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Fractions in second grade : a study using two types of manipulative materials

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**FRACTIONS IN SECOND GRADE:
A STUDY
USING TWO TYPES OF MANIPULATIVE MATERIALS**

**A Thesis
Presented to
The Faculty of the Division of Teacher Education
San José State University**

**In Partial Fulfillment
of the Requirements for the Degree
Master of Arts**

**by
Renée Irene Goularte
December, 1998**

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ABSTRACT

FRACTIONS IN SECOND GRADE: A STUDY USING TWO TYPES OF MANIPULATIVE MATERIALS

by Renée Irene Goularte

This thesis address the topic of the use of two different types of manipulative materials for teaching fractions to second grade students, and the students' achievement with fraction concepts.

The question of whether some types of manipulative materials might be more effective for teaching fraction concepts to young children is the underlying theme of this thesis. It discusses the development of mathematical concepts in children, examines the qualities of two types of manipulative materials used by two groups of students, describes the lessons in which students participated, and explains the assessment materials used. No significant difference in achievement was shown between the two groups of participating students.

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Chapter I

Introduction

Before children begin school, they may have many opportunities to begin constructing fraction concepts. When they break a cookie in half to share with a friend, cut their own birthday cake, or divide a bag of M&M's among three people, they are making parts out of a whole. These "real life" experiences away from school may appear to be very different from typical school-based fraction activities, even when three-dimensional objects are used for instruction. When we consider what children do with manipulative materials, however, rather than the manipulative materials themselves, the difference may be less evident. Do the type of objects used by children in part-whole activities affect their development of fraction concepts?

Fractions are relationships between parts and wholes (Van De Walle, 1994). They are numerical symbols for rational numbers, and include several sub-concepts. Ratios, measurements, decimals, quotients, parts of units and parts of sets are all types of fractions (Dorgan, 1994; Behr, Wachsmuth, Post & Lesh, 1984).

Between the ages of seven and twelve, most children develop the ability to understand part-whole relationships (Piaget, 1968). This understanding might be exemplified by children being able to describe separate parts within a whole set (Cowan, 1978). Pothier and Sawada (1983), for example, found that children between the ages of four and nine experience part-whole concepts in five distinct, sequential levels: sharing,

halving, evenness, oddness, and composition. Sharing and halving are particularly part of children's everyday, informal experiences.

The introduction of fraction concepts in the primary grades is supported by the California Department of Education (1992) and the National Council of Teachers of Mathematics (1989). NCTM states that all fraction work in the primary grades "should involve fractions that are useful in everyday life," that they "should draw on children's experiences in sharing," and that it is "critical in grades K-4 to develop concepts and relationships that will serve as a foundation for more advanced concepts and skills" (p. 57).

According to Behr, et al. (1984), "children have significant difficulty learning and applying concepts related to rational numbers" (p.323). Carpenter, Kepner, Corbitt, Lindquist, & Reys (1980) found that many 13-year-olds "have not learned basic fraction and decimal concepts" (p.10), and Bezuk and Cramer (1989) state that the third National Assessment of Educational Progress results "show an apparent lack of understanding of fractions by nine-, thirteen-, and seventeen-year-olds" (p. 156). Students often tend to count numbers of parts without regard to the whole those parts make up (Behr, et al., 1984; Mack, 1990; Peck & Jencks, 1981; Post, 1981). They frequently are unable to use logical reasoning to explain their fraction work (Peck & Jencks, 1981), rely on learned rules even when application of those rules is faulty (Carpenter, et al., 1980; Peck & Jencks, 1981; Post, 1981), lack the ability to make connections between models, symbols, and words (D'Ambrosio & Mendonca Campos, 1992; Post, 1981; Wearne & Hiebert, 1989), and are often unable to transfer their knowledge

of basic fraction concepts to new situations (Behr, et al., 1990; Owens & Menon, 1994).

Examining the difficulties children encounter in grasping part-whole relationships can help educators make decisions regarding effective instruction in fractions, especially if consideration is given to the stages of mathematical concept development within the stages of cognitive development, both of which move sequentially from the concrete to the abstract (Kroll, 1989; Langford & Sarullo, 1993, Piaget, 1968). Movement through these stages is governed in part by physical activity with concrete objects. In fact, according to Piaget (1977), mathematical structures “are never represented in the mind of the child except in the form of concrete manipulations, either physical or verbal” (p. 703).

Results of studies using a variety of objects for teaching fraction concepts have shown that using manipulative materials helps students develop quantitative understanding of fraction concepts and provides them with mental referents (Bezuk & Cramer, 1989). According to Behr, et al. (1984) and Owens and Menon (1994), for example, working with manipulative materials helps student use mental images to help them explain their strategies when working with parts and wholes. The use of manipulative materials for part-whole tasks gives students actual experience with concepts that textbook activities can only picture for them (Langford & Sarullo, 1993). In general, according to Kober (1991), work with concrete objects can help students apply their knowledge to what she calls “real-world situations.” What is meant by the currently frequently-

used phrase “real-world situations” seems to be interpreted as situations which children might encounter in their everyday lives.

The development of concepts of part-whole relationships requires many experiences over a long period of time (Rathmell & Huinker, 1989). The National Council of Teachers of Mathematics (1989) considers it “crucial that teachers use physical materials, diagrams, and real-world situations” (p. 57), and that an appropriate mathematics curriculum for young children “incorporates real-world contexts, children’s experiences, and children’s language in developing ideas” (p. 16). While primary children generally are not cognitively ready to do abstract work with fraction concepts (Piaget, 1968), the question for educators is whether some manipulative materials are more effective than others for fractions instruction.

Statement of the Problem

In selecting manipulative materials for teaching fraction concepts to young children, teachers have a variety of choices. They can use manufactured materials especially created to represent rational numbers, such as Fraction Circles or Fraction Tiles (Cuisenaire, 1997). They can choose to use more open-ended mathematics manipulative materials, such as base ten blocks or various counters. Or, they can use materials which are familiar to children in their everyday lives, such as food. While any of these materials can give children the opportunity to manipulate parts and wholes, do some provide more of a connection to children’s everyday experiences? Cutting food into parts may be a more familiar activity to a child than building fractional parts out of pre-made fraction

circles or sets of base ten blocks. According to Piaget (1970), the basis of logical and mathematical abstraction is drawn “not from the object that is acted upon, but from the action itself” (p. 16). If children develop mathematical concepts through their actions with objects rather than from the objects themselves, the type of manipulative materials used should make no difference. This study asked the question, “Is there a difference in achievement between two groups of second grade students when one group uses manufactured objects, in this case base ten blocks, and the other group uses food, in a ten week study of fractions?”

The purpose of the study was to compare second grade students’ achievement when two types of manipulative materials were used in the teaching of fraction concepts. It was hypothesized that there would be no difference in achievement between students using base ten blocks and students using food in learning fraction concepts.

Qualities of the Materials

The two types of manipulative materials used in this study were base ten blocks and food. These two types of materials presented several inherent differences, including the difference in how the children needed to begin their activity with the objects. The base ten blocks required children to use parts to create wholes which they again separated into parts, while the food presented children with intact wholes from which they created parts. Students in the blocks group moved from part to whole to part, while students in the food group moved from whole to part.

Qualities of the Materials

<u>Base Ten Blocks</u>	<u>Food</u>
Rectangles only	Rectangles, circles, free-form shapes
Wholes constructed by students from parts	Wholes presented to students intact
Wholes able to be returned to original state	Wholes unable to be returned to uncut state
Students counted pre-determined parts to establish denominator	Students cut parts to establish denominator
Size of parts always equal, as determined by blocks	Size of parts determined by child

Delimitations

This study did not attempt to assess the general, long-term benefits of the use of manipulative materials, nor did it attempt to compare differences in teacher styles or strategies, or to assess specific teaching methods.

Chapter II

Review of the Literature

Young children deal with fractional parts in their everyday lives, through sharing of food and objects, for example, well before formal schooling begins. Because the familiar, everyday items children experience at home may be very different from the manipulative materials used for instruction in school, children's at home experiences with fractions might seem, on the surface, to have little connection to the experiences which they may encounter when they begin school. This difference may not seem so great, however, when we consider the specific actions which children perform with these different materials.

Fractions are technically symbols which represent rational numbers. They have several different subconstructs. These subconstructs include equal shares, ratios, portions of sets, measurements, quotients, and decimals (Dorgan, 1994; Behr, et al., 1984). Essentially, fractions are expressions of relationships between parts and wholes (Van De Walle, 1994).

Fraction Instruction in Primary Grades

Primary grade mathematics curricula include the introduction of basic fraction concepts. This early introduction to fractions is supported by the California Department of Education (1992) and the National Council of Teachers of Mathematics (1989), both of which highly recommend the use of manipulative materials.

A review of some mathematics textbooks and programs used in the last decade in the state of California (Bohan, Clark, Kelleher & Thompson, 1995;

Burton, Fennell & Neufeld, 1995; Charles, Brummett, McDonald & Westley, 1995; Eicholz, O'Daffer, Fleenor, Charles & Young, 1989; Fennell, Reys, Reys, & Webb, 1988) shows that fractions have been presented to primary students in a variety of ways, including some use of manipulative materials along with activities which rely on the textbook page.

Mathematics Unlimited (Fennell, et al., 1988) introduces equal parts in first grade through the tracing of dotted lines that show equally-partitioned shapes, and provides punch-out fraction pieces for students to use to compare with circles and squares on textbook and worksheet pages. Second-grade activities include coloring sections of partitioned shapes or fractional parts of sets, circling shapes that show halves, thirds, and fourths, and writing fractions.

Addison Wesley Mathematics (Eicholz, et al., 1989) second-grade material has activities for coloring fractional parts of units and sets.

Math: Houghton Mifflin Mathematics (Bohan, et al., 1995) presents fraction concepts to second-grade students using pre-partitioned shapes, which are sometimes representative of food such as pizzas or candy bars. Activities include coloring fractional parts of these pictures, and circling fractions represented in illustrations.

Mathland (Charles, et al., 1995) presents fraction concepts to second grade through story problems for which they are asked to draw pictures and to use manipulative materials to explore fractional parts. Students are asked to partition shapes on their own.

Anytime Math (Burton, et al., 1995) introduces second grade students to part-whole relationships using manipulative materials such as Pattern Blocks

and Color Tiles (Cuisenaire, 1997). Children create a variety of wholes with these manipulative materials and record their manipulative work with drawings. They are required to identify, both orally and pictorially, fractional parts of wholes they create themselves.

Specific Difficulties With Fraction Concepts

Despite the fact that children probably have many experiences with beginning fraction concepts before they start school, and that fraction instruction begins in the early grades and continues to be presented throughout their school years, there are indications that many children do not fully understand part-whole relationships. The following studies show that many upper-grade students have a variety of specific difficulties understanding and applying fraction concepts.

Reliance on Whole Numbers

Students with whole number dominance are described, according to Behr, et al. (1984), as those who apply rules which center exclusively on the values of the denominators, and who fail to incorporate the inverse relationship between numerators and denominators. Post (1981) found that when asked to identify a fractional part of a set, fifty-eight percent of nine-year-old students tested simply counted the number of parts, rather than naming the fractional part. Mack (1990) found that sixth grade students, after separating whole units into parts, referred to them as numbers of pieces rather than fractional parts, suggesting to her that they were relying on whole number concepts rather than relating parts to wholes. Behr, et al. (1984) also found that students' thinking was dominated by their knowledge of ordering whole numbers. Most of the students in their study gave limited evidence of

awareness that both the numerator and denominator must be considered when judging the order or equivalence of fractions. After instruction, this whole number dominance diminished, but most students showed a tendency to “back-slide” into whole number thinking when faced with problem solving situations requiring them to apply concepts of order and equivalence. These studies suggest that students tend to ignore the relationship of parts to wholes when they are working with fractions.

Lack of Quantitative Understanding

That fractional parts of a whole are equal shares is one of the first fraction concepts taught to young children. A study by Peck and Jencks (1981), however, showed that even students as old as sixth grade generally do not demonstrate knowledge that the parts signified by the denominator need to be the same size, showing that they have little conceptual understanding of the meaning of fractions.

Lack of Connections Between Models and Symbols

Peck and Jencks (1981), asked sixth grade students to draw or use models to explain their answers for operations. They found that not more than ten percent of them were able to accurately indicate understanding of basic fraction concepts. Fewer than half knew that the subdivisions must be fair shares or were able to correctly draw representations of simple fractions. Wearne and Hiebert (1989) found that many students’ difficulties could be traced to an incomplete or nonexistent understanding of the written fraction symbol. In one study, fifty percent of nine-year-old children were unable to write the symbol for fractions given orally (Post, 1981), and in another study (D’Ambrosio & Mendonca Campos, 1992), students had difficulty when asked

to give a fractional name to represent shaded areas, even though they were able to use models to decide equivalencies and order. Similarly, Mack (1990) found that sixth-grade students were able to use their informal knowledge to correctly respond to questions based on real-world concepts, such as sectioning pizzas, but were often unable to solve the same problems when they were presented numerically. As shown by these studies, students often have difficulty making connections between fraction models and symbols.

Limited Ability to Transfer Knowledge to New Situations

Some researchers have found that even if students are able to develop adequate thinking to deal with questions about order and equivalence, they are often unable to apply their knowledge to questions which present new situations (Behr, et al., 1984). In a study with fourth, fifth, and sixth graders, Owens and Menon (1994) found that students who could identify fractional parts of sets made up of different-sized people had a great deal of difficulty solving the same problem when it was presented abstractly, with different-sized items in a set. It is apparent from these studies that students who have not developed solid number concepts of fractions sometimes use faulty or incorrect reasoning in problem-solving situations.

Reliance on Learned Rules

During a pretest for a study of pre-service teachers' understanding of students' thinking on fractions (D'Ambrosio & Mendonca Campos, 1992), several students did fairly well on the items involving operations with fractions, but showed less understanding on questions dealing with the conceptual understanding of fractions. Peck and Jencks (1981) found that many students tried to use rules they had previously learned, but often

misapplied these rules or remembered them incorrectly; furthermore, they could not tell they were doing so. Post (1981) found that students were able to use learned computational algorithms for operations, that they knew rote procedures for adding fractions, but that they had particular difficulty with unlike denominators. Similarly, Mack (1990) found that students' knowledge of procedures kept them from using their informal knowledge. In fact, students were more likely to trust their faulty procedures than their own informal knowledge of fraction concepts. In other studies (Carpenter, et al., 1980; Driscoll, 1984), students were found to have lower performance results on estimation exercises than on computation, indicating that they showed little understanding of underlying concepts. Furthermore, if they forgot the algorithm, they were not sufficiently able to reconstruct the algorithm or to fall back on intuitive models to solve even simple problems. These studies have shown that students often rely on learned rules in working with fraction problems, but that they sometimes show a lack of ability to connect concepts to these learned rules.

Children begin learning about fraction concepts through everyday activities before they begin school, and continue to work with part-whole relationships during the primary grades. However, students working with fractions in upper elementary grades have shown limited achievement and understanding of the quantitative relationships of parts to wholes. Why do so many students show such incomplete understanding of fraction concepts?

The Development of Fraction Concepts

Pothier and Sawada (1983) worked with children between kindergarten and grade three in an Alberta primary school. They describe the development

of fraction concepts as a series of five sequential levels: sharing, algorithmic halving, evenness, oddness, and composition. The first level, sharing, is not based on number concepts. An example of sharing is shown by Hunting & Sharpley (1988), who found that preschool children have a rudimentary conception of half, as demonstrated by their ability to systematically divide pieces of food. The second level, algorithmic halving, is the doubling of parts, as when children break a cookie in half and then break each half into halves. The third level, evenness, is considered by these researchers to be a breakthrough stage in the development of rational number concepts. This is when children begin to understand the idea that a fractional part has the concept of fairness, in that each fractional part of a unit is an equal share. When a child has learned to share equally, he is considered to have reached this level of understanding. The fourth level, oddness, is the recognition that using a median line for dividing a unit will not work, as when working with fifths or sevenths. Some of the children in this study were able to successfully perform tasks in which they were required to partition a shape into an odd number of parts, demonstrating a level beyond what these researchers consider the breakthrough stage in the understanding of fraction concepts. The fifth level, composition, deals with multiplicative actions such as bisecting parts, as when one-fourth is bisected into two-eighths.

Just as fraction concepts may develop in stages, a child's development of overall mathematical concepts also seems to proceed in stages, according to Langford & Sarullo (1993). These stages are nonverbal, listening and speaking, reading and writing words, and reading and writing symbol notations. In essence, children move from a concrete level during which they directly

manipulates objects, through a level when they manipulate mental images of those concrete objects, and on to an abstract level where they can manipulate symbols in place of those objects (Kroll, 1989).

