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A compartment method for space debris analysis

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**A Compartment Method
for
Space Debris Analysis**

A Thesis

Presented to

The Faculty of the Department of Physics

San Jose State University

In Partial Fulfillment

of the Requirements for the Degree

of Master of Science

by Arthur F. Hornsby

May 1998

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ABSTRACT

A Compartment Method for Space Debris Analysis

by Arthur F. Hornsby

We have developed and implemented a computational method for analyzing the distribution and collision potential of objects in earth orbit. We have tested the method for several trial orbits against the space object database maintained by the United States Space Command. We have displayed the time-averaged density of these objects within three different altitude bands in low Earth orbit, generated collision probability estimates, and made predictions as to where a collision event is most likely to occur.

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List of Acronyms and Abbreviations

CRRES	Combined Release and Radiation Effects Satellite
elsets	Two Line Mean Element Set
ESA	European Space Agency
KTG	Kinetic Theory of Gasses
km	Kilometer
LEO	Low Earth Orbit
NORAD	North American Aerospace Defense Command
RAAN	Right Ascension of Ascending Node
RCS	Radar Cross-Section
R/B	Rocket Body
RSO	Resident Space Object
SDP4	SGP Algorithm modified for deep space objects
SGP	Simplified General Perturbations
SGP4	Modified SGP Algorithm
SV	Space Vehicle
TEME	True Equator Mean Equinox
USSPACECOM	United States Space Command
USSR	Union of Soviet Socialist Republics
WGS	World Geophysical Survey
2LMES	Two Line Mean Element Set

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1. Introduction

1.1 Purpose of Present Work.

In recent years, governments and corporations have invested considerable sums to develop large constellations of low-earth satellites for national defense or telecommunications applications. A common risk inherent in these types of missions is that of collisions with other satellites or space debris. The most acute risk occurs at the highly-populated bands near 800 and 1000 km altitude.

Figure 1.1 Debris spatial density in Low Earth Orbit (LEO).

Analysis indicates that a high velocity collision in one of these bands would destroy not only the two objects directly involved, but could result in a process dubbed “collisional cascading,” in which the newly produced fragmentation debris results

in further collisions in a slow chain reaction, soon rendering the entire altitude band unusable for satellite missions (Kessler and Cour-Palais, 1978). Computing the probability of such an event is therefore of interest to scientists, government leaders, businessmen and environmentalists.

In general, predicting whether two satellites will collide requires accurate knowledge of their orbital state a short time prior to the anticipated collision. This knowledge must include a highly detailed force model which describes the perturbations from the two-body solution due primarily to Earth gravitational irregularities and atmospheric drag.

The obvious brute force method of calculating and comparing the ephemerides of every known orbiting object has a severe limitation in practice. Because of uncertainties in satellite state measurements and poor atmospheric drag estimates, satellite positions are only known to a specified tolerance. The largest positional uncertainty is along a satellite's flight path and is termed the in-track error. In-track error is typically on the order of tens of kilometers for all but the most closely tracked satellites in low-earth orbit. As they are less sensitive to drag forces, the cross-track and radial errors are generally smaller, often by an order of magnitude.

Methods that attempt to predict the probability of individual collision events are unusable in practice because a collision can rarely be predicted with a confidence of greater than one in a million (Mullikin et al., 1995). Thus, the problem of quantifying the space debris collision hazard is only solvable in a statistical sense.

This thesis presents a stochastic method for studying the collision potential within a particular altitude band, such as one planned to contain a constellation of satellites of low eccentricity. We have constructed a matrix of cubical compartments which physically span a specified altitude band and have calculated the space debris

density within each compartment over a time span of several days. In this way, we have created a database which consists of a number of compartments, each of which has an associated number proportional to the probability of collision within that compartment. This database allowed us to calculate the overall collision probabilities for several test satellites by propagating their orbital elements and averaging the collision potential of all compartments visited.

1.2 The Space Debris Environment.

For all but the last 40 years or so, the only objects penetrating the near-earth space environment have been meteoroids, traveling at inter-planetary speeds averaging 20 km/s. The environment within 2000 km of the earth's surface contained then as now some 200 kg of meteoroid mass, a number that fluctuates somewhat predictably with the seasons, as the earth orbits the sun. The vast bulk of near-earth meteoroids are represented by objects .01 cm in diameter and less (National Science and Technology Council, 1995).

During the last 4 decades however, mankind has executed over 2,000 spacecraft launches, creating a legacy of nearly 2,000,000 kg orbiting within 2000 km of earth. Most of this mass is contained in the 3,000 spent rocket stages and inactive payloads which inhabit this region and the 150 currently active satellites (National Science and Technology Council, 1995). There are also over 4,000 less massive objects which represent releases from spacecraft or the remnants of over 124 on-orbit fragmentation events. At least one of these fragmentation events, the breakup of a Russian Transit-type navigation satellite on July 24, 1981, is thought to have been caused by a collision with another orbiting object (Johnson and McKnight, 1987).

Although information about space debris is obtained from a variety of sources—

ground-based optical telescope observations, satellites retrieved from orbit such as the Solar Max, the Long Duration Exposure Facility and European Retrievable Carrier space missions, damaged windows from Shuttle flights, and returned hardware from the Hubble Space Telescope—our primary (and only continuous) source for information about the quantity, sizes, and orbits of space debris is the Space Surveillance Network of the United States Space Command (USSPACECOM, used interchangeably with NORAD in this thesis). Their ground-based radar data indicates that the current near-earth space environment (below 2000 km altitude) is composed of over 8000 pieces of orbiting matter larger than their sensitivity limit of about 10 cm. Of these, nearly half originated from the 120 or so fragmentation events mentioned above. These fragmentations are the result of launch vehicle residual propellants, deliberate breakups, and causes of unknown origin.

The orbital speed of objects near the earth is typically 7 km/s, however, since much of the near-earth debris is in polar orbits, the relative speed between two objects averages 10 km/s (Johnson and McKnight, 1987). At this speed, a 1 cm aluminum sphere could release a half megajoule of energy in a shock wave-induced fragmentation. The number of objects in the near earth space environment of diameter greater than 1 cm but less than 10 cm is a matter for speculation, but generally regarded to be between 3 and 9 times the trackable population (Klinkrad and Jehn, 1992). In addition, there is estimated to be 1000 kg in objects smaller than 1 cm and 300 kg in objects smaller than 0.1 cm (National Science and Technology Council, 1995). These smaller objects are themselves a cause for serious concern. Over the course of the first 70 US Space Shuttle flights, more than 60 windows required post-flight replacement, largely due to the small fractures resulting from collisions with millimeter-sized titanium paint flecks (Johnson and McKnight, 1987).

For debris less than .01 cm, erosion and surface pitting are the primary effects.

The sensitivity limit of the USSPACECOM network of ground based radars is determined mainly by the size of an object's radar cross-section (RCS) and its altitude above the earth. The RCS is not a precise measure of an object's size because different materials possess different radar reflectivities; however, it is a reasonable first approximation. At the lowest orbital altitudes, an object cannot be reliably detected and tracked unless its RCS exceeds 7 cm. This RCS minimum size threshold is, of course, larger at higher altitudes. It is important to note that objects no larger than 1 cm in diameter are capable of inflicting catastrophic damage to a satellite in a collision at the relative velocities encountered in low earth orbit, typically several kilometers per second. However, since little is known about the orbital states of this invisible population of space debris, we shall focus on those objects which can be tracked and leave the extrapolation to the untrackable population for other investigators to pursue.

