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Tundra constellation design and stationkeeping

Michael Joseph Bruno
San Jose State University

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TUNDRA CONSTELLATION DESIGN AND STATIONKEEPING

A Thesis

Presented to

The Faculty of the Department of Mechanical and Aerospace Engineering

San José State University

In Partial Fulfillment

of the Requirements for the Degree

Master of Science

By

Michael Joseph Bruno

May 2004

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APPROVED FOR THE DEPARTMENT OF MECHANICAL
AND AEROSPACE ENGINEERING

Dr. Raghu Agarwal, San José State University
Committee Chair

Dr. Dick Desautel, San José State University
Committee Member

Mr. Darren Stratemeier, Space Systems / Loral
Committee Member

Dr. Henry Pernicka, University of Missouri-Rolla
Advisor and Committee Member

APPROVED FOR THE UNIVERSITY

ABSTRACT

TUNDRA CONSTELLATION DESIGN AND STATIONKEEPING

By Michael J. Bruno

Constellations of satellites in Tundra orbits provide an innovative alternative to the increasingly crowded geostationary orbit belt. The Tundra constellation uses three or more spacecraft in inclined geosynchronous orbits. The nominal orbit design for the constellation must minimize any negative perturbation effects in order to provide affordable stationkeeping costs. This thesis describes the study of the Tundra orbit regime and design of constellations given a sample set of basic constraints. Frozen and partially frozen orbits are then sought from which to construct constellations allowing for reduced stationkeeping requirements. Perturbation effects from third-body and geopotential sources are quantified and used to select orbits that will provide the needed coverage while providing a reasonable propellant budget.

TABLE OF CONTENTS

1. INTRODUCTION.....	1
2. BASIC ORBIT DYNAMICS.....	4
2.1 KEPLERIAN ORBIT ELEMENTS	4
2.2 GROUND VISIBILITY	8
3. TUNDRA ORBIT CONSTELLATIONS	10
4. CHARACTERIZING THE ORBIT REGIME.....	14
5. SEEKING FROZEN ORBITS	29
6. CONSTELLATION DESIGN	38
6.1 NEARLY FROZEN ORBITS.....	39
6.2 CONSTELLATION PERFORMANCE	43
7. STATIONKEEPING.....	47
7.1 METHODOLOGY	47
7.2 PERFORMANCE EVALUATION	54
8. CONCLUSION	66
REFERENCES	68
APPENDIX A –FLOW DIAGRAMS	69
APPENDIX B – PROPAGATOR COMPARISON	75

LIST OF TABLES

Table 1. Tundra Constellation: Baseline Orbital Elements	13
Table 2. Frozen Orbit Algorithm Results	34
Table 3. Constellation Orbital Element Allowable Ranges	39
Table 4. “Frozen” Tundra Constellation Elements	41
Table 5. Representative Market Cities.....	44
Table 6. Orbit Maintenance Yearly ΔV “Frozen” Constellation	60
Table 7. Comparison Constellation Elements.....	60
Table 8. Orbit Maintenance Yearly ΔV Comparison Constellation	65
Table 9. Yearly ΔV Savings Per Spacecraft when Using the “Frozen” Constellation	65

LIST OF FIGURES

Figure 1. Keplerian Orbit Elements – Orbit Size and Shape	5
Figure 2. Keplerian Orbit Elements – Orbit Orientation ⁴	6
Figure 3. Azimuth and Elevation Reference Frame.....	9
Figure 4. Tundra Orbit Ground Trace.....	11
Figure 5. Element Deviations as a Function of Epoch.....	16
Figure 6. Element Deviations Due to SRP and Drag Perturbations	18
Figure 7. Effect of Various Perturbations on Semimajor Axis (kilometers/year)	20
Figure 8. Effect of Various Perturbations on Eccentricity (no units/year)	22
Figure 9. Effect of Various Perturbations on Inclination (degrees/year).....	23
Figure 10. Effect of Various Perturbations on Ascending Node (degrees/year)	25
Figure 11. Effect of Various Perturbations on Argument of Perigee (degrees/year).....	26
Figure 12. Element Deviations Due to All Perturbations	28
Figure 13. Search Results – J_2 /Sun/Moon.....	36
Figure 14. Search Results – All Perturbations Acting	37
Figure 15. Five-Year Propagation.....	42
Figure 16. US Market Elevation Coverage.....	45
Figure 17. European Market Elevation Coverage.....	46
Figure 18. ΔV_i Versus True Anomaly.....	53
Figure 19. Satellite 1: Orbit Maintenance Five-Year Propagation	56
Figure 20. Satellite 2: Orbit Maintenance Five-Year Propagation	57
Figure 21. Satellite 3: Orbit Maintenance Five-Year Propagation	58
Figure 22. Constellation Ω Correction Five-Year Propagation	59
Figure 23. Baseline 1: Orbit Maintenance Five-Year Propagation.....	61
Figure 24. Baseline 2: Orbit Maintenance Five-Year Propagation.....	62
Figure 25. Baseline 3: Orbit Maintenance Five-Year Propagation.....	63
Figure 26. Baseline Constellation Ω Correction Five-Year Propagation	64

1. INTRODUCTION

Since the launch of Sputnik on October 4, 1957, new satellites are being placed into orbit with ever increasing frequency. The orbit in which a satellite resides is based on the type of mission it is designed to fulfill. However, the most popular orbit for satellites, whether they be military, scientific or commercial is a geostationary orbit. In particular, this orbit has become the orbit of choice for commercial communications satellite operators.

Arthur C. Clarke first conceived of geostationary communications satellites in his 1945 article "Extra-Terrestrial Relays."¹ On July 26, 1963, Syncom 2, built by the Hughes Aircraft Company, was the first communications satellite to achieve a geostationary orbit (GSO). Since then, there have been hundreds of satellites launched into GSO. Today, there are over 270 operational satellites occupying GSO. Most of these satellites are built to operate for more than a decade and construction of more new satellites continues. As a result, the available space to place new satellites is growing sparse. In response to the increasing difficulty in obtaining orbital slots in GSO, some innovative companies are breaking away from the geo-satellite mold and are using constellations of satellites in inclined, eccentric, geosynchronous orbits, known as "Tundra" orbits from which to distribute their services.

The Tundra orbit constellation is a new idea in the commercial satellite world.² To date, Sirius Satellite Radio Inc. is the only commercial company operating satellites in this regime. In order to increase the commercial viability of this type of constellation, a

constellation design and stationkeeping method are desirable that minimize the propellant required for orbit maintenance while providing the needed communications coverage.

There has only been limited study of this orbit regime to date; in 1989, one early stationkeeping strategy was proposed.³ However the third-body effects of solar and lunar gravity were not addressed and may perturb an actual constellation into a less useful form. This study shows that these third-body perturbations greatly influence these orbits and study of their effects help determine an effective way to reduce stationkeeping requirements.

When the orbit of a satellite changes due to effects of perturbations, the spacecraft must occasionally perform a maneuver to restore the nominal orbit. Needing to perform maneuvers adds significant additional mass to a satellite: thrusters, propellant tanks, and the propellants themselves. While eliminating the need for maneuvers outright is unlikely, reducing their frequency can result in significant mass savings in the form of less propellant carried and smaller propellant tanks.

An effective way to reduce stationkeeping propellant is to reduce the number of orbit parameters requiring correction. This can either be achieved by allowing large tolerances on specific elements or by identifying a nominal orbit that is “frozen,” such that one or more elements do not vary greatly over time. The former is not a practical solution since the payload will typically require substantially smaller tolerances than would otherwise occur naturally. Frozen orbits however are routinely used in operations today. A constellation of frozen or nearly frozen orbits could be used to realize propellant savings for constellation operators.

