*} and drag coefficient C_D

Figure 4. Large sphere (8.5" diameter bowling ball) falling in still water with an initial velocity (5 ft/s) in the positive x-axis direction.

CONCLUSIONS

A non-dimensional equation set has been developed to described the planar motion, three degrees of freedom: x , y , and θ , of a rigid object within a simple flow field. The non-dimensional equations set assumes the existence of the aerodynamic parameters: lift coefficient, drag coefficient, and a moment coefficient; C_L , C_D , C_M , respectively. All the physical complexity of the flow field around the object is inherently buried in the lift, drag, and moment coefficients. Thus, to proceed with this solution approach, the appropriate aerodynamic coefficients must be known in advance.

From the observations of past investigators, the motion of an object may be one or a combination of several types including tipping or pitching, sliding, and rotation. This would be especially true for objects that resemble in a general manner the shape of a “wing” or airfoil section, such as a flat plate. Thus for an object of this type, the three degrees of freedom model may be insufficient.

RECOMMENDATIONS

The following activities for future work are recommended:

1. Focus the remaining activities on two groups of objects: 1) those dominated by drag forces (e.g., sphere like), and 2) those that have airfoil like shapes (e.g., disk).
2. Further research should be conducted to determine if any experimental data exists for the aerodynamic coefficients of general interest such as for square or rectangle plate and disks.
3. Experiments in a wind tunnel could provide data on aerodynamic coefficients beyond the normal range of interest (i.e., high angles of attack).

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Appendix A-1

TEST CASE #1 STEEL SPHERE (1" DIAMETER) FALLING IN STILL WATER

Table 6. Input Listing for Case #1: Small sphere (1" diameter)

```
&INPUT
ALPHA    = 0.0
AP        = 0.005454
AREF      = 0.005454
CD         = 1.0
CDAVE     = 1.0
CL         = 0.0
CM         = 0.0
DENF       = 62.4
DENO       = 490.
GZERO     = 32.2
H          = 0.01
INERTIA   = 0.000005341118
LD         = 0.04166
LM         = 0.04166
M          = 0
N          = 6
OBJDIM    = 0.08333
PI         = 0.31415926535E+01
TANKL     = 40.
VISCOS    = 2.09E-05
VOL        = 0.0003033330086
VFX       = 0.0
VFY       = 0.0
X          = 0.0
XLIM      = 1.0
Y(1)      = 0.0
Y(2)      = 0.0
Y(3)      = 0.0
Y(4)      = 0.0
Y(5)      = 0.0
Y(6)      = 2.35619
Y(7)      = 0.0
Y(8)      = 0.0
Y(9)      = 0.0
Y(10)     = 0.0
/
```

Table 7. Case #1 Output listing: Vx^* , Vy^* , ω^* , x^* , y^* , and θ^*

Time	Vx^*	Vy^*	ω^*	x^*	y^*	θ^*
0.01	5.04E-19	-0.44971	0	1.95E-21	-0.00226	2.35619
0.02	5.09E-18	-0.83275	0	2.76E-20	-0.00871	2.35619
0.03	1.61E-17	-1.11105	0	1.31E-19	-0.01847	2.35619
0.04	3.31E-17	-1.2906	0	3.74E-19	-0.03052	2.35619
0.05	5.46E-17	-1.39784	0	8.11E-19	-0.04399	2.35619
0.06	7.89E-17	-1.45903	0	1.48E-18	-0.0583	2.35619
0.07	8.05E-17	-1.49306	0	2.2E-18	-0.07307	2.35619
0.08	8.19E-17	-1.51171	0	2.93E-18	-0.0881	2.35619
0.09	8.32E-17	-1.52186	0	3.67E-18	-0.10327	2.35619
0.1	8.45E-17	-1.52735	0	4.43E-18	-0.11851	2.35619
0.11	8.57E-17	-1.53032	0	5.19E-18	-0.1338	2.35619
0.12	8.69E-17	-1.53193	0	5.97E-18	-0.14911	2.35619
0.13	8.8E-17	-1.53279	0	6.76E-18	-0.16444	2.35619
0.14	8.92E-17	-1.53326	0	7.56E-18	-0.17977	2.35619
0.15	9.04E-17	-1.53351	0	8.38E-18	-0.1951	2.35619
0.16	9.15E-17	-1.53365	0	9.2E-18	-0.21044	2.35619
0.17	9.27E-17	-1.53372	0	1E-17	-0.22578	2.35619
0.18	9.39E-17	-1.53376	0	1.09E-17	-0.24111	2.35619
0.19	9.5E-17	-1.53378	0	1.17E-17	-0.25645	2.35619
0.2	9.62E-17	-1.53379	0	1.26E-17	-0.27179	2.35619
0.21	9.74E-17	-1.5338	0	1.35E-17	-0.28713	2.35619
0.22	9.85E-17	-1.5338	0	1.44E-17	-0.30246	2.35619
0.23	9.97E-17	-1.5338	0	1.53E-17	-0.3178	2.35619
0.24	1.01E-16	-1.5338	0	1.62E-17	-0.33314	2.35619
0.25	1.02E-16	-1.53381	0	1.71E-17	-0.34848	2.35619
0.26	1.03E-16	-1.53381	0	1.81E-17	-0.36382	2.35619
0.27	1.04E-16	-1.53381	0	1.9E-17	-0.37915	2.35619
0.28	1.06E-16	-1.53381	0	2E-17	-0.39449	2.35619
0.29	1.07E-16	-1.53381	0	2.1E-17	-0.40983	2.35619
0.3	1.08E-16	-1.53381	0	2.2E-17	-0.42517	2.35619
0.31	1.09E-16	-1.53381	0	2.3E-17	-0.44051	2.35619
0.32	1.1E-16	-1.53381	0	2.4E-17	-0.45585	2.35619
0.33	1.11E-16	-1.53381	0	2.5E-17	-0.47118	2.35619
0.34	1.13E-16	-1.53381	0	2.6E-17	-0.48652	2.35619
0.35	1.14E-16	-1.53381	0	2.71E-17	-0.50186	2.35619
0.36	4E-17	-1.53381	0	2.79E-17	-0.5172	2.35619

