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Autonomous longitude and eccentricity control of geostationary spacecraft

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**AUTONOMOUS LONGITUDE AND ECCENTRICITY CONTROL OF
GEOSTATIONARY SPACECRAFT**

A Thesis

Presented to

The Faculty of the Department of Mechanical and Aerospace Engineering

San José State University

In Partial Fulfillment

Of the Requirements for the Degree

Master of Science

By

Brian Patrick Emma

December 2002

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ABSTRACT

AUTONOMOUS LONGITUDE AND ECCENTRICITY CONTROL OF GEOSTATIONARY SPACECRAFT

By Brian P. Emma

In order to lower the cost of maintaining satellites in orbit, spacecraft designers are seeking flight software that provides more autonomy. Longitude station-keeping and eccentricity are good candidates for autonomous control. The algorithm presented in this paper couples longitude control with eccentricity control. After the basic longitude control algorithm was written, a differential correction scheme was added resulting in an improved longitude error of ± 0.015 degrees longitude. Finally, longitude control and two-part maneuvers for eccentricity control were implemented. The result is good longitude control and eccentricity control to the mission constraints desired.

**In memory of
Brian's First Point of Aries
May he always lead me to the birds**

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CHAPTER 1 BACKGROUND

1.1 Introduction

In 1945 Arthur C. Clarke proposed that long-range and even global communications would be possible using spacecraft in geostationary orbits.¹ The first geostationary (GEO) spacecraft was launched in 1963, with the number of GEO spacecraft climbing to over 100 by 1980 and over 200 by 1990.² The need and desire for these spacecraft has only increased. They are easy to locate with ground antennas, always stay in the same area in the sky and it only takes one control node to maintain them (although multiple downlink stations spaced around the Earth make orbit determination easier and better). GEO spacecraft are also easily accessed by stationary (non-tracking) antennas so there can be many users with inexpensive antennas such as satellite television users. Communications is the primary use of GEO satellites, including both military and civilian applications, but there are a few scientific missions as well as Earth observing and DOD reconnaissance missions.

In order to lower the cost of maintaining satellites in orbit and increase their self-sufficiency, spacecraft designers are seeking flight software that provides more autonomy. Currently, orbit maintenance is performed by ground crews who determine the spacecraft orbit using some or all of the following: telemetry from the spacecraft, azimuth, elevation, range and/or range rate from the ground antenna. Next, if needed, orbit correction maneuvers are calculated and executed by either real-time ground commands or delayed commands sent by the ground crew. This method of orbit maintenance is labor and staff intensive. Flight software can incorporate the same basic

algorithms used by the ground crews for calculating the maneuvers and if done correctly can do it autonomously. At times, these maneuvers may need to be planned and/or executed on a weekend or in the middle of the night. If staff must be at the control node to do these activities then employers are forced to pay overtime and shift premiums. If the flight software is capable of performing these activities without human interventions, staff costs can be reduced. Moreover, for military operations, if a control node is temporarily compromised or destroyed it would be very advantageous to have satellites in orbit that will self maintain their orbits at least until another control node can be brought on-line.

Since the forces that perturb a geostationary spacecraft's Earth reference longitude are understood and predictable, longitude station-keeping is a good candidate for autonomous control. Controlling the behavior of eccentricity also falls into this category. Eccentricity is typically adjusted at the same time as longitude, so it is natural to combine these two corrections; moreover, every time a tangential or radial maneuver is performed, the eccentricity is affected.

The algorithm presented in this thesis couples longitude control with eccentricity control. The algorithm was developed in three phases. First, a plan for controlling longitude alone was developed and tested. After the basic longitude control algorithm was written, a differential corrections scheme was added using a more realistic model, resulting in an improved longitude control to ± 0.015 degrees longitude. This is well within the typical commercial GEO satellite longitude requirement of ± 0.05 degrees. Since the point at which the maneuver is performed to control longitude is relatively

unimportant, this maneuver was delayed until such a time in the orbit that it would not only correct the longitude error, but also help control eccentricity. This eccentricity control scheme is termed the “Sun-pointing-perigee” strategy. Finally, longitude control and two-part maneuvers for eccentricity control were implemented. The result is eccentricity control to the desired mission constraints as well as good longitude control that may be incorporated into an autonomous station-keeping process.

In 1997 Lockheed Martin started work on an autonomous longitude station-keeping algorithm for the flight software of the Space Based Infrared System (SBIRS) GEO (a Department of Defense (DoD) GEO satellite constellation). This was Lockheed Martin’s first attempt at autonomous station-keeping and it provided the motivation for much of this study. Due to mission constraints the SBIRS algorithm was very specific to the SBIRS mission. For purposes of this thesis, the research was taken further and a more general algorithm was developed, combining autonomous eccentricity control with the autonomous longitude control. The SBIRS program has decided to add autonomous eccentricity control to the flight software and the algorithm presented here provided the starting point for that work. In addition, this algorithm has been implemented in an analysis simulation tool for modeling the lifetime fuel usage for station-keeping of GEO satellites. The target application was the Lockheed Martin-built A2100 Bus.

1.2 Geostationary Orbits

A spacecraft in geostationary orbit does not move relative to the surface of the Earth. A geostationary orbit is also a mathematical abstraction. If the Earth was perfectly uniform in mass, and there were no other perturbing forces such as the gravity

from the Sun and Moon, a true geostationary orbit would be possible by injecting a spacecraft into a circular orbit with zero inclination at the proper radius. Unfortunately this is not possible for more than an instant due to these perturbing forces. The best that can be achieved is a nearly geostationary orbit, i.e.: low eccentricity, low inclination, and orbit period close to one sidereal day. The term geosynchronous is often confused with geostationary. Geosynchronous simply means “synchronous with the Earth’s rotational period.” Geosynchronous implies nothing about eccentricity or inclination. This thesis discusses nearly geostationary orbits in which the inclination and eccentricity are nearly zero.

When maintaining a satellite in a GEO orbit, the objective is to keep the spacecraft in about the same location (longitude) relative to the surface of the Earth. To do this, the period of the orbit must be close the Earth’s rotational period. “The Earth’s rotation rate ψ is known with a very high accuracy as follows: $\psi = 360.985647 \text{ deg/day} = 0.729211585 \cdot 10^{-4} \text{ rad/s}$,” (Soop, p. 7). The centrifugal force magnitude on a spacecraft orbiting at this rate would be

$$F_c = m\psi^2 r \quad (1.2-1)$$

where m is the mass of the spacecraft and r is the orbit radius. The attracting force magnitude between the Earth and the spacecraft is found with Newton’s Third Law

$$F_g = \frac{GmM}{r^2} \quad (1.2-2)$$

where M is the mass of the Earth. It is common to use the Earth central gravity constant $\mu = GM$ since it is known to a much higher accuracy than either G or M separately as

$$\mu = 398600.440 \text{ km}^3/\text{s}^2 \quad (\text{Soop, p.7}).$$

Equations (1.2-1) and (1.2-2) must be equal in order for the orbit to be circular and together can be solved for r as

$$r = \sqrt[3]{\frac{\mu}{\psi^2}}. \quad (1.2-3)$$

Substituting in the values for μ and ψ gives a GEO radius of $R_{GEO} = 42164200$ m. When higher order gravity terms are included the GEO radius is sometimes modified.

