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Sixth-grade students' use of schema knowledge in word problem solving

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SIXTH-GRADE STUDENTS' USE
OF SCHEMA KNOWLEDGE IN
WORD PROBLEM SOLVING

A Thesis

Presented to

The Faculty of the Division of Teacher Education
San Jose State University

In Partial Fulfillment
of the Requirement for the Degree
Master of Arts

by

Mary Howland

May 2001

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ABSTRACT

SIXTH-GRADE STUDENTS USE OF SCHEMA KNOWLEDGE IN WORD PROBLEM SOLVING

by Mary Howland

This thesis addresses the question of whether sixth-grade students in a Spanish two-way immersion program can use schema knowledge to help them solve mathematical word problems and whether there is a difference in performance between those students who are English-dominant and those students who are Spanish-dominant.

Twelve students were randomly chosen and asked to perform two tasks. In the first task the students were individually asked to sort twelve word problems and the categories formed were recorded. In the second task the students were asked to solve five word problems. They were asked to explain the processes they used to solve the problems. It was found that nine of the twelve students were able to use schema knowledge to sort and solve the word problems. English-dominant students did somewhat better than Spanish-dominant students. Female students did somewhat better than male students.

I would like to thank my advisors Dr. Sosniak, Dr. Pence, and Dr. Roddick.

Without their help this thesis could not have been completed.

This thesis is dedicated to the students of River Glen School.

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CHAPTER I

Introduction

Problem solving is an integral part of any mathematics program. There has been much debate as to what is a problem and what is problem-solving. In many mathematics textbooks, problems are defined as exercises or tasks that provide practice on a particular mathematical technique. Others suggest that true problems should be questions that are perplexing or difficult and that encourage the students to think creatively (Schoenfeld, 1992). This second definition of a problem means that the same set of words asking a question might be an exercise for one student, a problem for another student, and a completely impossible task for a third student. For example, if second grade students are asked how many cookies each person would have if there were 6 students and each one had seven cookies, a few might be able to do it as a simple exercise where they apply the rules of multiplication; others would be able to solve it as a problem, using diagrams and pictures to help them solve it. A few would give up, because they could not understand what should be done.

Many students can do computations, but cannot solve mathematical word problems (Kouba, et al., 1988). It seems that one of their major difficulties is that they do not understand what the problem is asking (Kroll & Miller, 1993). Researchers who are interested in cognitive factors that influence problem-solving often look at the importance of short-term and long-term memory and how good problem solvers are more likely than poor problem solvers to remember information about the structure of problems they have previously solved. This knowledge about the structure of the problem is sometime referred to as schema knowledge. The implication of this research is that the way to help students become good problem solvers is to help them learn schema or problem structures which they can then apply when solving problems (Kroll & Miller,

1993). In word problems these schema are the semantic structures that indicate the underlying mathematical structure of the problem.

Other researchers have stressed the linguistic and language factors which affect problem-solving. Researchers have looked at semantics, syntax, vocabulary, and sentence length. They have also looked at where the question being asked by the problem is positioned in the problem and whether that positioning is significant. Semantics, vocabulary, and syntax have been shown to be particularly significant (Gilbert, 1996; De Corte, Verschaffel & De Win, 1984; Riley, Greeno, & Heller, 1984). Most of the studies have looked at students' performance in the elementary grades (Carpenter et al, 1982). There have been far fewer studies at the middle school level. However, I have observed that many middle school students still have difficulties solving problems when the syntax, semantics, and vocabulary become more complex.

There have been several studies that look at whether being bilingual has a negative or positive effect on a student's ability to solve word problems (Secada, 1991). With the growing number of students in California schools coming from bilingual homes and with the controversies over the effectiveness of bilingual education, bilingualism and mathematical problem solving remains an important topic.

It is generally agreed that problem solving is important for effective mathematical performance, yet it is one of the most difficult areas to teach in elementary or middle school mathematics programs (Doby, 1992). How consistently and how well problem-solving is taught varies widely from teacher to teacher (Sowder & Sowder, 1989). In order to teach problem solving effectively, teachers need to know which are the areas that are of particular difficulty to the students. Many teachers are aware of the need for students to develop schema to help them solve problems. However, what has not been studied extensively is

whether schema knowledge learned in one language transfers to a second language and how fluency in the two languages affects mathematical problem solving.

Background and Purpose

It has been my observation as a Resource Specialist that many of my students who have a disability in the area of math have trouble understanding what is being asked for in a word problem. They cannot determine whether they should add, subtract, multiply, or divide, although they are able to do these operations when they are presented in purely computational form. They seem to have difficulties with the problems even when they are read the problem, which indicates that it is not simply a decoding problem. Therefore, I wanted to look at how students use the schema of a problem to help them solve it.

The school where I teach is in an urban area in California and has a special Spanish two-way immersion magnet program. Two-thirds of the students enter the school as English-dominant speakers, and one-third of the students enter the program as Spanish-dominant speakers. All the students receive the majority of their instruction in Spanish for kindergarten through second grade. In third grade English reading is introduced, and in the upper grades increasing amounts of time are spent on instruction in English. The majority of the day is in Spanish until sixth grade when the instruction changes so that a little more than half the day is taught in English. Mathematics instruction is in Spanish through fifth grade and then in English for sixth, seventh, and eighth grades. I wondered if students who had received all their formal mathematics instruction in Spanish would be able to transfer their knowledge of how to use the schema or semantic structures of mathematical word problems to English when they entered sixth grade.

The purpose of this research is to observe whether a randomly selected group of sixth grade students in this special magnet school can determine the

operation of given word problems using schema knowledge and whether there is a significant difference between those students who had English as their dominant language when entering kindergarten and those whose dominant language was Spanish. It is hoped that the results of this research can then aid the sixth grade math teacher in better transitioning students from the Spanish mathematics program that they have had through fifth grade to the English mathematics program that is introduced in sixth grade.

There are several questions which I hope to answer in this study. They are:

1. Can students use schema knowledge to sort mathematical word problems into whether they are subtraction or division problems?
2. If students can divide problems into whether they are division or subtraction, can they further classify problems within these two categories using schema knowledge?
3. Is there a difference between the students who started the program as English-dominant speakers and students who started the program as Spanish-dominant speakers but are now fluent in English?

The nature of the magnet program at the school where I work offers a unique opportunity to study the way sixth grade students use schema knowledge to help them solve mathematical word problems and how instruction in one language impacts the students' ability to use the semantic structure or schema of a problem to identify the underlying mathematical structure of the problem.

CHAPTER II

Review of Literature

Many researchers have studied the role of schema knowledge and second language acquisition in relation to students being able to solve mathematical word problems. The questions they have asked are similar to the ones being asked by the present study. The first part of this chapter will examine cognitive theory based on information processing models and then research on how students use schema to help them solve problems will be reviewed. In the second part language issues that relate to mathematical problem solving will be discussed and then research that shows the role of language and bilingualism in problem solving will be presented. Research that looks at other influences on a student's ability to solve problems will be examined so that those factors may be controlled for in the present research design.

Cognitive Theory

Cognitive theories that have their roots in Gestalt psychology emphasize the tendency of the mind to organize and integrate and to perceive situations, including problems, as total structures. These theories emphasize the structure of the problem and then the process of reorganization of the perceptual field which leads to insight. This insight leads to a solution of the problem (Loftus, 1970). Information-processing theories and theories that emphasize the learning and use of schema developed out of the Gestalt theories.

Information-processing theories have grown out of looking at how computers solve problems and comparing that process to what is done by human subjects. The basic notion behind the computer simulation approach is that complex thinking processes are built of elementary symbol manipulation processes. Simon (1975) describes the basic characteristics of the human information processing system as follows:

The system operates mostly serially, one process at a time, not in parallel fashion. (A partial exception must be made for recognition processes, which are probably carried out in parallel.) Its elementary processes take tens or hundreds of milliseconds. The inputs and outputs of these processes are held in a small short-term memory with a capacity of only a few symbols. The system has access to a practically infinite long-term memory, but the time required to store a symbol in that memory is of the order of seconds or tens of seconds. Access to long-term memory is obtained by recognizing stimuli, the recognizer serving as a sort of "index," or by associating from one item in memory to another. (p. 3)

As one can see, the information-processing theory of problem-solving involves looking at the role of memory in the problem-solving process. Mayer (1980) cites six characteristics that could be used to explain individual differences in problem-solving ability. They are (1) holding capacity of working memory, (2) type of code (visual, verbal, etc.) used in working memory, (3) speed of mental operations in working memory, (4) speed of search for target in long-term memory, (5) selective attention, and (6) pattern matching. Memory is an important component of problem solving, because to solve a problem efficiently one needs to be able to recognize the important features of a problem quickly, and then associate those features with possible solution steps.

The need to recognize the important features of a problem have been looked at by many researchers. They posit that semantic and knowledge schemata are central constructs in word problem understanding processes (Peled & Wittrock, 1990). The idea behind a schema theory is that people use patterns, derived from what they have learned previously, to guide their actions in problem solving as well as in other situations. The idea of schema has been used extensively when looking at how linguistic factors and problem structure affect a student's ability to solve mathematical word problems. The importance

of the idea of schema is that if problems are solved using schema then those schema may be taught.

Research Studies on the Role of Semantic Structure in Problem Solving

Much of the recent research on mathematical problem solving has focused on specific problem characteristics involving the relationships among quantities. The main findings of these studies is that the understanding of quantitative relationships in problems must take into account the conceptual structure of the problem (Riley, Greeno, & Heller, 1984). One of the ways that problems differ is in the semantic relations used to describe the problem situation. Semantic relations refers to conceptual knowledge about increases, decreases, combinations, and comparisons involving sets of objects. If students have learned a schema that can be used to recognize a particular problem type, they are able to solve the problem more efficiently.

Evidence for the influence of semantic structure in problem solution comes from studies showing that problems described by different semantic structures are not equally difficult, even when they require the same operation for solution. The basic procedure in these studies usually involves having children individually solve selected problems that are read to them by the experimenter. Memory and computational difficulties are kept at a minimum by reading the problems slowly, repeating them if necessary, and by restricting the size of the numbers in the problems such that sums are less than 15. In addition, concrete objects are often provided for children to use in solving the problems. Riley, Greeno, and Heller (1984) report that change problems were the easiest for students. Here is an example of a change problem:

Joe had 9 marbles.

He lost 5 marbles.

How many marbles does Joe have now?

Combine problems came next in the order of difficulty. An example of a

combine problem is:

Joe and Tom have 8 marbles altogether.

Joe has 3 marbles.

How many marbles does Tom have?

The most difficult type of problem for students is where students are asked to compare quantities. Here is an example of a compare problem:

Joe has 8 marbles.

Tom has 5 marbles.

How many more marbles does Joe have than Tom?

Within the change, combine, and compare categories there are subcategories that can involve a missing addend in the problem.

Bell, Fischbein, and Greer (1984) in their study of twelve- and thirteen-year old students found that there were different levels of difficulty in multiplication and division problems. Their study had two parts. In the first part students needed to write down the calculations needed to solve a word problem and in the second part they needed to write a story to go with a given calculation. In both parts of the study partition division problems were easier than quotient. Rate problems in each type were the most difficult type of problem for the students. An example of a partition problem is:

An electrician has 1400 m of electric cable in stock. He has to rewire 12 houses. How much cable can he use in each house?

An example of a quotient problem that was used is:

I am packing boxes of cassettes. I fit 24 cassettes in a box. I have 600 cassettes. How many boxes will I fill?

A partition-type rate problem that was used is:

A rowing crew covers a 3 km course in 7.2 minutes. How far did they row in one minute?

A quotient-type rate problem that was used is:

A pound of meat is priced at £2.56 in a shop window. A housewife pays £2.00 for a cut of meat. What should it weigh?

Bell et al. (1984) concluded that students see multiplication and division as algorithms and not structures, and that teachers need to explicitly teach structures such as partition and quotient.

Riley, Greeno, and Heller (1984) propose that three main kinds of knowledge are needed to solve these types of problems: (1) problem schemata for understanding the various semantic relations, (2) action schemata for representing the model's knowledge about actions involved in problem solving, and (3) strategic knowledge for planning solutions to problems.

Jitendra, Hoff, and Beck (1997) studied the effectiveness of a schema strategy on addition and subtraction word problem performance. The participants were four middle school students with learning disabilities who were selected because they were deemed to be poor word problem solvers as judged by their teachers. A comparison group of 21 normally achieving third graders were included for testing only. The four middle school students were taught to map the features of the word problem onto problem schemata diagrams, first for one-step problems and then for two-step problems. The students were taught to distinguish the unique features of each problem type (change, combine, and compare) in story situations that did not contain any unknown information. Students mapped the story situation onto schemata diagrams. Then the students were asked to map problems onto schemata diagrams when there was unknown information. A multiple-baseline design across students and across two behaviors was used. The results of this study indicated that the schema strategy led to increases in the percentage of correct operations and computation solutions to word problems. Follow-up testing was done after 2 weeks and then after 4 weeks and these results were maintained. Further, all four students' performance on the two-step word problems at the end of the study surpassed

that of the normative sample of third graders.

