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THREE-DIMENSIONAL IMAGE REGISTRATION

A Thesis

Presented to

The Department of Mechanical and Aerospace Engineering

San Jose State University

In Partial Fulfillment

Of the Requirements for the Degree

Master of Science

by

Miki Sode

December 2003

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ABSTRACT

THREE-DIMENSIONAL IMAGE REGISTRATION

by Miki Sode

The objective of this study is to quantify the performance of a three-dimensional image registration technique developed by Yan *et al.* (1998), and to develop general guidelines for its use. A series of studies was performed to quantify the accuracy and the precision of this algorithm. The characteristics of rod configuration of interest included the number of coplanar rods, the number of non-coplanar rods, and the number of contiguous slices included in the analysis. The x-ray energy level settings, the Computed Tomography (CT) slice thickness, and the image reconstruction thickness were the scan parameters tested. The rotations measured by the inclinometers were compared to those calculated by the algorithm to be within, approximately, $\pm 0.110^\circ$ in pitch and $\pm 0.094^\circ$ in roll, respectively. Slight but notable improvements in algorithm performance were observed as both numbers of coplanar and non-coplanar rods increased. X-ray energy level settings at 80 keV @ 250 mA demonstrated slightly better accuracy than 120 keV @ 190 mA. The CT slice thickness at 3.0 mm improved the algorithm performance compared to 1.0 mm. Image reconstruction thicknesses set equal to rather than smaller to the CT slice thickness improved algorithm performance.

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NOMENCLATURE

a = Normalized intercept

$\Delta\theta_{\text{inclinometer}}$ = Relative angular displacement between two CT image sets calculated from inclinometer measurements

$\Delta\theta_{\text{algorithm}}$ = Relative angular displacement between two CT image sets calculated by the algorithm

k = A constant

m = Normalized inclination

N = Number of rod sets analyzed

r = Equation of a line

r' = Equation of a transformed line

p = Probability value of a statistical hypothesis test (two-sided paired t-test)

R = Rotational matrix (3x3)

R^2 = A measure of a linear relationship between two sets of variables

T = Translational matrix (3x1)

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INTRODUCTION

Three-dimensional imaging tools and accurate methods of measuring bone density are essential for non-invasively studying bone adaptation due to long-duration space flight, changes in daily activity, aging, and interventional therapies. Three-dimensional image registration techniques allow images to be aligned with a previous image set in order to match the spatial location within a bone for accurate assessment of changes in bone density over time. Image registration is also useful for combining information in datasets from different imaging modalities (such as computed tomography (CT), magnetic resonance imaging (MRI), and positron emission tomography (PET)) or in temporal sequences for medical diagnosis, treatment and research (Heilbrun *et al.* 1994; Peters *et al.* 1996; Petti *et al.* 1994). For example, image registration is often used in clinical applications such as in stereotactic neurosurgery, in order to give surgeons information on brain structures during an operation (Grimson *et al.* 1996; Heilbrun *et al.* 1994; Lee *et al.* 2002; Peters *et al.* 1986, 1996).

All image registration methods determine the spatial transformation between two image-sets by mapping one image to the other. There are several registration methods based on internal landmarks (anatomical), external landmarks (frame-mounted fiducials), matching voxels, and mapping surfaces (Heilbrun *et al.* 1994; Lee *et al.* 2002; Maes *et al.* 1997; Peters *et al.* 1986; Petti *et al.* 1994; Yan *et al.* 1998).

Typical stereotactic frames are comprised of N- or V-shaped rigid frames with several fiducial markers. The fiducial marker is a solid substance placed inside a plastic stereotactic frame tube; it lights up in the CT scan for radiographic visualization. The

frame is mounted, for example, on the patient's head during a scan and fiducial markers situated at known three dimensional locations are identified in order to define the coordinate system of the three-dimensional image dataset (Heilbrun *et al.* 1994; Peters *et al.* 1986). A drawback of currently available frame-based image registration algorithms is the dependency on the mathematical model describing the configuration of the frame (Yan *et al.* (1998). Distortion of the frame will introduce image registration errors because the new shape is not exactly represented by the mathematical model (Yan *et al.* 1998). In addition, these methods only utilize a portion of the available spatial information (Maintz and Viergever 1998; Yan *et al.* 1998). Fixed frames may also not be convenient in size and shape for the object being scanned.

Yan *et al.* (1998) proposed an innovative registration algorithm that overcomes the disadvantages of current frame-based registration techniques. The algorithm assumes the object has undergone a rigid body transformation. This assumption is valid for registering a single bone. It only requires a set of two or more straight rods; therefore, (1) no mathematical model of the configuration, and (2) no a priori knowledge of position of the rods are necessary. The technique takes full advantage of spatial information obtained from a three-dimensional set of CT scan images. Assuming only that the rods are straight, the algorithm finds the equations of the rods from the three dimensional data in two data sets, then calculates the transformation between the two. A least-squares weighting scheme to determine the best-fit equation of the rods contributes to the high accuracy of the algorithm. Mathematically, this algorithm is in a determinant algebraic form, thus parametric optimization is not required.

This algorithm was previously tested only for its robustness against model mismatch in simulations done by Yan *et al.* (1998) and its performance was significantly better than Parallel-N and Point-Based techniques ($p \leq 0.05$). This image registration technique has not yet been tested experimentally for its performance. The performance of the algorithm can be described by its accuracy and precision. The accuracy of the algorithm describes how exact the algorithm can calculate the relative spatial transformation between two image sets compared to measured transformation values. Furthermore, precision illustrates the repeatability of the algorithm's calculation. The objective of this study is to quantify the accuracy and precision of this three-dimensional image registration algorithm proposed by Yan *et al.* (1998), and to develop general guidelines for using the proposed registration algorithm.

EXPERIMENTAL DESIGN

Computed Tomography (CT)

A computed tomography scanner is a widely used medical imaging device that uses x-rays to examine internal organs for visualization and diagnosis. The imaging component is comprised of a revolving x-ray beam source and a detector on the opposite side. The x-ray source and detector revolve in a ring around the patient to obtain x-ray projections (views) from all angles (360°). The patient lies on a horizontal table. In the new machines, the x-ray source and detector, and the table, move together so that the x-ray projection traces out a helical path along the scanned object. These machines are

called helical CT scanners. All scans were taken at the Stanford University Medical Center with the GE CT/I Hi Speed Advantage helical scanner (Figure 1).