The primary mechanism for cleansing the near earth environment of debris is atmospheric drag. This mechanism, however, is only effective at the lower altitudes since decay time is a function of atmospheric density and ballistic coefficient. While a typical satellite in circular orbit at 400 km altitude will decay and reenter from orbit in only a few months, a satellite at 800 km will take hundreds of years to reenter (Johnson and McKnight, 1987). Thanks to the worldwide recognition of this growing space debris problem, national space policies, and recent international agreements, most launch providers will now deboost their large rocket stages into rapidly decaying orbits whenever feasible. Teledesic and Iridium, the two recently-formed international companies proposing the largest commercial satellite constellations have, as a matter of practical economics as much as ecology, plans to deorbit

spent satellites as well. However, because launch rates continue to rise, the orbital population of space debris will continue to grow.

1.3 Thesis Outline.

This thesis proposes an original method for computing the collision potential of a satellite in Earth orbit. In Chapter 2 we will discuss the data used in this computation, present some of the analysis tools, and comment on some preliminary findings.

Chapter 3 shows how this data can be used to develop a debris density model of the space near Earth.

In Chapter 4, we look at the results of applying this model and what we can infer from some recent data.

Finally, in Chapter 5, we summarize our findings and suggest possible improvements and refinements which may be applied to our methods.

2. Available Data on Space Debris

2.1 Orbital Characteristics.

The fundamental orbit problem is a subclass of the two body problem in that it is concerned with the motion of a point mass which moves subject only to the gravitational attraction of a far more massive central body. Given the mass of the central body, the complete solution to this problem can be presented as a set of six mutually independent parameters and an epoch time. With these parameters, the position of the orbiting body can be determined at any future epoch time. This solution is referred to as Keplerian motion, as the moving point mass will strictly obey Kepler's three laws. Valid representations of these six parameters might be the six cartesian elements of position and velocity at epoch, or alternatively, the six Keplerian elements: semi-major axis (a), eccentricity (e), inclination (i), argument of pericenter (ω), right ascension of ascending node (Ω), and mean anomaly(θ). One crucial advantage in using Keplerian elements is that they are, by definition, constant in the two body problem. The one exception to this statement is that, as its name implies, the mean anomaly is not strictly constant; however, its rate of change is constant.

When one considers perturbations to this two body motion, more parameters are needed to accurately describe the motion, which is no longer precisely Keplerian. In the case of an earth satellite, which is subject to a variety of perturbing forces including that due to atmospheric drag, solar pressure, non-sphericity of the

earth, and third body gravitation, we must include estimates of these forces in our prediction of future orbital states.

In light of this, two common methods of representing the six orbital coefficients and associated perturbations have evolved. The first uses what are termed “osculating elements” in which the true orbital state is represented by a set of Keplerian elements at each instant of time. Because of perturbations, the Keplerian elements do not remain constant in time, but may experience periodic and secular variations. For instance, the ascending node will change in a secular fashion under the influence of the earth’s equatorial bulge. This is referred to as nodal regression. The semi-major axis will decrease due to atmospheric drag, but under certain conditions will increase due to solar pressure. The inclination will experience long-term periodic changes due to solar and lunar gravitational forces.

An osculating element set is particularly well suited for short-term propagation via the method of special perturbations. This is a numerical integration process whereby the equations of motion are integrated either directly, which known as Cowell’s method, or after transformation to a coordinate system based on a reference orbit, called Encke’s method. The result of these methods is a series of state vectors at specified times, whose accuracy depends on the correctness of the initial orbit and the perturbative force models, and degrades as integration step size and time from the initial orbit increase. Therefore, since the initial orbit is determined from a set of tracking observations, the best estimate of true orbital position and velocity is represented by the osculating element set epoched nearest the latest set of observations. Orbital elements then “age” from this time with the uncertainty generally expanding quadratically with time.

The second common representation of a satellite orbit uses a “mean” element

set, in which periodic and secular changes in each orbital parameter are fit over a specified time interval, usually on the order of several days. In this method, the best estimate of an object's position and velocity is no longer tied to a single epoch but is instead spread out over the specified timespan. In other words, a mean element set will not experience the very short-term fluctuations from a reference Keplerian orbit as the osculating set—or the true orbit—would. For this reason, an osculating element set is generally considered a more accurate description of a satellite's state at a specified point in time (Wertz, 1985). However, several important analytical solutions have been developed which utilize the mean element set construction. This is called the method of general perturbations and has the advantage of being far more efficient to compute. This is also the method by which the USSPACECOM catalogues all of the trackable objects in earth orbit.

2.2 The NORAD Two-Line Mean Element Set.

The United States Space Command (USSPACECOM, formerly NORAD, the North American Aerospace Defense Command) maintains general perturbations element sets on more than 8000 Resident Space Objects (RSOs). Each element set undergoes periodic refinement using the tens of thousands of daily observations from the Space Surveillance Network's worldwide system of radar facilities.

The typical method of distribution is the NORAD Two Line Mean Element Set (2LMES). These are element sets obtained by removing periodic perturbations from a satellite orbit in a systematic way. In order to make use of these element sets (or "elsets"), one must reconstruct the periodic variations in a way that complements the method used to remove them. In practice, this involves using one of the following algorithms: SGP, SGP4, and SDP4 (Hoots and Roehrich, 1980).

The SGP (Simplified General Perturbations) method (Hilton and Kuhlman, 1966) was developed for near earth satellites. It uses a simplification of the method of Kozai (Kozai, 1959) for its gravitational model and assumes that the drag effect on mean motion is linear in time. In addition, the drag effect on eccentricity is modeled so that perigee height remains constant. The method for reincorporation of perturbations is straightforward and simple. First the secular terms regarding the changes in Right Ascension and Argument of Perigee due to the Earth's oblateness are extracted from the input elset. Then long period perturbations due to the second and third zonal harmonics of the Earth are applied to the elements after they have been propagated to the target time. Following this, Kepler's equation is solved to relate Mean Anomaly to Eccentric Anomaly and produce cartesian coordinates. Finally, short period perturbations are added to give osculating elements. The details of this algorithm are presented in Appendix A.

SGP4 uses a simplification of the more extensive analytical theory of Lane and Cranford (Lane and Cranford, 1969) which was based on the solution of Brouwer (Brouwer, 1959) for its gravitational model. SGP4 uses a power density function for its atmospheric model. SDP4 is an extension of SGP4, intended for use with deep space satellites (Hujsak and Hoots, 1977).

This thesis will use the SGP method exclusively, notwithstanding the fact that the SGP4/SDP4 method is more accurate and more widely used. This decision was based solely on the fact that SGP is a simpler method to implement while still being completely compatible with the USSPACECOM method of producing 2LMES. However, it should be noted that improvements to the accuracy of the work presented in this thesis would benefit by simply upgrading the 2LMES propagation method from SGP to SGP4/SDP4.

A 2LMES is a Keplerian type of element set which consists of the following parameters:

t_0 = epoch time. The instant in time at which the orbital element set is valid.

n_0 = “mean” mean motion at epoch.

Mean motion at epoch describes the angular rate at which the satellite travels in its orbit. It is 2π divided by the period. Due to various perturbations, mean motion will fluctuate to some degree over several orbital revolutions. “Mean” mean motion is the average of this quantity over the time span of interest. The semi-major axis can be recovered from the “mean” mean motion at epoch by reconstructing the perturbations due to the earth’s oblateness. See Appendix A.

e_0 = “mean” eccentricity at epoch.

Eccentricity describes the deviation of the orbital path from circular. In terms of the semimajor axis a and semiminor axis b , $e = \sqrt{1 - \frac{b^2}{a^2}}$. When $b = a$, we have $e = 0$ which describes circular motion. $0 < e < 1$ describes an elliptical orbit path (and all of the satellites in the USSPACECOM catalog), and $e \geq 1$ describes a parabola or hyperbola, either of which would be associated with vehicles that have escaped earth orbit.

i_0 = “mean” inclination at epoch.

