1990

Quantum size effects of the electron conductivity in very thin wire

Yong Kyoo J Chang
San Jose State University

Follow this and additional works at: https://scholarworks.sjsu.edu/etd_theses

Recommended Citation

Chang, Yong Kyoo J, "Quantum size effects of the electron conductivity in very thin wire" (1990). *Master's Theses*. 3250.

DOI: <https://doi.org/10.31979/etd.7f3a-9res>

https://scholarworks.sjsu.edu/etd_theses/3250

This Thesis is brought to you for free and open access by the Master's Theses and Graduate Research at SJSU ScholarWorks. It has been accepted for inclusion in Master's Theses by an authorized administrator of SJSU ScholarWorks. For more information, please contact scholarworks@sjsu.edu.

INFORMATION TO USERS

The most advanced technology has been used to photograph and reproduce this manuscript from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each original is also photographed in one exposure and is included in reduced form at the back of the book.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

U·M·I

University Microfilms International
A Bell & Howell Information Company
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
313 761-4700 800 521-0600

Order Number 1340503

**Quantum size effects of the electron conductivity in very thin
wire**

Chang, Yong Kyoo Jason, M.S.

San Jose State University, 1990

U·M·I
300 N. Zeeb Rd.
Ann Arbor, MI 48106

**QUANTUM SIZE EFFECTS OF THE ELECTRON
CONDUCTIVITY IN VERY THIN WIRE**

**A THESIS
PRESENTED TO
THE FACULTY OF THE DEPARTMENT OF PHYSICS
SAN JOSE STATE UNIVERSITY**

**IN PARTIAL FULFILLMENT
OF THE REQUIREMENT FOR THE DEGREE
MASTER OF SCIENCE**

**BY
YONG KYOO J. CHANG
APRIL 1990**

APPROVED FOR THE DEPARTMENT OF PHYSICS

1. Karamjeet Arya
Dr. Karamjeet Arya, Physics Department, SJSU

2. Kiumars Parvin
Dr. Kiumars Parvin, Physics Department, SJSU

3. Alejandro Garcia
Dr. Alejandro Garcia, Physics Department, SJSU

APPROVED FOR THE UNIVERSITY

Serena H. Stanford

ABSTRACT

QUANTUM SIZE EFFECTS
OF THE ELECTRON CONDUCTIVITY IN VERY THIN WIRE
BY YONG KYOO J. CHANG

Metals are generally considered as good electrical conductors because the electrons are almost free to move under applied electric fields. The small resistance arises due to the elastic scattering of the conduction electrons by impurities in metals, by phonons at finite temperature and/or by the surface roughness on thin samples. When the electron's mean free path becomes comparable to the size of the sample, the resistivity due to the surface scattering becomes important. Recent technological advancement has made possible to make samples very thin compared with the electron mean free path. One can now grow samples of thickness comparable to the Fermi-wave length of the conduction electrons. In this case, it is necessary to consider the quantum motion of the electrons by solving the Schrodinger equation.

In this project, we have considered the case of very thin metal wire. The motion of electrons along the wire radius is quantized due to the finite thickness of the wire. Furthermore, the mean free path has contribution from both intra-band (within the same band) and inter-band (between the different bands) scattering. The latter corresponds to the quantum correction to the conductivity. We have calculated the mean free path of the electrons by treating random roughness in the first order perturbation. We also have calculated the quantum corrections to the conductivity for various wire thicknesses.

ACKNOWLEDGEMENTS

I would like to thank my thesis advisor Professor Karamjeet Arya for his guidance through out this project. I also would like to thank Professor Kiumars Parvin and Professor Alejandro Garcia for serving as members of the thesis committee. I am indebted to Minnie Raj for her valuable discussions and comments. I especially thank Heung-Rae Lee for his encouraging words.

Lastly, I would like to thank chairman of the Physics department Donald Strandburg for his valuable advice during my stay here at San Jose State University.

TABLE OF CONTENTS

ABSTRACT	-----	iii
ACKNOWLEDGEMENTS	-----	iv
LIST OF FIGURES	-----	vi
 1. INTRODUCTION	-----	 1
 2. THE CLASSICAL BOLTZMANN EQUATION FOR THE ELECTRICAL CONDUCTIVITY		
2.1. Electrical Conductivity and Ohm's Law	-----	4
2.2. The Boltzmann Transport Equation	-----	6
2.3. Free Electron Gas in Three Dimensions	-----	11
 3. ELECTRON CONDUCTIVITY IN A VERY THIN METAL WIRE		
3.1. Green's Function For Smooth Wire	-----	17
3.2. Green's Function For Rough Wire	-----	20
3.3. Electrical Conductivity	-----	27
 4. RESULTS AND CONCLUSIONS	-----	 29
 APPENDIX I	-----	 31
REFERENCES	-----	33

LIST OF FIGURES

Figure

1. (a) Schematic diagram of an infinite smooth wire
 (b) Schematic diagram of an infinite rough wire
- 2 Energy bands of an electron in a cylindrical wire
- 3 Total density of states
- 4 Conductivity σ as function of a^2
 - (a) $a \approx 1000 \text{ \AA}$
 - (b) a is between $50 \text{ \AA}$ and $150 \text{ \AA}$
 - (c) a is between $10 \text{ \AA}$ and $70 \text{ \AA}$