Stern (1993) conducted 6 experiments to investigate why solving compare problems is so difficult for students. She used 42 kindergartners, 190 first graders, and 15 second graders in Germany. In the first three experiments the students were asked to solve four to eight compare problems. The problems were presented individually to the students by a female research assistant. The first experiment looked at whether repeating the name was easier than substituting a pronoun in compare problems for kindergartners. The second experiment repeated the same procedure with first graders. No significant difference was found between the two types of problems. In Experiment 3 two different sets of objects were compared without reference to a person's ownership. To investigate the role of the personal pronoun, the researcher compared the solution rate of these two-set problems with the solution rate of the two-person problems. The results of this experiment were that performance was worse on two-set problems than on two-person problems. Therefore the data failed to show that the use of pronouns in the second sentence is the source of children's difficulty with compare problems. In Experiment 4 Stern looked at two classes of compare problems. One has an unknown compare set similar to the example given above for compare problems. The other type has an unknown reference set such as this one:

John has 5 marbles.

He has two marbles more than Peter.

How many marbles does Peter have?

Students were asked to restate the problem and then solve it. Stern found a close relation between solving a problem correctly and retelling it accurately. Children were much better at solving problems with an unknown compare set than problems with an unknown reference set. Experiment 5 looked at whether students understood that the statement "There are n more x than y " is the

same as "There are n fewer y than x ." This knowledge is necessary to solve unknown reference set problems by transforming them into unknown compare set problems. First grade students were asked whether the difference between two sets were valid or not. The results indicated that although the majority of the children could understand each statement of the form " x has n more objects than y " and " y has n objects fewer than x " they did not realize that the statements represented different sides of the same coin. Experiment 6 was designed to show that there is a relation between correctly performing the picture tasks of Experiment 5 and solving unknown reference set problems that is due not only to the difficulty of both tasks but also to common conceptual deficits in understanding the symmetry of comparison. The results confirmed this hypothesis. Overall, Stern concluded that the difficulties with compare problems with an unknown reference set came from students not understanding the symmetry of comparison. The awareness of several possibilities for putting the same event into different words is an important criterion for understanding a concept in mathematics.

De Corte, Verschaffel, and De Win (1985) studied the influence of rewording verbal problems on children's ability to solve them. Their observation is that experienced problem solvers process the word problem in a top-down way; that is, the processing is conceptually driven using semantic schemata. In less able and inexperienced children, however, the semantic schemata are not yet very well developed, so these children depend more on bottom-up or text driven processing to construct an appropriate problem representation. They hypothesized that especially for less able and inexperienced children, rewording verbal problems so that the semantic relations are made more explicit without affecting the underlying semantic relations and mathematical structure facilitates construction of a proper problem representation and, by extension, finding the correct solution. In their study, two series of six rather difficult word problems

were administered near the end of the school year to 4 first-grade classes and 4 second-grade classes, with a total of 89 and 84 children respectively. In Series A the problems were stated in the usual form in which they normally appear in first graders' textbooks. In Series B the same kind of problems were reformulated so that the semantic relations between the sets were stated more explicitly to be clearer to young children. De Corte, Verschaffel, and De Win found that for both grades and for the total group, the reworded problems of Series B were solved significantly better than the standard verbal problems of Series A. These results complement other research that has yielded evidence that semantic structure of word problems significantly influences the difficulty level of the problems and children's strategies applied to solve them.

Hong (1993) investigated how students' problem and mathematical models affected their performance in problem solving. The subjects of the study were fifth ($n=35$) and sixth ($n=49$) graders in Korea. These students were given seven word problems involving addition, subtraction, multiplication, and division that could be solved in two steps. Think-aloud protocol was used, and the students were pulled out of class to be tested individually. The students who represented the problems correctly either had rapid problem classification models indicating that the students were familiar with the types of problems and automatic categorization of the problem was possible, or restated the problems in their own words focusing on specific words and relating them to other information. In other words, the successful problem solvers were able to determine whether they needed to use addition, subtraction, multiplication, or division to solve the problem fairly quickly. Most students who could not solve the problem successfully had difficulties in constructing correct problem models while having little or no difficulty in constructing number sentences and computation. Therefore, this research again reinforces the idea that knowledge of semantic representation of problems is important to efficient and accurate

problem solving.

Quintero (1983) studied children's difficulties when solving multistep problems. She hypothesized that the difficulties arise from a lack of understanding of the concepts and relationships or to difficulties in planning and organizing the method of solution. The subjects were 12 students each selected from fifth, sixth, and seventh grades in Puerto Rico. They were selected randomly from four schools. There were 17 boys and 19 girls. The experimenter worked with one child at a time, and the session was audiotaped. Each child was asked to solve six word problems that involved ratios. The results of this study were that the majority of the children had difficulty with the two-step problems. Their difficulties, however, did not stem from the same source. This study provided evidence that the meaning of concepts and relationships in a two-step word problem is a major source of difficulty in solving the problem and that there are levels of understanding this meaning. Quintero found that the concept of ratio, which is not common to all types of word problems, was the key source of difficulty in solving the problems, and therefore showed the importance of having precise categories to characterize the word problems children are asked to solve.

Gliner (1989) has studied whether college students can think of related problems to help them solve given problems. The participants in her study were 133 students at an urban open-enrollment college. They were tested during class time, and came from different levels of math classes and beginning psychology classes. Thirteen mathematics word problems were used. There were four identified dimensions relating to the problems: mathematical structure, context, question form, and pseudostructure. The latter dimension pertained to a common unit of measurement, such as weight or time. The subjects were first given copies of the 13 word problems, each written on a separate sheet of paper. They were asked to sort the problems in groups based on similar mathematical

structure. After they completed the sorting task, they were given a randomly ordered list of the problems and asked to solve them, showing all their work. The results of this study were that there were obvious differences in the organization of the word problems for successful versus less successful problem solvers. When asked to sort problems based on mathematical structure, the more successful problem solvers recognized underlying mathematical relationships among the problems and were immediately able to organize the problems based on structural criteria. Less successful problem solvers did not see these underlying structural characteristics and, instead, organized the problems based on things such as question form, context, and common units of measurements. Again being able to recognize the underlying semantic representation of the problem aids efficient and accurate problem solving.

Hutchinson (1993) studied whether teaching problem representations to students with learning disabilities would improve their ability to solve algebra word problems. Twenty adolescents were selected to participate in the study from the population of LD students scheduled for small-group learning assistance classes in mathematics. A comparison group of 20 students was also selected. Pre- and post-tests consisting of the same set of 15 problems were given. Think-aloud protocols also were used for one problem. Posttest results for the comparison group were similar to the pretests. However, for the instructed group, posttests showed increased scores for the general questions. Greatest improvement was achieved on questions requiring an understanding of the roles of mathematical structure and surface structure in representing algebra problems. The students also showed improved awareness of the steps to take when solving an algebra word problem during the think-aloud posttest. The findings suggest that focusing LD students' attention on the mathematical structure of word problems removes the usual confusion between story line and mathematical relationships in word problems and is helpful for LD adolescents.

The posttest data revealed that the more problem types a student learned, the more correct choices that student made in deciding how the problem should be solved. These findings are consistent with other research that shows that good mathematical problem solvers may be inferred to have acquired schemata based on underlying problem structure and principles.

There have been several studies that look at factors that might interfere with a student's ability to use learned schemata to solve problems. Muth (1986) found that sixth grade students, solving word problems involving addition, subtraction, multiplication, and division of whole numbers and decimals, were better able to solve problems without extraneous information than with it. Muth (1992) conducted another study where 140 eighth grade students were given four versions of a 14-item word problem test. The four versions of the test were no extraneous information/no extra step, no extraneous information/extra step, extraneous information/no extra step, and extraneous information/extra step. Muth found that the presence of extraneous information and extra steps in the word problems hindered students' problem solving by interfering with their ability to identify the problem question and the appropriate rule for answering the question. Two common misconceptions that the students had were that you should use all of the numbers in a problem and that if the problem only has one number, you have to get another number from somewhere.

Some students have learned to rely on cue words to help them determine which schema to use. Parmar, Cawley, and Frazita (1996) showed that this strategy can interfere with problem solving when there are indirect statement, start unknown, or inconsistent language problems. They studied 210 students in third through eighth grades with 197 of these students having mild disabilities. These students were administered four sets of word problems that included problems with direct and indirect problem statements, problems with extraneous information, and problems with one versus two steps to reach a terminal solution.

These researchers concluded that students should not be taught cue words, but rather need experience with a variety of word problems that encourage analytic thinking and that enhance cognitive abilities.

De Corte and Verschaffel (1987) studied the effect of some non-semantic factors involved with problem structure. They were (1) the order of presentation of the two given numbers; (2) the order of presentation of the two given sets; and (3) the size of the difference between the two given numbers. They administered a paper-and-pencil test to 85 children and then interviewed individually a smaller group of 20 students. The collective test contained 20 elementary addition and subtraction word problems representing two different types of change problems and two different types of combine problems. Selected problems were modified to determine the effect of the non-semantic variable listed above. The collective test was given at the beginning of the school year, and then the group of 20 students was interviewed in the middle of the year using exactly the same set of word problems as before. During the interview the researcher read the problem aloud and the students were asked to solve the problem, to explain their strategy, and to build a material representation of the problem using puppets and bricks. The researchers found that in addition problems children's solution strategies are strongly influenced by the location of the smaller and the larger given number. However, there were no big differences in solution strategies depending on the size of the difference between the two given numbers. There was also only a small difference in solution strategies when the smaller number is given first in the combine problems. In subtraction problems solution strategies changed according to whether a bigger or smaller number was given first, but the size of the difference between the two numbers did not seem to make a difference. The researchers also found that the order of presentation of the given numbers is not the same for all semantic problem types.

Threadgill-Sowder (1983) looked at the influence of question placement

on problem solving. A set of 14 problems was given to 52 college students who were enrolled in intermediate algebra classes. The problems were randomly ordered, then written in two formats, one with the question given at the beginning, and the other with the question given at the end. What was found was that question placement apparently has no effect on the ability of students to solve word problems, regardless of length and complexity of the problems.

Research on the role of schema show that problems with different semantic structure vary in how difficult they are to solve. Factors that need to be controlled for in research examining semantic structure are the presence of extraneous information and the location of the smaller and larger numbers. Since two-step problems are more difficult to solve than are one-step problems, this factor needs to be controlled for as well.

Language Issues Related to Problem Solving

Language is essential to problem solving. Meijer and Riemersma (1986) state that the steps in problem solving can be listed as (a) the phase of encoding -- internal representation; (b) conception or representation of the problem; (c) solution of the problem; and (d) evaluation. Language skills are especially important to the first two steps. The encoding phase of the problem-solving process includes linguistic and semantic knowledge, as well as general and domain-specific knowledge. The conception or representation phase of the problem-solving requires students to translate from the English language into the mathematical language of numbers and symbols.

Cummins and Swain (1986) state that there is a difference between basic interpersonal (communication skills (BICS) and cognitive/academic language proficiency (CALP). Under this theory, children from lower socioeconomic levels may have developed a high BICS proficiency but a low CALP, and this low CALP contributes to difficulty in school, where CALP is stressed. In contrast to Cummins and Swain, Oller (1976) claimed that language proficiency is one skill.

Zepp (1989) suggests that Oller's factor may be one of the two factors suggested by Cummins and Swain. He goes on to state that the studies seem to show that that various abilities in mathematics and abilities in reading all load heavily onto a single factor, suggesting that many verbal and nonverbal skills may be simply surface features of what Oller called "a language factor deeper than speech." Zepp cites contemporary sociolinguistics who question the labels of "intelligence" and "cognitive ability" for this underlying factor. They suggest that what is really being measured is the ability to decontextualize a situation.

Cummins (1989) argues that proficiency in a child's first language can transfer to the second language, which implies that cognitive skills do not have to be relearned by the child in the second language. Thus a student who has developed math skills in his/her first language is likely to transfer these skills to the language s/he is learning. Therefore there should be no loss of math skills from one language to another in bilinguals.

The relationship of bilingualism to intelligence has been studied in the last 40 years. Peal and Lambert (1962) controlled for such factors as socioeconomic status, age, and degree of bilingualism. They found that bilinguals outperformed monolinguals, particularly in "cognitive flexibility," which might also be called divergent thinking. They also reported differences in concept formation, manipulation of figures, and completion of figures. Zepp (1989) explains that bilingualism leads to flexibility of thought because children who learn two languages have two world-views from which to consider any problem. The more multi-tiered the hierarchy of concepts, the greater is likely to be the ability to view concepts from many angles. The additional level of hierarchical structure that bilinguals have may be particularly important for mathematics learning.

Bilingualism may be of an advantage in mathematical problem solving in that bilinguals are often more aware of the grammatical structures of their languages than monolinguals. Ben-Zeev (1977) found that Hebrew-English

bilinguals were better than monolinguals at the analysis of sentence structure. It is this ability to distance oneself from a concept in order to analyze it abstractly, which Zepp (1989) states is important to mathematical analysis. This ability to analyze sentence structure might also help bilinguals decontextualize a situation which was mentioned previously when the idea of a language factor that underlies many verbal and non-verbal abilities was being discussed.

Vocabulary development and literal interpretation of the problem seems to be essential to problem solving. Bobrow (1968) developed a computer program that would translate mathematical word problems into equations. Johnson et al (1980) then used a version of Bobrow's procedure to teach students to solve algebra word problems. Their performance improved on tests of writing equations. Saville-Troike (1984) found that vocabulary was the most important linguistic component when learning academic content through a second language.