The width of the x-ray beam determines the CT slice thickness. Typical CT slice thicknesses are 1 mm and 3 mm, respectively. Thicker slices contain less scan noise but have lower spatial resolution. A two-dimensional image is “reconstructed” from the x-ray projection taken in a 360° view around the object in a cross-sectional plane. CT slice thickness and reconstructed image thickness are usually set to be equal in clinical settings. It is possible to reconstruct an image at a different thickness than the CT slice thickness. Interpolation is performed to obtain the x-ray projection in the image plane. A three-dimensional CT image of the body, or an object, is constructed from a series of contiguous two-dimensional images.

The x-ray attenuation is unique to a material, and is expressed in Hounsfield units (HU), or CT number. It is adjusted so that the water is at 0 HU, and air is at -1000 HU. For a reference, acrylic is at ~100 HU, and a cortical bone ranges from 1500 to 2000 HU, depending on density. A two-dimensional computed tomography image consists of 512 × 512 array of CT numbers (Figure 5(a)). A visual image is constructed from a gray-scale representation of the CT numbers.

Figure 1. GE CT/I HiSpeed Advantage Scanner at Stanford University Hospital.

Fixture

An acrylic cylindrical chamber with square ends was constructed, allowing glass rods to be placed inside at various orientations (Figure 2(a)). The rod placement for this study incorporated 16 rods in four different planes (right, left, top, and bottom) running parallel to scan direction (see also Figure 5(b)). Notice each rod was placed in various directions, inclinations, and spatial locations, so that the effect of the rod configuration and orientation on the algorithm performance could be studied by extracting various sets of rods. These configurations will be covered in detail in the Methodology section.

The rods and support structure surround another acrylic cylindrical fixture that was designed to hold a calcaneus (heel bone) (Figure 2(a)). This fixture was designed for future studies which will use rod registration as the “gold-standard” to test the accuracy of a surface-based registration of the calcaneus. Determination of the accuracy of the surface-based registration algorithm is, however, beyond the scope of this study.

Prior to scanning, the acrylic cylinder was filled with water to mimic *in vivo* scan conditions. The water increases image noise (reduces signal to noise ratio) and reduces the gradient in CT number between materials in the scan image. The square end of the chamber was attached to a rigid aluminum mounting plate bolted to the end of CT scanning table (Figure 2(b)). Figure 3 shows the aluminum base, outer acrylic cylinder, and inclinometer assembly without internal glass rods.

Figure 2. (a) The acrylic case encloses several straight glass rods and a mount holding an object (plastic calcaneus) enabling tests of proposed and surface-based registrations simultaneously. (b) The rigid aluminum mounting plate.

Figure 3. The assembly of the inclinometers, acrylic fixture, and the aluminum mount.

Inclinometers

The two inclinometers/tilt sensors (SX-015D-VIB, Advanced Orientation Systems, Inc., Linden, NJ) were rigidly mounted to the square end of the acrylic cylinder as shown in Figure 4. The inclinometers were perpendicular to each other and normal to Earth's gravity in order to measure the roll and pitch inclinations.

The inclinometer analog resolution is <5 arcsec, or $<0.001^\circ$, and its repeatability is $<0.002^\circ$. The sensor system incorporates a 12 bit analog to digital converter. The digital resolution is $<0.008^\circ$. The system was linearly calibrated in the range of $\pm 10^\circ$.

The manufacture's specification indicates a temperature sensitivity of the tilt sensor SX-015D-VIB. However, physical tests of the sensors under similar temperature

conditions as the CT room indicated sufficient temperature stability to carry out the experiments without thermal isolation or temperature compensation.

Figure 4. Two tilt sensors (SX-015D-VIB) placed perpendicularly to each other and rigidly attached to the fixture to measure the pitch and roll.

Glass Rods

Straight glass rods were obtained from Technical Glass Products, Inc. (Gonzales, LA). Fused quartz rods with a 4 mm diameter and a maximum bow of 1.3 mm per 1220 mm of length were used in this study. Glass has a high CT number (1500 HU) compared to water and soft tissues. The rods were, therefore, easily detected in CT images.

Rod-Based Registration Algorithm: Overview

The following is a summary of the registration algorithm proposed by Yan *et al.* (1998). The description is provided for basic understanding of the technique and requirements for the input and output of the algorithm.

Post-processing of the scans first uses the CT threshold values to detect the perimeter contour of the cross-section of each rod in each CT image (Figure 5(a)). From the elliptical contour outlines of rod cross sections, the algorithm automatically calculates the centroid of each rod cross-section in each slice for the entire set. A least squares weighing scheme for the straight line is fitted through those data points such that

$$\mathbf{r} = \mathbf{k}\mathbf{m} + \mathbf{a}$$

where $\mathbf{m}$ and $\mathbf{a}$ are the normalized inclination and intercept. These line parameters are obtained for every rod to produce three-dimensional line profiles (Figure 5(b)). A baseline set and another set that has undergone a rigid body transformation (rotation and translation) are obtained and fed into the algorithm.

Figure 5. (a) Example of a CT slice. In the center is a cross-section of a calcaneus. Since the glass rods have higher density than surroundings, their cross-sections light up enabling the algorithm to detect the edges and calculate the centroid. From contiguous slices of CT images, the line profiles of all rods are created as an input to the algorithm. (b) Example of three-dimensional line profiles obtained by tracking the centroids of straight glass rods over an entire set of CT slices. It shows a line profile of all 16 available rods. Note z-axis is the direction of the scan. Rods lie on four different planes: top, bottom, left and right.

The objective of the algorithm is to obtain the transformation that maps the rod line profiles of one dataset into the other dataset with the least overall estimation error.

Assuming rigid body transformations only, the transformation is linear and restricted to only translation and rotation. The equation of a transformed line (r') becomes

$$r' = kRm + Ra + T.$$

The algorithm outputs rotational and translational matrices (R , 3×3 , and T , 3×1 , respectively) that minimize the mismatch between mapped line profile and the second line profile. From the rotational matrix R , the Eulerian angular displacement in each rotational axis between two sets of images can be calculated.

Notice that rotational operation R is followed by translation T . In other words, rotation and translation are independent of each other, and translation is trivial compared to the rotation. Due to its inherit design, the accuracy of the registration algorithm is more sensitive to the performance of R , thus this is of particular interest. In this study, only the rotational transformation was prescribed to the fixture.

The algorithms were written in MatLab and ran on a Windows 2000 PC with a Pentium IV 2GHz processor and 1GB RAM.

METHODOLOGY

A series of CT scans of the fixture with 16 glass rods (as previously described) was taken on a GE CT/I HiSpeed Advantage scanner (single detector ring machine). The fixture was rotated and scanned in 5 orientations:

1. Baseline (no angular displacement) ("base"),
2. 5° of angular displacement in roll only ("roll 5"),
3. 5° of angular displacement in pitch only ("pitch 5"),

4. 5° of angular displacement each in both roll and pitch (“comb 5”), and
5. 1° of angular displacement in roll only (“roll 1”).