A four-step process is used to achieve a constellation design process providing reduced stationkeeping requirements. First, a numerical survey of the orbit regime is performed to characterize the behavior of these orbits. This will verify that frozen orbits exist and allow for the validation of assumption to simplify analytical and numerical modeling. Second is an analytic approach for identifying frozen orbits. Although the numerical survey provides a rough approximation of the initial conditions for frozen orbits, an analytic approach to determining the best initial conditions is desirable. Third is constellation design. A constellation can be assembled using frozen and “partially” frozen (discussed in detail later) orbits. Lastly is a quantitative evaluation of the stationkeeping requirements as well as the evaluation of various performance metrics of the constellation.

2. BASIC ORBIT DYNAMICS

This study is concerned with an application in the field of orbit dynamics.

Several basic concepts used are not explained in detail since many relevant published texts are readily available. However, to facilitate understanding of the application herein, a brief discussion of classical Keplerian orbit elements and the basic concepts of ground-to-satellite visibility are presented.

2.1 *Keplerian Orbit Elements*

Orbit elements provide a way of specifying the characteristics of an orbit, such as size and shape and energy. Keplerian orbit elements are the most common format for orbits to be specified. These elements are popular because they provide an intuitive way of defining the orbit such that at a glance, one can identify the distinctive characteristics of that orbit.

Traditionally, Keplerian elements define an orbit using six elements: the semimajor axis (a), eccentricity (e), inclination (i), right ascension of the ascending node (Ω), the argument of perigee (ω), and the true anomaly (v). The semimajor axis defines the size of the orbit while eccentricity defines the shape of the orbit as shown in Figure 1. Since Tundra orbits are elliptically shaped, eccentricity must lie in the range $0 \leq e < 1$.

Figure 1. Keplerian Orbit Elements – Orbit Size and Shape

Inclination defines the “tilt” of the orbit with respect to the equatorial plane. The right ascension of the ascending node defines where the orbit crosses the equator south to north. The right ascension of the ascending node is measured relative to the vernal equinox in the equatorial plane, positive to the east. Finally, the argument of perigee defines the location of the perigee point within the orbit plane. The argument of perigee is measured in the orbit plane from the ascending node positive in the direction of motion. These elements define the orientation of the orbit and are shown in Figure 2.

Figure 2. Keplerian Orbit Elements – Orbit Orientation⁴

The sixth element defines the instantaneous position of a satellite in that orbit. This sixth element can be either the true anomaly (ν), eccentric anomaly (E), or mean anomaly (M). The true anomaly is the angular position of the satellite as measured from perigee positive in the direction of motion. Mean anomaly is the fraction of an orbit period elapsed since perigee, computed using the average angular rate (mean motion),

expressed as an angle. Depending on the circumstances it can be more convenient to use either true or mean anomaly. True anomaly and mean anomaly can be used interchangeably and are related by the following two expressions.

$$\tan\left(\frac{\nu}{2}\right) = \sqrt{\frac{1+e}{1-e}} \tan\left(\frac{E}{2}\right) \quad (1)$$

$$M = E - e \sin E \quad (2)$$

where E is the eccentric anomaly.

The semimajor axis of the orbit describes its size and energy. The semimajor axis defines the period of the orbit, or the amount of time required to complete one orbit. The period of an orbit is defined by the following equation:

$$P = 2\pi \sqrt{\frac{a^3}{\mu}} \quad (3)$$

where μ is the gravitational constant of the body being orbited ($\mu_{earth}=398600 \text{ km}^3/\text{s}^2$). A geosynchronous orbit has a period equal to one sidereal day, 1436 minutes. Given the period of the orbit, the required semimajor axis can be calculated as $a = 42164.16 \text{ km}$.

Orbital eccentricity defines the shape of an orbit. An orbit with eccentricity of zero is a circular orbit. As eccentricity increases (but remains less than one) the orbit becomes more eccentric. When eccentricity is greater than zero, the radius becomes a function of true anomaly. The radius varies from a minimum, perigee ($\nu = 0^\circ$), to a maximum, apogee ($\nu = 180^\circ$). The more eccentric the orbit, the larger the variation in orbital radius during one revolution. The radius (r), eccentricity, semimajor axis and true anomaly are related through the “Orbit Equation”:

$$r = \frac{a(1-e^2)}{1+e \cos \nu} = a(1-e \cos E) \quad (4)$$

As the radius varies, so does the velocity. The velocity of a satellite at any given time is a function of the semimajor axis and the radius. The relationship between semimajor axis, radius and velocity (V) is given by

$$\frac{1}{2}V^2 - \frac{\mu}{r} = -\frac{\mu}{2a} \quad (5)$$

It can be seen that the velocity increases and decreases corresponding to the radius magnitude relative to the semimajor axis. In other words, when r is greater than a , the satellite moves slower, slowest at apogee, and when r is less than a , the satellite moves faster, fastest at perigee. For a geostationary orbit, the variations in velocity caused by eccentricity appear as a (slight) cyclical east/west motion when viewed from the surface of the Earth.

2.2 *Ground Visibility*

All satellites must communicate with the ground in one form or another. Many things can obstruct the view of a satellite from the ground, including buildings, trees, mountains, and the Earth itself. How easily visible a satellite is from a fixed point on the surface of the Earth influences the quality of the communications link to the satellite and thus can impact the size of the communications payload.

Visibility of a satellite from an observer on the surface of the Earth is typically quantified in a frame using azimuth and elevation angles. Shown in Figure 3, azimuth is the compass angle to the satellite, clockwise from north, while elevation is measured from the horizon, positive upwards. Generally, the higher the elevation angle, the better

the communications link because with higher elevation angles come fewer obstructions. Additionally, as the elevation angle decreases, the signal must propagate through more atmosphere further attenuating the signal.

Figure 3. Azimuth and Elevation Reference Frame

Theoretically, the best angle to view a satellite is at an elevation angle of 90° or straight up. Since satellites are generally moving with respect to the Earth, an elevation angle of 90° is a rare event. Satellites in orbits with significant velocities relative to the Earth's surface will transit a wide range of azimuth and elevation angles during the period of visibility. In the case of geostationary satellites their constant position over the Earth allows only minute changes in their azimuth and elevation; however due to their equatorial position, geostationary satellites generally have lower elevation angles to the major metropolitan areas of the world which lie at higher latitudes.

3. TUNDRA ORBIT CONSTELLATIONS

A Tundra orbit is not any single orbit, rather it is an orbit with specific characteristics. A Tundra orbit is characterized first by a geosynchronous period. This is similar to geostationary satellites because its orbital period matches the rotational rate of the Earth. The second key feature of a Tundra orbit is nonzero inclination. The nonzero inclination causes the satellite to travel into the northern and southern hemispheres up to latitudes equal to the inclination. Finally a nonzero orbital eccentricity, coupled with an argument of perigee equal to either 270° or 90° causes the satellite to spend more time in the northern or southern hemisphere respectively.

This type of orbit has many attractive features to a commercial operator. The geosynchronous period of the orbit allows each satellite to maintain its position over the same region of the world allowing operators to tailor their services to a specific market region. The higher inclinations allow the satellite to broadcast with higher elevation angles into that desired region. As the satellite travels into the higher latitudes the signals are received on the ground from overhead rather than from an acute angle. This feature mitigates the “urban jungle” effect in large cities as well as providing coverage into remote mountainous or heavily forested regions. A nonzero eccentricity coupled with an argument of perigee near 90° or 270° also allows the satellite to spend more time over the desired hemisphere, subsequently increasing the time the satellite spends over the desired region. An example ground trace of a Tundra orbit is shown in Figure 4.