Table 8. Case #1 Output listing: CD, CL, CM, RE, ARAD

Time	CD	CL	CM	RE	ARAD
0.01	0.447368	0	0	16798.15	1.570796
0.02	0.435134	0	0	30182.25	1.570796
0.03	0.43029	0	0	40418.61	1.570796
0.04	0.427879	0	0	47603.66	1.570796
0.05	0.426592	0	0	52257.61	1.570796
0.06	0.425894	0	0	55075.14	1.570796
0.07	0.425515	0	0	56701.60	1.570796
0.08	0.42531	0	0	57613.02	1.570796
0.09	0.425199	0	0	58114.99	1.570796
0.10	0.425139	0	0	58388.77	1.570796
0.11	0.425107	0	0	58537.30	1.570796
0.12	0.42509	0	0	58617.65	1.570796
0.13	0.42508	0	0	58661.04	1.570796
0.14	0.425075	0	0	58684.45	1.570796
0.15	0.425072	0	0	58697.08	1.570796
0.16	0.425071	0	0	58703.89	1.570796
0.17	0.42507	0	0	58707.56	1.570796
0.18	0.42507	0	0	58709.54	1.570796
0.19	0.425069	0	0	58710.60	1.570796
0.20	0.425069	0	0	58711.18	1.570796
0.21	0.425069	0	0	58711.49	1.570796
0.22	0.425069	0	0	58711.65	1.570796
0.23	0.425069	0	0	58711.74	1.570796
0.24	0.425069	0	0	58711.79	1.570796
0.25	0.425069	0	0	58711.82	1.570796
0.26	0.425069	0	0	58711.83	1.570796
0.27	0.425069	0	0	58711.84	1.570796
0.28	0.425069	0	0	58711.85	1.570796
0.29	0.425069	0	0	58711.85	1.570796
0.30	0.425069	0	0	58711.85	1.570796
0.31	0.425069	0	0	58711.85	1.570796
0.32	0.425069	0	0	58711.85	1.570796
0.33	0.425069	0	0	58711.85	1.570796
0.34	0.425069	0	0	58711.85	1.570796
0.35	0.425069	0	0	58711.85	1.570796
0.36	0.425069	0	0	58711.85	1.570796
0.37	0.425069	0	0	58711.85	1.570796
0.38	0.425069	0	0	58711.85	1.570796
0.39	0.425069	0	0	58711.85	1.570796
0.40	0.425069	0	0	58711.85	1.570796

Table 8. Case #1 Output listing -Continued

Time	CD	CL	CM	RE	ARAD
0.41	0.425069	0	0	58711.85	1.570796
0.42	0.425069	0	0	58711.85	1.570796
0.43	0.425069	0	0	58711.85	1.570796
0.44	0.425069	0	0	58711.85	1.570796
0.45	0.425069	0	0	58711.85	1.570796
0.46	0.425069	0	0	58711.85	1.570796
0.47	0.425069	0	0	58711.85	1.570796
0.48	0.425069	0	0	58711.85	1.570796
0.49	0.425069	0	0	58711.85	1.570796
0.50	0.425069	0	0	58711.85	1.570796
0.51	0.425069	0	0	58711.85	1.570796
0.52	0.425069	0	0	58711.85	1.570796
0.53	0.425069	0	0	58711.85	1.570796
0.54	0.425069	0	0	58711.85	1.570796
0.55	0.425069	0	0	58711.85	1.570796
0.56	0.425069	0	0	58711.85	1.570796
0.57	0.425069	0	0	58711.85	1.570796
0.58	0.425069	0	0	58711.85	1.570796
0.59	0.425069	0	0	58711.85	1.570796
0.60	0.425069	0	0	58711.85	1.570796
0.61	0.425069	0	0	58711.85	1.570796
0.62	0.425069	0	0	58711.85	1.570796
0.63	0.425069	0	0	58711.85	1.570796
0.64	0.425069	0	0	58711.85	1.570796
0.65	0.425069	0	0	58711.85	1.570796
0.66	0.425069	0	0	58711.85	1.570796
0.67	0.425069	0	0	58711.85	1.570796
0.68	0.425069	0	0	58711.85	1.570796
0.69	0.425069	0	0	58711.85	1.570796
0.70	0.425069	0	0	58711.85	1.570796
0.71	0.425069	0	0	58711.85	1.570796
0.72	0.425069	0	0	58711.85	1.570796
0.73	0.425069	0	0	58711.85	1.570796
0.74	0.425069	0	0	58711.85	1.570796
0.75	0.425069	0	0	58711.85	1.570796
0.76	0.425069	0	0	58711.85	1.570796
0.77	0.425069	0	0	58711.85	1.570796
0.78	0.425069	0	0	58711.85	1.570796
0.79	0.425069	0	0	58711.85	1.570796
0.80	0.425069	0	0	58711.85	1.570796

Table 8. Case #1 Output listing -Continued

Time	CD	CL	CM	RE	ARAD
0.81	0.425069	0	0	58711.85	1.570796
0.82	0.425069	0	0	58711.85	1.570796
0.83	0.425069	0	0	58711.85	1.570796
0.84	0.425069	0	0	58711.85	1.570796
0.85	0.425069	0	0	58711.85	1.570796
0.86	0.425069	0	0	58711.85	1.570796
0.87	0.425069	0	0	58711.85	1.570796
0.88	0.425069	0	0	58711.85	1.570796
0.89	0.425069	0	0	58711.85	1.570796
0.90	0.425069	0	0	58711.85	1.570796
0.91	0.425069	0	0	58711.85	1.570796
0.92	0.425069	0	0	58711.85	1.570796
0.93	0.425069	0	0	58711.85	1.570796
0.94	0.425069	0	0	58711.85	1.570796
0.95	0.425069	0	0	58711.85	1.570796
0.96	0.425069	0	0	58711.85	1.570796
0.97	0.425069	0	0	58711.85	1.570796
0.98	0.425069	0	0	58711.85	1.570796
0.99	0.425069	0	0	58711.85	1.570796
1.00	0.425069	0	0	58711.85	1.570796

