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Sensitivity of the models for predicting creep of concrete

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SENSITIVITY OF THE MODELS FOR PREDICTING
CREEP OF CONCRETE

A Thesis

Presented to

The Faculty of the Department of Civil and Environmental Engineering
San Jose State University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science

by

Hina Masood

December 2006

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ABSTRACT

SENSITIVITY OF THE MODELS FOR PREDICTING CREEP OF CONCRETE

by Hina Masood

This study attempts to evaluate the sensitivity of creep in concrete for various models. Creep causes increase in deflections in concrete structures and changes the stress distribution within elements. The benefits of an accurate prediction model of creep yield a safer design: a structure with minimum cracking, extended life, and durability. The error in determining the input parameters can produce significant changes in creep prediction values. These changes can be quantified by determining the sensitivity of the model used for creep prediction. In this study, six models are studied to ascertain the sensitivity of creep models.

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Abbreviations

A. Organizations - Acronyms

AASHTO	American Association of State Highway and Transportation Officials
ACI	The American Concrete Institute
CEB-FIP	The Committee Euro-International Du Beton/Federation International de la Precontrainte-Europe

B. General Abbreviations and Symbols

Type I	North American normal Portland cement – ASTM C150
Type II	North American moderate sulphate resistant cement – ASTM C150
Type III	North American high early strength cement – ASTM C150
R	European rapid hardening cement (Assumed equivalent to Type I cement)
SL	European slow hardening cement (Assumed equivalent to Type II cement)
RS	European rapid hardening high strength cement (Assumed equivalent to Type III cement)

C. Abbreviations and Symbols for Creep Models

C.1 The AASHTO 04 Model

E_c	=	Modulus of elasticity at 28 days [MPa] or [psi]
f_c	=	Average 28 day concrete compressive strength [MPa] or [psi]
H	=	Relative humidity [%]

$J(t, t_0)$	=	Creep compliance [$\times 10^{-6}/\text{MPa}$] or [$\times 10^{-6}/\text{psi}$]
k_c	=	Factor for the effect of volume-to-surface ratio of the component specified
k_f	=	Factor for the effect of concrete strength
t	=	Maturity of concrete (days)
t_i	=	Age of concrete when load is initially applied (days)
V/S	=	Volume-to-surface area ratio [mm] or [in.]
γ	=	Concrete unit weight [kg/m^3] or [lb/ft^3]
$\Phi(t, t_i)$	=	Creep coefficient

C.2 The ACI 209 Model

b	=	Constant in determining shrinkage strain, depends on curing method
c	=	Constant in determining shrinkage strain, depends on curing method
$C_c(t)$	=	Creep coefficient at time t
C_{cu}	=	Ultimate creep coefficient
E_{cmto}	=	Modulus of elasticity at age of loading [MPa] or [psi]
$f_c(t_0)$	=	Mean concrete compressive strength at age of loading [MPa] or [psi]
E_{cm28}	=	Average 28 day modulus of elasticity [MPa] or [psi]
f_{cm28}	=	Average 28 day concrete compressive strength [MPa] or [psi]

H	=	Relative humidity [%]
$J(t, t_0)$	=	Creep compliance [$\times 10^{-6}/\text{MPa}$] or [$\times 10^{-6}/\text{psi}$]
K_{ca}	=	Age at loading correction factor
K_{ch}	=	Relative humidity correction factor for creep
K_{cs}	=	Shape and size correction factor for creep
t	=	Age of concrete after casting [days]
t_c	=	Age of concrete drying commenced, usually taken as the age at the end of moist curing [days]
t_0	=	Age of concrete at loading [days]
V/S	=	Volume-to-surface area ratio [mm] or [in.]
Γ	=	Concrete unit weight [kg/m^3] or [lb/ft^3]

C.3 The B3 Model

a/c	=	Aggregate-to-cement ratio by weight
c	=	Cement content in concrete [kg/m^3] or [lb/ft^3]
$C_d(t, t_0, t_c)$	=	Compliance function for additional creep due to drying
$C_o(t, t_0)$	=	Compliance function for basic creep
D	=	Effective cross-section thickness [mm] or [in.]
E_{cm28}	=	Modulus of elasticity at 28 days [MPa] or [psi]
f_{cm28}	=	Average 28 day concrete compressive strength [MPa] or [psi]
H	=	Relative humidity [%]
$H(t)$	=	Spatial average of pore relative humidity within cross-section

$H(t_0)$	=	Spatial average of pore relative humidity within cross-section
$J(t, t_0)$	=	Creep compliance
K_s	=	Cross-section shape
$Q(t, t_0)$	=	Binomial integral
q_1	=	Instantaneous strain due to unit stress
q_2	=	Aging visco-elastic compliance
q_3	=	Non-aging visco-elastic compliance
q_4	=	Flow compliance
q_5	=	Drying creep compliance
$S(t)$	=	Time function for shrinkage
$S(t_0)$	=	Another time period for shrinkage
t	=	Age of concrete after casting [days]
t_c	=	Age of concrete drying commenced, usually taken as the age at the end of moist curing [days]
t_0	=	Age of concrete at loading [days]
T_{sh}	=	Shrinkage half time [days]
V/S	=	Volume-to-surface area ratio [mm] or [in.]
w	=	Water content in concrete [kg/m^3] or [lb/ft^3]
w/c	=	Water to cement ratio by weight
α_1	=	Type of cement correction factor on creep
α_2	=	Curing condition correction factor
γ	=	Concrete unit weight [kg/m^3] or [lb/ft^3]

ϵ_{shu} = Ultimate shrinkage strain [mm/mm or in./in.]

$\Phi(t, t_0)$ = Creep coefficient

C.4 The CEB 90 Model

E_{cm28} = Modulus of elasticity at 28 days [MPa] or [psi]

E_{cmto} = Modulus of elasticity at age of loading [MPa] or [psi]

f_{cm28} = Average 28 day concrete compressive strength [MPa] or [psi]

H = Relative humidity [%]

h_e = Effective thickness to account for volume/surface ratio [mm]
or [in.]

$J(t, t_0)$ = Creep compliance [$\times 10^{-6}$ /MPa] or [$\times 10^{-6}$ /psi]

S = Type of cement

t = Age of concrete after casting [days]

t_c = Age of concrete drying commenced, usually taken as the age at the
end of moist curing [days]

t_0 = Age of concrete at loading [days]

V/S = Volume-to-surface area ratio [mm] or [in.]

$\beta(t_0)$ = Age of concrete at loading factor on the notional creep coefficient

$\beta(f_{cm28})$ = Concrete strength factor on the notional creep

$\beta_c(t, t_0)$ = Coefficient describing development of creep with time after
loading

β_H = Relative humidity and notional member size, h_0 factor on creep

γ	=	Concrete unit weight [kg/m ³] or [lb/ft ³]
$\Phi(t, t_0)$	=	Creep coefficient at time t
Φ_0	=	Notional creep factor

C.5 The CEB 90-99 Model

E_{cm28}	=	Modulus of elasticity at 28 days [MPa] or [psi]
E_{cmto}	=	Modulus of elasticity at age of loading [MPa] or [psi]
f_{cm28}	=	Average 28 day concrete compressive strength [MPa] or [psi]
H	=	Relative humidity [%]
h_e	=	Effective thickness to account for volume/surface ratio [mm] or [in.]
$J(t, t_0)$	=	Creep compliance [$\times 10^{-6}$ /MPa] or [$\times 10^{-6}$ /psi]
s	=	Type of cement
T	=	Age of concrete after casting [days]
t_c	=	Age of concrete drying commenced, usually taken as the age at the end of moist curing [days]
t_0	=	Age of concrete at loading [days]
$t_{0,T}$	=	Age of concrete at loading adjusted according to the concrete temperature; for $T = 20^\circ\text{C}$, $t_{0,T}$ corresponds to t_0
V/S	=	Volume-to-surface area ratio [mm] or [in.]
α	=	Coefficient that depends on type of cement: -1 for SL, 0 for R and 1 for RS

$\beta(t_0)$	=	Age of concrete at loading factor on the notional creep coefficient
$\beta(f_{cm28})$	=	Concrete strength factor on the notional creep
$\beta_c(t, t_0)$	=	Coefficient describing development of creep with time after loading
β_H	=	Relative humidity and notional member size, h_0 factor on creep
$\Phi(t, t_0)$	=	Creep coefficient at time t
Φ_0	=	Notional creep factor

C.6 The GL 2000 Model

f_{cm28}	=	Average 28 day concrete compressive strength [MPa] or [psi]
V/S	=	Volume-to-surface area ratio [mm] or [in.]
H	=	Relative humidity [%]
E_{cm28}	=	Modulus of elasticity at 28 days [MPa] or [psi]
t	=	Age of concrete after casting [days]
t_0	=	Age of concrete at loading [days]
t_c	=	Age of concrete drying commenced, usually taken as the age at the end of moist curing [days]
$\Phi(t, t_0)$	=	Creep coefficient
$J(t, t_0)$	=	Creep compliance [$\times 10^{-6}$ /MPa] or [$\times 10^{-6}$ /psi]

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CHAPTER 1

Introduction

1.1 General

The design and serviceability of concrete and prestress structures require an accurate estimation of mechanical properties of materials. Creep causes increase in deflections in concrete structures and changes the stress distributions within elements. The design of concrete structure requires an accurate estimation of creep and shrinkage. The benefits of accurate prediction model of creep yield a safer design, a structure with minimum cracking, extended life, and increased durability.

Error in determining input parameters can produce significant change in creep prediction values. This change can be quantified by determining the sensitivity of each model which is used for creep prediction. This chapter will cover an overview of creep. The models used to predict creep, and the factors which affect creep, will also be discussed.

1.2 Definitions

The time dependent increase of strain in hardened concrete subjected to sustained stress is defined as “creep.”

“Creep coefficient” is defined as the ratio between creep strain and the instantaneous elastic strain.

“Creep compliance” represents the total-load dependent strain at time t produced by a unit constant stress that has been acting since time t_0 .

1.3 Concrete Creep

Creep refers to the time-dependent strain of material under constant applied loads in concrete member. Creep results in increases in deflection which can be several times of the elastic deflection. Creep effects depend on factors such as relative humidity, specimen size, age at loading and material strength.

There are two major types of concrete creep, basic and drying creep. Basic creep refers to the non-drying component of creep. It is a time dependent increase in strain under a constant stress without any moisture gain or loss. Drying creep, also called “Picket effect,” is a time dependent increase in strain under drying conditions, in excess of basic creep.

Figure 1.1: Creep Recovery

If a sustained load is removed, the strain decreases immediately. This instantaneous recovery is followed by a gradual decrease in strain, called creep recovery.

The experimental results such as Neville et al, shown above in Figure 1.1 shows that creep is not a fully reversible phenomena. Recoverable creep is generally within 10% to 20% of the total creep after 200 days of loading. The creep recovery is affected by time, temperature and relative humidity. The portion of irreversible creep increases with time under loading.

1.4 Sensitivity and Parameters Affecting Creep

Sensitivity of creep is presented as the percentile change in creep prediction value caused by change in input parameter. Equation (1) defines sensitivity as:

$$S(p) = \frac{[\varepsilon(p_1) - \varepsilon(p_2)]}{\varepsilon(p_1)} \times 100 \quad [1]$$

$S(p)$ = Creep sensitivity due to change of any input parameter p , such as relative humidity, V/S ratio, duration of loading or concrete strength

p_1 = Initial value of input parameter p

p_2 = Value of parameter p after change

$\varepsilon(p_1)$ = Predicted creep of concrete for the value of input parameter equal to p_1

$\varepsilon(p_2)$ = Predicted creep of concrete after change of input parameter from p_1 to p_2

The following parameters effect creep, which are discussed in detail in Chapter 2 for the six various models.

- Cement type
- Curing method
- Relative humidity
- Volume to surface (V/S) ratio
- Duration of loading
- Concrete strength

1.5 Objective

The objective of this thesis is to ascertain the creep prediction and sensitivity of creep for six models. The models are based on the AASHTO 2004, ACI 209, B3, CEB 90, CEB 90-99, and GL 2000 models. The study utilizes Equation (1) to derive sensitivity equations for the input parameters that affect the creep prediction as listed in Section 1.4. It also identifies the most sensitive parameters of creep sensitivity for each model and provides a comparison of the various parameters for all models.

CHAPTER 2

Literature Review of Existing Models

2.1 Introduction

Several models have been developed for estimating creep strain. The most widely used models are the AASHTO 04, ACI 209, CEB 90, and CEB 90-99. This study also includes the B3 model proposed by Bazant and Baweja 1995 [17] and GL 2000 model by Gardner [14]. The comparisons of various parameters used in the above models are discussed below.

2.2 AASHTO 04 Model

The AASHTO 04 is taken from Collins and Mitchell (1991). This method is based on the recommendation of the ACI committee 209, as modified by additional recently published data [6]. The creep coefficient is affected by: the compressive strength of concrete, the correction factors for the age at loading, element size effects, and relative humidity. The AASHTO 04 model can be applied to normal concrete with concrete compressive strengths less than 70 MPa. The maximum V/S ratio to determine the factor for the effect of the volume-to-surface ratio is based on 150 mm. Based on research presented by the PCI committee on prestress losses in 1975, creep remains nearly constant for V/S ratios greater than 150 mm. Therefore the V/S should be limited to 150 mm in equation [2-3].

2.2.1 Required Parameters for Creep Function

The computation of creep compliance depends on the following parameters:

- Age of concrete at loading in days
- Magnitude and duration of loads
- Modulus of elasticity of concrete at 28 days in MPa
- Relative humidity expressed as a decimal
- Volume to surface ratio in mm

2.2.2 Computation of Creep Compliance

The creep coefficient may be computed as follows:

$$\Phi(t, t_i) = 3.5k_c k_f \left(1.58 - \frac{H}{120} \right) t_i^{-0.118} \frac{(t - t_i)^{0.6}}{10.0 + (t - t_i)^{0.6}} \quad [2-1]$$

$$k_f = \frac{62}{42 + f'_c} \quad [2-2]$$

E_c = Modulus of elasticity of concrete at 28 days [MPa]

$\Phi(t, t_i)$ = Creep coefficient

f'_c = Average 28 day concrete compressive strength [MPa]

H = Relative humidity in decimal

k_c = Factor for the effect of the volume-to-surface ratio as given in Equation [2-3]

k_f = Factor for the effect of concrete strength

t = Maturity of concrete (days)

t_i = Age of concrete when load is initially applied (days)

$$k_c = \left[\frac{\frac{t}{26e^{0.0142(V/S)} + t}}{\frac{t}{45 + t}} \right] \left[\frac{1.80 + 1.77e^{-0.0213(V/S)}}{2.587} \right] \quad [2-3]$$

The compliance factor is computed as follows:

$$J(t, t_i) = \frac{1}{E_c} + \frac{\Phi(t, t_i)}{E_c} \quad [2-4]$$

2.3 ACI 209 Model

The ACI 209R-92 model is the current standard code model recommended by American Concrete Institute (ACI) which was reapproved in 1997 [1]. The ACI 209 model is based on an empirical model based on creep and shrinkage tests performed on 150 mm x 300 mm diameter cylinders. This model can be applied to type I and III Portland cement concrete with relative humidity ranging between 40% to 100%.

2.3.1 Required Parameters for Creep Function

The total load dependent strain is presented with compliance function, also called creep function, $J(t, t_0)$, which represents the total load dependent strain at time t produced by a unit constant stress that has been acting since time t_0 . The computation of creep compliance depends on the following parameters:

- Age of concrete at loading in days
- Curing method
- Modulus of elasticity of concrete at loading in MPa
- Relative humidity expressed as decimal

➤ Volume to surface ratio in mm

2.3.2 Computation of Creep Compliance

$$J(t, t_o) = \frac{1 + C_c(t)}{E_{cmto}} \quad [2-5]$$

$$E_{cmto} = 0.043(\gamma)^{3/2} \sqrt{f'_c(t_o)} \quad [2-6]$$

$C_c(t)$ = Creep coefficient at time t

E_{cmto} = Modulus of elasticity of concrete at age of loading (MPa)

γ = Concrete unit weight (kg/m³)

$f'_c(t_o)$ = Mean concrete compressive strength (MPa)

$$f'_c(t_o) = f_{cm28} \frac{t_o}{b + ct_o} \quad [2-7]$$

f_{cm28} = Average 28 day concrete compressive strength (MPa)

b and c constants according to Table 2.1:

Table 2.1 Constant b and c as a function of Cement Type and Curing Method

Type of Cement	Moist-Cured		Steam-Cured	
I	b = 4.0	c = 0.85	b = 1.0	c = 0.95
III	b = 2.3	c = 0.92	b = 0.7	c = 0.98

Creep coefficient, $C_c(t)$, can be determined as follows:

$$C_c(t) = \frac{t^{0.60}}{10 + t^{0.60}} C_{ca} K_{ch} K_{ca} K_{cs} \quad [2-8]$$

C_{ca} = Ultimate creep coefficient = 2.35

K_{ch} = Relative humidity correction factor for creep = $1.27 - 0.0067H$

K_{ca} = Age at loading correction factor = $1.25(t_o)^{-0.118}$

K_{cs} = Shape and Size correction factor for creep = $1.14 - 0.0035(V/S)$

2.4 B3 Model

The B3 model is complex and has a unique formulation. This model was developed by Bazant and Baweja at North Western University in 1997 [17]. An earlier version of this model was proposed in 1978 and was called the BP model. This model was further modified in 1991 and was named the BP-KX model. The current B3 model is a simpler version of the BP-KX model. The B3 model is only applicable for average compressive strength from 17.2 MPa to 69 MPa. The concrete mix should have a water-to-cement ratio between 0.3 to 0.85, cement contents between 160 kg/m³ to 719 kg/m³ and aggregate to cement ratio between 2.5 to 13.5. The age of concrete at loading must be greater than age of concrete after casting, i.e., $t_o \geq t_c$.

2.4.1 Required Parameters for Creep Function

The creep compliance depends on the following parameters for the B3 model:

- Age drying commenced, end of moist curing (days)
- Age of concrete at loading (days)
- Aggregate content in concrete (kg/m³)
- Cement content in concrete (kg/m³)
- Cement type I, II or III

- Concrete mean compressive strength at 28 days (MPa)
- Curing conditions
- Modulus of elasticity of concrete at 28 days (MPa)
- Relative humidity expressed as a decimal
- Shape of specimen
- Volume to surface ratio (mm)
- Water content in concrete (kg/m³)

2.4.2 Computation of Creep Compliance

The creep compliance $J(t, t_o)$ is computed as follows:

$$J(t, t_o) = [q_1 + C_o(t, t_o) + C_d(t, t_o, t_c)] \quad [2-9]$$

Where q_1 is the instantaneous strain given in (2-10), $C_o(t, t_o)$ is the compliance function for basic creep and $C_d(t, t_o, t_c)$ is the compliance function for drying creep.

$$q_1 = \frac{0.6}{E_{cm28}} \quad [2-10]$$

E_{cm28} = Modulus of elasticity of concrete in MPa at 28 days is given in Equation (2-11).

$$E_{cm28} = 4735 \sqrt{f_{cm28}} \quad [2-11]$$

The compliance function of the basic creep, $C_o(t, t_o)$, should be calculated as follows:

$$C_o(t, t_o) = q_2 Q(t, t_o) + q_3 \ln[1 + (t - t_o)^{0.1}] + q_4 \ln\left(\frac{t}{t_o}\right) \quad [2-12]$$

q_2 = Aging visco-elastic compliance parameter

$Q(t, t_o)$ = The binomial integral

q_3 = Nonaging visco-elastic compliance parameter

q_4 = Flow compliance parameter

t_o = Age of concrete at loading (days)

E_{cm28} = Modulus of elasticity of concrete at 28 days (MPa)

$$q_2 = 185.4 \times 10^{-6} (c)^{0.5} (f_{cm28})^{-0.9} \quad [2-13]$$

C is the cement content in kg/m^3

$$Q(t, t_o) = Q_f(t_o) \left[1 + \left(\frac{Q_f(t_o)}{Z(t, t_o)} \right)^{r(t_o)} \right]^{-1/r(t_o)} \quad [2-14]$$

$Q(t, t_o)$ is a binomial integral, which must be multiplied by the parameter q_2 to obtain the aging visco-elastic term. The three following equations can be used to approximate the binomial integral.

$$Q_f(t_o) = \frac{1}{0.086(t_o)^{2/9} + 1.21(t_o)^{4/9}} \quad [2-15]$$

$$Z(t, t_o) = \ln \left[1 + \frac{(t - t_o)^{0.1}}{\sqrt{t_o}} \right] \quad [2-16]$$

$$r(t_o) = 1.7(t_o)^{0.12} + 8 \quad [2-17]$$

q_3 is the nonaging visco-elastic compliance parameter and q_4 is the aging flow compliance parameter. These parameters are a function of the concrete mean compressive strength at 28 days, the cement content (c), the water to cement ratio (w/c) and the aggregate to cement ratio (a/c).

$$q_3 = 0.29q_2 \left(\frac{w}{c} \right)^4 \quad [2-18]$$

$$q_4 = 20.3 \left(\frac{a}{c} \right)^{-0.7} \times 10^{-6} \quad [2-19]$$

The compliance function for drying creep, $C_d(t, t_o, t_c)$ should be calculated as follows:

$$C_d(t, t_o, t_c) = q_5 \sqrt{e^{-8H(t)} - e^{-8H(t_o)}} \quad [2-20]$$

q_5 is the drying creep compliance parameter. This parameter is a function of the concrete mean compressive strength at 28 days, water content, cement type, method of curing and the ultimate shrinkage strain ϵ_{shu} .

$$q_5 = \frac{0.757 |\epsilon_{shu} \times 10^6|^{-0.6}}{f_{cm28}} \quad [2-21]$$

$$\epsilon_{shu} = -\alpha_1 \alpha_2 \left[0.019 w^{2.1} (f_{cm28}) - 0.28 + 270 \right] \times 10^{-6} \quad [2-22]$$

The value of α_1 and α_2 are given in Table 2.2 and Table 2.3

$H(t)$ and $H(t_o)$ are spatial averages of pore relative humidity. The $H(t)$ and $H(t_o)$ can be computed as follows:

$$H(t) = 1 - (1-h)S(t) \quad [2-23]$$

$$H(t_o) = 1 - (1-h)S(t_o) \quad [2-24]$$

$S(t)$ and $S(t_o)$ are the time function for shrinkage and τ_{sh} is the shrinkage half-time.

Table 2.2 Correction Factor α_1 as a Function of Cement Type

Type of Cement	α_1
I	1.00
II	0.85
III	1.10

Table 2.3 Correction Factor α_2 as a Function of Curing Type

Type of Curing	α_2
Steam cured	0.75
Water cured or H=100%	1.00
Sealed during curing	1.20

$$S(t) = \tanh \left[\left(\frac{t - t_c}{\tau_{sh}} \right)^{1/2} \right] \quad [2-25]$$

$$S(t_o) = \tanh \left[\left(\frac{t_o - t_c}{\tau_{sh}} \right)^{1/2} \right] \quad [2-26]$$

$$\tau_{sh} = 0.085t_c - 0.08(f_{cm28}) - 0.25[2k_s(V/S)]^2 \quad [2-27]$$

where k_s is the cross-section shape correction factor, given in Table 2.4.

Table 2.4 Correction Factor k_s as a Function of Cross-Section Shape

Cross-Section Shape	k_s
Infinite slab	1.00
Infinite cylinder	1.15
Infinite square prism	1.25
Sphere	1.30
Cube	1.55

2.5 CEB 90 Model

The CEB 90 model was developed by Comité Européen du Béton. CEB 90 [7] is similar to ACI 209, which computes the creep coefficient at certain time after loading.

However, CEB 90 computes the creep coefficient based on the modulus of elasticity of concrete at 28 days E_{cm28} . This model does not consider the effects of curing (i.e., the method or duration of curing) for the calculation of creep compliance. This model can be applied to concretes with relative humidity between 40% to 100%, at mean temperature ranges from 5 °C to 30 °C. This model is only applicable to normal weight concrete with an average compressive strength between 20 MPa to 90 MPa and for which the duration of curing is limited to a maximum of 14 days.

2.5.1 Required Parameters for Creep Function

In the CEB 90, the computation of creep compliance depends on the following parameters:

- Concrete mean compressive strength at loading (days)
- Modulus of elasticity of concrete at 28 days (MPa)
- Modulus of elasticity of concrete when loading commenced (MPa)
- Age of concrete at loading (days)
- Volume to surface ratio (mm)
- Relative humidity expressed as a decimal

2.5.2 Computation of Creep Compliance

The creep compliance can be computed as follows:

$$J(t, t_o) = \frac{1}{E_{cmto}} + \frac{\Phi(t, t_o)}{E_{cm28}} \quad [2-28]$$

E_{cmto} = Modulus of elasticity of concrete at age of loading (MPa)

E_{cm28} = Modulus of elasticity of concrete at 28 days (MPa)

$\Phi(t, t_0)$ = Creep coefficient = $\Phi_o * \beta_c(t, t_0)$

$$E_{cmto} = E_{cm28} e^{\left[0.5S \left(1 - \sqrt{\frac{28}{t_0}}\right)\right]} \quad [2-29]$$

S is a coefficient that depends on cement type as given in Table 2.5.

t_0 is the age of concrete at loading in days.