These four stages of mathematical concept development directly mirror the stages of cognitive development as described by Piaget (1968). From birth to about two years of age, children initially use their reflexes, then their senses, in reacting and responding to the environment. During early childhood, between approximately two years and seven years of age, they develop the ability to reconstitute past actions and anticipate future actions. Beginning at about age seven, children enter the concrete operational stage, when they begin new forms of organizing their thoughts. At this time, they can carry out actions mentally as well as physically. They are able to exhibit individual concentration when working alone, and they can also collaborate effectively in groups. This is the stage when children begin to consider the reversibility of their actions, and to consider two qualities simultaneously, such as recognizing that a particular object can be both bigger than and smaller than other objects (Piaget, 1970), or that a whole can be made up of parts (Piaget, 1965). According to Piaget (1977), "failure or lack of comprehension where any single link in the chain is concerned entails an increasing difficulty in following the succeeding links, so that the student who has failed to adapt at any point is unable to understand what follows and becomes increasingly doubtful of his ability" (p. 702). It follows, then, that students who do not develop quantitative understanding of basic fraction concepts are likely to have difficulty with more abstract concepts at a later time.

Using Manipulative Materials in Fractions Instruction

The construction of logical-mathematical knowledge, like a child's general cognitive development, is dependent not on the presence of objects themselves, but on the child's actions on his environment (Wadsworth, 1984). Learning takes place as a child interacts with physical materials (Resnick & Ford, 1981), manipulating and performing actions with various objects. According to Piaget (1970), "knowledge is abstracted from actions, from the coordination of actions, and not from objects" (p. 16). Building on the idea that a child's physical and social interaction with his environment, in the form of verbal exchange with others and internalization of words and actions, are "essential to mental development," (Piaget, 1968, p. 17), the use of manipulative materials in teaching children mathematical concepts is extensively supported by researchers (Suydam, 1984). Students who learn with manipulative materials are believed to be better able to cross the bridge to the abstract world of mathematical concepts and apply their knowledge to real situations (Kober, 1991).

The National Council of Teachers of Mathematics (1989) supports the use of physical materials, diagrams, and real-world situations in the teaching of fraction concepts, a strategy supported by researchers and other experts. Specifically, the National Council of Teachers of Mathematics suggests that using physical materials, diagrams, and real-world situations" will help children "relate their learning experiences to oral language and symbols" (p. 57), and recommends that the K-4 curriculum be "conceptually oriented" in order to enable children "to acquire clear and stable concepts by constructing meanings in the context of physical situations" (p. 17).

Piaget (1977) states that mathematical structures are never represented in the mind of the child except in the form of concrete manipulations. Along this line, researchers conclude that the use of manipulative materials is crucial in the development of students' understanding of fraction ideas, because they "help students construct mental referents that enable them to perform fraction tasks meaningfully" (Bezuk & Cramer, 1989, p. 158). When students use manipulative materials, they deal with the parts in creating and sometimes disassembling a whole.

Manipulative materials give students experiences in constructing mental images of fractions. Through explanations of their work, students who have used manipulative materials have shown quantitative understanding of fractions (Bezuk & Cramer, 1989). In one study, students being interviewed about their work often based their thinking on mental images of previous work with manipulative materials (Behr, et al., 1984). Owens & Menon (1994) found that sixth grade students who used manipulative materials during instruction often used them to model and develop fraction concepts, and to help explain their answers during interviews. If manipulative materials were not used, they often referred to pizzas and chocolate bars to represent and explain answers. Students who did not choose to use manipulative materials were more interested in completing as many problems as possible, believing that they already knew how to get answers. This seems to support the opinion in an article published by the Harvard Graduate School of Education (1993), that students who put their faith in procedures rather than common sense often ignore the concrete materials that could help them.

Manipulative materials, which appeal to several senses, can help students construct mental images which will enable them to perform fraction tasks meaningfully, allowing them to more easily cross the bridge to the abstract world of mathematical concepts, and to apply this knowledge to real world situations (Kober, 1991). However, it is important to remember that just using concrete materials is not enough to guarantee success, because the materials do not automatically carry mathematical meaning for students (Thompson & Lambdin, 1994). It is suggested by the National Council of Teachers of Mathematics (1989) that all fraction work at primary levels involve "fractions that are useful in everyday life, that is, fractions that can be easily modeled" (p. 57). Further, it is suggested that physical materials should be used for exploratory work in adding and subtracting basic fractions, and solving simple real-world problems, including problems with parts of sets in relation to division. Work with manipulative materials can help children construct a mathematical reality of their own, making mathematics a part of their lived-in world (Steffe & Wiegel, 1992).

Primary school children have the ability to learn fraction concepts, as evidenced by a comparison of their stages of mental development (Piaget, 1968) with the stages of general mathematical concept development (Langford & Sarullo, 1993; Kroll, 1989), and the stages of development of rational number ideas (Pothier & Sawada, 1983). Acquiring part-whole concepts requires many experiences with the concepts over an extended period of time (Rathmell & Huinker, 1989), while the use of manipulative materials in the teaching of fraction concepts in the primary grades is supported by the National Council of Teachers of Mathematics (1989), the California

Department of Education (1992), and by research. The question for teachers is, "Is there a best type of manipulative to use for the teaching of fractions to primary children?"

CHAPTER III

Methodology

Interest in children's development of concepts of the relationships of parts to wholes, and the presence of these part-whole relationships in the real world, as related to the development of general mathematics concepts within overall cognitive development, provided the initial impetus for this study, in which two groups of second grade students each performed different kinds of operations on two kinds of manipulative materials to build their understanding of fraction concepts. One group of students used base ten blocks during instruction. They built wholes from parts, then separated the whole into parts. The other group used food, in the form of sheets of cake, brownies, candy bars, and donuts, for example. This group separated a variety of given whole shapes into parts. Students in both groups were asked to identify and name fractional parts, while participating in teacher-directed activities which included showing and identifying parts of parts, combining fractional parts, and finding comparisons and equivalencies of fractional parts.

The study was conducted in a suburban, San José, California school, in a multi-age primary class comprised of seventeen first, twenty-four second, and twenty-two third-grade students. The class was team taught by two teachers in one large classroom. All twenty-four second-grade students in the class were involved in the study. Eighteen of the second-grade students were in their second year in the multi-age class; the other six students had entered the multi-age class in second grade. The students were of mixed socio-

economic backgrounds. Several students were identified as having special needs: five students met with the speech therapist twice a week for articulation exercises; three of these students also met with the resource specialist twice a week, where they received small group instruction in reading and writing skills. Three additional students had native languages which were other than English. None of the three had difficulty communicating in English; therefore, no translation of materials was necessary.

Procedure

Pretest Assessments

First, students were given Pretest A, in which they were to show, on a worksheet (Appendix A), how they would cut a round cake for three people, each having an equal-sized piece, and to explain their strategies.

Second, students were given Pretest B (Appendix B), a written test which included items which were reflective of the fraction work presented in the required district second-grade mathematics curriculum (Fennell, et al., 1988), as well as items which were not normally introduced to children until third, fourth, or fifth grade. Both teachers were available to help students read directions and examples to students, if necessary, but did not help students with problem-solving strategies.

Before Pretest A and Pretest B were scored, each test was assigned a number and students' names were removed from the papers. Pretest A and Pretest B were scored using the four-point Rubric For Problem Solving With Fractions (Appendix H), hereafter referred to as the Problem-Solving Rubric. Each item on the pretest was given an individual score; these scores were

added together, then combined with the interview score for a combined total pretest score. In order to check the objectivity of the rubric, four pretests were randomly selected and scored by another teacher. A comparison of results revealed a few scoring discrepancies, none of which varied more than one point on any one test item; these discrepancies were the result of the two scorers' different interpretations of the rubric wording. After the rubric language was clarified for further use, the tests were reviewed an additional time for accuracy in scoring. Using the combined pretest scores for the two pretests, students were assigned to one of two groups, in descending alternate order.

Pretest C required all students to participate in an open-ended individual writing and whole group oral language activity. The purpose of this task was to assess their prior knowledge of part-whole relationships as individuals, as members of Group A or Group B, and as a total group. Students were asked to write individually, before but without discussion, what they knew about fractions. After the papers were collected, students responded orally in the total, large group. Responses were charted and posted on the wall of the classroom. The written responses were scored using the Rubric For Writing About Fractions (Appendix K), hereafter referred to as the Writing Rubric.

Finally, students in both groups were asked to complete Worksheet 1 (Appendix C) and Worksheet 2 (Appendix D), which focused on a single skill, that of writing a fraction for a shaded diagram. Worksheets 1 and 2 were scored using the Rubric For Identifying Fractions, hereafter referred to as the Identification Rubric.

Instruction and Ongoing Assessments

Each group used a different type of manipulative in a series of twenty-nine activity-based lessons focusing on part-whole concepts. Group A used base ten blocks, while Group B used foods such as brownies, cake, granola bars, donuts, cookies, graham crackers, licorice whips, bananas, and candy bars. Overall objectives of the lessons included the exploration of relationships between parts and wholes and the establishment of connections between models, words, and symbols. Students were introduced to basic fraction concepts, including identifying and naming fractions, through lessons in which they were asked to find parts of parts (Neufeld, 1995), the concept behind multiplication of fractions. These basic concepts were reinforced in addition lessons, in which students combined fractional parts of units and sets. In comparison lessons, students used the concepts they had learned to extend their development of understanding of the relationships between different fractional parts.

Fractions instruction took place four days a week during the last hour of the day. The structure of the school day was such that students attended school on a staggered schedule, with the first and third grade students on a different schedule from second grade students. Second grade students were the only students in the classroom for the last hour of the day four days a week. The fraction lessons comprised half of the math period. During the other half of the period, students received textbook-based instruction in non-fraction mathematical concepts.

The lessons were taught by two teachers, who alternated teaching the two groups every two days to maintain balance in teacher/student

interaction. Fraction lessons took place in the classroom, while textbook lessons took place in the library, which was adjacent to, but not visible from, the classroom. Students and teachers changed places half-way through the hour; thus, the two groups were kept separated during fractions instruction, and were unable to hear or see the other group at any time during the math period. At the end of each lesson hour, Group B students ate the food which they had used during their lessons; identical snacks were distributed to all students in Group A at the end of their textbook lesson.

Lessons for the two instructional groups were written to be as similar as the manipulative materials would allow, with some differences dictated by the qualities of the materials being used by each group of children, who were involved in two very different processes. One group moved from part to whole to part, while the other group moved from whole to part. Lesson concepts and objectives were identical for both groups. The course of the lessons required students to respond to questions which were designed to help them learn to reflect on their actions with the manipulative materials, and to increase their understanding of the relationships between parts and wholes. Students were consistently required to explain their problem-solving strategies.

Lessons 1 through 7 focused on the relationships of parts to parts. These lessons required students to use manipulative materials to find relationships such as one-half of one-half, or one-third of one-half. Students using blocks, for example, were asked in one lesson to build a square with four blocks, to show half of the square, and then to show half of the half, while students using food were asked to cut a brownie in half and then to cut

one of the halves in half. Students were questioned about the numbers of parts in relation to the wholes, and introduced to the corresponding written fractions. Students participated in discussions about the meaning of fractional parts, specifically focusing on the number of parts in a whole and the parts as equal shares. Students also were asked to relate models, pictures and words to their corresponding symbols.

At the conclusion of the first seven lessons, students completed Worksheet 1 and Worksheet 2 a second time. Students then participated in Interview 1, which was used to help clarify information about their strategies in their beginning work with fractions. No fractions lessons took place during the interview week.

Lessons 8 through 20 focused on combining parts. These lessons required students to use their manipulative materials to explore the process of addition of fractional parts. Students using blocks, for example, were asked in one lesson to build a rectangle that showed fourths, separate the parts, and then combine parts to match equations such as $\frac{1}{4} + \frac{2}{4}$. The same lesson for students in the food group asked them to cut bread slices into fourths, then to combine parts to match equations. Students in both groups worked with like denominators first, then unlike denominators, such as thirds and fourths, or halves and thirds. The basic fraction skills of identifying and writing fractions were reinforced throughout this series of lessons.

At the conclusion of Lesson 20, students completed Worksheet 1 and Worksheet 2 a third time, and also completed Worksheet 3 (Appendix E), which required students to shade in parts of pre-drawn partitioned shapes, then to write the number and word for the fractional parts they had shaded.

Each student then participated in Interview 2, which was used to assess their understanding of combining parts. No lessons were held during this interview week.

Lessons 21 through 29 focused on comparisons and equivalencies. Students were asked to make comparisons of fractions with like and unlike denominators. Students using blocks, for example, built rectangles out of eight blocks, identified and separated $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{1}{8}$ from their rectangles, and compared the sizes of the identified parts, such as $\frac{1}{2}$ and $\frac{1}{4}$, $\frac{1}{4}$ and $\frac{1}{8}$, and $\frac{1}{2}$ and $\frac{2}{4}$. Students in the food group, in a corresponding lesson, cut slices of cheese into fourths, compared $\frac{1}{2}$ and $\frac{1}{4}$, then cut the fourths in half again to compare $\frac{1}{2}$ and $\frac{1}{4}$, $\frac{1}{4}$ and $\frac{1}{8}$, and $\frac{1}{2}$ and $\frac{2}{4}$. Students also participated in activities in which they arranged models of three different fractional amounts in order from smallest to largest. Finally, simple fraction addition problems with like denominators were used to introduce and create mixed numbers. Students using blocks were required to model equations like $\frac{3}{4} + \frac{3}{4}$ and $\frac{3}{4} + \frac{2}{4}$ using two rectangles; students using food were asked to cut balogna slices into fourths to model the same equations.

Post-Test Assessments

At the completion of all lessons, students completed Worksheets 1 and 2 a third time, and Worksheet 3 a second time. They also completed Worksheet 4 (Appendix F), which required them to show named fractional parts in a series of rectangles, and Worksheet 5 (Appendix G), which required them to partition circles, squares, and triangles to show halves, thirds, and fourths. Worksheets 1, 2, and 3 were scored using the Identifying Rubric; Worksheets 4 and 5 were scored using the Problem Solving Rubric.

After an overall review of multiplication, addition, and comparison concepts, students were given three post-tests.

First, in Post-Test A, students were asked to write a response to the prompt, "What I've Learned About Fractions." The responses were scored using the Writing Rubric.

Second, students took Post-Test B, the same written test which had been used for the pretest. Post-Test B was scored using the Problem Solving Rubric.

Post-Test C asked each student to show how he or she would cut a round cake for three people. During this interview, students were also asked to explain what a fraction was, to explain how fractions differed from whole numbers, to compare two fractional amounts, and to explain their thinking. The interview responses were scored using the Interview Rubric.

Manipulative Materials

The two different types of manipulative materials used for this study were base ten blocks for Group A and food for Group B.

The base ten blocks used by Group A required students to use rectangles. Whole rectangles were constructed from blocks by students before they were asked to consider parts of the whole rectangle. Wholes could be reconstructed in their original state after parts were separated. Students counted the blocks to establish the denominator and also to find numbers of parts of the whole rectangle. The parts were always equal in size, as determined by the size of the blocks.

The food used by Group B was presented in a variety of shapes, including rectangles, squares, and circles, as well as some irregular shapes.

Students cut the food and counted parts to establish the denominators and also to find numbers of parts of wholes. Most of the foods used lacked pre-determined parts and sizes of parts; therefore, students used their own concepts of size and quantity to decide where to cut the food.

Assessment Materials

Pretests / Post-Tests

The pretests and post-tests were developed from the curriculum which was written for the study. Some test items reflected activities and concepts which were normally included in the district mathematics curriculum for second grade (Fennell, et al., 1988). For example, students were asked to write fractions shown by diagrams with shaded parts, to show fractional parts in given shapes, and to identify the correct diagrams showing named fractional amounts. Also included on the pretest were items which required application of knowledge of fraction concepts, such as multiplication and addition of fractions, which are not typically introduced in the mathematics curriculum until third, fourth, and fifth grades.

Worksheets

Students were asked to complete a series of worksheets which were used for ongoing assessment of the development of understanding of basic fraction concepts. Worksheet 1 (Appendix A) and Worksheet 2 (Appendix B) were obtained from the second-grade Mathematics Unlimited workbooks (Fennell, et al., 1988), which the school district had used in previous years' instruction. These two worksheets required students simply to write the fractions for given shaded figures. These worksheets were administered

before lessons began and at the end of each of the three lesson sections, to provide a periodic grade level benchmark for assessing basic, grade-level student achievement on an ongoing basis. Worksheet 3 (Appendix E) was adapted from current second-grade Houghton Mifflin Mathematics materials (Bohan, et al., 1995) which had recently been adopted for use by the state of California. As originally published, the worksheet asked students to shade in given fractional parts of a predrawn, partitioned figure. The format of this worksheet was revised to allow for more open-ended responses by eliminating the numerical fractions so that students would need to choose how many parts to shade and then write the numerical symbol and the word for the fraction they had created. This worksheet was administered twice, after each of the second and third series of lessons. Worksheet 4 (Appendix F) was developed to measure the partitioning ability as well as understanding of the concept of parts of parts. It required students to show named fractional parts in a series of rectangles, to partition a rectangle to show one third of one half, and then to solve the same problem symbolically. Worksheet 5 (Appendix G) was developed to assess basic partitioning skill. It required students to partition squares, circles, and triangles into halves, thirds, and fourths. Students completed Worksheet 4 and Worksheet 5 once each, at the conclusion of the study.