Inclination is the angle between the orbital plane and the mean equator. The mean equator is used because the actual equatorial plane fluctuates slightly due to earth precession and nutation. A satellite with inclination near 90 degrees is termed “polar.”

ω_0 = “mean” argument of perigee at epoch.

Argument of perigee is the angular measure of the perigee from the point at which the orbital trace crosses the equator in the ascending direction.

M_0 = “mean” mean anomaly at epoch.

Mean anomaly is the angular position of the satellite in its orbital plane, measured from the argument of perigee.

Ω_0 = “mean” right ascension of ascending node at epoch.

The right ascension of ascending node (RAAN) is the angular distance between the first point of Aries (the earth-sun line at the vernal equinox) and the point at which the orbital trace crosses the equatorial plane in the ascending direction.

$\dot{n}_0/2$ = one-half the time derivative of the mean motion.

This is not a Keplerian element but is a necessary component of an SGP 2LMES for it describes the orbital decay due to atmospheric drag.

To propagate an element set via the NORAD SGP algorithm, several other inputs and constants are required:

t = time at which the propagated element set is required.

k_e = gravitational constant = $\sqrt{GM}$, where G is Newton’s gravitational constant and M is the mass of the earth.

J_2 = second zonal harmonic = .001082616 from the World Geophysical Survey 72 (WGS72). A measure of the flattening of the earth.

J_3 = third zonal harmonic of WGS72 = -2.53881×10^{-6} . A measurement of the asymmetry about the Earth’s equator.

The output vectors from the SGP algorithm are in cartesian elements in the true equator mean equinox of date frame (TEME) corresponding to the epoch year of 2000. This coordinate frame assumes that both precession and nutation corrections from January 1, 2000 at 0000 GMT have been applied.

2.3 The Trackable LEO Satellite Population.

Using a recent NORAD elset (2LMES) distribution, the following histograms were generated by culling the distribution for objects which traverse a specified altitude band. Figures 2.1 through 2.8 were developed using only elsets which penetrate below 2000 km altitude, which represent some 6806 objects out of the total of 8163 currently in the database.

Figures 2.1a,b. Eccentricity and Inclination Distributions in Low Earth Orbit (LEO).

Figure 2.1a shows that the vast majority of LEO objects are in near-circular orbits and therefore remain below 2000 km at all times. There are however, a small grouping of orbits with eccentricity near 0.7 which occupy LEO only near perigee. Such orbits have periods on the order of 700 minutes and spend only ≈ 30 minutes in the LEO region. Figure 2.1b displays the inclination distribution in

LEO and indicates that there are certain preferred inclinations, most notably near 65, 80 and 100 degrees. These plots are combined in Figure 2.2 which illustrates the interrelationships between eccentricity and inclination in this altitude regime. This 3D histogram and related contour plot reveals a grouping of objects with eccentricities between .5 and .75 and inclinations below 70 degrees.

The peak with eccentricity near .74 and inclination of 63.4 degrees is consistent with the element sets of Molniya orbits used for communications and strategic surveillance missions. Molniya orbits have perigees between 450 and 600 km in the Southern Hemisphere and apogees near 40,000 km in the Northern Hemisphere and make two complete revolutions per sidereal day, with an orbital period of 718 minutes. This great disparity in apogee and perigee imply that satellites in these orbits will spend the large majority of their time near apogee. Another important characteristic of an orbit at 63.4 degrees inclination is that the line of apsides does not rotate, which for the Molniya orbit means that perigee will stay in the Southern Hemisphere. This orbit was pioneered by the USSR to allow for continuous communications at its northernmost latitudes, which are not serviceable by geostationary satellites.

A characteristic of all earth satellites with high eccentricity and low perigee is that as the orbit of the satellite loses energy due to the relatively large atmospheric drag encountered at perigee, the apogee will tend to decrease far more quickly than the perigee, resulting in a circularization of the orbit, or a lowering of eccentricity. This can be seen in Figure 2.2b as the lighter-colored vertical streaks emanating down from $e \approx .74$ and at inclinations near 5, 25, 45, and 64 degrees. The two vertical streaks at $i \approx 5^\circ$ and $i \approx 25^\circ$ are likely spent transfer stages used to boost communication satellites to geosynchronous altitude. Also note in these diagrams

that most of the polar orbiting objects (that is, those with i near 90°) are near-circular orbits (e close to 0).

Figures 2.2a,b. Eccentricity dependence on inclination in LEO.

The Right Ascension of Ascending Node (RAAN) distribution in figure 2.3 shows no preferred right ascensions for LEO satellites. This important null result will be addressed in Chapter 3.

Figures 2.3. RAAN Distribution in Low Earth Orbit (LEO).

Figures 2.4 and 2.5 combine the RAAN histogram with eccentricity and inclination and indicate no discernable RAAN dependence on either parameter. The light-colored vertical streak in the RAAN/eccentricity contour plot (Fig 2.4b) at $e \approx .74$ is nearly constant over all RAANs, indicating a preponderance of objects with eccentricities near .74, but no obvious dependence on RAAN. A much wider vertical streak is also seen at $e = 0$, reiterating what was already shown in Figure 2.1a, that most objects in low earth orbit are in nearly circular orbits. A similar story holds for the RAAN/inclination plots of Figure 2.5a and 2.5b. Here we see many objects at inclinations near 70° and 96° , but with no discernable dependence on RAAN.

Figures 2.4a,b. RAAN dependence on eccentricity in LEO.

Figures 2.5a,b. RAAN dependence on inclination in LEO.

2.4 Accuracy Estimates Using NORAD Element Sets.

Accuracy studies on the general perturbations element sets published by NORAD are not publicly available, and we have therefore conducted a short investiga-

tion on several selected space objects in order to provide some estimate of the error underlying our data and its associated orbit propagation method.

It is not expected that all elsets describe the orbital states of their respective space objects to the same degree of accuracy. On the contrary, elset accuracy is a function of many disparate circumstances and characteristics including object ballistic number, solar activity, type of orbit, altitude above the earth's surface, number and quality of observations, etc. For example, an elset for an object with a high ballistic number (defined as $\frac{m}{C_D A}$ where m is mass, A is cross-sectional area in the direction of the velocity vector and C_D is drag coefficient) will likely produce more accurate predictions than one for an object with a low ballistic number, given the same type of orbit, quality of observations, and solar activity. However, we do not intend to extrapolate the results obtained from the few isolated elsets investigated here to determine the accuracy of the entire NORAD catalog. This investigation is meant merely to obtain a "ballpark" estimate of the accuracy of NORAD elsets in LEO.

The following objects were selected because they are known to be non-thrusting over the time interval of this study, their elsets have been updated by NORAD on a regular basis, and they are resident in LEO for at least some part of every orbital revolution:

Object 24838: IRIDIUM SV009, launched in June 1997, has a 780 km altitude near-circular, near-polar orbit.

Object 20712: CRRES, the Combined Release and Radiation Effects Satellite, was launched in 1990 and occupies a highly elliptical orbit ($e = .72$) with an apogee altitude of 34,578 km and perigee of 330 km at 18 degrees inclination.

Object 00877: Cosmos 44 R/B, is a Russian rocket body launched in 1964 which occupies a near circular, 700 km altitude orbit at 65 degrees inclination.

Our error estimation strategy consists of generating ephemeris for a particular elset over several orbital revolutions. This ephemeris is then compared to a second ephemeris similarly constructed for the same satellite, using a more recent elset. A time history of differences in relative position and velocity were compiled as well as a tabulation of positional differences in the intrack, crosstrack, and radial directions. These differences were computed using the following equations:

$$\Delta \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_0$$

$$\Delta r_{radial} = |\mathbf{r}_1| - |\mathbf{r}_0|$$

$$\Delta r_{intrack} = \frac{\Delta \mathbf{r} \cdot \dot{\mathbf{r}}_0}{|\dot{\mathbf{r}}_0|}$$

$$\Delta r_{crosstrack} = \frac{\Delta \mathbf{r} \cdot (\mathbf{r}_0 \times \dot{\mathbf{r}}_0)}{|\mathbf{r}_0 \times \dot{\mathbf{r}}_0|}$$

The IRIDIUM SV009 elsets selected had epochs on October 9, 1997, October 29, 1997 and November 3, 1997. Figure 2.6 graphically displays the comparisons.