Note that the above prescribed rotations are only approximate. In all cases, the inclinometer reading was taken as the actual rotation.

The 1° rotation in roll was to determine the sensitivity of the algorithm to small angular displacements. The yaw angle was not tested because of the difficulty in accurately determining absolute measurement since yaw occurs in the plane normal to gravity and inclinometers will not function in this direction. From the 5 sets of CT scan datasets in 5 different rotational positions, a total of 10 combinations of relative rotational displacements between the two CT datasets were obtained for the analysis (Table 1). The analyses were performed separately in three situations where there was an angular rotation in (a) roll only, (b) pitch only, and (c) both roll and pitch (combined).

Error in pitch and roll were measured and analyzed in all cases. For example, although the prescribed rotation may have been pitch = 0° and roll = 5° (combination #2, Table 1), a small angular displacement in pitch may have occurred because of the experimental setup.

Figure 6. Pitch, roll and yaw defined in respect to scan direction.

Combination	Set 1	Set 2	Pitch	Roll	Prescription Group
1	Base	Roll 1	0	1.0°	(a) Roll only
2	Base	Roll 5	0	5.0°	(a) Roll only
3	Base	Pitch 5	5.0°	0	(b) Pitch only
4	Base	Comb 5	5.0°	5.0°	(c) Combined
5	Roll 1	Roll 5	0	4.0°	(a) Roll only
6	Roll 1	Pitch 5	5.0°	1.0°	(c) Combined
7	Roll 1	Comb 5	5.0°	4.0°	(c) Combined
8	Roll 5	Pitch 5	5.0°	5.0°	(c) Combined
9	Roll 5	Comb 5	5.0°	0	(b) Pitch only
10	Pitch 5	Comb 5	0	5.0°	(a) Roll only

Table 1. Ten combinations of scans to be used in the studies with associated relative angular displacements and the corresponding prescription group. Values are only approximate.

All scan images were taken in helical mode with a field of view (FOV) of 18 cm and a 1 mm or 3 mm CT slice thickness. The scanner voltage and current settings were set at 120 keV and 190 mA, except for the *x-ray energy setting level settings* study. With

a field of view of 18 cm, an image size of 512 x 512 voxels leads to an in-plane voxel resolution of 0.35 mm x 0.35 mm ($1800 \text{ mm} / 512 = 0.35 \text{ mm}$). One scan set covering the entire fixture was approximately 100 mm in length. The number of slices in each dataset depends on the thickness of the image. For example, with CT images reconstructed every millimeter, a 100 mm scan length will have 100 images.

The relative angular displacement between the two CT image sets was calculated from inclinometer measurements ($\Delta\theta_{\text{inclinometer}}$). These were taken as the measured values. Assuming the algorithm's performance is consistent in any degree of angular rotation, the angular displacement in each axis calculated by the algorithm ($\Delta\theta_{\text{algorithm}}$) was compared to the corresponding measured value obtained from inclinometer measurements. A difference between the calculated values determined by the algorithm and the measured value measured by the inclinometer, namely $\Delta\theta_{\text{inclinometer}} - \Delta\theta_{\text{algorithm}}$, expressed in degrees, was recorded (for most cases) for each rotational axis.

The performance of the algorithm was characterized by the accuracy and the precision. The accuracy of the algorithm is the best estimate of a relative spatial transformation between two image sets determined by the algorithm with respect to the measured values. Particularly it was expressed in terms of an average value of all reported values ($\Delta\theta_{\text{inclinometer}} - \Delta\theta_{\text{algorithm}}$). The precision describes the repeatability of the algorithm calculation. The standard deviations of the differences between the measured and calculated relative angular displacements of the two image sets were reported as measures of the algorithm precision.

Since the yaw angle could not be measured accurately against the gravity, the accuracy of the algorithm in determining the yaw angle could not be assessed; therefore it was not included in this study.

Study protocols were constructed to determine the performance of the new three-dimensional registration technique developed by Yan *et al* (1998). Various factors may influence the performance of the rod registration:

- The configuration of glass rods (Number of coplanar and non-coplanar rods)
- Length of the rods inclined in the analysis
- CT parameters settings (x-ray beam energy, CT slice thickness, and image reconstruction thickness)

The following section discusses the series of studies performed to quantify the precision and accuracy of the algorithm in relation to these factors.

I. Configuration study

The numbers of coplanar and non-coplanar rods define the rod configuration. Many configurations of rod combinations with various spatial orientations can be extracted from the original scan image by selecting a reduced set of rods. These new line profiles of a rod configuration can be used to investigate the effect of each aspect of the rod configuration on the algorithm performance.

Number of coplanar rods

The addition of coplanar rods may give more information, resulting in improved accuracy and precision. The objective of this study was to investigate whether information from additional rods in a single plane affects the algorithm performance.

All 10 scan combinations were used and analyzed according to the corresponding three angular displacement groups, (a) roll only, (b) pitch only, and (c) combined (refer to Table 1). Only rods from one of the four planes were used in each analysis. Each of the four planes was analyzed separately. Due to the inherent limitations of fixture design, only two to five coplanar rods were available for each analysis (Figure 7).

(a) 2 non-parallel coplanar rods
(88 combinations analyzed)

(b) 3 coplanar rods
(52 combinations analyzed)

(c) 4 coplanar rods
(40 combinations analyzed)

(d) 5 coplanar rods
(8 combinations analyzed)

Figure 7. A sample of 4-rod configurations used for the number of coplanar rods study. A total of 188 combinations were analyzed.

Number of non-coplanar rods

While a set of two non-parallel coplanar rods is required to determine the rigid body rotation, the addition of rods in other planes may improve the algorithm performance. The objective of this study was to examine the effect of the number of non-coplanar rods on the algorithm performance.

All 10 scan combinations were used and analyzed according to the corresponding three angular displacement groups, (a) roll only, (b) pitch only, and (c) combined (refer to

Table 1). Only one rod was taken from a given plane in each analysis. Due to the inherent limitations of fixture design, only two to four planes (thus two to four rods) were tested (Figure 8).

Figure 8. A sample of 3-rod configurations used in the number of non-coplanar rods study. A total of 2236 combinations were analyzed.

Number of contiguous slices included (rod length)

The accuracy of a least squares fit through each rod centroid may improve if it is applied to longer span of the rod. If only a small number of slices are available, the accuracy of the least squares fit may be compromised. The objective of this analysis was

to determine the sensitivity of the algorithm performance to rod length, which is equivalent to the number of contiguous slices.