Student Interviews

Four student interviews were held during the course of the study.

Pretest A required students to show how they would cut a round cake for three people, so that each person would get the same amount with no

cake left over. The objective of this interview was to assess the students' levels of partitioning skill and their concept of equal shares.

Interview 1 consisted of two parts. In the first part, each student was asked to draw diagrams which showed fourths and thirds, to show one fourth and one third in their respective diagrams, and to write the fractions for what they had drawn. In the second part of the interview, students were given six unit blocks if they were in Group A or six peanuts if they were in Group B. They were asked to give the researcher half, to show one third of one half, and then answer the question, "What is one third of one half?" At each step, they were asked to explain their reasoning and responses. These interviews, which took place over the course of a week's time, were audio taped.

Interview 2 required students to draw two different diagrams of fifths, to show one fifth in one of the drawings, and write the fraction for their drawing. In the second part of the interview, students were asked to write and solve a given fraction addition problem with like denominators (fifths), and to explain their strategies. These interviews, which took place over the course of a week's time, were audio taped.

Post-test C was identical to Pretest A, in which students were asked to show how they would cut a round cake for three people. Particular attention was paid to the understanding of the need for equal-sized parts and the ability to recognize that a median line would not work for this task.

During all student interviews, notes were taken by the researcher about students' responses and drawings. Interview 1 and Interview 2 were audio taped. Although there was a set procedure which was followed by the interviewer, the interview questions were not always identical for each

interview, since student responses often required the interviewer to ask additional questions, to rephrase questions in order to help students understand the question being asked, or to clarify for the interviewer precisely what was meant by the student. Interview responses were scored using the four-point Rubric for Student Interviews (Appendix J), hereafter referred to as the Interview Rubric.

Scoring

Four rubrics were created for scoring the different types of student work. All rubrics were based on a scale of one to four.

A Problem-Solving Rubric (Appendix H) was used to score the written pretests and post-tests. This rubric was developed by combining information from an Analytic Scale for Problem Solving (Szetela & Nicol, 1992) with information regarding the four levels of partitioning capabilities described by Pothier and Sawada (1983). The rubric was scaled so that a score of four required logical and complete problem solving strategies, a high developmental level of partitioning capability, and solid understanding of the basic concepts, such as appropriate numbers of equal-sized parts within a whole unit. Scores of three and two reflected partially correct problem solving strategies, a mid-range developmental level of partitioning capability, and incomplete explanations or answers, while a score of one was given for items which students attempted but answered incorrectly, or which showed complete or nearly complete reliance on whole number concepts rather than rational number concepts. Items with no answers were given a score of zero.

An Identification Rubric (Appendix I) was developed to rate the variety of different types of errors students make when identifying and writing

fractions, which often reflect varying degrees of understanding of basic concepts. A score of four was given for work with no errors. Variations of reversals of the numerator and denominator were considered a high level of understanding, and therefore were given three points. If the student had counted shaded and unshaded parts separately, two points were given. If the student used a whole number instead of a fraction, and if that number was a correct number of parts shown, one point was given.

The Interview Rubric (Appendix J) was used to score interview responses. It was scaled so that students who performed the tasks with little hesitation and who were able to clearly explain their logical strategies received four points, while students with errors in their strategies and responses, or who had difficulty explaining their work, received three or two points, depending on the completeness and clarity of their strategies and responses. Students unable to explain what they had done, or who made many mathematical or procedural mistakes during the tasks, received one point for the interview task.

A Writing Rubric (Appendix K) was used to score students' written responses. This rubric specifically targets partitioning, evenness, and symbolic notations, as well as a student's ability to explain himself clearly in writing. Four points were given to students who wrote about all three concepts. Three points were given to students who wrote about any two concepts. Two points were given to students who wrote about only one of the concepts. One point was given to students who simply listed numerical examples or fractions, or drew a diagram with no written explanation.

CHAPTER IV

Analysis of Data

The purpose of this study was to compare achievement with and early development of understanding of fraction concepts in two groups of second grade students, relative to the type of manipulative materials used by each of the two groups. Group A received instruction with base ten blocks; Group B used food.

Pretest Assessments

Before instruction began, the students completed a series of pretest assessments designed to assess their understanding of fraction concepts. In Pretest A, each child was asked to show how he or she would cut a round cake equally for three people. Pretest B, a written pretest, included all the concepts to be taught. In Pretest C, students were asked to respond in writing to the prompt, "What Are Fractions?" Students also completed two worksheets which asked them to identify pre-drawn, pre-shaded figures, and to write the fractions for the diagrams.

Pretest A: Cutting a Round Cake for Three People

In individual interviews, students were asked to show, in a predrawn circle, how they would cut a cake so that three people could each have the same size piece. Thirds were selected in order to determine whether the children had reached the partitioning stage in which a unit cannot be divided using a median line. As shown in Table 1, only one child was able to partition the circle into three equal parts on the first try. Five children were able to

eventually partition into three equal parts by drawing lines, erasing and redrawing until they were satisfied with their results. Five children partitioned the circle into three unequal parts; two of these children started with a median line. Five children partitioned the circle into equal parts which were not thirds; two of these began with a median line. Two children created four unequal parts. Two children made three "cake" cuts into the circle without using the whole circle. Four children partitioned the circle into multiple unequal parts; one of these children tried many diameter lines, one made multiple triangular cuts from the circumference, one made multiple pie cuts beginning from the center, and one partitioned the circle into dozens of small sections, using numerous cross-hatched parallel lines. Five students began with a median line and were unable to successfully adjust their parts.

Table 1: Pretest A - Cutting a Round Cake for Three People
Types of Drawings

Type of Graphic Response	Number of students	ML
three equal parts on first try	1	0
three equal parts with multiple tries	5	1
three unequal parts	5	2
two, four, or more equal parts	5	2
four unequal parts	2	2
three equal cuts w/ unused portion	2	0
multiple unequal parts	4	1

Note: ML = Number of students beginning with a median line.

Pretest A responses were scored using the Interview Rubric, which rates the speed, clarity, and accuracy of students' explanations about their drawings. Six students received the highest possible score of 4 points, while eleven students received the lowest possible score of 1 point. Table 2 shows the points received by the number of students.

**Table 2: Pretest A: Cutting a Round Cake for Three People
Overall Scores**

score	4 points	3 points	2 points	1 point
students	6	1	2	11

Pretest B

Pretest B, a written pretest, included items which covered a variety of concepts, including identifying fractional parts, creating and naming fractional parts, writing fractions, and adding, multiplying, and comparing fractions. Pretest B was scored using the Problem Solving Rubric, which measures accuracy of responses, response quality, and quality of problem-solving strategies. Pretest B had a total possible score of 76 points. Scores ranged from a high of 51 points to a low of 21 points. Scores were divided into four equal ranges: high, medium-high, medium-low, and low. Table 3 shows the number of students whose scores fell within each range of scores:

Table 3: Pre-Test B Scores

score	High 58-76	M-High 39-57	M-Low 20-38	Low 1-19
students	0	3	21	0

Each child's Pretest A score and Pretest B score were added together. The total possible combined score of Pretest A and Pretest B was 80 points. The highest score was 55 points; the lowest was 23 points. Possible scores were divided into four equal ranges designated high, medium-high, medium-low, and low. Table 4 shows the number of students whose combined scores fell within each range of scores:

Table 4: Pretest A plus Pretest B
Combined Scores

score	High 61-80	M-High 41-60	M-Low 21-40	Low 1-20
students	0	4	20	0

Two instructional groups were created using the combined scores of Pretest A and Pretest B. Starting with the highest combined score, students were assigned to either Group A or Group B, in descending alternate order. Group A was selected to use base ten blocks for instruction; Group B was selected to use food for instruction. Table 5 shows the scores for the pretest assessments by number of students in each group.

After being assigned to Group A or Group B, students completed two worksheets which required them to identify fractional parts of predrawn shapes, and to write the fractions for the diagrams. Worksheet 1 (Table 6) contained eleven items, for a high possible score of 44 points. Two students scored 44 points; nineteen students received a low score of 11 points. Worksheet 2 (Table 7) contained six items, for a high possible score of 24 points. Four students scored 24 points; fifteen students received a low score of

6 points. The worksheet scores were divided into four equal ranges of high, medium-high, medium-low, and low.

Table 5: Pretest A and Pretest B Assessments

Score	High		Med-High		Med-Low		Low	
Group	A	B	A	B	A	B	A	B
Partitioning Task Cutting a Round Cake 4 points maximum	3	3	0	1	2	4	7	4
Written Pretest 76 points maximum	0	0	1	2	11	10	0	0
Combined Scores 80 points maximum	0	0	2	2	10	10	0	0

Table 6: Worksheet #1 - Identifying Predrawn Fractional Parts

score	High 34-44	Med-High 23-33	Med-Low 12-22	Low 1-11
Group A	1	1	0	10
Group B	1	2	0	9

Table 7: Worksheet #2 - Identifying Predrawn Fractional Parts

score	High 19-24	Med-High 13-18	Med-Low 7-12	Low 1-6
Group A	2	2	0	8
Group B	2	3	0	7

Pretest C: Individual Written Responses: What Are Fractions?

Group knowledge and understanding of fraction concepts was assessed by asking students to respond individually, in writing, to the question, "What Are Fractions?" The written responses were scored using the Writing Rubric, which requires, for a score of four points, a numerical or written example or explanation of fractions, an indication that fractions are parts of a whole unit or set, and an indication that fractional parts are equal sizes. No student in either group received a score of 4 points. One student in each group indicated at least two of the targeted items, for a score of 3 points. Four students in Group A and six students in Group B wrote about at least one of the items. Seven students in Group A and five students in Group B failed to write about any of the three items. Table 8 shows the number of students in each group who received each score.

Table 8: Pretest C: What Are Fractions?

Score	4 points	3 points	2 points	1 point
Group A	0	1	0	4
Group B	0	1	0	6

Ongoing Assessments

Lessons were divided into three sections. The first series of lessons presented parts of parts, the second series presented combinations of parts, and the third series presented comparisons of parts. At the end of each sequence of lessons, students completed fraction worksheets to assess their basic ability to identify graphic representations of fractions. Five different

worksheets requiring different types of tasks were administered. Worksheets were scored using the Identification Rubric or the Problem Solving Rubric. Maximum scores varied on each worksheet according to total number of items; scores were divided into four equal ranges.

Also at the end of each series of lessons, students were interviewed individually to assess their deeper understanding of part-whole relationships. Interviews were scored using the Interview Rubric. Scores were divided into four equal ranges designated high, medium-high, medium-low, and low.

Worksheets

Five different worksheets were administered at different times throughout the study. Each worksheet required different skills. The worksheets had various possible total numbers of points, but in all cases, the scores were grouped into four equal ranges which were considered to be high, medium-high, medium-low, and low scores. Worksheets were scored using the Identification Rubric or the Problem-Solving Rubric.

Worksheet 1 required students to identify fractional parts of predrawn, pre-shaded shapes, and to write the corresponding fractions. This worksheet was administered three times. The highest possible score was 44 points. Initially, two students in Group A and three students in Group B received scores in the highest range; only two students, both in Group B, received a high score of 44 points. Table 9 shows the number of students whose scores fell within the high, medium-high, medium-low, and low ranges for Worksheet 1 on four different occasions, including the pretest assessment.

Table 9: Scores for Worksheet 1 -Identifying Pre-Shaded Fractional Parts

Score	High 34-44 pts.		Med-High 23-33 pts.		Med-Low 12-22 pts.		Low 1-11 pts.	
Group	A	B	A	B	A	B	A	B
pre-assessment	1	1	1	2	0	0	10	9
after 1st set of lessons	2	3	0	1	0	0	10	8
after 2nd set of lessons	9	9	0	1	3	1	0	1
after last set of lessons	11	11	1	0	0	1	0	0

Worksheet 2 also required students to identify and write fractions for predrawn, preshaded shapes. This worksheet was administered three times. The highest possible score was 24 points. Initially, three students in Group A and two students in Group B received scores in the top range; only two students, both in Group A, received a high score of 24 points. Table 10 shows the number of students whose scores fell within the high, medium-high, medium-low, and low ranges for Worksheet 2 on four occasions, including the pretest assessment.

Table 10: Scores for Worksheet 2 -Identifying Pre-Shaded Fractional Parts

Score	High 19-24 pts.		Med-High 13-18 pts.		Med-Low 7-12 pts.		Low 1-6 pts.	
Group	A	B	A	B	A	B	A	B
pre-assessment	2	2	2	3	0	0	8	7
after 1st set of lessons	3	2	2	4	0	0	7	6
after 2nd set of lessons	7	9	3	1	2	2	0	0
after last set of lessons	10	11	2	1	0	0	0	0

Worksheet 3 allowed students to shade any number of parts of predrawn, pre-sectioned shapes, and then to write the fraction and the words for the fractional amounts they had created by shading the parts. Students were given this worksheet after they had completed the series of lessons on combining parts, and again after the series of lessons on comparing parts. Maximum score on Worksheet 3 was 24 points. Initially, three students in Group A and nine students in Group B received the highest possible score; when completed a second time, nine students in Group A and ten students in Group B received high scores of 24 points. Table 11 shows the number of students who received scores on Worksheet 3 which fell within the high, medium-high, medium-low, and low ranges on two different occasions.

Table 11: Scores for Worksheet 3 -
Shading and Identifying Fractional Parts in Predrawn Shapes

Score	High 19-24 pts.		Med-High 13-18 pts.		Med-Low 7-12 pts.		Low 1-6 pts.	
Group	A	B	A	B	A	B	A	B
after 2nd set of lessons	6	10	4	2	1	0	1	0
after last set of lessons	11	11	1	1	0	0	0	0

Worksheet 4 required students to partition predrawn rectangles into halves, thirds, and sixths, and then to shade in a given fractional amount. Students also needed to show one-third of one-half in a predrawn rectangle, and to solve the corresponding fraction multiplication problem as represented in symbolic form. This worksheet was administered only once, at

the end of the last series of lessons. The maximum overall score was 20 points. Two students in Group A and no students in Group B received maximum points. Table 12a shows how many students in each group had overall scores on Worksheet 4 which fell within the high, medium-high, medium-low, and low ranges. Table 12b shows the scores for each item by number of students in each group.

Table 12a: Overall Scores for Worksheet 4 -
Partitioning Rectangles and Multiplying Fractions

Score	High 16-20 pts.		Med-High 11-15 pts.		Med-Low 6-10 pts.		Low 1-5 pts.	
Group	A	B	A	B	A	B	A	B
after last set of lessons	2	2	7	8	1	2	2	0

Table 12b: Item Scores for Worksheet 4 -
Partitioning Rectangles and Multiplying Fractions

Score	4 points		3 points		2 points		1 point	
Group	A	B	A	B	A	B	A	B
partitioning halves	9	11	1	0	0	0	2	1
partitioning thirds	7	8	1	1	0	1	4	2
partitioning sixths	7	7	2	3	0	0	3	2
1/3 of 1/2 in a rectangle	2	2	1	0	3	7	6	3
multiplying $1/3 \times 1/2$	3	0	0	1	0	2	9	9

Worksheet 5 required students to show halves, thirds, and fourths in predrawn squares, circles, and triangles. The maximum possible score was 36

points. No student from either group received the maximum score, although one student from Group A received 35 points. Table 13 shows the number of students in each group whose scores fell within the high, medium-high, medium-low, and low ranges. Due to two absences, only eleven students from each group completed this worksheet.

Table 13: Scores for Worksheet 5
Creating Halves, Thirds, and Fourths in Predrawn Shapes

Score	High 28-36 pts.		Med-High 19-27 pts.		Med-Low 10-18 pts.		Low 1-9 pts.	
Group	A	B	A	B	A	B	A	B
after last set of lessons	5	10	6	0	0	1	0	0

Interviews

After each set of lessons, students were individually interviewed to assess their understanding of fraction concepts which had been presented during the lessons and the application of those concepts.

Interview 1 required students first to draw diagrams of fourths and thirds, to show one-fourth and one-third in their diagrams, and to write the fraction. Then they were asked to use base ten blocks (Group A) or peanuts (Group B) to show one-third of one-half, to name the resulting fraction, and to explain their reasoning.