1. INTRODUCTION

Metals are generally considered as good electrical conductors because the electrons are almost free to move under steady electric fields and these moving charges constitute current. The small resistance arises due to the elastic scattering of the conduction electrons by impurities in metals, by phonons at finite temperature, and/or by surface roughness of thin samples.¹⁻⁵ If the dimensions of the conducting specimen are very large compared with the mean free path of the conduction electron, the dominant contribution to the electrical resistivity comes from the impurities and the phonon scattering. In this case, by using classical Boltzmann theory and the free electron picture, one can show that the specific conductivity (conductivity / volume) is independent of the size and shape of the conducting sample and it obeys the well known Ohm's law.¹

However, when the electron's mean free path becomes comparable to the size of the sample, the surface scattering becomes equally important.^{2,3} The specific resistivity which is the reciprocal of the specific conductivity now depends on both the size and the shape of the material. These effects have been considered by various authors in the classical size effect regime.⁶⁻⁹ Classical size effects arise when the size of the sample (e.g., the thickness of the film or the diameter of the wire) approaches the bulk mean free path, and have largely been interpreted according to the theories of Fuchs⁸ and Sondheimer.⁹ These theories are based on the Boltzmann equation^{1,5} in which the surface is incorporated via boundary condition on the electron distribution function. In particular, the surface is characterized by a specularity parameter p according to the degree of scattering from the surface, with p lying between 0 (for completely diffuse) and 1 (for completely specular).

Recent technological advancement, however, has made it possible to make samples very thin compared with the electron mean free path.¹⁰⁻¹² In fact, one can grow samples of

thickness comparable to the Fermi-wave length of the conduction electrons. In this case, it is necessary to consider the quantum motion of the electrons by solving the Schrodinger equation.¹³ The quantized motion of the electron perpendicular to the direction of propagation can have important effects on the electron conductivity.¹⁴⁻¹⁷

There has been theoretical¹⁴⁻¹⁶ and experimental work^{10-12,18-20} in the recent past to study the quantum corrections to the electrical conductivity in very thin metal films. It has been shown that when the film thickness is large compared with the mean free path ℓ , due to the impurities, the conductivity is independent of the film thickness. However, when the film thickness d becomes comparable with ℓ , the effective mean free path approaches d . Then the conductivity goes as d . When the film thickness further decreases and becomes comparable to the Fermi-wave length λ , one finds quantum oscillation in the conductivity and its d dependence also changes.

In this project, we have considered similar studies in the case of a very thin metal wire. These calculations are possible because we can solve the Schrodinger equation for the electron in a smooth wire using cylindrical coordinates. This is essentially a square well potential problem¹³ with the potential $V(r) = 0$ inside the wire, and $V(r) = \infty$ outside the wire (see Fig. 1). The motion of the electron along the wire radius is quantized due to the finite thickness of the wire (Fig. 2). Furthermore, the electrical resistivity is assumed to be due to the elastic scattering of the electrons by random roughness present on the wire surface. We calculate the mean free path of the electron by treating random roughness in the first order perturbation. The mean free path has contribution due to both intra-band (in the same band) and inter-band (between different band) scattering. The latter corresponds to quantum correction to the conductivity. We have obtained the results for quantum corrections to the conductivity for various wire thicknesses.

In chapter two, we review the classical treatment using the Boltzmann equation for the electron transport. In chapter three, we give the solution of the Schrodinger equation for the electron in a thin metal wire using cylindrical coordinates. The mean free path is calculated using Green's function method. The results and conclusions are given in chapter 4.

2. THE CLASSICAL BOLTZMANN EQUATION FOR THE ELECTRICAL CONDUCTIVITY

When we apply electric field to the two ends of a metal piece, the free electrons in the metal start moving from higher potential side to the low potential side. During their motion, however, the electrons undergo scattering from impurities, lattice vibrations, etc. For a steady state current, the acceleration gained by the electrons due to the applied electric field is compensated by the loss of energy and momentum by the scattering. Thus the electrons move with a constant average drift speed v_d in the direction of the field. We can use this fact to derive an expression for the steady state current and hence, for the electrical conductivity. We here derive this expression, first by assuming a single electron picture. These results are simple to understand. We then consider many electron picture and include Fermi-Dirac distribution for the electrons. This is discussed by using the Boltzmann equation. Thus the expression derived is much more general.

2.1. Electrical Conductivity and Ohm's Law

We can calculate the drift speed in terms of the applied electric field E by using classical mechanics. When we apply the field to an electron with electrical charge $-e$ and mass M , a force $-eE$ will accelerate the electron by the Newton's second law of motion¹:

$$a = \frac{F}{M} = - \frac{eE}{M} . \quad (2.1)$$

However , an electron that has just undergone a typical collision will lose its memory of accumulated drift speed. Therefore, it will move off in a random direction. In the average time interval τ to the next collision, an average electron will change its velocity in the direction of $\mathbf{E}$ by an amount $a\tau$. We identify this with drift speed v_d and express

$$v_d = a\tau = \frac{eE\tau}{M} . \quad (2.2)$$

The average time τ is also known as the collision time.