Table 9. Case #1 Output listing: BETA, ADEG, BRACKET1, BRACKET2

Time	BETA	ADEG	BRACKET1	BRACKET2
0.01	1.570796	90	2.74E-17	0.447368
0.02	1.570796	90	2.66E-17	0.435134
0.03	1.570796	90	2.63E-17	0.43029
0.04	1.570796	90	2.62E-17	0.427879
0.05	1.570796	90	2.61E-17	0.426592
0.06	1.570796	90	-6.8E-17	0.425894
0.07	1.570796	90	-6.8E-17	0.425515
0.08	1.570796	90	-6.8E-17	0.42531
0.09	1.570796	90	-6.8E-17	0.425199
0.10	1.570796	90	-6.8E-17	0.425139
0.11	1.570796	90	-6.8E-17	0.425107
0.12	1.570796	90	-6.8E-17	0.42509
0.13	1.570796	90	-6.8E-17	0.42508
0.14	1.570796	90	-6.8E-17	0.425075
0.15	1.570796	90	-6.8E-17	0.425072
0.16	1.570796	90	-6.8E-17	0.425071
0.17	1.570796	90	-6.8E-17	0.42507
0.18	1.570796	90	-6.8E-17	0.42507
0.19	1.570796	90	-6.8E-17	0.425069
0.20	1.570796	90	-6.8E-17	0.425069
0.21	1.570796	90	-6.8E-17	0.425069
0.22	1.570796	90	-6.8E-17	0.425069
0.23	1.570796	90	-6.8E-17	0.425069
0.24	1.570796	90	-6.8E-17	0.425069
0.25	1.570796	90	-6.8E-17	0.425069
0.26	1.570796	90	-6.8E-17	0.425069
0.27	1.570796	90	-6.8E-17	0.425069
0.28	1.570796	90	-6.8E-17	0.425069
0.29	1.570796	90	-6.8E-17	0.425069
0.30	1.570796	90	-6.8E-17	0.425069
0.31	1.570796	90	-6.8E-17	0.425069
0.32	1.570796	90	-6.8E-17	0.425069
0.33	1.570796	90	-6.8E-17	0.425069
0.34	1.570796	90	-6.8E-17	0.425069
0.35	1.570796	90	-6.8E-17	0.425069
0.36	1.570796	90	2.6E-17	0.425069
0.37	1.570796	90	2.6E-17	0.425069
0.38	1.570796	90	2.6E-17	0.425069
0.39	1.570796	90	2.6E-17	0.425069
0.40	1.570796	90	-6.8E-17	0.425069
0.41	1.570796	90	-6.8E-17	0.425069

Table 9. Case #1 Output listing - Continued

Time	BETA	ADEG	BRACKET1	BRACKET2
0.42	1.570796	90	-6.8E-17	0.425069
0.43	1.570796	90	-6.8E-17	0.425069
0.44	1.570796	90	-6.8E-17	0.425069
0.45	1.570796	90	-6.8E-17	0.425069
0.46	1.570796	90	-6.8E-17	0.425069
0.47	1.570796	90	-6.8E-17	0.425069
0.48	1.570796	90	-6.8E-17	0.425069
0.49	1.570796	90	-6.8E-17	0.425069
0.50	1.570796	90	-6.8E-17	0.425069
0.51	1.570796	90	-6.8E-17	0.425069
0.52	1.570796	90	-6.8E-17	0.425069
0.53	1.570796	90	-6.8E-17	0.425069
0.54	1.570796	90	-6.8E-17	0.425069
0.55	1.570796	90	-6.8E-17	0.425069
0.56	1.570796	90	-6.8E-17	0.425069
0.57	1.570796	90	-6.8E-17	0.425069
0.58	1.570796	90	-6.8E-17	0.425069
0.59	1.570796	90	-6.8E-17	0.425069
0.60	1.570796	90	-6.8E-17	0.425069
0.61	1.570796	90	-6.8E-17	0.425069
0.62	1.570796	90	-6.8E-17	0.425069
0.63	1.570796	90	-6.8E-17	0.425069
0.64	1.570796	90	-6.8E-17	0.425069
0.65	1.570796	90	-6.8E-17	0.425069
0.66	1.570796	90	-6.8E-17	0.425069
0.67	1.570796	90	-6.8E-17	0.425069
0.68	1.570796	90	-6.8E-17	0.425069
0.69	1.570796	90	-6.8E-17	0.425069
0.70	1.570796	90	-6.8E-17	0.425069
0.71	1.570796	90	-6.8E-17	0.425069
0.72	1.570796	90	2.6E-17	0.425069
0.73	1.570796	90	2.6E-17	0.425069
0.74	1.570796	90	2.6E-17	0.425069
0.75	1.570796	90	2.6E-17	0.425069
0.76	1.570796	90	-6.8E-17	0.425069
0.77	1.570796	90	-6.8E-17	0.425069
0.78	1.570796	90	-6.8E-17	0.425069
0.79	1.570796	90	-6.8E-17	0.425069
0.80	1.570796	90	-6.8E-17	0.425069
0.81	1.570796	90	-6.8E-17	0.425069

Table 9. Case #1 Output listing - Continued

Time	BETA	ADEG	BRACKET1	BRACKET2
0.82	1.570796	90	-6.8E-17	0.425069
0.83	1.570796	90	-6.8E-17	0.425069
0.84	1.570796	90	-6.8E-17	0.425069
0.85	1.570796	90	-6.8E-17	0.425069
0.86	1.570796	90	-6.8E-17	0.425069
0.87	1.570796	90	-6.8E-17	0.425069
0.88	1.570796	90	-6.8E-17	0.425069
0.89	1.570796	90	-6.8E-17	0.425069
0.90	1.570796	90	-6.8E-17	0.425069
0.91	1.570796	90	-6.8E-17	0.425069
0.92	1.570796	90	-6.8E-17	0.425069
0.93	1.570796	90	-6.8E-17	0.425069
0.94	1.570796	90	-6.8E-17	0.425069
0.95	1.570796	90	-6.8E-17	0.425069
0.96	1.570796	90	-6.8E-17	0.425069
0.97	1.570796	90	-6.8E-17	0.425069
0.98	1.570796	90	-6.8E-17	0.425069
0.99	1.570796	90	-6.8E-17	0.425069
1.00	1.570796	90	-6.8E-17	0.425069