Interview 2 required students first to draw diagrams of fifths two different ways, to identify one-fifth in one of their diagrams, and to explain how they knew it was one-fifth. Then they were asked to write and solve the problem $1/5 + 3/5$, and to explain their strategies.

Interview responses were scored using the Interview Rubric, which rates accuracy and clarity of responses.

Interview 1

All students in both groups were easily able to draw a diagram to show one fourth, and were able to write the fractions for their diagrams. Most students responded that one fourth meant “one out of four” and that one-third meant “one out of three.” One student inverted the numerator and denominator when asked to write the fraction for one-fourth; when asked how she knew it said one fourth, she responded that she had seen it.

Six students in Group A and four students in Group B successfully drew a diagram to show thirds. Four students in Group A and five students in Group B drew diagrams which had three parts, but in which the parts were not of equal size. One student in each group drew a diagram in which the parts were of equal size, but the number of parts was incorrect. One student in Group B drew a diagram with the incorrect number of parts which were also not of equal size. One student in Group A was unable to complete the task, and one student in Group B did not respond to the interview question, so the interview was terminated.

When asked to show parts of parts using manipulative materials, six students in Group A were able to easily and immediately identify three blocks as half of six, but two were unable to explain why. Four students initially thought two blocks were half of six. One student did not understand the directions, and another questioned whether three blocks were one half or one third of six. Three students were able to identify one third of one half as one

sixth without prompting and were able to give a logical explanation for their answers. Six students were able to do so with prompting.

In Group B, eight students were able to easily and immediately identify three peanuts as half of six, but three of them were unable to explain why. One student was able to explain why after some initial hesitation. Two students wanted to break the peanuts in half. One student thought one peanut was half of six, using the justification that halves means “one piece of something.” Six students were able to identify one third of one half as one sixth without prompting, giving logical explanations for their answers. Three were able to do so with prompting. Three students in Group B were unable to identify one third of one half.

A summary of student responses in Interview 1 is available in Appendix L.

Interview 1 Scores

Interview responses received three different scores. First, they were scored for overall accuracy. Second, they were scored for the quality of students' verbal responses. Finally, the responses were analyzed with regard to the types of drawings the students created and the accuracy of those drawings. The Interview Rubric was used to score the interview responses. Table 14 shows the scores received by the number of students in each group for each of the two separate tasks in Interview 1.

Interview 1 Response Quality

Because students showed significant differences in the way they responded to interview questions, their responses were assessed according to how quickly, accurately, and clearly they responded. No points were received

if the student gave no response, requiring the interview to be terminated. A separate score was given for each portion of the interview. Table 15 shows the response quality scores received by the numbers of students in each group for each part of Interview 1.

Table 14: Overall Scores for Interview 1

Score	4 points		3 points		2 points		1 point	
Group	A	B	A	B	A	B	A	B
Creating and Identifying Fourths and Thirds	5	3	1	1	2	7	4	1
Finding One Third of One Half	1	4	2	0	7	2	1	6

Table 15: Response Quality Scores for Interview 1

Score	4 points		3 points		2 points		1 point	
Group	A	B	A	B	A	B	A	B
Creating Fourths	7	9	3	0	1	3	1	0
Identifying One Fourth	11	6	0	3	0	2	1	0
Creating Thirds	6	3	4	4	0	2	2	2
Identifying One Third	8	4	2	1	0	1	1	2
Finding One Half (of six items)	5	8	5	2	2	2	0	0
Finding One Third of One Half (of six items)	6	8	3	1	3	1	0	1
Naming One Sixth as One Third of One Half	8	7	0	3	2	0	2	0

Interview 1 Diagram Types

Students used a variety of strategies to solve the open-ended interview tasks which required them to draw a picture to show fourths, and then thirds, on a blank sheet of paper. All twelve students in Group A and nine students in Group B were able to successfully draw and explain fourths on their first try. Eight students in Group A and five students in Group B were able to successfully draw thirds on their first try. Table 16 shows how many students in each group drew each type of diagram when creating diagrams to show fourths and to show thirds.

Group A

To show fourths, six students in Group A drew diagrams using individual squares or other shapes. Three of these students drew small squares about the size of base ten unit blocks; one student drew separate rectangles to show parts; one student drew a square by attaching four or three separate squares; and one student drew pictures of four different items, saying it showed fourths because it was four things. Six students drew single shapes, which they all partitioned successfully. Three drew circles and three drew squares.

To show thirds, seven students drew diagrams using individual shapes; one of these students attached squares to each other while drawing them. Four other students drew a set of three small squares, one drew a set of three circles, and one drew a set of three rectangles. Five students drew single shapes and partitioned them. Of these five students, two drew circles, one

drew a square, one drew a rectangle, and one drew a triangle. The rectangle and one of the circles were partitioned into thirds successfully.

Group B

To show fourths, ten students in Group B drew unit shapes which they sectioned into fairly equal parts, when asked to draw diagrams showing fourths and then thirds. The remaining two students drew four separate squares to show fourths. One student initially said she did not know what to do, and finally decided to draw four squares with dolls in them when the word "fourths" was repeated to her three times.

To show thirds, students used a variety of strategies and unit shapes, including one "L" shape, which the student referred to as "a boot." This shape was sectioned fairly equally, and the student was able to equally section a rectangle drawn by the interviewer. Two students were unable to show thirds in a unit shape. One was able to state that thirds meant three parts, and at one point drew three circles to solve his dilemma, after having unsuccessfully tried to partition a rectangle, a circle, and a square. The other student's first attempt, in a circle, showed no attempt to show equal parts; his second attempt, in a rectangle drawn by the interviewer, was four sections, which he said showed thirds because if one were shaded, there would be three left.

Table 16: Interview 1 Diagram Types

Diagram Type	Group A Fourths	Group B Fourths		Group A Thirds	Group B Thirds
sectioned square unit	4	7		1	2
sectioned rectangle unit	0	1		2	3
sectioned circle unit	3	2		2	4
sectioned triangle	0	0		1	1
sectioned "L" shape	0	0		0	1
set of squares	3	1		3	0
set of rectangles	0	1		1	0
set of circles	0	0		1	0
sets inside of units	2	0		1	0
(unable to complete task)	0	0		0	1

Interview 2

Four students in Group A and one student in Group B were able to successfully draw diagrams which showed fifths, to identify and write one-fifth, and to explain their responses. Four students in each group tried to begin with a median line. Of these students, three students in Group A were able to successfully show fifths, two by adding an extension to the initial shape, one by erasing parts.

When orally given the addition problem to solve, four students in Group A and two students in Group B successfully completed the problem quickly and without any prompting. All students in both groups added the numerators correctly. Four students in Group A and six students in Group B added the denominators, resulting in an answer reflecting tenths rather than fifths. Appendix M shows a summary of student responses for Interview 2.

Interview 2 Scores

Students received three different scores for their responses during Interview 2, which had two main tasks. First, the responses to each part of the interview were scored separately for overall accuracy (Table 17). Second, students received six different scores for the quality of their verbal explanations for each of six sub-tasks (Table 18). Finally, students' responses were analyzed with regard to the types of drawings the students created and the accuracy of those drawings (Table 19). The Interview Rubric was used to score the interview responses.

Interview 2 Response Quality

Students' responses to the task for drawing fifths two different ways were assessed according to the speed and accuracy of their responses. Students responding quickly and accurately with no prompting received four points. Students hesitating noticeably before responding, who needed one prompt before giving accurate responses, who made minor mistakes, or who had unclear verbal responses, were given three points. If students needed more than one prompt, made many mistakes, or could not explain their work, they received two points. Students who began the task but could not complete it received one point.

Table 17: Overall Scores for Interview 2

Score	4 points		3 points		2 points		1 point	
Group	A	B	A	B	A	B	A	B
Creating and Identifying Fifths	4	1	4	3	4	8	0	0
Adding $1/5 + 3/5$	4	2	2	2	6	6	0	2

Table 18: Response Quality Scores for Interview 2

Score	4 points		3 points		2 points		1 point	
Group	A	B	A	B	A	B	A	B
Creating Fifths: 1st Diagram	4	5	5	6	3	1	0	0
Creating Fifths: 2nd Diagram	10	6	1	4	1	1	0	1
Identifying and Writing One Fifth	11	11	1	1	0	0	0	0
Writing Addition Problem $1/5 + 3/5$	10	12	2	0	0	0	0	0
Solving Addition Problem $1/5 + 3/5$	4	2	4	9	4	1	0	0
Explaining Strategy for Addition Problem $1/5 + 3/5$	5	7	5	3	2	2	0	0

Interview 2 Diagram Types

Students were asked to draw a diagram to show fifths, and then to do it again a different way. Five students in Group A and eight students in Group B successfully and quickly drew fifths in their first diagram. Nine students in Group A were able to successfully complete the task again using a different strategy, while only five students in Group B were able to do so.

Group A

In the first diagram, two students in Group A drew sets of five individual shapes to show fifths; one drew squares and the other drew rectangles. Of these students, only one drew equal-sized shapes. Ten students drew single shapes and then partitioned the shapes. Of these ten students, two

drew circles, four drew rectangles, and four drew squares; three of these students successfully drew a diagram that showed fifths.

In the second diagram, one student drew a set of five circles to show fifths, but the circles were unevenly-sized. Eleven students drew single shapes and then partitioned them. Three of these students drew circles, four drew rectangles, and four drew squares. Nine of these students successfully partitioned their shapes into fifths; one of them was a circle. One student who was successful with the first diagram was unsuccessful trying to draw fifths a different way. Five students who were unsuccessful with the first diagram were successful with the second.

Group B

In the first diagram, one student drew a set of five individual, attached squares to show fifths; all eleven other students drew single shapes which they then partitioned. Of these students, four drew circles, two drew squares, and five drew rectangles. Ten of these students attempted to partition their shapes using parallel lines, including two who had drawn circles; those who had drawn squares and rectangles were successful in creating equal sized parts.

In the second diagram, all students drew single shapes and then attempted to partition them. Four students drew circles, five drew squares, three drew rectangles. None of the students who drew circles were able to partition their shapes equally into fifths. Five students who had successfully drawn fifths the first time were unable to successfully draw fifths a different way. Three students who were unsuccessful the first time were successful the second time.

Table 19: Interview 2 Diagram Types

Diagram Type	Group A First	Group B First		Group A Second	Group B Second
sectioned square unit	4	2		4	5
sectioned rectangle unit	4	5		4	3
sectioned circle unit	2	4		3	4
sectioned triangle	0	0		0	0
set of squares	1	1		0	0
set of rectangles	1	0		0	0
set of circles	0	0		1	0

Post Test Assessments

At the conclusion of all lessons, three post test assessments were administered. Post-Test A asked students to write a response to the prompt, "What I've learned about fractions." Post-Test B was a written test that was identical to the written pretest. Post-Test C asked students to show how they would cut a cake for three people. Scores for all post-tests were divided into four equal ranges representing high, medium-high, medium-low, and low scores.

Post-Test A: Individual Written Responses: What I've Learned About Fractions

Understanding of fraction concepts was assessed by asking students to respond individually, in writing, to the prompt, "What I've Learned About Fractions." Responses were scored using the Writing Rubric. One student in Group B received a score of 4 points; no students in Group A did so. Six students in Group A and nine students in Group B received a score of 3 points. One student in Group A and no students in Group B received two

points. Five students in Group A and two students in Group B received one point. See Table 21 for the scores by number of students in each group.

Post-Test B: Written Post-Test

Post-test B, the written post-test, was scored using the Problem Solving Rubric. The highest possible score was 76 points. Scores in Group A ranged from a high of 73 points to a low of 29 points. One student in Group A did not take the post test. Scores in Group B ranged from a high of 68 points to a low of 39 points. See Table 21 for the scores by number of students in each group.

Post-Test C: Cutting a Round Cake For Three People

Post-Test C required students to show how they would cut a round cake so that three people could all have the same size piece. As shown in Table 20, three students in Group A and eight students in Group B were able to partition the circle into three relatively equal-sized parts. Three students in each group partitioned the circle into three parts which were not of equal size. Four students in Group A created partitions which showed four or more unequal parts; no students in Group B did so. One student in each group chose to make three pie-shaped cuts without using the whole cake. Five students in Group A and two students in Group B were unable to partition the circle successfully because they began with a median line and were unable to adjust their drawings.

Post-Test C responses were scored for speed, accuracy, and clarity, using the Interview Rubric. Three students in Group A and seven students in Group B received the highest possible score of four points on Post-Test C. See Table 21 for the scores by number of students in each group.

**Table 20: Post-Test C: Cutting a Round Cake for Three People
Scores by Group**

Graphic Responses	Group A	ML	Group B	ML
three equal parts on first try	2	0	5	0
three equal parts with multiple tries	1	0	3	1
three unequal parts	3	3	3	2
two, four, or more equal parts	0		0	
four unequal parts	2	2	0	
three equal cuts w/ unused portion	1	0	1	0
multiple unequal parts	2	2	0	

Note: ML = Number of students starting with median line

Combined Post-Test Scores

Scores from Post-Test B and Post-Test C were added together. The highest possible combined score was 80 points. Scores in Group A ranged from a high of 77 points to a low of 31 points. Scores in Group B ranged from a high of 72 points to a low of 42 points. Table 21 shows the combined scores from Post-Test B and Post-Test C, plus the scores for all post tests by number of students in each group, according to whether their scores fell into the high, medium-high, medium-low, or low range. Table 22 shows each student's individual score for each pretest and post-test assessment.

Table 21: Ranges of Scores for all Post-Tests

Group	A	B	A	B	A	B	A	B
Post-Test A "What I've learned about fractions." 4 points maximum	0	1	6	9	1	0	5	2
Post-Test B 76 points maximum	2	2	6	10	3	0	0	0
Post-Test C Cutting a Round Cake 4 points maximum	3	7	0	1	3	3	5	1
Combined Scores 80 points maximum	2	3	6	9	3	0	0	0

Table 22: Individual Student Scores for all Post-Test Assessments

Student	Pretest Assessments				Post-test Assessments			
	Cake Task A	Written Test B	Comb. A & B Scores	Writing Task C	Writing Task A	Written Test B	Cake Task C	Comb. B & C Scores
1A	4	51	55	3	3	73	4	77
2A	4	38	42	1	3	42	4	46
3A	2	37	39	-	3	43	2	45
4A	1	35	36	-	2	44	2	46
5A	4	29	33	1	3	60	4	64
6A	1	29	30	-	3	29	2	31
7A	1	28	29	-	1	45	1	46
8A	1	27	28	-	1	na	na	na
9A	1	26	27	-	1	46	1	47
10A	1	25	26	-	1	42	1	43
11A	2	23	25	1	3	38	1	39
12A	1	24	25	-	1	33	1	34
1B	2	41	43	1	3	54	4	58
2B	1	41	42	1	3	62	4	66
3B	4	34	38	1	3	68	4	72
4B	3	32	35	-	3	46	2	48
5B	2	31	33	1	1	39	3	42
6B	2	27	29	-	3	52	2	54
7B	4	24	28	3	3	57	4	61
8B	1	27	28	-	4	54	4	58
9B	1	26	27	-	3	42	1	43
10B	2	24	26	1	1	54	4	58
11B	4	21	25	1	3	47	2	49
12B	1	22	23	-	3	39	4	43

Post Test Analysis

Post-test B, the written post-test (Appendix B), was designed to assess achievement with a variety of fraction concepts of different levels, from basic fraction concepts to higher-order applications not normally introduced to second graders. The test consisted of nineteen items which included the following concepts and applications:

1. recognizing equal shares (1 item)
2. identifying given fractions in predrawn, preshaded figures (3 items)
3. creating given fraction quantities in blank shapes or sets (3 items)
4. creating given fraction quantities with no prompts (4 items)
5. comparing quantities for given fraction symbols (3 items)
6. comparing pictures of halves (1 item)
7. adding on to thirds to make one-whole (1 item)
8. subtracting parts of parts with like denominators (1 item)
9. adding fractions with unlike denominators (1 item)
10. multiplying fractions (1 item)

A post-test item analysis was used to determine whether there were any general similarities, differences, or patterns among or between the two groups. Table 23 shows the number of students in each group receiving high, medium-high, medium-low, and low scores, according to the concepts addressed by each post-test item. Items are clustered by concept, and are not listed in the order they occurred on the test.