Figure 4. Tundra Orbit Ground Trace

In order to provide continuous around-the-clock coverage to a region, a group of satellites in similar orbits is required. Since a satellite passes above and below the equator during each orbit, a portion of the orbit is not useable for service delivery. This unusable portion is defined as those times in the orbit when the elevation angle to the coverage area is less than zero, or in other words, when the satellite is below the horizon. In the case of Figure 4, this region corresponds roughly to the area below the equator. Thus, the minimum number of satellites required to provide around-the-clock coverage of a region is simply the integer number of satellites required such that sum of their coverage time is greater than 24 hours per day. However, this only provides the number of satellites for continuous coverage. Since one of the benefits of a Tundra orbit is high

elevation angle coverage, it follows that the more satellites there are in the constellation the higher the average elevation angle will be.

As a basis for this thesis, a baseline constellation of three orbits was selected. These orbits provide a foundation from which to gauge the relative benefit of one orbit versus another. These baseline orbits are shown in Table 1. The semimajor axis corresponds to a geosynchronous orbital period. The inclination allows the satellite to rise to higher latitudes where a majority of the population centers around the world are located. The nonzero eccentricity causes the satellite to pass through different portions of the orbit at different rates. The baseline eccentricity splits each orbit up into three eight hour blocks, from ascending node to apogee, apogee to descending node and from descending node through perigee and back to the ascending node. The argument of perigee is chosen to place apogee in the northern hemisphere. The ascending nodes were arbitrarily selected to define three evenly spaced orbit planes to provide a constellation of satellites in which at least two satellites will always be above the equator, capable of delivering services to a designated region. Lastly, the mean anomaly is time and date dependent and together with the ascending node defines the area over the Earth the satellite orbits.

Table 1. Tundra Constellation: Baseline Orbital Elements

Parameter	Value
Semimajor Axis (a)	42164.16 km
Inclination (i)	63.4°
Eccentricity (e)	0.2684
Argument of Perigee (ω)	270°
Right Ascension of the Ascending Node (Ω)	30°, 150°, 270°

4. CHARACTERIZING THE ORBIT REGIME

To date Tundra orbits have not yet been used significantly and thus there is not a great volume of data or analyses concerning the behavior of satellites in such orbits. This initial phase of research involves performing numerical simulations of the orbit regime and studying the independent and combined effects of various perturbations. The most significant perturbative accelerations were examined: Earth's geopotential, solar and lunar gravity, atmospheric drag, solar radiation pressure, as well as the epoch of evaluation.

The numerical simulations performed in this phase were conducted using the orbit propagator POHOP developed by the Jet Propulsion Laboratory. This propagator is useful for these studies because it allows all the perturbations of interest to be modeled either independently or in combination. Along with the solar and lunar gravitation, these simulations utilized the GEMT2 Earth gravity field model which allows the inclusion of tesseral harmonics up to degree and order 50,43.

Since this phase involves simulating the evolution of the orbit elements, it is not practical to perform extremely long duration simulations due to the excessive computing requirements. It is desired to minimize each simulation to a one-year duration. Thus it was necessary to investigate the possible independence of the orbital perturbation effects from the initial epoch of each propagation. It was hypothesized that the initial epoch of each simulation would not cause significant variations in the magnitudes of the orbit element deviations.

To test the hypothesis, 365-day simulations were performed on the baseline Tundra orbit from Table 1 using five initial epoch years, 2000, 2005, 2010, 2015 and 2020. A span of twenty years was selected to encompass a full luni-solar cycle. Each simulation was initialized on January first of the year of interest and the orbit element histories were recorded. The perturbations modeled in these simulations included an 8 by 8 Earth geopotential model and solar and lunar gravity. In order to capture the behavior of all possible orbit planes Ω was also varied from 30° to 360° at 30° intervals.

The results from the simulations are shown in Figure 5. This plot depicts the magnitude of the deviation from the initial value of each orbital element over the simulation span. The epoch year is shown along the abscissa, while the value of Ω is depicted along the ordinate. The color of the contour illustrates the increasing or decreasing deviation of each element.

Figure 5. Element Deviations as a Function of Epoch

It can be seen that the relative magnitude of the orbit element variations do fluctuate slightly from year to year; however it is clear that these variations are small compared to the effect of varying Ω . The horizontal bands following the epoch year axis show that for each value of Ω , the relative magnitude of variation is relatively small over the twenty-year time span. Thus for practical purposes the perturbation magnitude is independent of the epoch, validating the use of a one-year time span for all future simulations.

With independence from epoch established, two additional simplifying assumptions need to be validated. The perturbing forces due to atmospheric drag and solar radiation pressure (SRP) are believed to be negligible in this orbit regime. Neglecting these two forces will reduce the computational requirements of the simulations and will also simplify any analytical calculations.

To determine the influence of drag and SRP, one-year simulations were performed. Drag and SRP were each modeled independently of all other perturbations using arbitrary but typical satellite characteristics (mass = 2000 kg, area = 75 m², drag coefficient = 2.0). The results of the simulations are shown in Figure 6. Figure 6 shows clearly that these two perturbations have little if any effect on the orbital elements making it reasonable to neglect them from future consideration.

Figure 6. Element Deviations Due to SRP and Drag Perturbations

Characterizing the contributions of the three primary gravity perturbations (solar, lunar, and geopotential) allows for a narrower focus on how to achieve frozen orbits. To isolate the contributions of each individual perturbation they were modeled independently of each other. As before, 365-day simulations were performed. The ascending node value has already been shown to be a significant contributor to the magnitude of element variations and is again varied from 30° to 360° at 30° intervals. Inclination was chosen as the other parameter and was evaluated at values of 45° , 55° and 63.4° . As can be seen in Figures 2-6 it is clear that the three different gravity perturbations have a varying degree of impact on the different orbit elements.

Consider the deviations of semimajor axis shown in Figure 7. The semimajor axis is primarily affected by the Earth's gravity (Earth's geopotential was computed to degree 8 and order 8). The magnitude of variation appears dependent on the initial value of the ascending node. It is also of interest to note that the deviation magnitude is independent of inclination. At a given epoch, the ascending node together with the mean or true anomaly determines the position of the satellite over the surface of the Earth. It is well known from geostationary perturbation theory that the semimajor axis is perturbed almost exclusively by the tesseral terms of the geopotential⁵; and as such it comes as no surprise that the Tundra orbit regime behaves similarly. This leads to a conclusion that the semimajor axis is driven solely by the geopotential of the Earth and is a function of its sub-satellite longitude. Therefore semimajor axis is not a parameter that is suitable for control via a frozen orbit. It is however expected that it will respond to control methods similar to those used for geostationary satellites.

Figure 7. Effect of Various Perturbations on Semimajor Axis (kilometers/year)

Eccentricity and inclination, Figures 8 and 9 respectively, are perturbed greatest by the lunar gravity, however, effects from the solar gravitation are also significant. Additionally, the gravity of the Moon and Sun affect the orbit in the same “direction” at the same time. This will cause their combined effect to be additive creating an even larger overall deviation. However it can be seen that for certain values of Ω , the deviation of eccentricity and inclination are at a minimum. Thus it should be possible to find orbits that freeze either eccentricity, inclination, or both.