Figure 2.6a, b. IRIDIUM elset comparison.

Figure 2.6c. IRIDIUM elset comparison.

Figure 2.6a shows the differences in intrack, crosstrack, and radial distances between two IRIDIUM SV009 elsets which have epochs 20 days apart. This indicates that the error in the elset epoched on 09 Oct 97 (with respect to the position of the elset epoched on 29 Oct 97) is on the order of 8 km after 20 days, with most of the error occurring in the intrack component. Figure 2.6b expands the time scale to 10 days and we see that although the radial component of error remains at approximately 0.0 ± 1.8 km, the intrack component grows from 8 km to 14 km (± 3

km) (Crosstrack error was similar to the radial error, but is not shown in this graph in order to enhance clarity). This error analysis, however, is somewhat misleading in that we are estimating the error in an elset that has aged 20 days. Most elsets in the NORAD catalog are updated more frequently than every 20 days, and the results displayed in Figure 2.6c are more characteristic of a typical situation in which the elset has aged only 5 days. Here we compare the elset epoched at 29 Oct 97 with an elset epoched at 3 Nov 97 and see an intrack error after 5 days of approximately $200 \text{ m} \pm 700 \text{ m}$. Although we also see the error growing to approximately -1.0 km after 10 days, we must consider that the elsets will be updated by NORAD well before then.

Figure 2.7. Cosmos 44 R/B elset comparison.

The aging of the Cosmos 44 rocket body elset is evident in Figure 2.7 as the intrack error is in excess of 2 km at the epoch of the more recent elset. Although the error appears to be converging to zero, this is coincidental. What is important to note is the slope of the average intrack error which is on the order of 300 m/day. This is considerably larger than the 130 m/day intrack error slope of the IRIDIUM SV009 elsets estimated from Figure 2.6c. The larger errors for the Cosmos 44 rocket body

are possibly a result of greater atmospheric drag uncertainty at the lower Cosmos altitude. As with the IRIDIUM elsets, the radial error has a considerably narrower envelope, and does not appear to diverge on a timescale of several days. Crosstrack error, as before, is similar to the intrack error, and is omitted for clarity.

Figure 2.8a, b. CRRES elset comparison.

Due to the much larger energy and eccentricity e of the CRRES orbit, we see a marked difference in the characteristics of the comparison plots. With a semimajor axis length of 23,829 km, the CRRES vehicle requires approximately 10.2 hours to complete a single orbital revolution, which is apparent in Figures 2.8a and b, as the intrack, crosstrack and radial errors all exhibit this periodicity. The peaks in the trace of the crosstrack error (which can be identified as the trace with the lowest amplitude of the group, ranging between -6 km and 0 km) occur at perigee.

Reflecting on the data presented here on 3 sample objects from the NORAD catalog, we have seen that the accuracy of a NORAD elset is dependent on the type of orbit considered, with altitude as a primary influence. As we are concerned here with a only a rough approximation of NORAD accuracy rather than a precise measurement of it, we may conclude from the above data that, in general, elsets

for objects in LEO are accurate only on the order of a few kilometers after several days.

3. The Compartment Model

3.1 Unavoidable Error in Ephemeris Prediction.

In general, predicting whether or not two orbiting objects will collide requires accurate knowledge of their orbital state a short time prior to the time of their closest approach. Unfortunately, the state for any given satellite is rarely known to the accuracy required for a confident prediction. The accuracy of a state vector is a function of the quality and quantity of available observations and on the correctness of the various force models used in the orbit determination process. The largest perturbative forces acting on near earth satellites are gravitational asymmetry, atmospheric drag, and solar radiation pressure.

Although the geopotential is known to high precision, the same cannot be said of the atmosphere. Despite continuing research, solar activity remains largely unpredictable, and this results in atmospheric density prediction errors often on the order of 300 percent or more. Conversely, solar pressure, which is measured as a mean solar energy flux integrated over all wavelengths, is very constant, varying as the inverse square of the distance from the sun, and is predictable to within 0.5% (Wertz, 1985).

The process of propagating an estimated orbital state to some future epoch further degrades the accuracy of the estimate. In terms of a satellite's positional knowledge, most of the uncertainty is indicated as error along the orbital trace, commonly referred to as in-track error. Cross-track and radial errors are generally

smaller by an order of magnitude or more, as was shown in the previous section.

For a typical object tracked by the USSPACECOM, in-track error is often on the order of tens of kilometers. For the task of acquiring the object with a radar antenna, this is not usually a problem in that it merely results in an offset of a few seconds to a satellite's expected rise time over a tracking station. However, for the more demanding task of predicting a collision between two satellites, this error renders the problem nearly intractable. An approximation of the collision probability density in terms of the computed closest approach distance between two orbiting objects, assuming a normal distribution of positional uncertainty, can be written (Mullikin, et al., 1995) as

$$P(x) \approx \frac{d^2}{2\sigma^2} e^{\frac{-x^2}{2\sigma^2}}$$

where

d = collision cross sectional radius

σ = standard deviation in satellite positional knowledge

x = closest approach distance

This equation is valid only for d substantially less than σ , which is a reasonable assumption for our present work, considering the results of the analysis in section 2.4. Although we will not present derivation details of this equation here, it is worth noting that it is equivalent to the one-dimensional gaussian probability equation $P(x) = \frac{d}{\sigma\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma^2}}$ when one multiplies the right hand side by $\frac{d}{\sigma} \sqrt{\frac{\pi}{2}}$.

The pertinent point is that even when $x = 0$, the resultant probability of collision is

$$P_{maximum} = \frac{d^2}{2\sigma^2}$$

which, for a typical d of 2 meters and σ of 1 kilometer (as inferred from the analysis in section 2.4), results in a probability of one in a million. This is not particularly useful information for satellite operators. Clearly another way of analyzing the data is needed.

3.2 The KTG Approach to Satellite Collision Probability.

The unpredictability of satellite collisions suggests treating the problem using statistical mechanics. An order of magnitude collision probability estimate can be obtained using the kinetic theory of gasses (KTG) and the altitude distribution of the USSPACECOM data. We proceed as follows:

$$s = \frac{vt}{\rho\pi d^2 vt}$$

s = the mean free path of a satellite moving through a region of stationary objects

v = velocity

t = time interval

ρ = spatial density (number per unit volume)

d = collision cross sectional radius

Figure 3.1. Cylinder swept by object of equivalent radius.

We assume that the collision potential between two spherical objects of radii d_1 and d_2 is equivalent to the collision potential of one object of radius $d = d_1 + d_2$ with a point mass (Resnick and Halliday, 1977). Figure 3.1 depicts the cylindrical volume swept out by an object with an equivalent circular cross section of radius d as it traverses a distance vt . A collision will occur with any point mass inside the cylinder, as represented in the figure by the black dots. The mean free path is the average distance traveled between collisions and is simply the total distance vt divided by the number of point masses in the volume. The equation above replaces the number of point masses with spatial density times volume.