Table 23: Post-Test B Item Analysis

Concept / Application	4 points		3 points		2 points		1 point	
	A	B	A	B	A	B	A	B
equal shares	4	9	0	0	6	3	1	0
identifying shaded fractional parts	9	8	2	4	0	0	0	0
identifying shaded fractional parts	8	11	1	0	0	1	2	0
identifying shaded mixed number	1	4	0	0	6	5	4	3
creating fractions in given shapes	5	7	0	0	2	2	4	3
creating fraction in given shapes	6	10	0	0	2	1	3	1
creating fraction in given shapes	2	0	0	1	0	3	9	8
creating fraction in blank space (5/6)	3	12	1	0	3	0	4	0
creating fractions in blank space (8/8)	2	4	5	4	0	2	4	2
creating fraction in blank space (3/2)	0	3	0	0	3	2	8	7
creating part of part in blank space	2	2	0	1	6	5	3	4
comparing quantities: unlike denominators	6	6	0	0	0	0	5	6
comparing quantities: unlike denominators	3	6	0	0	6	5	2	1
comparing quantities with one whole	4	5	0	0	0	0	7	7
comparing halves: vert. vs. horiz.	6	6	0	0	1	5	4	1
adding on to make one-whole	2	3	0	0	5	5	4	4
adding fractions: unlike denominators	1	0	1	0	7	11	2	1
part of part (story problem)	1	1	1	2	8	6	1	3
multiplying fractions	3	2	1	0	5	10	2	0

More than half the students in both groups scored four points on two items which required simple identification of a regular fractional amount (not including mixed-numbers) in a predrawn, pre-shaded shape. Conversely, more than half the students in both groups scored only one point on an item which required students to shade in three-fifths of a set of ten circles. One noticeable difference in the number of students receiving a high score was on an item which required students to draw a picture which showed five-sixths in a blank space; only three students from Group A scored four points on this item, while twelve students in Group B did so.

Absences

It should be noted that the student absence rate during the study was vastly different for each group, as shown in Table 24. Students in Group A had a collective total of 68 absences, while students in Group B had a collective total of only 17 absences. In addition, one student in Group A, with no previous notice, moved to a different school the day before Post-Test B was administered.

Table 24: Number of Days Absent

Student #	1	2	3	4	5	6	7	8	9	10	11	12
Group A	0	11	4	1	2	15	18	6	4	7	0	0
Group B	1	0	1	2	4	1	1	3	3	1	0	0

Statistical Analysis Results

Chi-square analyses were used with different sets of data to determine whether there were relationships between the types of manipulative materials students used for fractions instruction and their achievement in different levels of fraction concepts and applications.

To determine whether there was a relationship between the types of manipulative materials students used and students' overall achievement with fraction concepts and applications, a chi-square analysis was conducted, using the data from the combined scores for Post-Test B, the written test, and Post-Test C, the cake partitioning task (Table 25). Scores were assigned a high or low level according to whether they fell above or below fifty percent of the total number of points possible. The relationship was shown to be not significant at the .05 level.

Table 25: Chi-square Analysis - Combined Post-Test Scores
Post-Test B and Post-Test C

score	High	Low
Group A	8	3
Group B	12	0

Chi-Square = 3.78; df: 1 Not Significant at p .05

To determine if there was a relationship between the types of manipulative materials students used and students' ability to use partitioning skills, a chi-square analysis was conducted, using the data from the Post-Test C, the cake partitioning task (Table 26). Scores were assigned a high or low level according to whether they fell above or below fifty percent of the total

number of points possible. The relationship was shown to be not significant at the .05 level.

Table 26: Chi-square Analysis - Scores for Post-Test C
Cutting a Round Cake for Three People

score	High	Low
Group A	3	8
Group B	8	4

Chi-Square = 3.57; df: 1; Not Significant at p .05

To determine if there was a relationship between the types of manipulative materials students used and students' performance when working with the relationships of parts to parts, a chi-square analysis was conducted, using the data from Interview 1 (Table 27). Scores were assigned a high or low level according to whether they fell above or below fifty percent of the total number of points possible. The relationship was shown to be not significant at the .05 level.

Table 27: Chi-square Analysis - Scores for Interview 1

score	High	Low
Group A	6	6
Group B	4	8

Chi-Square = .68; df: 1; Not Significant at p .05

To determine if there was a relationship between the types of manipulative materials students used and students' performance when working with the concept of combining parts, a chi-square analysis was

conducted, using the data from Interview 2 (Table 28). Scores were assigned a high or low level according to whether they fell above or below fifty percent of the total number of points possible. The relationship was shown to be not significant at the .05 level.

Table 28: Chi-square Analysis - Scores for Interview 2

score	High	Low
Group A	6	6
Group B	4	8

Chi-Square = 2.66; df: 1 Not Significant at p .05

To determine if there was a relationship between the types of manipulative materials students used and students' ability, without prompts, to recall and explain basic fraction concepts, a chi-square analysis was conducted, using the data from the open-ended written response to the prompt, "What I've learned about fractions." (Table 29). Scores were assigned a high or low level according to whether they fell above or below fifty percent of the total number of points possible. The relationship was shown to be not significant at the .05 level.

Table 29: Chi-square Analysis - Scores for Post-Test A
Written Response to "What I've learned about fractions."

score	High	Low
Group A	6	6
Group B	10	2

Chi-Square = 2.5; df: 1 Not Significant at p .05

To determine if there was a relationship between the types of manipulative materials students used and students' understanding of basic fraction concepts as demonstrated by their ability to partition given shapes into various fractional parts, a chi-square analysis was conducted, using the data from Worksheet 4 (Table 30). Scores were assigned a high or low level according to whether they fell above or below fifty percent of the total number of points possible. The relationship was shown to be not significant at the .05 level.

Table 30: Chi-square Analysis - Scores for Worksheet 4

score	High	Low
Group A	9	3
Group B	10	2

Chi-Square = .24; df: 1 Not Significant at p .05

To determine if there was a relationship between the types of manipulative materials students used and students' understanding of basic partitioning concepts as demonstrated by their ability to create given fractional parts in three different shapes, a chi-square analysis was conducted, using the data from the last administration of Worksheet 5 (Table 31). Scores were assigned a high or low level according to whether they fell above or below fifty percent of the total number of points possible. The relationship was shown to be not significant at the .05 level.

Table 31: Chi-square Analysis - Scores for Worksheet 5

score	High	Low
Group A	11	0
Group B	10	1

Chi-Square = 1.05; df: 1 Not Significant at p .05

To determine if there was a relationship between the types of manipulative materials students used and students' ability to identify fractional parts of predrawn, pre-shaded shapes, a chi-square analysis was conducted, using the data from the last administration of Worksheet 2 (Table 32). Scores were assigned a high or low level according to whether they fell above or below fifty percent of the total number of points possible. The relationship was shown to be not significant at the .05 level.

Table 32: Chi-square Analysis - Scores for Worksheet 2

score	High	Low
Group A	12	0
Group B	12	0

Chi-Square = 0; df: 1 Not Significant at p .05

Eight different chi-square analyses were conducted to differentiate between different types of data results. In addition to analyzing data measuring overall achievement in fractions, data results were analyzed which measured the understanding and application of several specific

fraction concepts: the relationships of parts to parts, the concept of combining parts, the concept of fair shares as equal-sized parts of a unit or set, and the concepts of evenness and oddness as demonstrated by partitioning skills. Also included in the analysis was the students' ability to recall and explain their knowledge of fraction concepts.

Not one of the chi-square analyses conducted showed a significant difference between the group using base ten blocks for fractions instruction and the group using food and other everyday, familiar items for fractions instruction.

Chapter V

Summary, Discussion, and Recommendations

This study focused on the use of two different types of manipulative materials for teaching fraction concepts to second-grade students. It was hypothesized that the type of manipulative materials used would be unrelated to achievement with fraction concepts.

Twenty-four second-grade students participated in a series of activity-based lessons dealing with fraction concepts. Group A, twelve students, used base ten blocks for instruction, while Group B, twelve students, used food. The lessons included activities dealing with part-whole relationships, including combining parts and comparing parts. During the lessons, students were required to identify fractional parts they created, to write fractions, and to discuss their work using vocabulary specific to fractions.

Students were divided into two groups of matched pairs according to their combined scores from two sections of a three-part pretest. Pretest A asked students to show how they would cut a cake for three people; Pretest B was a written test which included all concepts to be presented during the lessons; Pretest C was a writing task. During the study, students were periodically assessed for knowledge and application of specific concepts through worksheets and interviews. Overall achievement was assessed using a three-part post-test which mirrored the pretest, including a writing task (Post-Test A), a written test (Post-Test B) and a cake-cutting task (Post-Test C). All assessments were scored using four point rubrics. Individual students' scores were ranked within four quartile ranges designated high, medium-

high, medium-low, and low. For purposes of conducting chi-square analyses, high and medium-high scores were given a high ranking, while medium-low and low scores were given a low ranking. A chi-square analysis was conducted for the post-test results as well as for each assessment of targeted concepts.

Summary of Results

A variety of chi-square analyses were conducted to measure students' achievement with and understanding of fraction concepts. For each chi-square analysis, the df was 1. At a level of .05, the chi-square needed to be 3.84 or higher in order to show significant difference.

In the chi-square analysis comparing overall achievement between the two groups as measured by the combined scores of Post-Test B and Post-Test C, the result was a chi-square of 3.78, which was too low to indicate significant difference.

In the chi-square analysis comparing partitioning skill as shown in the Post-Test C, the result was a chi-square of 3.57, which was too low to indicate significant difference.

In a chi-square analysis comparing students' ability to show and explain parts of parts, as measured in Interview 1, the result was a chi-square of .68, which was too low to indicate significant difference.

In a chi-square analysis comparing students' ability to combine parts, name the resulting fraction, and explain their strategies, as measured in Interview 2, the result was a chi-square of 2.66, which was too low to indicate significant difference.

In the chi-square analysis comparing students' ability to recall and explain basic fraction concepts, as measured by Post-Test A, the result was a chi-square of 2.5, which was too low to indicate a significant difference between the two groups.

In a chi-square analysis comparing students' achievement with basic fraction concepts, as measured in Worksheet 4, the result was a chi-square of .24, which was too low to indicate a significant difference between the two groups.

In the chi-square analysis comparing students' partitioning ability, as measured in Worksheet 5, the result was a chi-square of 1.05, which was too low to indicate a significant difference.

In the chi-square analysis comparing students' ability to identify fractional parts, as measured in Worksheet 2, the result was a chi-square of 0, showing no difference between the two groups.

Whether analyzed for overall achievement or for performance with specific fraction concepts, a lack of significant difference between the two groups was shown in every chi-square analysis. However, a focus on the number of students receiving scores in the top quartile for specific tasks reveals some interesting similarities, differences, and patterns.

An analysis of individual test items in Post-Test B shows that both groups, as might have been expected, received the most number of high scores on test items which required them to simply identify fractions or to compare fraction quantities, and that both groups showed a concentration of medium-low and low scores in tasks requiring them to add on to a given fractional part to make a whole, to draw a diagram showing parts of parts, and

to perform symbolic fractions operations. One noticeable difference that emerged between the groups in the analysis of the written post-test was that more students using food were successful in drawing their own diagrams of fractional amounts than students using blocks.

Observations of individual tasks of various types revealed some differences between the two groups. For example, in Post-Test C, a median line, a line that divides an area in half down the middle, was initially used by more than twice as many students using blocks (seven students out of twelve) than students using food (three students out of twelve). This is especially interesting in light of the fact that in Pretest A, given before the groups were assigned, a median line was initially used by a total of only eight students out of twenty-four. Also interesting to note is that on Worksheet 4, which required students to create different fractional amounts in rectangles, scores for both groups were similar; however, on Worksheet 5, which required students to create fractional parts in different shapes, Group B, the food group, had more students receiving scores in the top quartile. One might suppose that working with manipulative materials in a variety of shapes was a benefit to students in Group B.

In three out of four tasks in Interview 1, Group A students had more high scores than Group B students when drawing and identifying parts of a whole, while Group B students had more high scores than Group A students when finding parts of parts with manipulative materials, a difference which did not show when a similar task was required on Worksheet 4, without manipulative materials. In Interview 2, more students using blocks (Group A) than students using food (Group B) received high scores when creating

fifths, but Group B students received more high scores for their explanations of their strategies and processes for adding fifths. Overall, in the interviews, Group A students appeared to outperform Group B students in basic tasks, while Group B students seemed to outperform Group A students in tasks which required more complex thinking. One might ask whether the variety of food items may have given Group B students more internal references on which to draw when asked to apply what they had learned.

A particular note of interest is that when students were asked to draw pictures which showed fourths and thirds in Interview 1, and then fifths in Interview 2, more students in the blocks group drew sets of shapes than students using food, rather than whole unit shapes. This observation was consistent for all four drawing tasks in the two interviews. One might assume that this occurred because Group A students' use of base ten blocks required them to consider parts first when constructing wholes.

Discussion

The results of the post-tests used for this study indicate that there was no significant difference between the two groups in regard to overall achievement with fraction concepts. However, some specific patterns, similarities, and differences were noticed in strengths and weaknesses within specific areas.

Students in both groups were highly successful when dealing with basic fraction concepts which are generally introduced in primary grades, such as shading in and identifying fractional parts in predrawn, pre-shaded diagrams. However, when they were required to perform more open-ended tasks, such as drawing diagrams with no prompts, solving complex problems

with manipulative materials, or giving written or oral responses, they showed less consistent success.

Both groups showed similar ability in partitioning rectangles into halves, thirds, and sixths. However, Group B students, those using food, showed less tendency to rely on a median line when partitioning; subsequently, they showed more success than Group A students when partitioning a variety of shapes into halves, thirds, and fourths.

Group A students were slightly more successful than Group B Students when working with symbols only in addition, subtraction, and multiplication problems, although neither group showed a particular strength in this area.

Students using food appeared to be more successful than students using blocks in open-ended tasks, in writing about fractions in open-ended writing assignments, and in explaining their strategies in both interviews.

Recommendations

This study showed no significant overall difference in two groups working with two different types of manipulative materials in learning fraction concepts, but it raised some questions in regard to the strategies and explanations used by children in solving problems dealing with part-whole relationships.

Generally, the students using blocks proceeded from part to whole to part, while the students using food proceeded from whole to part. The base-ten blocks restricted the students to using rectangles made up of units, required students to construct the shapes before they were able to consider parts, and allowed students to build and rebuild the same fractional amounts many times. Additionally, the very nature of the blocks meant that the

constructed rectangles were delineated into parts before students began considering the parts. On the other hand, the food presented students with a variety of predetermined shapes. Once the students cut the food, although they could generally reposition them into their original shapes, they could not fully reconstruct the items. Perhaps the specific differences and individual restrictions of the two types of manipulative materials added to the apparent differences in strengths between the two groups.

The group using the food showed more success with open-ended tasks and communicating their ideas. This group needed to decide for themselves how to cut a variety of differently-shaped foods into equal shares. Students in the blocks group, however, were required to separate the units according to the natural partitions created by the units themselves, so the sizes of shares were dictated by the units rather than the students. One might question whether working with a variety of manipulative materials gives students more inner resources when they need to draw their own models.

The group using base ten blocks showed slightly more success when working with abstract problems with symbols. The blocks group was only able to separate the units according to the units themselves, resulting in the parts always being equal, while students cutting up food could make many quantitative errors. The students using base ten blocks may have been more able to connect their consistently-sized equal shares to the numerals in the written symbolic fractions. Perhaps the inherent fixed nature of the units as opposed to the variety of shapes and sizes of food items created some kind of transition for students in making symbolic connections.

Does motivation make a difference in achievement? Parents of students using food and other familiar items reported that their students were showing them at home what they had done in school, using the same kinds of familiar items. Even though the blocks group was given a snack after every lesson that was identical to what the food group was using for creating parts, the group using food appeared more highly motivated and excited about the lessons, often coming into the classroom after recess and asking, "Are we doing fractions today?"

The highest number of absences by any one student in Group B was four absences, while three students in Group A were absent more than ten days each; one was absent eighteen days, one fifteen days, and one eleven days. One of these students received only one point more on the written post-test than on the written pretest, and another received the same score on the post-test as on the pretest. Essentially, Group A received fewer instructional hours than Group B; the possibility must be considered that this may have influenced their overall achievement.

The results of this study were inconclusive. The small size of the sample resulted in some unforeseen problems, especially with regard to the large number of absences of a few students in one group, and the fact that one student moved to a different school just before the post-test was administered. These two factors in such a small sample of students affected the results of the study.

This study attempted to simultaneously assess two different aspects of fraction work with second-grade students: achievement with fractions, and students' ability to show understanding of fraction concepts through open-

ended tasks and interviews. Because of the varying types of activities and assessment materials involved, there was a considerable amount and variety of data. It may have been more practical to use fewer assessment tools.

Fractions are simply part-whole relationships, but a complexity exists within the part-whole framework because of the levels and variety of concepts involved. This complexity, exemplified by the variety of questions raised by this study, reveals many possible directions for future research. Any of the questions raised could provide the impetus for further research in the area of primary students' understanding of fraction concepts.

Appendices

Appendix A: Cutting a Round Cake for Three People

Name _____ **Date** _____

This is a cake.