All 10 scan combinations were used according to the corresponding three angular displacement groups, (a) roll only, (b) pitch only, and (c) combined (refer to Table 1). Within each combination, configurations with two non-parallel coplanar rods were analyzed. Each dataset started with 202 slices that covers 100 mm (image reconstruction thickness = 0.5 mm). The length was reduced by 20 slices (10 mm) for each iteration (10 slices were taken from each end). A total of 10 iterations were performed, reducing the dataset down to 22 contiguous slices (11 mm scan length) in the last iteration.

II. Parametric study

Noise may alter the centroid location determination of each rod in each slice, resulting in erroneous three-dimensional line profiles and/or termination of the algorithm. The objective of this study was to determine the influence of scan parameters that affect image noise level on the algorithm performance. The parameters of particular interest are: x-ray beam energy level settings (voltage and current), CT slice thickness, and image reconstruction thickness.

The parameter studies required three baseline CT scans and three CT scans with a combination of 5° in roll and 5° in pitch (combination #4; see Table 1). The three sets of scan settings were

- 1) 80 keV @ 250 mA, CT slice thickness = 1.0 mm
- 2) 120 keV @ 190 mA, CT slice thickness = 1.0 mm

3) 120 keV @ 190 mA, CT slice thickness = 3.0 mm.

After scanning, the CT scan taken at 120 keV and CT slice thickness of 1.0 mm was retrospectively reconstructed at 0.5 mm and 1.0 mm; the CT scan taken at 120 keV and CT slice thickness of 3.0 mm was retrospectively reconstructed at 1.0 mm and 3.0 mm. The combinations of CT scan parameters used in the various studies are listed in Table 2.

Three rod configurations were used in the following studies: 1) all 16 available rods; 2) 4 non-coplanar rods, *i.e.*, one rod per plane; and 3) 2 non-parallel coplanar rods.

Parametric study	keV @ mA	CT Slice Thk. [mm]	Image Recon. Thk. [mm]
X-ray energy level settings (voltage & current)	80, 250	1.0	0.5
	120, 190	1.0	0.5
CT slice thickness	120, 190	1.0	1.0
	120, 190	3.0	1.0
Image reconstruction thickness	120, 190	1.0	0.5
	120, 190	1.0	1.0
	120, 190	3.0	1.0
	120, 190	3.0	3.0

Table 2. The CT scan parameter settings for the various parameter studies. These were taken at both baseline and combination of 5° in roll and 5° in pitch.

X-ray energy level settings (voltage and current)

Voltage (keV = kilo electron volts) directly controls the energy profile of the photons emitted. Higher energy photons pass more readily through tissues, producing more detected photons but reduce contrast between tissues. The CT tube current (mA = milliamps) controls photon flux density. A higher current produce more photons per unit time which reduces noise. Algorithm performance was compared between two commonly used voltage and current settings, 80 keV for 250 mA and 120 keV for 190 mA (see Table 2).

CT slice thickness

CT slice thickness is inversely related to the noise level. In clinical settings, a smaller CT slice thickness is encouraged to discriminate objects contained in the volume of interest. CT slice thickness also contributes to spatial resolution (edge definition). Since decreasing CT slice thickness introduces more noise, there exists a trade-off between spatial resolution (voxel size) and noise level. A voxel may be small but have too much noise that objects are not discernable.

The objective of this study was to examine the effect of CT slice thickness on the algorithm performance. Scans taken at 120 keV with CT slice thicknesses of 1.0 mm and 3.0 mm were compared (see Table 2).

Image reconstruction thickness

Images reconstructed at a smaller thickness than the CT slice thickness (beam width) have a lower signal to noise ratio caused by interpolation of x-ray projection data, but have higher spatial resolution (smaller voxels). On the other hand, images reconstructed at larger thicknesses have a greater signal to noise ratio and lower spatial resolution. A tradeoff exists between signal to noise and spatial resolution.

The objective of this study was to compare the algorithm performance in different image reconstruction thickness settings for a given CT slice thickness. With a CT slice thickness of 1.0 mm, the images were reconstructed at 0.5 mm and 1.0 mm; with a CT slice thickness of 3.0 mm, the images were reconstructed at 1.0 mm and 3.0 mm (see Table 2).

RESULTS

I. Configuration study

The major factors defining rod configuration are the number of coplanar rods and the number of non-coplanar rods. How each aspect of the rod configuration contributes to the algorithm performance is described by the results from each study.

Results of the configuration study are grouped in three prescription groups: roll, pitch, combined (refer to Table 1). Both roll and pitch displacement errors were presented for each of the three prescription groups. Linear regression analysis was used to determine the correlation between the algorithm performance and each criterion being tested.

Number of coplanar rods

(a) Prescription in roll only

Both the accuracy and the precision in determining the pitch displacement were constant regardless of the number of coplanar rods (Table 3 and Figure 9). A trend in decreasing the accuracy in roll displacement calculation was observed (Figure 9 (a)). The algorithm's precision in calculating the roll displacement, on the other hand, slightly improved as the number of coplanar rods increased (Figure 9(b)).

# of coplanar rods	N	$\Delta\theta_{\text{inclinometer}} - \Delta\theta_{\text{algorithm}} [^\circ]$	
		Pitch	Roll
2	88	-0.035 ± 0.031	-0.0003 ± 0.028
3	52	-0.036 ± 0.030	0.016 ± 0.027
4	40	-0.036 ± 0.031	0.010 ± 0.028
5	8	-0.036 ± 0.031	0.017 ± 0.024

Table 3. Effect of the number of coplanar rods on the algorithm performance in case of angular prescription in roll only.

Figure 9. Effect of the number of coplanar rods on the algorithm (a) accuracy and (b) precision in case of angular prescription in roll only.

(b) Prescription in pitch only

A trend of slight improvement in the accuracies of both pitch and roll was observed as the number of coplanar rods was increased (Table 4 and Figure 10(a)). The precisions in determining pitch and roll displacements were unaffected by the number of coplanar rods (Table 4 and Figure 10(b)).

# of coplanar rods	N	$\Delta\theta_{\text{inclinometer}} - \Delta\theta_{\text{algorithm}} [^\circ]$	
		Pitch	Roll
2	44	-0.054 ± 0.022	0.011 ± 0.027
3	26	-0.050 ± 0.022	0.005 ± 0.029
4	20	-0.046 ± 0.023	-0.005 ± 0.027
5	4	-0.046 ± 0.023	-0.003 ± 0.027

Table 4. Effect of the number of coplanar rods on the algorithm performance in case of angular prescription in pitch only.

Figure 10. Effect of the number of coplanar rods on the algorithm (a) accuracy and (b) precision in case of angular prescription in pitch only.