Table 2.5 Coefficient S as Function of Cement Type

Cement Type	European Type	US Type	S
Slow hardening	SL	II	0.38
Normal/rapid hardening	R	I	0.25
Rapid hardening high strength	RS	III	0.20

Creep coefficient, $\Phi(t, t_0)$, can be computed from the following equation:

$$\Phi_o = \Phi_{RH} \beta(f_{cm28}) \beta(t_0) \quad [2-30]$$

Φ_{RH} = Relative humidity factor on the notional creep coefficient, which is given as:

$$\Phi_{RH} = 1 + \frac{1 - H/100}{0.46 \sqrt[3]{h_e/100}} \quad [2-31]$$

$\beta(f_{cm28})$ = Concrete strength factor on notional creep coefficient, which is given as:

$$\beta(f_{cm28}) = \frac{5.3}{\sqrt{f_{cm28}/10}} \quad [2-32]$$

$\beta(t_0)$ = Age of concrete at loading factor on notional creep coefficient, which is given as:

$$\beta(t_o) = \frac{1}{0.1 + t_o^{0.2}} \quad [2-33]$$

The development of creep with time, $\beta_c(t, t_o)$, after loading can be computed using the following equation:

$$\beta_c(t, t_o) = \left(\frac{t - t_o}{\beta_H + t - t_o} \right)^{0.3} \quad [2-34]$$

$$\beta_H = 1.5h_e \left[1 + (0.012H)^{18} \right] + 250 \leq 1500 \text{ days} \quad [2-35]$$

2.6 CEB 90-99 Model

The CEB 90-99 is a modification to the CEB 90 [8]. The modification is due to the type of cement and the corrections due to mean concrete compressive strength at 28 days age.

2.6.1 Required Parameters for Creep Function

In CEB 90-99, the computation of creep compliance depends on the following parameters:

- Concrete mean compressive strength at loading (days)
- Modulus of elasticity of concrete at 28 days (MPa)
- Modulus of elasticity of concrete when loading commenced (MPa)
- Age of concrete at loading (days)
- Volume to surface ratio (mm)
- Relative humidity expressed as a decimal

2.6.2 Computation of Creep Compliance

The creep compliance can be computed as follows:

$$J(t, t_o) = \frac{1}{E_{cmto}} + \frac{\Phi(t, t_o)}{E_{cm28}} \quad [2-36]$$

E_{cmto} = Modulus of elasticity of concrete at age of loading (MPa)

E_{cm28} = Modulus of elasticity of concrete at 28 days (MPa)

$\Phi(t, t_o)$ = Creep coefficient = $\Phi_o * \beta_c(t, t_o)$

$$E_{cmto} = E_{cm28} e^{\left[0.5S \left(1 - \sqrt{\frac{28}{t_o}}\right)\right]} \quad [2-37]$$

S is a coefficient that depends on cement type and compressive strength as given in Table 2.6.

t_o is the age of concrete at loading in days.

Creep coefficient, $\Phi(t, t_o)$, can be computed from the following equation:

$$\Phi_o = \Phi_{RH} \beta(f_{cm28}) \beta(t_o) \quad [2-38]$$

Φ_{RH} = Relative humidity factor on the notional creep coefficient, which is given as:

$$\Phi_{RH} = 1 + \frac{1 - H/100}{0.46 \sqrt[3]{h_e/100}} \quad [2-39]$$

Table 2.6 Coefficient S as a Function of Cement Type and Compressive Strength

f_{cm28} (MPa)	Type of Cement	S
≤ 60	Rapidly hardening high strength (RS or III)	0.20
	Normal and rapidly hardening (R, I)	0.25
	Slow hardening (SL, II)	0.38
> 60	All types	0.20

Φ_{RH} = Relative humidity factor on the notional creep coefficient, which is given as:

$$\begin{aligned}\Phi_{RH} &= \left[1 + \frac{1 - H/100}{0.46\sqrt[3]{h_e/100}} \alpha_1 \right] \alpha_2 \\ \alpha_1 &= \left[\frac{35}{f_{cm28}} \right]^{0.7} \\ \alpha_2 &= \left[\frac{35}{f_{cm28}} \right]^{0.2}\end{aligned}\quad [2-40]$$

$\beta(f_{cm28})$ = Concrete strength factor on notional creep coefficient, which is given as:

$$\beta(f_{cm28}) = \frac{5.3}{\sqrt{f_{cm28}/10}} \quad [2-41]$$

$\beta(t_o)$ = Age of concrete at loading factor on notional creep coefficient, which is given as:

$$\beta(t_o) = \frac{1}{0.1 + t_o^{0.2}} \quad [2-42]$$

$$\text{where, } t_o = t_{o,T} \left[\frac{9}{2 + t_{o,T}^{1.2}} + 1 \right]^\alpha \geq 0.5 \text{ days} \quad [2-43]$$

t_o = Age of concrete at loading (days)

$t_{o,T}$ = Age of concrete at loading adjusted according to the concrete temperature; for $T =$

20 °C, $t_{o,T}$ corresponds to t_o

α = Coefficient that depends on type of cement:

= -1 for slowly hardening cement

= 0 for normal or rapidly hardening cement

= 1 for rapidly hardening high-strength cement

The development of creep with time, $\beta_c(t, t_o)$, after loading can be computed using the following equation:

$$\beta_c(t, t_o) = \left(\frac{t - t_o}{\beta_H + t - t_o} \right)^{0.3} \quad [2-44]$$

$$\beta_H = 1.5h_e \left[1 + (0.012H)^{18} \right] + 250\alpha_3 \leq 1500\alpha_3 \quad \text{days} \quad [2-45]$$

$$\alpha_3 = \left[\frac{35}{f_{cm28}} \right]^{0.5} \quad [2-46]$$

2.7 GL 2000 Model

The GL 2000 model is a modified model which was originally presented by Gardner and Zhao in 1993 [14]. This model's method simplifies the computation of creep compliances to provide a more rational design office procedure. This model is limited to average concrete compressive strengths between 16 MPa to 82 MPa, water to cement ratio between 0.40 to 0.60, and relative humidity between 20 and 100 percent. The volume to surface ratio must be larger than 19 mm and the age of concrete at loading must be greater than or equal to the age of concrete drying commenced.

2.7.1 Required Parameters for Creep Function

The computation of creep compliance depends on the following parameters:

- Age of concrete at loading (days)
- Concrete mean compressive strength at 28 days (MPa)
- Concrete mean compressive strength when loading commenced (MPa)

- Modulus of elasticity of concrete at loading (MPa)
- Relative humidity expressed as a decimal
- Volume to surface ratio (mm)

2.7.2 Computation of Creep Compliance

The compliance $J(t, t_o)$ is calculated using the following equation.

$$J(t, t_o) = \frac{1}{E_{cmto}} + \frac{\Phi(t, t_o)}{E_{cm28}} \quad [2-47]$$

E_{cmto} = Modulus of elasticity of concrete at loading (MPa)

E_{cm28} = Modulus of elasticity of concrete at 28 days (MPa)

$\Phi(t, t_o)$ = Creep coefficient

$$E_{cmto} = 3500 + 4300\sqrt{f_{cmto}} \quad [2-48]$$

Where f_{cmto} = Concrete mean compressive strength at loading (MPa), which can be determined as follows:

$$f_{cmto} = f_{cm28} \frac{t^{3/4}}{a + bt^{3/4}} \quad [2-49]$$

Coefficients a and b are related to the cement type as given in Table 2.7.

$$E_{cm28} = 3500 + 4300\sqrt{f_{cm28}} \quad [2-50]$$

Creep coefficient, $\Phi(t, t_o)$, can be calculated as follows:

$$\Phi(t, t_o) = \Phi(t_c) \left[2 \left(\frac{(t - t_o)^{0.3}}{(t - t_o)^{0.3} + 14} \right) + \left(\frac{7}{t_o} \right)^{0.5} \left(\frac{t - t_o}{t - t_o + 7} \right)^{0.5} + 2.5(1 - 1.086h^2) \left(\frac{t - t_o}{t - t_o + 0.12(V/S)^2} \right)^{0.5} \right] \quad [2-51]$$

$$\text{If } t_o = t_c \text{ then } \Phi(t_c) = 1 \quad [2-52]$$

$$\text{If } t_o > t_c \text{ then } \Phi(t_c) = \left[1 - \left(\frac{t_o - t_c}{t_o - t_c + 0.12(V/S)^2} \right)^{0.5} \right]^{0.5} \quad [2-53]$$

Table 2.7 Coefficients a and b as a Function of Cement Type

Cement Type	European Type	US Type	a	b
Slow hardening	SL	II	3.40	0.72
Normal/rapid hardening	R	I	2.80	0.77
Rapid hardening high strength	RS	III	1.00	0.92

CHAPTER 3

Sensitivity of Models

3.1 Introduction

The sensitivity of the AASHTO 04, ACI 209, B3, CEB 90, CEB 90-99, and GL 2000 models are presented in this chapter. The input parameters utilized for these models are summarized in Table 3.1. The limitations of input parameters for each model are shown in Table 3.2. The Equation [1] is used to ascertain the sensitivity of each model for each input parameter listed in Table 3.1. The sensitivity equations for each model are presented next.

Table 3.1 Input Parameter Characteristics for Different Models

Input parameter p	AASHTO 04	ACI 209	B3	CEB 90	CEB 90-99	GL 2000
28-day comp. strength	x	x	x	x	x	x
Duration at loading	x	x	x	x	x	x
Relative humidity	x	x	x	x	x	x
Specimen shape	----	----	x	----	----	----
Specimen size	x	x	x	x	x	x
Type of cement	----	x	x	x	x	x
Type of curing	----	x	x	----	----	----
Water content	----	----	x	----	----	----
Total Input Parameters	4	6	8	5	5	5

Table 3.2 Limitations of Input Parameters of Analyzed Models

Input Parameter p	AASHTO 04	ACI 209	B3	CEB 90	CEB 90-99	GL 2000
Aggregate to cement ratio	----	----	2.5-13.5	----	----	----
f_{cm28} [MPa] [ksi]	16-70 (2.3-10.2)	----	17.2-69 (2.5-10.0)	20-90 (2.9-13.0)	20-90 (2.9-13.0)	16-82 (2.3-11.9)
Relative humidity [%]	40-100	40-100	40-100	40-100	40-100	20-100
t_0 or t_c (Moist Cured)	≥ 7 days	≥ 7 days	$t_0 \geq t_c$	< 14 days	< 14 days	$t_0 \geq t_c \geq 1$
t_0 or t_c (Steam Cured)	≥ 7 days	≥ 1 days	$t_0 \geq t_c$	< 14 days	< 14 days	$t_0 \geq t_c \geq 1$
Type of Cement:						
European	R, RS	R, RS	R, SL, RS	R, SL, RS	R, SL, RS	R, SL, RS
American	I, III	I, III	I, II, III	I, II, III	I, II, III	I, II, III
Water to cement ratio	0.45-0.58	----	0.35-0.85	----	----	----

3.2 AASHTO 04 Model

3.2.1 AASHTO 04 Model — Relative Humidity

Sensitivity of the model due to change of relative humidity (H) can be determined as follows:

$$S(H) = \left[1 - \left(\frac{\Phi_{RH(2)}}{\Phi_{RH(1)}} \right) \right] \times 100\% \quad [3-1]$$

where $\Phi_{RH} = \text{CONST}(1) + \text{CONST}(2) \left(1.58 - \frac{H}{120} \right)$

$$\text{CONST}(1) = \frac{1}{E_c}$$

$$\text{CONST}(2) = \frac{3.5K_c K_f t_i^{-0.118}}{E_c} \left(\frac{(t-t_i)^{0.6}}{(10.0 + (t-t_i)^{0.6})} \right)$$

This model can be applied to type I, II and III Portland cement concrete with relative humidity ranging between 40% to 100%.

3.2.2 AASHTO 04 Model — Shape and Size Correction Factor

Sensitivity due to specimen size is similar to the sensitivity due to relative humidity. Sensitivity of the model due to change of specimen shape and size (i.e., volume to surface ratio (V/S)) can be determined as follows:

$$S(V/S) = \left[1 - \frac{k_{c(2)}}{k_{c(1)}} \right] \times 100\% \quad [3-2]$$

where $k_c = \text{CONST}(1) + \text{CONST}(3) \left[\frac{\frac{t}{26e^{0.0142(V/S)} + t}}{\frac{t}{45 + t}} \right] \left[\frac{1.80 + 1.77e^{-0.0213(V/S)}}{2.587} \right]$

$$\text{CONST}(1) = \frac{1}{E_c}$$

$$\text{CONST}(3) = \frac{3.5K_c K_f t_i^{-0.118}}{E_c} \left(\frac{(t-t_i)^{0.6}}{(10.0 + (t-t_i)^{0.6})} \right)$$

The maximum V/S ratio considered in k_c is 150 mm. Based on the research presented by the PCI Committee for prestress losses in 1975, creep remains nearly constant for V/S ratios greater than 150 mm. Therefore, V/S should be limited to 150 mm in computing the k_c .

3.2.3 AASHTO 04 Model — Age at Loading

Sensitivity of the model due to change at loading can be determined as follows:

$$S(t-t_i) = \left[1 - \frac{S(t_i)_{(2)}}{S(t_i)_{(1)}} \right] \times 100\% \quad [3-3]$$

where
$$S(t_i) = \text{CONST}(1) + \text{CONST}(4) \left(\frac{(t-t_i)^{0.6} t_i^{-0.118}}{10.0 + (t-t_i)^{0.6}} \right)$$

$$\text{CONST}(1) = \frac{1}{E_c}$$

$$\text{CONST}(4) = \frac{3.5K_c K_f}{E_c} \left(1.58 - \frac{H}{120} \right)$$

This model can be applied to type I, II and III Portland cement concrete with $t_i \geq 7$ days for moist cured concrete and $t_i \geq 1$ day for steam-cured concrete.

3.2.4 AASHTO 04 Model — Concrete Strength (f_{cm28}) at 28 days

Sensitivity of the model due to change of concrete strength at 28 days can be computed as follows:

$$S(f'_c) = \left[1 - \left(\frac{K_{f(2)}}{K_{f(1)}} \right)^{1/2} \right] \times 100\% \quad [3-4]$$

where $K_f = \text{CONST}(1) + \text{CONST}(5) \left(\frac{62 (f'_c)^{0.5}}{42 + f'_c} \right)$

$$\text{CONST}(1) = \frac{1}{E_c}$$

$$\text{CONST}(5) = \frac{3.5K_c}{E_c} \left(1.58 - \frac{H}{120} \right) t_i^{-0.118} \left(\frac{(t - t_i)^{0.6}}{10.0 + (t - t_i)^{0.6}} \right)$$

3.3 ACI 209 Model

3.3.1 ACI 209 Model — Cement and Curing Type

Sensitivity of the model due to change of cement and/or curing type can be determined as follows:

$$S(\text{cement/curing type}) = \left[1 - \left(\frac{f'_c(t_o)_{(1)}}{f'_c(t_o)_{(2)}} \right)^{1/2} \right] \times 100\% \quad [3-5]$$

where $f'_c(t_o) = \frac{t_o}{b + ct_o}$

This model can be applied to type I and III of Portland cement concrete where b and c are the constant for cement type and method of curing.

3.3.2 ACI 209 Model — Relative Humidity

Sensitivity of the model due to change of relative humidity (H) can be determined as follows:

$$S(H) = \left[1 - \frac{K_{ch(2)}}{K_{ch(1)}} \right] \times 100\% \quad [3-6]$$

where $K_{ch} = \text{CONST}(1) + \text{CONST}(2)(1.27 - 0.0067H)$

$$\text{CONST}(1) = \frac{1}{E_{cmto}}$$

$$\text{CONST}(2) = \left(\frac{t^{0.6}}{10 + t^{0.6}} \right) \left(\frac{C_{ca} K_{ca} K_{cs}}{E_{cmto}} \right)$$

This model can be applied to type I and III of Portland cement concrete with relative humidity ranging between 40% to 100%.

3.3.3 ACI 209 Model — Shape and Size Correction Factor

Sensitivity of the model due to change of specimen shape and size (i.e., volume to surface ratio (V/S)) can be determined as follows:

$$S(V/S) = \left[1 - \frac{K_{cs(2)}}{K_{cs(1)}} \right] \times 100\% \quad [3-7]$$

where $K_{cs} = \text{CONST}(1) + \text{CONST}(3)[1.14 - 0.0035(V/S)]$

$$\text{CONST}(1) = \frac{1}{E_{cmto}}$$

$$\text{CONST}(3) = \left(\frac{t^{0.6}}{10 + t^{0.6}} \right) \left(\frac{C_{ca} K_{ca} K_{ch}}{E_{cmto}} \right)$$

This model can be applied to type I and III of Portland cement concrete with a V/S ratio up to 325 mm.

3.3.4 ACI 209 Model — Age at Loading

Sensitivity of the model due to change at loading can be determined as follows:

$$S(t_o) = \left[1 - \frac{K_{ca(2)}}{K_{ca(1)}} \right] \times 100\% \quad [3-8]$$

where $K_{ca} = \text{CONST}(1) + \text{CONST}(4)(t_o)^{-0.118}$

$$\text{CONST}(1) = \frac{1}{E_{cmto}}$$

$$\text{CONST}(4) = 1.25 \left(\frac{t^{0.6}}{10 + t^{0.6}} \right) \left(\frac{C_{ca} K_{ch} K_{cs}}{E_{cmto}} \right)$$

This model can be applied to type I and III of Portland cement concrete with $t_o \geq 7$ days for moist cured concrete and $t_o \geq 1$ day for steam-cured concrete.

3.3.5 ACI 209 Model — Concrete Strength (f_{cm28}) at 28 days

Sensitivity of the model due to change of concrete strength at 28 days can be computed as follows:

$$S(f_{cm28}) = \left[1 - \left(\frac{f_{cm28(1)}}{f_{cm28(2)}} \right)^{1/2} \right] \times 100\% \quad [3-9]$$

3.4 B3 Model

3.4.1 B3 Model — Cement / Curing Type

Sensitivity of the model due to change of cement type can be determined as follows:

$$S(\text{cement type}) = \left[\frac{C_d(t, t_o, t_c)_{(1)} - C_d(t, t_o, t_c)_{(2)}}{\text{CONST}_c + C_d(t, t_o, t_c)_{(1)}} \right] \times 100\% \quad [3-10]$$

where $\text{CONST}_c = q_1 + C_o(t, t_o)$

This model can be applied to type I, II and III of Portland cement concrete where α_1 is constant for cement type.

3.4.2 B3 Model — Relative Humidity

Sensitivity of the model due to change of relative humidity (H) can be determined

as follows:

$$S(H) = \left[\frac{C_d(t, t_o, t_c)_{(1)} - C_d(t, t_o, t_c)_{(2)}}{CONST_c + C_d(t, t_o, t_c)_{(1)}} \right] \times 100\% \quad [3-11]$$

where $CONST_c = q_1 + C_o(t, t_o)$

This model can be applied to type I, II, and III Portland cement concrete with relative humidity ranging between 40% to 100%.

3.4.3 B3 Model — Shape and Size Correction Factor

Sensitivity due to specimen size is similar to the sensitivity due to relative humidity. Sensitivity of the model due to change of specimen shape and size (i.e., volume to surface ratio (V/S)) can be determined as follows:

$$S(H) = \left[\frac{C_d(t, t_o, t_c)_{(1)} - C_d(t, t_o, t_c)_{(2)}}{CONST_c + C_d(t, t_o, t_c)_{(1)}} \right] \times 100\% \quad [3-12]$$

where $CONST_c = q_1 + C_o(t, t_o)$

3.4.4 B3 Model — Age at Loading

Sensitivity of the model due to change at loading can be determined as follows:

$$S(V/S) = \left[\frac{\{C_o(t, t_o) + C_d(t, t_o, t_c)\}_{(1)} - \{C_o(t, t_o) + C_d(t, t_o, t_c)\}_{(2)}}{q_1 + \{C_o(t, t_o) + C_d(t, t_o, t_c)\}_{(1)}} \right] \times 100\% \quad [3-13]$$

This model can be applied to type I, II, and III Portland cement concrete with $t_o \geq 7$ days for moist cured concrete and $t_o \geq 1$ day for steam-cured concrete.

3.4.5 B3 Model — Concrete Strength (f_{cm28}) at 28 days

In the B3 model, concrete strength at 28 days modified all parameters for computing the creep compliance. Sensitivity of the model due to change of concrete strength at 28 days can be computed as follows:

$$S(f_{cm28}) = \left[1 - \frac{J(t, t_o)_{(2)}}{J(t, t_o)_{(1)}} \right] \times 100\% \quad [3-14]$$

3.5 CEB 90 Model

3.5.1 CEB 90 Model — Cement Type

Sensitivity of the model due to change of cement type can be determined as follows:

$$S(\text{cement type}) = \left[1 - \frac{\alpha_{c(2)}}{\alpha_{c(1)}} \right] \times 100\% \quad [3-15]$$

where
$$\alpha_c = \frac{1}{e^{[0.5S(1-(28/t_o)^{0.5})]}}$$

This model can be applied to type I, II, and III of Portland cement concrete where S is constant for cement type.

3.5.2 CEB 90 Model — Relative Humidity

Sensitivity of the model due to change of relative humidity (H) can be determined as follows:

$$S(H) = \left[1 - \frac{\text{CONST}(1) + \text{CONST}(2)(\Phi_{RH(2)}\beta_{c(2)})}{\text{CONST}(1) + \text{CONST}(2)(\Phi_{RH(1)}\beta_{c(1)})} \right] \times 100\% \quad [3-16]$$

where $\Phi_{RH} = 1 + \frac{(1-H/100)}{0.46(h_e/100)^{1/3}}$

$$\beta_c(t, t_o) = \frac{(t - t_o)^{0.3}}{[\beta_H + (t - t_o)]^{0.3}}$$

$$\beta_H = 1.5[1 + (0.012H)^{18}]h_e + 250 \leq 1500 \text{ days}$$

$$\text{CONST}(1) = \frac{1}{E_{cmto}}$$

$$\text{CONST}(2) = \frac{\beta(f_{cm28})\beta(t, t_o)}{E_{cm28}}$$

This model can be applied to type I, II, and III Portland cement concrete with relative humidity ranging between 40% to 100%.

3.5.3 CEB 90 Model — Shape and Size Correction Factor

Sensitivity due to specimen size is similar to the sensitivity due to relative humidity. Sensitivity of the model due to change of specimen shape and size (i.e., volume to surface ratio (V/S)) can be determined as follows:

$$S(H) = \left[1 - \frac{\text{CONST}(1) + \text{CONST}(2)(\Phi_{RH(2)}\beta_{c(2)})}{\text{CONST}(1) + \text{CONST}(2)(\Phi_{RH(1)}\beta_{c(1)})} \right] \times 100\% \quad [3-17]$$

where $\Phi_{RH} = 1 + \frac{(1-H/100)}{0.46(h_e/100)^{1/3}}$

$$\beta_c(t, t_o) = \frac{(t - t_o)^{0.3}}{[\beta_H + (t - t_o)]^{0.3}}$$

$$\beta_H = 1.5[1 + (0.012H)^{18}]h_e + 250 \leq 1500 \text{ days}$$

$$\text{CONST}(1) = \frac{1}{E_{cmto}}$$

$$\text{CONST}(2) = \frac{\beta(f_{\text{cm}28})\beta(t, t_o)}{E_{\text{cm}28}}$$

3.5.4 CEB 90 Model — Age at Loading

Sensitivity of the model due to change at loading can be determined as follows:

$$S(t - t_o) = \left[1 - \frac{\alpha_{t(2)}}{\alpha_{t(1)}} \right] \times 100\% \quad [3-18]$$

where

$$\alpha_t = \text{CONST}(3) \left(\frac{1}{0.1 + t_o^{0.2}} \right) \left(\frac{t - t_o}{\beta_H + (t - t_o)} \right)^{0.3} + \frac{1}{e^{[0.5S(1 - (28/t_o)^{0.5})]}}$$

$$\text{CONST}(3) = \Phi_{\text{RH}} \beta(f_{\text{cm}28})$$

This model can be applied to type I and III Portland cement concrete with $t_o \geq 7$ days for moist cured concrete and $t_o \geq 1$ day for steam-cured concrete.

3.5.5 CEB 90 Model — Concrete Strength ($f_{\text{cm}28}$) at 28 days

Sensitivity of the model due to change of concrete strength at 28 days can be computed as follows:

$$S(f_{\text{cm}28}) = \left[1 - \left(\frac{\text{CONST}(1) + \text{CONST}(4)\beta(f_{\text{cm}28})_{(2)}}{\text{CONST}(1) + \text{CONST}(4)\beta(f_{\text{cm}28})_{(1)}} \right) \right] \times 100\% \quad [3-19]$$

$$\text{CONST}(1) = \frac{1}{E_{\text{cm}to}} \quad \text{and} \quad \text{CONST}(4) = \frac{\Phi_{\text{RH}} \beta(f_{\text{cm}28}) \beta(t, t_o)}{E_{\text{cm}28}}$$

3.6 CEB 90-99 Model

3.6.1 CEB 90-99 Model — Cement Type

Sensitivity of the model due to change of cement type can be determined as follows:

$$S(\text{cement type}) = \left[1 - \frac{\alpha_{c(2)}}{\alpha_{c(1)}} \right] \times 100\% \quad [3-20]$$

where
$$\alpha_c = \frac{1}{e^{[0.5S(1-(28/t_o)^{0.5})]}}$$

This model can be applied to type I, II, and III of Portland cement concrete where S is constant for cement type.