**There are three people
in the family.**

**Show how you would
cut the cake so that
everyone would get the
same amount.**

(Neufeld, 1995)

Appendix B: Pretest B / Post-Test B

Name _____ Date _____

Circle the fraction that tells what part of the figure is shaded:

$$\frac{10}{3}$$

$$\frac{3}{7}$$

$$\frac{3}{10}$$

Solve:

$$\frac{1}{4} \times \frac{2}{3} = \boxed{}$$

Show thirds on the square:

What is the shaded amount?

Color three-fifths of the marbles:

What fractional part of the square is shaded?

Appendix B: Pretest B / Post-Test B

Is $\frac{1}{2} + \frac{1}{3}$ less or
more than one whole?

less

more

Draw a picture which shows
five-sixths.

If you had half a cake and
you ate one half of that, how
much cake would be left?

Show one fourth in the
rectangle:

Solve:

$$\frac{1}{3} + \frac{2}{6} =$$

Draw a picture that shows
one third of one half:

If this is two-thirds, draw what
you would need to have one
whole:

Appendix B: Pretest B / Post-Test B

Circle the figure if it shows eighths:

Circle the fraction that is greatest:

$$\frac{3}{4}$$

$$\frac{2}{5}$$

$$\frac{1}{10}$$

Draw a picture which shows eight eighths:

Which is more, $\frac{1}{2}$ or $\frac{3}{8}$?

Which is more?

Draw a picture which shows three halves:

Appendix C: Worksheet 1

Name _____ Date _____

Write the fraction to show what part is shaded.

Appendix D: Worksheet 2

Draw a line.

Match the fraction with the picture that shows the correct number of shaded parts.

$\frac{5}{8}$

$\frac{1}{2}$

$\frac{3}{4}$

$\frac{1}{4}$

Write the fraction that names the shaded part.

Appendix E: Worksheet 3

Name _____

Appendix F: Worksheet 4

Name _____

Show halves in the rectangle.

Shade $\frac{1}{2}$

Show thirds in the rectangle.

Shade $\frac{2}{3}$

Show sixths in the rectangle.

Shade $\frac{4}{6}$

Show one-third of one-half in the rectangle.

Solve:

$$\frac{1}{3} \times \frac{1}{2} =$$

Appendix G: Worksheet 5

Name _____

Show halves:

Show thirds:

Show fourths:

Appendix H

Rubric for Problem Solving With Fractions

4	<ul style="list-style-type: none">• Shows complete understanding of the problem.• Has a plan that can lead to a correct solution.• Has a correct solution.• Diagrams show correct numbers of parts.• Parts are of equal size.• Fractions are written correctly
3	<ul style="list-style-type: none">• Misinterprets minor part of the problem.• Has a substantially correct procedure with a minor omission or procedural error.• Has incorrect solution due to procedural error.• Diagrams show correct numbers of unequal parts.• Sizes of parts are approximately accurate.• Fractions are written correctly.
2	<ul style="list-style-type: none">• Misinterprets major part of the problem.• Has partially correct procedure with major fault.• Has a computational error.• Has incorrect or partial answer.• Diagrams show correct numbers of parts.• Sizes of parts are inaccurate or uneven.• Shows reliance on whole number concepts.• Fractions are written incorrectly.
1	<ul style="list-style-type: none">• Completely misinterprets the problem.• Has a totally inappropriate plan.• Has wrong answer based on inappropriate plan.• Diagrams show incorrect numbers of parts.• Sizes of parts are inaccurate or uneven.• Shows reliance on whole number concepts.• Fractions are written incorrectly.

Renée Goularte 1996; ref. Szetela & Nicol, 1992; Pothier & Sawada, 1983

Appendix I

Rubric for Identifying Fractions

4	<ul style="list-style-type: none">• Fractions written correctly.
3	<ul style="list-style-type: none">• Reversal of numerator and denominator, or• Numerator = shaded plus denominator = unshaded, or• Numerator = unshaded plus denominator correct
2	<ul style="list-style-type: none">• Attempt to write fraction, but little evidence of counting parts: one or both numbers incorrect.• Parts counted separately; no relationship to whole.
1	<ul style="list-style-type: none">• Reliance on whole numbers: answer reflects counting parts, but no attempt to write fraction.

Renée Goularte 1996

Appendix J

Rubric For Student Interviews

4	<ul style="list-style-type: none">• Quickly and accurately completes all parts of the task with minimal hesitation.• Needs no prompting.• Explains strategies clearly and logically, using mathematical language appropriate and meaningful to the tasks.• Refers to numbers and evenness of parts.
3	<ul style="list-style-type: none">• Completes most parts of the task correctly.• Needs minimal prompting.• Explanations of strategies contain ambiguities or incorrect references, or use mathematical language which may not be appropriate or meaningful for the tasks.• Refers to numbers or evenness of parts.
2	<ul style="list-style-type: none">• Completes some parts of the task incorrectly.• Needs repeated prompting.• Gives partially correct responses.• Explanations are unclear, contain errors or ambiguities, or do not completely explain task responses.• Makes no reference to evenness of parts.
1	<ul style="list-style-type: none">• Most task responses are incorrect.• Needs repeated prompting.• Explanations do not completely explain task responses, or are not given.

Renée Goularte 1996

Appendix K

Rubric for Writing About Fractions

4	<ul style="list-style-type: none">• Lists or gives explanation of all of the following:<ul style="list-style-type: none">– partitioning: parts of a unit or set– equal sizes of fractional parts of unit or set– numerical example or diagram• Explanation is clear, organized, and accurate.
3	<ul style="list-style-type: none">• Lists or gives explanation of two of the following:<ul style="list-style-type: none">– partitioning: parts of a unit or set– equal sizes of fractional parts of unit or set– numerical example or diagram• Explanation is clear and accurate.
2	<ul style="list-style-type: none">• Lists, gives explanation of, or refers to one of the following:<ul style="list-style-type: none">– partitioning: parts of a unit or set– equal sizes of fractional parts of unit or set– numerical example or diagram• Explanation is accurate.
1	<ul style="list-style-type: none">• Gives numerical examples of fractions only or• Gives diagram of fractional parts only or• Explanation is inaccurate.

Renée Goularte 1996

Appendix L: Interview 1 Summaries

Interview 1 consisted of a diagramming task and a manipulative task. In the diagramming task, students were first asked to draw a picture which showed fourths, to identify one fourth in their drawing, and to write the fraction. Then they were asked to draw a picture which showed thirds, to identify one-third in their drawing, and write the fraction. In the manipulative task, students in Group A used base-ten blocks, while students in Group B used peanuts. Given six items, students were asked to show half, then to show what they would do if they had two friends and they were to give each friend a third of what was left, and finally to tell what fraction of the whole they had left. At all steps, students were asked to justify their work, usually with the question, "How do you know?"

Students with identified special needs have been identified as RSP (Resource Specialist Program), Speech (Speech Therapist), and LEP (Limited English Proficient).

Group A Summaries

Student #1A

The student drew and sectioned circles for both parts of the diagramming task, calling the circle "some kind of cake or something." She worked quickly and carefully, making sure the three pieces of the circle were equal by starting in the center. She knew that the denominator signified the total number of parts, which she called "the mother fraction," a term learned at home. Both fractions were written quickly and accurately. (Score: 4 points)

In the manipulative task, the student did not understand the directions at first. After the directions were rephrased, she quickly manipulated the blocks and identified one third of one half as one sixth. (Score: 4 points)

Student #2A

In the diagramming task, the student drew and sectioned a circle to show fourths, calling each part a half. To show thirds, she drew a circle with three sections, carefully trying to make the parts even. When asked to show thirds in a rectangle, she had more success. The student knew that the denominator signified the total number of parts, and she correctly wrote both fractions. (Score: 3 points)

In the manipulative task, she quickly indicated three when asked for half, but had trouble identifying fractional parts, wanting to cut one block in half to give me a third. Starting over, and with prompting, she was able to

identify one third of one half as one sixth, justifying the response by referring to the total number of blocks. (Score: 2 points)

Student #3A (RSP)

The student showed minor hesitation at each direction, but needed no prompting to complete the diagram tasks by drawing and sectioning a square into fourths and a rectangle into thirds. He knew that the denominator signified the total number of parts, and correctly wrote both fractions. (Score: 4 points)

In the manipulative task, he quickly identified three blocks as half of six, and then each remaining block as one third of one half, but identified the remaining block as one half of the total. (Score: 2 points)

Student #4A

In the diagramming tasks, the student drew small, adjacent, touching squares to form units made of smaller parts. She wanted to erase a square and move it away from the created unit in order to show one fourth. She knew that the denominator signified the total number of parts, and quickly wrote both fractions correctly. (Score: 4 points)

In the manipulative task, she immediately arranged the six unit blocks into a 2x3 array, then gave the interviewer two when she was asked to show half, explaining that if you were to cut one in half, it would be two pieces. After being questioned about equal shares, she started over and identified three as one half of six. She was quickly able to identify one third of half as one sixth, reviewing the procedure in her explanation. (Score: 3 points)

Student #5A

In the diagramming task, the student quickly drew and evenly sectioned a square into four parts, then explained the meaning of fourths as "one out of four." When asked to show one fourth, she pointed to one section which she had shaded, but when asked about an unshaded section, identified it as "one third." To show thirds, she drew a triangle with a line across the middle and another line down to the base, stating that thirds means "one out of three." Asked to try a rectangle, she hesitated slightly, then evenly sectioned it with two vertical lines. She knew that the denominator signified the total number of parts, but had difficulty explaining the quantitative value of shaded and unshaded parts, referring to one shaded part as one fourth, and an unshaded part as one third. She correctly wrote both fractions. (Score: 2 points)

In the manipulative task, the student took three when asked to show half, explaining that each set of three was one third. When asked what she had been asked to show, she said "one third," then "three," and then "one half." She identified one block as one third, then, when reminded of the

whole amount, changed her answer to one sixth. When the question was stated symbolically, she identified one third of one half as one sixth of the total amount, with an explanation that "all of them together is one sixth."
(Score: 1 point)

Student #6A

The student drew sets of squares to show fourths and thirds. She shaded one of the squares in each drawing to identify one-fourth and one-third, stating that it was "one out of four" and "one out of three." After minor hesitation, she was able to write both fractions correctly without prompting.
(Score: 4 points)

In the manipulative task, the student was unsure whether three blocks was one half or one third, and in fact identified the total amount as one sixth. When it was pointed out that she had three blocks, she was not sure whether she had one-half or one-third, but decided on one third. With prompting ("There are three, and each one is a") she responded that she had one-sixth. After much step-by-step questioning which focused on total numbers of parts, she was eventually decided that one block was one sixth of the whole.
(Score: 1 point)

Student #7A

In the diagramming task, the student drew one large square plus a set of four separate rectangles for fourths, saying, "fourths are one big thing, turning out to be four things split together," indicating the large square as the whole, and the four rectangles as the fourths. When asked to show fourths in the large square, he drew three vertical lines, with little regard to equal spacing. To show thirds, he drew a set of three large rectangles, saying that one piece was one-third. He knew that one fourth was one piece, but wrote "1 4th is" when asked to write the fraction. (Score: 2 points)

In the manipulative task, when asked to tell the fraction for one block, the student said it was, "one block," even when reminded that there were six blocks all together. After much prompting, he said one block was one sixth. He quickly identified three blocks as half, and also very quickly identified one of the three as one third of the half, but he was unable to identify one third of one half as one sixth, calling it "one block," even with prompting. (Score: 2 points)

Student #8A

In the diagramming task, when asked to draw a picture of fourths, the student showed confusion, talking about twelve units with two in each row. She drew a 2x3 array of separated squares. When asked to show fourths in one square, drew the same array within a square. She was unable to explain what fourths meant. When asked to count the parts, she counted incorrectly,

showing no one to one correspondence, as "1, 2, 3, 4, 1, 2, 3, 4" for six squares. She self-initiated a recount, counted six, recognizing her mistake, and drawing a new array which was 2x4. She explained fourths as two gumdrops for each of four people. In diagramming thirds, she drew a 2x3 array, saying "two in a row, two in a row, two in a row." She was unable to identify one fourth or one third of the respective arrays, after shading incorrect numbers of squares. When asked to write the fractions, she wrote them both correctly. (Score: 1 point)

In the manipulative task, she took away three when asked for half, and was able to identify one block as one sixth of the set. She took away two more when asked to take one third for each of two people, then identified one third of one half as one sixth of the total, explaining that, "I had three and two people came, and I gave them two, and I had one left." (Score: 3 points)

Student #9A (RSP / Speech)

The student showed a great deal of hesitation when given initial directions in the diagramming task, so it was suggested that he start with a "box." He drew a rectangle with four small squares inside, explaining that he thought they were fourths. He repeated the procedure with thirds. He was able to write both fractions easily, but could not explain the relationship between the denominator and total number of parts. (Score: 1 point)

In the manipulative task, the student first took two blocks when asked to show half. When asked how he knew it was half, pointing out that there were two in one pile and four in the other, he changed his response to three blocks, saying that there were "half here, and half here" and that they were "the same." Identified one of six blocks as one half of the set, until reminded that there were six all together, when he changed his response to one sixth. He was unable to identify one third of one half as one sixth; he said it was one half, because "there's nothing there." He was unable to explain what he meant. (Score: 1 point)

Student #10A (RSP / Speech)

In the diagramming task, the student drew four unrelated items (a square, a bird, a McDonald's "M," and a circle) arranged in a 2x2 array, saying her drawing showed fourths because it was four things. When the interviewer drew a square for her, she drew four circles inside, with dots in the circles. Finally, when given another square and asked to show how she would cut it for four people, she evenly partitioned it, drawing a dot in each section, and explaining while she worked that one would be put on the ground for the dog. Asked to tell how she knew it was fourths, she began talking about the dots. With prompting, she shaded in one section to show how much one person would get. She drew three circles to show thirds, erasing and redrawing to make them equal in size, and then shaded in one

circle when asked to color one-third. When asked what one third was, she said it was "one out of three," and was able to write the fraction correctly. (Score: 1 point)

In the manipulative task, she said that one block was "one out of six." It took several tries and many prompts for her to be able to identify six as half the blocks, repeatedly wanting to take away two instead. When asked what half meant, she said, "half means half and you have a cake and half equals parts." She quickly took away two blocks when she was asked how she would give one third of her half to each of two people. She also quickly identified one third of one half as one sixth, pointing out that I had five and she had one, and "last time it was six, and if you took away one and there was five, it would be one-sixth." (Score: 1 point)

Student #11A (Speech)

In the diagramming task, the student quickly drew a circle "with a cross in it" to show fourths. To show one fourth, instead of shading one section, he drew another square and sectioned it into three uneven parts, saying there were four fourths in the circle and three fourths in the square. When asked to show thirds, he hesitated, then repeated the same drawing of a square that he had drawn to show fourths. He said fourths meant that it had "four squares... whatever... four halves," and that thirds meant, "there's one, and there's two." He did not write either fraction. (Score: 1 point)

In the manipulative task, he took away three when asked for half, but was confused whether halves meant that it's equal or not equal. When asked to show one third of one half, he picked up one block but could not explain why, stating only that "it's one-third of one-half." When asked what the fraction was for that one block he said it was "one-half of one-fourth." Even with much prompting, he could not explain that the block was one of a total of six blocks. (Score: 1 point)

Student #12A (LEP)

In the diagramming task, the student drew sets of small, separate squares to show fourths and thirds, then circled one of the squares in each set to show one-fourth and one-third. He correctly wrote both fractions, and knew that the denominator signified the number of parts. (Score: 4 points)

In the manipulative task, he arranged the blocks in a 2x3 array and then gave away two when asked for half. When asked to explain his thinking, he took the blocks back and did nothing. He was unable to explain what halves meant. With the blocks arranged randomly, he gave away three when asked for half, justifying the action by referring to the two resulting sets being the same. He showed one block when asked to show one third of one half, saying it was one out of three. When asked what fraction one third of one

half was, he again said it was one out of three, and then one third. He did not consider all six blocks. (Score: 2 points)

Group B Summaries

Student #1B

In the diagramming tasks, the student first drew and sectioned a square for fourths. Then she started to draw a giant circle which was going to be too large for the paper, so, with prompting, she drew a rectangle which was sectioned with a vertical line and then a horizontal line from the center of the vertical line, resulting in three fairly even parts. When asked to try it another way, she evenly sectioned a rectangle with two vertical lines. When asked if size mattered, she said, "sort of, because they have to be even." When asked to write the fraction, she first wrote "1 fourth," so the request was rephrased to reflect the desire for a number, and she wrote "4th." (Score: 2 points)

In the diagramming task, she performed very quickly, without hesitation at any step. She gave away three when asked for half, then showed one peanut when asked to show one-third of one-half, saying it was one sixth, justifying the response by referring to the total number of peanuts. (Score: 4 points)