Semantic mastery of both the symbols and the language they represent is crucial to success in mathematical problem solving. To develop a high level of semantic understanding, students must engage in a variety of experiential activities (Gilbert, 1996). These experiences can be used to develop vocabulary, semantic structures, and comprehension of the problem situation. When semantic structures are in place, students make fewer mistakes and often invent their own methods for finding solutions (Quintero, 1985). Research seems to indicate semantic structures learned in one language can transfer to a second language.

Research Studies Involving Linguistic and Language Factors

Many researchers have studied various language factors as they relate to a student's ability to solve mathematical word problems. Some have been concerned with how the syntactic or semantic structure of a problem influences the difficulty of the problem. Others have examined whether a student's

bilingualism has a positive or negative effect on his/her ability to solve mathematical word problems.

Linville (1976) studied whether syntactical (involving word order, verb tense, etc.) or semantic (involving meaning, vocabulary, etc.) was more important in determining the difficulty of a word problem. Linville devised problems which could be classified as having easy or hard syntax, and easy or hard vocabulary. These scheme led to four categories of problems: easy-easy, easy-hard, hard-easy, hard-hard. The only significant result that was found was that problems became harder as their syntax and vocabulary increased. It could not be determined whether the syntax alone or the vocabulary alone contributed to the problem-solving difficulty, nor could it be concluded that the combination of syntax and vocabulary was a stronger deterrent than either one in isolation. All that could be said was that the linguistic variables somehow contributed to difficulty in problem solving.

MacGregor and Price (1999) studied whether an association exists between students' algebra learning and their awareness of symbols and syntax, the two components of metalinguistic awareness that seem to have algebraic analogs. Metalinguistic awareness enables the language user to reflect on the structural and functional features of text as an object. Symbol awareness would involve the student knowing that numerals, letters, and other mathematical signs can be treated as symbols detached from real-world referents. Syntax awareness would involve the recognition of well-formedness in algebraic expressions (e.g., knowing that $2x = 10 = x+5$ is well formed whereas $2x=10= 5$ is not well formed) and the ability to make judgments about how syntactic structure controls both meaning and the making of inferences.

In their first study MacGregor and Price (1999) examined how 1236 students in Melbourne in grades 4 through 10 and in their first to fourth years of algebra learning performed on 42 test items. The test items consisted of 13

items that assessed awareness of symbols and syntax in nonalgebraic contexts, 18 simple algebra items, 4 basic literacy items, and 7 easy arithmetic problems that checked students' understanding of the four operations. They found that all the students who obtained maximum or near maximum scores on algebra items also obtained maximum or near maximum scores on language items. There were a number of students with a high language score and a low algebra score, but no students with a low language score and a high algebra score.

In their second study MacGregor and Price (1999) increased the scope and difficulty of the language items so that the scores would be more normally distributed. This new test was given to 340 students in grades 8 through 10 in one school in a middle-class suburb of Melbourne. The correlations between language and algebra scores were found to be positive for each grade level. In the new data there are no very high language scores for students who had very low algebra scores. High language scores tended to be associated with high algebra scores. There were very few instances of a student's having a very low language score and a high algebra score.

MacGregor and Price (1999) conclude that the reason some students fail to learn the algebra taught in introductory courses might be that they have poorly developed metalinguistic awareness. This conscious awareness of language structures and the ability to manipulate those structures may be manifestations of deeper cognitive processes that also underlie the understanding of algebraic notation. It is possible that bilingual students who have developed CALP (Cummins & Swain, 1986) in both their first and second languages might be able to manipulate these structures better than monolingual students, because they have developed a better awareness of language structures in the course of becoming bilingual.

Are language minority students who are not proficient in English forced into a pattern of failure simply because they do not yet understand the language

in which mathematics problems and concepts are embedded when those problems are in English? Kaplan and Patino (1996) studied the effectiveness of a communicative approach on mathematical problem solving. The participants in this study were sixth grade Spanish-dominant students who had a wide range of English competence. A pretest and posttest were administered with an eight-week instructional period in between. The instructional period involved daily lessons that consisted of a linguistic warm-up to problems where vocabulary and the situational context of the problem were discussed before any mathematics was addressed. Then the problems were broken down into natural grammatical phrases which taught the students a technique for word problem analysis. Then the semantic structure of the problems was analyzed using graphic representations, and students were encouraged to restate the problems in their own words. Posttest results showed that students significantly increased their ability to solve mathematical word problems in English. The question might be raised, however, as to whether the improvement stemmed from the students' greater familiarity with problem schema or from an increased knowledge of English grammatical structures and vocabulary used in the problems.

Cohen (1994) studied which language students used in completing mathematical problem-solving tasks in an immersion classroom. Thirty-two students were selected from third through sixth grade at a Spanish full-immersion school in St. Paul. The students were interviewed individually. The findings revealed that for the immersion students English seemed at times to play a more prominent role in their internal language environment than did Spanish. In responding to word problems in math that were written in Spanish, the fifth and sixth graders in the study reported favoring English in their cognitive processing and were also observed to be doing so. This same process might be assumed to be taking place when students who are dominant in Spanish are asked to solve word problems in English.

Adetula (1990) studied whether presenting word problems in a student's first language affected performance. Previous research by Adetula (1985) found that learning mathematics is hampered if given in a foreign language, because the children are forced to learn an increased number of new words in order to be able to think and express themselves entirely in the foreign language for mathematical purposes. Cuevas (1984) also found that a child whose native language is not English will have problems with the language constructions used in the mathematics classroom. Adetula (1990) suggests that mathematics should be taught in a child's first language. In this study Adetula (1990) examined how 48 fourth grade students from four schools in Nigeria performed on a test asking them to solve 10 addition and subtraction word problems presented in English, and 10 parallel problems presented in their native language. Half of the children were first given the problems in English before their native language. The process was reversed for the other half. The results indicated that the students generally performed better when problems were presented in their native language than when presented in English language. Given the results of this study, a question that needs to be addressed in research is whether students in a two-way immersion program who are learning mathematics in their second language are at a disadvantage when compared with students who are learning the same material in their first language.

Secada (1991) studied bilingualism and its effect on problem-solving ability. There has been research such as that by Aiken (1971) linking Hispanic students' underachievement in mathematics to limitations in their English proficiency. However, Fernandez and Nielsen (1986) found that Hispanic and white bilingual high school seniors outperformed Hispanic and white monolingual students in general mathematics achievement, even after adjusting for a broad range of social and demographic variables. Secada examined whether there was a relationship between competence in a given language and the ability to

solve arithmetic problems in that language. In this study forty-five Hispanic first graders in four classes at two schools in Chicago were given ten story problems to solve in both English and Spanish. English language proficiency was also determined using the Language Assessment Scales, preschool version. Secada found that children, despite their differing levels of language proficiency, tended to be consistent in their ability to solve problems across their languages. Children who could solve problems in one language tended to be able to do so in their other language, and if they could not solve problems in one language, then they could not in the other. The Hispanic bilingual children's arithmetic problem solving was similar to that of their monolingual peers. There was limited support for the claim that degree of bilingualism is related to enhanced problem solving.

As one can see there is no clear consensus as to whether being bilingual helps a student with mathematical problem solving or puts the student at a disadvantage. There is also disagreement as to whether students do better if mathematical problem solving is taught in their primary language versus in their second language if the grammatical structures and vocabulary in the problems are familiar.

Research Involving Other Factors

Besides semantic structure and linguistic factors, researchers have found the context in which a problem is presented and time allowed to solve a problem to be significant in mathematical problem solving. Davis-Dorsey, Ross, and Morrison (1991) examined what the impact of personalizing mathematical word problems and rewording them for explicitness was with second grade and fifth grade students. Eight forms of a 16 item problem-solving instrument was designed for each grade. Each form contained four instances of four problem types (2 types of change problems, a combine problems, and a compare problem). Each of the 16 problems on a given form was represented as one of four wording-context variations: standard wording-standard context, standard

wording-personalized context, rewording-standard context, rewording-personalized context. All problems were rotated through each of these four variations on alternate forms. When problems were reworded, it increased the explicitness of the relationship between the numbers in the problem. Context personalization was accomplished by changing the names of problem protagonists and supporting referents in standard-context problems to include parallel information that had been gleaned from biographical questionnaires that had been filled out by the students. The results of this study showed that younger, less experienced problem solvers (the second-graders) performed best when personalization and rewording were used in combination. That is personalization was not beneficial on standard-wording problems, whereas rewording was not beneficial on standard-context problems. By comparison, older students (the fifth graders) benefited from personalization on both reworded and (especially) standard problems, but not from rewording for either context. One of the implications of this study for older multicultural students is that they may do better at solving problems that reflect their lives and culture.

Lopez and Sullivan (1992) studied the effect of personalization with a group of 123 seventh-grade Mexican-American students. A biographical inventory was given to the subjects to determine personal background and interests. The subjects were divided into three roughly equal groups to receive two 55-minutes periods of instruction on division of whole numbers. One group received instruction using non-personalized worksheets, one group received individually personalized worksheets, and the third group received worksheets that reflected the most commonly occurring first name for all students, the brand name of the soda listed by the most students as their favorite, etc. The research revealed a significant effect for personalization on student achievement on two-step problems, the more difficult problems in the instructional unit. Students in the two personalized treatments also scored higher on one-step problems than

students in the non-personalized treatment, although not significantly so. Understanding of the problem situation is critical for successful problem solving in mathematics. Personalized problems may contribute to greater understanding, because the places and events in the problems are more familiar. This research also found that subjects under individualized personalization had significantly more positive attitudes than those under the other two treatments.

Jordan and Montani (1997) studied problem-solving success in third-grade special education students who only had specific mathematics disabilities and those who had more general academic weaknesses. A control group of students without learning disabilities was also studied. Each group of students was given a set of story problems and a set of number-fact problems in each of two conditions: timed and untimed. All the problems were read aloud. Results showed that students who had specific mathematics disabilities performed worse than the control group on the timed test but not on the untimed test, while the students with more general academic weaknesses performed more poorly in both conditions. The difference between the timed and untimed conditions resulted from the use of backup strategies. Both groups of special education students used backup strategies such as finger counting more than the control group. The difference between the two special education groups was that the group with a specific mathematical disability was more likely to use the backup strategy accurately allowing them to perform as well as the control group in an untimed situation, while the special education students with more general academic weaknesses executed the backup strategies less skillfully. The significance of this study is that the special education students correctly perceived the semantic structure of the problem. They just needed more time or more accurate use of backup strategies to solve the problem.

As one can see there has been considerable research on factors that affect success in problem solving. Language seems to play a crucial role in a

student's ability to solve word problems in math as does a student's knowledge of the semantic structure or schema of a problem. Some research seems to indicate that language proficiency for second-language learners is a critical factor in whether those students are successful in learning math in the second language. There is other research that supports the idea that if students are able to solve problems in their first language, they are able to solve problems in their second language. The present study, by using one-step problems without extraneous information and vocabulary and contexts familiar to the students, hopes to determine whether the semantic structures learned in one language will transfer to problems presented in another language when students are fluent in both languages. This study also hopes to determine whether there is a difference between learning those semantic structures in one's primary language or learning them in a second language and then using them.

CHAPTER III

Methodology

This study examined whether sixth grade students in a special magnet program in an urban area in California were able to identify given word problems as subtraction or division based on the underlying mathematical structures or schema of the problems. The schema was represented in the semantic structures of the problems. It also asked whether there was a difference between the performance of students who started the program in kindergarten as English-dominant and those students who started the program as Spanish-dominant.

The study consisted of two tasks, both of which examined whether students used the schema of the problems to identify the underlying mathematical structure of the problems. The first involved sorting twelve word problems into piles according to how the students saw the problems as being similar. The second task asked the students to solve five word problems and then explain how they solved the problems. The examiner recorded the students' responses and analyzed them as to whether the students referred to the schema of the problems in their explanations.

Participants

The participants in this study were sixth grade students at a Spanish Two-way Immersion magnet school in an urban area in California. The students had attended the program since kindergarten and entered the program with a score of 4 or 5 on the pre-LAS in their dominant language and a score of 1 or 2 on the pre-LAS in their second language. The students are now considered to be orally fluent in English, having scored a 4 or 5 on the LAS in English. The pre-LAS is a oral language test given to students before they enter kindergarten or during their kindergarten year. The Language Assessment Scales (LAS) is the test given to older students to test their oral fluency in a language. A score of 4 or 5 on the

pre-LAS or the LAS is considered to be a score that indicates fluency in the language being tested. In the sixth grade there were a total of 57 students. Thirty-five of these students had been enrolled in the program since kindergarten and had entered with a score of 4 or 5 in their dominant language and with a score of 1 or 2 in their non-dominant language.

All sixth grade students at the school were given a basic math computations test during the second week of school. All the participants in this study were able to pass the section involving adding and subtracting two digit numbers with regrouping and the section that involved multiplying and dividing by one digit with at least 80% accuracy. These two sections included ten problems.

A random sample of six students of those students who met the above criteria and started school as English-dominant was chosen as well as a random sample of 6 students who met the above criteria and started school as Spanish-dominant. Four of the students who started school as Spanish-dominant were female and two were male. Three of the students who started school as English-dominant were female and three were male. One student was in the Resource Specialist Program as well as received Speech and Language Services. No other students received special education services.