1.3 Requirements

For a GEO orbit, unlike any other orbit, the spacecraft's location in the orbit is defined by its Earth reference longitude, or more commonly referred to as just longitude. This is unlike any other orbit because in all other orbits the spacecraft is moving relative to the surface of the Earth, often at a high rate. But a GEO orbiting spacecraft remains stationary or drifts very slowly. Drifting to the east or west is called longitude drift and this drift is what this study attempts to control. Drifting north or south is caused by nonzero inclination and is not addressed by this study.

When the parameters of a mission are being collected, a longitude requirement is defined. This longitude requirement is usually defined by a maximum number of degrees in Earth reference longitude the spacecraft is allowed to drift from its assigned longitude station. As was stated earlier, it is impossible to maintain the spacecraft in exactly one location over the Earth so it is contained to a region or a neighborhood about its assigned longitude station. The longitude requirement for a commercial GEO satellite is on the order of ± 0.05 degrees.

As soon as the spacecraft arrives at its longitude station, perturbing forces start to pull it away and cause it to drift. Because zero eccentricity is also a mathematical abstraction, the nonzero eccentricity causes a daily longitude libration. This libration is caused by the fact that the spacecraft is moving faster as it goes through perigee than it is when it goes through apogee. This is not considered drift because it oscillates back and forth with no daily net change in longitude. When determining if the longitude requirement is met, this daily libration must be taken into consideration, but when controlling longitude, only the long period mean longitude is examined. The instantaneous longitude error due to the orbit eccentricity will be dealt with by controlling eccentricity. Due to this eccentricity driven longitude libration, the longitude requirement often drives the eccentricity requirement. The maximum allowed eccentricity for a commercial GEO communication satellite is usually on the order of 10^{-3} .

When controlling longitude, there are a number of limits to consider. Start with the basic requirement by assuming ± 0.05 degrees. This defines a region about the station longitude where it is considered acceptable for the spacecraft to be. The spacecraft is never allowed outside these limits. There needs to be a buffer zone inside these limits to allow for errors in the orbit determination (OD), burn error, longitude libration due to eccentricity and other short term perturbations. For this reason another set of limits must be defined inside the requirement limits. The requirement limits at ± 0.05 degrees are called “hard limits.” The second set of limits set inside the hard limits are called “soft limits.”

CHAPTER 2 LONGITUDE CONTROL

2.1 Frame of Reference

The plane containing the orbit of the Earth around the Sun is called the ecliptic plane. The Earth orbits the Sun in a near circle with an aphelion of 152 million km and a perihelion of 147 million km and a sidereal period of one year (365.2422 days). The equatorial plane of the Earth is tilted by 23.442 degrees from the ecliptic plane. The x - y -plane of the coordinate system used for GEO orbit analysis is coincident with the Earth's equatorial plane. The intersection of the ecliptic plane and equatorial plane define the x -axis with positive x pointing towards the *First Point of Aries* or the *Vernal Equinox* commonly marked with the Aries head, γ . A right-handed orthogonal frame is then defined with the y -axis perpendicular to the x -axis in the equatorial plane and the z -axis pointing through the North Pole. This quasi-inertial frame is a standard for GEO analysis and is part of the *Mean Equatorial Geocentric System of Date* "MEGSD."

"The *Mean of Date* in the name of MEGSD means that it is transformed from the inertial system only by the precession of the Earth's rotational axis but uses the mean value of zero for the other more or less periodic transformations" (Soop, p. 16). "The precession of the Earth is the secular effect of the gravitational attraction from the Sun and the planets on the equatorial bulge of the Earth. The main effect is a rotation of the mean-of-date system in the negative sense on the ecliptic plane by one turn in 26000 years, which is equivalent to 0.014 degrees per year" (Soop, p. 15). There are other systems that can be used for GEO orbit analysis but the MEGSD system has the advantage that it can be assumed the Earth rotates about the z -axis only for all analysis

except for high-precision orbit determination. The rotation of the Earth defines the angular velocity that the GEO spacecraft needs to match. The Earth rotates in the positive direction about the z-axis. The only transformation now needed is between the x - y - z coordinate system defined above and the surface of the Earth. Locations on the surface of the Earth as well as the locations of satellites in GEO orbit are defined by longitude and latitude. Longitude starts with zero at the prime meridian (originally drawn through the Royal Greenwich Observatory in England) and increases positively in the eastern direction in increment of degrees or radians for computational purposes. Latitude starts with zero at the equator and increases positively in the northern direction in increments of degrees. Longitude generally has a range from 0° to $+360^\circ$ and latitude generally ranges from -90° to $+90^\circ$.

2.2 Modeling Longitude for Control

When controlling spacecraft referenced to Earth longitudes, the motion can be modeled as one-dimensional. It is assumed that the orbit has low inclination so north/south (cross track) motion is ignored. Low eccentricity is also assumed so radial motion and daily longitude vibrations can be ignored (effects of undesired components of station-keeping maneuvers along the radial direction *are* considered later). In addition, the longitude “vibration” caused by nonzero eccentricity is ignored because the daily mean longitude is being controlled as opposed to the instantaneous longitude. The only other motion is in-track motion or east/west drift. The spacecraft is moving quite fast relative to inertial space but ideally it is not moving much relative to the Earth’s surface. For longitude control only east/west motion is monitored and accounted for.

The primary natural force causing longitude acceleration is the Earth's non-uniform mass distribution giving rise to a non-uniform gravitational potential. This perturbative force comes from the tesseral terms of the Earth gravity model and is known as "tesseral resonance." Tesseral resonance acts in the same way as a very low constant magnitude tangential thrust. The magnitude and direction of the acceleration varies with longitude, so the drift time between burns may vary from station to station. The sign of the acceleration determines the direction of the acceleration. Figure 1 shows the longitude acceleration as a function of longitude. In this study a positive acceleration is assumed to point east. A negative acceleration, or west acceleration, is analyzed with the same equations but opposite sign.

Figure 1. Tesseral Acceleration as a Function of East Longitude.

This problem can be modeled in one dimension since only longitude is of concern. It is also assumed that the longitude acceleration will remain constant because

the spacecraft will remain in a relatively small neighborhood about the assigned longitude station. With constant acceleration, the graph of longitude as a function of time is parabolically shaped. Figure 2 shows a sketch of a desired longitude history with station-keeping when positive tesseral acceleration acts.

Figure 2. Nominal Longitude History.

The parabolic longitude history can be modeled as

$$\ddot{\lambda} = C_1$$

where $\ddot{\lambda}$ is the longitude acceleration due to tesseral forces and C_1 is the constant value of that acceleration determined by the tesseral model incorporating the following terms:

C_{22} , S_{22} , C_{33} , S_{33} , C_{21} , S_{21} and J_2 . Integrating once gives

$$\dot{\lambda} = C_1 t + C_2$$

where $\dot{\lambda}$ is the current longitude drift rate and C_2 is the original longitude drift rate $\dot{\lambda}_0$.