Student #2B

In the diagramming task, the student quickly drew and sectioned a rectangle to show fourths, and then drew and sectioned a circle to show thirds, taking care to make the parts even each time. She hesitated in answering questions, as though afraid to make a mistake. She wrote both fractions correctly, and knew that the denominator signified the total number of parts. (Score: 4 points)

In the manipulative task, she gave away three peanuts very quickly, questioningly identifying one half as "six thirds?" When the question was repeated, with a prompt that thirds would mean three all together, she responded again, "six thirds?" She was able to identify one peanut as one sixth, with the explanation that there were six all together. (Score: 2 points)

Student #3B

The student showed some initial hesitation in the diagramming task, but then quickly drew and evenly sectioned one large square into fourths. When showing thirds, she first drew a square which she sectioned in half, and then bisected one of the halves, saying, "I made one half and then another half." She said that thirds meant it had three parts. When asked if there was anything else, she said, "Not that I can think of." When given a

circle, she partitioned it evenly into three sections. She compared the differences between the two, pointing out that the parts in the circle were "kind of even." She wrote both fractions correctly. (Score: 2 points)

In the manipulative task, she worked very quickly at each step, showing three peanuts as half and then one peanut as a third of a half. She identified one-third of one-half as one-sixth, saying it was one out of six. (Score: 4 points)

Student #4B

In the diagramming task, the student drew four rectangles to show fourths and three circles to show thirds. When asked to show thirds in a square, he tried a median line first, and recognized that he couldn't do it after dividing it in half. Asked to try a rectangle, he drew a diagonal line, then bisected one of the resulting triangles. When asked to use a circle, he started with a median line, then bisected one half. He explained that the rectangle and circle showed thirds because they had three parts. The student showed no concern for equal sizes of parts. He correctly wrote both fractions. (Score: 2 points)

In the manipulative task, he took away three peanuts when asked for half of the six, but had difficulty identifying one-third of half or any other fractional parts, even with prompting. When asked what he would have left if he gave one-third to each of two people, he first said "none," then "one," then "one-sixth." All responses needed much prompting. (Score: 1 point)

Student #5B

In the diagramming task showing fourths, the student needed much prompting to begin, including a suggestion to begin with a square, which she drew and then stopped working, and did not continue even with prompting. She was asked to listen carefully for a number in the word "fourths," and asked to show what she could do to the square to show fourths, at which point she wanted to circle the corners of the square. She was finally able to complete the task when asked to pretend the square was a brownie. She said one person would eat "one half," but sectioned it with four equal parts. To show thirds, she drew a circle and sectioned it slightly unevenly, after some prompting. When asked to shade in how much one person would get, she shaded in one section, calling it one half. She was able to write both fractions correctly without hesitation. (Score: 2 points)

After some initial hesitation and slight prompting, the student identified three peanuts as half of six in the manipulative task, saying it was three and three. Was able to show one third of one half as one peanut. After giving me two thirds of her half, she identified one sixth as the fraction of peanuts left, referring to the original total in saying that when I gave her some there were six. (Score: 4 points)

Student #6B (Speech)

In the diagramming task, the student quickly drew and evenly sectioned a circle into fourths, but could not clearly explain his drawing, confusing the concept of fourths with that of one fourth: when asked which piece was one fourth, he first said, "the whole thing," and then "four pieces." His explanations were vague and confusing. To show thirds, he drew and fairly evenly partitioned an "L" shape, but did not respond when asked to explain how it showed thirds. When asked to try it with a different shape, he did not respond. When given a square, he evenly sectioned it with two vertical lines. When asked what thirds meant, he responded, "one piece." He showed much confusion about the relationship between the total number of parts and individual parts. He wrote both fractions correctly. (Score: 2 points)

In the manipulative task, the student gave away three peanuts when asked for half, but had a hard time explaining why it was half, saying only that he had given away three, and, after repeated prompting, that "three plus three equals six." He knew that one third of one half was "one piece of a peanut," but first translated that as one-fourth, and when challenged, changed his response to one-sixth because "we had three sixths." He eventually said that one-third of one-half was one-sixth, because there were six pieces in all. (Score: 1 point)

Student #7B (LEP)

The student showed confidence in the diagramming task, working quickly to draw and evenly section two circles to show fourths and then thirds. When asked how one diagram showed thirds, he counted the parts, and indicated that he knew the one shaded part was one third by comparing the two unshaded parts to the one shaded part. He correctly wrote both fractions, and knew that the denominator signified the total number of parts. (Score: 4 points)

In the manipulative task, he gave away one peanut when asked for half, saying halves means "one piece of something." On a second attempt, when asked to give each of us half, gave away one and kept one, and put the rest aside. He needed prompting ("You have to use all the peanuts.") to show halves as three each, but said he had three halves. After referring to the parts in his drawings and the numbers of peanuts, he said that one peanut would be one sixth, but when asked the question again, changed his answer to "three," and then "one third," and then "one sixth," finally justifying this response by referring to the total number of peanuts. He successfully identifies one peanut as one third of one half, saying that the fraction was "six" and then "one sixth." (Score: 1 point)

Student #8B

In the diagramming task, the student easily and quickly drew and sectioned a square into four parts, explaining that it was "one piece, and one piece, and one piece, and one piece." Further questioning clarified his meaning as indicating four parts. In trying to draw thirds, he first used a circle, with two "cuts" close to the circumference, leaving vastly unequal shares. When asked to use a rectangle, he sectioned it with three horizontal lines, making four parts, explaining that it showed thirds because if one were shaded, three would be left. When it was pointed out that there were four parts altogether, he had no response. He did not write either fraction. (Score: 2 points)

In the manipulative task, he quickly gave away three when asked for half. When asked for one third of the half, he gave away the remaining three, and was unable to explain his reasoning. When the question was rephrased, asking how many one-third would be if he had three peanuts to start with, he said "all of them," and again could not explain the reasoning. The interview was stopped. (Score: 1 point)

Student #9B

The student did not know how to begin the diagramming task, even after a square was drawn for him; he decided to shade in the whole square. When asked how that showed fourths, he then partitioned it evenly. He drew a new square with one part shaded to show one fourth. To show thirds, he drew another square, sectioned into fourths, with three parts shaded, unable to relate the total number of parts to the fractional amount. He needed prompting to be able to write the fractions, but read "one third $\frac{1}{3}$ " as " $\frac{1}{4}$." (Score: 1 point)

In the manipulative task, he started to break a peanut in half to show half, but gave away three when the directions were repeated. He gave away two more when asked to give away one-third each to two people. He said that what was left was one fifth, because there was one and five. When asked what the fraction would be for one out of the whole sixth, he said, "one sixth." (Score: 2 points)

Student #10B

In the diagramming task, the student very confidently drew and evenly sectioned a square, which he called "a brownie," to show fourths, saying he knew it was fourths because there were four parts. He shaded one section to show one fourth. To show thirds, he drew a rectangle with one vertical line which partitioned off approximately one third of the whole. He was able to justify this as thirds with an explanation which included indicating with his finger an imaginary line which would bisect the remaining two thirds. When

asked to shade one-third, he drew the other line in and shaded the center section. He correctly wrote both fractions. (Score: 4 points)

In the manipulative task, when asked to show half, the student gave away three peanuts, saying "if you gave away three, you would have three left." He knew that the fraction for one peanut would be one sixth, and after a few mistakes in responses, very quickly picked up one peanut when asked to show one third of one half, justifying his response by referring to the total number of peanuts. (Score: 4 points)

Student #11B

In the diagramming task, the student worked quickly to draw and evenly section a square by cutting it "in half and half again," but saying only that you needed to look at the parts to see fourths, making no mention of the number or size of parts, even with prompting and rephrasing of the question. He tried to show thirds in a triangle by drawing a line across the middle, then another one down the center. When asked to try showing thirds in a circle, he sectioned it fairly evenly, beginning with a 'V' at the top center. He correctly wrote both fractions. (Score: 3 points)

In the manipulative task, he gave away three when asked for half of six peanuts. After being reminded that there were six all together, he identified one-third of one-half as one-sixth, but could not justify his response in any way; he gave no explanation. (Score: 1 point)

Student #12B (LEP)

In the diagramming task, the student was eager to start, but did not remember how to do fourths. Needed much prompting, including a suggestion to listen for a number in the word "fourths." After writing a 4, she drew a set of four separate squares with "dolls" in them, explaining that it showed fourths because it was four things. In writing the fraction, she inverted the numerator and denominator, explaining that she "had seen it that way." She did not do a diagram of thirds. (Score: 2 points)

In the manipulative task, she asked if she could break the peanuts when I asked for half. When told she could not break them, she gave away one, then, when told to use all the peanuts, she said she would give other people one each. Finally, when told just the two of us would get peanuts, she gave away three and kept three. When asked who had half, she pointed to the ones which she gave away. With prompting, she said she had half, also. She was unable to identify the fractional part represented by one peanut. The interview was stopped (Score: 1 point)

Appendix M: Interview 2 Summaries

Interview 2 consisted of a diagramming task and a computation task. In the diagramming task, students were first asked to draw a picture which showed fifths, to identify one fifth and write the fraction. Then they were asked to draw a picture that showed fifths a different way, and again to identify one fifth. In the computation task, students were orally given the problem "one-fifth plus three-fifths" and asked to write and solve the problem. At all steps, students were asked to justify their thinking, usually with the question, "How do you know?"

Students with identified special needs have been identified as RSP (Resource Specialist Program), Speech (Speech Therapist), and LEP (Limited English Proficient).

Group A Summaries

Student #1A

In the diagramming task, the student quickly and accurately drew a circle with five sections, starting with a dot in the middle. Recognizing that the parts were not exactly equal, she began to erase, having a little difficulty explaining what she wanted to do. Asked to show fifths another way, she quickly drew a rectangle with five equal sections, giving a clear explanation, referring to number and equality of parts. Asked to try a circle again, she accurately drew a circle with five equal parts, and compared this circle to the first one. To show one fifth, she quickly wrote the fraction " $\frac{1}{5}$ " in one of the sections. (Score: 4 points)

The student wrote and solved the addition problem quickly, with a clear explanation which included the words "numerator" and "denominator." (Score: 4 points)

Student #2A

In the diagramming task, the student drew a rectangle with ten parts, indicating five sets of two to show fifths, saying the rectangle had "ten fifths." Her second try was a rectangle with five equal parts, which she compared with her first drawing, saying that they both showed fifths. To show one fifth, she shaded one part and wrote the fraction quickly and correctly. Asked to try showing fifths with a different shape, she drew a square with a smaller square sectioned off inside, saying she would draw five little squares, but partitioned the square into fourths, and decided she couldn't complete the task. (Score: 3 points)

In the addition task, she quickly and correctly wrote the problem. She wanted to draw a 5x5 array to find the denominator, showing confusion as to whether the denominator in the sum would be 10 or 5. She needed much prompting to solve the problem correctly. (Score: 2 points)

Student #3A

In the diagramming task, the student quickly and accurately drew a square equally partitioned with vertical lines. On his second try, he drew a square partitioned into fourths, then added an extra section onto one side. He shaded one section to show one fifth and correctly wrote the fraction with no hesitation. (Score: 4 points)

In the addition task, the student wrote the problem quickly, then referred to his drawings to help him solve and explain the problem. With slight prompting, he shaded parts of the square to find the correct answer, explaining that "the denominator is the same." (Score: 4 points)

Student #4A

In the diagramming task, the student first drew a rectangle with slightly unequal parts. When asked what fifths means, she replied, "five equal parts," and adjusted the lines to make equal parts. On her second try, she drew a square with four vertical lines. To show fifths, she shaded in one section and correctly wrote the fraction. (Score: 4 points)

In the addition task, the student quickly wrote the problem, then solved it quickly and accurately, referring to her drawings in her explanation, which included the information that since both denominators were five, the denominator in the answer "should be five." (Score: 4 points)

Student #5A

In the diagramming task, the student first drew a rectangle out of ten squares. When questioned, she stated that one could "split it in five parts," drawing lines to separate two parts at a time. She wanted to take one away to show one-fifth. Asked to show fifths another way, she drew a rectangle partitioned into five equal parts. She shaded one part to show one-fifth, then wrote the fraction quickly and accurately. (Score: 3 points)

The student wrote the addition problem quickly and accurately, immediately giving the correct answer, explaining that "first you have to look at the denominator, and they're the same," and proceeded to explain each step of the addition process. (Score: 4 points)

Student #6A

In the diagramming task, the student drew a "box" with "little squares in it," equally partitioned into sixths; she counted the parts and erased one of the sections. Asked to try it another way, she drew a circle sectioned into six

equal parts, and erased one of the parts to show five parts. In her explanation, she counted the parts, compared the different shapes of the parts, and said that size doesn't matter, but that fifths means "five things." She wanted to remove one of the parts to show fifths, but, with prompting, shaded one part, then wrote the fraction quickly and easily. (Score: 3 points)

In the addition task, the student wrote the problem with no hesitation. In her explanation of how to solve it, she made confusing references to drawing a square and taking parts out. With prompting and reference to the addition symbol, she articulated very clearly that it was for adding, and solved the problem accurately, saying, "because one plus three is four, and it's actually fifths, so you put five at the bottom." (Score: 3 points)

Student #7A

In the diagramming task, the student drew five separate attached rectangles, one much wider than the rest. His explanation made no reference to parts of a whole nor to size, referring instead to "five blocks." Asked to show fifths a different way, he drew five circles of relatively equal size, again referring to five things as showing fifths. After questioning him about fairness, he decided that size mattered, and, with prompting, adjusted the size of the widest section of the rectangle. He wanted to draw an arrow pointing to one section to show one fifth; with prompting, he shaded one section and wrote the fraction quickly and accurately. (Score: 2 points)

The student had some difficulty writing the addition problem from oral directions; he needed the directions repeated, and then needed prompting to include the equal sign. After reading the problem aloud, he said the answer was $4/10$, saying, "it's three wholes... one more equals four wholes, and five plus five equals ten." Prompted to use his rectangle drawing to help solve the problem, he recognized the answer as four fifths, but was unable to explain why. (Score: 2 points)

Student #8A

In the diagramming task, the student first drew a square but did no partitioning. She needed much prompting to continue; she said she needed arrays and said, "like one fifth plus one half or one fourth," and wanted to write the fraction instead of draw the diagram. Finally, she drew a rectangle which she had trouble partitioning because she said she knew they had to be equal sizes. Asked to try it another way, she quickly reproduced the same drawing rotated 45 degrees. She shaded one section to show one-fifth, referring to coloring parts and past lessons to try to explain her reasoning, finally saying, "because it's the same as one fifth." She wrote the fraction quickly and accurately. (Score: 2 points)

The student wrote the addition problem incorrectly at first, but recognized her own mistake and asked for the numbers again. She added the

numerators correctly, then wanted to add the denominators. When it was suggested that she use her drawing, she shaded in parts of the rectangle to correspond to the addition problem, but was unable to connect the drawing to the addition problem even with prompting, referring to the parts as "squares" rather than fifths. (Score: 2 points)

Student #9A (RSP / Speech)

In the diagramming task, the student made two attempts, first with a square and the with a circle, each sectioned into fourths. He knew they were not fifths because they had only four parts. On a third attempt, he drew a rectangle sectioned with vertical lines into fifths. Asked to try one more time, he drew a circle partitioned into five vertical sections, saying it was fifths because it had five parts. He said, "they have to be the same size," and noted that the rectangle parts were equal while the circle parts were not. He shaded one part of the rectangle to show fifths, then wrote the fraction quickly and accurately. (Score: 2 points)

In the addition task, he wrote the problem quickly and accurately, but hesitated before attempting to solve the problem. When prompted, he referred to his drawing, saying that he needed to add three fifths. Asked what he should do, he said he would shade two more because one was already shaded in, focusing on a total of three-fifths rather than adding on three fifths. With prompting to compare the problem with the drawing, he said that the three fifths were "using" the one fifth. He showed confusion when asked what to do next, and needed prompting to complete the problem accurately. (Score: 2 points)

Student #10A (RSP / Speech)

In the diagramming task, the student quickly and accurately drew fifths with a square and one "added-on" section. For her second diagram she drew a circle partitioned into fourths, three of which she bisected, then she erased part of the circle, leaving five equal-sized parts. When asked if size mattered, she said it has "to be the same," because "then it won't be fifths" because "one would take longer to color in, and one would take less to color in." She shaded one part to show one fifth, then wrote the fraction quickly and accurately. (Score: 4 points)

The student wrote the addition problem quickly and accurately from verbal directions. She added the denominators, then the numerators, and put a "+" after the numerator answer. She needed help to recognize that there was nothing more to add. With prompting, she used her drawing to help with the problem-solving process, but she could not relate the diagram work to the addition problem, even with repeated prompting. She said the answer was $4/10$, because "5 and 5 is 10." (Score: 2 points)

Student #11A (Speech)