Task 1 and Task 2

This study consisted of two tasks. The first task involved the students being given twelve cards with word problems typed on them and asked to sort them. They were then asked to explain why they sorted them in the way that they did. The problems consisted of six subtraction problems and six division problems. As was detailed in the literature, certain problem features can affect student performance in problem solving. For example, the presence or absence of extraneous information and extra steps (Muth, 1992) interfere with a students' ability to solve word problems. Lopez and Sullivan (1992) found that familiar names and situations lead to better problem solving. Care was taken to design

the questions to eliminate extraneous factors that might affect how students sorted the problems. The problems were all single step problems and all used one large number with two or three digits and a one digit number. The larger number in each problem could be evenly divided by the smaller number. The problems were matched so that if there was a division problem that mentioned cassette tapes there was a subtraction problem that mentioned cassette tapes, etc. The problems all used names that were familiar to the students with an equal number of male and female names. One division problem and one subtraction problem used the pronoun I. All the questions in the problems were put at the end of the problem.

Different types of mathematical structures or schema were used in the different subtraction and division problems. In the subtraction questions there were two that used the combine schema, two that used the change schema, and two that used the compare schema. In the division questions there were three that were partition questions and three that were quotient questions. Two of the partition questions involved finding rate. The questions were randomly ordered and printed on cards. A list of the questions used is in Appendix A.

On Task 2 students were asked to solve five word problems. The problems were printed on cards and presented to the students one at a time. The students were given a sheet of paper to use to solve the problems. The presentation of the questions was randomly ordered. After solving the problems the students were instructed to explain how they solved the problems as though they were talking to a second grader. The problems in Task 2 consisted of three subtraction questions and two division questions. The three subtraction problems included one example each of change, combine, and compare problems, and the division problems had one example each of partition and quotient problems. As in Task 1 all the problems involved one larger number and one smaller number. The problems used are listed in Appendix B.

Procedure

1. Students were randomly selected using the criteria outlined in the participants section.
2. A parent permission slip was sent home, explaining the study and asking the parents if their child could participate.
3. Students were individually asked to sort the cards that were part of Task 1 into piles according to any similarities that were seen. The examiner used the following direction: "Please sort the cards into piles according to how they are alike." The cards were read to the students if necessary. The examiner recorded which cards were put together.
4. The students were then asked to explain how they sorted the cards. Their responses were tape recorded. If a student's response was unclear, clarifying questions were asked, such as "You said you put these problems in the same pile because they had the same things in them. What do you mean by 'the same things'?"
5. If the students correctly sorted the problems into those that required division and those that required subtraction, they were asked to sort the subtraction cards again into smaller piles according to how they were similar and then the division cards into smaller piles according to how they were similar. The students' responses were recorded by the examiner.
6. The students were then asked to explain how they sorted the cards. Their responses were tape recorded. If a student's response was unclear, clarifying questions were asked.
7. On Task 2 students were shown one card at a time and asked to solve the problem. They were given a sheet of graph paper to use for their calculations. After they solved the problem they were asked to explain how they solved the problem as though they were explaining it to a second grader. Their responses were tape recorded.

8. If needed, clarifying questions were asked the student such as

- * How did you know that it was division (or subtractions, etc.)?
- * Why did you do the problem in the way that you did it?

9. After the student was done explaining how he or she did the problem, the following questions were asked:

- * Was there any part of the problem that gave you difficulty?
- * Were there any words or phrases that you didn't understand?

10. Students were thanked for their participation and a pen such as is often used in school was offered to them for their help.

Data Analysis

After completing the interviews, the data was analyzed in a variety of ways. First, how the students sorted the cards on Task 1 was examined. The categories into which the students sorted the problems, the number of groups which were made, which problems were placed together, and the labels which the students gave their categories were noted. The results were compiled into tables. The results were then evaluated to determine if the students seemed to be using the schema of the problems to help them determine into which groups the cards should be sorted.

The researcher then examined the students' performance on Task 2. The first question asked was whether the problems in Task 2 were solved correctly and if they weren't, was the correct operation used in the attempt to solve the problem. The researcher then looked at how the students solved the problems and how they explained their solutions. It was noted whether the students seemed to use the schema of the problems when explaining the problems or whether they relied on some other method of solution.

The students' performance and statements on Task 1 and Task 2 were then compared. It was noted whether the same students did well on Task 1 and Task 2 or were there some students that did well on one of the tasks, but not the

other. Their explanations of how they completed the two tasks were analyzed and compared. Did the students refer to the schema of the problems in both tasks, one of them, or neither of them? What methods did the students use when sorting the cards and solving the problems? Were the methods similar?

Finally, the students' performance on Task 1 and Task 2 was looked at in relation to whether they started the magnet program as English-dominant or Spanish-dominant. How many cards in Task 1 were sorted correctly into categories indicating subtraction and division was tabulated, and the mean scores and standard deviations were determined for students who were English-dominant and students who were Spanish-dominant. The scores were then compared and interpreted. Because there seemed to be a difference in the performance of the female students in comparison to the performance of the male students, the mean scores and standard deviations of these two groups were compared and interpreted as well.

CHAPTER IV

Results

Introduction

Many middle school students have problems with mathematical word problem solving. One of the difficulties that they have is understanding what the problem is asking them to do. Researchers (Riley, Greeno, & Heller, 1984; Peled & Wittrock, 1990) have studied the extent to which the semantic structure of a problem (and the schema which those semantic structures represent) influence how students solve mathematical word problems. The study presented here is asking whether sixth grade students at an urban K - 8 school can interpret the semantic structures of word problems and use their knowledge of the mathematical structure to sort word problems into categories. Another question being asked was whether there is a significant difference in students' ability to use the semantic structures in English. Students who had English as their dominant language when entering kindergarten and those whose dominant language was Spanish were compared. The mathematics instruction of all the students in the study was in Spanish from kindergarten through fifth grade.

This study consisted of two parts. In the first part the students were asked to sort twelve mathematical word problems into piles according to any student perceived similarities. The students were then asked to explain the categories into which the cards were placed. The word problems consisted of six subtraction problems and six division problems. The subtraction problems consisted of two change problems, two combine problems, and two compare problems (Cummins et al, 1988). The six division problems consisted of three quotient problems and three partition problems (Bell, Fischbein, & Greer, 1984). Two of the partition problem involved finding rate.

In the second part of the study the students were asked to solve five mathematical word problems and then to explain their method of solution. Three of the problems were subtraction with one example each of the subtypes of change problems, combine problems, and compare problems. The remaining two problems were division problems, one of the partition subtype and one of the quotition subtype. None of the problems involved finding rate.

Student Performance on Task 1

Task 1 involved the students sorting the 12 word problems listed in Appendix A into groups. Student sorting categorizations were analyzed according to general patterns in both student perceived dimensions and in mathematical structures. Results are reported in relation to patterns found across the 12 students and to specific patterns found in individual responses. Significant trends are highlighted.

In Task 1 students described their categories in a variety of ways. As one can see on Table 1 (on the following page), students divided the word problems into two to six groups. Some students labeled their groups by mathematical structures such as subtraction, division, multiplication, or addition. Other students referenced the context of the problem, using such labels as money, candy, or phrases such as how much more. Some students sorted the division and subtraction problems separately while other students mixed the two types of problems.

Table 2 shows the totals of those piles that were sorted correctly according to mathematical structure. The number of groups that were made by each student is shown. The number of problems that were actually labeled division and subtraction is indicated. Some students labeled the subtraction and division problems by their inverse operations, addition and multiplication, and these groupings are also shown.

Five students were able to sort the cards in Task 1 correctly, and these

Table 1: Labels Given Groups - Task 1

Subject #	# of groups	Group label	Problems	Types of problems
1	5	1. Division	B I H D J L	2 partition div., 1 quotient div., 2 combine sub., 1 change sub.
		2. Multiplication	C K	1 compare sub., 1 partition div.
		3. How fewer	G	1 compare sub.
		4. Estimation	E	1 change sub.
		5. Div. - not money	A F	2 quotient div.
2	2	1. Subtraction	J G D C L E	all six subtraction problems
		2. Division	K H B F A I	all six division problems
3	5	1. Difference	A L C J E	1 combine sub., 2 change sub., 1 compare sub., 1 quotient div.
		2. Times	F H	2 quotient div.
		3. Division - money	I B D	2 partition div. (rate problems), 1 combine sub.
		4. Minus	G	1 compare sub.
		5. Division	K	1 partition div.
4	3	1. Dividing	F J L A K C B	3 partition div., 2 quotient div., 2 combine sub., 1 change sub., 1 compare sub.
		2. Adding	H	1 quotient div.
		3. More or less	E G	1 change sub., 1 compare sub.

Table 1 (continued): Labels Given to Groups - Task 1

Subject #	# of groups	Group label	Problems	Types of problems
5	3	1. Minus	C J L D E G	2 combine sub., 2 change sub., 2 compare sub.
		2. Dividing	A K B	2 partition div., 1 quotient div.
		3. Timesing	H I F	1 partition div., 2 quotient div.
6	3	1. How much more	J B G H	1 change sub., 1 compare sub., 1 partition div., 1 quotient div.
		2. How much someone can make	I D K F	1 combine sub., 2 partition div., 1 quotient div.
		3. How to divide them into a certain amount	E L C A	1 combine sub., 1 change sub., 1 compare sub., 1 quotient div.
7	6	1. Subtraction	J E	2 change sub.
		2. Division	F H	2 quotient div.
		3. Other division	K A	1 partition div., 1 quotient div.
		4. Multiplication	I B	2 partition div. (both rate probl.)
		5. Addition	C G	2 compare sub.
		6. Other subtraction	D L	2 combine sub.
8	4	1. Division	A E K F	1 partition div., 2 quotient div., 1 change sub.
		2. How much money	I B D	2 partition div. (both rate probl.) 1 combine sub.
		3. How many more	G J H	1 change sub., 1 compare sub., 1 quotient div.
		4. How much to make the number in total	C L	1 combine sub., 1 compare sub.

Table 1 (continued): Labels Given Groups - Task 1

Subject #	# of groups	Group label	Problems	Types of problems
9	4	1. Seeds	G H	1 compare sub., 1 quotient div.
		2. Cassettes	J B	1 change sub., 1 partition div.
		3. Take away	I D	1 combine sub., 1 partition div.
		4. Candy	K A L E F C	1 partition div., 2 quotient div., 1 combine sub., 1 change sub., 1 compare sub.
10	3	1. Division	K F B I A	3 partition div., 2 quotient div.
		2. Subtracting	D C J L G E	2 combine sub., 2 change sub., 2 compare sub.
		3. Multiplication	H	1 quotient div.
11	2	1. Subtraction	D L E C G J	2 change sub., 2 combine sub., 2 compare sub.
		2. Division	B K F H A I	3 partition div., 3 quotient div.
12	3	1. Take away	E D C G	1 combine sub., 1 change sub., 2 compare sub.
		2. Dividing	J F I B	2 partition div., 1 quotient div., 1 change sub.
		3. Take away - how many would he put in each thing	A K H L	1 partition div., 2 quotient div., 1 combine sub.

Table 2: Sorting by Mathematical Structure - Task 1

Subject	# of groups	Called Sub.*	Sub. called	Called Division**	Division called	Comments:
1	5	none	addition	5/6	multiplication 1(K)	3 sub. prob. labeled as div.(D,J,L), 3 sub. prob. labeled by words in problems or other
2+	2	6/6		6/6		
3	5	5/6		3/6	2 (F, H)	1 div. prob. (A) called sub. 1 sub. prob. (D) called div.
4	3	none		5/6		4 sub. prob. (C,D,J,L) called div. 2 sub. prob. labeled by words in problems, 1 div. (H) called adding
5+	3	6/6		3/6	3 (F, H, I)	
6	3	none		1/6		2 groups labeled by words in problem, 3 sub. prob. (C, E, L) called division

+ those students who sorted the cards completely correctly
 * also minus, take away, *finding the difference*; Combine (D, L), Change (E,J), Compare (C,G)
 ** Partition (B, I, K), Quotition (A, F, H)

Table 2 (continued): Sorting by Mathematical Structure - Task 1

Subject	# of groups	Called Sub.*	Sub. called	Called Division**	Division called	Comments:
7+	6	4/6	addition 2 (C, G)	4/6	2 (B, I)	3 sub. groups corresponded to subtypes, 2 div. problems that were called mult. were rate prob.
8	4	none		3/6		3 div. prob. (B, H, I) and 5 sub. prob. (C, D, G, J, L) were labeled by words in prob., 1 sub. prob. (E) called division
9	4	1/6		none		1 div. prob. (I) called sub. most prob. labeled by words in problems
10+	3	6/6		5/6	1 (H)	
11+	2	6/6		6/6		
12	3	4/6		6/6		2 sub. prob. (J, L) called div.

+ those students who sorted the cards completely correctly
 * also minus, take away, *finding the difference* ; Combine (D, L), Change (E, J), Compare (C, G)
 ** Partition (B, I, K), Quotition (A, F, H)

five students were identified by a plus (+) on Table 2. As can be seen from the table, variations exist across correctly sorted piles. These variations were a result of the way in which students subdivided across and within the operations. Two of the students (#2 and #11) made two piles of the cards in Task 1 and called those piles subtraction and division. Two students (#5 and #10) sorted the cards into one pile labeled subtraction or minus, but divided the division problems into those that were mostly easily solved by division and those that were solved by using multiplication. Those problems that were seen as being more easily solved by division involved larger numbers than those that were seen as being most easily solved by multiplication. The remaining student (#7) also divided the division problems into those most easily solved by division and those most easily solved by multiplication. However, she also put the two subtraction problems that compare two quantities into a pile that she said would most easily be solved by addition or adding on. This student (#7) was the only student who matched the groups she made of subtraction problems into the different classes of subtraction problems. Two of the groups she made of the division problems also matched the different types of division problems. She made a third group of division problems which combined one problem of each type. The two division problems which she said were most easily solved by multiplication were of the partition type and involved finding the rate.