(c) Prescription in both pitch and roll (combined)

The algorithm performance in roll displacement calculation was weakly related to the number of coplanar rods; however, trends in decreasing both the accuracy and the precision were observed (Table 5 and Figure 11). The accuracy in pitch slightly increased as the number of coplanar rods increased, while the precision remained unaffected (Table 5 and Figure 11).

# of coplanar rods	N	$\Delta\theta_{\text{inclinometer}} - \Delta\theta_{\text{algorithm}}$ [°]	
		Pitch	Roll
2	88	-0.077±0.039	-0.011±0.019
3	52	-0.073±0.039	0.041±0.037
4	40	-0.070±0.039	0.027±0.030
5	8	-0.070±0.039	0.050±0.037

Table 5. Effect of the number of coplanar rods on the algorithm performance in case of angular prescription in both pitch and roll (combined).

Figure 11. Effect of the number of coplanar rods on the algorithm (a) accuracy and (b) precision in case of angular prescription in both pitch and roll (combined).

The number of coplanar rods had a strong correlation ($R^2 \sim 0.89$) with pitch displacement calculation. The accuracy in pitch displacement was slightly increased as the number of coplanar rods increased; the precision on the other hand, remained unaffected. Overall the algorithm performance in roll displacement calculation had weaker correlation the number of coplanar rods. Consequently the behavior of the algorithm performance in roll was inconclusive.

Number of non-coplanar rods

(a) Prescription in roll only

The algorithm precision in determining the pitch displacement was significantly correlated to the number of non-coplanar rods ($R^2 \cong 1.00$; $p < 0.01$). As the number of non-coplanar rods increased, the precision in pitch slightly improved (Table 6 and Figure 12(b)).

# of non-coplanar rods	N	$\Delta\theta_{\text{inclinometer}} - \Delta\theta_{\text{algorithm}} [^\circ]$	
		Pitch*	Roll
2	376	-0.036 ± 0.031	0.026 ± 0.020
3	960	-0.036 ± 0.030	0.019 ± 0.023
4	900	-0.036 ± 0.030	0.018 ± 0.023

Table 6. Effect of the number of non-coplanar rods on the algorithm performance in case of angular prescription in roll only. * $R^2 > 0.95$; $p < 0.01$.

Figure 12. Effect of the number of non-coplanar rods on the algorithm (a) accuracy and (b) precision in case of angular prescription in roll only.

(b) Prescription in pitch only

A significant correlation ($R^2 \cong 1.00$; $p < 0.01$) was found between the algorithm precision in pitch displacement and the number of non-coplanar rods (Table 7 and Figure 13). As the number of non-coplanar rods increased, the algorithm precision in pitch slightly improved. A trend in the increasing algorithm accuracy in pitch was observed with increasing number of non-coplanar rods. On the contrary, the algorithm accuracy in roll and the number of non-coplanar rods were inversely related slightly. The algorithm precision in roll remained unaffected.

# of non-coplanar rods	N	$\Delta\theta_{\text{inclinometer}} - \Delta\theta_{\text{algorithm}} [^\circ]$	
		Pitch*	Roll
2	188	-0.047 ± 0.020	0.029 ± 0.026
3	480	-0.047 ± 0.020	0.0330 ± 0.026
4	450	-0.046 ± 0.019	0.035 ± 0.026

Table 7. Effect of the number of non-coplanar rods on the algorithm performance in case of angular prescription in pitch only. * $R^2 > 0.95$; $p < 0.01$.

Figure 13. Effect of the number of non-coplanar rods on the algorithm (a) accuracy and (b) precision in case of angular prescription in pitch only.

(c) Prescription in both pitch and roll (combined)

Table 8 and Figure 14 show that the algorithm accuracy in roll displacement was significantly related to the number of non-coplanar rods ($R^2 > 0.95$; $p < 0.01$). The algorithm accuracy in both pitch and roll displacements demonstrated a slight tendency in improvement as the number of non-coplanar rods increased. While the algorithm precision in pitch remained unaffected, a slight trend of increasing precision was observed in roll displacement.

# of non-coplanar rods	N	$\Delta\theta_{\text{inclinometer}} - \Delta\theta_{\text{algorithm}} [^\circ]$	
		Pitch	Roll*
2	376	-0.070 ± 0.040	0.096 ± 0.045
3	960	-0.070 ± 0.040	0.087 ± 0.038
4	900	-0.069 ± 0.040	0.078 ± 0.037

Table 8. Effect of the number of non-coplanar rods on the algorithm performance in case of angular prescription in both pitch and roll (combined). * $R^2 > 0.95$; $p < 0.01$.

Figure 14. Effect of the number of non-coplanar rods on the algorithm (a) accuracy and (b) precision in case of angular prescription in both pitch and roll (combined).

The algorithm precision in determining the pitch displacement was significantly correlated ($R^2 \cong 1.00$; $p < 0.01$) to the number of non-coplanar rods when the rotational prescription between two CT datasets was in the single axis only (pitch or roll). The algorithm accuracy in roll displacement was also significantly correlated ($R^2 > 0.95$; $p < 0.01$) when the rotational prescription was in both pitch and roll.

Number of contiguous slices included (rod length)

(a) Prescription in roll only

Both the algorithm accuracy and precision slightly improved with the increasing number of contiguous slices included in both pitch and roll displacement determinations (see Table 9 and Figure 15). The algorithm performance stabilized around 30 % of the total number of slices (that is 62 slices or a scan length of 31 mm) in general.

# of slices included	% of length	N	$\Delta\theta_{\text{inclinometer}} - \Delta\theta_{\text{algorithm}} [^\circ]$	
			Pitch	Roll
22	11%	76	-0.067 ± 0.060	0.732 ± 0.960
42	21%	76	-0.055 ± 0.023	-0.241 ± 0.396
62	31%	76	-0.042 ± 0.026	0.049 ± 0.075
82	41%	76	-0.038 ± 0.030	0.011 ± 0.113
102	50%	76	-0.038 ± 0.030	0.011 ± 0.064
122	60%	76	-0.036 ± 0.030	-0.012 ± 0.017
142	70%	76	-0.036 ± 0.031	0.001 ± 0.034
162	80%	76	-0.036 ± 0.031	-0.018 ± 0.039
182	90%	76	-0.036 ± 0.031	-0.001 ± 0.031
202	100%	76	-0.035 ± 0.031	-0.003 ± 0.025

Table 9. Effect of the number of contiguous slices included on the algorithm performance when 2 non-parallel coplanar rods were used and angular prescription was only in roll.

Figure 15. Effect of the number of contiguous slices included on the algorithm (a) accuracy and (b) precision when 2 non-parallel coplanar rods were used and angular prescription was only in roll.