3.6.2 CEB 90-99 Model — Relative Humidity

Sensitivity of the model due to change of relative humidity (H) can be determined as follows:

$$S(H) = \left[1 - \frac{\text{CONST}(1) + \text{CONST}(2)(\Phi_{RH(2)}\beta_{c(2)})}{\text{CONST}(1) + \text{CONST}(2)(\Phi_{RH(1)}\beta_{c(1)})} \right] \times 100\% \quad [3-21]$$

where
$$\Phi_{RH} = 1 + \frac{(1-H/100)\alpha_1}{0.46(h_e/100)^{1/3}}$$

$$\beta_c(t, t_o) = \frac{(t - t_o)^{0.3}}{[\beta_H + (t - t_o)]^{0.3}}$$

$$\beta_H = 1.5 \left[1 + (0.012H)^{18} \right] h_e + 250\alpha_3 \leq 1500\alpha_3 \quad \text{days}$$

$$\text{CONST}(1) = \frac{1}{E_{cmto}}$$

$$\text{CONST}(2) = \frac{\beta(f_{cm28})\beta(t, t_o)}{E_{cm28}}$$

This model can be applied to type I, II, and III Portland cement concrete with relative humidity ranging between 40% to 100%.

3.6.3 CEB 90-99 Model — Shape and Size Correction Factor

Sensitivity due to specimen size is similar to the sensitivity due to relative humidity. Sensitivity of the model due to change of specimen shape and size (i.e., volume to surface ratio (V/S)) can be determined as follows:

$$S(H) = \left[1 - \frac{\text{CONST}(1) + \text{CONST}(2)(\Phi_{RH(2)}\beta_{e(2)})}{\text{CONST}(1) + \text{CONST}(2)(\Phi_{RH(1)}\beta_{e(1)})} \right] \times 100\% \quad [3-22]$$

where $\Phi_{RH} = 1 + \frac{(1 - H/100)\alpha_1}{0.46(h_e/100)^{1/3}}$

$$\beta_e(t, t_o) = \frac{(t - t_o)^{0.3}}{[\beta_H + (t - t_o)]^{0.3}}$$

$$\beta_H = 1.5 \left[1 + (0.012H)^{18} \right] h_e + 250\alpha_3 \leq 1500\alpha_3 \quad \text{days}$$

$$\text{CONST}(1) = \frac{1}{E_{cmto}}$$

$$\text{CONST}(2) = \frac{\beta(f_{cm28})\beta(t, t_o)}{E_{cm28}}$$

3.6.4 CEB 90-99 Model — Age at Loading

Sensitivity of the model due to change at loading can be determined as follows:

$$S(t - t_o) = \left[1 - \frac{\alpha_{t(2)}}{\alpha_{t(1)}} \right] \times 100\% \quad [3-23]$$

where
$$\alpha_t = \text{CONST}(3) \left(\frac{1}{0.1 + t_o^{0.2}} \right) \left(\frac{t - t_o}{\beta_H + (t - t_o)} \right)^{0.3} + \frac{1}{e^{[0.5S(1-(28/t_o)^{0.5})]}}$$

$$\text{CONST}(3) = \Phi_{RH} \beta(f_{cm28})$$

This model can be applied to type I and III Portland cement concrete with $t_o \geq 7$ days for moist cured concrete and $t_o \geq 1$ day for steam-cured concrete.

3.6.5 CEB 90-99 Model — Concrete Strength (f_{cm28}) at 28 days

Sensitivity of the model due to change of concrete strength at 28 days can be computed as follows:

$$S(f_{cm28}) = \left[1 - \left(\frac{\text{CONST}(1) + \text{CONST}(4) \beta(f_{cm28})_{(2)}}{\text{CONST}(1) + \text{CONST}(4) \beta(f_{cm28})_{(1)}} \right) \right] \times 100\% \quad [3-24]$$

$$\text{CONST}(1) = \frac{1}{E_{cmto}}$$

$$\text{CONST}(4) = \frac{\Phi_{RH} \beta(f_{cm28}) \beta(t, t_o)}{E_{cm28}}$$

3.7 GL 2000 Model

3.7.1 GL 2000 Model — Cement Type

Sensitivity of the model due to change of cement type can be determined as follows:

$$S(\text{cement type}) = \left[1 - \frac{J(t, t_o)_{(2)}}{J(t, t_o)_{(1)}} \right] \times 100\% \quad [3-25]$$

This model can be applied to type I, II, and III of Portland cement concrete where

a and b are the constants for cement type for computing f_{cmto} .

3.7.2 GL 2000 Model — Relative Humidity

Sensitivity of the model due to change of relative humidity (H) can be determined as follows:

$$S(H) = \left[1 - \frac{J(t, t_o)_{(2)}}{J(t, t_o)_{(1)}} \right] \times 100\% \quad [3-26]$$

This model can be applied to type I, II, and III Portland cement concrete with relative humidity ranging between 20% to 100%.

3.7.3 GL 2000 Model — Shape and Size Correction Factor

Sensitivity due to specimen size is similar to the sensitivity due to relative humidity. Sensitivity of the model due to change of specimen shape and size (i.e., volume to surface ratio (V/S)) can be determined as follows:

$$S(V/S) = \left[1 - \frac{J(t, t_o)_{(2)}}{J(t, t_o)_{(1)}} \right] \times 100\% \quad [3-27]$$

3.7.4 GL 2000 Model — Age at Loading

Sensitivity of the model due to change at loading can be determined as follows:

$$S(t - t_o) = \left[1 - \frac{J(t, t_o)_{(2)}}{J(t, t_o)_{(1)}} \right] \times 100\% \quad [3-28]$$

This model can be applied to type I, II, and III Portland cement concrete with $t_o \geq 7$ days for moist cured concrete and $t_o \geq 1$ day for steam-cured concrete.

3.7.5 GL 2000 Model — Concrete Strength (f_{cm28}) at 28 days

Sensitivity of the model due to change of concrete strength at 28 days can be computed as follows:

$$S(f_{cm28}) = \left[1 - \frac{J(t, t_o)_{(2)}}{J(t, t_o)_{(1)}} \right] \times 100\% \quad [3-29]$$

CHAPTER 4

Comparison of Results

4.1 Introduction

This chapter will provide the results of sensitivity of the AASHTO 04, ACI 209, B3, CEB 90, CEB 90-99, and GL 2000 models. The creep coefficients were evaluated for various parameters such as relative humidity, volume to surface ratio (V/S), specimen age at loading, and concrete strength at loading.

The creep compliance is computed by changing only one parameter and keeping the remaining parameters constant. The sensitivity of creep is presented as the percentile change in creep prediction caused by change in input parameter. The sensitivity of creep is presented in line graphs where sensitivity is computed by keeping one input parameter at its initial value. This produces a positive percentile change in sensitivity when the initial value of input parameter is less than the value of the parameter after change. However, sensitivity becomes negative when the initial value of the input parameter is greater than the value of the parameter after change. For example, a decrease from 60% relative humidity (used as the initial input parameter) to 40% relative humidity will produce a negative sensitivity, while sensitivity will become positive when relative humidity is increased from 60% to 80%.

4.2 AASHTO 04 Model

4.2.1 AASHTO 04 Model — Cement Type/Curing Type

The sensitivity of creep in the AASHTO 04 model does not depend on cement type or curing method, as computation of the creep coefficient does not consider the cement type and/or curing type.

4.2.2 AASHTO 04 Model — Relative Humidity

Figures 4.1a and 4.1b show that the sensitivity due to relative humidity has a linear relationship on sensitivity of creep for the AASHTO 04 model. The sensitivity due to change in relative humidity is always positive when the initial value of relative humidity is 40%. The change in sensitivity becomes negative when the initial value of relative humidity is more than 40% and the change in humidity is less than the initial value of relative humidity.

Figure 4.1a Sensitivity of the AASHTO 04 Model Due to Change in Relative Humidity

Figure 4.1b Sensitivity of the AASHTO 04 Model Due to Change in Relative Humidity

4.2.3 AASHTO 04 Model — Shape and Size Correction Factor

Figures 4.2a and 4.2b show the sensitivity of the AASHTO 04 model due to change in V/S ratio. The increase in specimen size changes the sensitivity of creep in nonlinear fashion for the AASHTO 04 model.

Figure 4.2a Sensitivity of the AASHTO 04 Model Due to Change in V/S Ratio

Figure 4.2b Sensitivity of the AASHTO 04 Model Due to Change in V/S Ratio

The maximum V/S ratio considered in development of k_c (i.e., as a factor for the effect of V/S ratio) was 150 mm. Based on the research presented by the PCI committee in 1975, creep remains constant for V/S ratios greater than 150 mm. Therefore, V/S should be limited to 150 mm for AASHTO 04 code.

4.2.4 AASHTO 04 Model — Age at Loading

The sensitivity due to age at loading is shown in Figures 4.3a and 4.3b. The age at loading has a linear relationship to sensitivity and, the sensitivity increases gradually. The sensitivity due to age at loading is greater at 28 days.

Figure 4.3a Sensitivity of the AASHTO 04 Model Due to Age at Loading

Figure 4.3b Sensitivity of the AASHTO 04 Model Due to Age at Loading

4.2.5 AASHTO 04 Model — Concrete Strength (f_{cm28}) at 28 days

Figures 4.4a and 4.4b show the sensitivity of the AASHTO 04 model for concrete strengths at 28 days. The concrete strength also has a nonlinear effect on the sensitivity

of creep for the AASHTO 04 model, and modulus of elasticity at 28 days has a direct relationship with concrete strength.

Figure 4.4a Sensitivity of the AASHTO 04 Model Due to Concrete Strength at 28 Days

Figure 4.4b Sensitivity of the AASHTO 04 Model Due to Concrete Strength at 28 Days

4.3 ACI 209 Model

4.3.1 ACI 209 Model — Cement and Curing Types

A comparison of sensitivities for cement type and curing type is shown in Table 4.1 and in Table 4.2. The ACI 209 model is less sensitive to cement type and/or curing type, as the sensitivity due to change of cement type, or change of curing type, is less than 1%. Negative sensitivity represents an increase in creep compliance when the type of cement is changed from type I to type III cement. Similarly, the creep compliances are increased when curing type is changed from moist cured (M) to steam cured (S) for type III cement, which also produces a negative sensitivity.

Table 4.1 Sensitivity for Cement Type — ACI 209 Model

Cement Type	Curing Type [M/S]	$J(t, t_0) \times 10^{-6} / \text{MPa}$	Sensitivity in %
I	M	51.73	0.00
III	M	51.97	-0.47
I	S	51.54	0.00
III	S	52.04	-0.97

Table 4.2 Sensitivity for Curing Type — ACI 209 Model

Cement Type	Curing Type [M/S]	$J(t, t_0) \times 10^{-6} / \text{MPa}$	Sensitivity in %
I	M	51.73	0.00
I	S	51.54	0.36
III	M	51.97	0.00
III	S	52.04	-0.14

4.3.2 ACI 209 Model — Relative Humidity

The sensitivity of ACI 209 model follows the same linear relationship for all cement types; Figures 4.5a and 4.5b show the change in the sensitivity for type I cement.

The relative humidity correction factor (K_{ch}) is a linear function of humidity (i.e., $K_{ch} = 1.27 - 0.00678H$), and ranges between 1.0 to 0.6 for relative humidity between 40-100 %.

Figure 4.5a Sensitivity of the ACI 209 Model Due to Change in Relative Humidity

Figure 4.5b Sensitivity of the ACI 209 Model Due to Change in Relative Humidity

4.3.3 ACI 209 Model — Shape and Size Correction Factor

The sensitivity of the ACI 209 model due to change in V/S ratio is shown in Figures 4.6a and 4.6b.

Figure 4.6a Sensitivity of the ACI 209 Model Due to Change in V/S Ratio

Figure 4.6b Sensitivity of the ACI 209 Model Due to Change in V/S Ratio

The sensitivity due to V/S ratio is also a linear function due to the shape and size correction factor, which affects the sensitivity of the ACI 209 model. However, the sensitivity increases rapidly for large V/S ratios and ACI 209 model is not applicable beyond V/S ratios of 325 mm.

4.3.4 ACI 209 Model — Age at Loading

Figures 4.7a and 4.7b show that the age at loading has a nonlinear relationship to sensitivity. The sensitivity for all cement types and curing types increases when structure is loaded at various loading ages. The variation of sensitivity of steamed cured concrete are smaller compared to moist cured concrete.

Figure 4.7a Sensitivity of the ACI 209 Model Due to Age at Loading

Figure 4.7b Sensitivity of the ACI 209 Model Due to Age at Loading

4.3.5 ACI 209 Model — Concrete Strength (f_{cm28}) at 28 days

Figures 4.8a and 4.8b show that the sensitivity due to change in concrete strength is increased nonlinearly as creep compliance is inversely proportional to concrete strength.

Figure 4.8a Sensitivity of the ACI 209 Model Due to Concrete Strength at 28 Days

Figure 4.8b Sensitivity of the ACI 209 Model Due to Concrete Strength at 28 Days

4.4 B3 Model

4.4.1 B3 Model — Cement /Curing Type

The sensitivity of the B3 model due to change of cement type or curing type is relatively small. A comparison of sensitivity for cement type and curing type is documented in Table 4.3 and Table 4.4.

Table 4.3 Sensitivity for Cement Type — B3 Model

Cement Type	Curing Type [M/S]	$J(t,t_0) \times 10^{-6} / \text{MPa}$	Sensitivity in %
I	M	43.3	0.00
II	M	43.8	-1.18
III	M	43.3	0.64

Table 4.4 Sensitivity for Curing Type — B3 Model

Cement Type	Curing Type [M/S]	$J(t, t_0) \times 10^{-6} / \text{MPa}$	Sensitivity in %
I	M	43.3	0.00
I	S	44.2	-2.16
I	SL	42.8	3.28

4.4.2 B3 Model — Relative Humidity

The spatial average of pore relative humidity within a cross-section for the B3 model has a linear relationship with relative humidity. Therefore, the sensitivity due to relative humidity changes remains linear. The sensitivity of the B3 model follows the same linear relationship for all cement types. The change in sensitivity for type I cement is shown in Figures 4.9a and 4.9b.

Figure 4.9a Sensitivity of the B3 Model Due to Change in Relative Humidity

Figure 4.9b Sensitivity of the B3 Model Due to Change in Relative Humidity

4.4.3 B3 Model — Shape and Size Correction Factor

The sensitivity of the B3 model due to change in the V/S ratio is shown in Figures 4.10a and 4.10b.

Figure 4.10a Sensitivity of the B3 Model Due to Change in V/S Ratio

Figure 4.10b Sensitivity of the B3 Model Due to Change in V/S Ratio

The sensitivity due to V/S ratio is also a nonlinear function due to shrinkage half time, which affects the sensitivity of the B3 model. The sensitivity increases rapidly for V/S ratio equal to 100 and then the sensitivity gradually reduces for larger V/S ratios.

4.4.4 B3 Model — Age at Loading

Figures 4.11a and 4.11b show that the age at loading has a nonlinear relationship to sensitivity. The sensitivity for all cement types increases rapidly when loads are applied at the initial stage. The sensitivity increases until 15 days and then the rate of change in sensitivity decreases after 21 days.

Figure 4.11a Sensitivity of the B3 Model Due to Age at Loading

Figure 4.11b Sensitivity of the B3 Model Due to Age at Loading

4.4.5 B3 Model — Concrete Strength (f_{cm28}) at 28 days

Figures 4.12a and 4.12b show that the sensitivity due to change in concrete strength has a nonlinear effect in the B3 model. Concrete strength between 30 to 45 MPa

Figure 4.12a Sensitivity of the B3 Model Due to Concrete Strength at 28 Days

Figure 4.12b Sensitivity of the B3 Model Due to Concrete Strength at 28 Days

has an increasing affect on sensitivity and the sensitivity change decreases for concrete strength beyond 45 MPa.

4.5 CEB 90 Model

4.5.1 CEB 90 Model — Cement Type

Table 4.5 shows the comparison of sensitivity for cement type for the CEB 90 model. The sensitivity of the CEB 90 model due to change in cement type is negligible.

Table 4.5 Sensitivity for Cement Type — CEB 90 Model

Cement Type	$J(t,t_0) \times 10^{-6} / \text{MPa}$	Sensitivity in %
I	42.9	0.00
II	42.8	0.26
III	43.0	-0.10

4.5.2 CEB 90 Model — Relative Humidity

Figures 4.13a and 4.13b show the change in sensitivity for type I cement. The sensitivity of the CEB 90 model varies linearly for all cement types due to changes in relative humidity.

Figure 4.13a Sensitivity of the CEB 90 Model Due to Change in Relative Humidity

Figure 4.13b Sensitivity of the CEB 90 Model Due to Change in Relative Humidity

4.5.3 CEB 90 Model — Shape and Size Correction Factor

The sensitivity of the CEB 90 model due to change in V/S ratio can be observed from Figures 4.14a and 4.14b. The increase in specimen size changes the sensitivity of creep in nonlinear fashion as $\Phi_{RH} = 1 + (1 - H/100) / [0.46 * (h_e/100)^{1/3}]$ directly affects the creep coefficients.

Figure 4.14a Sensitivity of the CEB 90 Model Due to Change in V/S Ratio

Figure 4.14b Sensitivity of the CEB 90 Model Due to Change in V/S Ratio

The nonlinear variation of creep is more evident for this model when the V/S ratio is less than 200 mm. Beyond 200 mm, the variation in sensitivity moves more closely towards a linear relationship.

4.5.4 CEB 90 Model — Age at Loading

The sensitivity for various cement types is more prominent between seven to twenty-one days of loading as shown in Figures 4.15a and 4.15b. The age at loading has a nonlinear relationship on sensitivity. The sensitivity for all cement types increases rapidly when loads are applied at the initial stage.

Figure 4.15a Sensitivity of the CEB 90 Model Due to Age at Loading

Figure 4.15b Sensitivity of the CEB 90 Model Due to Age at Loading

4.5.5 CEB 90 Model — Concrete Strength (f_{cm28}) at 28 days

The sensitivity of the CEB 90 model for various concrete strengths at 28 days is shown in Figures 4.16a and 4.16b

Figure 4.16a Sensitivity of the CEB 90 Model Due to Concrete Strength at 28 Days

Figure 4.16b Sensitivity of the CEB 90 Model Due to Concrete Strength at 28 Days

Concrete strength also has a nonlinear effect on sensitivity in the CEB 90 model and the modulus of elasticity at 28 days shows a direct relationship to concrete strength.

4.6 CEB 90-99 Model

4.6.1 CEB 90-99 Model — Cement Type

The sensitivity of the CEB 90-99 model due to change in cement type is zero at 28 days. The sensitivity due to cement type at age 35 days is documented in Table 4.6.

Table 4.6 Sensitivity for Cement Type at 35 days

Cement Type	$J(t, t_0) \times 10^{-6} / \text{MPa}$	Sensitivity in %
RS	44.56	0.00%
R	45.05	-1.11%
SL	45.56	-2.25%

4.6.2 CEB 90-99 Model — Relative Humidity

Figures 4.17a and 4.17b show the change in sensitivity of type R cement for the CEB 90-99 model. The sensitivity of the CEB 90-99 model varies linearly for all cement type due to change in relative humidity.

Figure 4.17a Sensitivity of the CEB 90-99 Model Due to Change in Relative Humidity

Figure 4.17b Sensitivity of the CEB 90-99 Model Due to Change in Relative Humidity

4.6.3 CEB 90-99 Model — Shape and Size Correction Factor

Figures 4.18a and 4.18b show the sensitivity of CEB 90-99 model due to change in V/S ratio. The increase in specimen size changes the sensitivity of creep in nonlinear fashion as $\Phi_{RH} = [1+(1-H/100)\alpha_1/[0.46*(h_c/100)^{1/3}]] \alpha_3$ directly affects the creep coefficients.

Figure 4.18a Sensitivity of the CEB 90-99 Model Due to Change in V/S Ratio

Figure 4.18b Sensitivity of the CEB 90-99 Model Due to Change in V/S Ratio

The nonlinear variation of creep is more evident in this model where the V/S ratio is less than 200 mm. Beyond 200 mm, the variation in sensitivity is closer to a linear relationship. The sensitivity of CEB 90-99 model due to change in V/S ratio is shown in Figures 4.18a and 4.18b.

4.6.4 CEB 90-99 Model — Age at Loading

Figures 4.19a and 4.19b show that the age at loading has a nonlinear relationship to sensitivity. The sensitivity for all cement types increases rapidly when loads are applied at the initial stage. The sensitivity for various cement types is more prominent between seven to twenty-one days of loadings.

Figure 4.19a Sensitivity of the CEB 90-99 Model Due to Age at Loading

Figure 4.19b Sensitivity of the CEB 90-99 Model Due to Age at Loading

4.6.5 CEB 90-99 Model — Concrete Strength (f_{cm28}) at 28 days

The sensitivity of the CEB 90-99 model for various concrete strengths is shown in Figures 4.20a and 4.20b.

Figure 4.20a Sensitivity of the CEB 90-99 Model Due to Concrete Strength at 28 Days

Figure 4.20b Sensitivity of the CEB 90-99 Model Due to Concrete Strength at 28 Days

The concrete strength also has a nonlinear effect on sensitivity in the CEB 90-99 model, and the modulus of elasticity at 28 days shows a direct relationship to concrete strength.

4.7 GL 2000 Model

4.7.1 GL 2000 Model — Cement /Curing Type

Comparisons of sensitivity for cement type at loading age of 28 days and 35 days are documented in Table 4.7 and Table 4.8. The GL 2000 model does not consider the curing type and therefore there is no change in creep compliance. The sensitivity of the GL 2000 model is also very small due to change in cement type, especially when load is applied at 28 days.

Table 4.7 Sensitivity for Cement Type at Loading Age of 28 days

Cement Type	$J(t, t_0) \times 10^{-6} / \text{MPa}$	Sensitivity in %
I	52.2	0.00
II	52.2	0.02
III	52.2	-0.05

Table 4.8 Sensitivity for Cement Type at Loading Age of 35 days

Cement Type	$J(t, t_0) \times 10^{-6} / \text{MPa}$	Sensitivity in %
I	29.0	0.00
II	28.9	0.39
III	29.4	-1.15

4.7.2 GL 2000 Model — Relative Humidity

Figures 4.21a and 4.21b show the sensitivity of the GL 2000 model for type I cement. The relative humidity has a quadratic function for computing creep factor. Therefore, the sensitivity due to relative humidity produces a nonlinear response.

Figure 4.21a Sensitivity of the GL 2000 Model Due to Change in Relative Humidity

Figure 4.21b Sensitivity of the GL 2000 Model Due to Change in Relative Humidity

4.7.3 GL2000 Model — Shape and Size Correction Factor

Figures 4.22a and 4.22b show the sensitivity of GL 2000 model due to change in V/S ratio.

Figure 4.22a Sensitivity of the GL 2000 Model Due to Change in V/S Ratio

Figure 4.22b Sensitivity of the GL 2000 Model Due to Change in V/S Ratio

The sensitivity due to V/S ratio is also a nonlinear function due to the quadratic effect of V/S ratio for creep factor. The sensitivity increases rapidly for V/S ratios less than 150 mm and then the sensitivity gradually lessens for larger V/S ratios.

4.7.4 GL 2000 Model — Age at Loading

Figures 4.23a and 4.23b show that the age at loading has a nonlinear relationship on sensitivity for the GL 2000 model. The sensitivity for all cement types increases rapidly when loads are applied at the initial stage. The sensitivity increases until 15 days and then the rate of change of sensitivity decreases after 21 days.

Figure 4.23a Sensitivity of the GL 2000 Model Due to Age at Loading

Figure 4.23b Sensitivity of the GL 2000 Model Due to Age at Loading

4.7.5 GL 2000 Model — Concrete Strength (f_{cm28}) at 28 days

Figures 4.24a and 4.24b show that the sensitivity due to change in concrete strength has a nonlinear effect in the GL 2000 model. Concrete strength between 30 to

45 MPa has an increasing effect on sensitivity, and sensitivity decreases for concrete strength beyond 45 MPa.

Figure 4.24a Sensitivity of the GL 2000 Model Due to Concrete Strength at 28 Days

Figure 4.24b Sensitivity of the GL 2000 Model Due to Concrete Strength at 28 Days

CHAPTER 5

Discussion and Conclusions

5.1 Introduction

The sensitivity of six different creep models is evaluated. The studied models are the AASHTO 04, the ACI 209 model, the B3 model, the CEB 90 model, the CEB 90-99 model, and the GL 2000 model. The computations for sensitivity of creep for each model are presented in Appendix A to F. The following conclusions are made from this study:

5.1.1 AASHTO 04 Model

- 1) The AASHTO 04 model is the most simplified model which has only 4 input parameters to compute creep in concrete.
- 2) The AASHTO 04 model does not account the cement type and/or the curing type for computing creep of concrete.
- 3) The sensitivity of the AASHTO 04 model due to change in relative humidity has a linear relationship.
- 4) The AASHTO 04 model limits the V/S ratio to 150 mm and the sensitivity of creep remains constant for V/S ratio > 150 mm.
- 5) The sensitivity due to age at loading has nonlinear effect.
- 6) The concrete strength has largest effect on sensitivity for the AASHTO 04 model when compared with other input parameters. The sensitivity of creep rate is higher when concrete strength is lower.