Figure 8. Effect of Various Perturbations on Eccentricity (no units/year)

Figure 9. Effect of Various Perturbations on Inclination (degrees/year)

The ascending node deviation is primarily driven by the Earth's gravity field and, not surprisingly, its magnitude is a function of the orbit inclination. These results, as shown in Figure 10, are consistent with the behavior of lower Earth orbits, where the inclination of the orbit drives the rate at which Ω recesses due to oblateness (J_2). The Sun and the Moon also cause the ascending node to recess. Their gravities have a surprisingly uniform effect on the recession rate; only near Ω values of 180° does the third-body effect decrease. Because the various perturbations always cause the ascending node to recess, it is not a viable candidate for frozen orbits. However, the variations in the recession rate due to either inclination or the initial node location may become an important factor for constellation design. Since the satellites in a constellation must maintain relative plane separation, if the orbit planes recess at differential rates then significant stationkeeping efforts would be required in order to maintain the appropriate separation.

The argument of perigee is a good candidate for control by a frozen orbit. As Figure 11 shows, Earth's gravity causes perigee to advance while the Sun and Moon predominantly cause perigee to recede. The amount of advance caused by Earth's gravity is related to the inclination of the orbit and is unaffected by Ω , while the deviation caused by the Moon and the Sun are a function of both Ω and inclination. The combination of orbital elements that balance these competing forces, should it exist, will freeze perigee.

Figure 10. Effect of Various Perturbations on Ascending Node (degrees/year)

Figure 11. Effect of Various Perturbations on Argument of Perigee (degrees/year)

It is now clear that the Earth, solar and lunar gravity forces each affect one or more orbit element significantly. It can thus be concluded that these three gravity forces are the dominant forces for this orbit regime. Balancing their respective positive and negative variations should allow for one or more of the orbital elements to remain nearly fixed over an extended period of time. A final survey was performed with all three perturbations active to better visualize these orbit element combinations. As before, 365-day simulations were performed with Ω varied from 30° to 360° at 30° intervals and inclination varied from 30° to 70° at 10° intervals. The resulting orbit element deviations are illustrated in Figure 12.

It can be seen in Figure 12 that all the orbital elements, except for Ω , have regions where their yearly deviations are quite small. Of particular interest is the argument of perigee. For all values of Ω , there is a corresponding value of inclination that should ostensibly freeze the perigee point. This has the potential to create significant propellant savings since the argument of perigee can be costly to correct in large, high-energy orbits such as the Tundra-type. Inclination and eccentricity also have regions of low variation; as anticipated, they are correlated to the value of Ω . These simulations show that there are initial orbit conditions that will allow eccentricity, inclination and or the argument of perigee to be “frozen.”

Figure 12. Element Deviations Due to All Perturbations

5. SEEKING FROZEN ORBITS

The results presented in the previous chapter provide a rough numerical assessment of the effects of the most significant perturbations for a Tundra-type orbit. However, a method by which to identify specific orbital elements for the orbits of the constellation needs to be provided. In order to reduce stationkeeping costs, a frozen or near-frozen orbit is sought that minimizes the changes to eccentricity and argument of perigee.

The orbit characterization simulations summarized in Chapter 4 show that the Earth, solar and lunar gravitation are the primary perturbations affecting this regime. A model was sought that would include these effects yet be simple enough to allow the analytic identification of frozen orbits. Kwok presents a doubly-averaged potential function for third-body perturbations expressed in planet-equator coordinates.⁶ Using planet-equator coordinates allows the third-body potential to be easily added to the geopotential function. In order to eliminate the singularity when eccentricity is zero, Kwok uses semi-equinocial elements h and k with

$$h = e \sin(\omega) \tag{6}$$

$$k = e \cos(\omega) \tag{7}$$

The remaining classical orbital elements semimajor axis (a), inclination (i), and right ascension of the ascending node (Ω) are then used with h and k . The needed equations of motion⁶ using the semi-equinocial elements are

$$\frac{dh}{dt} = \frac{\sqrt{1-e^2}}{na^2} \frac{\partial R}{\partial k} - \frac{k \cot i}{na^2 \sqrt{1-e^2}} \frac{\partial R}{\partial i} \quad (8)$$

$$\frac{dk}{dt} = -\frac{\sqrt{1-e^2}}{na^2} \frac{\partial R}{\partial h} + \frac{h \cot i}{na^2 \sqrt{1-e^2}} \frac{\partial R}{\partial i} \quad (9)$$

$$\frac{di}{dt} = \frac{\cot i}{na^2 \sqrt{1-e^2}} \left(k \frac{\partial R}{\partial h} - h \frac{\partial R}{\partial k} \right) - \frac{1}{na^2 \sqrt{1-e^2} \sin i} \frac{\partial R}{\partial \Omega} \quad (10)$$

$$\frac{d\Omega}{dt} = \frac{1}{na^2 \sqrt{1-e^2} \sin i} \frac{\partial R}{\partial i} \quad (11)$$

where R is the disturbing potential function. This function, due to the doubly-averaged third-body perturbation, is given by Kwok as

$$\begin{aligned} R = & \frac{\mu'}{a'} \left(\frac{a}{a'} \right)^2 \frac{1}{(1-e'^2)^{3/2}} \left\{ \left(\frac{3}{4} \sin^2 i' - \frac{1}{2} \right) \left(\frac{3}{4} \sin^2 i (1-k^2+h^2) - \frac{1}{2} \left(1 + \frac{3}{4} k^2 + \frac{3}{4} h^2 \right) \right) \right. \\ & + \frac{3}{8} \sin 2i' \left(5kh \sin \Delta \sin i + \frac{1}{2} \sin 2i \cos \Delta (1-k^2+4h^2) \right) \\ & \left. + \frac{3}{8} \sin 2i' \left(\frac{1}{2} \cos 2\Delta (5(k^2-h^2) + \sin^2 i (1-k^2+4h^2)) - 5kh \cos i \sin 2\Delta \right) \right\} \end{aligned} \quad (12)$$

where the variables with primes refer to the perturbing body, μ' is the gravitational parameter of the perturbing body, Ω' is the ascending node of the perturbing body and $\Delta = \Omega - \Omega'$. This expanded potential has been averaged over the mean anomaly of both the spacecraft orbit *and* the perturbing body, then truncated at first order. Thus an implicit assumption is that the orbital elements of the spacecraft vary slowly during one orbit of the perturbing body. The partial derivatives needed by the equations of motion in equations (8) through (11) are then

$$\begin{aligned}
\frac{\partial R}{\partial h} = & \frac{\mu'}{a'} \left(\frac{a}{a'} \right)^2 \frac{1}{(1-e'^2)^{3/2}} \left\{ \frac{3}{2} h \left(\frac{3}{4} \sin^2 i' - \frac{1}{2} \right) (4 \sin^2 i - 1) \right. \\
& + \frac{3}{8} \sin 2i' \sin i (5k \sin \Delta + 8h \cos i \cos \Delta) \\
& \left. - \frac{3}{8} \sin^2 i' (h \cos 2\Delta (1 + 4 \cos^2 i) + 5k \cos i \sin 2\Delta) \right\}
\end{aligned} \tag{13}$$

$$\begin{aligned}
\frac{\partial R}{\partial k} = & \frac{\mu'}{a'} \left(\frac{a}{a'} \right)^2 \frac{1}{(1-e'^2)^{3/2}} \left\{ -\frac{3}{2} k \left(\frac{3}{4} \sin^2 i' - \frac{1}{2} \right) (\sin^2 i + 1) \right. \\
& + \frac{3}{8} \sin 2i' \sin i (5h \sin \Delta + 2k \cos i \cos \Delta) \\
& \left. + \frac{3}{8} \sin^2 i' (k \cos 2\Delta (4 + \cos^2 i) - 5h \cos i \sin 2\Delta) \right\}
\end{aligned} \tag{14}$$