Errors in sorting reflect student connections and confusions among subtraction, division, multiplication, and division. Two students (#3 and #12) were able to sort some of the cards in Task 1 into piles labeled subtraction and division. However, they also mislabeled some problems. Subject #3 accurately identified 5 of the subtraction cards in Task 1 as subtraction. He labeled three of the division cards as division and stated that two other division problems needed to be solved by multiplying. He mislabeled one division problem as subtraction and one subtraction problem as division. Subject #12 had one group which she

called take away. She put 4 subtraction problems into this group, including the two compare problems, one change problem, and one combine problem. She then had another group which she also called take away, but which she went on to explain was really more like how many piles the person could make. Two of the division problems in this pile were of the quotient type and one was of the partition type. The subtraction problem which she put in this pile was of the combine type. Her remaining pile she called division, placing three division problems in this pile and one subtraction problem of the change type.

Three students (#1, #4, and #8) were able to sort some of the division problems correctly, but none of the subtraction problems were sorted into a category that was labeled subtraction. Subject #1 sorted the cards into five groups. She labeled two of the groups division, one involving money and one which did not involve money. The division group with money had six cards in it. Three of these problems involved money and three did not. There were three subtraction problems in this group and three division problems. Both problems using the subtraction change schema were included in this group. The division group that did not include money correctly included two division problems. She also made a group which she called multiplication which included one subtraction problem and one division problem. The other two groups contained one card each and were labeled "involving estimation" and "involving finding some fewer." She did not identify any of the problems as subtraction.

Subject #4 sorted most of the cards into a large pile that he called division. It included 5 division problems and four subtraction problems. When asked why he put most of the cards into this pile, he just said that they were all alike and that they were division. He placed one division problem in a pile which he called adding and two subtraction problems in a pile that were those that "were how many more or how many less a person has." Only one of these problems (G) was a compare problem. The other problem (E) was a change problem.

Subject #8 sorted the cards into four groups. One group was called division and included three division problems and one subtraction problem. The remaining three division problems and five subtraction problems were sorted into groups that were labeled by the words contained within the problems such as “how many more” or that they involved money.

Two subjects (#6 and #9) sorted the cards by similarities in the wording on the cards. Subject #6 sorted the cards by similarities which she saw in the wording of the questions. She made three groups. One of the groups contained equal numbers of division and subtraction problems. One group had more subtraction problems and one group had more division problems, but no group was labeled by the mathematical process used to solve the problems. Subject #9 sorted 10 of the cards by categories based on items listed in the problems such as candy, seeds, and cassettes. One group he labeled “take away” but included one subtraction and one division problem. Subject #9 is a Spanish-dominant student who is in the Resource Specialist Program and receives Speech and Language services. He was the only student who asked to have the cards read to him.

Interpretation of Results on Task 1

What was being looked for in Task 1 was whether students could use the semantic structure or schema of a problem to determine the underlying mathematical structure and thus sort the problems by the operations needed to solve them. Five students were able to sort the cards correctly by operation. They appeared to be able to use the semantic structure of the problems to help them sort the cards into groups. Subject #2 explained it this way:

Researcher: So how did you sort them?

Subject #2: I think this is subtracting. He had 56 candies and he gave away 8 so you take 8 away from 56 and all these are the same. You subtract. And these well you have to divide. Seven girls paid \$42 to the movies, so you

have to divide to see how much each girl paid. These are all like that.

Subject #7 put the subtraction problems into three different piles which corresponded to the subtraction subtypes of compare, combine, and change. She labeled the compare problems addition, indicating that she would find the answer by looking for the missing addend. When asked why she made the different groups of subtraction, she indicated that she didn't really know.

Subject #7: This one (pause) this one you have to use subtraction.

Researcher: And are there any ways that these subtraction differ from your other subtraction group? Is there any reason you didn't put them in the same pile?

Subject #7: I just wasn't thinking that they could go in the same pile.

Subject #7 also made different groups of division that corresponded to the different types of division problems. She made two piles which she labeled division, and then she designated the two rate problems as problems that could most easily be solved by multiplication. She was unable to explain why she made two different division piles.

Researcher: Why did you put these division in a different pile than these other divisions?

Subject #7: I did think (pause) I didn't think about that.

(subject shakes head and indicates that she didn't really know why she made two different piles and goes on to indicate the third pile)

Subject #7: These two here are a lot of different ways you could solve this one, but I would solve it with multiplication (pause) because they're easy.

Researcher: And how did you solve them with multiplication?

Subject #7: Well, right here (showing card) you would just (pause) it's almost like division, but you would have to see 7 times what is 42.

When the other students were asked to sort the subtraction and division into smaller piles or other piles, none of them could see other divisions that could

be made. None of the other students recognized the subtraction and division subtypes.

Two students (#3 and #12) were able to sort most of the cards into piles that indicated a knowledge of their mathematical structure. Subject #3 put one division problem in with the subtraction and one subtraction problem in with the division, but other than that he was able to sort the problems correctly. Subject #12 made one subtraction pile which included four subtraction problems and two division piles which included six division problems and two subtraction problems. One of the division piles she called division. The other pile she wasn't sure what to call it. She initially said that it was take-away, but that it was different from the other take-aways. She explained the difference like this:

Subject #12: Well, these are like how many he would have left (pointing to first pile which she labeled take away) and these are like how many he would put in each thing (pointing to second pile).

Subject #12 seems to see the second pile as division that involves repeated subtraction and thus similar to take-aways. She seems to be able to use the semantic structure of the problems to help her recognize the underlying mathematical structures of the problems.

Not all of the students were able to identify the mathematical structures of the problems by using the semantic structures.. Five students (#1, #4, #6, #8, and #9) had considerable difficulty sorting the problems. Four of these students (#1, #6, #8, and #9) made groupings of the cards in Task 1 that were based on words in the cards that were not necessarily significant for solving the problems. Subjects #1 and #8 made piles according to whether the problems involved money or not. Subject #6 made piles according to whether the problem asked how much or how much more. Subject #9 made piles according to whether the problems discussed seeds, candy, or cassettes. Subject #4 saw most of the problems as division. He made one pile that he said involved asking how many

more or how many fewer a person had; however, that pile had one problem (G) which asked “how many fewer,” but the other problem (E) was a simple change problem that just asked “how many does he have left.” From their performance on Task 1, it might be surmised that these five students have trouble using the semantic structure or schema in a problem to recognize the underlying mathematical operations; however, when we look at their performance on Task 2, we’ll see that’s not the case for all of these students.

Student Performance on Task 2

Task 2 involved the students solving five word problems and then explaining how they solved them. As can be seen on Table 3, nine of the students were able complete Task 2 accurately. Three students (#1, #3, and #6) completed three of the problems in Task 2 accurately but missed the other two. Subject #1 divided on four of the problems and subtracted on one. Her computations were accurate. Subject #3 chose the correct operation needed to solve the problems, but made mistakes in calculations, stating that 54 take away 9 is 43 and that 108 divided by 9 was 19. He was able to pass the computation test given to all sixth graders with 90% accuracy; so it is unclear why he had problems with computation on Task 2. Subject #6 missed one division problem because she multiplied and one subtraction problem because when she saw two girls, she doubled the amount that needed to be subtracted.

Three of the students who correctly solved all the problems on Task 2 made use of alternative procedures in solving the problems. Subject #4 used his fingers to solve some of the calculations. Subject #5 wrote the multiplication tables to help her solve the division problems. Subject #12 drew pictures to solve the quotient division problem and tried to draw a picture to solve the partition division problem.

Interpretation of Results on Task 2

Nine out of the twelve students were able to complete the problems in

Table 3: Problems Correct in Task 2

Subject	Subtraction Correct	Division Correct	Total Correct	Comments:
1	1/3	2/2	3/5	Divided on 4 problems, subtracted on one
2	3/3	2/2	5/5	
3	2/3	1/2	3/5	chose correct operation, but made mistakes in calculations
4	3/3	2/2	5/5	needed to count on fingers to solve problems
5	3/3	2/2	5/5	solved division problems by writing multiplication tables
6	2/3	1/2	3/5	multiplied instead of divided on 1 division problem
7	3/3	2/2	5/5	
8	3/3	2/2	5/5	
9	3/3	2/2	5/5	
10	3/3	2/2	5/5	
11	3/3	2/2	5/5	
12	3/3	2/2	5/5	drew pictures for quotient division, tried to draw a picture for partition division, but couldn't

Task 2 correctly; however, their manner of solving the problems and their explanations about how they solved the problems indicate differences in their ability to use the semantic structures of the problems to help them determine which operation they should use to solve the problems.

Nine of the students (#2, #3, #4, #5, #7, #8, #10, #11, and #12) referred to the wording in the problems or the semantic structure of the problem as helping them determine how to solve the problem when they were making their explanations. An example that subject #11 gave is to problem 2C (see appendix):

Researcher: Could you explain how you might solve this problem to a second grader? How did you solve it?

Subject #11: In the question it says that Miguel has 56 pictures so I wrote 56, and then it says that he threw 8 of them away. So I wrote the 8 under the 56, and then it says that he threw them away so he got rid of them. So that's why it's subtraction. Then you do the problem.

Subject #7 explained how she knew to divide in question 2B this way:

Researcher: And how did you know it was division?

Subject #7: I knew it was division because when you read the question (pause) it's just the way they have it set up (pause) they could have it set up a different way and it would have to be something else.

Subject #10 answered the question about how she knew to divide in question 2B in this way:

Researcher: How did you know it was division?

Subject #10: You put 9 books in each pile and there are 108 books. It says, "How many piles will she make?" So that gives me the clue. She wants to put them in piles of 9 and there are 108 books. So you divide.

When Subject #7 and Subject #10 said that it was "the way they have it set up" or "that gives me the clue," it's an indication that they are recognizing the semantic structures of the problems as giving them the information they need to

determine which operation to use and to solve the problems correctly.

Not all of the subjects explained in detail that the words in the problem gave them the information that they needed to determine how they were going to solve the problem; however, they were able to use the semantic structure of the problem to determine how they should solve it. Here's an example of an explanation given by Subject #4 to question 2A:

Researcher: Could you explain how you might solve this problem to a second grader? How did you solve it?

Subject #4: Well, I divided 8 into 72.

Researcher: And why did you do that?

Subject #4: 'Cause it says here that he gave an equal number of marbles to each of his friends, so I knew that I had to divide 72 into 8.

It should also be noted that several of the subjects made a mistake in expressing the order of operations as Subject #4 did above ("divide 72 into 8"); however, no pattern could be seen in when the subjects made this mistake or in which subjects had a tendency to do it.

Subjects #3 also referred to schema in his explanations but was only able to solve three of the problems in Task 2 correctly. However, his mistakes were in computation. He set the problems up using the correct operation. He was able to pass the computation test given to all sixth graders with 90% accuracy; so it is unclear why he had problems with computation on Task 2. Here is the explanation that he gave to question 2B which indicates that he understands the semantic structure of the problem even though his answer is incorrect:

Subject #3: Okay, there were a 108 books and she has to put all of those books into (pause) she needs to get nine books in a pile and she needs to keep doing that and she has 108 books. She must divide 9 into 108 to see how many piles of books that she'll have in total.

Researcher: And what was the answer that you came up with?

Subject #3: 19

Subject # 12 solved all the problems in Task 2 correctly and was able to use the semantic structures in the problems to help her decide which operation to use; however, her processes differed between subtraction and division. Here is the explanation that Subject #12 gave for question 2E:

Researcher: Could you explain how you might solve this problem to a second grader? How did you solve it?

Subject #12: It says that two girls paid \$42 for CD's. One girl paid \$ 7. How much did the other girl pay? So they paid \$42 and one girl paid \$ 7. The other girl paid 35.

Researcher: How did you know it was subtraction?

Subject #12: Because it says here, how much did the other girl pay? That's why the problem is subtraction.

Division gave Subject #12 more problems. She understood what the problem was saying, but she needed to draw pictures to help her find the answers. A copy of her answer sheet is reproduced on the following page. Here is the explanation she gave for how she solved question 2B:

Researcher: Could you explain how you might solve this problem to a second grader? How did you solve it?

Subject #12: It says here that she had 108 books and I had to make 9 piles. I take 12 piles and I counted them or times them and I got 108. (subject drew groups of 9 lines)

Researcher: What I saw you do was count out the sticks. How many did you first make?

Subject #12: I made 11.

Researcher: Why did you make 11 at first?

Subject #12: First I was just putting the little sticks and then I started counting them.

Worksheet with Drawing Used by Subject # 12 - Task 2

Subject #12 was able to make a drawing to illustrate division as repeated subtraction, but she was not able to draw the partition division problem 2A. She ended up just dividing, because she recognized the problem as division even though she was unable to make a picture to represent it.

Subjects #6 referred to schema when she explained how she solved the subtraction problems, but not when she was explaining how she solved the division problems. Subject #6 got only 3 of the problems in Task 2 correct. She multiplied on one division problem and doubled what she was supposed to subtract on one combine subtraction problem. Here is an example of the explanation that subject #6 gave for problem 2C:

Researcher: Could you explain how you might solve this problem to a second grader? How did you solve it?