(b) Prescription in pitch only

The algorithm performance in both pitch and roll displacements hit a plateau at around 50% of the total number of slices (102 contiguous slices) was included (see Table 10 and Figure 16). It did not necessarily improve with the increasing number of contiguous slices. The algorithm accuracy in pitch displacement determination hit the maximum degree of error, $\Delta\theta_{\text{inclinator}} - \Delta\theta_{\text{algorithm}} = -0.249^\circ$, at 30% and gradually decreased.

# of slices included	% of length	N	$\Delta\theta_{\text{inclinator}} - \Delta\theta_{\text{algorithm}} [^\circ]$	
			Pitch	Roll
22	11%	38	-0.049±0.022	-0.132±0.630
42	21%	38	-0.038±0.0003	-0.179±0.531
62	31%	38	-0.033±0.013	-0.249±0.038
82	41%	38	-0.039±0.022	-0.164±0.113
102	50%	38	-0.040±0.020	-0.113±0.080
122	60%	38	-0.045±0.021	-0.084±0.010
142	70%	38	-0.045±0.021	-0.051±0.036
162	80%	38	-0.046±0.021	-0.043±0.046
182	90%	38	-0.047±0.022	-0.001±0.032
202	100%	38	-0.054±0.023	-0.023±0.023

Table 10. Effect of the number of contiguous slices included on the algorithm performance when 2 non-parallel coplanar rods were used and angular prescription was only in pitch.

Figure 16. Effect of the number of contiguous slices included on the algorithm (a) accuracy and (b) precision when 2 non-parallel coplanar rods were used and angular prescription was only in pitch.

(c) Prescription in both pitch and roll (combined)

The algorithm performance stabilized at roughly 50 % of the total number of slices (102 contiguous slices) (see Table 11 and Figure 17). The only exception was that the algorithm accuracy in pitch gradually increased with the increasing number of contiguous slices included; however, no significant correlation was found.

# of slices included	% of length	N	$\Delta\theta_{\text{inclinometer}} - \Delta\theta_{\text{algorithm}} [^\circ]$	
			Pitch	Roll
22	11%	76	-0.088±0.038	-0.588±1.243
42	21%	76	-0.046±0.057	-0.313±0.226
62	31%	76	-0.042±0.051	-0.282±0.094
82	41%	76	-0.052±0.051	-0.219±0.070
102	50%	76	-0.051±0.051	-0.140±0.038
122	60%	76	-0.055±0.050	-0.076±0.010
142	70%	76	-0.055±0.050	-0.043±0.029
162	80%	76	-0.056±0.050	-0.033±0.019
182	90%	76	-0.058±0.049	-0.002±0.025
202	100%	76	-0.065±0.048	-0.026±0.019

Table 11. Effect of the number of slice included on the algorithm performance when 2 non-parallel coplanar rods were used and angular prescription was in both pitch and roll (combined).

Figure 17. Effect of the number of contiguous slices included on the algorithm (a) accuracy and (b) precision when 2 non-parallel coplanar rods were used and angular prescription was in both pitch and roll (combined).

No significant correlation between the algorithm performance and the number of contiguous slices included was found. Both the accuracy and precision of the algorithm were significantly related to the number of contiguous slices included according to the small p values (< 0.05). On the other hand, the linear regression showed various R^2 values (ranged from 0.003 to 0.853 for the accuracy and 0.009 to 0.550 for the precision), implying that the number of contiguous slices included was not the only factor contributing to the algorithm performance.

The algorithm performance hit a plateau roughly about 40-50% of the total number of slices (around 82-102 slices for a scan of 202 slices) were included. In the cases where fewer slices were included, both the accuracy and the precision fluctuated.

Generally, both the accuracy and the precision increased as the number of slices was increased, except for the pitch accuracy. These trends were also observed in a case where a configuration of 4 non-coplanar rods was used.

II. Parametric study

Results of the configuration study are presented separately in three rod configuration types: 1) all 16 rods, 2) 4 non-coplanar rods, *i.e.*, one rod per plane, and 3) 2 non-parallel coplanar rods. Both roll and pitch displacements were presented for each rod configuration type except for all 16 rods. Whether a tested parameter significantly influences the algorithm performance was examined by a two-sided paired t-test at 95% confidence interval.

X-ray energy level settings (voltage and current)

The accuracy of the algorithm in determining both pitch and roll displacements was slightly better in the lower x-ray beam energy level setting of 80 keV @ 250 mA than at 120 keV @ 190 mA in all three rod configurations tested. This trend was significant ($p < 0.01$) particularly when a rod configuration of 4 coplanar rods was used (Table 11(b) and Figure 18 (a)).

Voltage [keV]	N	$\Delta\theta_{\text{inclinometer}} - \Delta\theta_{\text{algorithm}} [^\circ]$	
		Pitch	Roll
80	1	-0.107	0.097
120	1	-0.112	0.099

(a) A configuration of 16 rods

Voltage [keV]	N	$\Delta\theta_{\text{inclinometer}} - \Delta\theta_{\text{algorithm}} [^\circ]$	
		Pitch	Roll
80	225	-0.108±0.004*	0.106±0.091*
120	225	-0.112±0.004	0.109±0.090

(b) A configuration of 4 non-coplanar rods

Voltage [keV]	N	$\Delta\theta_{\text{inclinometer}} - \Delta\theta_{\text{algorithm}} [^\circ]$	
		Pitch	Roll
80	22	-0.116±0.024	0.002±0.266
120	22	-0.118±0.019	0.001±0.256

(c) A configuration of 2 non-parallel coplanar rods

Table 12. Effect of the x-ray beam energy settings (keV & mA) on the algorithm performance in the rod configurations with (a) all 16 rods, (b) 4 non-coplanar rods, and (c) 2 non-parallel coplanar rods. * $p < 0.01$, two-tailed paired t-test.

Figure 18. Effect of the x-ray beam energy settings (keV & mA) on the algorithm performance in the rod configurations with (a) 4 non-coplanar rods and (b) 2 non-parallel coplanar rods.

CT slice thickness

The algorithm accuracy was slightly but significantly ($p < 0.01$) better with larger CT slice thickness at 3 mm when both configurations with 4 non-coplanar rods and 2

non-parallel coplanar rods were used (Table 13 and Figure 19). The same trend was also observed for a configuration with all 16 rods (Table 13 (a)).