Table 10. Case #1 Output listing: MAGOBJ, UOBJX, UOBJY

Time	MAGOBJ	UOBJX	UOBJY
0.01	0.43884	2.67E-18	-1
0.02	0.78849	9.89E-18	-1
0.03	1.055907	1.84E-17	-1
0.04	1.243612	2.9E-17	-1
0.05	1.365193	4.15E-17	-1
0.06	1.438799	5.57E-17	-1
0.07	1.481289	1.15E-16	-1
0.08	1.505099	1.17E-16	-1
0.09	1.518213	1.18E-16	-1
0.1	1.525365	1.19E-16	-1
0.11	1.529245	1.2E-16	-1
0.12	1.531344	1.21E-16	-1
0.13	1.532478	1.22E-16	-1
0.14	1.533089	1.23E-16	-1
0.15	1.533419	1.23E-16	-1
0.16	1.533597	1.24E-16	-1
0.17	1.533693	1.25E-16	-1
0.18	1.533745	1.26E-16	-1
0.19	1.533773	1.26E-16	-1
0.2	1.533788	1.27E-16	-1
0.21	1.533796	1.28E-16	-1
0.22	1.5338	1.29E-16	-1
0.23	1.533802	1.29E-16	-1
0.24	1.533804	1.3E-16	-1
0.25	1.533804	1.31E-16	-1
0.26	1.533805	1.32E-16	-1
0.27	1.533805	1.32E-16	-1
0.28	1.533805	1.33E-16	-1
0.29	1.533805	1.34E-16	-1
0.3	1.533805	1.35E-16	-1
0.31	1.533805	1.35E-16	-1
0.32	1.533805	1.36E-16	-1
0.33	1.533805	1.37E-16	-1
0.34	1.533805	1.38E-16	-1
0.35	1.533805	1.39E-16	-1
0.36	1.533805	2.61E-17	-1
0.37	1.533805	4.44E-17	-1
0.38	1.533805	-5.1E-17	-1
0.39	1.533805	3.22E-17	-1
0.4	1.533805	5.05E-17	-1
0.41	1.533805	1.16E-16	-1
0.42	1.533805	1.16E-16	-1

Table 10. Case #1 Output listing - Continued

Time	MAGOBJ	UOBJX	UOBJY
0.43	1.533805	1.17E-16	-1
0.44	1.533805	1.18E-16	-1
0.45	1.533805	1.19E-16	-1
0.46	1.533805	1.19E-16	-1
0.47	1.533805	1.2E-16	-1
0.48	1.533805	1.21E-16	-1
0.49	1.533805	1.22E-16	-1
0.5	1.533805	1.22E-16	-1
0.51	1.533805	1.23E-16	-1
0.52	1.533805	1.24E-16	-1
0.53	1.533805	1.25E-16	-1
0.54	1.533805	1.25E-16	-1
0.55	1.533805	1.26E-16	-1
0.56	1.533805	1.27E-16	-1
0.57	1.533805	1.28E-16	-1
0.58	1.533805	1.29E-16	-1
0.59	1.533805	1.29E-16	-1
0.6	1.533805	1.3E-16	-1
0.61	1.533805	1.31E-16	-1
0.62	1.533805	1.32E-16	-1
0.63	1.533805	1.32E-16	-1
0.64	1.533805	1.33E-16	-1
0.65	1.533805	1.34E-16	-1
0.66	1.533805	1.35E-16	-1
0.67	1.533805	1.35E-16	-1
0.68	1.533805	1.36E-16	-1
0.69	1.533805	1.37E-16	-1
0.7	1.533805	1.38E-16	-1
0.71	1.533805	1.38E-16	-1
0.72	1.533805	2.6E-17	-1
0.73	1.533805	4.43E-17	-1
0.74	1.533805	-5.1E-17	-1
0.75	1.533805	3.21E-17	-1
0.76	1.533805	5.03E-17	-1
0.77	1.533805	1.15E-16	-1
0.78	1.533805	1.16E-16	-1
0.79	1.533805	1.17E-16	-1
0.8	1.533805	1.18E-16	-1
0.81	1.533805	1.19E-16	-1
0.82	1.533805	1.19E-16	-1
0.83	1.533805	1.2E-16	-1

Table 10. Case #1 Output listing - Continued

Time	MAGOBJ	UOBJX	UOBJY
0.84	1.533805	1.21E-16	-1
0.85	1.533805	1.22E-16	-1
0.86	1.533805	1.22E-16	-1
0.87	1.533805	1.23E-16	-1
0.88	1.533805	1.24E-16	-1
0.89	1.533805	1.25E-16	-1
0.9	1.533805	1.25E-16	-1
0.91	1.533805	1.26E-16	-1
0.92	1.533805	1.27E-16	-1
0.93	1.533805	1.28E-16	-1
0.94	1.533805	1.28E-16	-1
0.95	1.533805	1.29E-16	-1
0.96	1.533805	1.3E-16	-1
0.97	1.533805	1.31E-16	-1
0.98	1.533805	1.31E-16	-1
0.99	1.533805	1.32E-16	-1
1.00	1.533805	1.33E-16	-1

Table 11. Case #1 Output listing: MAGVREL, URELX, and URELY

Time	MAGVREL	URELX	URELY
0.01	0.43884	-2.7E-18	1
0.02	0.78849	-9.9E-18	1
0.03	1.055907	-1.8E-17	1
0.04	1.243612	-2.9E-17	1
0.05	1.365193	-4.1E-17	1
0.06	1.438799	-5.6E-17	1
0.07	1.481289	-1.2E-16	1
0.08	1.505099	-1.2E-16	1
0.09	1.518213	-1.2E-16	1
0.1	1.525365	-1.2E-16	1
0.11	1.529245	-1.2E-16	1
0.12	1.531344	-1.2E-16	1
0.13	1.532478	-1.2E-16	1
0.14	1.533089	-1.2E-16	1
0.15	1.533419	-1.2E-16	1
0.16	1.533597	-1.2E-16	1
0.17	1.533693	-1.2E-16	1
0.18	1.533745	-1.3E-16	1
0.19	1.533773	-1.3E-16	1
0.2	1.533788	-1.3E-16	1
0.21	1.533796	-1.3E-16	1
0.22	1.5338	-1.3E-16	1
0.23	1.533802	-1.3E-16	1
0.24	1.533804	-1.3E-16	1
0.25	1.533804	-1.3E-16	1
0.26	1.533805	-1.3E-16	1
0.27	1.533805	-1.3E-16	1
0.28	1.533805	-1.3E-16	1
0.29	1.533805	-1.3E-16	1
0.3	1.533805	-1.3E-16	1
0.31	1.533805	-1.4E-16	1
0.32	1.533805	-1.4E-16	1
0.33	1.533805	-1.4E-16	1
0.34	1.533805	-1.4E-16	1
0.35	1.533805	-1.4E-16	1
0.36	1.533805	-2.6E-17	1
0.37	1.533805	-4.4E-17	1
0.38	1.533805	5.05E-17	1
0.39	1.533805	-3.2E-17	1
0.4	1.533805	-5E-17	1
0.41	1.533805	-1.2E-16	1
0.42	1.533805	-1.2E-16	1