We can thus find the average current J , which is the total amount of charge passing through a unit area of cross-section at any time. We have

$$J = -nev_d \quad (2.3)$$

where n is the electron number density, i.e., number of electrons per unit volume.

From Eqs. (2.2) and (2.3), we have

$$J = \left(\frac{e^2 n \tau}{M} \right) E . \quad (2.4)$$

Comparing this with Ohm's law, i.e.,

$$J = \sigma E,$$

we find the expression for electrical conductivity σ as

$$\sigma = \frac{e^2 n \tau}{M} . \quad (2.5)$$

Since the electrical resistivity ρ is defined as the reciprocal of the conductivity, we find

$$\rho = M / ne^2t .$$

We would like to mention here that σ in gaussian units has dimensions of frequency (sec^{-1}).

2.2. The Boltzmann Transport Equation

It is well known that electrons in a metal obey Fermi-Dirac distribution, which at very high temperature, reduces to Maxwell distribution. The transport equation is derived essentially by calculating the change in this distribution by the externally applied field.⁵

Let us assume that the electron distribution in a metal at time t is given by the classical distribution function $f(\mathbf{r}, \mathbf{v}; t)$ where $\mathbf{r}$ and $\mathbf{v}$ denote the position and velocity in the $\mathbf{r} - \mathbf{v}$ phase space. By definition,

$$f(\mathbf{r}, \mathbf{v}; t) d^3\mathbf{r} d^3\mathbf{v} = \text{number of particles in } d^3\mathbf{r} d^3\mathbf{v} .$$

Under the applied electric field $\mathbf{E}$, let $f(\mathbf{r}, \mathbf{v}; t)$ changes to $f(\mathbf{r} + d\mathbf{r}, \mathbf{v} + d\mathbf{v}, t + dt)$. We denote this change in the distribution function by

$$df(\mathbf{r}, \mathbf{v}; t) = f(\mathbf{r} + d\mathbf{r}, \mathbf{v} + d\mathbf{v}; t + dt) - f(\mathbf{r}, \mathbf{v}; t) . \quad (2.6)$$

For transport studies, we need steady state situation which in our case is obtained through the collision. Therefore,

$$\frac{df(\mathbf{r}, \mathbf{v}; t)}{dt} = \left(\frac{df}{dt} \right)_{\text{coll}}, \quad (2.7)$$

or

$$\frac{\partial f(\mathbf{r}, \mathbf{v}; t)}{\partial t} + \mathbf{v} \cdot \vec{\nabla}_{\mathbf{r}} f(\mathbf{r}, \mathbf{v}; t) + \frac{d\mathbf{v}}{dt} \cdot \vec{\nabla}_{\mathbf{v}} f(\mathbf{r}, \mathbf{v}; t) = \left[\frac{\partial f}{\partial t} \right]_{\text{collisions}}. \quad (2.8)$$

We can write the right hand side of Eq. (2.8) as

$$\left[\frac{\partial f}{\partial t} \right]_{\text{coll}} = - \frac{f - f_0}{\tau} \quad (2.9)$$

where f_0 is the equilibrium distribution function and τ is the average collision time or the relaxation time. The τ need not be a constant, but may depend, for example, on the velocity. If $\mathbf{v}$ is the mean velocity of those electrons to which τ refers, then the corresponding mean free path ℓ is defined by $\ell = \mathbf{v}\tau$. Thus by equating (2.8) and (2.9) we obtain the Boltzmann equation in the relaxation time approximation :

$$\frac{\partial f(\mathbf{r}, \mathbf{v}; t)}{\partial t} + \mathbf{v} \cdot \vec{\nabla}_{\mathbf{r}} f(\mathbf{r}, \mathbf{v}; t) + \frac{d\mathbf{v}}{dt} \cdot \vec{\nabla}_{\mathbf{v}} f(\mathbf{r}, \mathbf{v}; t) = - \frac{f(\mathbf{r}, \mathbf{v}; t) - f_0}{\tau}. \quad (2.10)$$

We now consider the distribution of particles moving in steady state with non-zero particle concentration gradient. If the concentration gradient is along the x-axis of rectangular Cartesian coordinates, then the transport equation becomes

$$v_x \frac{df(x, v_x)}{dx} = - \frac{f(x, v_x) - f_0}{\tau} \quad (2.11)$$

or

$$f(x, v_x) = f_0 - v_x \tau \frac{df_0}{dx} \quad (2.12)$$

Equation (2.12) determines the distribution function f . In the linear transport theory, we solve it by replacing df/dx by df_0/dx on the right hand side :

$$f(x, v_x) = f_0 - v_x \tau \frac{df_0}{dx} \quad (2.13)$$

The current density J_x is given by

$$J_x = e \int v_x f(x, v_x) N(\epsilon) d\epsilon \quad (2.14)$$

where $N(\epsilon)$ is the density of state. Using (2.13) for f , we have

$$J_x = e \int v f_0 N(\epsilon) d\epsilon - e \int v_x^2 \tau \frac{df_0}{dx} D(\epsilon) d\epsilon \quad (2.15)$$

The first term on the right hand side of Eq. (2.15) vanishes because it corresponds to the current in equilibrium state. Thus we have

$$J_x = - e \int v_x^2 \tau \frac{df_0}{dx} N(\epsilon) d\epsilon \quad (2.16)$$

For equilibrium distribution function f_0 , we assume Fermi-Dirac statistics for electrons.