Integrating once again gives

$$\lambda(t) = \frac{1}{2} C_1 t^2 + C_2 t + C_3$$

For initial conditions specified as $\lambda(0) = \lambda_0$ and $\dot{\lambda}(0) = \dot{\lambda}_0$, then

$$C_2 = \dot{\lambda}_0$$

$$C_3 = \lambda_0$$

$$\lambda(t) = \frac{1}{2} \ddot{\lambda} t^2 + \dot{\lambda}_0 t + \lambda_0 \quad (2.2-1)$$

Time t_0 is usually taken as zero for convenience, so that $\lambda_{\max}$ in the figure is equal to λ_0 .

Equation (2.2-1) is the primary equation used in the design of the following algorithm, which computes a tangential maneuver to return the spacecraft orbit to the appropriate location on Figure 2. Equation (2.2-1) is rearranged to obtain initial drift rate in terms of initial position, final position, acceleration and time as

$$\dot{\lambda}_0 = \frac{\lambda_{final} - \lambda_0 - \frac{1}{2} \ddot{\lambda} t_f^2}{t_f} \quad (2.2-2)$$

Ideally $\lambda_0 = \lambda_{\max}$ and $\lambda_{final} = \lambda_{\max}$, however, both angles will vary due to operational considerations (e.g., a maneuver may be performed a day early if the nominal date falls on a holiday).

This equation can be used to determine the initial velocity required to begin a parabolic longitude station keeping cycle (as in Figure 2) that after t days will arrive at λ_{final} . The drift time between burns is a user input and usually driven by mission requirements and satellite operations complex (SOC) logistics. The longer the time between burns, the farther the spacecraft will drift from the longitude station. A typical value on the order of three to seven days between burns is nominal.

An east-west station-keeping ΔV is not required every day. The regularity of ΔV s will vary as the longitude station varies since each longitude has a different longitude acceleration. The magnitude of the ΔV will also vary from maneuver to maneuver depending on where the spacecraft is with respect to the station and magnitude of its current drift rate.

Once the desired drift rate $(\dot{\lambda}_0)$ is known, it can be subtracted from the current drift rate to produce a delta drift rate $(\Delta\dot{\lambda})$. The desired drift rate is not zero. It is the required drift rate to put the spacecraft on the path shown in Figure 2.

$$\Delta\dot{\lambda} = \dot{\lambda}_{desired} - \dot{\lambda}_{current} \quad (2.2-3)$$

To determine this maneuver, an equation is derived relating change in $\dot{\lambda}$ and ΔV .

Starting with the energy equation

$$E = \frac{1}{2}v^2 - \frac{\mu}{r} = \frac{-\mu}{2a}$$

and linearizing about a circular GEO orbit gives (by taking the differential)

$$v dv = \frac{1}{2} \frac{\mu}{a^2} da$$

Which can be rearranged as

$$da = 2 \frac{va^2}{\mu} dv \quad (2.2-4)$$

Now v , the velocity, and n , mean motion for a circular GEO orbit are

$$v = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{\mu}{a}}$$

$$n = \text{mean motion} = \sqrt{\frac{\mu}{a^3}}$$

Then v is substituted into equation (2.2-4) and simplified with mean motion n to give

$$\begin{aligned} da &= 2 \frac{\sqrt{\frac{\mu}{a}} a^2}{\mu} dv = 2 \frac{\sqrt{\frac{\mu a^3}{a}}}{\mu} dv = 2 \frac{\sqrt{\mu a^3}}{\mu} dv = 2 \sqrt{\frac{a^3}{\mu}} dv = \frac{2}{n} dv \\ \therefore da &= \frac{2}{n} dv \end{aligned}$$

Next the period of a circular orbit is

$$P = 2\pi \sqrt{\frac{a^3}{\mu}} \Rightarrow \frac{2\pi}{P} = \sqrt{\frac{\mu}{a^3}}$$

The drift rate, D , is now introduced which is the difference between the orbital period and the Earth's rotation rate. A perfectly geosynchronous satellite has zero drift rate. The rotation rate of the Earth is subtracted from the mean motion to obtain drift rate as

$$\frac{2\pi}{P} - \frac{2\pi}{24hr} = \sqrt{\frac{\mu}{a^3}} - \frac{2\pi}{24hr} \Rightarrow D = \sqrt{\frac{\mu}{a^3}} - \frac{2\pi}{24hr}$$

Now take the differential of D and simplify using n as before to obtain

$$\begin{aligned} dD &= \frac{-3}{2} \sqrt{\frac{\mu}{a^5}} da = \frac{-3}{2} \frac{\sqrt{\frac{\mu}{a^3}}}{a} da = \frac{-3}{2} \frac{n}{r} da \quad (\text{again assuming a circular orbit}) \\ \therefore dD &= \frac{-3}{2} \frac{n}{r} da \end{aligned} \tag{2.2-5}$$

Now combine equations (2.2-4) and (2.2-5) to give

$$dD = \frac{-3}{2} \frac{n}{r} \frac{2}{n} dv \Rightarrow dD = \frac{-3}{r} dv \tag{2.2-6}$$

Drift rate, D , is the same as $\dot{\lambda}$ (rate of change of longitude), so the above is rewritten in terms of $\dot{\lambda}$ and ΔV is substituted for dv to give

$$\Delta \dot{\lambda} = \frac{-3}{r} \Delta V \quad (2.2-7)$$

Equation (2.2-7) can be solved for ΔV in order to get a ΔV in terms of $\Delta \dot{\lambda}$; recall the definition of $\Delta \dot{\lambda}$ from equation (2.2-3). Since this is a model of a geostationary orbit, the radius of a geostationary orbit replaces r in the final equation to give

$$\Delta V = \frac{-r_{geo}}{3} \Delta \dot{\lambda} \quad (2.2-8)$$

Equation (2.2-8) is used when estimating the magnitude of a tangential longitude station-keeping maneuver.

The time between maneuvers and a maximum longitude limit is determined during mission design and used as inputs to the algorithm. When this predetermined time has passed since the last longitude station-keeping maneuver or the spacecraft longitude has reached the predetermined limit, equation (2.2-2) is used to determine a new drift rate as

$$\dot{\lambda}_{new} = \frac{\lambda_{max} - \lambda_{current} - \frac{1}{2} \ddot{\lambda} (\Delta t)^2}{\Delta t} \quad (2.2-9)$$

(where Δt is the predetermined time between burns), and this new drift rate is used in equation (2.2-7) to calculate the appropriate tangential ΔV as

$$\Delta V_{tangential} = \frac{-r_{geo}}{3} (\dot{\lambda}_{new} - \dot{\lambda}_{current}). \quad (2.2-10)$$

2.3 Differential Corrections

There are many forces perturbing the longitude of a GEO orbit. The model used in the previous section only accounts for the tesseral forces. All of the other perturbing forces are much smaller than the tesseral forces. Instead of attempting to include these perturbations in the model when computing a station-keeping maneuver, experience demonstrated that it is better to calculate a maneuver with the method described above (using the original, more simple model) and then improve it with differential corrections while using a more realistic model by accounting for the next two largest forces. The more realistic model used in this study included lunar and solar gravity models along with the tesseral model incorporating the following terms: C_{22} , S_{22} , C_{33} , S_{33} , C_{21} , S_{21} and J_2 . The solar and lunar locations were determined with equations from the Astronomical Almanac^{3,4}.