5.1.2 ACI 209 Model

- 7) The effect of cement type or curing type has least effect on sensitivity of creep for the ACI 209 model. The percentage change in sensitivity of creep due to change in cement type or curing type is less than 1%.
- 8) The change in sensitivity of creep is a linear function of relative humidity for the ACI 209 model.
- 9) The ACI 209 model is highly sensitive for volume to surface ratio when compared with other input parameters used for this model. The sensitivity of the ACI 209 model due to change in V/S ratio has a linear relationship to the shape and size correction factor. The shape and size correction factor for ACI 209 model becomes negative when V/S ratio exceeds 325 mm which suggests limiting the V/S ratio for this model.
- 10) The age at loading of the ACI 209 model has a nonlinear relationship to sensitivity of creep. The sensitivity of creep increases when age at loading is increased. The change in sensitivity of steamed cured concrete is smaller compared to moist cured concrete.
- 11) The sensitivity of creep of the ACI 209 model is increased nonlinearly, as creep compliance is inversely proportional to concrete strength.

5.1.3 B3 Model

- 12) The B3 model is the most complex model which accounts for several input parameters to compute creep coefficient. Age at loading has the most significant

impact on sensitivity of creep when compared with other input parameters for this model which is followed by the change in sensitivity due to change in concrete strength.

- 13) The cement type or method of curing has the least effect on sensitivity when compared with other input parameters for this model. The overall variation due to change in cement type is less than 2% while change in sensitivity due to change in curing method is less than 4%.
- 14) The sensitivity due to relative humidity follows the linear relationship for all cement types.
- 15) The sensitivity of B3 model due to V/S ratio has a nonlinear function due to shrinkage half time which is defined as follow:

$$\tau_{sh} = 0.085t_c - 0.08(f_{cm28}) - 0.25[2k_s(V/S)]^2$$
- 16) The sensitivity due to age at loading and due to change in concrete strength has a nonlinear effect in the B3 model. The sensitivity of creep rate is higher for concrete strength less than 45 MPa.

5.1.4 CEB 90 Model

- 17) Sensitivity determined by various input parameters is very close to each other except for cement types for the CEB 90 model. The change in age at loading has the largest effect on sensitivity of creep for the CEB 90 model.
- 18) The change in relative humidity has a linear effect on the sensitivity of the CEB 90 model.

- 19) The change in V/S ratio, age at loading and concrete strength has nonlinear effects on sensitivity.

5.1.5 CEB 90-99 Model

- 20) The CEB 90-99 model is not sensitive to cement type when model is loaded at 28 days and sensitivity due to change in cement type is increased by 2% when structure is loaded at 35 days. The SL cement type is more sensitive when compared with the R or RS cement type.
- 21) The sensitivity of creep varies linearly due to change in relative humidity for the CEB 90-99 model.
- 22) The increase in specimen sizes i.e., V/S ratio changes the sensitivity of creep in nonlinear fashion. The nonlinear variation is more evident in this model where V/S ratio is less than 200 mm.
- 23) The characteristic behavior of the CEB 90-99 model is similar to the CEB 90 model. However, the CEB 90-99 model is less sensitive due to correction factor for concrete strength and age of concrete at loading adjusted according to the concrete temperature for $T = 20\text{ }^{\circ}\text{C}$.
- 24) The CEB 90-99 model also includes new parameters for cement type, concrete strength and age of concrete at loading adjusted to the concrete temperature which does affect the sensitivity of concrete when structure is loaded after 28 days.

- 25) The CEB 90-99 now includes several correction factors for concrete strength and these correction factors also affect the sensitivity of concrete when concrete strength is different than 35 MPa.

5.1.6 GL 2000 Model

- 26) The GL 2000 model produces almost similar sensitivity of creep as the B3 model. However, the formulation for GL 2000 model is simplified.
- 27) The age at loading and concrete strength are the most influencing parameters for the sensitivity of creep.
- 28) The sensitivity due to relative humidity and V/S ratio also has significant contribution while cement type has least impact on sensitivity.

5.1.7 Comparisons of Creep Models

The following conclusions were made based on the above study.

- i) The change in sensitivity of creep due to change in relative humidity is linear for all models except for the GL 2000 which has a nonlinear relationship.
- ii) The ACI 209 model is the only model which causes a linear variation due to change in V/S ratio. All other model has nonlinear effects on sensitivity of creep due to change in V/S ratio.
- iii) All creep models have good agreements in predicting the change in sensitivity due to change in concrete strength as the variation due to change in concrete strength is less than 10%.

There are two methods adopted to compare the sensitivity of the creep for all models as presented in Table 5.1, Table 5.2, Table 5.3, and Table 5.4. In the first method, an integer index which ranges from 1 to 6 is given to all models for each input parameter as shown in Table 5.1 and Table 5.3. In the second method, normalized index for sensitivity of creep is obtained for each input parameter as shown in Table 5.2 and Table 5.4.

- iv) Table 5.1 shows that the B3 model is the most sensitive model followed by the ACI 209 model while AASHTO 04 model is the least sensitive model for predicting the sensitivity of creep based on the integer index method.
- v) Table 5.2 presents the normalized index for all input parameters and this table also shows that B3 is the most sensitive creep model followed by the ACI 209 model while AASHTO 04 is the least sensitive creep model.
- vi) The results from Table 5.1 and Table 5.2 have similar conclusions although two different indexing methods were adopted.
- vii) The change in sensitivity of creep due to change in cement type or change in curing method is very small for all models. The variation due to change in cement type is less than 2% while change in curing method produces less than 4% variations. The above data suggests that creep models can be simplified by removing the cement type and curing method for creep models. Table 5.3 and Table 5.4 present the integer index and normalized index without considering the cement type or curing type.
- viii) The sensitivity of creep data was further analyzed by removing the parameters due to change of cement type or curing type. Table 5.3 shows the integer index without

considering the cement type or curing type. This table also suggests that the B3 and the CEB 90 model is the most sensitive model for creep while the AASHTO 04 model is the least sensitive model.

ix) Table 5.4 shows the normalized index without considering the sensitivity of creep due to cement type or curing type. The normalized index without cement type or curing type is contradicting to the computed sensitivity as normalized index shows that the ACI 209 as most sensitive model while GL2000 is the least sensitive model.

A comparison of change in sensitivity of creep expressed in percentage for various input parameters is presented below:

Table 5.1 Integer Scaling of Sensitivity of Creep

Input parameter	AASHTO 04	ACI 209	B3	CEB 90	CEB 90-99	GL 2000
28 day comp. strength	4 [†] (38.46)	1 (29.29)	4 (38.05)	2 (32.25)	2 (32.79)	3 (33.60)
Duration at loading	2 (28.75)	1 (14.14)	6 (45.40)	3 (34.50)	4 (35.94)	5 (42.41)
Relative humidity	1 (13.17)	3 (19.19)	2 (17.80)	5 (27.22)	4 (22.46)	4 (22.79)
Specimen size	5 (29.59)	6 (46.00)	2 (10.15)	4 (21.72)	3 (19.69)	1 (9.07)
Type of cement	0	1 (0.97)	2 (1.37)	0 (0.00)	0 (0.00)	0 (0.05)
Type of curing	0	1 (0.36)	2 (3.57)	0	0	0
Total	12	15	18	14	13	13

[†] Scaling – 1 means less sensitive and 6 means most sensitive to input parameters

() – Maximum percentage of sensitivity between initial and final input parameter

Table 5.2 Normalized Scaling of Sensitivity of Creep

Input parameter	AASHTO 04	ACI 209	B3	CEB 90	CEB 90-99	GL 2000
28 day comp. strength	1.3 [†] (38.46)	1.0 (29.29)	1.3 (38.05)	1.1 (32.25)	1.1 (32.79)	1.1 (33.60)
Duration at loading	1.8 (28.75)	1.0 (14.14)	3.2 (45.40)	2.4 (34.50)	2.5 (35.94)	3.0 (42.41)
Relative humidity	1.0 (13.17)	1.5 (19.19)	1.4 (17.80)	2.1 (27.22)	1.7 (22.46)	1.7 (22.79)
Specimen size	3.3 (29.59)	5.1 (46.00)	1.1 (10.15)	2.4 (21.72)	2.2 (19.69)	1.0 (9.07)
Type of cement	0	1.0 (0.97)	1.4 (1.37)	0 (0.00)	0 (0.00)	0 (0.05)
Type of curing	0	1.0 (0.36)	9.9 (3.57)	0	0	0
Total	7.4	10.6	18.3	8.0	7.5	6.8

Table 5.3 Integer Scaling of Sensitivity of Creep (Without cement type or curing type)

Input parameter	AASHTO 04	ACI 209	B3	CEB 90	CEB 90-99	GL 2000
28 day comp. strength	4 [†] (38.46)	1 (29.29)	4 (38.05)	2 (32.25)	2 (32.79)	3 (33.60)
Duration at loading	2 (28.75)	1 (14.14)	6 (45.40)	3 (34.50)	4 (35.94)	5 (42.41)
Relative humidity	1 (13.17)	3 (19.19)	2 (17.80)	5 (27.22)	4 (22.46)	4 (22.79)
Specimen size	5 (29.59)	6 (46.00)	2 (10.15)	4 (21.72)	3 (19.69)	1 (9.07)
Total	12	11	14	14	13	13

[†] Scaling – 1 means less sensitive and 6 means most sensitive to input parameters

() – Maximum percentage of sensitivity between initial and final input parameter

Table 5.4 Normalized Scaling of Sensitivity of Creep (Without cement type or curing type)

Input parameter	AASHTO 04	ACI 209	B3	CEB 90	CEB 90-99	GL 2000
28 day comp. strength	1.3 [†] (38.46)	1.0 (29.29)	1.3 (38.05)	1.1 (32.25)	1.1 (32.79)	1.1 (33.60)
Duration at loading	1.8 (28.75)	1.0 (14.14)	3.2 (45.40)	2.4 (34.50)	2.5 (35.94)	3.0 (42.41)
Relative humidity	1.0 (13.17)	1.5 (19.19)	1.4 (17.80)	2.1 (27.22)	1.7 (22.46)	1.7 (22.79)
Specimen size	3.3 (29.59)	5.1 (46.00)	1.1 (10.15)	2.4 (21.72)	2.2 (19.69)	1.0 (9.07)
Total	7.4	8.6	7.0	8.0	7.5	6.8

5.2 Recommendations for Future Research

Further experiments are required to test the validity of each model. The sensitivity of each model should also be verified with the RILEM data bank.

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APPENDIX A — Formulation of AASHTO 04 Model

f_c = Average 28 day concrete compressive strength [MPa] or [psi]

V/S = Volume-to-surface area ratio [mm] or [in.]

γ = Concrete unit weight [kg/m³] or [lb/ft³]

H = Relative humidity [%]

E_c = Modulus of elasticity at 28 days [MPa] or [psi]

$E_c = 0.043 * \gamma^{1.5} * (f_c)^{1/2}$

t = Maturity of concrete (days)

t_i = Age of concrete when load is initially applied (days)

k_f = Factor for the effect of concrete strength

$k_f = 62/[42 + f_c]$

k_c = Factor for the effect of volume-to-surface ratio of the component specified

$k_c = [(t/(26 e^{(0.0142 * V/S)} + t)/(t/(45+t)))][(1.80+1.77e^{(-0.0213 * V/S)})/2.587]$

$\Phi(t, t_i)$ = Creep coefficient

$\Phi(t, t_i) = 3.5K_c K_f (1.58 - H/120) t_i^{-0.118} [(t - t_i)^{0.6}/(10.0 + (t - t_i)^{0.6})]$

$J(t, t_o)$ = Creep compliance [$\times 10^{-6}$ /MPa] or [$\times 10^{-6}$ /psi] = $\Phi(t, t_o)/E_c + 1/E_c$

Table A-1 Sensitivity of the AASHTO 04 Model Due to Change in Relative Humidity

f_c MPa	V/S mm	γ kg/m ³	H %	E_c MPa	t_i days	t days	k_f	k_c	$\Phi(t, t_i)$	$J(t, t_i) \times 10^{-6}$ /MPa	Sensitivity for Relative Humidity in %			
											RH = 40	RH = 60	RH = 80	RH = 100
50.0	37.5	2400	40	35750	28	35	0.674	1.013	0.489	41.65	0.00	-4.59	-9.62	-15.17
50.0	37.5	2400	60	35750	28	35	0.674	1.013	0.423	39.82	4.39	0.00	-4.81	-10.11
50.0	37.5	2400	80	35750	28	35	0.674	1.013	0.358	37.99	8.78	4.59	0.00	-5.06
50.0	37.5	2400	100	35750	28	35	0.674	1.013	0.293	36.16	13.17	9.18	4.81	0.00

Figure A-1 Sensitivity of the AASHTO 04 Model Due to Change in Relative Humidity

Table A-2 Sensitivity of the AASHTO 04 Model Due to Change in V/S Ratio

f'_c MPa	V/S mm	γ kg/m ³	H %	E_c MPa	t_i days	t	k_r	k_c	$\Phi(t, t_i)$	$J(t, t_i) \times 10^{-6}$ /MPa	Sensitivity for V/S Ratio in %			
											V/S=37.5 mm	V/S=100 mm	V/S=125 mm	V/S=150 mm
50.0	37.5	2400	50	35750	28	35	0.674	1.013	0.618	45.26	0.00	-27.79	-35.66	-42.02
50.0	100.0	2400	50	35750	28	35	0.674	0.436	0.266	35.41	21.75	0.00	-6.16	-11.13
50.0	125.0	2400	50	35750	28	35	0.674	0.316	0.193	33.36	26.28	5.80	0.00	-4.69
50.0	150.0	2400	50	35750	28	35	0.674	0.228	0.139	31.87	29.59	10.02	4.48	0.00

Figure A-2 Sensitivity of the AASHTO 04 Model Due to Change in V/S Ratio

Table A-3 Sensitivity of the AASHTO Model Due to Age at Loading

f'_c MPa	V/S mm	γ kg/m ³	H %	E_c MPa	t_i days	t days	k_f	k_c	$\Phi(t, t_i)$	$J(t, t_i) \times 10^{-6}$ /MPa	Sensitivity for Duration of Loading in %			
											7 Days	14 Days	21 Days	28 Days
50.0	37.5	2400	50	35750	7	35	0.674	1.013	1.271	63.52	0.00	-10.42	-22.03	-40.36
50.0	37.5	2400	50	35750	14	35	0.674	1.013	1.056	57.53	9.44	0.00	-10.51	-27.11
50.0	37.5	2400	50	35750	21	35	0.674	1.013	0.861	52.05	18.05	9.51	0.00	-15.02
50.0	37.5	2400	50	35750	28	35	0.674	1.013	0.618	45.26	28.75	21.33	13.06	0.00

Figure A-3 Sensitivity of the AASHTO Model Due to Age at Loading

Table A-4 Sensitivity of the AASHTO 04 Model Due to Concrete Strength at 28 days

f'_c MPa	V/S mm	γ kg/m ³	H	E_c MPa	t_i days	t days	k_r	k_c	$\Phi(t, t_i)$	$J(t, t_i) \times 10^{-6}$ /MPa	Sensitivity for Concrete Strength in %			
											30 MPa	40 MPa	50 MPa	60 MPa
30.0	37.5	2400	50%	27691	28	35	0.861	1.013	0.790	64.624	0.00	-22.04	-42.80	-62.51
40.0	37.5	2400	50%	31975	28	35	0.756	1.013	0.693	52.955	18.06	0.00	-17.01	-33.16
50.0	37.5	2400	50%	35750	28	35	0.674	1.013	0.618	45.256	29.97	14.54	0.00	-13.81
60.0	37.5	2400	50%	39162	28	35	0.608	1.013	0.557	39.766	38.46	24.90	12.13	0.00

Figure A-4 Sensitivity of the AASHTO 04 Model Due to Concrete Strength at 28 days

APPENDIX B — Formulation of ACI 209 Model

Curing Type = M for moist cured concrete and S for steam cured concrete

b = Constant in determining shrinkage strain depends on curing method

c = Constant in determining shrinkage strain depends on curing method

V/S = Volume-to-surface area ratio [mm] or [in.]

γ = Concrete unit weight [kg/m³] or [lb/ft³]

H = Relative humidity [%]

t = Age of concrete after casting [days]

t₀ = Age of concrete at loading [days]

t_c = Age of concrete drying commenced, usually taken as the age at the end of moist curing [days]

f_{cm28} = Average 28 day concrete compressive strength [MPa] or [psi]

f_c(t₀) = Mean concrete compressive strength at age of loading [MPa] or [psi]

f_c(t₀) = f_{cm28} * [t₀ / (b + c * t₀)]

E_{cmto} = Modulus of elasticity at age of loading [MPa] or [psi]

E_{cmto} = 0.043(γ)^{3/2} * f_c(t₀)^{1/2}

C_{cu} = Ultimate creep coefficient

K_{ca} = Age at loading correction factor

K_{ca} = 1.25 * (t₀)^{-0.118}

K_{ch} = Relative humidity correction factor for creep

K_{ch} = 1.27 - 0.0067 * H

K_{cs} = Shape and size correction factor for creep

$$K_{cs} = 1.14 - 0.0035 \cdot (V/S)$$

$C_c(t)$ = Creep coefficient at time t

$$C_c(t) = (t^{0.60}/(10+t^{0.60})) \cdot C_{cu} \cdot K_{ch} \cdot K_{ca} \cdot K_{cs}$$

$J(t, t_0)$ = Creep compliance [$\times 10^{-6}$ /MPa] or [$\times 10^{-6}$ /psi]

$$J(t, t_0) = (1 + C_c(t)) / E_{cmto}$$

Table B-1 Sensitivity of the ACI 209 Model Due to Change in Cement/Curing Type

Cement Type	Curing Type [†]	b	c	f_{cm28} MPa	V/S mm	γ kg/m ³	H %	t_0 days	t_c days	t days
I	M	4.00	0.85	50.0	37.5	2400	50	28	7	35
III	M	2.30	0.92	50.0	37.5	2400	50	28	7	35
I	S	1.00	0.95	50.0	37.5	2400	50	28	3	35
III	S	0.70	0.98	50.0	37.5	2400	50	28	3	35

$f_c(t_0)$ MPa	C_{cu}	E_{cmto} MPa	K_{ca}	K_{ch}	K_{cs}	$f_c(t_0)$ MPa	E_{cmto} MPa	$C_c(t)$
50.4	2.35	35878	0.844	0.935	1.009	50.4	35878	0.856
49.9	2.35	35711	0.844	0.935	1.009	49.9	35711	0.856
50.7	2.35	36008	0.844	0.935	1.009	50.7	36008	0.856
49.8	2.35	35660	0.844	0.935	1.009	49.8	35660	0.856

$J(t, t_0) \times 10^{-6}$ /MPa	Sensitivity for Cem. Type (%)	Sensitivity for Curing Type (%)
51.73	-0.47	0.36
51.97		
51.54		
52.05	-0.97	-0.14

[†] — M is Moist and S is Steam cured

Table B-2 Sensitivity of the ACI 209 Model Due to Change in Relative Humidity

Cement Type	Curing Type [†]	b	c	f_{cm28} MPa	V/S mm	γ kg/m ³	H %	t_o days	t_c days	t days
I	M	4.00	0.85	50.0	37.5	2400	40	28	7	35
I	M	4.00	0.85	50.0	37.5	2400	60	28	7	35
I	M	4.00	0.85	50.0	37.5	2400	80	28	7	35
I	M	4.00	0.85	50.0	37.5	2400	100	28	7	35
III	M	2.30	0.92	50.0	37.5	2400	40	28	7	35
III	M	2.30	0.92	50.0	37.5	2400	60	28	7	35
III	M	2.30	0.92	50.0	37.5	2400	80	28	7	35
III	M	2.30	0.92	50.0	37.5	2400	100	28	7	35
I	S	1.00	0.95	50.0	37.5	2400	40	28	3	35
I	S	1.00	0.95	50.0	37.5	2400	60	28	3	35
I	S	1.00	0.95	50.0	37.5	2400	80	28	3	35
I	S	1.00	0.95	50.0	37.5	2400	100	28	3	35
III	S	0.70	0.98	50.0	37.5	2400	40	28	3	35
III	S	0.70	0.98	50.0	37.5	2400	60	28	3	35
III	S	0.70	0.98	50.0	37.5	2400	80	28	3	35
III	S	0.70	0.98	50.0	37.5	2400	100	28	3	35

$f_c(t_o)$ MPa	C_{cu}	E_{cmto} MPa	K_{ca}	K_{ch}	K_{cs}	$f_c(t_o)$ MPa	E_{cmto} MPa	$C_c(t)$	$J(t, t_o)$ 10 ⁻⁶ /MPa	Sensi- tivity RH (%)
50.4	2.35	35878	0.844	1.002	1.009	50.4	35878	0.917	53.44	0.00
50.4	2.35	35878	0.844	0.868	1.009	50.4	35878	0.795	50.02	6.40
50.4	2.35	35878	0.844	0.734	1.009	50.4	35878	0.672	46.60	12.80
50.4	2.35	35878	0.844	0.600	1.009	50.4	35878	0.549	43.18	19.19
49.9	2.35	35711	0.844	1.002	1.009	49.9	35711	0.917	53.69	-6.84
49.9	2.35	35711	0.844	0.868	1.009	49.9	35711	0.795	50.25	0.00
49.9	2.35	35711	0.844	0.734	1.009	49.9	35711	0.672	46.82	6.84
49.9	2.35	35711	0.844	0.600	1.009	49.9	35711	0.549	43.38	13.67
50.7	2.35	36008	0.844	1.002	1.009	50.7	36008	0.917	53.25	-14.67
50.7	2.35	36008	0.844	0.868	1.009	50.7	36008	0.795	49.84	-7.34
50.7	2.35	36008	0.844	0.734	1.009	50.7	36008	0.672	46.43	0.00
50.7	2.35	36008	0.844	0.600	1.009	50.7	36008	0.549	43.03	7.34
49.8	2.35	35660	0.844	1.002	1.009	49.8	35660	0.917	53.76	-23.75
49.8	2.35	35660	0.844	0.868	1.009	49.8	35660	0.795	50.32	-15.84
49.8	2.35	35660	0.844	0.734	1.009	49.8	35660	0.672	46.88	-7.92
49.8	2.35	35660	0.844	0.600	1.009	49.8	35660	0.5493	43.44	0.00

[†] — M is Moist and S is Steam cured

Figure B-1 Relative Humidity Correction Factor

Figure B-2 Sensitivity of the ACI 209 Model Due to Change in Relative Humidity

Table B-3 Sensitivity of the ACI 209 Model Due to Change in V/S Ratio

Cement Type	Curing Type [†]	b	c	f _{cm28} MPa	V/S mm	γ kg/m ³	H	t ₀ days	t _c days	t days
I	M	4.00	0.85	50.0	37.5	2400	50%	28	7	35
I	M	4.00	0.85	50.0	100.0	2400	50%	28	7	35
I	M	4.00	0.85	50.0	200.0	2400	50%	28	7	35
I	M	4.00	0.85	50.0	325.0	2400	50%	28	7	35
III	M	2.30	0.92	50.0	37.5	2400	50%	28	7	35
III	M	2.30	0.92	50.0	100.0	2400	50%	28	7	35
III	M	2.30	0.92	50.0	200.0	2400	50%	28	7	35
III	M	2.30	0.92	50.0	325.0	2400	50%	28	7	35
I	S	1.00	0.95	50.0	37.5	2400	50%	28	3	35
I	S	1.00	0.95	50.0	100.0	2400	50%	28	3	35
I	S	1.00	0.95	50.0	200.0	2400	50%	28	3	35
I	S	1.00	0.95	50.0	325.0	2400	50%	28	3	35
III	S	0.70	0.98	50.0	37.5	2400	50%	28	3	35
III	S	0.70	0.98	50.0	100.0	2400	50%	28	3	35
III	S	0.70	0.98	50.0	200.0	2400	50%	28	3	35
III	S	0.70	0.98	50.0	325.0	2400	50%	28	3	35

f _c (t ₀) MPa	C _{cu}	E _{cmto} MPa	K _{ca}	K _{ch}	K _{cs}	f _c (t ₀) MPa	E _{cmto} MPa	C _c (t)	J(t,t ₀) x 10 ⁻⁶ /MPa	Sensi- tivity V/S (%)
50.4	2.35	35878	0.844	0.935	1.009	50.4	35878	0.856	51.73	0.00
50.4	2.35	35878	0.844	0.935	0.790	50.4	35878	0.670	46.56	10.00
50.4	2.35	35878	0.844	0.935	0.440	50.4	35878	0.373	38.28	26.00
50.4	2.35	35878	0.844	0.935	0.002	50.4	35878	0.002	27.93	46.00
49.9	2.35	35711	0.844	0.935	1.009	49.9	35711	0.856	51.97	-11.11
49.9	2.35	35711	0.844	0.935	0.790	49.9	35711	0.670	46.77	0.00
49.9	2.35	35711	0.844	0.935	0.440	49.9	35711	0.373	38.46	17.78
49.9	2.35	35711	0.844	0.935	0.002	49.9	35711	0.002	28.06	40.00
50.7	2.35	36008	0.844	0.935	1.009	50.7	36008	0.856	51.54	-35.14
50.7	2.35	36008	0.844	0.935	0.790	50.7	36008	0.670	46.39	-21.62
50.7	2.35	36008	0.844	0.935	0.440	50.7	36008	0.373	38.14	0.00
50.7	2.35	36008	0.844	0.935	0.002	50.7	36008	0.002	27.83	27.03
49.8	2.35	35660	0.844	0.935	1.009	49.8	35660	0.856	52.04	-85.20
49.8	2.35	35660	0.844	0.935	0.790	49.8	35660	0.670	46.84	-66.68
49.8	2.35	35660	0.844	0.935	0.440	49.8	35660	0.373	38.51	-37.04
49.8	2.35	35660	0.844	0.935	0.002	49.8	35660	0.002	28.10	0.00