Subject #6: By subtracting 8 from 56, because it says....Can I read it?

Researcher: Sure

Subject #6: Miguel has 56 pictures. He throws 8 of them away. How many does he have left?

Researcher: What lets you know it was subtraction?

Subject #6: Because it says how many does he have left.

In contrast here is the explanation that subject #6 gave to how question 2B was solved:

Researcher: Could you explain how you might solve this problem to a second grader? How did you solve it?

Subject #6 By multiplying the number of books to....the number of books in a pile.

Researcher: And why did you do that?

Subject #6: To get the answer of how many piles she should make.

Subject #1 also was not able to solve two of the problems in Task 2. She divided on two of the subtraction problems. Her explanations of how she solved

the problems basically amounted to her stating what operation she used and then walking the researcher through the steps to do the operation. She did not refer to the wording of the problem at all as an aid in helping her decide how she was going to solve the problem. None of her explanations indicated that she was able to use the semantic structure or schema of the problem to help her solve the problems.

Subject #9 was able to complete all the problems in Task 2 correctly, but did not seem to use the semantic structures of the problems to help him. He looked at what the answer should be to determine which operation to use rather than using the schema in the problem. Here is the explanation that he gave for question A:

Researcher: Could you explain how you might solve this problem to a second grader? How did you solve it?

Subject #9: I divided....8 goes into 72 and 1 times and then I put 72 underneath and then I'm done.

Researcher: How did you decide that you needed to divide rather than subtract?

Subject #9: Because if I subtract I would get 64.

Researcher: And what would be wrong with that?

Subject #9: He has 8 friends and he only has 72 marbles.

Researcher: So how did you know that you needed to divide to get the right answer.

Subject #9: Because he only has 72 marbles and 8 friends, so if I subtract he could only give one 64 and the rest wouldn't get (finished sentence in a mumbled voice)

Task 2 indicates that all but three of the subjects were able to use the semantic structure or schema of a problem to determine how they should solve it. Two of the students that did not use it (#1 and #9) entered school as Spanish-

dominant students. The other student (#6) entered school as English-dominant.

Comparison of Student Performance on Task 1 and Task 2

Sorting the 12 cards in Task 1 into those that required subtraction and those that required division proved to be a harder task for the students than correctly solving the five problems on Task 2. As one can see on Table 4, five of the students were able to sort the cards by subtraction and division accurately, while nine of the students were able to solve all the problems on Task 2 correctly. All of the five students (#2, #5, #7, #10, and #11) who were able to sort the cards correctly were able to complete all the problems in Task 2 accurately.

Three students were not able to solve all the problems on Task 2 correctly. Two of the students (#1 and #6) who missed two of the problems on Task 2 also had difficulties sorting the problems in Task 1. Subject #1 did not sort any of the subtraction problems correctly and missed two of the subtraction problems on Task 2. She was able to sort all of the division problems into piles labeled division and did not miss any of the division problems on Task 2. When she was sorting cards in Task 1, she put three subtraction problems into a pile labeled division and when she was solving the problems in Task 2, she tried to solve two subtraction problems by using division. Subject #6 had problems sorting both the subtraction and the division problems and made mistakes solving both subtraction and division problems on Task 2 because she did not conceptualize the problem accurately. The third student who had trouble solving the problems on Task 2 was Subject #3. Subject #3 was able to sort the cards on Task 1 with only two mistakes. He made mistakes on two of the problems in Task 2; however, his mistakes on Task 2 involved calculation errors rather than choosing the incorrect operation to solve the problem.

There were three students (#4, #8, and #9) who were able to solve all the problems on Task 2 correctly, but who had difficulties sorting the cards in Task 1. All three of these students included categories when sorting cards for Task 1 that

Table 4: Performance on Task 1 and Task 2

Subject	Sub. Problems		Div. Problems		Div. Problems		Total		Total Correct minus errors *
	Correct Task 2	Correct Task 1	Correct Task 2	Correct Task 1	Correct Task 2	Correct Task 1	Correct Task 2	Correct Task 1	
1	1/3	0/6	2/2	6/6	3/5	2/12			
2+	3/3	6/6	2/2	6/6	5/5	12/12			
3	2/3	5/6	1/2	5/6	3/5	8/12			
4	3/3	0/6	2/2	5/6	5/5	1/12			
5+	3/3	6/6	2/2	6/6	5/5	12/12			
6	2/3	0/6	1/2	1/6	3/5	0/12			
7+	3/3	6/6	2/2	6/6	5/5	12/12			
8	3/3	0/6	2/2	3/6	5/5	2/12			
9	3/3	1/6	2/2	0/6	5/5	0/12			
10+	3/3	6/6	2/2	6/6	5/5	12/12			
11+	3/3	6/6	2/2	6/6	5/5	12/12			
12	3/3	4/6	2/2	6/6	5/5	8/12			

+ - those students who sorted the cards in Task 1 completely correctly.
 * items that were included in a division or subtraction pile inappropriately
 are subtracted from the number that were correct.

were based on the wording of the problems rather than the mathematical structures of the problems.

Like Subjects #4, #8, and #9, Subject #12 was able to solve all the problems on Task 2 accurately. However, in contrast to Subjects #4, #8, and #9, she only made two mistakes when she sorted the cards in Task 1. She included two subtraction problems in two groups where the other problems were all division.

Interpretation of Student Performance on Task 1 and Task 2 Together

In general those students who were able to sort the cards well on Task 1 were able to solve the problems on Task 2 accurately. The only exception to this statement was Subject #3 who only wrongly sorted two of the cards on Task 1, but missed two of the problems on Task 2. However, Subject #3 chose the correct operation when trying to solve the problems on Task 2. He just made mistakes in calculation. He seems to understand the semantic structures of the problems.

Of the five students who did poorly on Task 1, two of them (#1 and #6) also did poorly on Task 2 indicating a poor grasp of the semantic structure or schema of the problems. The remaining three students (#4, #8, and #9) were able to complete all the problems on Task 2 correctly, but had trouble sorting the cards into groups indicating subtraction and division. Subjects #4 and #8 had no problems understanding what to do on Task 2. They were able to read the problem, write a number sentence that solved it, and do the computations accurately. When sorting the problems on Task 1, they both called most of the problems division, including in that category some subtraction problems. When asked to explain why they made the piles that they did, both subjects just said that the problems were alike and were division. These students performance on Task 2 indicated a good sense of what schema needed to be used to solve the problems. Their poor sorting ability might have therefore stemmed from poor

reading of the problems or poor memory of the information contained in the problems.

Range of Mathematical Understanding

How the students sorted and solved the problems in Task 1 and Task 2 indicated a range of mathematical understanding. They varied in their knowledge of the mathematical structure represented in the problems or their schema knowledge. There were differences in the students' ability to categorize the problems and then to subcategorize them. They varied in their ability to make connections across operations. In this section I am going to discuss three students (Subjects #7, #9, and #12) who represent three different levels of mathematical understanding. Subject #7 represents the most sophisticated level of understanding and will be discussed first. Subject #9 represents the least sophisticated understanding and will be discussed last.

The performance of Subject #7 on Task 1 and Task 2 indicated a deep understanding of the semantic and underlying mathematical structures of the problems. Subject #7 was one of the subjects that sorted all the division and subtraction problems correctly. She also completed all the problems in Task 2 accurately. Subject #7 was the only student who sorted the subtraction problems in Task 1 into the different categories of subtraction - change, combine, and compare. When asked to explain why she sorted the cards into different piles labeled subtraction, she made the following comment:

Researcher: Are there any way that these subtraction differ from your other subtraction group? Is there any reason you didn't put them in the same pile?

Subject #7: I just wasn't thinking that they could go in the same pile.

The two change subtraction problems and the two combine subtraction problems were called subtraction. She called the two compare subtraction problems addition, indicating that they could be solved by adding on. Neither of these

problems would normally be considered missing addend problems. It could be that she is used to solving compare subtraction problems as missing addend problems so that when she saw the words *more* and *fewer*, it reminded her of other problems where she solved compare subtraction problems by adding on.

Similarly, Subject #7 made different piles for division, including one pile that separated out the problems that involved finding rate, but she could not explain her reasoning. Although she is able to use the differing semantic structures of the cards to sort them into different categories, she does not have the vocabulary or experience to explain her reasoning. However, her understanding of the semantic differences in the wording of the cards is fairly advanced.

Subject #7's understanding of multiplication as the inverse operation of division is also clearly evident.

Researcher: And how did you solve them with multiplication?

Subject #7: Well, right here (showing card with problem 1) you would just (pause) it's almost like division, but you would have to see 7 times what is 42.

Subject #7 also used multiplication to solve another division problem, 2A from Task 2:

Researcher: Could you explain how you might solve this problem to a second grader? How did you solve it?

Subject #7: It says that Juan has 72 marbles. He wants to give an equal number of marbles to each of his 8 friends, and the way you would solve this problem is by long division. You have to see how many times that 8 goes into 72 which means you have to use multiplication (pause) as 9 times 8 is 72 because it goes in 9 times.

Subject #7 did not seem to have any problems sorting or solving the problems in the present study. When asked if any part of the problems gave her difficulty, she responded that she sometimes had to read the problems over

again so that she would know what to do; otherwise she had no problems.

While Subject # 7 had a good conceptualization of the basic mathematical structures based on the semantic structures of the word problems, Subject #12 had an intuitive grasp of the semantic structures in the problems, but her understanding was not as deep as Subject # 7's. On the sorting problems she could not subdivide the subtraction problems into categories, and her sorting of the division problems mixed the two categories. By labeling one pile of division problems as take away, but saying that they were also division, she indicated that she had an understanding of division as repeated subtraction. However, on Task 2 she could not draw a picture of the partition division problems (see drawing on page 51). She could draw a picture of the quotition division problem where she needed to make 12 piles of 9 books. This problem is an example of a division problem that can be solved by repeated subtraction, and on Task 1 she showed that she thought of division as repeated subtraction. Quotition division problems are easier problems to model with pictures than partition division problems. Subject #12 has enough of an understanding of the schema or semantic structures of the word problems to recognize their mathematical structures, but she does not yet have a deep enough conceptualization to see the more subtle differences in the problems.

Subject #9 is a student who represents limited mathematical understanding. He was able to solve the problems in Task 2 by using the answers to check if he had chosen the correct operation. As was reported before, when asked why he divided on question 2A, he stated:

Subject #9: Because if I subtract I would get 64.

Researcher: And what would be wrong with that?

Subject #9: He has 8 friends and he only has 72 marbles.

Researcher: So how did you know that you needed to divide to get the right answer.

Subject #9: Because he only has 72 marbles and 8 friends, so if I subtract he could only give one 64 and the rest wouldn't get (finished sentence in a mumbled voice)

On question 2D when asked why he knew to subtract he stated:

Subject #9: Because I thought that divided would be less and times wouldn't make sense because it says how many more.

Subject #9 had the beginning of schema knowledge for take-away subtraction problems. On question 2C, he stated:

Subject #9: I subtract 56 minus 8 and I got for an answer 48.

Researcher: And how did you know that was how you should do it?

Subject #9: It says he threw away 8.

Subject #9 might just have been using the cue words of "threw away" to determine that question 2C was a subtraction problem, but it shows the beginning step of using schema knowledge to solve the word problem rather than solving it by just eliminating answers that don't make sense.

Subject #9 sorted the problems in Task 1 by categories that did not relate to the mathematical operations needed to solve the problems. He sorted them by items in the problems such as seeds, candy, and cassettes. He labeled one category "take-away," but included one subtraction problem and one division problem in that category. The division problem that he included was one that is a partition-type division problem which would not normally be solved by repeated subtraction. Subject #9 sorted the problems on Task 1 and solved the problems on Task 2 mainly by relating them to real-life situations. He did not show an extensive understanding of schema knowledge, but he does have a way of making sense out of word problems. It should be noted that Subject #9 was the only student who asked to have the problems read to him, so part of his problems with sorting the cards in Task 1 might have stemmed from his poor reading abilities. He is an example of a student who can compute accurately, but

does not yet have a good conceptualization of the mathematical structures of different kinds of problems.

Subjects #7, #9, and #12 represent the range of mathematical understanding that one can find in a sixth grade classroom. One can see that understanding the semantic structures that represent the underlying mathematical structures is a developmental process. In Subject #7, we see that understanding fairly well developed, less so in Subject #12, and just beginning to develop in Subject #9.

Performance in Relation to Language Dominance

Students who entered kindergarten as English-dominant did significantly better on Task 1 than the students who entered kindergarten as Spanish-dominant. Table 5 shows divides the students into which students were Spanish-dominant and which were English-dominant. Three of the students (#5, #7, and #11) who were able to sort the cards completely accurately were English-dominant while two of them (#2 and #10) were Spanish-dominant. In addition to this difference between students who entered school and students who entered school as Spanish-dominant, the overall performance of the two groups favored the English-dominant students. Looking at Table 5 labeled Sorted By Language - Task 1, there are four columns that should be compared. These are Columns E, G, I, and K. Columns E and I examine the items that are labeled correctly minus any errors made by placing inappropriate items in the piles. In other words if a student sorted four subtraction cards in a pile labeled subtraction, but also included one division card in that pile, his total score would be three (four correct subtraction minus the one division). Columns G and K record the groupings of subtraction and division problems by whatever name was used by the students minus again any inappropriate items included in those piles. I felt the resulting scores would give me more accurate numbers to use to determine the means and the standard deviations of the different groups.