In formulating the differential corrections algorithm, it is first assumed that the ratio of the error in the final longitude ($\delta\lambda(t_f)$) to the error in the ΔV ($\delta\Delta V$) is the same as the ratio of a change in the final longitude ($\partial\lambda(t_f)$) to a change in the ΔV ($\partial\Delta V$), or

$$\frac{\delta\lambda(t_f)}{\delta\Delta V} = \frac{\partial\lambda(t_f)}{\partial\Delta V} \quad (2.3-1)$$

thus

$$\delta\Delta V = \frac{\partial\Delta V}{\partial\lambda(t_f)} \delta\lambda(t_f) \quad (2.3-2)$$

The term $\delta\lambda(t_f)$ is determined through simulation using a calculated $\delta\lambda(t_f)$ from equation (2.2-10) and the more realistic model, but the $\frac{\partial\Delta V}{\partial\lambda(t_f)}$ ratio is found (approximately) analytically through the finite difference formula

$$\frac{\partial\Delta V}{\partial\lambda(t_f)} \approx \frac{\Delta V_2 - \Delta V_1}{\lambda_2(t_f) - \lambda_1(t_f)} \quad (2.3-3)$$

As shown in Appendix A, this finite difference may be approximated as $\frac{-r_{geo}}{3t_f}$.

The differential corrections equation needed is then

$$\delta\Delta V = \frac{-r_{geo}}{3t_f} \delta\lambda(t_f) \quad (2.3-4)$$

The way in which differential corrections would be implemented by flight software to correct the calculated ΔV as well as the way it was implemented in this research is as follows:

1. The thruster control module of the flight software is instructed to calculate a longitude control maneuver by the main module.
2. A maneuver is calculated using equations (2.2-9) and (2.2-10) and the current orbit parameters, (Recall: equation (2.2-9) has a λ_{max} term).
3. Next the flight software propagates the orbit parameters forward in time to the next longitude control maneuver.
4. The propagated orbit parameters are analyzed and λ_{final} is compared to λ_{max} to determine the error in λ_{final} , $(\delta\lambda(t_f))$.

5. $\delta\lambda(t_f)$ is inserted into equation (2.3-2) to determine the correction to ΔV (i.e., $\delta\Delta V$).

6. $\delta\Delta V$ is then added to the originally calculated ΔV and this renders the corrected

$$\Delta V. \quad \Delta V_{corrected} = \Delta V_{original} + \delta\Delta V$$

One iteration was ample, as additional iterations did not significantly improve the results.

The flow chart in Figure 3 summarizes the proposed steps flight software would take to perform longitude station-keeping with differential corrections

Figure 3. Flow Chart for Longitude Station-Keeping Only.

CHAPTER 3 ECCENTRICITY CONTROL

3.1 Eccentricity Considerations and Equations

The eccentricity vector lies in the MEGSD equatorial plane and therefore can be defined by only two vector components. Its magnitude is equal to the eccentricity of the orbit and its direction is from the center of the orbit toward periapsis. The ec unit vector points along the x -axis of the MEGSD frame and the es unit vector points along the y -axis. Mathematically the components of the eccentricity vector are defined as

$$ec = e \cos(\Omega + \omega) \quad (3.1-1)$$

$$es = e \sin(\Omega + \omega) \quad (3.1-2)$$

where $\Omega + \omega = \Pi$ and is the longitude of perigee.

Now eccentricity can be analyzed and controlled as a vector. Since it is a vector there are two aspects of eccentricity to be concerned with: magnitude and direction.

Unlike longitude, which is negligibly affected by a small radial maneuver, eccentricity is affected by both tangential and radial maneuvers. The location of the spacecraft in the orbit when the maneuver takes place is also important. The effects on the eccentricity vector from a maneuver can be quantified as

$$\Delta \mathbf{e} = \langle \Delta ec, \Delta es \rangle$$

$$\Delta ec = \sin(S_b) \frac{\Delta V_r}{V} + 2 \cos(S_b) \frac{\Delta V_t}{V} \quad (3.1-3)$$

$$\Delta es = -\cos(S_b) \frac{\Delta V_r}{V} + 2 \sin(S_b) \frac{\Delta V_t}{V} \quad (3.1-4)$$

where S_b is the sidereal angle of the spacecraft at the time of the maneuver. (See Appendix B for a derivation of equations (3.1-3) and (3.1-4).) The sidereal angle is measured from the First Point of Aries to the spacecraft in the orbit plane. Combining equations (3.1-3) and (3.1-4) gives the magnitude of the eccentricity change as

$$\Delta e_{total} = \frac{1}{V} \sqrt{4\Delta V_t^2 + \Delta V_r^2} \quad (3.1-5)$$

It is apparent from equations (3.1-3) and (3.1-4) that a tangential ΔV will have twice the effect on eccentricity as a radial ΔV . For this reason, maneuvers performed for the sake of adjusting eccentricity should always be tangential if possible. Due to spacecraft design or mission constraints, though, sometimes purely tangential maneuvers are not possible. This is discussed in more detail in Section 3.4.

3.2 Solar Radiation Pressure

The greatest perturbing force affecting eccentricity is solar radiation pressure (SRP). SRP is not to be confused with solar wind, which consists of particles, mostly protons but also electrons and ions, and has an effect on a GEO orbiting spacecraft many magnitudes less than the effect of SRP. SRP is electromagnetic radiation, mostly in the visible and neighboring wavelengths. The term used to refer to the power output of the Sun is the solar constant. “The mean radiation intensity near the Earth is about 1.4 kW/m². It changes by only between 0.1% and 0.2% due to variations in the solar activity, whereas the annual change in the Sun-Earth distance contributes to a variation by $\pm 3.3\%$.” (Soop, p. 91)

SRP causes a force on the spacecraft parallel to the solar rays; therefore, the direction of the perturbation on the orbit done on any given day will be reversed six months later. Unfortunately the perturbation may be too large to wait a half a year for the resulting effect to reverse, or in other words, the amplitude of the SRP effects may be too large to wait for it to correct itself. Some missions can take full advantage of the cyclic nature of the SRP perturbations on eccentricity but for most GEO missions eccentricity must be at least partially controlled.