In the diagramming task, the student first drew a circle which was partitioned into five vertical sections, adjusting the lines to make the sections equal in width, but saying that size didn't matter, because "if I give you a big one, and then these small ones, it would be one whole." Asked to show fifths a different way, he drew a square, partitioned it into four vertical sections, then added another section to make five parts. When asked to compare his two diagrams, his explanations did not refer specifically to parts of the whole, but focused on the overall shapes: "...one has corners and one does not." After prompting and repeated questioning about fair shares, he decided that the circle did not show fifths but the square did, based on equality of parts. He shaded one section of the square to show one-fifth, and wrote the fraction quickly and accurately. (Score: 2 points)

The student wrote the addition problem quickly and accurately. Although he needed little prompting to solve it correctly, his explanations were confusing, making references to shading in parts of his drawings or using arrays. He quickly recognized that the denominator would be five, because "they were both the same," but was unable to clearly explain the addition process for the numerators. (Score: 3 points)

Student #12A (LEP)

In the diagramming task, the student drew five separate small squares to show fifths, calling them "boxes" and justifying them as fifths because there were five of them. Asked to show fifths another way, and prompted to use a rectangle after hesitation, he drew a square with five parts nearly equal in size, saying that size didn't matter, until he was questioned about fairness. Asked to try again, he drew another square, trying to partition it equally. He wanted to take one away to show one fifth and was unable to think of another way to do so, finally needing prompting to shade one section. He explained quickly that it was one fifth because one out of five. He wrote the fraction quickly and accurately. (Score: 3 points)

In the addition task, the student wrote the problem quickly and accurately, then hesitated before proceeding. When prompted, he said, "I was thinking to make a 5x5 array." When the problem was read aloud to him, and when he was prompted through the steps of using the diagrams to help him, was able to complete the process accurately. He referred to the drawing when explaining his answer, but could not explain the answer when referring only to the symbolic problem. (Score: 2 points)

Group B Summaries

Student #1B

In the diagramming task, the student drew a rectangle and used the pencil to gauge imaginary lines before deciding how to partition the shape; she then drew lines and self-corrected by erasing and adjusting the lines, having trouble making five equal sections. Asked to try again with a new shape, she drew a rectangle with four sections which she immediately recognized as fourths. With prompting to find the easiest way to make fifths, she added another section onto the rectangle. Asked to show fifths one more different way, she tried three different circles partitioned into sections with vertical lines. She recognized that they were not equal only because they were different widths, making no reference to overall size of the parts in her explanation. She shaded one section of the rectangle to show one fifth, and wrote the fraction quickly and accurately. (Score: 2 points)

The student wrote the addition problem quickly and accurately, but hesitated before explaining that she could "get something three times." Prompted to look at the problem again, she referred back to her rectangle diagram, shading three more parts with no further prompting, then writing $\frac{4}{5}$ as the sum. Her explanation was clear and accurate. (Score: 3 points)

Student #2B

In the diagramming task, the student quickly drew a circle with a median line, saying she was "going to cut one half into thirds" and proceeding to partition with radiating lines, resulting in two fourths and three sixths, which she said were "almost the same size." Asked to try it a different way, she drew a rectangle partitioned into five fairly equal vertical sections, making repeated adjustments to create equal parts. In her explanation, she referred to the rectangle as "one set" and "one shape of it." She shaded one section of the rectangle to show one fifth, then wrote the fraction quickly and accurately. (Score: 2 points)

In the addition task, the student wrote the problem quickly and accurately, hesitated, then added the denominators to get 10, because "my teacher taught me that you look at the two denominators and then you get one of the denominator or the other one.... denominators... and the two denominators are five and five, so I get five objects five times...." When challenged, she was unable to explain her work, and said she could not add the numerators without knowing what the denominator was supposed to be. With prompting to use the rectangle to help solve the problem, she recognized the correct answer to the problem. (Score: 1 point)

Student #3B

In the diagramming task, the student drew a square partitioned with vertical lines, making adjustments so that the sections would be equal widths, saying she knew it was fifths because it was "five equal parts." Asked to show fifths another way, she drew a circle in which she began with a median line which she immediately erased, trying many adjustments of lines radiating from the center in an attempt to partition the circle equally. With prompting, she tried a third time, using a circle with a median line with lines radiating off, creating five unequal-sized parts, saying that "some of them are like a little bigger than the rest of them." Asked what she could do to solve the problem, she explained that she could make some sections bigger and some smaller. She shaded one section of the square to show one fifth, and wrote the fraction quickly and accurately. (Score: 3 points)

The student wrote the addition problem with verbal directions, then worked the problem quickly and accurately, saying, "three plus one is four, and then five is five because you don't really need to do anything with the denominator. (Score: 4 points)

Student #4B

In the diagramming task, the student drew a circle partitioned into five vertical sections, saying he was "going to split it up into five fifths," referring more than once to the number of lines when asked how it showed fifths. With prompting, he recognized that it wasn't fifths because of the unequal sizes of parts. On a second try, he drew a square partitioned into four sections, then added another section, stating that they were fifths because they were all the same size. He shaded one section to show one-fifth, and wrote the fraction quickly and accurately. (Score: 2 points)

In the addition task, the student wrote and solved the problem quickly and accurately, explaining that there was "five at the bottom, and one at the top, and three." (Score: 4 points)

Student #5B

In the diagramming task, the student drew individual, attached squares, positioned in an "L" shape. Asked to do it another way, she drew a circle with what she called "pizza cuts." She began partitioning the circle with a median line and could not figure out how to get five equal parts. Asked what fifths meant, she said, "it's five pieces." When asked if size mattered, she said, "yeah, a little," because "if I draw a circle, and I draw fifths... the pieces wouldn't really be fifths." Asked to try one more time, she used a rectangle, but needed some prompting to help her get equal parts without completely erasing everything each time. To show one fifth, she shaded one part of the rectangle, explaining her strategy only by saying, "I shaded one-

fifth in" and "because I shaded it in." She wrote the fraction quickly and accurately. (Score: 2 points)

The student wrote the addition problem quickly and accurately, added the denominators, then added the numerators. When prompted to use her rectangle to help her, she said the correct answer was four-fifths, because "they're shaded in." (Score: 2 points)

Student #6B (Speech)

In the diagramming task, the student first drew a circle partitioned with vertical lines, carefully checking the parts for equal width. Asked about the size of the parts, he said that they were not equal because some were shorter, and therefore the drawing did not show fifths. On a second try, he drew a square partitioned equally with vertical lines. Asked to do it another way, he tried a circle again, which was partitioned into six fairly equal "pie" sections. After two more tries, he was able to fairly accurately partition a circle into five equal sections. He shaded one part of the square to show one fifth, and wrote the fraction quickly and accurately. (Score: 3 points)

In the addition task, wrote the problem accurately from oral directions. To solve it, he first added the denominators, then stopped, then needed prompting to find the correct numerator. With further prompting, he used his drawings to work through the problem. He recognized that his answer was incorrect, and changed the denominator to 5. (Score: 2 points)

Student #7B (LEP)

In the diagramming task, the student drew a rectangle partitioned into five equal parts, saying that he needed five parts "because it's supposed to be fifths," and that "three plus two equals five," while pointing to the parts in the rectangle. Asked to show fifths another way, he drew a square with six parts, then erased to self-correct the mistake. He pointed to one part of the rectangle to show one fifth, then shaded it when prompted to do so, and clearly explained that it was one shaded part out of five parts. He wrote the fraction quickly and accurately. (Score: 4 points)

In the addition task, the student wrote the problem quickly and accurately, hesitated for several seconds, then wrote four-tenths as the sum. His explanation for this included a reference to four parts being shaded, but he made no reference to adding the denominators. Prompted to use one of the diagrams to help solve the problem, he recognized the answer as four-fifths, and explained the answer by referring to the diagram, noting that there were not ten parts there. (Score: 2 points)

Student #8B

In the diagramming task, the student drew a square partitioned into five equal vertical sections, but called them "one-fifth" and could not explain

what he meant. On his second try, he drew a rectangle, which he quickly partitioned into five equal horizontal sections. He shaded in one section to show one fifth, explaining that there were "five equal parts and I shaded in one-fifth." He wrote the fraction quickly and accurately. (Score: 3 points)

In the addition task, he first wrote one of the numbers incorrectly, but self-corrected it. He hesitated in solving the problem, wanting to make a 4x5 array to find the denominator, but had no reason for this strategy. When shown the two fives in the denominators, he hesitated again. Prompted to use the diagram to help, he was able to correctly solve the problem, made a small error in reading the answer, and again self-corrected his mistake. (Score: 2 points)

Student #9B

In the diagramming task, the student drew a rectangle evenly partitioned into five sections. On the second try he hesitated, then drew a square with eight parts. Asked to try again, he drew a square and partitioned it with crossed diagonal lines into four parts, then hesitated after starting to draw a vertical line down the middle, stating that it would have six parts. On a fourth try, he drew a circle partitioned into six fairly equal parts; he knew it was not fifths because "they're not equal" and "there's not five pieces." He used his first diagram to shade one section to show one fifth, but wondered aloud whether he should shade in more parts. He showed confusion over whether it was one half or one fifth. Asked what half means, he said, "If you have five pieces and you shade one it's one half." He wrote the fraction quickly and correctly. (Score: 2 points)

In the addition task, the student wrote the problem quickly and made a verbal reference to the denominators being the same, but hesitated before proceeding. With slight verbal prompting, he gave a correct answer, referring only to the numerators when explaining his answer. (Score: 3 points)

Student #10B

In the diagramming task, the student drew a circle partitioned with vertical lines into five parts, trying to make the parts equal widths. He said fifths means "five equal parts," and that "equal means the sum." In further discussion of the concept of equality in relation to sharing, he said that equal means "one half" which means that "if you have ten pieces, and there's ten more, and you give ten to your friend, that's one half." After referring to the parts of the circle, he drew a rectangle with five equal sections, after self-correcting to adjust one section which was thinner than the others. He shaded one section to show one fifth. He began to write "one" when asked to write the fraction, then "15" when asked to write the number, then, with prompting, wrote the fraction accurately. (Score: 2 points)

In the addition task, the student wrote the problem without hesitating, but wanted to use an array to solve it because the numerators were different. When prompted to use his rectangle drawing to help solve the problem, he referred to the parts of the rectangle as "four pieces" and "equal pieces" and "candy bars." When encouraged to solve the problem by thinking about candy bars, he explained that "if I had five pieces and I ate four pieces of it, then I'll have one more left, and that makes it five," and wrote four fifths for the answer. (Score: 1 point)

Student #11B

In the diagramming task, the student drew a rectangle with five equal parts by adding a section onto one with four parts. Asked to show fifths another way, he drew a circle with a median line, a diagonal line, and a horizontal line, resulting in six unequal parts. He said, "I've got sixths and I want fifths," and that fifths means, "there's, like, five pieces." He did not try to change it. He shaded one part of the rectangle to show one fifth, and wrote the fraction correctly and quickly. (Score: 2 points)

The student wrote the addition problem accurately and quickly, but wanted to multiply the denominators: "look at the denominator...and... I'll get five...five pieces of cookies....five times." When prompted to use his rectangle to help solve the problem, he could not explain the process he used, even when referring to and comparing the diagram with the algorithm, because he appeared to make no connection between the shaded parts of his diagram and the numerals in the numerators. He needed much prompting and questioning to get through the addition process. His explanations were confusing and unclear. (Score: 2 points)

Student #12B (LEP)

In the diagramming task, the student first said she didn't remember how to draw fifths. After much prompting and questioning, during which she referred to fifths as five things, she eventually said that fifths meant "five pieces," then drew a rectangle partitioned with vertical lines, making many adjustments for equality. Asked to show fifths another way, she tried many different configurations of lines in a rectangle, starting with a median line, but was unsuccessful. She shaded one part of the first rectangle to show fifths, and wrote the fraction quickly and accurately. (Score: 2 points)

She wrote the addition problem with no hesitation, and quickly wrote four-tenths as the answer. Prompted to use her first drawing to show how to solve the problem, she first shaded an additional two parts and needed prompting to see that she needed to shade one more. When she recognized her mistake, she realized she had "the wrong answer" and explained her error by referring to the drawing. (Score: 2 points)

Appendix N: Pretest and Post-test Assessment Open-Ended Writing Samples

For the pretest assessment writing task, the open-ended prompt, "What Are Fractions?" was written on the chalkboard. Students were asked to write whatever they knew about fractions, to think about where they might have seen fractions, and how they might be used. For the post-test assessment writing task, the prompt, "What I've Learned About Fractions" was written on the board and read aloud. Students were encouraged to write as much as they could about what they had learned about fractions.

The samples selected show a range of scores for the pretest writing assessment, with each selected students' post-test assessment writing included for comparison.

All writing samples have been transcribed from the originals; spelling, mechanics, and grammar have been left as written by students.

Group A

Student 1A

You could see fractions in your mathbook. Fractions could help you divide. If you had 5 slices of cake and you had three guests and each of the guests wanted $\frac{1}{3}$, how are you to cut it? Fractions, of course! Fractions also might be able to help you with your job. (Pretest Score: 3 points)

I learned lots of things about fractions! I learned $\frac{1}{2} = \frac{2}{4}$. I'll show you how: 1.) 4 by 2 array. 2.) Another 4 by 2 array. 3.) Do as below (note: shows diagram of two $\frac{2}{4}$ arrays, separated in sequence into two sets of two and one set of four). 4.) Compare. We do arrays depending on the denominator. (The denominator is the lower part of the fraction). Fractions could be used in bakerys. A costumer could say, "Give me $\frac{5}{15}$ of that blue cheesecake." Fractions also can be used in road signs, like this (note: includes drawing with $1\frac{1}{2}$ miles). The upper part of the fraction (the numerator) is how many pieces you took. The denominator is the number of pieces altogether. For example, $\frac{5}{10}$ would be 5 out of 10. (Post-test Score: 3 points)

Student 4A

I see fractions on the incredible Eqachens chart.
Fractions can be in numbers or in words. (No Pretest Score)

What I have learned about fractions is that sometimes the bottom fraction is bigger or smaller. (note: gives examples of $1/12$ or $1/3$, inside a box). Sometimes fractions are colored in so we remember what thing we were doing (note: drawing of a circle with one-half shaded in, with label $1/2$). You could use fractions in Incredible Equations. The bottom number is called the denominator. (Post-test Score: 2 points)

Student 5A

I have seen them when we took a test. There are $1/2$, $2/2$, $1/4$, $1/3$, and $1/5$ and others. Fractions are kind of hard. (Pretest Score: 1 point)

Some fractions are $2/2$, $8/6$, $1/4$ Some could be $400/500$. It depends on how many blocks you're using. There could be thousands in fractions. There are denominators and numerators. Here is a problem with fractions $1/2 + 1/2 = 1$ whole. $400/500$ means 400 out of 500. (Post-test Score: 3 points)

Student 11A

Fractions are like one half they have numbers. (Pretest Score: 1 point)

Fractions can be big some times.

They help you learn.

When you write it you put a number on top, a line in the middle and then a number on the bottom. Fractions count a full number. Numbers are adding things. (Post-test Score: 3 points)

Group B

Student 1B

You can see fractions on problems and when you do math. Fractions are like $1/2$ and $3/3$. You mostly see them on math test. They could be used at school and home. (Pretest Score: 1 point)

I've learned that fractions have to have two numbers like $1/2$. You can use them in bakeries. Fractions are two numbers. A denominator and a numerator. Fractions are used in math. When you have a problem like $1/2 + 1/5 =$ you can't just add across you have to ask yourself if I draw a square I shade in one half and then another half I would have one whole. (Post-test Score: 3 points)

Student 7B

Fractions is a math thing. Fractions is like two in a half and three in a half. Fraction is like if you cut a apple and you cut it three twos. (Pretest Score: 3 points)

What I've Learned About Fractions is that there are mix nummber they are like you had some apples there were two half in two apple and you had one half left you would have 2 1/2. mix nummber are more then one hole if you had one hole you will not have a mix nummber. What I've Learned about Fraction is that denommnater are at the bottom and the lammnater is at the top. (Post-test Score: 3 points)

Student 8B

if you were doing a mathprolbem in the mathproblem (No Pretest Score)

I learnd That all fractions has two numbers and a line in the midle. Some Fractions end up into a mixed number. If you have $5/8 + 5/8 =$ it wouldn't be $10/16$ becuse you dont oweys ad the boddom nummber. $1/4$ means 4 eqal parts and a 4th means 1 out of 4 eqal parts. (Post-test Score: 4 points)

Student 11B

Fractions are like one half two thirds. fractios are very hard when (Pretest Score: 1 point)

I learned lote of things about fractions like $1/4$ is like a (drawing of a partitioned square with one section shaded) and one is shaeted in then that waned mean $1/4$. and a misked number a misked number is lik one hole and sum left over. (Post-test Score: 3 points)

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IMAGE EVALUATION TEST TARGET (QA-3)

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