Table 5: Sorted by Language - Task 1

Subject	Lang.	M / F	B	C	D	E	F	G	H	I	J	K	L
			Total	called sub.*	Total	Minus Errors	Sub. Prob. grouped together under any name	All Sub. minus errors	Total called div.*	Total Minus Errors	Div. Prob. grouped together under any name	All Div. minus errors	Overall Correct E + I
1	Sp.	F	0	0	0	0	0	0	6	2	6	2	2
2	Sp.	F	6	6	6	6	6	6	6	6	6	6	12
8	Sp	M	0	0	0	0	4	3	3	2	5	3	2
9	Sp	M	1	0	0	0	1	0	0	0	0	0	0
10	Sp	F	6	6	6	6	6	6	6	6	6	6	12
12	Sp	F	4	4	4	4	4	4	6	4	6	4	8
Totals			17	16	21	19	21	19	27	20	29	21	36
Ave.			2.83	2.67	3.5	3.17	3.5	3.17	4.5	3.33	4.83	3.5	6
SD			2.86	3.01	2.51	2.71	2.51	2.71	2.51	2.51	2.4	2.35	5.37

* Subtraction, minuw, take-away, finding the difference, plus adding
 ** Division, dividing, plus multiplication

Table 5 (continued): Sorted by Language - Task 1

A	B	C	D	E	F	G	H	Total	Total	Div. Prob.	All Div.	Overall
Subject	Lang.	M / F	Total	called	Sub. Prob.	minus	called	Minus	Errors	grouped	minus	Correct
			sub.*	Errors	together	errors	div.*	Errors		together	errors	E + I
					under					under		
					any name					any name		
3	Eng	M	5	4	5	4	5	4		5	4	8
4	Eng.	M	0	0	2	2	5	1		5	1	1
5	Eng.	M	6	6	6	6	6	6		6	6	12
6	Eng	F	0	0	5	2	0	0		5	3	0
7	Eng.	F	4	4	6	6	6	6		6	6	12
11	Eng	F	6	6	6	6	6	6		6	6	12
Totals			21	20	30	26	28	23		33	26	45
Ave.			3.5	3.33	5	4.33	4.66	3.83		5.5	4.33	7.5
SD			2.81	2.73	1.55	1.97	2.33	2.71		0.55	2.07	5.64

* Subtraction, minus, take-away, finding the difference, plus adding

** Division, dividing, plus multiplication

On the subtraction problems, the English-dominant students had a mean score of 4.33 with a standard deviation of 1.97 while the Spanish-dominant students had a mean score of 3.17 with a standard deviation of 2.71 in Column G (all subtraction problems minus errors). In Column E (total problems called subtraction minus errors) the English-dominant students had a mean score of 3.33 with a standard deviation of 2.73 while the Spanish-dominant students had a mean score of 2.67 with a standard deviation of 3.01. The higher standard deviation in the Spanish-dominant sample is a result of the fact that two of the Spanish-dominant students did not sort any of the subtraction problems accurately while all of the English-dominant students were able to sort at least two of the subtraction problems as subtraction.

On the division problems, the English-dominant students had a mean score of 4.33 with a standard deviation of 2.07 while the Spanish-dominant students had a mean score of 3.5 with a standard deviation of 2.35 on Column K (all division problems minus errors). In Column I (total problems called division minus errors) the English-dominant students had a mean score of 3.83 with a standard deviation of 2.71 while the Spanish-dominant students had a mean score of 3.33 with a standard deviation of 2.51.

On Task 1 the students who entered kindergarten as English-dominant students were able to sort the cards into categories based on the correct operation needed to solve the problem more accurately than the students who entered kindergarten as Spanish-dominant. Looking at Column L (overall correct) on the chart which shows the totals for sorting problems into both subtraction and division categories correctly, one can see that the Spanish-dominant students had a mean score of 6 on Task 1 with a standard deviation of 5.37 while the English-dominant students had a mean score of 7.5 with a standard deviation of 5.64. There were three Spanish-dominant students who did poorly on Task 1 while there were only two English-dominant students who had trouble sorting the cards on Task 1.

Performance in Relation to Gender

There were 7 females students who participated in the study and 5 male students. Female students were able to sort the problems on Task 1 significantly better than the male students. Four of the students who were able to sort the problems perfectly were female (#2, #7, #10, and #11) and one was male (#5). Three of the male students and two of the female students had a great deal of difficulty sorting the cards on Task 1.

Looking at the chart labeled Sorted by Male and Female - Task 1, there are four columns that should be compared. These are Columns E, G, I, and K and are the same ones that were compared when we were looking at the students by language. Columns E and I examine the items that are labeled correctly minus any errors made by placing inappropriate items in the piles. In other words if a student sorted four subtraction cards in a pile labeled subtraction, but also included one division card in that pile, his total score would be three (four correct subtraction minus the one division). Columns G and K record the groupings of subtraction and division problems by whatever name was used by the students minus again any inappropriate items included in those piles.

Table 9: Sorted by Male and Female - Task 1

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Subject	Lang.	M / F	Total called sub.*	Total Minus Errors	Sub. Prob. grouped together under any name	All Sub. minus errors	Total called div.*	Total Minus Errors	Div. Prob. grouped together under any name	All Div. minus errors	Overall Correct E + I														
1	Sp.	F	0	0		0	6	2	6	2	2														
2	Sp.	F	6	6		6	6	6	6	6	12														
6	Eng	F	0	0		2	0	0	5	3	0														
7	Eng.	F	4	4		6	6	6	6	6	12														
10	Sp	F	6	6		6	6	6	6	6	12														
11	Eng	F	6	6		6	6	6	6	6	12														
12	Sp	F	4	4		4	6	4	6	4	8														
Totals			26	26		30	36	30	41	33	58														
Ave.			3.71	3.71		4.29	5.14	4.29	5.86	4.71	8.29														
SD			2.69	2.69		2.43	2.27	2.43	0.38	1.7	5.22														

* Subtraction, minus, take-away, finding the difference, plus adding
 ** Division, dividing, plus multiplication

Table 9 (continued): Sorted by Male and Female - Task 1

Subject	Lang.	M / F	Total called sub.*	Total Minus Errors	Sub. Prob. grouped together under any name	All Sub. minus errors	Total called div.*	Total Minus Errors	Div. Prob. grouped together under any name	All Div. minus errors	Overall Correct E + I
3	Eng	M	5	4		5	4	4	5	4	8
4	Eng.	M	0	0	2	5	1	5	5	1	1
5	Eng.	M	6	6	6	6	6	6	6	6	12
8	Sp	M	0	0	4	3	2	5	5	3	2
9	Sp	M	1	0	1	0	0	0	0	0	0
Totals			12	10	18	15	13	21	21	14	23
Ave.			2.4	2	3.6	3	2.6	4.2	4.2	2.8	4.6
SD			2.88	2.83	2.07	2.24	2.41	2.39	2.39	2.39	5.18

* Subtraction, minus, take-away, finding the difference, plus adding

** Division, dividing, plus multiplication

As in the discussion of the difference based on language dominance in kindergarten, I felt that the resulting scores would give me more accurate numbers to use to determine the means and the standard deviations of the different groups.

On the subtraction problems, the female students had a mean score of 4.29 with a standard deviation of 2.43 in Column G (all subtraction problems minus errors). In Column E (subtraction problems called subtraction minus errors) the female students had a mean score of 3.71 with a standard deviation of 2.69 in their ability to sort subtraction problems. Subject #1 got none of the subtraction problems correct. The male students on the subtraction problems had a mean score of 3 with a standard deviation of 2.24 in Column G. On Column E the male students had a mean score of 2 with a standard deviation of 2.83. Subject #9 got none of the subtraction problems correct. As one can see, in both Column E and Column G, the females had higher mean scores indicating that they were better able to sort subtraction problems into appropriate categories.

On the division problems as shown in Column K (all division problems minus errors), the female students had a mean score of 4.71 with a standard deviation of 1.7. All of the female students were able to sort some of the division problems correctly; therefore, the standard deviation is less than in the subtraction problems. In Column I (division problems called division minus errors) the female students had a mean score of 4.29 with a standard deviation of 2.43. The male students on the division problems as shown in Column K had a mean score of 2.8 with a standard deviation of 2.39. Subject #9 sorted none of the problems correctly and Subject #4 only sorted one of the problems correctly when his mistakes were subtracted which made the standard deviation for the male students greater than for the female students. In Column I the male students had a mean score of 4.29 with a standard deviation of 2.43. Overall looking at the mean scores in Column I and Column K, one can see that the female students were much more able to sort the division problems in Task 1 into appropriate categories.

Looking at Column L on the chart, one can see that the mean score of the female students for correctly sorting the subtraction and division problems into appropriate piles is 8.29 while the mean score of the male students is 4.6 which indicates that the female students were much more able to sort the problems into appropriate categories than the male students.

One purpose of the present study was to observe whether a randomly selected group of students in a special Spanish two-way immersion program can determine the operation asked for in a given word problem using schema knowledge. The students selected had all passed a basic math computations test where they demonstrated their ability to add and subtract two digit numbers with regrouping and multiply and divide by one digit with 80% accuracy. The results of this study indicate that the majority of the students was able to use the semantic structure or schema of the problems to help them solve the problems.

A second purpose of the study was to determine whether there was a difference in performance between students who entered the program in

kindergarten as English-dominant and those students who entered as Spanish-dominant. The results show that there were differences in performance on Task 1 between students who entered school as English-dominant and those who entered school as Spanish-dominant, but students in both groups were able to use their knowledge of the schema of a problem to understand the mathematical operations needed to solve the problem.

An unexpected result of this study was that female students performed better on the two tasks than male students. Of the three students that seemed to have the most problems using schema, two were female (#1 and #6) and one was male (#9). However, a higher percentage of the female subjects than male subjects did extremely well on both Task 1 and Task 2. Both the male and female subjects were able to use the semantic structures of the problems to help them decide which mathematical operation to use.

CHAPTER V

Conclusions

Three questions were asked by this study. The questions were:

1. Can students use schema knowledge to sort mathematical word problems into whether they are subtraction or division problems?
2. If students use schema knowledge to sort mathematical word problems into whether they are subtraction or division, can they classify problems within these two categories using schema knowledge?
3. Is there a difference between the students who started the program as English-dominant speakers and students who are now fluent in English, but who started the program as Spanish-dominant speakers?

The results of two tasks were used to determine the answers to these questions. The first task involved sorting 12 word problems by the operations used to solve the problems. The second task involved actually solving five word problems and explaining the processes used.

Use of Schema Knowledge in Word Problem Solving

The first question asked by this study was whether students can use schema knowledge to sort mathematical word problems into whether they are subtraction or division problems. Looking at how the student sorted the cards as well as the explanations used to describe how the problems were solved on Task 2, one can conclude that the majority of the subjects in this study were able to use schema knowledge to determine which operation to use to solve a word problem involving subtraction or division. Five of the subjects were able to sort the word problems accurately by subtraction and division and solve all the problems in Task 2 without difficulty. Their explanations included reference to the schema in the problem, and they used schema knowledge to help them solve the problems in Task 2. Four other subjects had some ability to use schema

knowledge to solve problems. Two of these subjects could only use the schema in the problems in subtraction. They needed to use other techniques to determine which operation to use in division. Only three subjects did not use schema knowledge to determine what operation to use when solving mathematical word problems. These results indicate that schema knowledge learned in Spanish does indeed transfer to English and that students in sixth grade in an immersion program do use schema knowledge to solve mathematical word problems.

The results suggest that students' ability to use schema knowledge follows a developmental progression. Researchers (De Corte et al, 1985; Gilner, 1989; Hong, 1993; Quintero, 1983) observed differences between proficient problem solvers and less able problem solvers. The results of the present study showed patterns similar to their research.

Hong (1993) found that the majority of the students who represented mathematical word problems correctly had rapid classification models, indicating that they had a good understanding of the schema of the problems. There were a few students in Hong's study who drew diagrams as a way to understand the word problems. These students represented a middle category where they could classify the problems, but did not have the rapid classification ability of the most proficient students. Most students who had problems with recognizing the correct problem model, in other words recognizing the schema of the problem, were able to construct and solve number sentences without problems. In the present study, the five most proficient students had the ability to quickly recognize which process was needed and sort the cards accordingly. Subject #7 was an example of a proficient problem solver as defined in Hong's study. In the present study Subject #12 was an example of a student who fit Hong's middle category. She was able to correctly represent the mathematical word problems, but attempted to draw diagrams to help her understand what was being asked for

in the problem. Subject # 9 was an example of a student who could construct a number sentence and do the calculations, but who was not able to classify the problems rapidly using schema knowledge. He is similar to the students in Hong's study who were not proficient problem solvers.

De Corte et al. (1985) found that experienced problem solvers processed word problems in a top-down way; that is, the processing is conceptually driven using schema knowledge. However, less able problem solvers depended on a bottom-up or text driven processing to construct an appropriate problem representation. The less able problem solvers looked for cue words or phrases in the problems to help them solve the problems. Gilner (1989) also found that when students were asked to sort cards with mathematical word problems, the more successful problem solvers were able to organize the problems on structural criteria or schema. Less successful problem solvers did not see these underlying structures and, instead, organized the problems based on things such as question form, context, and common units of measurements. Similar results were found in the present study. Six students who were able to solve all the problems in Task 2 successfully also sorted the problems in Task 1 on the basis of the mathematical structures. Two students who were less proficient problem-solvers on Task 2 sorted the problems on Task 1 on the basis of context or question form. Four of the students in the present study showed more mixed results. Three students were able to solve all the problems in Task 2 correctly, but had problems sorting the problems in Task 1. One student sorted the problems in Task 1 on the basis of mathematical structure, but made computation errors when solving the problems on Task 2. Since Gilner had a much larger sample size and used multidimensional scaling techniques to obtain her results, rather than focusing on individual performance, it is unclear whether some of her subjects also showed mixed results.