Slice. Thk. [mm]	Recon. Thk. [mm]	N	$\Delta\theta_{\text{inclinometer}} - \Delta\theta_{\text{algorithm}} [^\circ]$	
			Pitch	Roll
1.0	1.0	1	-0.111	0.100
3.0	1.0	1	-0.105	0.095

(a) A configuration of 16 rods

Slice. Thk. [mm]	Recon. Thk. [mm]	N	$\Delta\theta_{\text{inclinometer}} - \Delta\theta_{\text{algorithm}} [^\circ]$	
			Pitch	Roll
1.0	1.0	225	-0.112±0.004	0.110±0.095
3.0	1.0	225	-0.105±0.004*	0.103±0.089*

(b) A configuration of 4 non-coplanar rods

Slice. Thk. [mm]	Recon. Thk. [mm]	N	$\Delta\theta_{\text{inclinometer}} - \Delta\theta_{\text{algorithm}} [^\circ]$	
			Pitch	Roll
1.0	1.0	22	-0.118±0.020	-0.003±0.260
3.0	1.0	22	-0.111±0.018*	-0.008±0.293

(c) A configuration of 2 non-parallel coplanar rods

Table 13. Effect of CT slice thickness on the algorithm performance in the rod configurations with (a) all 16 rods, (b) 4 non-coplanar rods, and (c) 2 non-parallel coplanar rods. * $p < 0.01$, two-tailed paired t-test.

Figure 19. Effect of CT slice thickness on the algorithm performance in the rod configurations with (a) 4 non-coplanar rods, and (b) 2 non-parallel coplanar rods.

Image reconstruction thickness

Improvement of the algorithm accuracy in pitch displacement was observed when the image reconstruction and the CT slice thicknesses were set equal, regardless of the CT slice thickness (1.0 mm or 3.0 mm) (Table 14). This trend was significant for a configuration of 4 non-coplanar rods ($p < 0.01$) with both CT slice thicknesses (Table 14(b)), but not for a configuration of 2 non-parallel coplanar rods.

In contrast, the algorithm accuracy in roll displacement decreased if the image reconstruction and the CT slice thicknesses were set equal. The algorithm precision, on the other hand, was a trade-off, when images were reconstructed at the same thickness as the CT slice thickness regardless of rod configuration.

Slice Thk. [mm]	Recon. Thk. [mm]	N	$\Delta\theta_{\text{inclinometer}} - \Delta\theta_{\text{algorithm}} [^\circ]$	
			Pitch	Roll
1.0	0.5	1	-0.112	0.099
1.0	1.0	1	-0.111	0.100
3.0	1.0	1	-0.105	0.095
3.0	3.0	1	-0.104	0.096

(a) A configuration of 16 rods

Slice Thk. [mm]	Recon. Thk. [mm]	N	$\Delta\theta_{\text{inclinometer}} - \Delta\theta_{\text{algorithm}} [^\circ]$	
			Pitch	Roll
1.0	0.5	225	-0.112 $\pm$ 0.004	0.109 $\pm$ 0.090
1.0	1.0	225	-0.112 $\pm$ 0.004*	0.110 $\pm$ 0.095
3.0	1.0	225	-0.105 $\pm$ 0.004	0.103 $\pm$ 0.089
3.0	3.0	225	-0.105 $\pm$ 0.004*	0.106 $\pm$ 0.111

(b) A configuration of 4 non-coplanar rods

Slice Thk. [mm]	Recon. Thk. [mm]	N	$\Delta\theta_{\text{inclinometer}} - \Delta\theta_{\text{algorithm}} [^\circ]$	
			Pitch	Roll
1.0	0.5	22	-0.118 $\pm$ 0.019	0.001 $\pm$ 0.256
1.0	1.0	22	-0.118 $\pm$ 0.019	-0.003 $\pm$ 0.260
3.0	1.0	22	-0.111 $\pm$ 0.018	-0.008 $\pm$ 0.293
3.0	3.0	22	-0.113 $\pm$ 0.022	-0.019 $\pm$ 0.322

(c) A configuration of 2 non-parallel coplanar rods

Table 14. Effect of the image reconstruction thickness on the algorithm performance in the rod in the rod configurations with (a) all 16 rods, (b) 4 non-coplanar rods, and (c) 2 non-parallel coplanar rods. * $p < 0.05$, two-tailed paired t-test.

Figure 20. Effect of the image reconstruction thickness on the algorithm performance in the rod configurations with (a, b) 4 non-coplanar rods, and (c, d) 2 non-parallel coplanar rods.

DISCUSSION

The objective of this thesis was to quantify the accuracy and precision of a new three-dimensional image registration algorithm proposed by Yan *et al.* (1998), and to develop general guidelines for using the proposed registration algorithm for future applications. The method was tested by designing and scanning a fixture incorporating multiple glass rods and highly accurate inclinometers to evaluate algorithm performance.

One important advantage of this algorithm is that there is no need for a priori mathematical model of the rod configuration is required. Because of this algorithm flexibility, various rod configurations and scan parameters were investigated to evaluate their effect on the algorithm performance. The characteristics of rod configuration of interest included the number of coplanar rods, the number of non-coplanar rods, and the number of contiguous slices included in the analysis. The x-ray energy level settings, the CT slice thickness, and the image reconstruction thickness were the scan parameters tested.

In general, for all conditions and factors examined, the algorithm calculates the relative rotational displacement between the two CT datasets in good agreement with the physically measured values determined by the inclinometers. The overall rotational error in this study was similar in magnitude for both directions studied; the rotations measured by the inclinometers were calculated by the algorithm within approximately $\pm 0.110^\circ$ in pitch and $\pm 0.094^\circ$ in roll. Although no large effects were observed with the different factors tested, some trends were observed that may be useful in the future application of the method.

Slight but notable improvements in both the accuracy and the precision were observed as the number of coplanar rods increased. A trend towards improved accuracy and precision as the number of non-coplanar rods increased was observed as well. In particular, both the precision of the pitch displacement and the accuracy of the roll displacement between the two CT datasets demonstrated a small but statistically significant improvement with increased number of non-coplanar rods. The number of

slices fed into the algorithm also showed an improvement in the algorithm performance although the improvement seemed to hit a plateau at around 40% of available scan length.

The scan parameter studies also showed some small but interesting trends. Results at 80 keV @ 190mA were slightly better in the accuracy and the precision than 120 keV @ 250mA. The CT slice thickness study revealed that both the accuracy and the precision were slightly but significantly better ($p < 0.05$) with the larger CT slice thickness at 3.0 mm than 1.0 mm. Image reconstruction thicknesses set identically to the CT slice thickness resulted in slightly but significantly better accuracy compared to scan sets reconstructed at thicknesses smaller than the CT slice thickness.

In order to further evaluate general algorithm performance, an additional analysis with all 16 rods was performed for all 10 scan combinations (see Table 1); the scans were taken with the settings of 120 keV @ 190mA, 1 mm CT slice thickness, and 0.5 mm image reconstruction thickness. The results of these analyses are reported in Table 15.