Thus

$$f_0(\epsilon) = \left[\exp \left(\frac{\epsilon - \mu}{k_b T} \right) + 1 \right]^{-1} \quad (2.17)$$

where μ is the chemical potential, T is temperature and k_b is the Boltzmann constant.

By the chain rule, the derivative of f_0 with respect to x becomes

$$\frac{df_0(\epsilon)}{dx} = \frac{\partial f_0(\epsilon)}{\partial \mu} \frac{\partial \mu}{\partial x} \quad (2.18)$$

Also at $T = 0$ K, f_0 is discontinuous at $\epsilon = \epsilon_f$. Hence at very low temperature, we can approximate

$$\frac{\partial f_0}{\partial \mu} \approx \delta(\epsilon - \mu(T = 0 \text{ K})) = \delta(\epsilon - \epsilon_f)$$

where we have identified the chemical potential μ at zero temperature by the Fermi energy ϵ_f . Therefore the current density Eq. (2.16) is

$$J_x = -e\tau \frac{\partial \mu}{\partial x} \int v_x^2 \delta(\epsilon - \epsilon_f) N(\epsilon) d\epsilon \quad (2.20)$$

Furthermore, assuming $|v_x| = |v_y| = |v_z|$, one can replace $(v_x)^2 = v^2 / 3$ in Eq. (2.20),

where v is the average velocity. Using this and performing the integration in Eq. (2.20)

with the help of the δ function, we find

$$J_x = -e\tau \frac{v_f^2}{3} N(\epsilon_f) \frac{d\mu}{dx} \quad (2.21)$$

where v_f is the fermi velocity. Furthermore, as we defined earlier, the mean free path becomes

$$\ell = v_f \tau. \quad (2.22)$$

Also we use this to define the diffusion coefficient

$$D = \ell v_f / 3. \quad (2.23)$$

Using Eq. (2.22) and Eq. (2.23), we obtain

$$J_x = -eD N(\epsilon_f) \frac{d\mu}{dx}. \quad (2.24)$$

To derive an expression for conductivity, we assume that the electrons are in a steady state under the external electric field E . Then, the chemical potential μ is the sum of two parts : $\mu = \mu_{\text{int}} + \mu_{\text{ext}}$ where μ_{ext} is the potential energy per particle in the external potential, and μ_{int} is the internal chemical potential defined as the chemical potential that would be present if the external potential is zero. An equilibrium condition $\mu_1 = \mu_2$ can be expressed as $\Delta\mu_{\text{ext}} + \Delta\mu_{\text{int}} = 0$ or $\Delta\mu_{\text{ext}} = -\Delta\mu_{\text{int}}$. Moreover, if we suppose that the distribution function was in equilibrium before the electric field was turned on, then in the absence of the external field $\Delta\mu_{\text{ext}}$ would be zero. Consequently the $\Delta\mu_{\text{int}}$ has to be zero, too. Hence the chemical potential we are considering in the case of electrical conduction is completely due to the external potential. The non-equilibrium chemical potential difference $\Delta\mu_{\text{ext}} = \mu_1 - \mu_2$ on the charge carrier e is established by the potential difference $\Delta\phi$. Thus $\Delta\mu_{\text{ext}} = e\Delta\phi$. Therefore

$$\frac{d\mu}{dx} = \frac{d\mu_{\text{ext}}}{dx} = e \frac{d\phi}{dx} = -eE_x \quad (2.25)$$

From Eq. (2.24) and Eq. (2.25) we obtain

$$J_x = e^2 D N(\epsilon_f) E_x \quad (2.26)$$

Using the definition of Ohm's law

$$\mathbf{J} = \sigma \mathbf{E}, \quad (2.27)$$

we find the expression for conductivity σ as

$$\sigma = e^2 D N(\epsilon_f) \quad (2.28)$$

where D is the diffusion coefficient, $N(\epsilon_f)$ is the density of states for electrons at Fermi energy, and e is the electron charge.

Equation (2.28) is quite general although we derived it for three dimensional system. For finite size systems, for example, a metal film or metal wire the only modification we have to make is to calculate the density of state $N(\epsilon_f)$ and the diffusion coefficient D for the corresponding geometry.