[†] — M is Moist and S is Steam cured

Figure B-3 Shape and Size Correction Factor

Figure B-4 Sensitivity of the ACI 209 Model Due to Change in V/S Ratio

Table B-4 Sensitivity of the ACI 209 Model Due to Change in Duration of Loading

Cement Type	Curing Type [†]	b	c	f _{cm28} MPa	V/S mm	γ kg/m ³	H	t ₀ days	t _c days	t days
I	M	4.00	0.85	50.0	37.5	2400	50%	7	7	35
I	M	4.00	0.85	50.0	37.5	2400	50%	14	7	35
I	M	4.00	0.85	50.0	37.5	2400	50%	21	7	35
I	M	4.00	0.85	50.0	37.5	2400	50%	28	7	35
III	M	2.30	0.92	50.0	37.5	2400	50%	7	7	35
III	M	2.30	0.92	50.0	37.5	2400	50%	14	7	35
III	M	2.30	0.92	50.0	37.5	2400	50%	21	7	35
III	M	2.30	0.92	50.0	37.5	2400	50%	28	7	35
I	S	1.00	0.95	50.0	37.5	2400	50%	7	3	35
I	S	1.00	0.95	50.0	37.5	2400	50%	14	3	35
I	S	1.00	0.95	50.0	37.5	2400	50%	21	3	35
I	S	1.00	0.95	50.0	37.5	2400	50%	28	3	35
III	S	0.70	0.98	50.0	37.5	2400	50%	7	3	35
III	S	0.70	0.98	50.0	37.5	2400	50%	14	3	35
III	S	0.70	0.98	50.0	37.5	2400	50%	21	3	35
III	S	0.70	0.98	50.0	37.5	2400	50%	28	3	35

f _c (t ₀) MPa	C _{cu}	E _{cmto} MPa	K _{ca}	K _{ch}	K _{cs}	f _c (t ₀) MPa	E _{cmto} MPa	C _c (t)	J(t,t ₀) x 10 ⁻⁶ /MPa	Sensitivity t ₀ (%)
35.2	2.35	29985	0.994	0.935	1.009	35.2	29985	1.008	66.97	0.00
44.0	2.35	33546	0.916	0.935	1.009	44.0	33546	0.929	57.50	14.14
48.1	2.35	35047	0.873	0.935	1.009	48.1	35047	0.885	53.80	19.67
50.4	2.35	35878	0.844	0.935	1.009	50.4	35878	0.856	51.73	22.76
40.0	2.35	31994	0.994	0.935	1.009	40.0	31994	1.008	62.76	0.00
46.1	2.35	34332	0.916	0.935	1.009	46.1	34332	0.929	56.18	10.48
48.6	2.35	35233	0.873	0.935	1.009	48.6	35233	0.885	53.51	14.74
49.9	2.35	35711	0.844	0.935	1.009	49.9	35711	0.856	51.97	17.20
45.8	2.35	34197	0.994	0.935	1.009	45.8	34197	1.008	58.72	0.00
49.0	2.35	35373	0.916	0.935	1.009	49.0	35373	0.929	54.53	7.13
50.1	2.35	35792	0.873	0.935	1.009	50.1	35792	0.885	52.68	10.29
50.7	2.35	36008	0.844	0.935	1.009	50.7	36008	0.856	51.54	12.22
46.3	2.35	34400	0.994	0.935	1.009	46.3	34400	1.008	58.37	0.00
48.5	2.35	35225	0.916	0.935	1.009	48.5	35225	0.929	54.76	6.19
49.3	2.35	35514	0.873	0.935	1.009	49.3	35514	0.885	53.09	9.05
49.8	2.35	35660	0.844	0.935	1.009	49.8	35660	0.856	52.04	10.84

[†] — M is Moist and S is Steam cured

Figure B-5 Age at Loading Correction Factor

Figure B-6 Sensitivity of the ACI 209 Model Due to Age at Loading

Table B-5 Sensitivity of the ACI 209 Model Due to Concrete Strength

Cement Type	Curing Type [†]	b	c	f _{cm28} MPa	V/S mm	γ kg/m ³	H	t ₀ days	t _c days	t days
I	M	4.00	0.85	30.0	37.5	2400	50%	28	7	35
I	M	4.00	0.85	40.0	37.5	2400	50%	28	7	35
I	M	4.00	0.85	50.0	37.5	2400	50%	28	7	35
I	M	4.00	0.85	60.0	37.5	2400	50%	28	7	35
III	M	2.30	0.92	30.0	37.5	2400	50%	28	7	35
III	M	2.30	0.92	40.0	37.5	2400	50%	28	7	35
III	M	2.30	0.92	50.0	37.5	2400	50%	28	7	35
III	M	2.30	0.92	60.0	37.5	2400	50%	28	7	35
I	S	1.00	0.95	30.0	37.5	2400	50%	28	3	35
I	S	1.00	0.95	40.0	37.5	2400	50%	28	3	35
I	S	1.00	0.95	50.0	37.5	2400	50%	28	3	35
I	S	1.00	0.95	60.0	37.5	2400	50%	28	3	35
III	S	0.70	0.98	30.0	37.5	2400	50%	28	3	35
III	S	0.70	0.98	40.0	37.5	2400	50%	28	3	35
III	S	0.70	0.98	50.0	37.5	2400	50%	28	3	35
III	S	0.70	0.98	60.0	37.5	2400	50%	28	3	35

f _c (t ₀) MPa	C _{cu}	E _{cmt0} MPa	K _{ca}	K _{ch}	K _{cs}	f _c (t ₀) MPa	E _{cmt0} MPa	C _c (t)	J(t,t ₀) x /MPa	Sensitivity for f _{cm28} (%)
30.2	2.35	27791	0.844	0.935	1.009	30.2	27791	0.856	66.78	0.00
40.3	2.35	32090	0.844	0.935	1.009	40.3	32090	0.856	57.84	13.40
50.4	2.35	35878	0.844	0.935	1.009	50.4	35878	0.856	51.73	22.54
60.4	2.35	39302	0.844	0.935	1.009	60.4	39302	0.856	47.22	29.29
29.9	2.35	27662	0.844	0.935	1.009	29.9	27662	0.856	67.09	-15.47
39.9	2.35	31941	0.844	0.935	1.009	39.9	31941	0.856	58.10	0.00
49.9	2.35	35711	0.844	0.935	1.009	49.9	35711	0.856	51.97	10.56
59.9	2.35	39120	0.844	0.935	1.009	59.9	39120	0.856	47.44	18.35
30.4	2.35	27891	0.844	0.935	1.009	30.4	27891	0.856	66.54	-29.10
40.6	2.35	32206	0.844	0.935	1.009	40.6	32206	0.856	57.63	-11.80
50.7	2.35	36008	0.844	0.935	1.009	50.7	36008	0.856	51.54	0.00
60.9	2.35	39444	0.844	0.935	1.009	60.9	39444	0.856	47.05	8.71
29.9	2.35	27622	0.844	0.935	1.009	29.9	27622	0.856	67.19	-41.42
39.8	2.35	31896	0.844	0.935	1.009	39.8	31896	0.856	58.19	-22.47
49.8	2.35	35660	0.844	0.935	1.009	49.8	35660	0.856	52.04	-9.54
59.7	2.35	39064	0.844	0.935	1.009	59.7	39064	0.856	47.51	0.00

[†] — M is Moist and S is Steamed cured

Figure B-7 Sensitivity of the ACI 209 Model Due to Concrete Strength at 28 Days

APPENDIX C — Formulation of B3 Model

Cement Type = I, II and III

Curing Type = M-Moist Cured Concrete, S-Steam Cured and SL-Sealed Concrete

f_{cm28} = Average 28-day concrete compressive strength [MPa] or [psi]

V/S = Volume-to-surface area ratio [mm] or [in.]

γ = Concrete unit weight [kg/m³] or [lb/ft³]

H = Relative humidity [%]

E_{cm28} = Modulus of elasticity at 28 days [MPa] or [psi] = $4735 \cdot (f_{cm28})^{1/2}$

t = Age of concrete after casting [days]

t_o = Age of concrete at loading [days]

t_c = Age of concrete drying commenced, usually taken as the age at the end of moist curing [days]

c = Cement content in concrete [kg/m³] or [lb/ft³]

w/c = Water to cement ratio by weight

a/c = Aggregate-to-cement ratio by weight

α_1 = Type of cement correction factor on creep

α_2 = Curing condition correction factor

w = Water content in concrete [kg/m³] or [lb/ft³]

q_1 = Instantaneous strain due to unit stress = $0.6/E_{cm28}$

q_2 = Aging visco-elastic compliance = $185.4(c)^{0.5} (f_{cm28})^{-0.9} \times 10^{-06}$

q_3 = Non-aging visco-elastic compliance = $0.29 \times (w/c)^4 \cdot (q_2)$

q_4 = Flow compliance = $20.3 (a/c)^{-0.7} \times 10^{-06}$

$$Q_f(t_0) = \text{Solve for Binomial integral} = [0.086(t_0)^{2/9} + 1.21(t_0)^{4/9}]^{-1}$$

$$r(t_0) = 1.7(t_0)^{0.12} + 8$$

$$Z(t, t_0) = (t_0)^{-0.5} \ln[1 + (t - t_0)^{0.1}]$$

$$Q(t, t_0) = \text{Binomial Integral} = Q_f(t_0)[1 + Q_f(t_0)^{r(t_0)} / (Z(t, t_0)^{r(t_0)} - 1)^{r(t_0)}]$$

$$C_o(t, t_0) = \text{Compliance function for basic creep}$$

$$C_o(t, t_0) = q_2 Q(t, t_0) + q_3 \ln[1 + (t - t_0)^{0.1}] + q_4 \ln(t/t_0)$$

$$\epsilon_{shu} = \text{Ultimate shrinkage strain [mm/mm or in/in]} = -\alpha_1 \alpha_2 [(0.019(w)^{2.1} (f_{cm28})^{-0.28}) + 270] \\ \times 10^{-6} \text{ mm/mm}$$

$$q_5 = \text{Drying creep compliance} = 0.757(f_{cm28})^{-1} |\epsilon_{shu} \times 10^6|^{-0.6}$$

$$K_t = 0.085(t_c)^{-0.08} (f_{cm28})^{-0.25} \text{ and } K_s = \text{Cross-section shape}$$

$$D = \text{Effective cross-section thickness [mm] or [in]} = 2(V/S)$$

$$T_{sh} = \text{Shrinkage half time [days]} = K_t (K_s D)^2$$

$$S(t) = \text{Time function for shrinkage} = \tanh(\text{SQRT}((t - t_c)/T_{sh}))$$

$$H(t) = \text{Spatial average of pore relative humidity within cross-section}$$

$$H(t) = 1 - [(1 - H/100) * S(t)]$$

$$S(t_0) = \text{Another time period for shrinkage} = \tanh(\text{SQRT}((t_0 - t_c)/T_{sh}))$$

$$H(t_0) = \text{Spatial average of pore relative humidity within cross-section}$$

$$H(t_0) = 1 - [(1 - H/100) * S(t_0)]$$

$$C_d(t, t_0, t_c) = \text{Compliance function for additional creep due to drying}$$

$$C_d(t, t_0, t_c) = q_5 \{e^{-8H(t)} - e^{-8H(t_0)}\}^{1/2}$$

$$J(t, t_0) = \text{Creep Compliance} = q_1 + C_o(t, t_0) + C_d(t, t_0, t_c) \times 10^{-6} \text{ MPa}$$

$$\Phi(t, t_0) = \text{Creep Coefficient} = E_{cm28} * J(t, t_0) - 1$$

Table C-1 Sensitivity of the B3 Model Due to Change in Cement Type and Curing Type

Cement Type	Curing Type	f_{cm28} MPa	V/S mm	γ kg/m ³	H %	E_{cm28} MPa	t_o days	t_c days	t	c kg/m ³	w/c	a/c	α_1	α_2	w kg/m ³
I	M	50.0	37.5	2400	50	33482	28	7	35	462.04	0.45	3.73	1.00	1.00	207.92
II	M	50.0	37.5	2400	50	33482	28	7	35	462.04	0.45	3.73	0.85	1.00	207.92
III	M	50.0	37.5	2400	50	33482	28	7	35	462.04	0.45	3.73	1.10	1.00	207.92
I	S	50.0	37.5	2400	50	33482	28	7	35	462.04	0.45	3.73	1.00	0.75	207.92
II	S	50.0	37.5	2400	50	33482	28	7	35	462.04	0.45	3.73	0.85	0.75	207.92
III	S	50.0	37.5	2400	50	33482	28	7	35	462.04	0.45	3.73	1.10	0.75	207.92
I	SL	50.0	37.5	2400	50	33482	28	7	35	462.04	0.45	3.73	1.00	1.20	207.92
II	SL	50.0	37.5	2400	50	33482	28	7	35	462.04	0.45	3.73	0.85	1.20	207.92
III	SL	50.0	37.5	2400	50	33482	28	7	35	462.04	0.45	3.73	1.10	1.20	207.92

q_1	q_2	q_3	q_4	$Q_f(t_o)$	r (t _o)	$\epsilon_{shu} \times 10^{-6}$ mm/mm	q_5	K_t	K_s	D mm	T_{sh}	S(t _o)	H(t _o)
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-738.4	2.88E-04	0.02736	1.0	75.00	153.88	0.353	0.823
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-627.6	3.17E-04	0.02736	1.0	75.00	153.88	0.353	0.823
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-812.2	2.72E-04	0.02736	1.0	75.00	153.88	0.353	0.823
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-553.8	3.42E-04	0.02736	1.0	75.00	153.88	0.353	0.823
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-470.7	3.77E-04	0.02736	1.0	75.00	153.88	0.353	0.823
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-609.2	3.23E-04	0.02736	1.0	75.00	153.88	0.353	0.823
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-886.1	2.58E-04	0.02736	1.0	75.00	153.88	0.353	0.823
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-753.2	2.84E-04	0.02736	1.0	75.00	153.88	0.353	0.823
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-974.7	2.44E-04	0.02736	1.0	75.00	153.88	0.353	0.823

Continued Table C-1...

Z (t, t ₀)	Q (t, t ₀)	C ₀ (t, t ₀)	S(t)	H(t)	C _d (t, t ₀ , t _c) x10 ⁻⁶ MPa	Φ(t, t ₀)	J(t, t ₀) x 10 ⁻⁶ /MPa	Sensitivity for Cement Type, (%)	Sensitivity for Curing Type, (%)
0.150	0.148	2.04E-05	0.402	0.799	4.973	0.450	43.31	0.00	0.00
0.150	0.148	2.04E-05	0.402	0.799	5.482	0.467	43.82	-1.18	0.00
0.150	0.148	2.04E-05	0.402	0.799	4.696	0.441	43.03	0.64	0.00
0.150	0.148	2.04E-05	0.402	0.799	5.909	0.481	44.24	0.00	-2.16
0.150	0.148	2.04E-05	0.402	0.799	6.515	0.502	44.85	-1.37	-2.36
0.150	0.148	2.04E-05	0.402	0.799	5.581	0.470	43.91	0.74	-2.06
0.150	0.148	2.04E-05	0.402	0.799	4.457	0.433	42.79	0.00	3.28
0.150	0.148	2.04E-05	0.402	0.799	4.914	0.448	43.25	-1.07	3.57
0.150	0.148	2.04E-05	0.402	0.799	4.210	0.425	42.54	0.58	3.12

Table C-2 Sensitivity of the B3 Model Due to Change in Relative Humidity

Cement Type	Curing Type	f _{cm28} MPa	V/S mm	γ kg/m ³	H %	E _{cm28} MPa	t ₀ days	t _c days	t	c kg/m ³	w/c	a/c	α ₁	α ₂	w kg/m ³
I	M	50.0	37.5	2400	40	33482	28	7	35	462.04	0.45	3.73	1.00	1.00	207.92
I	M	50.0	37.5	2400	60	33482	28	7	35	462.04	0.45	3.73	1.00	1.00	207.92
I	M	50.0	37.5	2400	80	33482	28	7	35	462.04	0.45	3.73	1.00	1.00	207.92
I	M	50.0	37.5	2400	100	33482	28	7	35	462.04	0.45	3.73	1.00	1.00	207.92
II	S	50.0	37.5	2400	40	33482	28	7	35	462.04	0.45	3.73	0.85	0.75	207.92
II	S	50.0	37.5	2400	60	33482	28	7	35	462.04	0.45	3.73	0.85	0.75	207.92
II	S	50.0	37.5	2400	80	33482	28	7	35	462.04	0.45	3.73	0.85	0.75	207.92
II	S	50.0	37.5	2400	100	33482	28	7	35	462.04	0.45	3.73	0.85	0.75	207.92
III	SL	50.0	37.5	2400	40	33482	28	7	35	462.04	0.45	3.73	1.10	1.20	207.92
III	SL	50.0	37.5	2400	60	33482	28	7	35	462.04	0.45	3.73	1.10	1.20	207.92
III	SL	50.0	37.5	2400	80	33482	28	7	35	462.04	0.45	3.73	1.10	1.20	207.92
III	SL	50.0	37.5	2400	100	33482	28	7	35	462.04	0.45	3.73	1.10	1.20	207.92

Continued Table C-2....

q_1	q_2	q_3	q_4	$Q_r(t_0)$	$r(t_0)$	$\epsilon_{shu} \times 10^{-6}$ mm/mm	q_5	K_t	K_s	D mm	T_{sh}	$S(t_0)$	$H(t_0)$
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.1818	10.5358	-738.4	2.88E-04	0.027	1.0	75.00	153.88	0.353	0.788
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.1818	10.5358	-738.4	2.88E-04	0.027	1.0	75.00	153.88	0.353	0.859
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.1818	10.5358	-738.4	2.88E-04	0.027	1.0	75.00	153.88	0.353	0.929
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.1818	10.5358	-738.4	2.88E-04	0.027	1.0	75.00	153.88	0.353	1.000
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.1818	10.5358	-470.7	3.77E-04	0.027	1.0	75.00	153.88	0.353	0.788
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.1818	10.5358	-470.7	3.77E-04	0.027	1.0	75.00	153.88	0.353	0.859
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.1818	10.5358	-470.7	3.77E-04	0.027	1.0	75.00	153.88	0.353	0.929
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.1818	10.5358	-470.7	3.77E-04	0.027	1.0	75.00	153.88	0.353	1.000
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.1818	10.5358	-974.7	2.44E-04	0.027	1.0	75.00	153.88	0.353	0.788
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.1818	10.5358	-974.7	2.44E-04	0.027	1.0	75.00	153.88	0.353	0.859
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.1818	10.5358	-974.7	2.44E-04	0.027	1.0	75.00	153.88	0.353	0.929
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.1818	10.5358	-974.7	2.44E-04	0.027	1.0	75.00	153.88	0.353	1.000

Z(t,t ₀)	Q(t,t ₀)	C _o (t,t ₀)	S(t)	H(t)	C _d (t,t ₀ ,t _c) x10 ⁻⁶ MPa	$\Phi(t,t_0)$	J(t,t ₀) x 10 ⁻⁶ /MPa	Sensitivity for Relative Humidity (%)			
								RH = 100			
								RH = 40	RH = 60	RH = 80	RH = 100
0.150	0.148	2.04E-05	0.402	0.759	6.339	0.496	44.68	0.00	-5.97	-10.76	-16.53
0.150	0.148	2.04E-05	0.402	0.839	3.823	0.412	42.16	5.63	0.00	-4.53	-9.97
0.150	0.148	2.04E-05	0.402	0.920	1.997	0.350	40.33	9.72	4.33	0.00	-5.21
0.150	0.148	2.04E-05	0.402	1.000	0.000	0.284	38.34	14.19	9.07	4.95	0.00
0.150	0.148	2.04E-05	0.402	0.759	8.304	0.562	46.64	0.00	-7.60	-13.89	-21.66
0.150	0.148	2.04E-05	0.402	0.839	5.008	0.451	43.35	7.07	0.00	-5.84	-13.06
0.150	0.148	2.04E-05	0.402	0.920	2.616	0.371	40.95	12.20	5.52	0.00	-6.82
0.150	0.148	2.04E-05	0.402	1.000	0.000	0.284	38.34	17.80	11.55	6.39	0.00
0.150	0.148	2.04E-05	0.402	0.759	5.366	0.463	43.70	0.00	-5.12	-9.18	-14.00
0.150	0.148	2.04E-05	0.402	0.839	3.236	0.392	41.57	4.87	0.00	-3.86	-8.44
0.150	0.148	2.04E-05	0.402	0.920	1.690	0.340	40.03	8.41	3.72	0.00	-4.41
0.150	0.148	2.04E-05	0.402	1.000	0.000	0.284	38.34	12.28	7.78	4.22	0.00

Figure C-1 Sensitivity of the B3 Model Due to Change in Relative Humidity

Figure C-2 Sensitivity of the B3 Model Due to Change in Relative Humidity

Figure C-3 Sensitivity of the B3 Model Due to Change in V/S Ratio

Figure C-4 Sensitivity of the B3 Model Due to Change in V/S Ratio for Type I Cement

Table C-3 Sensitivity of the B3 Model Due to Change in Volume-to-Surface (V/S) Ratios

Cement Type	Curing Type	f_{cm28} MPa	V/S mm	γ kg/m ³	H %	E_{cm28} MPa	t_o days	t_c days	t	c kg/m ³	w/c	a/c	α_1	α_2	w kg/m ³
I	M	50.0	37.5	2400	50	33482	28	7	35	462.04	0.45	3.73	1.00	1.00	207.92
I	M	50.0	100.0	2400	50	33482	28	7	35	462.04	0.45	3.73	1.00	1.00	207.92
I	M	50.0	200.0	2400	50	33482	28	7	35	462.04	0.45	3.73	1.00	1.00	207.92
I	M	50.0	325.0	2400	50	33482	28	7	35	462.04	0.45	3.73	1.00	1.00	207.92
II	M	50.0	37.5	2400	50	33482	28	7	35	462.04	0.45	3.73	0.85	1.00	207.92
II	M	50.0	100.0	2400	50	33482	28	7	35	462.04	0.45	3.73	0.85	1.00	207.92
II	M	50.0	200.0	2400	50	33482	28	7	35	462.04	0.45	3.73	0.85	1.00	207.92
II	M	50.0	325.0	2400	50	33482	28	7	35	462.04	0.45	3.73	0.85	1.00	207.92
III	M	50.0	37.5	2400	50	33482	28	7	35	462.04	0.45	3.73	1.10	1.00	207.92
III	M	50.0	100.0	2400	50	33482	28	7	35	462.04	0.45	3.73	1.10	1.00	207.92
III	M	50.0	200.0	2400	50	33482	28	7	35	462.04	0.45	3.73	1.10	1.00	207.92
III	M	50.0	325.0	2400	50	33482	28	7	35	462.04	0.45	3.73	1.10	1.00	207.92

q_1	q_2	q_3	q_4	$Q_t(t_o)$	$r(t_o)$	$\epsilon_{shu} \times 10^{-5}$ mm/mm	q_5	K_t	K_s	D mm	T_{sh}	$S(t_o)$	$H(t_o)$
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-738.4	2.88E-04	0.027	1.0	75.00	153.88	0.353	0.823
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-738.4	2.88E-04	0.027	1.0	200.00	1094.28	0.138	0.931
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-738.4	2.88E-04	0.027	1.0	400.00	4377.12	0.069	0.965
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-738.4	2.88E-04	0.027	1.0	650.00	11558.34	0.043	0.979
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-627.6	3.17E-04	0.027	1.0	75.00	153.88	0.353	0.823
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-627.6	3.17E-04	0.027	1.0	200.00	1094.28	0.138	0.931
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-627.6	3.17E-04	0.027	1.0	400.00	4377.12	0.069	0.965
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-627.6	3.17E-04	0.027	1.0	650.00	11558.34	0.043	0.979
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-812.2	2.72E-04	0.027	1.0	75.00	153.88	0.353	0.823
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-812.2	2.72E-04	0.027	1.0	200.00	1094.28	0.138	0.931
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-812.2	2.72E-04	0.027	1.0	400.00	4377.12	0.069	0.965
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-812.2	2.72E-04	0.027	1.0	650.00	11558.34	0.043	0.979

Continued Table C-3...