$$\begin{aligned}
\frac{\partial R}{\partial i} = & \frac{\mu'}{a'} \left(\frac{a}{a'} \right)^2 \frac{1}{(1-e'^2)^{3/2}} \left\{ \left(\frac{3}{4} \sin^2 i' - \frac{1}{2} \right) \left(\frac{3}{2} \sin i \cos i (1 - k^2 + 4h^2) \right) \right. \\
& + \frac{3}{8} \sin 2i' ((2 \cos^2 i - 1) \cos \Delta (1 - k^2 + 4h^2) + 5kh \cos i \sin \Delta) \\
& \left. + \frac{3}{8} \sin 2i' (\sin i \cos i (1 - k^2 + 4h^2) \cos 2\Delta + 5kh \sin i \sin 2\Delta) \right\}
\end{aligned} \tag{15}$$

$$\begin{aligned}
\frac{\partial R}{\partial \Omega} = & \frac{\mu'}{a'} \left(\frac{a}{a'} \right)^2 \frac{1}{(1-e'^2)^{3/2}} \left\{ \frac{3}{8} \sin 2i' \sin i (5kh \cos \Delta - \cos i \sin \Delta (1 - k^2 + 4h^2)) \right. \\
& - \frac{3}{8} \sin^2 i' \sin 2\Delta ((1 - h^2 + 4k^2) - \cos 2i (1 - k^2 + 4h^2)) \\
& \left. - \frac{15kh}{4} \sin^2 i' \cos i \cos 2\Delta \right\}
\end{aligned} \tag{16}$$

These are summed with the potential derivatives due to J_2 (higher order geopotential terms may also be similarly added) to the equations of motion. The disturbing potential derivatives due to J_2 only are

$$\frac{\partial R}{\partial h} = \frac{3}{2} \mu J_2 \frac{R_{\oplus}^2}{a^3} \frac{1 - \frac{3}{2} \sin^2 i}{(1 - e^2)^{5/2}} h \quad (17)$$

$$\frac{\partial R}{\partial k} = \frac{3}{2} \mu J_2 \frac{R_{\oplus}^2}{a^3} \frac{1 - \frac{3}{2} \sin^2 i}{(1 - e^2)^{5/2}} k \quad (18)$$

$$\frac{\partial R}{\partial i} = -\frac{3}{2} \mu J_2 \frac{R_{\oplus}^2}{a^3} \frac{\sin i \cos i}{(1 - e^2)^{3/2}} h \quad (19)$$

$$\frac{\partial R}{\partial \Omega} = 0 \quad (20)$$

where $R_{\oplus}$ is the radius of the Earth.

With this model available in the convenient reference of Earth's equatorial plane, attention is turned to seeking frozen orbits. The approach begins by fixing the semimajor axis at the geosynchronous value (42,164.16 km) and the argument of perigee at 270° . Then eccentricity, inclination, and right ascension of the ascending node are varied from their baseline values in Table 1 to seek frozen orbits. In particular, orbits are sought such that eccentricity and argument of perigee are frozen, or at least vary slowly so that they may be easily controlled with stationkeeping. In the semi-equinoctial elements the frozen orbit requirement is then

$$\frac{dh}{dt} = 0 \quad (21)$$

$$\frac{dk}{dt} = 0 \quad (22)$$

Since the argument of perigee $\omega = 270$ degrees, then $h = -e$ and $k = 0$. The equations of motion (8) and (9) then simplify to

$$\frac{dh}{dt} = \frac{\sqrt{1-e^2}}{na^2} \frac{\partial R}{\partial k} \quad (23)$$

$$\frac{dk}{dt} = -\frac{\sqrt{1-e^2}}{na^2} \frac{\partial R}{\partial h} - \frac{e \cot i}{na^2 \sqrt{1-e^2}} \frac{\partial R}{\partial i} \quad (24)$$

Setting equations (23) and (24) to zero in an attempt to freeze the orbit requires that

$$\frac{\partial R}{\partial k} = 0 \quad (25)$$

$$\left(1-e^2\right) \frac{\partial R}{\partial h} + e \cot i \frac{\partial R}{\partial i} = 0 \quad (26)$$

The MATLAB[®] function “fmins” was used to minimize the root sum squared value of equations (25) and (26) using the baseline elements of Table 1 as initial values. At the end of the search by “fmins,” the root sum square was very small (3.5072×10^{-9}) and effectively zero. Thus, roots were found that satisfy equations (25) and (26) and are shown in Table 2. Note that these values correspond to regions of small variances in Figure 12, as expected.

Table 2. Frozen Orbit Algorithm Results

Parameter	Initial Value	Solved Value
Inclination (i)	63.5°	62.3136°
Eccentricity (e)	0.2684	0.3440
Argument of Perigee (ω)	270°	270°
Right Ascension of the Ascending Node (Ω)	30°	8.6657°

In order to test whether these “improved” elements actually froze the orbit, additional simulations were run. First, a simple test run was conducted with the perturbation model in the simulator including only effects from J_2 and luni-solar third-body gravitation. Since this corresponds to the same model in which the frozen elements were determined, the orbit should remain “fairly” frozen (it was not expected that the orbit would be *exactly* frozen, since the potential function was doubly-averaged and truncated at first order – thus the force model used is not an exact representation). Figure 13 shows the results of this simulation presented as the orbit elements deviations from their initial values. This figure shows that the “improved” elements do reduce the drift rate of eccentricity and argument of perigee. While not truly frozen, the orbit behavior is clearly improved.

Next, a simulation was run that tests the improved orbit using a more realistic perturbation environment. The geopotential model was expanded to 42 by 42 and drag and solar radiation pressure were included (arbitrary but typical satellite characteristics were used as before: mass = 2000 kg, area = 75 m², drag coefficient = 2.0). Figure 14

shows that the orbit behavior is nearly identical to that shown in Figure 13. Thus the improved orbital elements should provide a practical nominal orbit for use in a Tundra constellation.

Figure 13. Search Results – J_2 /Sun/Moon

Figure 14. Search Results – All Perturbations Acting

6. CONSTELLATION DESIGN

Tundra constellations have many possible configurations, typically ranging from three to six satellites. A three-satellite configuration was arbitrarily chosen for this initial study. All orbits were chosen as geosynchronous, with inclinations above 30° and nonzero eccentricity. This combination of elements allows the constellation to provide the benefits mentioned in the Introduction.

Part of the nominal orbit design process is defining the operational requirements of the constellation. A Tundra constellation would likely be used to provide a telecommunications service to one of the major population centers of the world. A majority of these centers are located in the northern hemisphere at latitudes greater than 30° and less than 70° . Thus the argument of perigee of each orbit must be near 270° and the inclination constrained to a region between 30° and 70° to provide an orbit with apogee over desired regions.

Another key consideration is the duration of time the satellite is in view of its customers. If the eccentricity is too small, the satellite will not provide coverage for a sufficient period of time because the apogee loiter time will be too short. Alternately, if eccentricity is too large, the satellite will cross into the Van Allen radiation belts. Thus, the eccentricity is constrained to values roughly between 0.13 and 0.50. These selections correspond to a fourteen-hour dwell time near apogee and a perigee height just above the outer Van Allen belt (the high proton region, in particular) respectively.⁷

The right ascension of the ascending node defines the orientation of each orbit plane. Each satellite requires its own unique plane and, in order to maintain proper

phasing, they must be separated by $360^\circ/n$, where n is the number of satellites in the constellation. For simplicity, the ascending node of the first satellite defines the ascending node for the remaining satellites in the constellation by subsequently adding $360^\circ/n$ for each satellite.

The parameter ranges defined as allowable are summarized in Table 3.

Table 3. Constellation Orbital Element Allowable Ranges

Parameter	Value
Semimajor Axis (a)*	42164 km
Inclination (i)	30° - 70°
Eccentricity (e)	0.13 – 0.50
Argument of Perigee (ω)	270°
Right Ascension of the Ascending Node (Ω)	$\Omega_1, \Omega_1+120^\circ, \Omega_1+240^\circ$

* The semimajor axis is varied from the value given in Table 3 to better match the rotational rate of the Earth when taking into account the presence of perturbations.