However, gas molecules and satellites are never stationary. To account for the motion of the objects we recognize that the v in the numerator represents the inertial velocity v_i of the satellite while the v in the denominator denotes the relative velocity v_r between the satellite and the objects (Resnick and Halliday, 1977). Therefore,

$$s = \frac{v_i t}{\rho \pi d^2 v_r t}$$

Since the number density of objects is small, the probability of collision is the total path length $v_i t$ divided by the mean free path:

$$PC = \frac{v_i t}{s}$$

or, substituting for s ,

$$PC = \rho \pi d^2 v_r t$$

This is the familiar kinetic theory of gases collision probability equation. For spacecraft in low earth orbit, v_i is about 7 km/s while v_r averages 10 km/s (Johnson and McKnight, 1987). We shall produce empirical evidence to support this assertion in Chapter 4. The spatial density ρ can be read directly from Figure 1.1 for a specific altitude. The collision cross-section d^2 varies considerably over the USSPACECOM database, from $\approx 40 \text{ cm}^2$ for the smallest tracked objects to hundreds of square meters for the largest objects such as the MIR space station. In practice, the cross-sectional area is not precisely known, however it can be estimated from NORAD radar cross-section measurements. Regardless, as we are concerned here with statistical collision probability over a timespan of days rather than the probability for a single collision event, use of an average cross-section will be adequate. Johnson and McKnight estimate the average radar cross-section for all space objects to be 4.42 m^2 (Johnson and McKnight, 1987). Although this is not the same as the physical cross-section and includes objects at higher altitudes, we shall use this estimate as a reasonable approximation.

By substituting typical values into the above equation, we calculate the probability of collision for a 3 meter diameter satellite in circular orbit at 800 km to be 4×10^{-5} over the course of a year. This compares well with the numbers derived

by other investigators using significantly more sophisticated estimations (Johnson and McKnight, 1987).

3.3 Compartment Model Construction.

As a refinement of the kinetic theory of gases approach, we subdivide the altitude band under consideration into a set of cubical compartments (see Figure 3.2) and calculate the density of debris objects in each cube. The size of each cube is an important consideration, and should be on the same order of magnitude as the mean positional uncertainty of the element sets in the USSPACECOM database. In Figure 3.2a, there are 530 cubes, the edges of each cube measuring ≈ 1000 km. This is considerably larger than the optimal size which, taking into account the uncertainty estimates from Chapter 2, would suggest a cube edge length of less than 10 kilometers. However, this more preferred size would require some 5 million cubes to span the same altitude band in LEO, and a considerable increase in computer resources. For the purposes of this thesis, 530 cubes will be adequate, however more accurate results will likely be obtained by increasing the number of cubes and decreasing the cube size.

Figures 3.2a, b. Subdivision of an altitude band into cubical compartments.
X, Y, and Z in thousands of kilometers.

Figure 3.2b is a cutaway to show a sphere representing the Earth inside the cluster of cubes. Although the Earth underneath them rotates, the cubical compart-

ments remain fixed in a local inertial frame. The x axis points to the First Point of Aries in accordance with the True Equator Mean Equinox of date coordinate frame, as used by the SGP algorithm.

Construction of the compartments was accomplished with the following simple code segment written in the C programming language:

```
int x, y, z, r = 7, d = 5;
for(x = -r; x <= r; x++)
for(y = -r; y <= r; y++)
for(z = -r; z <= r; z++)
if(x*x + y*y + z*z >= r*r - d && x*x + y*y + z*z <= r*r + d)
printf(outputfile, "%d %d %d \n", x, y, z);
```

The code consists of 3 nested “for” loops which increment values for the integers x, y, and z through the range $-r$ to $+r$. It tests for the condition where $x^2 + y^2 + z^2$ is within the range $r^2 - d$ and $r^2 + d$. If the condition is met, the coordinates are saved. The resultant 530 sets of coordinates are used as the locations of the cubes in Figure 3.2a. These locations are multiplied by a scale factor, in this case 1000 km, to cover a radius band near 7000 km. In order to avoid any “holes” in the coverage, a band thickness of $\sqrt{d} = 2.24$, or ± 2240 km was required. Clearly, this degree of coarseness is rather large considering the precision of the NORAD element sets, but was the highest practical resolution possible given the available computer resources.

Population of each of the compartments with a density number was accomplished by generation of a cartesian ephemeris, via the SGP algorithm, for every object in the NORAD database, excluding those with a perigee height above the altitude band of interest or an apogee height below it. The ephemeris time step t_i used was 60 seconds over a timespan t_s of 1 day. Each ephemeris point was

compared against the compartment coordinates to determine if it was contained within any compartment. If so, a counter associated with that compartment was incremented by one.

This procedure resulted in a one-dimensional array of 530 numbers, each associated with the number of “hits” recorded for a specific compartment. These numbers n_j were converted to a space debris spatial density number σ_j by

$$\sigma_j = \frac{n_j t_i}{V t_s}$$

where V is the volume of a compartment, in this case $(1000km)^3$.

The spatial density σ_j was then used to redraw figure 3.2a, with the shading of the j^{th} compartment corresponding to the relative density $\frac{\sigma_j}{\sigma_M}$, where σ_M represents the maximum of all the σ_j values. Figure 3.3a shows the compartment model shaded according to density, and Figure 3.3b shows the relative density scale. Higher density indicates greater collision potential within that compartment.

Figure 3.3a, b Compartment relative densities.

4. Results and Discussion

4.1 Space Object Density at 3 Altitude Bands.

Figure 4.1 was generated using the NORAD element set catalog from October 1997, filtered to include only objects which traverse the 790-810 km altitude band. It displays the 530-cube compartment model at nine different viewing angles. Each cube is shaded according to its computed space object density.

Figure 4.1. Compartments at 800 km from different viewing angles.

Each successive illustration in Figure 4.1 shows a view which is rotated clockwise from the one before, by approximately 32° about the z axis. The shading of each compartment was intended to be directly proportional to that compartment's

space object density, relative to the density of all the other compartments. This density is in turn proportional to the product of the number of orbiting objects which traverse the compartment and their dwell time within the compartment. However, in order to increase the contrast between the light and dark regions in the printed copy of this thesis, shading was enhanced by squaring the density number. No contrast enhancement was necessary for the images as displayed on a computer screen.

Study of Figure 4.1 reveals a region of higher object density offset slightly from the north pole. Examination of the actual data (see Appendix B) shows there to be another similar high density region near the south pole, precisely opposite the northern high density region, but not visible in the figure. Further consideration of Figure 4.1 indicates areas of moderately higher density connecting these two regions. These high density areas seem to form a band spanning the model, in a plane approximately orthogonal to the line $(0,0,0)(5,5,3)$. This suggests that there may be a small number of satellites in circular orbits which pass through the same compartments on a regular basis. There are two orbits that would fit this profile: one has its Right Ascension of Ascending Node (Ω) $\approx 135^\circ$ and inclination (i) $\approx 82^\circ$. The more likely candidate has $\Omega \approx 315^\circ$ and $i \approx 98^\circ$. This is an example of an orbit which is *sun-synchronous*, that is, its nodal regression matches the Earth's orbital rate about the Sun of $.9856^\circ$ per day. Furthermore, at $\Omega \approx 315^\circ$, it will constantly follow the dawn/dusk terminator, as this value of Ω results in an orbit whose ground trace follows twilight on November 7th. This is expected to be a popular orbit for a variety of reasons: a constant view of the sun for power and scientific instrumentation, absence of the heating/cooling cycles associated with frequent eclipses, advantageous lighting for Earth atmospheric studies, and so on.

However, Figures 4.2 through 4.4 suggest that this is probably too simplistic a notion. Figure 4.2 indicates that there are several dozen objects which combine to produce this dark band.

Figure 4.2. Number of compartment model intrusions per space object.

Figure 4.2 also shows that over the course of 24 hours, over 55,000 individual intrusions have occurred somewhere in the compartment model by a total of 370 objects. Of these, some 50 objects are responsible for over 400 intrusions each. These objects must be in near circular orbits to remain in the 800 km altitude band and produce approximately half of the total number of intrusions. However, the right ascensions of these objects are not neatly situated at 315° and 135° , as shown in Figures 4.3 and 4.4.