Table 11. Case #1 Output listing - Continued

Time	MAGVREL	URELX	URELY
0.43	1.533805	-1.2E-16	1
0.44	1.533805	-1.2E-16	1
0.45	1.533805	-1.2E-16	1
0.46	1.533805	-1.2E-16	1
0.47	1.533805	-1.2E-16	1
0.48	1.533805	-1.2E-16	1
0.49	1.533805	-1.2E-16	1
0.5	1.533805	-1.2E-16	1
0.51	1.533805	-1.2E-16	1
0.52	1.533805	-1.2E-16	1
0.53	1.533805	-1.2E-16	1
0.54	1.533805	-1.3E-16	1
0.55	1.533805	-1.3E-16	1
0.56	1.533805	-1.3E-16	1
0.57	1.533805	-1.3E-16	1
0.58	1.533805	-1.3E-16	1
0.59	1.533805	-1.3E-16	1
0.6	1.533805	-1.3E-16	1
0.61	1.533805	-1.3E-16	1
0.62	1.533805	-1.3E-16	1
0.63	1.533805	-1.3E-16	1
0.64	1.533805	-1.3E-16	1
0.65	1.533805	-1.3E-16	1
0.66	1.533805	-1.3E-16	1
0.67	1.533805	-1.4E-16	1
0.68	1.533805	-1.4E-16	1
0.69	1.533805	-1.4E-16	1
0.7	1.533805	-1.4E-16	1
0.71	1.533805	-1.4E-16	1
0.72	1.533805	-2.6E-17	1
0.73	1.533805	-4.4E-17	1
0.74	1.533805	5.06E-17	1
0.75	1.533805	-3.2E-17	1
0.76	1.533805	-5E-17	1
0.77	1.533805	-1.2E-16	1
0.78	1.533805	-1.2E-16	1
0.79	1.533805	-1.2E-16	1
0.8	1.533805	-1.2E-16	1
0.81	1.533805	-1.2E-16	1
0.82	1.533805	-1.2E-16	1
0.83	1.533805	-1.2E-16	1

Table 11. Case #1 Output listing - Continued

Time	MAGVREL	URELX	URELY
0.84	1.533805	-1.2E-16	1
0.85	1.533805	-1.2E-16	1
0.86	1.533805	-1.2E-16	1
0.87	1.533805	-1.2E-16	1
0.88	1.533805	-1.2E-16	1
0.89	1.533805	-1.2E-16	1
0.9	1.533805	-1.3E-16	1
0.91	1.533805	-1.3E-16	1
0.92	1.533805	-1.3E-16	1
0.93	1.533805	-1.3E-16	1
0.94	1.533805	-1.3E-16	1
0.95	1.533805	-1.3E-16	1
0.96	1.533805	-1.3E-16	1
0.97	1.533805	-1.3E-16	1
0.98	1.533805	-1.3E-16	1
0.99	1.533805	-1.3E-16	1
1.00	1.533805	-1.3E-16	1

Figure 5. Small sphere (1" diameter) falling in still water: Vy^* and y^*

Figure 6. Small sphere (1" diameter) falling in still water: V_{y^*} and C_D

Figure 7. Small sphere (1" diameter) falling in still water: Reynolds number.

Appendix A-2

TEST CASE #2 LARGE SPHERE (8.5" DIAMETER BOWLING BALL) FALLING IN STILL WATER

**Table 12. Input Listing for Case #2: Large sphere
(8.5" diameter bowling ball)**

```

&INPUT
ALPHA    = 0.0
AP        = 0.394063
AREF      = 0.394063
CD         = 1.0
CDAVE     = 1.0
CL         = 0.0
CM         = 0.0
DENF       = 62.43
DENO       = 85.9822
GZERO      = 32.2
H          = 0.005
INERTIA    = 0.041585
LD         = 0.3541667
LM         = 0.3541667
M          = 0
N          = 6
OBJDIM     = 0.7083333
PI         = 0.31415926535E+01
TANKL      = 40.
VISCOS     = 3.229E-05
VOL         = 0.186085
VFX        = 0.0
VFY        = 0.0
X          = 0.0
XLIM       = 1.0
Y(1)       = 0.0
Y(2)       = 0.0
Y(3)       = 0.0
Y(4)       = 0.0
Y(5)       = 0.0
Y(6)       = 2.35619
Y(7)       = 0.0
Y(8)       = 0.0
Y(9)       = 0.0
Y(10)      = 0.0
/

```

Figure 8. Large sphere (8.5" diameter bowling ball) falling in still water V_{y^*} and C_D

Figure 9. Large sphere (8.5" diameter bowling ball) falling in still water V_{y^*} and y^*

Figure 10. Large sphere (8.5" diameter bowling ball) falling in still water: Reynolds Number

**Table 13. Input Listing for Case #2: Large sphere
(8.5" diameter bowling ball) with an initial side velocity
of 5 ft/s**

```

&INPUT
ALPHA    = 0.0
AP       = 0.394063
AREF     = 0.394063
CD       = 1.0
CDAVE    = 1.0
CL       = 0.0
CM       = 0.0
DENF     = 62.43
DENO     = 85.9822
GZERO    = 32.2
H        = 0.005
INERTIA  = 0.041585
LD       = 0.3541667
LM       = 0.3541667
M        = 0
N        = 6
OBJDIM   = 0.7083333
PI       = 0.31415926535E+01
TANKL    = 40.
VISCOS   = 3.229E-05
VOL      = 0.186085
VFX      = 0.0
VFX      = 0.0
VFX      = 0.0
X        = 0.0
XLIM     = 1.0
Y(1)     = 5.0
Y(2)     = 0.0
Y(3)     = 0.0
Y(4)     = 0.0
Y(5)     = 0.0
Y(6)     = 2.35619
Y(7)     = 0.0
Y(8)     = 0.0
Y(9)     = 0.0
Y(10)    = 0.0
/

```

Figure 11. Large sphere (8.5" diameter bowling ball) falling in still water with an initial velocity (5 ft/s) in the positive x-axis direction: horizontal velocity, V_{x^*}