6.1 Nearly Frozen Orbits

With the allowable constellation parameters defined, frozen or nearly frozen orbits are sought that have orbital elements within the ranges defined in Table 3 but that still meet mission objectives. The orbits used for the constellation are selected in a two-step process. The first step is to use the analytic equations (18-19) to determine at least one frozen orbit. The second step is to complete the constellation using orbits spaced at the specified ascending node interval, given in Table 3, using orbits that have had the orbit element variations reduced as much as possible, or nearly frozen.

Using the frozen orbit search results in Table 2 the ascending node values of the two “companion” orbits then become defined at the intervals given in Table 3. Since “fmins” optimizes the function over all possible values of the ascending node, searching for the companion orbits is further constrained because the remaining ascending node values are then fixed. A constrained, one-dimensional optimizer is used to determine the remaining two companion orbits. In particular, Brent’s method⁸ is employed to determine ideal values of e and i for the companion orbits.

Building on the knowledge gained from the orbit regime characterization (as described in Chapter 4), it was recognized that minimizing variation in eccentricity was predominately a function of ascending node. Since the ascending nodes for the companion orbits are predetermined, i became the best candidate to vary using Brent’s method to determine the ideal inclination to minimize the drift in the argument of perigee. The methodology involves stepping through small increments of e while evaluating values of i . When the minimum values of equations (18) and (19) are determined (typically on the order of $10^{-5} \text{ km}^2/\text{sec}^2$), the result is a nearly frozen companion orbit.

Frozen and nearly frozen orbits were determined to create a three-satellite Tundra orbit constellation. The “Frozen Tundra” constellation elements are shown in Table 4.

Table 4. “Frozen” Tundra Constellation Elements

Element	Sat 1	Sat 2	Sat 3
a^*	42158.7 km	42153.5 km	42153.5 km
i	62.3136°	48.3078°	48.0590°
e	0.3440	0.1300	0.1300
ω	270°	270°	270°
Ω	8.6657°	127°	249°
M^{**}	127.73°	322.62°	205.88°

* The semimajor axes were selected such that the mean satellite longitude remained near the desired value for one year.

** Epoch of January 1, 2004, 00:00 UT.

To verify the frozen behavior of these orbits, five-year element histories were generated without implementing any corrective maneuvers. Each orbit was propagated using the POHOP orbit propagator with an 8 by 8 geopotential model as well as with solar and lunar third-body effects. The orbit element histories for each satellite are shown in Figure 15.

Figure 15. Five-Year Propagation

It is interesting to note that the *initial* eccentricity for both companions, Satellites 2 and 3, drifted to the minimum allowable eccentricity during the search for nearly frozen orbits. When consulting the results of the orbit regime characterization this is not surprising. It can be seen in Figure 5 that for the ascending nodes selected, eccentricity variation over time is reduced as the initial value of e correspondingly became smaller.

6.2 Constellation Performance

Once a constellation is designed, its ability to perform its mission must be evaluated. In the case of commercial Tundra orbit constellations, that equates to the quality of coverage the constellation provides. The key coverage parameter is elevation angle with the best coverage supplied at elevation angles above 60° . To evaluate how well the Frozen Tundra constellation performs, sample markets of the United States and Europe are examined. These markets are given priority for their commercial marketability; however, the coverage assessment could be applied to any potential market. Representative cities within each region provide a benchmark of coverage quality. The cities evaluated for the two market cases are summarized in Table 5.

Table 5. Representative Market Cities

Market	City	Latitude (°N)	Longitude (°E)
US	Miami	25.783	-79.717
US	Boston	42.367	-70.967
US	Vancouver	49.200	-123.170
US	Los Angeles	34.050	-117.767
Europe	Madrid	40.260	-3.420
Europe	London	51.570	0.280
Europe	Athens	38.000	23.380
Europe	Moscow	55.450	37.370

To verify the quality of the constellation coverage, the elevation angle to various cities was evaluated. To do this, the elevation angle to each satellite over a single day was computed. A Keplerian propagation with no perturbations was executed for each satellite over one orbit. During that orbit, at each time step, the elevation angle was calculated for the latitude and longitude of each city.

A key component in coverage quality is the mean longitude (the average sub-satellite longitude over an orbit); all satellites in the constellation have identical mean longitudes. When the argument of perigee equals 270° , the ground track can be bisected about the mean satellite longitude, with apogee and perigee located at the mean longitude. Thus the mean longitude relative to the region of interest can bias coverage east or west. In the case of the US market, the mean longitude is selected to be 90° west

longitude; for the European example, the mean longitude is 17° east. Figures 16 and 17 show the constellation composite maximum elevation angle time history for one day for the various cities.

Figure 16. US Market Elevation Coverage

Figure 17. European Market Elevation Coverage

Figures 16 and 17 show that the goal of 60° elevation angle coverage is nearly met. In Figure 16 there is a region at around 300 minutes where the elevation dips fairly low for the two southern-most markets. This region corresponds to when Satellite 1 is at apogee. Because this orbit has the highest inclination, the southern-most cities suffer degraded elevation angle coverage as Satellite 1 reaches apogee. The other regions of short-term elevation angle coverage correspond to the transition from one satellite providing the highest elevation angles to another.

7. STATIONKEEPING

Once a complete constellation has been assembled its stationkeeping requirements can be evaluated. Stationkeeping, or orbit maintenance, is the process of performing a maneuver or several maneuvers to change the orbit elements in a desired manner. In the case of a constellation of satellites the primary goal of stationkeeping is to maintain the integrity of the constellation.

7.1 *Methodology*

It has been observed for Tundra orbits that nearly all the orbit elements have long period variations that exceed a reasonable tolerance after one year or more. Thus the orbital elements are corrected using a traditional “bang-bang” control method. A maneuver interval of once per year is arbitrarily selected. To perform this stationkeeping evaluation, an orbit propagator based in MATLAB[®] was used. The propagator utilizes Cowell’s method for the long-term prediction of spacecraft behavior subject to perturbations and compares favorably to the POHOP propagator used previously. The MATLAB-based propagator was favored for this phase because it allowed the necessary stationkeeping algorithms to be built around the propagator. In this way after each propagation period, stationkeeping maneuvers are automatically performed. Following the completion of the maneuvers, the propagation is resumed and the process is repeated until completion.

The nominal orbits are defined as described in the previous chapters. Each orbit is propagated at one-year intervals for a total of five years. The propagation uses an 8 by

8 geopotential model (higher order terms have minimal effects on the results but significantly slow computational time), solar and lunar gravity. At the conclusion of each propagation period, the need for stationkeeping maneuvers is evaluated. The need for maneuvers is determined by comparing the propagated orbit to the nominal one. If a given element has varied from the nominal value more than its defined tolerance it is reset to its nominal value.

A Tundra orbit constellation requires the elements Ω , a and e be maintained precisely in order to maintain the integrity of the constellation and the cohesiveness of operations. The remaining two parameters, ω and i , are less critical to the actual operations of the constellation and do not need to be maintained as precisely. However, ω and i do impact the quality of the coverage and may not be ignored. Taking these considerations into account, allowable orbit element deviations are defined such that, when exceeded, a corrective maneuver is needed.