Figure 4.3. RAAN distribution at 800 Km Altitude.

Figure 4.3 shows a small peak in the distribution for objects with $\Omega \approx 100^\circ$, and another smaller one at $\Omega \approx 315^\circ$, but they are not pronounced enough to fully explain the density distribution of Figure 4.1. It was thought that this discrepancy could be resolved by considering the RAAN distribution at 800 Km of polar orbiting objects with eccentricity near 0. However, Figure 4.4a shows that the small peak in RAAN from Figure 4.3 nearly disappears when we filter out all objects with $e > .2$. Figure 4.4b shows a further filtering of the distribution by retaining only those objects with $e < .1$ and i between 95° and 105° , which are necessary conditions for a circular, sun-synchronous orbit at 800 Km altitude. Figure 4.4b shows only two objects with circular, sun-synchronous orbital characteristics at 315° .

Figure 4.4a. RAAN distribution at 800 Km for $e < .2$ and $70^\circ < i < 110^\circ$.

Figure 4.4b. RAAN distribution at 800 Km for $e < .1$ and $95^\circ < i < 105^\circ$.

Figure 4.5a. Eccentricity distribution at 800 Km.

Figure 4.5b. Inclination distribution at 800 Km.

Figure 4.5a shows the eccentricity distribution for a 10 Km band centered at 800 km to be about the same as the general LEO distribution as previously seen in Figure 2.1. However, Figure 4.5b shows the inclination distribution in this band

is heavily weighted toward three distinct inclinations—over half of all the satellites which inhabit this region are within 2 degrees of 64, 82, or 98 degrees inclination.

Figures 4.6–4.8 show the compartment densities for 10 km bands centered at 600, 800, and 1000 km altitude. The dark band is present for each altitude in the same place. It should be noted that there is a fair amount of overlap in each altitude band because of the 1000 km size of the compartments, however, because the population of each band is filtered by apogee and perigee to screen out those objects which cannot cross that specific altitude band, the overlap is confined to objects with eccentricities greater than .025, at which the apogee and perigee difference is approximately 400 km. 50 percent of the catalogued objects at 800 km meet this criteria.

Figure 4.6. 600 Km distribution at 133°, 166°, and 199° Right Ascension.

Figure 4.7. 800 Km distribution at 133°, 166°, and 199° Right Ascension.

Figure 4.8. 1000 Km distribution at 133°, 166°, and 199° Right Ascension.

Figure 4.9 shows line graphs of the boxes at each altitude sorted by the number of intrusions. They are shown here in a reduced size for ease of comparison, however they are reproduced in full size in Appendix B. They show distinct peaks in the distributions with the sharpest peak occurring in the 600 km band.

Figure 4.9. Intrusion Distribution at 600, 800, 1000 Km.

The table below each graph shows the number of intrusions for selected boxes at each altitude over the 1 day span in which data was generated. The boxes tabulated represent the 10 highest and 10 lowest intrusion counts for each altitude band. Also included in the tables are the intrusion counts for the three boxes in which velocity data was recorded: boxes 91, 140, and 383. The tables reveal that there is not only a sharp peak in the distribution over the north pole, but there is one of equal magnitude over the south pole as well. These are the regions where

collisions are most likely to occur. Given the high number of polar and circular orbits as witnessed in the preceding figures, this is as expected, since the poles are where all circular polar orbits intersect. Interestingly, judging by the 10 highest intrusion counts, the south pole apparently has higher density than the north pole at 600 km altitude, while at 1000 km the north pole has the higher density. At 800 km, they are nearly equal.

4.2 Velocity Distributions.

Figure 4.10. Location of Velocity Compartments.

In order to gain more insight into the object distributions in LEO, we have investigated the distributions of object velocities in the three compartments depicted above. The orientation of the model corresponds to that of the center drawings in Figure 4.8. All of the compartments are located inside the high density region discussed in previous sections. Boxes 91 and 383 are near the equator, on opposite sides of the earth. Box 140 is situated near the north pole.

Figures 4.11a.

Figure 4.11b.

Velocity Distributions for Box 140.

Figure 4.11a shows the distribution of velocities in the x, y, and z directions for box 140. These three graphs are combined to produce Figure 4.11b but are shown here for comparison, as an aid to separating the individual traces. In the combined graphs, the most probable velocities are annotated if they are considerably more

prominent than the rest of the distribution. For instance, in Figure 4.11b we see that, by far, the most probable velocity in the z-direction is -2.8 km/s. If the most probable velocity stands out significantly from the rest, it gives an indication of some preferred orbital parameter, such as RAAN, for a substantial number of objects as they pass through that particular box.

	Intrusions	% of Max	VxAvg	VyAvg	VzAvg	VMagAvg
B91_600	1507	23.8	-2.66	-2.07	-0.48	7.73
B91_800	2713	39.2	-1.79	-1.38	-0.90	7.60
B91_1000	2829	30.9	-1.80	-1.30	-1.41	7.54
B140_600	5330	84.3	-1.99	-5.01	-1.90	7.55
B140_800	5869	84.8	-0.80	-3.91	-1.22	7.51
B140_1000	5496	60.1	1.44	-3.21	-0.22	7.42
B383_600	1307	20.7	2.89	2.39	0.23	7.73
B383_800	2382	34.4	1.82	1.63	1.38	7.58
B383_1000	2692	29.4	1.74	1.58	1.50	7.53

Figure 4.12. Velocity Distributions at 600, 800, 1000 Km.

Figure 4.11b is reproduced as the center graph in Figure 4.12, where the combined graphs for each of the 3 boxes at 600, 800 and 1000 km altitude are shown.

These graphs are presented here on one page for ease of comparison; they are also printed full size in Appendix C.

The middle graph in the top row shows the velocity distribution for Box 140 at 600 km altitude. In this graph, the most probable velocities are -4.0, -5.9, and -3.0 in the x, y, and z directions. The chart shows that the average velocities for this box are skewed from the most probable velocities in the positive direction: -1.99, -5.01 and -1.90 in x, y, and z.

Each box shows at least one most probable velocity that clearly stands out from the others regardless of altitude. Box 91, for example shows two prominent z-components, one at -7.3 km/s and the other at +7.2. This suggests that in this box, which is located in the equatorial plane and within the high density band, a head-on collision is more likely than a box chosen at random. Box 383 shows similar results, and the two taken together reveal that there is a preferred direction in the band, in the negative z direction in Box 91 and in the positive z direction in box 383, which is also in the equatorial plane, nearly 180 degrees from the position of Box 91. This gives further credence to the assertion that the most significant contributors to the high density band are satellites in dawn/dusk sun-synchronous orbits, at RAAN $\approx 313^\circ$.

4.3 Collision Probability Estimation.

Three circular trial orbits were selected at each of the three altitude bands to test the collision prediction capabilities of the CM model. In each altitude band, the first orbit was selected to reside continuously within the high density band, the second to intersect with the band at the poles, and the third to intersect with it at the equator. The results are tabulated in Table 4.1.

	Collision Probability Results			
	Total Intrusions	Total Boxes	Spacial Density	Probability
T601	2461	986	2.50E-09	6.10E-08
T602	2669	518	5.15E-09	1.26E-07
T603	1146	715	1.60E-09	3.91E-08
T801	2337	910	2.57E-09	6.27E-08
T802	1917	429	4.47E-09	1.09E-07
T803	6917	710	1.76E-09	4.30E-08
T1001	2464	677	3.64E-09	8.89E-08
T1002	1819	425	4.28E-09	1.05E-07
T1003	1424	673	2.12E-09	5.17E-08

Table 4.1. Collision Prediction Results.