The angular displacement calculated by the algorithm was generally in good agreement with the measured inclinometer values for both pitch and roll. A plot of the calculated angular displacement in roll against the measured angular displacement (by the inclinometers) is shown in Figure 21. Regression analysis confirmed this agreement between expected and measured values with a highly significant correlation ($R^2 = 0.99$, $p < 0.01$). When the combinations are analyzed according to the three angular prescription groups ((a) roll only, (b) pitch only, (c) combination of roll and pitch), there are no significant differences in the performance of the algorithm (see Table 16).

Combination	Expected $\Delta\theta_{\text{inclinometer}} [^\circ]$		Calculated $\Delta\theta_{\text{algorithm}} [^\circ]$	
	Pitch	Roll	Pitch	Roll
1	-0.017	-0.997	-0.018	-0.992
2	-0.036	-4.993	-0.035	-4.984
3	4.861	-0.014	4.829	0.032
4	4.899	-4.989	4.788	-4.890
5	-0.019	-3.996	-0.033	-3.992
6	4.878	0.983	4.864	1.019
7	4.916	-3.992	4.830	-3.903
8	4.897	4.979	4.879	4.995
9	4.935	0.004	4.875	0.073
10	0.038	-4.975	-0.042	-4.922

Table 15. Results with all 16 available rods taken at the standard scan setting (120 keV, 1 mm CT slice thickness, and 0.5 mm image reconstruction thickness) for the 10 scan combinations.

Group	N	$\Delta\theta_{\text{inclinometer}} - \Delta\theta_{\text{algorithm}} [^\circ]$	
		Pitch	Roll
(a) Roll only	4	-0.018 $\pm$ 0.008	0.022 $\pm$ 0.032
(b) Pitch only	2	-0.046 $\pm$ 0.020	0.058 $\pm$ 0.016
(c) Combined	4	-0.058 $\pm$ 0.049	0.060 $\pm$ 0.041

Table 16. Results with all 16 available rods taken at the standard scan setting (120 keV, 1 mm CT slice thickness, and 0.5 mm image reconstruction thickness) grouped by the type of rotational prescription between two CT datasets.

Linear Regression of the Algorithm Performance (Roll)

Figure 21. Linear regression of the algorithm performance from datasets with all 16 rods included in the algorithm input. The calculated angular displacement in roll against the measured angular displacement (by the inclinometers) is plotted.

Future studies and recommendations

The effect of all possible configuration and scan parameter combinations on algorithm performance are impossible to test. This study was intended to establish useful guidelines for practical use of the algorithm in likely research and clinical settings.

The scan resolution is another factor that may affect algorithm performance. Resolution can be manipulated by the scan field of view (FOV). The FOV is usually set to tightly fit the region of interest (*e.g.* the cross-section of our fixture) in order to have the maximum resolution of the scanned objects. In this experiment, the FOV was set at 18 cm to encompass the vertical shift at the beginning and the end of the fixture when tilted 5° in pitch. Increasing the FOV would decrease the definition of edges of objects; decreasing the FOV would decrease the length of the rods available for analysis (an alternate way to control the resolution of the rods would be to use rods of different diameters). The effects of these types of changes were not evaluated in the current set of experiments, but may be of interest in future work.

CONCLUSIONS

From these studies, the following were found as general guidelines for using the three-dimensional image registration proposed by Yan *et al.* (1998) in verifying algorithm performance:

1. The algorithm performance improves slightly as the number of coplanar rods is increased.

2. The algorithm performance improves as the number of non-coplanar rods is increased.
3. The algorithm performance improves and become stable when at least 40-50% of the total number of slices is used in the algorithm.
4. The x-ray energy level setting of 80 keV @ 250 mA results in significantly better algorithm performance than 120 keV @ 190 mA.
5. The CT slice thickness of 3.0 mm results in significantly better algorithm performance than 1.0 mm when reconstructed at 1.0 mm.
6. The image reconstruction thickness that is equal to the CT slice thickness results in significantly better algorithm performance.

The parameters investigated were particularly geared toward future studies using this rod registration as the “gold-standard” to test the accuracy of a surface-based registration of the calcaneus. However, wide applicability is one of the strengths of this registration algorithm. It is hoped that the results of these experiments will also aid in the application of the method for use with existing fiducial frame techniques, or registration in and/or between different imaging modalities.

REFERENCES

- Grimson, W.E.L., G.J. Ettinger, S.J. White, T. Lozano-Perez, W.M. Wells III, and R. Kikinis. 1996. An automatic registration method for frameless stereotaxy, image guided surgery and enhanced reality visualization. *IEEE Trans. Med. Imaging* **15**(2) (April): 129-139.
- Heilbrun, M.P., S. Koehler, P. MacDonald, V. Siemionow, and W. Peters. 1994. Preliminary experience using an optimized three-point transformation algorithm for spatial registration of coordinate systems: a method of noninvasive localization using frame-based stereotactic guidance systems. *J Neurosurg* **81** (November): 676-682.
- Lee, S., G. Fichtinger, and G.S. Chirikjian. 2002. Numerical algorithms for spatial registration of line fiducials from cross-sectional images. *Am. Assoc. Phys. Med.* **29**(8) (August): 1881-1891.
- Maes, F., A. Collignon, D. Vandermeulen, G. Marchal, and P. Seutens. 1997. Multimodality image registration by maximization of mutual information. *IEEE Trans. Med. Imag.* **16**(2) (April): 187-197.
- Maintz, J.B. and M.A. Viergever. 1998. A survey of medical image registration. *Med. Image Anal.* **2**(1) (March): 1-36.
- Peters, T., J.A. Clark, A. Olivier, E.P. Marchand, G. Mawko, M. Dieumegarde, L.V. Muresan, and R. Ethier. 1986. Integrated stereotaxic imaging with CT, MR imaging, and digital subtraction angiography. *Med. Phys.* **161**(3) (December): 821-826.
- Peters, T., B. Davey, P. Munger, R. Comeau, A. Evans, and A. Olivier. 1996. Three-dimensional multimodal image-guidance for neurosurgery. *IEEE Trans. Med. Imaging* **15**(2) (April): 121-128.
- Petti, P.L., M.C. Kessler, T. Fleming, and S. Pitluck. 1994. An automated image-registration technique based on multiple structure matching. *Med. Phys.* **21**(9) (September): 1419-1426.
- Yan, C.H., R.T. Whalen, G.S. Beaupré, T.S. Sumanaweera, S.Y. Yen, and S. Napel. 1998. A new frame-based registration algorithm. *Med. Phys.* **25**(1) (January): 121-128.