The tolerances selected for each element are designed to ensure the constellation's integrity. Sample (estimated) values are used in this study. The magnitude of a must be maintained near the synchronous radius in order for the satellite to remain over the desired coverage region; a tolerance of 1 km is used. The eccentricity e must be kept near its design value or the transition from one satellite to another (as one rises and the other sets) would be adversely affected. The tolerance on e was defined as ± 0.005 . This variation on e corresponds to a maximum 4.5 minute variation on the time spent in the coverage region, and would thus be a total of 9 minutes for two satellites. The value of ω

must also be maintained closely, for as ω increases or decreases the satellite transition is affected, as is the quality of the elevation angle coverage. The tolerance on ω was selected as 1° . Inclination is a key parameter in each orbit remaining frozen. The combination of i and Ω define the magnitude of the drift of ω . Since Ω regresses slowly, i is maintained to within 1° of the nominal value.

In the case of Ω , a specific value is not the concern, but the relative motion of Ω of all the satellites must be controlled. For this study, Ω for all satellites is controlled to the mean Ω -regression rate of the constellation. The key to this method is to utilize the semimajor axis, the orbital period, to maintain the satellite's mean longitude rather than a specific ascending node value. Varying the orbital period accordingly allows the satellite to counteract any effect the regressing node has on the mean longitude.

In order to maintain the broader focus of this study on the overall stationkeeping costs and to simplify the stationkeeping computations, the mean longitude is not maintained. Instead, The initial value of a accounts for both the tesseral harmonic perturbation on semimajor axis⁵ as well as the affect of nodal regression. The initial value of a selected results in the mean longitude drifting away from, approximately 6° , then back to its initial value during the one year propagation periods between stationkeeping events. If a tighter tolerance on mean longitude was desired, more frequent maneuvers to adjust the semimajor axis could be performed at a minimal cost.

Once it is determined a maneuver is needed, the required ΔV must be calculated. Meirovitch's formulation⁹ expressing changes in orbit elements due to impulsive

maneuvers provides a convenient means by which to compute the ΔV . These changes are given as

$$\Delta a = \frac{2}{n} \left[\Delta V_r \frac{e \sin \nu}{\sqrt{1-e^2}} + \Delta V_t \frac{a}{r} \sqrt{1-e^2} \right] \quad (27)$$

$$\Delta e = \frac{\sqrt{1-e^2}}{na} \left[\Delta V_r \sin \nu + \Delta V_t \frac{a}{er} \left(1-e^2 - \frac{r^2}{a^2} \right) \right] \quad (28)$$

$$\begin{aligned} \Delta \omega = & -\frac{\sqrt{1-e^2}}{nae} \left[\Delta V_r \cos \nu - \Delta V_t \left(1 + \frac{r}{a(1-e^2)} \right) \sin \nu \right. \\ & \left. + \Delta V_n \frac{re \cot i \sin(\omega + \nu)}{a(1-e^2)} \right] \end{aligned} \quad (29)$$

$$\Delta i = \Delta V_n \frac{r \cos(\omega + \nu)}{na^2 \sqrt{1-e^2}} \quad (30)$$

$$\Delta \Omega = \Delta V_n \frac{r \sin(\omega + \nu)}{na^2 \sqrt{1-e^2} \sin i} \quad (31)$$

where r is the radial distance to the satellite from Earth's center, n is the mean motion, and ν is the true anomaly. The impulses are expressed in component form as ΔV_t , ΔV_r and ΔV_n along transverse, radial, and orbit normal directions, respectively.

Since most satellites perform maneuvers either normally or transversely within the orbit plane (and not radially), the radial terms are now omitted from equations (27) through (31). Furthermore, since the magnitude of the needed correction to each orbit

element is known, the respective required normal or transverse ΔV for each can be determined as

Normal ΔV (ΔV_n):

$$\Delta V_n = \Delta i \frac{na^2 \sqrt{1-e^2}}{r \cos(\omega + \nu)} \quad (32)$$

$$\Delta V_n = \Delta \Omega \frac{na^2 \sqrt{1-e^2} \sin i}{r \sin(\omega + \nu)} \quad (33)$$

$$\Delta V_n = -\Delta \omega \frac{na^2(1-e^2)}{\sqrt{1-e^2}(r \cot i \sin(\omega + \nu))} \quad (34)$$

Transverse ΔV (ΔV_t):

$$\Delta V_t = \Delta a \frac{nr}{2a\sqrt{1-e^2}} \quad (35)$$

$$\Delta V_t = \Delta e \frac{ner}{\sqrt{1-e^2} \left(1-e^2 - \frac{r^2}{a^2}\right)} \quad (36)$$

$$\Delta V_t = \Delta \omega \frac{nae}{\sqrt{1-e^2} \left(1 + \frac{r}{a(1-e^2)}\right) \sin \nu} \quad (37)$$

Equations (32) through (37) show that each element is corrected with either a normal or transverse ΔV maneuver. Due to the nonzero eccentricity, the effectiveness of ΔV_n is affected by the true anomaly of the maneuver. Thus, it is important to specify the location of each maneuver.

Three parameters are affected by ΔV_n : Ω , i and ω . The large inclination of a Tundra orbit makes correcting ω using ΔV_n inefficient (resulting from the $\cot(i)$ term in equation (34)). Therefore, the effect of ΔV_n on ω is not used to find the true anomaly; rather the (small) change in ω due to ΔV_n is calculated using equation (29) and added to the needed $\Delta\omega$ correction, which is then corrected using the ΔV_t component as described below.

The normal-maneuver true anomaly (v_n) is defined as the place where the desired Δi and $\Delta\Omega$ can be achieved simultaneously using one ΔV_n application. This can be determined by setting equation (32) equal to (33) and solving for v_n , resulting in

$$v_n = \arctan\left(\frac{\Delta\Omega \sin i}{\Delta i}\right) - \omega \quad (38)$$

The ΔV_t component is used to correct the elements a , e and ω . Typically, the ΔV_t required to obtain the needed Δe and $\Delta\omega$ is significantly larger than what is required for Δa . Additionally, when only considering ΔV_t , Δa is affected only slightly by the true anomaly of the maneuver while the semimajor axis increases with a positive ΔV_t and decreases with a negative ΔV_t . Therefore, it is not practical to use a maneuver performed at a single true anomaly, because the (single) maneuver needed to correct e and ω would not result in the proper correction to the semimajor axis

Equations (36) and (37) are functions of true anomaly and behave like cosecant and secant functions respectively. Thus, there are two distinct true anomalies at which the desired Δe and $\Delta\omega$ can be achieved for the same ΔV_t . This condition is illustrated in

Figure 18 for an arbitrary Tundra orbit given sample Δe and $\Delta \omega$ values. Since performing all the required ΔV_t at a single true anomaly will change semimajor axis more than is required, the necessary ΔV_t must be split between the two true anomalies such that the desired Δa is also achieved. The transverse-maneuver true anomalies are determined numerically by stepping true anomaly in small increments (first from 0° - 180° then from 180° - 360°) and evaluating the ratio of equations (36) and (37). The transverse-maneuver true anomaly is defined where the ratio is nearly equal to one, as illustrated in Figure 18 by the intersection points that occur twice per orbit.

Figure 18. ΔV_t Versus True Anomaly

A similar method is used to determine the amount of ΔV_t that is performed at each true anomaly. A scaling factor F (0-100%) is applied to the first maneuver, performed at

the first true anomaly, and the change of orbit elements is calculated. Given the adjusted orbit elements, the second maneuver performs the remaining ΔV_t at the second true anomaly. Since the maneuvers at the two true anomalies are in opposite directions, the semimajor axis is both increased and decreased; summing produces the residual Δa . A simplified algorithm is as follows:

$$\Delta a_1 \sim F \Delta V_t \quad (39)$$

$$\Delta a_2 \sim (1-F) \Delta V_t \quad (40)$$

$$\Delta a = \Delta a_1 + \Delta a_2 \quad (41)$$

If Δa is not close enough to the needed change, F is adjusted and a new Δa is computed. The loop continues until a value for F is found that achieves the needed correction to the semimajor axis. Appendix A includes a more detailed flowchart illustrating the algorithm.