Z (t,t ₀)	Q (t,t ₀)	C ₀ (t,t ₀)	S(t)	H(t)	C _d (t,t ₀ ,t _c) x10 ⁻⁶ MPa	Φ(t,t ₀)	J(t,t ₀) x 10 ⁻⁶ /MPa	Sensitivity for V/S ratio in (%), V/S =			
								37.5 mm	100 mm	200 mm	325 mm
0.150	0.148	2.04E-05	0.402	0.799	4.973	0.450	43.31	0.00	-7.23	-9.37	-10.27
0.150	0.148	2.04E-05	0.159	0.921	2.053	0.352	40.39	6.74	0.00	-1.99	-2.84
0.150	0.148	2.04E-05	0.080	0.960	1.263	0.326	39.60	8.56	1.96	0.00	-0.83
0.150	0.148	2.04E-05	0.049	0.975	0.938	0.315	39.28	9.32	2.76	0.82	0.00
0.150	0.148	2.04E-05	0.402	0.799	5.482	0.467	43.82	0.00	-7.93	-10.29	-11.30
0.150	0.148	2.04E-05	0.159	0.921	2.263	0.359	40.60	7.34	0.00	-2.19	-3.12
0.150	0.148	2.04E-05	0.080	0.960	1.393	0.330	39.73	9.33	2.14	0.00	-0.91
0.150	0.148	2.04E-05	0.049	0.975	1.034	0.318	39.37	10.15	3.03	0.90	0.00
0.150	0.148	2.04E-05	0.402	0.799	4.696	0.441	43.03	0.00	-6.85	-8.86	-9.72
0.150	0.148	2.04E-05	0.159	0.921	1.939	0.349	40.28	6.41	0.00	-1.89	-2.69
0.150	0.148	2.04E-05	0.080	0.960	1.193	0.324	39.53	8.14	1.85	0.00	-0.78
0.150	0.148	2.04E-05	0.049	0.975	0.885	0.313	39.22	8.86	2.62	0.78	0.00

Table C-4 Sensitivity of the B3 Model Due to Change in Age of Loading

Cement Type	Curing Type	f _{cm28} MPa	V/S mm	γ kg/m ³	H %	E _{cm28} MPa	t ₀ days	t _c days	t days	c kg/m ³	w/c	a/c	α1	α2	w kg/m ³
I	M	50.0	37.5	2400	50%	33482	7	7	35	462.04	0.45	3.73	1.00	1.00	207.92
I	M	50.0	37.5	2400	50%	33482	14	7	35	462.04	0.45	3.73	1.00	1.00	207.92
I	M	50.0	37.5	2400	50%	33482	21	7	35	462.04	0.45	3.73	1.00	1.00	207.92
I	M	50.0	37.5	2400	50%	33482	28	7	35	462.04	0.45	3.73	1.00	1.00	207.92
II	M	50.0	37.5	2400	50%	33482	7	7	35	462.04	0.45	3.73	0.85	1.00	207.92
II	M	50.0	37.5	2400	50%	33482	14	7	35	462.04	0.45	3.73	0.85	1.00	207.92
II	M	50.0	37.5	2400	50%	33482	21	7	35	462.04	0.45	3.73	0.85	1.00	207.92
II	M	50.0	37.5	2400	50%	33482	28	7	35	462.04	0.45	3.73	0.85	1.00	207.92
III	M	50.0	37.5	2400	50%	33482	7	7	35	462.04	0.45	3.73	1.10	1.00	207.92
III	M	50.0	37.5	2400	50%	33482	14	7	35	462.04	0.45	3.73	1.10	1.00	207.92
III	M	50.0	37.5	2400	50%	33482	21	7	35	462.04	0.45	3.73	1.10	1.00	207.92
III	M	50.0	37.5	2400	50%	33482	28	7	35	462.04	0.45	3.73	1.10	1.00	207.92

q ₁	q ₂	q ₃	q ₄	Q _r (t ₀)	r(t ₀)	ε _{shu} x10 ⁻⁶ mm/mm	q _s	K _t	K _s	D mm	T _{sh}	S(t ₀)	H(t ₀)
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.333	10.147	-738.4	2.88E-04	0.027	1.0	75.00	153.88	0.000	1.000
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.246	10.333	-738.4	2.88E-04	0.027	1.0	75.00	153.88	0.210	0.895
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.206	10.450	-738.4	2.88E-04	0.027	1.0	75.00	153.88	0.293	0.854
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-738.4	2.88E-04	0.027	1.0	75.00	153.88	0.353	0.823
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.333	10.147	-627.6	3.17E-04	0.027	1.0	75.00	153.88	0.000	1.000
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.246	10.333	-627.6	3.17E-04	0.027	1.0	75.00	153.88	0.210	0.895
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.206	10.450	-627.6	3.17E-04	0.027	1.0	75.00	153.88	0.293	0.854
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-627.6	3.17E-04	0.027	1.0	75.00	153.88	0.353	0.823
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.333	10.147	-812.2	2.72E-04	0.027	1.0	75.00	153.88	0.000	1.000
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.246	10.333	-812.2	2.72E-04	0.027	1.0	75.00	153.88	0.210	0.895
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.206	10.450	-812.2	2.72E-04	0.027	1.0	75.00	153.88	0.293	0.854
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-812.2	2.72E-04	0.027	1.0	75.00	153.88	0.353	0.823

Z (t,t ₀)	Q (t,t ₀)	C _o (t,t ₀)	S(t)	H(t)	C _d (t,t ₀ ,t _c) x10 ⁻⁶ MPa	Φ(t,t ₀)	J(t,t ₀) x 10 ⁻⁶ /MPa	Sensitivity for Age at Loading (%)			
								7 Days	14 Days	21 Days	28 Days
0.330	0.310	5.07E-05	0.402	0.799	10.546	1.651	79.17	0.00	-29.47	-54.64	-82.80
0.229	0.221	3.46E-05	0.402	0.799	8.638	1.048	61.15	22.76	0.00	-19.44	-41.20
0.182	0.178	2.63E-05	0.402	0.799	7.025	0.714	51.20	35.33	16.28	0.00	-18.21
0.150	0.148	2.04E-05	0.402	0.799	4.973	0.450	43.31	45.30	29.18	15.41	0.00
0.330	0.310	5.07E-05	0.402	0.799	11.626	1.687	80.25	0.00	-29.36	-54.58	-83.14
0.229	0.221	3.46E-05	0.402	0.799	9.522	1.077	62.04	22.70	0.00	-19.49	-41.58
0.182	0.178	2.63E-05	0.402	0.799	7.745	0.738	51.92	35.31	16.31	0.00	-18.48
0.150	0.148	2.04E-05	0.402	0.799	5.482	0.467	43.82	45.40	29.37	15.60	0.00
0.330	0.310	5.07E-05	0.402	0.799	9.960	1.631	78.59	0.00	-29.52	-54.67	-82.61
0.229	0.221	3.46E-05	0.402	0.799	8.157	1.031	60.67	22.79	0.00	-19.42	-40.99
0.182	0.178	2.63E-05	0.402	0.799	6.635	0.701	50.81	35.35	16.26	0.00	-18.06
0.150	0.148	2.04E-05	0.402	0.799	4.696	0.441	43.03	45.24	29.07	15.30	0.00

Figure C-5 Sensitivity of the B3 Model Due to Age at Loading

Figure C-6 Sensitivity of the B3 Model Due to Age at Loading for Cement Type I

Figure C-7 Sensitivity of the B3 Model Due to Concrete Strength at 28 days

Figure C-8 Sensitivity of the B3 Model Due to Concrete Strength at 28 days

Table C-5 Sensitivity of the B3 Model Due to Change in Concrete Strength

Cement Type	Curing Type	f_{cm28} MPa	V/S mm	γ kg/m ³	H %	E_{cm28} MPa	t_0 days	t_c days	t	c kg/m ³	w/c	a/c	α_1	α_2	w kg/m ³
I	M	30.0	37.5	2400	50	25935	28	7	35	462.04	0.45	3.73	1.00	1.00	207.92
I	M	40.0	37.5	2400	50	29947	28	7	35	462.04	0.45	3.73	1.00	1.00	207.92
I	M	50.0	37.5	2400	50	33482	28	7	35	462.04	0.45	3.73	1.00	1.00	207.92
I	M	60.0	37.5	2400	50	36677	28	7	35	462.04	0.45	3.73	1.00	1.00	207.92
II	M	30.0	37.5	2400	50	25935	28	7	35	462.04	0.45	3.73	0.85	1.00	207.92
II	M	40.0	37.5	2400	50	29947	28	7	35	462.04	0.45	3.73	0.85	1.00	207.92
II	M	50.0	37.5	2400	50	33482	28	7	35	462.04	0.45	3.73	0.85	1.00	207.92
II	M	60.0	37.5	2400	50	36677	28	7	35	462.04	0.45	3.73	0.85	1.00	207.92
III	M	30.0	37.5	2400	50	25935	28	7	35	462.04	0.45	3.73	1.10	1.00	207.92
III	M	40.0	37.5	2400	50	29947	28	7	35	462.04	0.45	3.73	1.10	1.00	207.92
III	M	50.0	37.5	2400	50	33482	28	7	35	462.04	0.45	3.73	1.10	1.00	207.92
III	M	60.0	37.5	2400	50	36677	28	7	35	462.04	0.45	3.73	1.10	1.00	207.92

q_1	q_2	q_3	q_4	$Q_t(t_0)$	$r(t_0)$	$\epsilon_{shu} \times 10^{-6}$ mm/mm	q_5	K_t	K_s	D mm	T_{sh}	$S(t_0)$	$H(t_0)$
2.31E-05	1.87E-04	2.22E-06	8.08E-06	0.182	10.536	-810.4	4.54E-04	0.0311	1.0	75.00	174.85	0.333	0.833
2.00E-05	1.44E-04	1.71E-06	8.08E-06	0.182	10.536	-768.6	3.51E-04	0.0289	1.0	75.00	162.71	0.345	0.828
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-738.4	2.88E-04	0.0274	1.0	75.00	153.88	0.353	0.823
1.64E-05	1.00E-04	1.19E-06	8.08E-06	0.182	10.536	-715.1	2.45E-04	0.0261	1.0	75.00	147.03	0.361	0.820
2.31E-05	1.87E-04	2.22E-06	8.08E-06	0.182	10.536	-688.9	5.00E-04	0.0311	1.0	75.00	174.85	0.333	0.833
2.00E-05	1.44E-04	1.71E-06	8.08E-06	0.182	10.536	-653.3	3.87E-04	0.0289	1.0	75.00	162.71	0.345	0.828
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-627.6	3.17E-04	0.0274	1.0	75.00	153.88	0.353	0.823
1.64E-05	1.00E-04	1.19E-06	8.08E-06	0.182	10.536	-607.8	2.70E-04	0.0261	1.0	75.00	147.03	0.361	0.820
2.31E-05	1.87E-04	2.22E-06	8.08E-06	0.182	10.536	-891.5	4.28E-04	0.0311	1.0	75.00	174.85	0.333	0.833
2.00E-05	1.44E-04	1.71E-06	8.08E-06	0.182	10.536	-845.5	3.32E-04	0.0289	1.0	75.00	162.71	0.345	0.828
1.79E-05	1.18E-04	1.40E-06	8.08E-06	0.182	10.536	-812.2	2.72E-04	0.0274	1.0	75.00	153.88	0.353	0.823
1.64E-05	1.00E-04	1.19E-06	8.08E-06	0.182	10.536	-786.6	2.31E-04	0.0261	1.0	75.00	147.03	0.361	0.820

Continued Table C-5...

Z (t,t ₀)	Q (t,t ₀)	C _o (t,t ₀)	S(t)	H(t)	C _d (t,t ₀ ,t _c) x10 ⁻⁶ MPa	Φ(t,t ₀)	J(t,t ₀) x 10 ⁻⁶ /MPa	Sensitivity for Concrete Strength (%)			
								30 MPa	40 MPa	50 MPa	60 MPa
0.150	0.148	3.13E-05	0.380	0.810	7.341	0.602	61.76	0.00	-22.32	-42.60	-61.32
0.150	0.148	2.46E-05	0.393	0.804	5.896	0.512	50.49	18.25	0.00	-16.58	-31.88
0.150	0.148	2.04E-05	0.402	0.799	4.973	0.450	43.31	29.87	14.22	0.00	-13.13
0.150	0.148	1.76E-05	0.411	0.795	4.325	0.404	38.28	38.01	24.17	11.60	0.00
0.150	0.148	3.13E-05	0.380	0.810	8.093	0.621	62.51	0.00	-22.34	-42.65	-61.41
0.150	0.148	2.46E-05	0.393	0.804	6.500	0.530	51.09	18.26	0.00	-16.60	-31.93
0.150	0.148	2.04E-05	0.402	0.799	5.482	0.467	43.82	29.90	14.24	0.00	-13.15
0.150	0.148	1.76E-05	0.411	0.795	4.768	0.420	38.73	38.05	24.20	11.62	0.00
0.150	0.148	3.13E-05	0.380	0.810	6.933	0.591	61.35	0.00	-22.30	-42.56	-61.26
0.150	0.148	2.46E-05	0.393	0.804	5.569	0.502	50.16	18.24	0.00	-16.56	-31.85
0.150	0.148	2.04E-05	0.402	0.799	4.696	0.441	43.03	29.86	14.21	0.00	-13.12
0.150	0.148	1.76E-05	0.411	0.795	4.085	0.395	38.04	37.99	24.16	11.60	0.00

APPENDIX D — Formulation of CEB 90 Model

S = Type of cement

f_{cm28} = Average 28-day concrete compressive strength [MPa] or [psi]

V/S = Volume-to-surface area ratio [mm] or [in.]

h_e = Effective thickness to account for volume/surface ratio [mm] or [in.]

γ = Concrete unit weight [kg/m³] or [lb/ft³]

H = Relative humidity [%]

E_{cm28} = Modulus of elasticity at 28 days [MPa] = $21500 (f_{cm28}/10)^{1/3}$

t = Age of concrete after casting [days]

t_o = Age of concrete at loading [days]

t_c = Age of concrete drying commenced, usually taken as the age at the end of moist curing [days]

E_{cmto} = Modulus of elasticity at age of loading [MPa] = $E_{cm28} (e^{0.5*S(1-((28/t_o)^{(1/2)})})}$

$\beta(f_{cm28})$ = Concrete strength factor on the notional creep coefficient = $5.3/(f_{cm28}/10)^{1/2}$

$\beta(t_o)$ = Age of concrete at loading factor on the notional creep coefficient = $1/(0.1 + t_o^{0.2})$

Φ_o = Notional creep factor = $(\Phi_{RH}) * \beta(f_{cm28}) * \beta(t_o)$

β_H = Relative humidity, H, and notional member size, h_e factor on creep

$\beta_H = 1.5[1 + (0.012H)^{18}](h_e) + 250 \leq 1,500$ days

$\beta_c(t, t_o)$ = Coefficient describing development of creep with time after loading

$\beta_c(t, t_o) = (t - t_o)^{0.3} / [\beta_H + (t - t_o)]^{0.3}$

$\Phi(t, t_o)$ = Creep coefficient at time t = $\Phi_o * \beta_c(t, t_o)$

$J(t, t_o)$ = Creep compliance [$\times 10^{-6}$ /MPa] or [$\times 10^{-6}$ /psi] = $\Phi(t, t_o) / E_{cm28} + 1 / E_{cmto}$

Table D-1 Sensitivity of the CEB 90 Model Due to Change in Cement Type

At $t_0 = 28$ days

Cement Type	a	b	S	h_e mm	f_{cm28} MPa	V/S mm	γ kg/m ³	H %	t_0 days	t_c days	t days	E_{cm28} MPa
I	2.8	0.77	0.25	75.0	50.0	37.5	2400	50	28	7	35	36764
II	3.4	0.72	0.38	75.0	50.0	37.5	2400	50	28	7	35	36764
III	1.0	0.92	0.20	75.0	50.0	37.5	2400	50	28	7	35	36764

E_{cmto} MPa	Φ_{RH}	$\beta(f_{cm28})$ MPa	$\beta(t_0)$	Φ_0	β_H	$\beta_c(t, t_0)$	$\Phi(t, t_0)$	$J(t, t_0) \times 10^{-6}$ /MPa	Sensitivity Cement Type (%)
36764	2.196	2.370	0.488	2.543	362.5	0.304	0.774	48.24	0.00
36764	2.196	2.370	0.488	2.543	362.5	0.304	0.774	48.24	0.00
36764	2.196	2.370	0.488	2.543	362.5	0.304	0.774	48.24	0.00

At $t_0 = 35$ days

Cement Type	a	b	S	h_e mm	f_{cm28} MPa	V/S mm	γ kg/m ³	H %	t_0 days	t_c days	t days	E_{cm28} MPa
I	2.8	0.77	0.25	75.0	50.0	37.5	2400	50	32	7	35	36764
II	3.4	0.72	0.38	75.0	50.0	37.5	2400	50	32	7	35	36764
III	1.0	0.92	0.20	75.0	50.0	37.5	2400	50	32	7	35	36764

E_{cmto} MPa	Φ_{RH}	$\beta(f_{cm28})$ MPa	$\beta(t_0)$	Φ_0	β_H	$\beta_c(t, t_0)$	$\Phi(t, t_0)$	$J(t, t_0) \times 10^{-6}$ /MPa	Sensitivity Cement Type (%)
37062	2.196	2.370	0.476	2.479	362.5	0.237	0.587	42.94	0.00
37218	2.196	2.370	0.476	2.479	362.5	0.237	0.587	42.83	0.26
37003	2.196	2.370	0.476	2.479	362.5	0.237	0.587	42.99	-0.10

Table D-2 Sensitivity of the CEB 90 Model Due to Change in Relative Humidity

Cement Type	a	b	S	h_e mm	f_{cm28} MPa	V/S mm	γ kg/m ³	H %	t_o days	t_c days	t days	E_{cm28} MPa
I	2.8	0.77	0.25	75.0	50.0	37.5	2400	40	28	7	35	36764
I	2.8	0.77	0.25	75.0	50.0	37.5	2400	60	28	7	35	36764
I	2.8	0.77	0.25	75.0	50.0	37.5	2400	80	28	7	35	36764
I	2.8	0.77	0.25	75.0	50.0	37.5	2400	100	28	7	35	36764
II	3.4	0.72	0.38	75.0	50.0	37.5	2400	40	28	7	35	36764
II	3.4	0.72	0.38	75.0	50.0	37.5	2400	60	28	7	35	36764
II	3.4	0.72	0.38	75.0	50.0	37.5	2400	80	28	7	35	36764
II	3.4	0.72	0.38	75.0	50.0	37.5	2400	100	28	7	35	36764
III	1.0	0.92	0.20	75.0	50.0	37.5	2400	40	28	7	35	36764
III	1.0	0.92	0.20	75.0	50.0	37.5	2400	60	28	7	35	36764
III	1.0	0.92	0.20	75.0	50.0	37.5	2400	80	28	7	35	36764
III	1.0	0.92	0.20	75.0	50.0	37.5	2400	100	28	7	35	36764

Continued Table D-2 ...

E_{cmto} MPa	Φ_{RH}	$\beta(f_{cm28})$ MPa	$\beta(t_o)$	Φ_o	β_H	$\beta_c(t, t_o)$	$\Phi(t, t_o)$	$J(t, t_o) \times 10^{-6}$ /MPa	Sensitivity for RH (%)
36764	2.436	2.370	0.488	2.820	362.5	0.304	0.858	50.54	0.00
36764	1.957	2.370	0.488	2.266	362.5	0.304	0.689	45.95	9.07
36764	1.479	2.370	0.488	1.712	362.5	0.304	0.521	41.37	18.15
36764	1.000	2.370	0.488	1.158	362.5	0.304	0.352	36.78	27.22
36764	2.436	2.370	0.488	2.820	362.5	0.304	0.858	50.54	0.00
36764	1.957	2.370	0.488	2.266	362.5	0.304	0.689	45.95	9.07
36764	1.479	2.370	0.488	1.712	362.5	0.304	0.521	41.37	18.15
36764	1.000	2.370	0.488	1.158	362.5	0.304	0.352	36.78	27.22
36764	2.436	2.370	0.488	2.820	362.5	0.304	0.858	50.54	0.00
36764	1.957	2.370	0.488	2.266	362.5	0.304	0.689	45.95	9.07
36764	1.479	2.370	0.488	1.712	362.5	0.304	0.521	41.37	18.15
36764	1.000	2.370	0.488	1.158	362.5	0.304	0.352	36.78	27.22

Figure D-1 Relative Humidity Correction Factor

Figure D-2 Sensitivity of the CEB 90 Model Due to Change in Relative Humidity

Table D-3 Sensitivity of the CEB 90 Model Due to Change in V/S Ratio

Cement Type	a	b	S	h _e mm	f _{cm28} MPa	V/S mm	γ kg/m ³	H %	t ₀ days	t _c days	t days	E _{cm28} MPa
I	2.8	0.77	0.25	75.0	50.0	37.5	2400	50	28	7	35	36764
I	2.8	0.77	0.25	200.0	50.0	100.0	2400	50	28	7	35	36764
I	2.8	0.77	0.25	400.0	50.0	200.0	2400	50	28	7	35	36764
I	2.8	0.77	0.25	650.0	50.0	325.0	2400	50	28	7	35	36764
II	3.4	0.72	0.38	75.0	50.0	37.5	2400	50	28	7	35	36764
II	3.4	0.72	0.38	200.0	50.0	100.0	2400	50	28	7	35	36764
II	3.4	0.72	0.38	400.0	50.0	200.0	2400	50	28	7	35	36764
II	3.4	0.72	0.38	650.0	50.0	325.0	2400	50	28	7	35	36764
III	1.0	0.92	0.20	75.0	50.0	37.5	2400	50	28	7	35	36764
III	1.0	0.92	0.20	200.0	50.0	100.0	2400	50	28	7	35	36764
III	1.0	0.92	0.20	400.0	50.0	200.0	2400	50	28	7	35	36764
III	1.0	0.92	0.20	650.0	50.0	325.0	2400	50	28	7	35	36764

Continued Table D-3 ...

E _{cmto} MPa	Φ _{RH}	β(f _{cm28}) MPa	β(t ₀)	Φ ₀	β _H	β _c (t, t ₀)	Φ(t, t ₀)	J(t, t ₀) x 10 ⁻⁶ /MPa	Sensitivity V/S Ratio (%)
36764	2.196	2.370	0.488	2.543	362.5	0.304	0.774	48.24	0.00
36764	1.863	2.370	0.488	2.157	550.0	0.269	0.580	42.98	10.91
36764	1.685	2.370	0.488	1.950	850.0	0.236	0.461	39.74	17.62
36764	1.582	2.370	0.488	1.832	1225.0	0.212	0.388	37.76	21.72
36764	2.196	2.370	0.488	2.543	362.5	0.304	0.774	48.24	0.00
36764	1.863	2.370	0.488	2.157	550.0	0.269	0.580	42.98	10.91
36764	1.685	2.370	0.488	1.950	850.0	0.236	0.461	39.74	17.62
36764	1.582	2.370	0.488	1.832	1225.0	0.212	0.388	37.76	21.72
36764	2.196	2.370	0.488	2.543	362.5	0.304	0.774	48.24	0.00
36764	1.863	2.370	0.488	2.157	550.0	0.269	0.580	42.98	10.91
36764	1.685	2.370	0.488	1.950	850.0	0.236	0.461	39.74	17.62
36764	1.582	2.370	0.488	1.832	1225.0	0.212	0.388	37.76	21.72

Figure D-3 Shape and Size Correction Factor

Figure D-4 Sensitivity of the CEB 90 Model Due to Change in V/S Ratio

Table D-4 Sensitivity of the CEB 90 Model Due to Change in Duration of Loading

Cement Type	a	b	S	h_e mm	f_{cm28} MPa	V/S mm	γ kg/m ³	H %	t_o days	t_c days	t days	E_{cm28} MPa
I	2.8	0.77	0.25	75.0	50.0	37.5	2400	50	7	7	35	36764
I	2.8	0.77	0.25	75.0	50.0	37.5	2400	50	14	7	35	36764
I	2.8	0.77	0.25	75.0	50.0	37.5	2400	50	21	7	35	36764
I	2.8	0.77	0.25	75.0	50.0	37.5	2400	50	28	7	35	36764
II	3.4	0.72	0.38	75.0	50.0	37.5	2400	50	7	7	35	36764
II	3.4	0.72	0.38	75.0	50.0	37.5	2400	50	14	7	35	36764
II	3.4	0.72	0.38	75.0	50.0	37.5	2400	50	21	7	35	36764
II	3.4	0.72	0.38	75.0	50.0	37.5	2400	50	28	7	35	36764
III	1.0	0.92	0.20	75.0	50.0	37.5	2400	50	7	7	35	36764
III	1.0	0.92	0.20	75.0	50.0	37.5	2400	50	14	7	35	36764
III	1.0	0.92	0.20	75.0	50.0	37.5	2400	50	21	7	35	36764
III	1.0	0.92	0.20	75.0	50.0	37.5	2400	50	28	7	35	36764

Continued Table D-4 ...