The force exerted on the spacecraft is proportional to the effective cross-section, C , of the spacecraft. The reflectivity coefficient, ϵ , of this cross-sectional area is also important and the force is proportional to $(1+\epsilon)$ where $0 < \epsilon < 1$ and

$$F = PC(1 + \epsilon) \quad (3.2-1)$$

P is the pressure from the Sun and can be found by dividing the above mentioned radiation intensity near the Earth by the speed of light to obtain a pressure of $P = 4.56\text{e-}6$ N/m². Dividing the right hand side of equation (3.1-1) by the mass of the spacecraft gives acceleration as

$$\frac{dV}{dt} = \frac{PC(1 + \epsilon)}{m} \quad (3.2-2)$$

As can be seen by studying equation (3.2-2), an important relationship exists between spacecraft mass, effective cross-section and reflectivity and the magnitude of the SRP perturbation on a particular spacecraft. A large, light spacecraft is affected more by SRP than is a small, heavy one. GEO spacecraft often require significant electric power and therefore have very large solar arrays. This in turn creates a high effective cross-

section to mass ratio and a high SRP effect. The effective cross-section to mass ratio is often denoted by sigma as

$$\sigma = \frac{C(1+\varepsilon)}{m} \text{ which leads to } \frac{dV}{dt} = P\sigma \quad (3.2-3)$$

When the σ value of a spacecraft is greater than about 0.01 to 0.02 m²/kg (Soop, p. 95), SRP is the dominating force in perturbing the eccentricity vector.

SRP is always pushing the spacecraft away from the Sun and hence imparting a ΔV parallel to the solar radiation. This creates a change in the eccentricity vector δe perpendicular to the solar rays (see Appendix C for evidence of this). Because the Earth is orbiting the Sun, the direction of the solar rays change throughout the year, and so does the direction of the δe vector. An equation that approximates the eccentricity vector as a function of time with initial condition e_o is (Soop, p. 95)

$$e(t) = e_o + \frac{3P\sigma Y}{4\pi V} \begin{pmatrix} \cos s_s \\ \sin s_s \end{pmatrix} \quad (3.2-4)$$

where Y is the orbital period of the Sun equal to one year.

Left unmaintained, the eccentricity vector (e) will rotate about its origin once over the course of one year as shown in Figure 4. This rotation of e is caused mainly by solar radiation pressure. By specifying proper initial conditions, the eccentricity magnitude will change very little throughout the year as the eccentricity vector rotates following the Sun. These initial conditions include perigee at or near the Earth-Sun line and an eccentricity magnitude suited to the σ value of the spacecraft. The magnitude of eccentricity in this case may not be acceptable though. As stated earlier, the eccentricity

requirement for a commercial geostationary communications satellite is usually on the order of 10^{-4} . If this requirement cannot be met with a passive approach, active control is required.

Figure 4. Eccentricity Vector Under Natural Perturbations for Eleven Months (Long Period Mean).

One method of mitigating the effect of the SRP perturbation is to perform longitude control maneuvers and even momentum unloading maneuvers at locations in the orbit where the resultant ΔV will be opposite in direction of the perturbation caused by the SRP. Generally these types of maneuvers have a large tangential component. The energy already being expended to control longitude or momentum unloading may also be used to control changes to eccentricity caused by SRP. If the maneuver is a prograde maneuver, or increases the velocity of the spacecraft, it should be executed at 6 a.m. local satellite time (LST). A prograde maneuver executed at 6 a.m. LST will point in the opposite direction as the current perturbation caused by SRP. If it is a retrograde

maneuver, or decreases the velocity of the spacecraft, it should be executed at 6 p.m. LST. Implementing the station-keeping process in this open-loop manner takes advantage of orbit maneuvers already being performed.

3.3 Sun-Pointing-Perigee Strategy with Single Longitude Maneuvers

Another method of eccentricity control is to attempt to keep the eccentricity vector pointing in the neighborhood of the Earth-Sun line while maintaining an acceptable eccentricity magnitude. This is known as a “Sun-pointing-perigee” strategy and its general application is described in detail by Soop. The Sun-pointing-perigee strategy is widely used in controlling the eccentricity of commercial GEO satellites and the roots of the strategy developed for this thesis have their origins with earlier studies by orbit maintenance personnel from a Lockheed Martin satellite control complex. The basic set of rules originate from the control complex, but the logic in applying them in an autonomous algorithm was developed during the course of this research. Sun-pointing-perigee is generally attempted using just the ΔV from the longitude control maneuvers. When controlling longitude, the maneuver may be executed at any point on the orbit as long as the needed tangential ΔV magnitude, calculated in equation (2.2-10), is obtained. If the spacecraft thruster configuration is expected to produce an undesired radial component of ΔV along with the tangential ΔV , the radial component can be determined or estimated compared to the overall maneuver. The resulting magnitudes of the tangential and radial component ΔV s from a planned longitude control maneuver can be inserted into equation (3.1-5) to determine the magnitude of the Δe resulting from this longitude correction maneuver. Regardless of where in the orbit the maneuver is

performed, Δe will have the same magnitude. The approach from here is to plan the timing of the maneuver such that it will best benefit eccentricity. The desire is to rotate e to lie just ahead of the Earth-Sun line. If Δe is not long enough to rotate e all the way back to the Earth-Sun line then e is rotated as far as possible towards the Earth-Sun line without reducing the magnitude of e . If there is more Δe than is required to rotate e ahead of the Earth-Sun line then the remaining Δe is used to reduce the magnitude of e after rotating it. Figure 5 shows a sketch demonstrating this strategy.

Figure 5. Sun-Pointing-Perigee Method.

The flow chart in Figures 6, 7 and 8 summarize the proposed steps flight software would take to perform longitude station-keeping with differential corrections and passive eccentricity control using the Sun-Pointing-Perigee Strategy.

Figure 6. Flow Chart for Longitude Station-Keeping with Sun-Pointing-Perigee-Method.

Figure 6. Flow Chart for Longitude Station-Keeping with Sun-Pointing-Perigee-Method (Page 2).

Figure 6. Flow Chart for Longitude Station-Keeping with Sun-Pointing-Perigee-Method (Page 3).

3.4 Eccentricity and Longitude Control with Two-Part Maneuvers

Some missions require more precise handling of e than may be achievable using the above described approach. For these missions two-part eccentricity maneuvers are required. A two-part maneuver can be planned that will result in the desired longitude drift rate, the desired eccentricity magnitude, and the desired orientation of e . A two-part maneuver is technically a Hohmann transfer. Two maneuvers are performed spaced at 180 degrees on the orbit. Given sufficient fuel and adequate thruster configuration, e can be rotated to any orientation and magnitude. The first maneuver, executed at sidereal angle S_{b1} (the calculation of S_{b1} is shown later in equation (3.4-13)), raises perigee to the desired radius while the second maneuver, executed at sidereal $S_{b2} = S_{b1} + \pi$, lowers apogee. These maneuvers, designated ΔV_1 and ΔV_2 , result in eccentricity changes Δe_1

and $\Delta \mathbf{e}_2$ respectively. They can also be performed in the opposite order as long as the total $\Delta \mathbf{e}$ desired is the sum of $\Delta \mathbf{e}_1$ and $\Delta \mathbf{e}_2$.