Figure 12. Large sphere (8.5" diameter bowling ball) falling in still water with an initial velocity (5 ft/s) in the positive x-axis direction: vertical velocity, V_y^*

Figure 13. Large sphere (8.5" diameter bowling ball) falling in still water with an initial velocity (5 ft/s) in the positive x-axis direction: location within the plane: (y^* vs. x^*)

Appendix A-3

Additional Simulation

**Large Sphere (8.5" Diameter) falling in a
horizontal flow velocity field**

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Figure A3-1

Large sphere (8.5" diameter bowling ball) falling in water
with a velocity of 5 ft/s in the positive x direction:
 V_x^* and V_y^*

Figure A3-2

Large sphere (8.5" diameter bowling ball) falling in water with a velocity of 5 ft/s in the positive x direction: x^* and y^* as a function of t^*

Figure A3-3. Large sphere (8.5" diameter bowling ball) falling in water with a velocity of 5 ft/s in the positive x direction: object location x^* vs. y^*

Figure A3-4. Large sphere (8.5" diameter bowling ball) falling in water with a velocity of 2 ft/s in the positive x direction: Reynolds number and C_D

Figure A3-5

Large sphere (8.5" diameter bowling ball) falling in water with a velocity of 2 ft/s in the positive x direction:
 V_x^* and V_y^*

Figure A3-6 Large sphere (8.5" diameter bowling ball) falling in water with a velocity of 2 ft/s in the positive x direction: x^* and y^* as a function of t^*

Figure A3-7. Large sphere (8.5" diameter bowling ball) falling in water with a velocity of 2 ft/s in the positive x direction: object location x^* vs. y^*

Figure A3—8. Large sphere (8.5" diameter bowling ball) falling in water with a velocity of 2 ft/s in the positive x direction: Reynolds number and C_D

Appendix A-4
FORTRAN PROGRAM LISTING

AEROCOEF

```
      SUBROUTINE AEROCOEF (RE,CD,CL,CM,ALPHA)
C
C
      DOUBLE PRECISION RE, CD, CL, CM, ALPHA
C
      IF(RE .LE. 0.0)THEN
        CD = DBLE(10000.)
      ELSE
        ONE = DBLE(24.) / RE
        TWO = DBLE(6.0) / (DBLE(1.0) + DSQRT(RE) )
        CD = ONE + TWO + DBLE(0.4)
      END IF
C
      CL = DBLE(0.0)
      CM = DBLE(0.0)
C
C
C
      RETURN
      END
```

ATTACK

```
      SUBROUTINE ATTACK (UOBJX,UOBJY,UFLOWX,UFLOWY,MAGVOBJ
+ ,MAGVREL,ARAD,ADEG,PI)
C
C
C      REAL MAGVOBJ, MAGVREL
      DOUBLE PRECISION UOBJX, UOBJY, UFLOWX, UFLOWY,
+ MAGVOBJ, MAGVREL, ARAD, ADEG, PI, VODOTVF,
+ DENOM, RATIO

C
C      VODOTVF = (UOBJX*UFLOWX) + (UOBJY*UFLOWY)
      DENOM    = MAGVOBJ * MAGVREL
      IF (DENOM) 10,10,20
C
10  CONTINUE
C
      RATIO    = 0.
      GO TO 30
C
20  CONTINUE
C
      RATIO    = VODOTVF / DENOM
C
30  CONTINUE
C
      ARAD     = DACOS(RATIO)
      ADEG     = ( DBLE(180.) / PI ) * ARAD
C
C
C      RETURN
      END
```

BRACKET

```
      SUBROUTINE BRACKET (CD,CL,BRACKET1,BRACKET2,BETA)
C
C
      DOUBLE PRECISION BETA, BRACKET1, BRACKET2, T1, T2,
+      CD, CL
C
      T1 = DSIN (BETA)
      T2 = DCOS (BETA)
C
C
      BRACKET1 = ( CD * T2 ) - ( CL * T1 )
      BRACKET2 = ( CD * T1 ) + ( CL * T2 )
C
C
C
      RETURN
      END
```


FLOWVEL

```
      SUBROUTINE FLOWVEL (Y,VFX,VFY,UFLOWX,UFLOWY,MAGFLOW)
C
C
      DIMENSION Y(10)
C      REAL MAGFLOW
      DOUBLE PRECISION
VXSQ,VYSQ,VFX,VFY,SUMSQ,UFLOWX,UFLOWY,
      +                MAGFLOW, Y
C
C
C   FOR NOW VFX AND VFY ARE SET TO ZERO UNTIL A FUNCTION
C   TABLE IS MADE 5-23-93 RSD
C
C
      VXSQ  = VFX * VFX
      VYSQ  = VFY * VFY
      SUMSQ = VXSQ + VYSQ
C
C
      IF (SUMSQ) 10,10,20
C
C
10  CONTINUE
C
      UFLOWX = 0.
      UFLOWY = 0.
      MAGFLOW = 0.
      GO TO 30
C
20  CONTINUE
C
      MAGFLOW = DSQRT(SUMSQ)
      UFLOWX = VFX/MAGFLOW
      UFLOWY = VFY/MAGFLOW
      GO TO 30
C
30  CONTINUE
C
      RETURN
      END
```