The first column in Table 4.1 is an identifier for the orbit type. The types are identified as follows:

T601: Type 1, 600 km altitude circular, $\Omega \approx 310^\circ$, $i = 99^\circ$, therefore residing perpetually in the high density band.

T602: Type 2, 600 km altitude circular, $\Omega \approx 220^\circ$, $i = 99^\circ$, thereby intersecting with the high density band only at the poles.

T603: Type 3, 600 km altitude circular, $i \approx 0^\circ$.

Of course, T801 signifies a Type 1 orbit at 800 km altitude, etc. The second column is the total number of objects which passed through *any* of the boxes which the trial orbit passed through. The total number of these boxes is tabulated in the

third column. The next column is the spatial density, or the average number of other objects per km^3 that the test orbit “sees” over its trial run. Finally, the last column combines these numbers into a collision probability number based on the kinetic theory of gases model as discussed in section 3.2. The collision probability calculation assumes an average relative velocity of 10 km/s and a collision cross-section of 3 meters.

Although all of the probability numbers are small, the Type 2 orbits which intersect with the high density band over the poles computed the highest collision probabilities in all three altitude bands. This was an unexpected outcome, and contrary to our intuition. The Type 2 orbits also (unexpectedly!) passed through the fewest boxes by a substantial margin which may partially explain the higher probability numbers but also may indicate a systematic error in the model. Analysis of the Type 2 orbit data showed that they passed directly through the high density polar regions on every pass, and missed much of the equatorial regions by slipping between the edges of the middle cubes in a fortuitous fashion.

5. Conclusion

Space debris analysis is a relatively new but important scientific and engineering challenge. Dire consequences—the irreversible pollution of near-earth space rendering vast regions unusable and hazardous—are inevitable if spacefaring nations ignore the ramifications of their rapid and incessant technological colonization of space. New techniques must be developed to help understand the severity of the problems and provide clues to their eventual mitigation.

In this thesis, we have presented a tool for the study of the space debris environment, based on the concept of compartmentalizing an altitude band in near-earth space. We have applied this tool to the 8,000-plus space object orbital element sets collected and distributed by NORAD. We were then able to display the time-averaged density of these objects within three different altitude bands in low Earth orbit and show that this data can be used to generate collision probability estimates and make predictions as to where a collision event is most likely to occur. We have also looked at time-averaged velocity data and have correlated it with plausible space object orbital scenarios.

Designers of satellite constellations could apply the results presented here by avoiding use of the heavily populated polar regions in low earth orbit or, if this is not feasible, at least made aware of the increased probability of collision with such orbits. Deboosting satellites from these orbits at the end of their operational lifetimes should be given high priority.

We have by no means exhausted all possible refinements and avenues of use for this tool—the areas for future work are plentiful. Accuracy and computation speed were two of the biggest drivers for the development of the methods presented in this thesis. However, both can be enhanced enormously with only moderate additional work and expense.

For instance, use of actual velocity information should replace the average velocity in the KTG calculation of Section 3.2. This would greatly reduce the probability of collision for two objects whose relative velocities remain low because of similarities in their orbits. This would be the least expensive and most easily implemented modification to the present work, and should provide a substantial improvement in collision probability calculations.

In addition, the fact that we predicted higher collision probabilities for the Type 2 orbits of Chapter 4 points up the shortcomings of using an excessively coarse spatial model. Despite this, the compartment model was able to make order of magnitude predictions for collision probability and indicate areas of high object density. Certainly a finer model using cube sizes on the order of the uncertainty in the elset data would substantially improve our results. Thus, another significant area for improvement would be to augment resolution by increasing the number of cubes and decreasing their volumes by at least an order of magnitude. This can be made possible by enlisting more powerful computers but also by investigating more efficient data processing algorithms by which the compartments are populated. Specifically, a Newton-Raphson or other root-finding method could be employed to process the element sets of objects in highly elliptical orbits which spend only a small fraction of their time at any one altitude. Obviously, much processing time could be saved by performing only minimal processing on those parts of an eccentric

orbit which are not at the altitude of interest.

However, the use of the compartment method is not restricted to the compartmentalization of an entire altitude band, or to time spans on the order of days as was done in this thesis. Clearly, one can focus on a specific volume of interest—such as the polar regions in LE0—or on a specific span of time when a collision appears most probable. For these more focused studies, the use of the more accurate SGP4 propagator, as mentioned in section 2.2, should replace the SGP propagator used here.

Additional work could be done comparing how compartment densities change over the course of months or years, and studying how the high density polar regions evolve and what seasonal or other dependencies can be inferred.

Other compartment shapes could be investigated. While a cube cannot perfectly fill a sphere or any other shape except for another regular parallelepiped, it is perfectly general. A collection of them can fill any volume to any arbitrary degree of precision. However, an alternative strategy might be to use compartments made up of layers of tiles of various symmetrical shapes within the altitude band of interest. This would require far fewer tiles than cubes to ward off coverage holes in addressing a specific altitude band. On the other hand, a tile model would require a certain imaginative problem-solving with regards to coordinate transformations and would not be immune to the edge effects experienced with cubes. Still, it is a viable candidate for refining the compartment method.

The ultimate value of this thesis is not in the specific numbers produced here for collision prediction, but rather in the general method used to arrive at collision probability numbers. The approach presented here is essentially a brute-force method which only recently—with the advent of ever more powerful and inexpensive

computers—has become feasible.

It is today both possible and economical to model large regions of space with enough compartments (cubes or tiles) to attain the kilometer or hectometer resolutions required by the collision prediction problem. The method proffered in this thesis, and refinements thereof, can aid in the discovery of new insights and relationships in and about the space debris environment.

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All of the computations, illustrations and report preparation in this thesis were done on a 25 MHz black and white NeXTstation by NeXT Computer Inc. For compiling and debugging all of the C programs, I used Project Builder Release 3.0, v14.0, and the GNU debugger from the Free Software Foundation. Spreadsheets and histograms were prepared using *Improv* and Presentation Builder Release 1.0. by Lotus Development Corporation. Three-dimensional histograms and the Cube-Matrix model were developed using *Mathematica* by Wolfram Research Inc. All illustrations were incorporated in the thesis as Encapsulated PostScript files, developed by Adobe Systems Inc. For the report itself, I used CTeX, NeXT Version 3.141 and TeXview Release 3.0 by Radical Eye Software.

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A. The SGP Propagator.

The Simplified General Perturbations (SGP) algorithm, being a general perturbations propagator, represents a closed analytical solution to the equations of motion. A NORAD Two-Line Mean Element Set (2LMES) is input to the propagator, and the cartesian elements of the orbit at any future time are output. The precise details of how this algorithm is derived is beyond the scope of this thesis, however a short description of how the algorithm was implemented will be given here. This description borrows heavily from SpaceTrack Report No. 3 (Hoots and Roehrich, 1980). The classic derivation is well presented in the paper by Kozai (Kozai, 1959). All parameters are as defined in Section 2.2.