2.3. Free Electron Gas in Three Dimensions

As an example, we now give explicit expression for the electrical conductivity σ by calculating the density of states $N(\epsilon)$ and the diffusion coefficient D for free electron gas in three dimensions.¹

The Schrodinger equation describing the motion of free electron in three dimensions is

$$-\frac{\hbar^2}{2M} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \Psi_{\mathbf{k}}(\mathbf{r}) = E_{\mathbf{k}} \Psi_{\mathbf{k}}(\mathbf{r}) \quad (2.29)$$

Assuming electrons to be confined in a cube of edge L , the normalized wave function $\Psi_{\mathbf{k}}(\mathbf{r})$ can be written as

$$\Psi_{\mathbf{k}}(\mathbf{r}) = \frac{1}{L^{3/2}} \exp(i\mathbf{k} \cdot \mathbf{r}) \quad (2.30)$$

The values of $\mathbf{k}$ can be obtained by using the periodic boundary condition, i.e.,

$$\Psi(x+L, y, z) = \Psi(x, y, z)$$

which gives

$$\begin{aligned} k_x &= 2\pi n_x / L \\ k_y &= 2\pi n_y / L \\ k_z &= 2\pi n_z / L \end{aligned} \quad (2.31)$$

where n_x, n_y, n_z are positive or negative integers. Using Eq. (2.29) and Eq. (2.30), we find

$$E_{\mathbf{k}} = -\frac{\hbar^2}{2M} [k_x^2 + k_y^2 + k_z^2] \quad (2.32)$$

In the ground state of a system of N free electrons, the occupied orbitals may be represented as points inside a sphere in $\mathbf{k}$ space. The energy at the surface of the sphere is the Fermi energy; the wave vectors at the Fermi surface have a magnitude k_f such that

$$E_f = \frac{\hbar^2}{2M} k_f^2 \quad (2.33)$$

From (2.33) we see that there is one allowed wave vector - that is, one distinct triplet of quantum numbers k_x, k_y, k_z - for the volume element $(2\pi/L)^3$ of k space. Thus in the sphere of volume $4\pi k_f^3/3$, the total number of orbitals is

$$2 \frac{4\pi k_f^3/3}{(2\pi/L)^3} = \frac{V}{3\pi^2} k_f^3 = N \quad (2.34)$$

where the factor 2 on the left is due to electron spin; there are two degenerate states- spin up and spin down. Thus

$$k_f = (3\pi^2 n)^{1/3} \quad (2.35)$$

where $n = N/V$ is the electron density. Also from Eq. (2.33),

$$E_f = \frac{\hbar^2}{2M} (3\pi^2 n)^{2/3} \quad (3.36)$$

This relates the Fermi energy to the electron density n . The electron velocity v_f at the Fermi energy is

$$v_f = \frac{\hbar k_f}{M} = \frac{\hbar}{M} (3\pi^2 n)^{1/3} \quad (2.37)$$

We now find an expression for the density of states $N(\epsilon)$, that is, the number of orbitals per unit energy per unit volume. We use (2.36) for the total number of orbitals of energy less than or equal to ϵ :

$$n = \frac{1}{3\pi^2} \left(\frac{2M\epsilon}{\hbar^2} \right)^{3/2} \quad (2.38)$$

$$N(\epsilon) = \frac{dn}{d\epsilon} = \frac{1}{2\pi^2} \left(\frac{2M}{\hbar^2} \right)^{3/2} \epsilon^{1/2} \quad (2.39)$$

Equation (2.39) gives the well known relation that $N(\epsilon)$ varies as $\epsilon^{1/2}$ in three dimensions. From Eq. (2.38) and Eq. (2.39), one can also write

$$N(\epsilon) = 3n / 2\epsilon \quad (2.40)$$

By using (2.40) for $N(\epsilon_f)$ and for the diffusion coefficient

$$D = \frac{v_f \ell}{3} = \frac{v_f^2 \tau}{3} = \frac{2\epsilon_f \tau}{3M} \quad (2.41)$$

in Eq. (2.28), we find

$$\sigma = \frac{ne^2\tau}{M} \quad (2.42)$$

which is the same as derived in Eq. (2.5).

It is instructive to note that the typical values of σ for copper at room temperature is approximately $5 \times 10^{17} \text{ sec}^{-1}$. Using electron density $n = 8.5 \times 10^{22} \text{ cm}^{-3}$ in (2.42)

we find relaxation time $\tau \approx 2 \times 10^{-14}$ sec. Furthermore, taking $v_f = 1.56 \times 10^8$ cm/sec for Cu, we find mean free path $\ell = v_f \tau = 3 \times 10^{-6}$ cm at room temperature.

3. ELECTRON CONDUCTIVITY IN A VERY THIN METAL WIRE

In the previous chapter, using the Boltzmann equation, we derived a general expression for electrical conductivity σ in terms of the diffusion coefficient D and the electron density of states $N(\epsilon_F)$. We also used this expression in the case of electron gas in three dimensions. The relaxation time τ which could be due to scattering by impurities and/or lattice vibrations was introduced phenomenologically. However, we assumed that the corresponding mean free path ℓ is much smaller than the dimensions of the system. Thus the expression derived for the conductivity does not show any classical or quantum size effects.

In this chapter, we consider electron transport in the metal wire such that the mean free path ℓ is comparable or less than the wire radius. Furthermore, we assume random roughness present on the wire surface to be responsible for the elastic scattering of the electrons. We explicitly calculate the corresponding mean free path in terms of roughness parameters. For simplicity, we neglect any contribution to the scattering by impurities or by lattice vibrations, i.e., very pure metal and close to the absolute zero degree temperature.