7.2 *Performance Evaluation*

There are two primary metrics considered in assessing the stationkeeping methodology: were the orbital element tolerances maintained and how much ΔV was required to maintain them? Figures 19 through 21 show that a , e , i and ω were corrected after each one-year propagation period only when their value exceeded their respective tolerance.

It is clearly seen that all of the elements do not require yearly correction, thus reducing ΔV . In each plot, the various maneuvers can be seen as the vertical line, or “spike” within the element time history. A spike appearing in only one element and not

another is evidence that the maneuver true anomaly corresponds to a position such that only one of the various elements needs to be corrected. Similarly, the large spikes in the semimajor axis history correspond to when eccentricity or argument of perigee (or both) are corrected. The algorithm to proportionally split ΔV_t is evidenced by the semimajor axis always being reset to the desired value as well.

Figure 22 illustrates the ascending node variation of each satellite over time with respect to the constellation mean Ω -regression rate, 4.67° per year. The variation of each orbit's ascending node from mean is seen as each satellite diverges from zero. A maneuver to adjust the ascending node to correspond to the mean regression rate is similarly represented by the vertical line in the time history. All three orbit planes vary less than a quarter of a degree a year keeping the required corrections small and further helping to reduce ΔV .

Figure 19. Satellite 1: Orbit Maintenance Five-Year Propagation

Figure 20. Satellite 2: Orbit Maintenance Five-Year Propagation

Figure 21. Satellite 3: Orbit Maintenance Five-Year Propagation

Figure 22. Constellation Ω Correction Five-Year Propagation

The magnitude and frequency of corrections translates directly to required orbit maintenance ΔV . To provide a clearer picture of the true costs over time, the total ΔV was averaged to provide a yearly ΔV requirement. The yearly ΔV requirements directly impact the cost in both mass of the propellant(s) and the propulsion subsystem. The yearly ΔV requirements for each satellite in the constellation are summarized in Table 6.

Table 6. Orbit Maintenance Yearly ΔV “Frozen” Constellation

	ΔV_n (m/s)	ΔV_t (m/s)	Total ΔV (m/s)
Satellite 1	7.2631	4.6052	11.8683
Satellite 2	18.2506	13.3735	31.6241
Satellite 3	16.6418	12.5346	29.1764
Average	14.0518	10.1711	24.2229

As a more meaningful assessment of the constellation design, a comparison baseline constellation using the values taken from Table 1 with orbit elements as shown in Table 7 was also used to simulate orbit maintenance requirements. Figures 23 through 25 show the corrections to a , e , i and ω of the baseline orbits while Figure 26 illustrates each satellite’s Ω variation over time with respect to the mean Ω -regression rate of the baseline constellation, 3.57° per year.

Table 7. Comparison Constellation Elements

Element	Sat 1	Sat 2	Sat 3
a	42164.2 km	42164.2 km	42164.2 km
i	63.4°	63.4°	63.4°
e	0.2684	0.2684	0.2684
ω	270°	270°	270°
Ω	30°	150°	270°
M^*	101.02°	281.33°	185.99°

* Epoch of January 1, 2004, 00:00 U.T..

Figure 23. Baseline 1: Orbit Maintenance Five-Year Propagation

Figure 24. Baseline 2: Orbit Maintenance Five-Year Propagation

Figure 25. Baseline 3: Orbit Maintenance Five-Year Propagation

Figure 26. Baseline Constellation Ω Correction Five-Year Propagation

The values of e and ω of the baseline orbits vary more rapidly over time than those of the frozen constellation resulting in a requirement for yearly correction of both increasing their required ΔV . The required ΔV is further increased because the Ω -regression rates between satellites are more varied. Table 8 summarizes the required ΔV for this constellation's maintenance.

Table 8. Orbit Maintenance Yearly ΔV Comparison Constellation

	ΔV_n (m/s)	ΔV_t (m/s)	Total ΔV (m/s)
Satellite 1	29.7365	10.2430	39.9796
Satellite 2	47.1633	46.4372	93.6005
Satellite 3	14.4301	36.0858	50.5159
Average	30.4433	30.9220	61.3653

It can be seen that there is significant reduction in the required ΔV for orbit maintenance using the frozen constellation. The frozen constellation can reduce ΔV requirements by over 37 m/s per year per spacecraft. It is notable that the most significant savings occur in the transverse direction. This results from the significant reduction in required ω and e corrections due to the “frozen” nature of the orbits. A summary of the orbit maintenance ΔV requirements is presented in Table 9.

Table 9. Yearly ΔV Savings Per Spacecraft when Using the “Frozen” Constellation

ΔV_n (m/s)	ΔV_t (m/s)	Total ΔV (m/s)
16.3915	20.7509	37.1424

8. CONCLUSION

The Tundra orbit regime can be a commercially viable alternative to geostationary orbit. These orbits can be made even more desirable by utilizing frozen and nearly frozen orbits as a basis for constellation design. Use of these orbits result in significant reductions in the required orbit maintenance ΔV while nearly achieving the desired coverage goals. These savings can be returned to the operator in the form of lower procurement and operating costs.

Nearly frozen orbits in the Tundra regime have been identified and can be of use to future mission designers. A constellation designed with such orbits provides an operator with significant propellant savings over the lifetime of the constellation and thus significant cost savings. The method for searching for frozen orbits described here uses the doubly averaged perturbation model in identifying useful Tundra orbits. Adaptation of the frozen orbit search allows the discovery of partially frozen or nearly frozen orbits with which a complete constellation can be designed.

This study examines only one Tundra constellation configuration. Although significant orbit maintenance benefits are shown, the coverage limitations of the three-satellite constellation and the limits on the allowable orbit elements are also exhibited. The quality of coverage could be improved by either increasing the minimum allowable eccentricity or by adding additional satellites to the constellation. Increasing the minimum eccentricity would ensure a greater percentage of time spent in the hemisphere of interest. This would likely come at a relatively small cost in additional ΔV relative to

the baseline constellation. Similarly, the coverage could also be improved by the addition of one or more satellites.

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APPENDIX A –FLOW DIAGRAMS

A-1. Partially Frozen Orbit Search Using Brent's Method

A-2. Stationkeeping Simulation – Propagation

A-3. Stationkeeping Simulation – Normal ΔV

A-4. Stationkeeping Simulation – Tangential ΔV

A-5. Stationkeeping Simulation – Δa Via Proportional ΔV_t

Figure A-1. Partially Frozen Orbit Search Using Brent's Method

Figure A-2. Stationkeeping Simulation - Propagation

Figure A-3. Stationkeeping Simulation – Normal ΔV

Figure A-4. Stationkeeping Simulation – Tangential ΔV

Figure A-5. Stationkeeping Simulation – Δa Via Proportional ΔV_t

APPENDIX B – PROPAGATOR COMPARISON

Two orbit propagators were used in this thesis. Each propagator was better suited for a particular portion of the analysis. To validate that each propagator would produce similar ephemerides, identical initial conditions were propagated one-year and the orbit element time histories compared. Figures B-1 through B-5 show the difference in the orbit element time histories for a , e , i , Ω and ω . The orbit propagators compare favorably.

Figure B-1. Semimajor Axis Propagation Comparison

Figure B-2. Eccentricity Propagation Comparison

Figure B-3. Inclination Propagation Comparison

Figure B-4. Right Ascension of the Ascending Node Propagation Comparison

Figure B-5. Argument of Perigee Propagation Comparison