PROGRAM LPA

```

C
C
C-----
C
C      THIS PROGRAM CALCULATES THE DYNAMIC MOTION OF AN
C      OBJECT FALLING IN A GIVEN VELOCITY FIELD.  THE OBJECT
C      IS ASSUMED RIGID WITH PLANER MOTION (THREE DEGREES
C      OF FREEDOM).  THE FOLLOWING VARIABLES ARE USED WITHIN
C      THE PROGRAM:
C
C      N = NUMBER OF DIFFERENTIAL EQUATIONS TO BE SOLVED (SET
C           BY THE USER) .
C
C      Y = ARRAY OF N DEPENDENT VARIABLES (WITH INITIAL VALUES
C           SET BY THE USER) .
C
C      F = ARRAY OF THE N DERIVATIVES OF THE VARIABLES Y [THE
C           USER MUST PROVIDE AN EXPRESSION IN THE MAIN PROGRAM
C           FOR THE CALCULATION OF EACH F(I)]
C
C      X = INDEPENDENT VARIABLE (INITIALIZED BY THE USER) .
C
C      H = STEP SIZE DELTA X (SET BY THE USER) .
C
C      M = INDEX USED IN THE SUBROUTINE WHICH MUST BE SET EQUAL
C           TO ZERO BY THE USER BEFORE FIRST CALL.
C
C      K = INTEGER FROM THE SUBROUTINE WHICH IS USED AS THE
C           ARGUMENT OF A COMPUTED GO TO STATEMENT IN THE MAIN
C           PROGRAM, SUCH AS GO TO (10,20),K.  STATEMENT 10
C           CALCULATES THE DERIVATIVES F(I) AND STATEMENT 20
C           PRINTS THE ANSWERS, X AND Y(I) .
C-----
C
C      DIMENSION Y(10), F(10)
C
C      REAL LM,LD,INERTIA,MAGVREL,MAGVOBJ,MAGFLOW,MAGOBJ
C
C      DOUBLE PRECISION ALPHA, BRACKET1, BRACKET2, AP, AREF,
C      +CD, CDAVE, CL, CM, DENF, DENO, COEF1, COEF2, COEF3,
C      +COEF4, ARAD, ADEG, BETA, F, GZERO, H, LM, LD,
C      +INERTIA, MAGVREL,MAGVOBJ, MAGFLOW, MAGOBJ, PI, OBJDIM,
C      +RE, TANKL, VISCOS, VOL, VFX, VFY, VTERM, UFLOWX, UFLOWY,
C      +UOBJX, UOBJY, URELX, URELY, Y, X, XLIM
C
C      NAMEDLIST /INPUT/ ALPHA, AP, AREF, CD, CDAVE,CL,CM,
C      +          DENF, DENO, GZERO, H, INERTIA, LD, LM,
C      +          M, N, OBJDIM, PI, TANKL, VISCOS, VOL, VFX, VFY,
C      +          X, XLIM, Y
C
C
C
C
C

```

```

C
C
C
OPEN(UNIT=10,FILE='B:\DATAIN.DAT',STATUS='OLD')
OPEN(UNIT=20,FILE='B:\OUTPUT.DAT',STATUS='OLD')
OPEN(UNIT=15,FILE='B:\OUTPUT2.DAT',STATUS='OLD')
OPEN(UNIT=16,FILE='B:\OUTPUT3.DAT',STATUS='OLD')
OPEN(UNIT=17,FILE='B:\OUTPUT4.DAT',STATUS='OLD')
OPEN(UNIT=18,FILE='B:\OUTPUT5.DAT',STATUS='OLD')
C
C
READ(UNIT=10,NML=INPUT)
WRITE(UNIT=20,NML=INPUT)
C
C
CALL SETUP (COEF1,COEF2,COEF3,COEF4,LM,LD,INERTIA,
+ DENF,DENO,TANKL,AREF,VOL,AP,GZERO)
C
WRITE(20,50) COEF1
50 FORMAT(2X,'COEF1 = ',2X,E10.5)
C
WRITE(20,60) COEF2
60 FORMAT(2X,'COEF2 = ',2X,E10.5)
C
WRITE(20,70) COEF3
70 FORMAT(2X,'COEF3 = ',2X,E10.5)
C
WRITE(20,80) COEF4
80 FORMAT(2X,'COEF4 = ',2X,E10.5)
C
VTERM = DSQRT(((2*VOL*32.2)/(CDAVE*AP*DENF))*(DENO-DENF)))
C
WRITE(20,90) VTERM
90 FORMAT(2X,'VTERM = ',2X,E10.5)
C
8 CONTINUE
IF(X - XLIM)6,6,7
6 CONTINUE
C
CALL FLOWVEL (Y,VFX,VFY,UFLOWX,UFLOWY,MAGFLOW)
C
CALL VOBJECT (Y,MAGOBJ,UOBJX,UOBJY)
C
CALL RELVEL (VFX,VFY,URELX,URELY,MAGVREL,Y)
C
CALL REYNOLDS (VTERM,MAGVREL,DENF,OBJDIM,VISCOS,RE)
C
CALL ATTACK (UOBJX,UOBJY,UFLOWX,UFLOWY,MAGVOBJ,MAGVREL,
+ ARAD,ADEG,PI)
C
CALL AEROCOEF (RE,CD,CL,CM,ARAD)
C
CALL ROTATE (URELX,URELY,BETA)
C
CALL BRACKET (CD,CL,BRACKET1,BRACKET2,BETA)
C
CALL RUNGE (N,Y,F,X,H,M,K)
C
GO TO (10,20),K
C

```

```

10 CONTINUE
C
  F(1) = 0.5*COEF1*COEF2*BRACKET1*MAGVREL*MAGVREL
  F(2) = ( (0.5*COEF1*COEF2*BRACKET2*MAGVREL*MAGVREL)
&        - ( (CDAVE/2.0)*COEF1*COEF3) )
  F(3) = COEF4*CM*MAGVREL*MAGVREL
  F(4) = Y(1)
  F(5) = Y(2)
  F(6) = (TANKL/LD) * Y(3)
  GO TO 6
C
20 CONTINUE
C
  WRITE(20,5) X, (Y(I), I = 1, N)
5  FORMAT(2X, F15.8, 2X, 6E14.7)
C
  WRITE(15,9) X, CD, CL, CM, RE, ARAD
  WRITE(16,9) X, BETA, ADEG, BRACKET1, BRACKET2
  WRITE(17,9) X, MAGOBJ, UOBJX, UOBJY
  WRITE(18,9) X, MAGVREL, URELX, URELY
C
9  FORMAT(2X, 6E14.7)
C
C
  GO TO 8
C
7 CONTINUE
C
  STOP
  END

```

READIN

```
      SUBROUTINE READIN (DENF,DENO,TANKL,AREF,VOL,CD,CL,CM,  
+CDAVE,AP,LM,LD,INERTIA,GZERO,Y)  
C  
C  
      DIMENSION Y(6)  
      REAL LM,LD,INERTIA  
C  
C  
      READ(10,5) DENF,DENO, TANKL,AREF,VOL,CD  
      READ(10,5) CL,CM,CDAVE,AP,LM,LD  
      READ(10,6) INERTIA,GZERO  
5  FORMAT (6E10.5)  
6  FORMAT (2E10.5)  
C  
C  
C  
      RETURN  
      END
```