Once the current and desired orbit elements are calculated, the longitude and eccentricity corrections can be determined. The required $\Delta \dot{\lambda}$ can be calculated as detailed in the previous sections and corrected with the differential corrections method. As was stated earlier, the sidereal angle of this maneuver is not important for longitude control. It also does not matter if the maneuver is performed in one or two-parts. All that matters for longitude control is that the tangential components of the maneuvers performed sum to the right hand side of equation (2.2-8) as

$$\Delta V_{\text{tangential sum}} = \Delta V_{1 \text{ tangential}} + \Delta V_{2 \text{ tangential}} = \frac{-R_{geo}}{3} \Delta \dot{\lambda} \quad (3.4-1)$$

The required $\Delta \mathbf{e}$ is simply found by subtracting the desired or final $\mathbf{e}$, which is determined during mission design, from the current $\mathbf{e}$

$$\Delta \mathbf{e} = \langle \Delta ec, \Delta es \rangle = \mathbf{e}_{\text{final}} - \mathbf{e}_{\text{current}} \quad (3.4-2)$$

This gives both the magnitude and direction of $\Delta \mathbf{e}$. The right ascension of $\Delta \mathbf{e}$, needed later, is determined as the angle between the First Point of Aries and $\Delta \mathbf{e}$ as

$$\Delta e_{\text{right ascension}} = \arctan \frac{\Delta ec}{\Delta es} \quad (3.4-3)$$

With equation (3.4-2) $\Delta \mathbf{e}$ is determined. Noting that S_{b1} and S_{b2} are π radians apart (since $S_{b2} = S_{b1} + \pi$ when performing a Hohmann-type transfer the resulting $\Delta \mathbf{V}_1$ and $\Delta \mathbf{V}_2$ will either both be oriented along the spacecraft velocity, or one may be opposite to the velocity. The directions depend upon the required changes to apogee and perigee and

thus the eccentricity vector. If both $\Delta \mathbf{e}_1$ and $\Delta \mathbf{e}_2$ are smaller in magnitude than $\Delta \mathbf{e}$, then $\Delta \mathbf{e}$, $\Delta \mathbf{e}_1$ and $\Delta \mathbf{e}_2$ are all parallel, i.e.: point in the same direction and hence have the same right ascension. If $\Delta \mathbf{e}_1$ is larger than $\Delta \mathbf{e}$, then $\Delta \mathbf{e}_1$ is parallel to $\Delta \mathbf{e}$ and $\Delta \mathbf{e}_2$ is antiparallel, i.e., the right ascension of $\Delta \mathbf{e}_2$ is equal to the right ascension of $\Delta \mathbf{e}_1$ plus π radians. And if $\Delta \mathbf{e}_2$ is larger than $\Delta \mathbf{e}$, then $\Delta \mathbf{e}_2$ is parallel to $\Delta \mathbf{e}$ and $\Delta \mathbf{e}_1$ is antiparallel. In all cases, the vector sum of $\Delta \mathbf{e}_1$ and $\Delta \mathbf{e}_2$ always equals $\Delta \mathbf{e}$.

If there are no radial components to the maneuvers then the magnitude of $\Delta \mathbf{e}$ is (from equations (3.1-3) and (3.1-4))

$$\Delta e_{\text{tangential}} = \frac{2}{V} (\Delta V_{1 \text{ tangential}} - \Delta V_{2 \text{ tangential}}) \quad (3.4-4)$$

but typically a radial component must be accounted for because some thruster configurations do not allow for a tangential maneuver without a radial component. Considering one maneuver at a time, define $R_{\Delta e}$ as the ratio of the change in $\mathbf{e}$ due to both radial and tangential components of a single maneuver to the change in $\mathbf{e}$ due to the tangential component only

$$R_{\Delta e} \equiv \frac{\Delta e_{\text{total}}}{\Delta e_{\text{tangential}}} \quad (3.4-5)$$

From equations (3.1-3), (3.1-4) and (3.1-5) we know

$$\Delta e_{\text{total}} = \frac{\sqrt{4\Delta V_{\text{tangential}}^2 + \Delta V_{\text{radial}}^2}}{V} \quad (3.4-6)$$

$$\Delta e_{\text{tangential}} = \frac{2\Delta V_{\text{tangential}}}{V} \quad (3.4-7)$$

Now combining equations (3.4-5)-(3.4-7) gives

$$R_{\Delta e} = \sqrt{1 + \left(\frac{\Delta V_{\text{radial}}}{2\Delta V_{\text{tangential}}} \right)^2} \quad (3.4-8)$$

An equation for Δe is known (magnitude only) taking into account both the tangential and radial components of two maneuvers by substituting equation (3.4-4) into equation (3.4-5) to give

$$\Delta e = \frac{2R_{\Delta e}}{V} (\Delta V_{1 \text{ tangential}} - \Delta V_{2 \text{ tangential}}) \quad (3.4-9)$$

Now recall equation (2.2-8) (expanded for clarity as (2.2-8a))

$$\Delta V_{\text{tangential sum}} = \Delta V_{1 \text{ tangential}} + \Delta V_{2 \text{ tangential}} = \frac{-r_{geo}}{3} \Delta \dot{\lambda} \quad (2.2-8a)$$

giving two equations and two unknowns, equations (2.2-8a) and (3.4-9) and unknowns

$\Delta V_{1 \text{ tangential}}$ and $\Delta V_{2 \text{ tangential}}$. The ΔV s can then be solved for as

$$\Delta V_{1 \text{ tangential}} = \frac{V\Delta e}{4R_{\Delta e}} - \frac{r_{geo}}{6} \Delta \dot{\lambda} \quad (3.4-10)$$

$$\Delta V_{2 \text{ tangential}} = \Delta V_{\text{tangential sum}} - \Delta V_{1 \text{ tangential}} \quad (3.4-11)$$

At this point the magnitudes and components of the maneuvers are known, but their locations are not yet determined. Equation (3.1-3) or (3.1-4) can be numerically solved to determine the locations, however an analytic result can be found from a geometric argument as follows. Figure 10 illustrates this geometry. The sidereal location of a maneuver can be determined by constructing a perpendicular line segment to the tangential component of the ΔV .

Figure 7. Determination of Maneuver Location.

The $\Delta \mathbf{V}$ vector is first located using the angle between the $\Delta \mathbf{V}$ and the $\Delta \mathbf{e}$ vector. Referring back to equations (3.1-3) and (3.1-4), note that as the sidereal angle of the maneuver changes, the magnitude of the resultant $\Delta \mathbf{e}$ does not change and neither does the relative orientation of $\Delta \mathbf{e}$ to $\Delta \mathbf{V}$. In order to determine the $\Delta \mathbf{V}$ orientation, the angle between $\Delta \mathbf{V}$ and $\Delta \mathbf{e}$ is needed. A simple case is shown and is easily generalized. With this information, the sidereal angles can be determined where the maneuvers should be executed to produce a $\Delta \mathbf{e}$ in the desired direction.

Assume a $\Delta \mathbf{V}$ with unity magnitude is executed at $S_b = 0$ that includes a small radial component inwardly directed.