In Section 3-1, we first solve the Schrodinger equation for the electron motion in a smooth wire, i.e., without any roughness at the surface. Instead of solving for the wave function and the energy eigen values, we solve the Schrodinger equation for the one electron Green's function. The poles of the Green's function correspond to the energy eigen values. The Green's function method is more suitable to calculate the relaxation time or the mean free path due to the roughness scattering. We discuss these results in section 3-3.

3.1. Green's Function for Smooth Wire

Consider an electron moving in an infinite wire of radius a with smooth surface. The potential is assumed to be zero inside the wire and infinite outside. (see Fig. 1.a) The electron motion is described by the Schrodinger equation.

$$\left(\frac{p^2}{2M} + V(\mathbf{r}) \right) \Psi(\mathbf{r}) = E \Psi(\mathbf{r}) \quad (3.1)$$

where

$$\begin{aligned} V(\rho, \phi, z) &= 0 & \rho < a \\ V(\rho, \phi, z) &= \infty & \rho \geq a \end{aligned}$$

and M is the rest mass of the electron. The corresponding equation for the Green's function G^0 is

$$\left[\frac{\hbar^2}{2M} \nabla^2 + (E - V(\mathbf{r})) \right] G^0(\mathbf{r}, \mathbf{r}'; E) = \delta(\mathbf{r} - \mathbf{r}') \quad (3.2)$$

Since the potential $V(\mathbf{r})$ has cylindrical symmetry, we use the cylindrical coordinates system, i.e., $\mathbf{r} = (\rho, \phi, z)$, to solve Eq. (3.2). In cylindrical coordinate^{21,22}

$$\nabla^2 = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}, \quad (3.3)$$

and

$$\begin{aligned}
\delta(\mathbf{r} - \mathbf{r}') &= \frac{1}{\rho} \delta(\rho - \rho') \delta(\phi - \phi') \delta(z - z') \\
&= \frac{1}{\rho} \delta(\rho - \rho') \left(\frac{1}{2\pi} \right) \sum_{m=-\infty}^{\infty} e^{im(\phi - \phi')} \left(\frac{1}{2\pi} \right) \int_{-\infty}^{\infty} e^{ik(z - z')} dk
\end{aligned} \quad (3.4)$$

We also make a similar expansion for the Green's function

$$G^0(\mathbf{r}, \mathbf{r}'; E) = \left(\frac{1}{2\pi} \right)^2 \sum_{m=-\infty}^{\infty} G_{k,m}^0(\rho, \rho'; E) e^{im(\phi - \phi')} \int_{-\infty}^{\infty} e^{ik(z - z')} dk \quad (3.5)$$

Using (3.3) - (3.5) in Eq. (3.2), we find for ρ and ρ' inside the wire,

$$\left[\frac{d}{d\rho} \left(\rho \frac{d}{d\rho} \right) + \alpha^2 \rho - \frac{M^2}{\rho} \right] G_{k,m}^0(\rho, \rho'; E) = \delta(\rho - \rho') \quad (3.6)$$

where

$$\alpha^2 = \frac{2M}{\hbar^2} E - k^2 \quad (3.7)$$

To solve Eq. (3.6), we note that the corresponding homogeneous equation

$$\left[x^2 \frac{d^2}{dx^2} + x \frac{d}{dx} + (x^2 - m^2) \right] U(x) = 0 \quad (3.8)$$

with $x = \alpha\rho$, is a Bessel's differential equation. The two solutions of Eq. (3.8) are Bessel function $J_m(x)$ and Neumann function $N_m(x)$.^{21,22} Since we know the solution of the

homogeneous equation (3.8), it is straight forward to solve Eq. (3.6) for the Green's function G^0 by using the standard method.^{22,23} We find

$$G_{km}^0(\rho, \rho'; E) = \frac{\pi M}{h^2 J_m(\alpha a)} J_m(\alpha \rho) [J_m(\alpha \rho') N_m(\alpha a) - J_m(\alpha a) N_m(\alpha \rho')] ,$$

$$\rho < \rho' \leq a$$
(3.9)

and

$$G_{km}^0(\rho, \rho'; E) = \frac{\pi M}{h^2 J_m(\alpha a)} J_m(\alpha \rho') [J_m(\alpha \rho) N_m(\alpha a) - J_m(\alpha a) N_m(\alpha \rho)]$$

$$\rho' < \rho \leq a$$
(3.10)

The complete Green's function $G^0(\rho, \rho'; E)$ is then given by Eq. (3.5).

Note that the energy eigen values of Eq. (3.1) or (3.2) are determined by the poles of the Green's function $G_{km}^0(\rho, \rho'; E)$. From Eq. (3.9) and (3.10), we see that these poles are given by

$$J_m(\alpha a) = 0$$
(3.11)

Eq. (3.11) has infinite solutions for each m . We denote them by $x_{mn} = \alpha_{mn} a$, where m and n are integers. For the first few integer values of n , the first three roots are :^{22,24}

$$\begin{aligned}
m = 0, & \quad x_{0n} = 2.405, 5.520, 8.654 \\
m = 1, & \quad x_{1n} = 3.832, 7.016, 10.173 \\
m = 2, & \quad x_{2n} = 5.136, 8.417, 11.620 .
\end{aligned} \tag{3.12}$$

For higher roots, the asymptotic formula

$$x_{mn} \approx n\pi + (m - 1/2) \pi/2 \tag{3.13}$$

gives adequate results. The energy eigen values are then given by (3.7)

$$E_{kmn} = \frac{\hbar^2}{2M} \left(\frac{x_{mn}^2}{a^2} + k^2 \right) . \tag{3.14}$$

These energy levels are plotted schematically in Fig. 2.