RELVEL

```
      SUBROUTINE RELVEL (VFX,VFY,URELX,URELY,MAGVREL,Y)
C
C
      DIMENSION Y(10)
C      REAL MAGVREL
      DOUBLE PRECISION VFX, VFY, URELX, URELY, MAGVREL, Y,
+          SUMSQ, VRELX, VRELY
C
      VRELX = VFX - Y(1)
      VRELY = VFY - Y(2)
      SUMSQ = ( VRELX * VRELX ) + ( VRELY * VRELY )
C
      IF (SUMSQ)10,10,20
C
10  CONTINUE
C
      URELX = DBLE(0.)
      URELY = DBLE(0.)
      MAGVREL = DBLE(0.)
      GO TO 30
C
20  CONTINUE
C
      MAGVREL = DSQRT(SUMSQ)
      URELX = VRELX / MAGVREL
      URELY = VRELY / MAGVREL
C
30  CONTINUE
C
C
      RETURN
      END
```

REYNOLDS

```
      SUBROUTINE REYNOLDS
(VTERM,MAGVREL,DENF,OBJDIM,VISCOS,RE)
C
C
      DIMENSION Y(10)
      DOUBLE PRECISION VTERM,MAGVREL,DENF,OBJDIM,VISCOS,RE
C
C
C
C
      RE
= (DABS (MAGVREL*VTERM) *DENF*OBJDIM) / (VISCOS*DBLE (32.2) )
C
C
C
C
      RETURN
      END
```

ROTATE

```
      SUBROUTINE ROTATE (URELX, URELY, BETA)
C
C
C
C
      DOUBLE PRECISION URELX, URELY, BETA
C
C
      BETA = DACOS ( (URELX * DBLE(1.) ) )
C
C
C
      RETURN
      END
```


RUNGE

```

C      SUBROUTINE RUNGE (N,Y,F,X,H,M,K)
C      THIS ROUTINE PERFORMS RUNGE-KUTTA CALCULATION
C      BY GILLS METHOD.  TAKEN FROM APPENDIX C OF
C      WHITE'S VISCOUS FLUID FLOW TEXT.
C
C      DIMENSION Y(10), F(10), Q(10)
C
C      *****  ADDED 11-30-93  *****
C
C      DOUBLE PRECISION Y, F, X, H
C
C      *****  ADDED 11-30-93  *****
C
C      M = M + 1
C      GO TO (1,4,5,3,7), M
1 DO 2 I = 1, N
  Q(I) = 0
2 CONTINUE
  A = DBLE(0.5)
  GO TO 9
3 A = DBLE(1.7071067811865475244)
C
C      IF YOU NEED MORE ACCURACY, USE
C      A = 1.7071067811865475244
C
4 X = X + DBLE(0.5) * H
5 DO 6 I = 1, N
  Y(I) = Y(I) + A*(F(I)*H - Q(I))
  Q(I) = DBLE(2.0)*A*H*F(I) + (DBLE(1.0) - DBLE(3.0)
+      *A)*Q(I)
C
6 CONTINUE
  A = DBLE(0.29289323188134524756)
C
C      IF YOU NEED MORE ACCURACY, SET
C      A = 0.29289323188134524756
C
C      GO TO 9
7 DO 8 I = 1, N
  Y(I) = Y(I) + H*F(I)/DBLE(6.0) - Q(I)/DBLE(3.0)
8 CONTINUE
  M = 0
  K = 2
  GO TO 10
9 K = 1
10 CONTINUE
  RETURN
  END

```

SETUP

```
      SUBROUTINE SETUP
      (COEF1, COEF2, COEF3, COEF4, LM, LD, INERTIA,
      +DENF, DENO, TANKL, AREF, VOL, AP, GZERO)
C
C      REAL  LM, LD, INERTIA
      DOUBLE PRECISION COEF1, COEF2, COEF3, COEF4, LM, LD,
      +                      INERTIA, DENF, DENO, TANKL, AREF,
      +                      VOL, AP, GZERO
C
C      COEF1 = ( DENF / DENO )
      COEF2 = (TANKL * AREF ) / VOL
      COEF3 = (TANKL * AP ) / VOL
      COEF4 =
      (DENF*TANKL*LM*LD*AREF) / (DBLE(2.0)*GZERO*INERTIA)
C
C      WRITE(20,5) COEF1
      5 FORMAT(2X, 'COEF1 = ', 2X, E10.5)
C
C      WRITE(20,6) COEF2
      6 FORMAT(2X, 'COEF2 = ', 2X, E10.5)
C
C      WRITE(20,7) COEF3
      7 FORMAT(2X, 'COEF3 = ', 2X, E10.5)
C
C      WRITE(20,8) COEF4
      8 FORMAT(2X, 'COEF4 = ', 2X, E10.5)
C
C      RETURN
      END
```

VOBJECT

```
      SUBROUTINE VOBJECT (Y,MAGOBJ,UOBJX,UOBJY)
C
C
      DIMENSION Y(10)
C      REAL MAGOBJ
      DOUBLE PRECISION Y, MAGOBJ, UOBJX, UOBJY, VXSQ, VYSQ,
+      SUMSQ
C
C      VXSQ = Y(1) * Y(1)
      VYSQ = Y(2) * Y(2)
      SUMSQ = VXSQ + VYSQ
C
      IF (SUMSQ) 10,10,20
C
10  CONTINUE
      UOBJX = DBLE(0.)
      UOBJY = DBLE(0.)
      MAGOBJ = DBLE(0.)
      GO TO 30
C
20  CONTINUE
C
      MAGOBJ = DSQRT(SUMSQ)
      UOBJX = Y(1) / MAGOBJ
      UOBJY = Y(2) / MAGOBJ
C
30  CONTINUE
C
C
C
      RETURN
      END
```

WRITEOUT

```
      SUBROUTINE WRITEOUT
      (DENF,DENO,TANKL,AREF,VOL,CD,CL,CM,
      +CDAVE,AP,LM,LD,INERTIA,GZERO,Y)
C
C
      DIMENSION Y(6)
      REAL LM,LD,INERTIA
C
C
      WRITE(20,5) DENF,DENO, TANKL,AREF,VOL,CD
      WRITE(20,5) CL,CM,CDAVE,AP,LM,LD
      WRITE(20,6) INERTIA,GZERO
      WRITE(20,5) Y(1), Y(2), Y(3), Y(4), Y(5), Y(6)
5  FORMAT (6E10.5)
6  FORMAT (2E10.5)
C
C
C
      RETURN
      END
```