Let: $\Delta \mathbf{V} = \Delta \mathbf{V}_{\text{tangential}} + \Delta \mathbf{V}_{\text{radial}}$ where $|\Delta \mathbf{V}_{\text{tangential}}| = t$ and $|\Delta \mathbf{V}_{\text{radial}}| = r$

The sidereal angle of $\Delta \mathbf{V}$ is zero, so from equations (3.1-3) and (3.1-4)

$$\Delta \mathbf{e} = \langle \Delta e_c, \Delta e_s \rangle = \left\langle \frac{2t}{V}, \frac{-r}{V} \right\rangle \text{ and } \Delta \mathbf{V} = \langle -r, t \rangle$$

Now the dot product is used with $\Delta \mathbf{V}$ and $\Delta \mathbf{e}$ to find the angle between the two vectors as

$$\Delta \mathbf{V} \bullet \Delta \mathbf{e} = |\Delta \mathbf{V}| |\Delta \mathbf{e}| \cos \theta \Rightarrow \theta = \arccos \left[\frac{-3rt}{\sqrt{r^4 5r^2 t^2 + 4t^4}} \right] \quad (3.4-12)$$

The right ascension of $\Delta \mathbf{e}_1$, calculated with equation (3.4-3) and the logic in the paragraph following equation (3.4-3), may now be added to θ from equation (3.4-12) to render the right ascension of $\Delta \mathbf{V}_1$, denoted γ . The angle γ is the measure of the angle from the First Point of Aries to the $\Delta \mathbf{V}_1$ vector, not the location in the orbit where the maneuver is to take place. One more angle, ϕ , is required and is the angle between the $\Delta \mathbf{V}$ and its tangential component (recall that due to the particular thruster configuration, a small radial component to the maneuver may occur). Figure 10 shows the relationship between γ , ϕ and S_{b1} . The angle S_{b1} is the sidereal angle of the maneuver and is calculated as

$$\begin{aligned} S_{b1} &= \gamma - \phi + 3\frac{\pi}{2} \quad \text{for prograde maneuvers} \\ S_{b1} &= \gamma - \phi + \frac{\pi}{2} \quad \text{for retrograde maneuvers} \end{aligned} \quad (3.4-13)$$

And S_{b2} is, as stated earlier, $S_{b1} + \pi$.

Figure 8. Angles Needed for Determining the Sidereal Angle of a Maneuver.

The flow chart in Figures 11, 12 and 13 summarize the proposed steps flight software would take to perform longitude station-keeping with differential corrections and active eccentricity control using Two-Part Burns.

Figure 9. Flow Chart for Longitude Station-Keeping with Eccentricity Control Using Two-Part Burns.

Figure 9. Flow Chart for Longitude Station-Keeping with Eccentricity Control Using Two-Part Burns (Page 2).

Figure 9. Flow Chart for Longitude Station-Keeping with Eccentricity Control Using Two-Part Burns (Page 3).

CHAPTER 4 RESULTS

Spacecraft parameters from a commercial communication satellite were used for this analysis. Initial orbit elements were chosen with zero inclination, Earth reference longitude of 35 degrees east longitude, orbit epoch of March 21, 2006, eccentricity of 0.0003 and an argument of perigee of zero. Although testing and analysis were done with a variety of initial conditions, the figures presented here refer only to the above specifications.

Figure 10 shows a longitude history of one year with a longitude station of 35° east longitude. Both longitude and eccentricity are controlled with single maneuvers (computed from equations (2.2-9) and (2.2-10)) spaced at seven days intervals. Note that the spacecraft longitude never exceeds ± 0.015 degrees, well within the ± 0.05 degree requirement.

Figure 10. Station Keeping Longitude History Using Eccentricity Control.

Figure 11 shows two longitude histories. The broken line displays longitude control without differential corrections implemented and the solid line includes differential corrections. Note the consistency of the longitude when the maneuvers are computed using differential corrections. Also note the pattern of the depth of the parabolas, alternating shallow and deep. In this simulation, the longitude correction maneuvers were performed every seven days or one quarter of the Moon's cycle and the lunar gravity is causing this depth variation.

Figure 11. Longitude History with and without Differential Corrections.

Figure 12 shows an eccentricity history over five years with the Sun-pointing-perigee strategy employed. The axes of this graph are coincident with the MEGDS equatorial plane. The broken line circle shows the eccentricity limit of 0.0003. This circle is often referred to as an eccentricity control circle. Notice that the eccentricity vector makes five rotations about the origin, one per year, as it follows the Earth-Sun line with the magnitude contained by the eccentricity control circle.

Figure 12. Eccentricity History with Sun-Pointing Perigee Strategy Employed.

Figure 13 shows an eccentricity history over one year with two-part maneuvers. Like Figure 12, the axes of this graph are also coincident with the MEGDS equatorial plane. Each “tic” mark along the large circle results from the temporary change in eccentricity during the period between the two maneuvers. Notice that eccentricity is very tightly controlled. With two-part maneuvers the eccentricity vector can be controlled to almost any desired magnitude and direction. With this control scheme, the eccentricity control circle does not have to be centered at the origin, as is required with some co-location schemes.

Figure 13. Eccentricity History Using Two-Part Maneuvers.

CHAPTER 5 CONCLUSIONS

By developing rules based on known orbit equations and how ground station crews currently control longitude and eccentricity, an algorithm has successfully been written to perform the task of controlling longitude and eccentricity autonomously aboard a spacecraft. The algorithms developed form the basis for flight software being prepared for a geostationary spacecraft scheduled to launch in the next few years.

Inclination is an orbit parameter that has not been controlled in this algorithm. When inclination control maneuvers are performed, errors in the maneuvers often change the longitude drift rate. These errors need to be corrected usually within one orbit or even as soon as a half an orbit after the maneuver. Methods of combining inclination control with longitude and eccentricity control is one area of possible future research. Planning inclination maneuver at or about the time eccentricity and longitude drift is to be corrected with a follow up eccentricity and longitude control maneuver may be a good approach. Other methods should be explored.

Another area of possible related research comes from the fact that this algorithm assumes the use of instantaneous ΔV s. Continuous/low thrust propulsion is becoming more popular with current satellites. Low thrust propulsion uses less fuel for the same magnitude ΔV and therefore can extend the life of a mission leading to more revenue on commercial missions and more service out of military missions. Since continuous/low thrust would likely require a different approach (continuous/low thrust maneuvers cannot be modeled as instantaneous), future work could develop modifications to the methods presented here when continuous thrusting systems are used.

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APPENDIX A Differential Correction Derivation

This appendix provides a derivation of the analytic approximation for the $\frac{\partial \lambda(t_f)}{\partial \Delta V}$ term used in the differential corrections algorithm. Starting with the finite difference formula

$$\frac{\partial \lambda(t_f)}{\partial \Delta V} \approx \frac{\lambda_2(t_f) - \lambda_1(t_f)}{\Delta V_2 - \Delta V_1} \quad (\text{A-1})$$

The difference in λ_1 and λ_2 is computed using equation (2.2-1) with the initial drift rate perturbed by $\Delta \dot{\lambda}$ to give

$$\lambda(t) = \frac{1}{2} \ddot{\lambda}^2 + (\dot{\lambda}_0 + \Delta \dot{\lambda})t + \lambda_0. \quad (\text{A-2})$$

Recalling equation (2.2-7) and substituting in equation (A-2) gives

$$\lambda_1(t_f) = \frac{1}{2} \ddot{\lambda}^2 + \left(\dot{\lambda}_0 - \frac{3}{r_{geo}} \Delta V_1 \right) t_f + \lambda_0 \quad (\text{A-3})$$

$$\text{Defining } \Delta V_2 = \Delta V_1 + \delta \quad (\text{A-4})$$

and replacing ΔV_1 in equation (A-3) with ΔV_2 gives $\lambda_2(t_f)$ as

$$\lambda_2(t_f) = \frac{1}{2} \ddot{\lambda}^2 + \left(\dot{\lambda}_0 - \frac{3}{r_{geo}} (\Delta V_1 + \delta) \right) t_f + \lambda_0 \quad (\text{A-5})$$