Quintero (1983) found that students differed in their understanding of the

structure of problems as represented by the semantic structure (schema knowledge) and that different types of problems result in different levels of difficulty. In the present study, besides the students showing a range in their ability to use the schema knowledge, they clearly showed that it was more difficult for them to use schema knowledge with division problems than with subtraction problems. There were five students who sorted the problems in Task 1 accurately. Besides these five, Subject #3 and Subject #12 were able to sort most of the subtraction problems into categories that clearly indicated subtraction. The remaining subjects sorted the cards into groups that combined division and subtraction problems. For example, Subject #1 had a category labeled division that contained three division problems and three subtraction problems.

Riley, Greeno, and Heller (1984) stated that change, combine, and compare subtraction problems varied in difficulty with change problems being the easiest and compare problems being the most difficult. Looking at mistakes that were made in sorting in the present study, the students were most likely to categorize combine problems under the incorrect mathematical structure (usually division) while compare problems were the most likely to be put in categories based on the wording of the problem rather than in a category labeled by a mathematical operation. Change problems were the least likely to be miscategorized.

Bell, Fischbein, and Greer (1984) found that partition division problems were easier for students to identify than quotient division problems. Looking at mistakes that were made in sorting in the present study, students were more likely to sort quotient division problems under subtraction or addition than partition problems. Quotient problems were also more likely to be sorted into a category labeled by the question form. Partition problems were more likely to be mislabeled by the context in the problem. On Task 2 there was no pattern in

relation to the types of problems that were missed; however, Subject #12 clearly illustrated with her drawing that quotient division problems are easier to model than partition division problems.

The results of Task 1 demonstrated how the subjects conceptualized division and subtraction. The second question asked by this study was whether students who could sort problems by division and subtraction could classify the problems within these two categories. Only one student sorted the problems in Task 1 by the problem subcategories discussed in the literature such as combine, change, and compare subtraction problems or quotient and partition division problems. However, the students divided the problems into other categories that were related to the schema they had of the problems. Several students saw division as the inverse of multiplication and in their sorting indicated that multiplication could be used to solve simple division problems. Other students saw division as repeated subtraction and confused it with the regular subtraction process. Subject #4 saw at least one division problem as repeated addition. Subject # 7 was able to distinguish and sort together the subcategories of subtraction and division. She was the only student who saw addition as the inverse of subtraction and thought of addition as a way to solve two subtraction problems. As we teach the different processes, we need to be aware that they can be conceptualized in many different ways and that students need to be aware of the variety of approaches that can be used to solve a mathematical word problem. The semantic structures or schema of a problem represent the underlying mathematical structures, but which mathematical structures will be accessed will vary according to how the various operations are conceptualized.

Language Dominance

The last question asked by this study was whether the use of schema knowledge in solving mathematical word problems differed between those students who started the program as English-dominant and those who are now

fluent in English, but who started the program as Spanish-dominant in kindergarten. Researchers such as Cummins (1989) have supported the idea that schema learned in one language can transfer to a second language. Secada (1991) found if Hispanic first graders were able to solve mathematical word problems in Spanish, they could solve them in English and vice versa. The results of the present study seem to support these researchers; however, the degree of fluency in the second language does seem to be a factor.

Six of the subjects in this study entered school as Spanish-dominant and six of them entered school as English-dominant. The results of this study show that those subjects who entered school as English-dominant were better able to sort the cards in Task I correctly when compared with the subjects who entered school as Spanish-dominant. Of the five students who were able to sort the cards accurately into division and subtraction, three were English-dominant and two were Spanish-dominant. Two of the students who did not use schema knowledge to help them solve mathematical word problems entered school as Spanish-dominant. There was also more variation as shown by the standard deviations in the performance of the Spanish-dominant students than in the performance of the English-dominant students. Thus one can conclude that although schema knowledge learned in Spanish does transfer to English, this transfer is most easily accomplished in those students whose proficiency in English is greatest which in this study would be those students who started the immersion program as English-dominant students.

Gender Differences

One result of this study that was not predicted was difference that was shown between the performance of the girls and that of the boys. The girls were able to sort the cards in Task I much better than the boys. Four of the subjects who were able to sort the cards accurately into division and subtraction were girls. Only one was a boy. However, there was a much greater range of

performance by the girls in sorting subtraction problems than with the boys as shown by the standard deviations in the scores of both groups. On Task 2 three of the female subjects showed problems using schema knowledge in their explanations of how they solved the word problems. Only one male had considerable difficulty using schema knowledge. More of the male students fell in the middle where they could use schema knowledge, but did not use it consistently. Marshall (1986) has linked the fact that sixth grade girls appear to be stronger on certain types of problem-solving tasks to the fact that girls are more likely to learn a set of rules for solving problems and then will apply those rules more consistently than boys. She also found that when girls do not know how to solve a problem, they are more likely to use guessing strategies than boys. Because the problems in this study were fairly obvious in their format and thus able to be solved by following a set of rules, the female subjects possibly did better than the male subjects because they were more consistent in applying those rules. The fact that females use guessing strategies more often than males when they do not know how to solve a problem might explain why the female subjects had a tendency to sort the cards and solve the problems accurately or they had difficulty with the tasks.

Summary

The most significant finding in this study was that sixth grade students in a two-way Spanish immersion program do use schema knowledge in English even though their mathematics instruction was in Spanish through fifth grade. Even in the beginning of the year, they are able to use the schema knowledge that they developed in Spanish to help them solve mathematical word problems in English. However, the students who are most proficient in English are able to use schema knowledge in English better than those who are less proficient. This study also found that girls are able to use schema knowledge better than boys.

Because of the interviewing techniques used, this study also was able to look in depth at how students used schema knowledge to solve mathematical word problems. There was a range in students' abilities to use schema knowledge, and the differences in the students' abilities seemed to indicate a developmental progression. Even those students who were able to sort the problems in Task 1 accurately, conceptualized the problems differently. Division was sometime seen as multiplication or repeated subtraction. Subtraction was seen by one student as addition. The majority of the students used schema knowledge when determining how to solve mathematical word problems.

Questions for Future Research

Generally, the results of this study suggest that students who learn schema represented by the semantic structures in word problems in one language can then use that knowledge to solve mathematical word problems in a second language. The fact that the students in this study had attended the same school in a two-way Spanish immersion program since kindergarten and that all of their classroom mathematics instruction had been in Spanish provided a unique opportunity to study the question of whether students who learned schema knowledge in one language could use it in a second language. However, since no testing was done to determine what the students' schema knowledge was in Spanish, it is unclear to what extent the understanding of the schema in English was affected by knowledge of English versus original knowledge of schema in Spanish. It would be interesting to conduct a second study that would compare schema knowledge of entering sixth graders in Spanish with their ability to use schema knowledge in English when solving mathematical word problems. Also, this study focused on sixth grade, but what happens to students as they receive more of their math instruction in English. Do the differences disappear or do the same differences continue between those

students who started school as English-dominant and those students who started school as Spanish-dominant?

In this study the female subjects were able to use schema knowledge to sort word problems better than the male subjects. This difference might have been a result of better reading skills or some other factor. Further research comparing reading skills to ability to sort word problems using schema knowledge would be needed to determine whether reading ability was indeed a factor in the better performance of the female subjects. Also, looking at a correlation of reading skill and ability to solve mathematical word problems could provide important information, especially if a larger number of subjects was included in the sample. How does reading ability relate to the ability to use schema knowledge as the students progress through the middle school math curriculum? As the problems become more complex, does reading ability

- become more important in students' ability to solve mathematical word problems?

The present study focused on schema knowledge in relation to solving subtraction and division problems. One would expect similar results if addition and multiplication problems were added, but further research would be needed to determine whether the pattern of results would remain the same. More in depth questioning might lead to a better understanding of how the students are conceptualizing the problems and how they relate the semantic structures of the problems to their conceptualizations. Only one student was able to subdivide the subtraction and division problems into subcategories that matched the categories set forth in the literature. Is this knowledge only useful for teachers in planning how they are going to teach schema for mathematical word problem solving or is it useful for students as well?

The present study showed that students conceptualized problems in a various manners. In other words, the semantic structures of the problems

seemed to represent different underlying mathematical structures to different students. I feel that research that looked at whether students could explain the same problem in a number of different ways would yield useful information.

There is definitely a continuum that students go through in developing their ability to use schema knowledge in solving word problems. A study that looked at the same students over time would be able to document how a student's ability to use schema knowledge changed. It would also be interesting to observe if there is a developmental process in how students choose different representations to explain what the structure of a problem is. For example, pictures could be used to show different models of subtraction or division and students could be asked to choose the most appropriate one for how the problem was structured. More complex problems could also be used, including ones that required more steps.

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APPENDIX A

Problems Used in Task 1

- A. Andy is putting candy in bags. He puts 7 pieces in each bag. He has 63 pieces of candy. How many bags will he fill? (Division - quotient)
- B. Paul paid \$96 for 8 cassette tapes. How much did 1 tape cost? (Division - Partition - rate)
- C. I have 72 pieces of chocolate candy. I have 6 more than my brother. How many pieces of chocolate candy does my brother have? (Subtraction - compare)
- D. Two boys have \$108. One of the boys has \$9. How much does the other boy have? (Subtraction - combine)
- E. John has 56 candies. He gave 8 of them to a friend. How many does he have left? (Subtraction - change)
- F. I am making cakes. On each cake I will put 6 chocolate candies. If I have 36 chocolate candies, how many cakes can I make? (Division - quotient)
- G. Marco has 84 corn seeds. David has 7 seeds. How many fewer seeds does David have than Marco? (Subtraction - compare)
- H. Sandy is planting corn seeds. In the first minute she plants 6 seeds. How long will it take her to plant 48 seeds? (Division - quotient)
- I. 7 girls paid \$42 to go to the movies. How much did each girl pay? (Division - Partition - rate)
- J. Sally had 54 cassette tapes. She listened to some of them. There are 9 cassettes left that she has not listened to. How many cassette tapes did she listen to? (Subtraction - change)
- K. María has 72 candies. She wants to give an equal number of candies to each of her 8 friends. How many candies should she give each friend? (Division - Partition)
- L. Cathy and Debra have 64 pieces of candy. Cathy has 8 pieces of candy. How many pieces does Debra have? (Subtraction - combine)

APPENDIX B

Problems Used in Task 2

2A Juan has 72 marbles. He wants to give an equal number of marbles to each of his 8 friends. How many marbles should he give each friend? (Division - partition)

2B Teresa is putting books into piles. She puts 9 books in each pile. There are 108 books. How many piles will she make? (Division - quotient)

2C Miguel has 56 pictures. He throws 8 of them away. How many does he have left? (Subtraction - change)

2D I have 54 stickers. I have 9 more stickers than my friend. How many stickers does my friend have? (Subtraction - compare)

2E Two girls paid \$42 for CD's. One girl paid \$7. How much did the other girl pay? (Subtraction - combine)

APPENDIX C

Agreement to Participate in Research

Responsible Investigator: Mary Howland, Resource Specialist
Title of Project: Language and Mathematical Word Problem Solving

1. My child has been asked to participate in a research study investigating the relationship of primary language dominance, second language acquisition, and mathematical word problem solving in English. There are no discernible risks for your child associated with this study.
2. My child will be asked to sort 12 mathematical word problems by type. He/she will then be given 5 other problems that he/she will be asked to solve describing his/her strategies for solution. His/her comments about how he/she solved the problems will be audiotaped. My child will be asked to stay after school one day in order to participate in this study. The amount of time needed to finish the task will be 30 minutes to an hour. Transportation home will be provided if requested.
3. The results of this study will be used to aid the teachers at (school name) in improving their teaching of mathematical word problem solving in English.
4. The results of this study will be published as a Master's Thesis to be stored in the San Jose State University library and a summary of the results will be made available to the teachers at (school name). No information that could identify individual subjects participating in this study will be included in any report of this study.
5. Your child will receive a thank you note with a pencil or pen as is commonly used in school after participating in this study.
6. Questions about the research may be addressed to Mary Howland, Resource Specialist at (school name), (phone numbers). Complaints about the research may be presented to (principal's name) at (school name), (phone number). Questions about research, subject's rights, or research-related injury may be presented to Nabil Ibrahim, Ph.D., Associate Vice President of Graduate Studies and Research, at (408)924-2480.
7. No service of any kind, to which your child is otherwise entitled, will be lost or jeopardized if he or she does not participate in this study.
8. Consent to participate in this study is voluntary. A student may refuse to participate in the study or in any part of the study. If a student decides to participate in the study, he or she is free to withdraw at any time without prejudice to the student's relations with (school name) or its staff.
9. Please sign and date both copies of this form. The second copy is yours to keep.

My child _____ has my permission to participate in this study. I acknowledge that the signature of the Mary Howland on this document indicates agreement to include my child in the research and attestation that I have been fully informed of my child's rights.

Parent signature _____ date _____ phone number _____

Parent Mailing Address

Mary Howland, Resource Specialist	date
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