The first step in the SGP 2LMES orbit prediction process is to recover the mean semi-major axis a_0 from the mean mean motion n_0 , the mean eccentricity e_0 , and the mean inclination i_0 . This is accomplished by reconstructing the periodic variations due to the Earth's oblateness, where J_2 is the second zonal harmonic, $k_e = \sqrt{GM}$, and a_e is the equatorial Earth radius:

$$a_1 = \left(\frac{k_e}{n_0}\right)^{2/3}$$

$$\delta_1 = \frac{3}{2} \frac{J_2(a_e)^2}{a_1^2} \frac{(1 - \frac{3}{2} \sin^2 i_0)}{(1 - e_0^2)^{3/2}}$$

$$a_0 = a_1 \left(1 - \frac{1}{3} \delta_1 - \delta_1^2 - \frac{134}{81} \delta_1^3\right)$$

The semi-latus rectum p_0 and perigee distance p_0 (earth radii) can then be computed as

$$p_0 = a_0(1 - e_0^2)$$

$$q_0 = a_0(1 + e_0)$$

The secular rates of change (in radians per minute) of ascending node $\dot{\Omega}$ and argument of perigee $\dot{\omega}$ are then approximated by

$$\dot{\Omega} = -\frac{3}{2} \frac{J_2 a_e^2}{p_0^2} n_0 \cos i_0$$

$$\dot{\omega} = \frac{3}{4} \frac{J_2 a_e^2}{p_0^2} n_0 (4 - 5 \sin^2 i_0)$$

The mean mean argument of latitude at epoch U_0 is derived from

$$U_0 = M_0 + \omega_0 - 2\pi$$

This completes the initialization of the SGP elements. From these, elements at any future time can be computed. The first step is to apply the terms in $\dot{\Omega}$, $\dot{\omega}$, $\dot{n}_0$ (the change in mean motion due to atmospheric drag), and of course, t .

$$\Delta t = (t - t_0)/60$$

(in minutes)

$$\Omega = \Omega_0 + \dot{\Omega} \Delta t$$

$$\omega = \omega_0 + \dot{\omega} \Delta t$$

$$U = U_0 + (n_0 + \dot{\omega})\Delta t + \left(\frac{\dot{n}_0}{2}\right)\Delta t^2$$

$$n = n_0 + 2\left(\frac{\dot{n}_0}{2}\right)\Delta t$$

$$a = a_0 \left(\frac{n_0}{n}\right)^{2/3}$$

$$e = 1 - \frac{q_0}{a}$$

$$p = a(1 - e^2)$$

To these, the long-period zonal effects must be added as follows:

$$A = e \cos \omega$$

$$B = e \sin \omega - \frac{1}{2} \frac{J_3}{J_2} \frac{\sin i_0}{p} a_e$$

$$U_1 = U + \frac{1}{2} \left(-\frac{1}{2} \frac{J_3}{J_2} \frac{\sin i_0}{p} a_e \right) A \left(\frac{3 + 5 \cos i_0}{1 + \cos i_0} \right)$$

$$e_1 = \sqrt{A^2 + B^2}$$

$$p_1 = a(1 - e_1^2)$$

At this point we must iterate to find a solution to Kepler's equation.

$$U_{i+1} = U_i + \Delta U_i$$

where

$$\Delta U_i = \frac{U_1 - B \cos U_i + A \sin U_i - U_i}{1 - A \cos U_i - B \sin U_i}$$

Iteration continues until the value of ΔU_i is less than the Kepler's equation convergence criteria E_6 . At this point we set $U = U_i$.

The following partially osculating terms are then computed

$$e \cos E = A \cos U + B \sin U$$

$$e \sin E = A \sin U - B \cos U$$

$$u = U + 2 \tan^{-1} \left(\frac{e \sin E}{1 + \sqrt{1 - e_1^2} - e \cos E} \right)$$

$$r = a(1 - e \cos E)$$

$$\dot{r} = k_e \frac{\sqrt{a}}{r} e \sin E$$

$$\dot{r} f = k_e \frac{\sqrt{p_1}}{r}$$

The short period perturbations are then added to give osculating quantities

$$r_k = r + \frac{1}{4} \frac{J_2}{p_1} a_e^2 \sin^2 i_0 \cos 2u$$

$$u_k = u - \frac{1}{8} \frac{J_2}{p_1} a_e^2 (6 - 7 \sin^2 i_0) \sin 2u$$

$$\Omega_k = \Omega + \frac{3}{4} \frac{J_2}{p_1} a_e^2 \cos i_0 \sin 2u$$

$$i_k = i_0 + \frac{3}{4} \frac{J_2}{p_1} a_e^2 \sin i_0 \cos i_0 \cos 2u$$

Unit orientation vectors are calculated by

$$\bar{U} = \bar{M} \sin u_k + \bar{N} \cos u_k$$

$$\bar{V} = \bar{M} \cos u_k - \bar{N} \sin u_k$$

where

$$\bar{M} = \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} -\sin \Omega_k \cos i_k \\ \cos \Omega_k \cos i_k \\ \sin i_k \end{pmatrix}$$

$$\bar{N} = \begin{pmatrix} N_x \\ N_y \\ N_z \end{pmatrix} = \begin{pmatrix} \cos \Omega_k \\ \sin \Omega_k \\ 0 \end{pmatrix}$$

The osculating position and velocity vectors at the requested time t are then given in units of earth radii and minutes.

$$\bar{r} = r_k \bar{U}$$

$$\dot{\bar{r}} = \dot{r} \bar{U} + r \dot{f} \bar{V}$$

B. Intrusion Distribution Graphs.

	Selected CM Boxes			
	695-705 Km Altitude 7 Nov 97			
	x	y	z	Density
293	1	0	-7	6324
291	1	-1	-7	5598
249	0	-2	-7	5441
331	2	1	-7	5379
140	-3	-2	6	5343
209	-1	-2	-7	5232
251	0	-1	-7	4658
295	1	1	-7	4398
174	-2	-3	6	4112
297	1	2	-7	4048
91	-5	5	0	1512
383	4	-6	0	1313
70	-5	-2	-4	1
39	-6	0	-3	1
35	-6	-1	-3	0
143	-3	0	-6	0
207	-1	-3	-6	0
105	-4	-2	-5	0
240	0	-6	-3	0
171	-2	-4	-5	0
98	-4	-5	-2	0
64	-5	-4	-2	0

Figure 4.9a. Intrusion Distribution at 600 Km.

	Selected CM Boxes			
	795-805 Km Altitude 7 Nov 97			
	x	y	z	Intrusions
297	1	2	-7	6917
174	-2	-3	6	6901
295	1	1	-7	6682
140	-3	-2	6	5877
331	2	1	-7	5825
220	-1	3	6	5705
210	-1	-2	7	5673
257	0	2	-7	5508
138	-3	-3	6	5473
178	-2	0	7	5378
91	-5	5	0	2717
383	4	-6	0	2391
98	-4	-5	-2	1
39	-6	0	-3	1
105	-4	-2	-5	0
64	-5	-4	-2	0
70	-5	-2	-4	0
171	-2	-4	-5	0
207	-1	-3	-6	0
35	-6	-1	-3	0
143	-3	0	-6	0
240	0	-6	-3	0

Figure 4.9b. Intrusion Distribution at 800 Km.

	Selected CM Boxes			
	995-1005 Km Altitude 7 Nov 97			
	x	y	z	Density
216	-1	1	7	9151
176	-2	-1	7	9076
178	-2	0	7	8494
256	0	1	7	8111
220	-1	3	6	7610
254	0	0	7	7607
294	1	0	7	7265
260	0	3	6	7186
148	-3	2	6	6913
206	-1	-4	6	6808
140	-3	-2	6	5502
91	-5	5	0	2832
383	4	-6	0	2703
35	-6	-1	-3	0
39	-6	0	-3	0
64	-5	-4	-2	0
207	-1	-3	-6	0
171	-2	-4	-5	0
70	-5	-2	-4	0
98	-4	-5	-2	0
105	-4	-2	-5	0
240	0	-6	-3	0
143	-3	0	-6	0

Figure 4.9c. Intrusion Distribution at 1000 Km.

C. Velocity Distribution Graphs.

Figure 4.11a. Box 91 Velocity Distributions at 600, 800, 1000 Km.

Figure 4.11b. Box 140 Velocity Distributions at 600, 800, 1000 Km.

Figure 4.11c. Box 383 Velocity Distributions at 600, 800, 1000 Km.

IMAGE EVALUATION TEST TARGET (QA-3)

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