The different bands defined by m and n correspond to the quantized electron motion perpendicular to the z - direction. Moreover, the separation between different bands is inversely proportional to the square of the wire radius. However, along the z - direction, the solution is plane wave and the corresponding energy is a parabolic curve.

3.2. Green's Function For Rough Wire

We now consider electron motion in a wire with roughness present on its surface. Let $\xi(\phi, z)$ defines the rough surface profile, (Fig.1b) so that

$$\begin{aligned}
V(\mathbf{r}) &= 0 & \rho < \xi(\phi, z) + a \\
&= \infty & \rho \geq \xi(\phi, z) + a .
\end{aligned}$$

(3.15) The Schrodinger equation for the Green's function $G(\mathbf{r}, \mathbf{r}'; E)$ is

$$\left[\frac{\hbar^2}{2M} \nabla^2 + E - V(\mathbf{r}) \right] G(\mathbf{r}, \mathbf{r}'; E) = \delta(\mathbf{r} - \mathbf{r}') \quad (3.16)$$

Using Green's theorem²²

$$\begin{aligned} & \int_V \left[G(\mathbf{r}, \mathbf{r}''; E) \nabla^2 G_0(\mathbf{r}'', \mathbf{r}'; E) - G_0(\mathbf{r}'', \mathbf{r}'; E) \nabla^2 G(\mathbf{r}, \mathbf{r}''; E) \right] d^3x'' \\ &= \oint_S \left[G(\mathbf{r}, \mathbf{r}''; E) \frac{\partial}{\partial n} G_0(\mathbf{r}'', \mathbf{r}'; E) - G_0(\mathbf{r}'', \mathbf{r}'; E) \frac{\partial}{\partial n} G(\mathbf{r}, \mathbf{r}''; E) \right] d\sigma'' \end{aligned} \quad (3.17)$$

and Eq. (3.2), we can change the differential equation (3.16) into an integral equation:¹⁴

$$G(\mathbf{r}, \mathbf{r}'; E) = G_0(\mathbf{r}, \mathbf{r}'; E) - \frac{\hbar^2}{2M} \oint_S G_0(\mathbf{r}'', \mathbf{r}'; E) \frac{\partial}{\partial n} G(\mathbf{r}, \mathbf{r}''; E) d\sigma'' \quad (3.18)$$

Note that in Eq. (3.17) and (3.18), s denotes integration over the surface of the rough wire and $\partial/\partial n$ denotes the differentiation along the normal to the rough surface.

Assuming small surface roughness, i.e., for small ξ , we can expand $G^0(\mathbf{r}'', \mathbf{r}'; E)$ in Eq. (3.18) around $\rho = a$, i.e.,

$$G^0(\mathbf{r}'', \mathbf{r}'; E) = G^0(\rho'', \phi'', z'', \mathbf{r}'; E) \Big|_{\rho''=a} + \xi(\phi'', z'') \frac{\partial}{\partial \rho} G_0(\rho'', \phi'', z'', \mathbf{r}'; E) \Big|_{\rho''=a} \quad (3.19)$$

Using (3.19) in (3.18) and after simplification we obtain

$$G(\mathbf{r}, \mathbf{r}'; E) = G^0(\mathbf{r}, \mathbf{r}'; E)$$

$$- \frac{\hbar^2 a}{2M} \int d\phi'' dz'' \left[G^0(\mathbf{r}, \mathbf{r}''; E) \frac{\overleftarrow{\partial}}{\partial \rho''} \right]_{\rho''=a} \xi(\phi'' z'') \left[\frac{\overrightarrow{\partial}}{\partial \rho''} G(\mathbf{r}'', \mathbf{r}'; E) \right]_{\rho''=a} . \quad (3.20)$$

$G(\mathbf{r}, \mathbf{r}'; E)$ is a function of surface profile ξ . For a random roughness, we need to take an ensemble average over the distributions of ξ . Assuming Gaussian distribution, i.e.,

$$\langle \xi \rangle = 0 \quad (3.21)$$

$$\langle \xi(\phi, z) \xi(\phi', z') \rangle = \delta^2 g(\phi - \phi', z - z') \quad (3.22)$$

where δ is the root mean square roughness amplitude and g gives the correlation at two points on the surface of the wire. By using (3.21) and (3.22), we can now obtain average Green's function $\langle G \rangle$ from Eq. (3.20). Since we have assumed small roughness, we make a series expansion of (3.20) and retain only the lowest order nonvanishing term in ξ . Also, we note that $\langle G(\mathbf{r}, \mathbf{r}'; E) \rangle = \langle G(\mathbf{r} - \mathbf{r}'; E) \rangle$.