E_{cmto} MPa	Φ_{RH}	$\beta(f_{cm28})$ MPa	$\beta(t_o)$	Φ_o	β_H	$\beta_c(t, t_o)$	$\Phi(t, t_o)$	$J(t, t_o) \times 10^{-6}$ /MPa	Sensitivity t_o (%)
32445	2.196	2.370	0.635	3.304	362.5	0.454	1.499	71.58	0.00
34909	2.196	2.370	0.557	2.900	362.5	0.418	1.213	61.64	13.88
36060	2.196	2.370	0.516	2.686	362.5	0.372	1.000	54.94	23.25
36764	2.196	2.370	0.488	2.543	362.5	0.304	0.774	48.24	32.60
30403	2.196	2.370	0.635	3.304	362.5	0.454	1.499	73.65	0.00
33982	2.196	2.370	0.557	2.900	362.5	0.418	1.213	62.42	15.24
35700	2.196	2.370	0.516	2.686	362.5	0.372	1.000	55.22	25.02
36764	2.196	2.370	0.488	2.543	362.5	0.304	0.774	48.24	34.50
33266	2.196	2.370	0.635	3.304	362.5	0.454	1.499	70.82	0.00
35273	2.196	2.370	0.557	2.900	362.5	0.418	1.213	61.35	13.38
36200	2.196	2.370	0.516	2.686	362.5	0.372	1.000	54.83	22.57
36764	2.196	2.370	0.488	2.543	362.5	0.304	0.774	48.24	31.88

Figure D-5 Modulus of Elasticity at t_o

Figure D-6 Sensitivity of the CEB 90 Model Due to Age at Loading

Table D-5 Sensitivity of the CEB 90 Model Due to Change in Concrete Strength

Cement Type	a	b	S	h_e mm	f_{cm28} MPa	V/S mm	γ kg/m ³	H %	t_o days	t_c days	t days	E_{cm28} MPa
I	2.8	0.77	0.25	75.0	30.0	37.5	2400	50	28	7	35	31008
I	2.8	0.77	0.25	75.0	40.0	37.5	2400	50	28	7	35	34129
I	2.8	0.77	0.25	75.0	50.0	37.5	2400	50	28	7	35	36764
I	2.8	0.77	0.25	75.0	60.0	37.5	2400	50	28	7	35	39068
II	3.4	0.72	0.38	75.0	30.0	37.5	2400	50	28	7	35	31008
II	3.4	0.72	0.38	75.0	40.0	37.5	2400	50	28	7	35	34129
II	3.4	0.72	0.38	75.0	50.0	37.5	2400	50	28	7	35	36764
II	3.4	0.72	0.38	75.0	60.0	37.5	2400	50	28	7	35	39068
III	1.0	0.92	0.20	75.0	30.0	37.5	2400	50	28	7	35	31008
III	1.0	0.92	0.20	75.0	40.0	37.5	2400	50	28	7	35	34129
III	1.0	0.92	0.20	75.0	50.0	37.5	2400	50	28	7	35	36764
III	1.0	0.92	0.20	75.0	60.0	37.5	2400	50	28	7	35	39068

Continued Table D-5 ...

E_{cmto} MPa	Φ_{RH}	$\beta(f_{cm28})$ MPa	$\beta(t_o)$	Φ_o	β_H	$\beta_c(t, t_o)$	$\Phi(t, t_o)$	$J(t, t_o) \times 10^{-6}/\text{MPa}$	Sensitivity $f_{cm28}(\%)$
31008	2.196	3.060	0.488	3.283	362.5	0.304	0.999	64.460	0.00
34129	2.196	2.650	0.488	2.843	362.5	0.304	0.865	54.645	15.23
36764	2.196	2.370	0.488	2.543	362.5	0.304	0.774	48.244	25.16
39068	2.196	2.164	0.488	2.321	362.5	0.304	0.706	43.674	32.25
31008	2.196	3.060	0.488	3.283	362.5	0.304	0.999	64.460	0.00
34129	2.196	2.650	0.488	2.843	362.5	0.304	0.865	54.645	15.23
36764	2.196	2.370	0.488	2.543	362.5	0.304	0.774	48.244	25.16
39068	2.196	2.164	0.488	2.321	362.5	0.304	0.706	43.674	32.25
31008	2.196	3.060	0.488	3.283	362.5	0.304	0.999	64.460	0.00
34129	2.196	2.650	0.488	2.843	362.5	0.304	0.865	54.645	15.23
36764	2.196	2.370	0.488	2.543	362.5	0.304	0.774	48.244	25.16
39068	2.196	2.164	0.488	2.321	362.5	0.304	0.706	43.674	32.25

Figure D-7 Sensitivity of the CEB 90 Model Due to Concrete Strength at 28 days

APPENDIX E — Formulation of CEB 90-99 Model

Cement Type = Rapid hardening high strength (RS), Normal and rapidly hardening (R)
Slow Hardening (SL)

s = Type of cement

f_{cm28} = Average 28 day concrete compressive strength [MPa] or [psi]

V/S = Volume-to-surface area ratio [mm] or [in.]

h_e = Effective thickness to account for volume/surface ratio [mm] or [in.]

H = Relative humidity [%]

t = Age of concrete after casting [days]

$t_{0,T}$ = Age of concrete at loading adjusted according to the concrete temperature;

for T = 20 °C $t_{0,T}$ corresponds to t_0

t_c = Age of concrete drying commenced, usually taken as the age at the end of moist
curing [days]

t_0 = Age of concrete at loading [days] = $t_{0,T}[9/(2+(t_{0,T})^{1.2})+1]^\alpha \geq 0.5$ days

α = Coefficient that depends on type of cement = -1 for SL, 0 for R and 1 for RS

$\alpha_1 = [35/f_{cm28}]^{0.7}$

$\alpha_2 = [35/f_{cm28}]^{0.2}$

$\alpha_3 = [35/f_{cm28}]^{0.5}$

E_{cm28} = Modulus of elasticity at 28 days [MPa] or [psi] = $21500(f_{cm28}/10)^{1/3}$

E_{cmto} = Modulus of elasticity at age of loading [MPa] or [psi]

$E_{cmto} = E_{cm28}\{E^{0.5*S(1-((28/t_0)^{0.5}))}\}$

$\Phi_{RH} = [1 + (1-H/100)\alpha_1 / [0.46*(h_e/100)^{1/3}]]\alpha_2$

$\beta(f_{cm28})$ = Concrete strength factor on the notional creep coefficient

$$\beta(f_{cm28}) = 5.3/(f_{cm28}/10)^{1/2}$$

$$\beta(t_0) = \text{Age of concrete at loading factor on the notional creep coefficient}$$

$$\beta(t_0) = 1/(0.1+t_0^{0.2})$$

$$\Phi_0 = \text{Notional creep factor} = (\Phi_{RH}) * \beta(f_{cm28}) * \beta(t_0)$$

$$\beta_H = \text{Relative humidity, H, and notional member size, } h_0 \text{ factor on creep}$$

$$\beta_H = 1.5[1+(0.012H)^{18}](h_e) + 250\alpha_3 \leq 1,500\alpha_3 \text{ days}$$

$$\beta_c(t, t_0) = \text{Coefficient describing development of creep with time after loading}$$

$$\beta_c(t, t_0) = [(t - t_0)/(\beta_H + t - t_0)]^{0.3}$$

$$\Phi(t, t_0) = \text{Creep coefficient at time } t = \Phi_0 * \beta_c(t, t_0)$$

$$J(t, t_0) = \text{Creep compliance [X10}^{-6}\text{/MPa] or [X10}^{-6}\text{/psi]} = \Phi(t, t_0) / E_{cm28} + 1 / E_{cmto}$$

Table E-1 Sensitivity of the CEB 90-99 Model Due to Change in Cement Type

At $t_0 = 28$ days

Cement Type	S	h_e mm	f_{cm28} MPa	α	α_1	α_2	α_3	V/S mm	H %	t_0	$t_{0,T}$ days	t_c days	t days
RS	0.20	75.0	50	1	0.78	0.93	0.84	37.5	50	32.46	28	7	28
R	0.25	75.0	50	0	0.78	0.93	0.84	37.5	50	28.00	28	7	28
SL	0.38	75.0	50	-1	0.78	0.93	0.84	37.5	50	24.15	28	7	28

E_{cm28} MPa	E_{cmto} MPa	Φ_{RH}	$\beta(f_{cm28})$ MPa	$\beta(t_0)$	Φ_0	β_H	$\beta_c(t, t_0)$	$\Phi(t, t_0)$	$J(t, t_0) \times 10^{-6}/\text{MPa}$	Sensitivity for Cem. Type (%)
36764	36764	1.799	2.370	0.475	2.025	321.67	0.0	0	27.20	0.00
36764	36764	1.799	2.370	0.488	2.083	321.67	0.0	0	27.20	0.00
36764	36764	1.799	2.370	0.502	2.142	321.67	0.0	0	27.20	0.00

At $t_0 = 35$ days

Cement Type	S	h_e mm	f_{cm28} MPa	α	α_1	α_2	α_3	V/S mm	H %	t_0	$t_{0,T}$ days	t_c days	t days
RS	0.20	75.0	50	1	0.78	0.93	0.84	37.5	50	32.46	28	7	35
R	0.25	75.0	50	0	0.78	0.93	0.84	37.5	50	28.00	28	7	35
SL	0.38	75.0	50	-1	0.78	0.93	0.84	37.5	50	24.15	28	7	35

E_{cm28} MPa	E_{cmto} MPa	Φ_{RH}	$\beta(f_{cm28})$ MPa	$\beta(t_0)$	Φ_0	β_H	$\beta_c(t, t_0)$	$\Phi(t, t_0)$	$J(t, t_0) \times 10^{-6}/\text{MPa}$	Sensitivity for Cem. Type (%)
36764	36764	1.799	2.370	0.475	2.025	321.67	0.315	0.638	44.56	0.00%
36764	36764	1.799	2.370	0.488	2.083	321.67	0.315	0.656	45.05	-1.11%
36764	36764	1.799	2.370	0.502	2.142	321.67	0.315	0.675	45.56	-2.25%

Table E-2 Sensitivity of the CEB 90-99 Model Due to Change in Relative Humidity

Cement Type	S	h_e mm	f_{cm28} MPa	α	α_1	α_2	α_3	V/S mm	H %	t_0	$t_{0,T}$ days	t_c days	t days
RS	0.20	75.0	50	1	0.78	0.93	0.84	37.5	40	32.46	28	7	35
RS	0.20	75.0	50	1	0.78	0.93	0.84	37.5	60	32.46	28	7	35
RS	0.20	75.0	50	1	0.78	0.93	0.84	37.5	80	32.46	28	7	35
RS	0.20	75.0	50	1	0.78	0.93	0.84	37.5	100	32.46	28	7	35
R	0.25	75.0	50	0	0.78	0.93	0.84	37.5	40	28.00	28	7	35
R	0.25	75.0	50	0	0.78	0.93	0.84	37.5	60	28.00	28	7	35
R	0.25	75.0	50	0	0.78	0.93	0.84	37.5	80	28.00	28	7	35
R	0.25	75.0	50	0	0.78	0.93	0.84	37.5	100	28.00	28	7	35
SL	0.38	75.0	50	-1	0.78	0.93	0.84	37.5	40	24.15	28	7	35
SL	0.38	75.0	50	-1	0.78	0.93	0.84	37.5	60	24.15	28	7	35
SL	0.38	75.0	50	-1	0.78	0.93	0.84	37.5	80	24.15	28	7	35
SL	0.38	75.0	50	-1	0.78	0.93	0.84	37.5	100	24.15	28	7	35

E_{cm28} MPa	E_{cmto} MPa	Φ_{RH}	$\beta(f_{cm28})$ MPa	$\beta(t_0)$	Φ_0	β_H	$\beta_c(t, t_0)$	$\Phi(t, t_0)$	$J(t, t_0) \times 10^{-6}/\text{MPa}$	Sensitivity for Rel. Humidity (%)
36764	36764	1.973	2.370	0.475	2.220	321.67	0.315	0.700	46.23	0.00
36764	36764	1.625	2.370	0.475	1.830	321.67	0.315	0.577	42.88	7.24
36764	36764	1.278	2.370	0.475	1.439	321.67	0.315	0.453	39.53	14.49
36764	36764	0.931	2.370	0.475	1.048	321.67	0.315	0.330	36.18	21.73
36764	36764	1.973	2.370	0.488	2.284	321.67	0.315	0.720	46.78	0.00
36764	36764	1.625	2.370	0.488	1.882	321.67	0.315	0.593	43.33	7.36
36764	36764	1.278	2.370	0.488	1.480	321.67	0.315	0.466	39.89	14.73
36764	36764	0.931	2.370	0.488	1.078	321.67	0.315	0.340	36.44	22.09
36764	36764	1.973	2.370	0.502	2.349	321.67	0.315	0.740	47.33	0.00
36764	36764	1.625	2.370	0.502	1.935	321.67	0.315	0.610	43.79	7.49
36764	36764	1.278	2.370	0.502	1.522	321.67	0.315	0.480	40.25	14.97
36764	36764	0.931	2.370	0.502	1.109	321.67	0.315	0.349	36.70	22.46

Figure E-1 Relative Humidity Correction Factor

Figure E-2 Sensitivity of the CEB 90-99 Model Due to Change in Relative Humidity

Table E-3 Sensitivity of the CEB 90-99 Model Due to Change in Volume-to-Surface (V/S) Ratio

Cement Type	S	he mm	f _{cm28} MPa	α	α_1	α_2	α_3	V/S mm	H %	t ₀	t _{0,T} days	t _c days	t days
RS	0.20	75.0	50	1.00	0.78	0.93	0.84	37.5	50	32.46	28	7	35
RS	0.20	200.0	50	1.00	0.78	0.93	0.84	100.0	50	32.46	28	7	35
RS	0.20	400.0	50	1.00	0.78	0.93	0.84	200.0	50	32.46	28	7	35
RS	0.20	650.0	50	1.00	0.78	0.93	0.84	325.0	50	32.46	28	7	35
R	0.25	75.0	50	0.00	0.78	0.93	0.84	37.5	50	28.00	28	7	35
R	0.25	200.0	50	0.00	0.78	0.93	0.84	100.0	50	28.00	28	7	35
R	0.25	400.0	50	0.00	0.78	0.93	0.84	200.0	50	28.00	28	7	35
R	0.25	650.0	50	0.00	0.78	0.93	0.84	325.0	50	28.00	28	7	35
SL	0.38	75.0	50	-1.00	0.78	0.93	0.84	37.5	50	24.15	28	7	35
SL	0.38	200.0	50	-1.00	0.78	0.93	0.84	100.0	50	24.15	28	7	35
SL	0.38	400.0	50	-1.00	0.78	0.93	0.84	200.0	50	24.15	28	7	35
SL	0.38	650.0	50	-1.00	0.78	0.93	0.84	325.0	50	24.15	28	7	35

E _{cm28} MPa	E _{cmto} MPa	Φ_{RH}	$\beta(f_{cm28})$ MPa	$\beta(t_0)$	Φ_0	β_H	$\beta_c(t, t_0)$	$\Phi(t, t_0)$	J(t, t ₀) x 10 ⁻⁶ /MPa	Sensitivity for V/S Ratio (%)
36764	36764	1.7990	2.370	0.475	2.025	321.665	0.315	0.638	44.56	0.00
36764	36764	1.5570	2.370	0.475	1.753	509.165	0.275	0.482	40.32	9.51
36764	36764	1.4279	2.370	0.475	1.607	809.165	0.240	0.386	37.69	15.42
36764	36764	1.3537	2.370	0.475	1.524	1184.165	0.214	0.326	36.08	19.04
36764	36764	1.7990	2.370	0.488	2.083	321.665	0.315	0.656	45.05	0.00
36764	36764	1.5570	2.370	0.488	1.803	509.165	0.275	0.496	40.69	9.67
36764	36764	1.4279	2.370	0.488	1.653	809.165	0.240	0.397	37.99	15.69
36764	36764	1.3537	2.370	0.488	1.567	1184.165	0.214	0.336	36.33	19.36
36764	36764	1.7990	2.370	0.502	2.142	321.665	0.315	0.675	45.56	0.00
36764	36764	1.5570	2.370	0.502	1.854	509.165	0.275	0.510	41.08	9.84
36764	36764	1.4279	2.370	0.502	1.700	809.165	0.240	0.408	38.29	15.95
36764	36764	1.3537	2.370	0.502	1.612	1184.165	0.214	0.345	36.59	19.69

Figure E-3 Shape and Size Correction Factor

Figure E-4 Sensitivity of the CEB 90-99 Model Due to Change in V/S Ratio

Table E-4 Sensitivity of the CEB 90-99 Model Due to Change in Duration of Loading

Cement Type	S	h_e mm	f_{cm28} MPa	α	α_1	α_2	α_3	V/S mm	H %	t_0	$t_{0,T}$ days	t_c days	t days
RS	0.20	75.0	50	1	0.78	0.93	0.84	37.5	50	15.04	7	7	35
RS	0.20	75.0	50	1	0.78	0.93	0.84	37.5	50	18.90	14	7	35
RS	0.20	75.0	50	1	0.78	0.93	0.84	37.5	50	25.65	21	7	35
RS	0.20	75.0	50	1	0.78	0.93	0.84	37.5	50	32.46	28	7	35
R	0.25	75.0	50	0	0.78	0.93	0.84	37.5	50	10.00	7	7	35
R	0.25	75.0	50	0	0.78	0.93	0.84	37.5	50	14.00	14	7	35
R	0.25	75.0	50	0	0.78	0.93	0.84	37.5	50	21.00	21	7	35
R	0.25	75.0	50	0	0.78	0.93	0.84	37.5	50	28.00	28	7	35
SL	0.38	75.0	50	-1	0.78	0.93	0.84	37.5	50	6.65	7	7	35
SL	0.38	75.0	50	-1	0.78	0.93	0.84	37.5	50	10.37	14	7	35
SL	0.38	75.0	50	-1	0.78	0.93	0.84	37.5	50	17.19	21	7	35
SL	0.38	75.0	50	-1	0.78	0.93	0.84	37.5	50	24.15	28	7	35

E_{cm28} MPa	E_{cmto} MPa	Φ_{RH}	$\beta(f_{cm28})$ MPa	$\beta(t_0)$	Φ_0	β_H	$\beta_c(t, t_0)$	$\Phi(t, t_0)$	$J(t, t_0) \times 10^{-6}/MPa$	Sensitivity for Duration of Loading (%)
36764	33266	1.799	2.370	0.572	2.441	321.67	0.469	1.145	61.19	0.00
36764	35273	1.799	2.370	0.526	2.244	321.67	0.433	0.971	54.76	10.51
36764	36200	1.799	2.370	0.497	2.118	321.67	0.386	0.816	49.83	18.57
36764	36764	1.799	2.370	0.475	2.025	321.67	0.315	0.638	44.56	27.18
36764	32445	1.799	2.370	0.635	2.706	321.67	0.469	1.269	65.33	0.00
36764	34909	1.799	2.370	0.557	2.375	321.67	0.433	1.028	56.60	13.36
36764	36060	1.799	2.370	0.516	2.200	321.67	0.386	0.848	50.80	22.24
36764	36764	1.799	2.370	0.488	2.083	321.67	0.315	0.656	45.05	31.04
36764	30403	1.799	2.370	0.703	2.997	321.67	0.469	1.405	71.12	0.00
36764	33982	1.799	2.370	0.589	2.513	321.67	0.433	1.088	59.01	17.03
36764	35700	1.799	2.370	0.536	2.285	321.67	0.386	0.881	51.97	26.92
36764	36764	1.799	2.370	0.502	2.142	321.67	0.315	0.675	45.56	35.94

Figure E-5 Sensitivity of the CEB 90-99 Model Due to Age at Loading

Figure E-6 Sensitivity of the CEB 90-99 Model Due to Concrete Strength at 28 days

Table E-5 Sensitivity of the CEB 90-99 Model Due to Change in Concrete Strength

Cement Type	S	h_e mm	f_{cm28} MPa	α	α_1	α_2	α_3	V/S mm	H %	t_0	$t_{0,T}$ days	t_c days	t days
RS	0.20	75.0	30	1	1.11	1.03	1.08	37.5	50	32.46	28	7	35
RS	0.20	75.0	40	1	0.91	0.97	0.94	37.5	50	32.46	28	7	35
RS	0.20	75.0	50	1	0.78	0.93	0.84	37.5	50	32.46	28	7	35
RS	0.20	75.0	60	1	0.69	0.90	0.76	37.5	50	32.46	28	7	35
R	0.25	75.0	30	0	1.11	1.03	1.08	37.5	50	28.00	28	7	35
R	0.25	75.0	40	0	0.91	0.97	0.94	37.5	50	28.00	28	7	35
R	0.25	75.0	50	0	0.78	0.93	0.84	37.5	50	28.00	28	7	35
R	0.25	75.0	60	0	0.69	0.90	0.76	37.5	50	28.00	28	7	35
SL	0.38	75.0	30	-1	1.11	1.03	1.08	37.5	50	24.15	28	7	35
SL	0.38	75.0	40	-1	0.91	0.97	0.94	37.5	50	24.15	28	7	35
SL	0.38	75.0	50	-1	0.78	0.93	0.84	37.5	50	24.15	28	7	35
SL	0.38	75.0	60	-1	0.69	0.90	0.76	37.5	50	24.15	28	7	35

E_{cm28} MPa	E_{cmto} MPa	Φ_{RH}	$\beta(f_{cm28})$ MPa	$\beta(t_0)$	Φ_0	β_H	$\beta_a(t, t_0)$	$\Phi(t, t_0)$	$J(t, t_0) \times 10^{-6}/MPa$	Sensitivity for Concrete Strength (%)
31008	31008	2.406	3.060	0.475	3.496	382.53	0.299	1.047	66.01	0.00
34129	34129	2.035	2.650	0.475	2.560	346.35	0.308	0.790	52.43	20.57
36764	36764	1.799	2.370	0.475	2.025	321.67	0.315	0.638	44.56	32.50
39068	39068	1.634	2.164	0.475	1.679	303.44	0.321	0.538	39.38	40.35
31008	31008	2.406	3.060	0.488	3.596	382.53	0.299	1.077	66.98	0.00
34129	34129	2.035	2.650	0.488	2.633	346.35	0.308	0.812	53.09	20.73
36764	36764	1.799	2.370	0.488	2.083	321.67	0.315	0.656	45.05	32.73
39068	39068	1.634	2.164	0.488	1.727	303.44	0.321	0.554	39.77	40.62
31008	31008	2.406	3.060	0.502	3.698	382.53	0.299	1.107	67.97	0.00
34129	34129	2.035	2.650	0.502	2.708	346.35	0.308	0.835	53.77	20.88
36764	36764	1.799	2.370	0.502	2.142	321.67	0.315	0.675	45.56	32.96
39068	39068	1.634	2.164	0.502	1.776	303.44	0.321	0.569	40.17	40.89

APPENDIX F — Formulation for GL 2000 Model

Cement Type = I, II or III

f_{cm28} = Average 28 day concrete compressive strength [MPa] or [psi]

V/S = Volume-to-surface area ratio [mm] or [in.]