Subtracting equation (A-3) from equation (A-5) gives

$$\lambda_2(t_f) - \lambda_1(t_f) = -\frac{3}{r_{geo}} \delta \cdot t_f. \quad (\text{A-6})$$

From equation (A-4)

$$\Delta V_2 - \Delta V_1 = \delta, \quad (\text{A-7})$$

thus

$$\frac{\partial \lambda(t_f)}{\partial \Delta V} = \frac{-3t_f}{r_{geo}} \quad (\text{A-8})$$

Hence, the differential correction equation needed is

$$\delta \Delta V = \frac{-r_{geo}}{3t_f} \delta \lambda(t_f). \quad (\text{A-9})$$

APPENDIX B $\delta \mathbf{e}$ Due to $\Delta \mathbf{V}$ Derivation

The instantaneous change in the eccentricity vector due to a “small” $\Delta \mathbf{V}$ applied along a geostationary orbit can be found by taking the differential of the eccentricity vector

$$\mathbf{e} = \frac{1}{\mu}(\mathbf{V} \times \mathbf{h}) - \hat{\mathbf{r}} \quad (\text{B-1})$$

to obtain

$$\delta \mathbf{e} = \frac{1}{\mu}(\delta \mathbf{V} \times \mathbf{h} + \mathbf{V} \times \delta \mathbf{h}) - \delta \hat{\mathbf{r}} \quad (\text{B-2})$$

where $\mathbf{e}$ represents the eccentricity vector, μ is the gravitational parameter, $\mathbf{V}$ is the velocity of the spacecraft, $\mathbf{h}$ is the specific angular momentum of the spacecraft, and $\hat{\mathbf{r}}$ is the unit vector along the radial direction. Since the location of the spacecraft does not change during the application of the instantaneous maneuver, $\delta \hat{\mathbf{r}} = 0$.

To further simplify equation (B-2), the direction perpendicular to $\hat{\mathbf{r}}$ within the orbit plane and in the general direction of the spacecraft velocity is defined as $\hat{\boldsymbol{\theta}}$, while the direction along the angular momentum vector is defined as $\hat{\mathbf{h}}$ to complete the right-handed rotating frame. Then the position vector of the spacecraft is defined as

$$\mathbf{r} = r\hat{\mathbf{r}} \quad (\text{B-3})$$

while the velocity is designated

$$\mathbf{V} = V_r\hat{\mathbf{r}} + V_t\hat{\boldsymbol{\theta}} \quad (\text{B-4})$$

Substituting equations (B-3) and (B-4) into equation (B-2) and recalling that $\mathbf{h} = \mathbf{r} \times \mathbf{V}$ so that $\delta \mathbf{h} = \mathbf{r} \times \delta \mathbf{V}$ (applying $\delta \hat{\mathbf{r}} = 0$ again) results in

$$\delta \mathbf{e} = \frac{r}{\mu} \left[(2V_t \Delta V_t) \hat{\mathbf{r}} - (V_t \Delta V_r + V_r \Delta V_t) \hat{\boldsymbol{\theta}} \right] \quad (\text{B-5})$$

The unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ can be expressed in terms of the MEGSD frame using the sidereal angle of the spacecraft at the time of the maneuver as

$$\begin{aligned} \hat{\mathbf{r}} &= \cos S_b \hat{\mathbf{x}} + \sin S_b \hat{\mathbf{y}} \\ \hat{\boldsymbol{\theta}} &= -\sin S_b \hat{\mathbf{x}} + \cos S_b \hat{\mathbf{y}} \end{aligned} \quad (\text{B-6})$$

Substituting equation (B-6) into equation (B-5) produces

$$\begin{aligned} \delta \mathbf{e} = \frac{r}{\mu} & \left[(2V_t \Delta V_t \cos S_b + V_t \Delta V_r \sin S_b + V_r \Delta V_t \sin S_b) \hat{\mathbf{x}} \right. \\ & \left. + (2V_t \Delta V_t \sin S_b - V_t \Delta V_r \cos S_b - V_r \Delta V_t \cos S_b) \hat{\mathbf{y}} \right] \end{aligned} \quad (\text{B-7})$$

For a geostationary orbit, V_r is small and can be ignored to first order. Similarly, V_t is approximately equal to V and $\frac{r}{\mu} \equiv \frac{1}{V^2}$ for small eccentricities. Using these

approximations in equation (B-7) gives the final result as

$$\delta \mathbf{e} = \left(2 \frac{\Delta V_t}{V} \cos S_b + \frac{\Delta V_r}{V} \sin S_b \right) \hat{\mathbf{x}} + \left(2 \frac{\Delta V_t}{V} \sin S_b - \frac{\Delta V_r}{V} \cos S_b \right) \hat{\mathbf{y}} \quad (\text{B-8})$$

APPENDIX C $\delta \mathbf{e}$ Due to Solar Radiation Pressure Derivation

Denote the sidereal angle (recall: an angle measured from the x -axis in the MEGSD plane) of the projection of the Sun's direction onto the orbital plane by s_s .

When the spacecraft is at the sidereal angle s , the two components in the tangential and radial directions of the velocity differential dV from the radiation pressure become, respectively:

$$dV_t = \sin(s - s_s)dV \quad ; \quad dV_r = -\cos(s - s_s)dV \quad (\text{C-1})$$

From equations (3.1-3) and (3.1-4) a differential equation for the eccentricity vector $\mathbf{e}$ is obtained as

$$\frac{d\mathbf{e}}{dt} = \frac{2}{V} \begin{pmatrix} \cos s \\ \sin s \end{pmatrix} \frac{dV_t}{dt} + \frac{1}{V} \begin{pmatrix} \sin s \\ -\cos s \end{pmatrix} \frac{dV_r}{dt} \quad (\text{C-2})$$

By integrating this equation over the time interval $\delta t = 2\pi/\psi = 1$ sidereal day, after replacing $\delta t = \delta s/\psi$, the mean drift rate of $\mathbf{e}$ is obtained. Since the drift is very small it is not of interest to consider the details of its variation over intervals shorter than a day.

$$\begin{aligned} \frac{\delta \mathbf{e}}{\delta t} &= \frac{P\sigma}{2\pi V} \int_0^{2\pi} \left[2 \begin{pmatrix} \cos s \\ \sin s \end{pmatrix} \sin(s - s_s) - \begin{pmatrix} \sin s \\ -\cos s \end{pmatrix} \cos(s - s_s) \right] ds = \\ &\frac{3P\sigma}{2V} \begin{pmatrix} -\sin s_s \\ \cos s_s \end{pmatrix} \end{aligned} \quad (\text{C-3})$$

This result shows that the drift is in the direction $+90^\circ$ from the direction to the Sun (Soop p. 93 – 94).