H = Relative humidity [%]

E_{cm28} = Modulus of elasticity at 28 days [MPa] or [psi]

t = Age of concrete after casting [days]

t_o = Age of concrete at loading [days]

t_c = Age of concrete drying commenced, taken as the age at the end of moist curing
[days]

E_{cm28} = Modulus of elasticity at age of loading [MPa] = $3500 + 4300f_{cm28}^{1/2}$

$f_{cmto} = f_{cm28} [t_o^{3/4}/(a+b(t_o)^{3/4})]$

$E_{cmto} = 3500 + 4300 f_{cmto}^{1/2}$

$\Phi(t_c) = [1 - \{(t_o - t_c)/(t_o - t_c + 0.12(V/S)^2)\}^{0.5}]^{0.5}$

$\Phi(t, t_o) = \Phi(t_c)[2(t - t_o)0.3/((t - t_o)^{0.3} + 14) + (7/t_o)^{0.5}((t - t_o)/(t - t_o + 7))^{0.5}$
 $+ 2.5(1 - 1.086h^2)\{(t - t_o)/(t - t_o + 0.12(V/S)_2)\}^{0.5}]$

$J(t, t_o) =$ Creep compliance [$X10^{-6}/\text{MPa}$] or [$X10^{-6}/\text{psi}$] = $1/E_{cmto} + \Phi(t, t_o)/E_{cm28}$

Table F-1 Sensitivity of the GL 2000 Model Due to Change in Cement Type

Cement Type	a	b	f_{cm28} MPa	V/S mm	H %	t_o days	t_c days	t days
I	2.80	0.77	50	37.5	50	28	7	35
II	3.40	0.72	50	37.5	50	28	7	35
III	1.00	0.92	50	37.5	50	28	7	35

E_{cm28} MPa	f'_{cmto} MPa	E_{cmto} MPa	$\Phi(t_c)$	$\Phi(t, t_o)$	$J(t, t_o) \times 10^{-6}/\text{MPa}$
33906	50.00	33905	0.817	0.771	52.240
33906	50.03	33916	0.817	0.771	52.230
33906	49.89	33873	0.817	0.771	52.268

Table F-2 Sensitivity of the GL 2000 Model Due to Change in Relative Humidity

Cement Type	a	b	f_{cm28} MPa	V/S mm	H %	t_o days	t_c days	t days
I	2.80	0.77	50	37.5	20	28	7	35
I	2.80	0.77	50	37.5	40	28	7	35
I	2.80	0.77	50	37.5	60	28	7	35
I	2.80	0.77	50	37.5	80	28	7	35
I	2.80	0.77	50	37.5	100	28	7	35
II	3.40	0.72	50	37.5	20	28	7	35
II	3.40	0.72	50	37.5	40	28	7	35
II	3.40	0.72	50	37.5	60	28	7	35
II	3.40	0.72	50	37.5	80	28	7	35
II	3.40	0.72	50	37.5	100	28	7	35
III	1.00	0.92	50	37.5	20	28	7	35
III	1.00	0.92	50	37.5	40	28	7	35
III	1.00	0.92	50	37.5	60	28	7	35
III	1.00	0.92	50	37.5	80	28	7	35
III	1.00	0.92	50	37.5	100	28	7	35

E_{cm28} MPa	f'_{cmto} MPa	E_{cmto} MPa	$\Phi(t_c)$	$\Phi(t, t_o)$	$J(t, t_o) \times 10^{-6}/\text{MPa}$	Sensitivity due to RH (%), RH =				
						20	40	60	80	100
33906	50.00	33906	0.817	0.864	54.98	0.00	-2.93	-8.22	-16.61	-29.52
33906	50.00	33906	0.817	0.811	53.41	2.85	0.00	-5.14	-13.29	-25.83
33906	50.00	33906	0.817	0.723	50.80	7.60	4.89	0.00	-7.75	-19.68
33906	50.00	33906	0.817	0.599	47.15	14.25	11.73	7.20	0.00	-11.07
33906	50.00	33906	0.817	0.439	42.45	22.79	20.53	16.45	9.97	0.00
33906	50.00	33906	0.817	0.864	54.98	0.00	-2.93	-8.22	-16.61	-29.52
33906	50.00	33906	0.817	0.811	53.41	2.85	0.00	-5.14	-13.29	-25.83
33906	50.00	33906	0.817	0.723	50.80	7.60	4.89	0.00	-7.75	-19.68
33906	50.00	33906	0.817	0.599	47.15	14.25	11.73	7.20	0.00	-11.07
33906	50.00	33906	0.817	0.439	42.45	22.79	20.53	16.45	9.97	0.00
33906	50.00	33906	0.817	0.864	54.98	0.00	-2.93	-8.22	-16.61	-29.52
33906	50.00	33906	0.817	0.811	53.41	2.85	0.00	-5.14	-13.29	-25.83
33906	50.00	33906	0.817	0.723	50.80	7.60	4.89	0.00	-7.75	-19.68
33906	50.00	33906	0.817	0.599	47.15	14.25	11.73	7.20	0.00	-11.07
33906	50.00	33906	0.817	0.439	42.45	22.79	20.53	16.45	9.97	0.00

Figure F-1 Sensitivity of the GL 2000 Model Due to Change in Relative Humidity

Figure F-2 Sensitivity of the GL 2000 Model Due to Change in V/S Ratio

Table F-3 Sensitivity of the GL 2000 Model Due to Change in V/S Ratio

Cement Type	a	b	f _{cm28} MPa	V/S mm	H %	t ₀ days	t _c days	t days
I	2.80	0.77	50	37.5	50	28	7	35
I	2.80	0.77	50	100	50	28	7	35
I	2.80	0.77	50	200	50	28	7	35
I	2.80	0.77	50	325	50	28	7	35
II	3.40	0.72	50	37.5	50	28	7	35
II	3.40	0.72	50	100	50	28	7	35
II	3.40	0.72	50	200	50	28	7	35
II	3.40	0.72	50	325	50	28	7	35
III	1.00	0.92	50	37.5	50	28	7	35
III	1.00	0.92	50	100	50	28	7	35
III	1.00	0.92	50	200	50	28	7	35
III	1.00	0.92	50	325	50	28	7	35

Continued Table F-3 ...

E _{cm28} MPa	f _{cmto} MPa	E _{cmto} MPa	Φ(t _c)	Φ(t, t ₀)	J(t, t ₀) x 10 ⁻⁶ /MPa	Sensitivity due to V/S Ratio (%) V/S =			
						37.5	100	200	325
33906	50.00	33906	0.817	0.771	52.24	0.00	-6.03	-8.78	-9.97
33906	50.00	33906	0.932	0.670	49.27	5.69	0.00	-2.59	-3.72
33906	50.00	33906	0.966	0.628	48.02	8.07	2.53	0.00	-1.10
33906	50.00	33906	0.979	0.611	47.50	9.07	3.59	1.09	0.00
33906	50.00	33906	0.817	0.771	52.24	0.00	-6.03	-8.78	-9.97
33906	50.00	33906	0.932	0.670	49.27	5.69	0.00	-2.59	-3.72
33906	50.00	33906	0.966	0.628	48.02	8.07	2.53	0.00	-1.10
33906	50.00	33906	0.979	0.611	47.50	9.07	3.59	1.09	0.00
33906	50.00	33906	0.817	0.771	52.24	0.00	-6.03	-8.78	-9.97
33906	50.00	33906	0.932	0.670	49.27	5.69	0.00	-2.59	-3.72
33906	50.00	33906	0.966	0.628	48.02	8.07	2.53	0.00	-1.10
33906	50.00	33906	0.979	0.611	47.50	9.07	3.59	1.09	0.00

Table F-4 Sensitivity of the GL 2000 Model Due to Change in Duration of Loading

Cement Type	a	b	f_{cm28} MPa	V/S mm	H %	t_0 days	t_c days	t days
I	2.80	0.77	50	37.5	50	7	7	35
I	2.80	0.77	50	37.5	50	14	7	35
I	2.80	0.77	50	37.5	50	21	7	35
I	2.80	0.77	50	37.5	50	28	7	35
II	3.40	0.72	50	37.5	50	7	7	35
II	3.40	0.72	50	37.5	50	14	7	35
II	3.40	0.72	50	37.5	50	21	7	35
II	3.40	0.72	50	37.5	50	28	7	35
III	1.00	0.92	50	37.5	50	7	7	35
III	1.00	0.92	50	37.5	50	14	7	35
III	1.00	0.92	50	37.5	50	21	7	35
III	1.00	0.92	50	37.5	50	28	7	35

Continued Table F-4 ...

E_{cm28} MPa	f'_{cmto} MPa	E_{cmto} MPa	$\Phi(t_c)$	$\Phi(t, t_0)$	$J(t, t_0) \times 10^{-6}/\text{MPa}$	Sensitivity for Duration of Loading (%) in Days			
						7	14	21	28
33906	35.20	29010	1.000	1.907	90.70	0.00	-26.68	-47.45	-73.63
33906	43.22	31769	0.895	1.360	71.60	21.06	0.00	-16.40	-37.06
33906	47.37	33096	0.850	1.061	61.51	32.18	14.09	0.00	-17.75
33906	50.00	33906	0.817	0.771	52.24	42.41	27.04	15.08	0.00
33906	33.11	28243	1.000	1.907	91.64	0.00	-27.29	-48.63	-75.42
33906	42.03	31375	0.895	1.360	71.99	21.44	0.00	-16.77	-37.82
33906	46.88	32941	0.850	1.061	61.66	32.72	14.36	0.00	-18.03
33906	50.00	33906	0.817	0.771	52.24	42.99	27.44	15.27	0.00
33906	43.39	31824	1.000	1.907	87.65	0.00	-24.56	-43.51	-67.79
33906	47.25	33058	0.895	1.360	70.37	19.72	0.00	-15.21	-34.71
33906	48.93	33577	0.850	1.061	61.08	30.32	13.20	0.00	-16.92
33906	50.00	33906	0.817	0.771	52.24	40.40	25.77	14.48	0.00

Figure F-3 Sensitivity of the GL 2000 Model Due to Age at Loading

Figure F-4 Sensitivity of the GL 2000 Model Due to Age at Loading for Cement Type I

Table F-5 Sensitivity of the GL 2000 Model Due to Change in Concrete Strength

Cement Type	a	b	f_{cm28} MPa	V/S mm	H %	t_o days	t_c days	t days
I	2.80	0.77	30	37.5	50	28	7	35
I	2.80	0.77	45	37.5	50	28	7	35
I	2.80	0.77	60	37.5	50	28	7	35
I	2.80	0.77	75	37.5	50	28	7	35
II	3.40	0.72	30	37.5	50	28	7	35
II	3.40	0.72	45	37.5	50	28	7	35
II	3.40	0.72	60	37.5	50	28	7	35
II	3.40	0.72	75	37.5	50	28	7	35
III	1.00	0.92	30	37.5	50	28	7	35
III	1.00	0.92	45	37.5	50	28	7	35
III	1.00	0.92	60	37.5	50	28	7	35
III	1.00	0.92	75	37.5	50	28	7	35

Continued Table F-5 ...

E_{cm28} MPa	f_{cmto} MPa	E_{cmto} MPa	$\Phi(t_c)$	$\Phi(t, t_o)$	$J(t, t_o) \times 10^{-6}/\text{MPa}$	Sensitivity due to Concrete Strength (%) in MPa			
						30	45	60	75
27052	30.00	27052	0.817	0.771	65.47	0.00	-19.57	-36.06	-50.60
32345	45.00	32345	0.817	0.771	54.76	16.36	0.00	-13.80	-25.95
36808	60.00	36808	0.817	0.771	48.12	26.50	12.12	0.00	-10.68
40739	75.00	40739	0.817	0.771	43.48	33.60	20.60	9.65	0.00
27052	30.00	27052	0.817	0.771	65.47	0.00	-19.57	-36.06	-50.60
32345	45.00	32345	0.817	0.771	54.76	16.36	0.00	-13.80	-25.95
36808	60.00	36808	0.817	0.771	48.12	26.50	12.12	0.00	-10.68
40739	75.00	40739	0.817	0.771	43.48	33.60	20.60	9.65	0.00
27052	30.00	27052	0.817	0.771	65.47	0.00	-19.57	-36.06	-50.60
32345	45.00	32345	0.817	0.771	54.76	16.36	0.00	-13.80	-25.95
36808	60.00	36808	0.817	0.771	48.12	26.50	12.12	0.00	-10.68
40739	75.00	40739	0.817	0.771	43.48	33.60	20.60	9.65	0.00

Figure F-5 Sensitivity of the GL2000 Model Due to Concrete Strength at 28 days

APPENDIX G

Conversion of Creep Models in US unit

G.1 AASHTO 04 Model — Computation of Creep Compliance

The creep coefficient may be computed as follows:

$$\psi(t, t_i) = 3.5k_c k_f \left(1.58 - \frac{H}{120} \right) t_i^{-0.118} \frac{(t - t_i)^{0.6}}{10.0 + (t - t_i)^{0.6}} \quad [G-1]$$

$$k_f = \frac{1}{0.67 + \frac{f'_c}{9}} \quad [G-2]$$

H = Relative humidity expressed as a decimal

k_c = Factor for the effect of the volume-to-surface ratio as given in Equation [G-3]

k_f = Factor for the effect of concrete strength

t = Maturity of concrete (days)

t_i = Age of concrete when load is initially applied (days)

$$k_c = \left[\frac{\frac{t}{26e^{0.36(V/S)} + t}}{\frac{t}{45 + t}} \right] \left[\frac{1.80 + 1.77e^{-0.54(V/S)}}{2.587} \right] \quad [G-3]$$

The compliance factor is computed as follows:

$$J(t, t_i) = \frac{1}{E_c} + \frac{\psi(t, t_i)}{E_c} \quad [G-4]$$

G.2 ACI 209 Model — Computation of Creep Compliance

$$J(t, t_o) = \frac{1 + C_c(t)}{E_{cmto}} \quad [G-5]$$

$$E_{cmto} = 33(\gamma)^{3/2} \sqrt{f'_c(t_o)} \quad [G-6]$$

$C_c(t)$ = Creep coefficient at time t

E_{cmto} = Modulus of elasticity at age of loading (psi)

γ = Concrete unit weight (lb/ft³)

$f'_c(t_o)$ = Mean concrete compressive strength (psi)

$$f'_c(t_o) = f_{cm28} \frac{t_o}{b + ct_o} \quad [G-7]$$

f_{cm28} = Average 28 day concrete compressive strength (psi)

b and c constants according to Table G.1:

Table G.1 Constants b and c as a function of Cement Type and Curing Method

Type of Cement	Moist-Cured		Steam-Cured	
I	$b = 4.0$	$c = 0.85$	$b = 1.0$	$c = 0.95$
III	$b = 2.3$	$c = 0.92$	$b = 0.7$	$c = 0.98$

Creep coefficient, $C_c(t)$, can be determined as follows:

$$C_c(t) = \frac{t^{0.60}}{10 + t^{0.60}} C_{ca} K_{ch} K_{ca} K_{cs} \quad [G-8]$$

C_{ca} = Ultimate creep coefficient = 2.35

K_{ch} = Relative humidity correction factor for creep = $1.27 - 0.0067H$

K_{ca} = Age at loading correction factor = $1.25(t_o)^{-0.118}$

K_{cs} = Shape and size correction factor for creep = $1.14 - 0.09(V/S)$

G.3 B3 Model — Computation of Creep Compliance

The creep compliance $J(t, t_o)$ is computed as follows:

$$J(t, t_o) = [q_1 + C_o(t, t_o) + C_d(t, t_o, t_c)] \quad [G-9]$$

Where q_1 is the instantaneous strain given in (G-10), $C_o(t, t_o)$ is the compliance function for basic creep, and $C_d(t, t_o, t_c)$ is the compliance function for drying creep.

$$q_1 = \frac{0.6 \times 10^6}{E_{cm28}} \quad [G-10]$$

E_{cm28} = Modulus of elasticity of concrete in psi at 28 days, as given in Equation (G-11).

$$E_{cm28} = 57000 \sqrt{f_{cm28}} \quad [G-11]$$

The compliance function of the basic creep, $C_o(t, t_o)$, should be calculated as follows:

$$C_o(t, t_o) = q_2 Q(t, t_o) + q_3 \ln[1 + (t - t_o)^{0.1}] + q_4 \ln\left(\frac{t}{t_o}\right) \quad [G-12]$$

q_2 = Aging visco-elastic compliance parameter

$Q(t, t_o)$ = The binomial integral

q_3 = Nonaging visco-elastic compliance parameter

q_4 = Flow compliance parameters

t_o = Age of concrete at loading (days)

E_{cm28} = Modulus of elasticity at 28 days (psi)

$$q_2 = 451.1(c)^{0.5}(f_{cm28})^{-0.9} \quad [G-13]$$

c is the cement content in kg/m^3

$$Q(t, t_o) = Q_f(t_o) \left[1 + \left(\frac{Q_f(t_o)}{Z(t, t_o)} \right)^{r(t_o)} \right]^{-1/r(t_o)} \quad [G-14]$$

$Q(t, t_o)$ is a binomial integral, which must be multiplied by the parameter q_2 to obtain the aging visco-elastic term. The three following equations can be used to approximate the binomial integral.

$$Q_f(t_o) = \frac{1}{0.086(t_o)^{2/9} + 1.21(t_o)^{4/9}} \quad [G-15]$$

$$Z(t, t_o) = \ln \left[1 + \frac{(t - t_o)^{0.1}}{\sqrt{t_o}} \right] \quad [G-16]$$

$$r(t_o) = 1.7(t_o)^{0.12} + 8 \quad [G-17]$$

q_3 is the nonaging visco-elastic compliance parameter and q_4 is the aging flow compliance parameter. These parameters are a function of the concrete mean compressive strength at 28 days, the cement content (c), the water to cement ratio (w/c) and the aggregate to cement ratio (a/c).

$$q_3 = 0.29q_2 \left(\frac{w}{c} \right)^4 \quad [G-18]$$

$$q_4 = 0.14 \left(\frac{a}{c} \right)^{-0.7} \quad [G-19]$$

The compliance function for drying creep, $C_d(t, t_o, t_c)$, should be calculated as follows:

$$C_d(t, t_o, t_c) = q_5 \sqrt{e^{-8H(t)} - e^{-8H(t_o)}} \quad [G-20]$$

q_5 is the drying creep compliance parameter. This parameter is a function of the concrete mean compressive strength at 28 days, water content, cement type, method of curing and the ultimate shrinkage strain ϵ_{shu} .

$$q_5 = \frac{7.57 \times 10^5 |\epsilon_{shu}|^{-0.6}}{f_{cm28}} \quad [G-21]$$

$$\epsilon_{shu} = -\alpha_1 \alpha_2 \left[26w^{2.1}(f_{cm28}) - 0.28 + 270 \right] \times 10^{-6} \quad [G-22]$$

The value of α_1 and α_2 are given in Table G.2 and Table G.3.

$H(t)$ and $H(t_0)$ are spatial averages of pore relative humidity. The $H(t)$ and $H(t_0)$ can be computed as follows:

$$H(t) = 1 - (1-H)S(t) \quad [G-23]$$

$$H(t_0) = 1 - (1-H)S(t_0) \quad [G-24]$$

$S(t)$ and $S(t_0)$ are the time functions for shrinkage and τ_{sh} is the shrinkage half-time.

Table G.2 Correction Factor α_1 as a Function of Cement Type

Type of Cement	α_1
I	1.00
II	0.85
III	1.10

Table G.3 Correction Factor α_2 as a Function of Curing Type

Type of Curing	α_2
H < 98%	0.75
H = 100%	1.00
98% ≤ H ≤ 100%	1.20

$$S(t) = \tanh \left[\left(\frac{t - t_c}{\tau_{sh}} \right)^{1/2} \right] \quad [G-25]$$

$$S(t_o) = \tanh \left[\left(\frac{t_o - t_c}{\tau_{sh}} \right)^{1/2} \right] \quad [G-26]$$

$$\tau_{sh} = 190.8(t_o)^{-0.08}(f_{cm28})^{-0.25} [2k_s(V/S)]^2 \quad [G-27]$$

where k_s is the cross-section shape correction factor, given in Table G.4.

Table G.4 Correction Factor k_s as a Function of Cross-Section Shape

Cross-Section Shape	k_s
Infinite slab	1.00
Infinite cylinder	1.15
Infinite square prism	1.25
Sphere	1.30
Cube	1.55

G.4 CEB 90 Model — Computation of Creep Compliance

The creep compliance can be computed as follows:

$$J(t, t_o) = \frac{1}{E_{cmto}} + \frac{\Phi(t, t_o)}{E_{cm28}} \quad [G-28]$$

E_{cmto} = Modulus of elasticity at age of loading (psi)

E_{cm28} = Modulus of elasticity at 28 days (psi)

$\Phi(t, t_0)$ = Creep coefficient

$$E_{cmto} = E_{cm28} e^{\left[0.5S \left(1 - \sqrt{\frac{28}{t_0}} \right) \right]} \quad [G-29]$$

S is a coefficient that depends on cement type as given in Table G.5.

Table G.5 Coefficient S as Function of Cement Type

Cement Type	European Type	US Type	S
Slow hardening	SL	II	0.38
Normal/rapid hardening	R	I	0.25
Rapid hardening high strength	RS	III	0.20

Creep coefficient, $\Phi(t, t_0)$, can be computed from the following equation:

$$\Phi_o = \Phi_{RH} \beta(f_{cm28}) \beta(t_o) \quad [G-30]$$

Φ_{RH} = Relative humidity factor on the notional creep coefficient, which is given as:

$$\Phi_{RH} = 1 + \frac{1 - H/100}{0.46 \sqrt[3]{h_e/4}} \quad [G-31]$$

$\beta(f_{cm28})$ = Concrete strength factor on notional creep coefficient, which is given as:

$$\beta(f_{cm28}) = \frac{5.3}{\sqrt{f_{cm28}/1450}} \quad [G-32]$$

$\beta(t_o)$ = Age of concrete at loading factor on notional creep coefficient, which is given as

$$\beta(t_o) = \frac{1}{0.1 + t_o^{0.2}} \quad [G-33]$$

The development of creep with time after loading, $\beta_c(t, t_o)$, can be computed using the following equation:

$$\beta_c(t, t_o) = \left(\frac{t - t_o}{\beta_H + t - t_o} \right)^{0.3} \quad [G-34]$$

$$\beta_H = 150 \frac{h_e}{4} [1 + (0.012H)^{18}] + 250 \leq 1500 \text{ days} \quad [G-35]$$

G.5 CEB 90-99 Model — Computation of Creep Compliance

The creep compliance can be computed as follows:

$$J(t, t_o) = \frac{1}{E_{cmto}} + \frac{\Phi(t, t_o)}{E_{cm28}} \quad [G-36]$$

E_{cmto} = Modulus of elasticity at age of loading (psi)

E_{cm28} = Modulus of elasticity at 28 days (psi)

$\Phi(t, t_o)$ = Creep coefficient

$$E_{cmto} = E_{cm28} e^{\left[0.5S \left(1 - \sqrt{\frac{28}{t_o}} \right) \right]} \quad [G-37]$$

S is a coefficient that depends on cement type as given in Table G.6.

Creep coefficient, $\Phi(t, t_o)$, can be computed from the following equation:

$$\Phi_o = \Phi_{RH} \beta(f_{cm28}) \beta(t_o) \quad [G-38]$$

Φ_{RH} = Relative humidity factor on the notional creep coefficient, which is given as:

$$\Phi_{RH} = 1 + \frac{1 - H / 100}{0.46 \sqrt[3]{h_e / 4}} \quad [G-39]$$

Table G.6 Coefficient S as Function of Cement Type and Compressive Strength

f_{cm28} (psi)	Type of Cement	S
≤ 60	Rapidly hardening high strength (RS or III)	0.20
	Normal and rapidly hardening (R, I)	0.25
	Slow hardening (SL, II)	0.38
> 60	All types	0.20

Φ_{RH} = Relative humidity factor on the notional creep coefficient, which is given as:

$$\begin{aligned} \Phi_{RH} &= \left[1 + \frac{1 - H / 100}{0.46 \sqrt[3]{h_e / 4}} \alpha_1 \right] \alpha_2 \\ \alpha_1 &= \left[\frac{5076.14}{f_{cm28}} \right] \\ \alpha_2 &= \left[\frac{5076.14}{f_{cm28}} \right]^{0.2} \end{aligned} \quad [G-40]$$

$\beta(f_{cm28})$ = Concrete strength factor on notional creep coefficient, which is given as:

$$\beta(f_{cm28}) = \frac{5.3}{\sqrt{f_{cm28} / 1450}} \quad [G-41]$$

$\beta(t_o)$ = Age of concrete at loading factor on notional creep coefficient, which is given as

$$\beta(t_o) = \frac{1}{0.1 + t_o^{0.2}} \quad [G-42]$$

$$\text{where,} \quad t_o = t_{o,T} \left[\frac{9}{2 + t_{o,T}^{1.2}} + 1 \right]^\alpha \geq 0.5 \text{ days} \quad [G-43]$$

t_o = Age of concrete at loading (days)

$t_{0,T}$ = Age of concrete at loading adjusted according to the concrete temperature; for $T =$

20 °C, $t_{0,T}$ corresponds to t_0

α = Coefficient that depends on type of cement:

= -1 for slowly hardening cement

= 0 for normal or rapidly hardening cement

= 1 for rapidly hardening high-strength cement

The development of creep with time after loading, $\beta_c(t, t_0)$, can be computed using the following equation:

$$\beta_c(t, t_0) = \left(\frac{t - t_0}{\beta_H + t - t_0} \right)^{0.3} \quad [G-44]$$

$$\beta_H = 150 \frac{h_e}{4} \left[1 + (0.012H)^{18} \right] + 250\alpha_3 \leq 1500\alpha_3 \quad \text{days} \quad [G-45]$$

$$\alpha_3 = \left[\frac{5076.14}{f_{cm28}} \right]^{0.5} \quad [G-46]$$

G.6 GL 2000 Model — Computation of Creep Compliance

The compliance $J(t, t_0)$ is calculated using the following equation.

$$J(t, t_0) = \frac{1}{E_{cmto}} + \frac{\Phi(t, t_0)}{E_{cm28}} \quad [G-47]$$

E_{cmto} = Modulus of elasticity of concrete at loading (psi)

E_{cm28} = Modulus of elasticity of concrete at 28 days (psi)

$\Phi(t, t_o)$ = Creep coefficient

$$E_{cmto} = 500,000 + 52,000\sqrt{f_{cmto}} \quad [G-48]$$

Where f_{cmto} = concrete mean compressive strength at loading (psi), which can be determined as follows:

$$f_{cmto} = f_{cm28} \frac{t^{3/4}}{a + bt^{3/4}} \quad [G-49]$$

Coefficient a and b are related to the cement type as given in Table G.7.

$$E_{cm28} = 500,000 + 52,000\sqrt{f_{cm28}} \quad [G-50]$$

Table G.7 Coefficient a and b as Function of Cement Type

Cement Type	European Type	US Type	a	b
Slow hardening	SL	II	3.40	0.72
Normal/rapid hardening	R	I	2.80	0.77
Rapid hardening high strength	RS	III	1.00	0.92

Creep coefficient, $\Phi(t, t_o)$, can be calculated as follows:

$$\Phi(t, t_o) = \Phi(t_c) \left[2 \left(\frac{(t-t_o)^{0.3}}{(t-t_o)^{0.3} + 14} \right) + \left(\frac{7}{t_o} \right)^{0.5} \left(\frac{t-t_o}{t-t_o + 7} \right)^{0.5} + 2.5(1 - 1.086h^2) \left(\frac{t-t_o}{t-t_o + 97(V/S)^2} \right)^{0.5} \right] \quad [G-51]$$

$$\text{If } t_o = t_c \text{ then } \Phi(t_c) = 1 \quad [G-52]$$

$$\text{If } t_o > t_c \text{ then } \Phi(t_c) = \left[1 - \left(\frac{t_o - t_c}{t_o - t_c + 97(V/S)^2} \right)^{0.5} \right]^{0.5} \quad [\text{G-53}]$$