So, we use fourier transformation as defined earlier in (3.5) :

$$\langle G(\mathbf{r} - \mathbf{r}'; E) \rangle = \left(\frac{1}{2\pi} \right)^2 \sum_{m=-\infty}^{\infty} \int G_{k, m}(\rho, \rho'; E) e^{ik(z-z')} e^{im(\phi-\phi')} dk . \quad (3.23)$$

Thus from Eq. (3.20), after some simplifications, we find

$$G_{k, m}(\rho, \rho'; E) = \frac{G_{km}^0(\rho, \rho'; E)}{1 - \Sigma(E)\tilde{G}(k, m; E)} \quad (3.24)$$

where

$$\Sigma(E) = g \left(\frac{\delta}{a} \right)^2 \left(\frac{1}{2\pi} \right)^2 \sum_m \int dk' \tilde{G}(k, m; E), \quad (3.25)$$

and

$$\tilde{G}(k, m; E) = \alpha a \frac{J'_m(\alpha a)}{J_m(\alpha a)}. \quad (3.26)$$

In writing Eq. (3.25), we have assumed the correlation function g to be independent of ϕ and z . Note that for smooth wire, the energy eigen values are given by the poles of G_{km}^0 in Eq. (3.9) or (3.10). However, for a rough wire, the energy eigen values are given by the poles of G_{km} in Eq. (3.24). Using Eq. (3.9), (3.10) and (3.26) in (3.24), we find that the poles of G_{km} are given by

$$J_m(\alpha a) - \alpha a \Sigma J'_m(\alpha a) = 0. \quad (3.27)$$

For small Σ , i.e., for small roughness, Eq. (3.27) can be written as

$$J_m[\alpha a(1 - \Sigma)] = 0. \quad (3.28)$$

The solutions of Eq. (3.28) are given by

$$\alpha a(1 - \Sigma) = x_{mn} \quad (3.29)$$

where x_{mn} correspond to the zeros of the Bessel function. Using (3.29) in (3.7), we find the energy eigen values for a rough wire

$$E_{kmn} = E_{kmn}^0 + \frac{\hbar^2}{Ma^2} x_{mn}^2 \Sigma \quad (3.30)$$

Note that Σ has both real and imaginary part as we can see in Eq. (3.25). The real part gives a shift in the energy values due to roughness and imaginary part gives a finite width. More over, the imaginary part is related to the relaxation time τ due to the elastic scattering of electrons by the roughness. In fact, in transport theory we need only to calculate τ which can be written as

$$\frac{1}{\tau_{mn}} = \frac{\hbar}{2Ma^2} x_{mn}^2 \Sigma_{Im}, \quad (3.31)$$

where Σ_{Im} denotes the imaginary part of Σ in Eq. (3.25). We should note that the relaxation time for electrons moving in different bands is different, and this is indicated by the indices m and n .

We now give the explicit expression of the imaginary part of Σ . Note that in Eq. (3.25), $\tilde{G}$ has poles given by $J_m(\alpha a) = 0$. Using²¹

$$\frac{1}{f(x) + i\eta} = P\left(\frac{1}{f(x)}\right) + i\pi\delta(f(x)) \quad (3.32)$$

we can write

$$\Sigma_{Im} = g\left(\frac{\delta}{a}\right)^2 \frac{1}{4\pi} \sum_m \int dk \alpha a J'_m(\alpha a) \delta(J_m(\alpha a)) \quad (3.33)$$

Using the properties of δ - function :¹³

$$\delta(f(x)) = \sum_n \frac{1}{f'(x_n)} \delta(x - x_n) \quad (3.34)$$

where x_n corresponds to the zeros of $f(x)$;

$$\delta(x^2 - x_n^2) = \frac{1}{2x} \delta(x - x_n) + \frac{1}{2x} \delta(x + x_n), \quad (3.35)$$

Eq. (3.33) reduces to

$$\Sigma_{Im} = g\left(\frac{\delta}{a}\right)^2 \frac{1}{2\pi} \sum_{m,n} \int dk \cdot \alpha^2 \delta[\alpha^2 - \alpha_{mn}^2] \quad (3.36)$$

From Eq. (3.7) and (3.14), we have

$$\alpha^2 - \alpha_{mn}^2 = \frac{2M}{\hbar^2} (E - E_{kmn}) \quad (3.37)$$

Defining density of states

$$N_{mn}(E) = \frac{1}{2\pi} \frac{dk}{dE_{kmn}} \quad (3.38)$$

we can perform the integration in Eq. (3.36). Thus we have

$$\Sigma_{Im} = g\left(\frac{\delta}{a}\right)^2 \sum_{m,n} \frac{\hbar^2 \alpha_{mn}^2}{2M} N_{mn}(E_f) \quad (3.39)$$

We can calculate the density of states $N_{mn}(E)$ at Fermi energy from Eq. (3.38) and (3.14). Hence we obtain

$$N_{mn}(E_f) = \frac{M}{2\pi\hbar^2 k_{fmn}} \quad (3.40)$$

where

$$k_{fmn} = \left(\frac{2ME_f}{\hbar^2} - \alpha_{mn}^2 \right)^{1/2} \quad (3.41a)$$

In Fig. 3, we have plotted the total density of states

$$N(E) = \sum_{m, n} N_{mn}(E) \quad (3.41b)$$

It shows peaks corresponding to different bands. Also, for a given band, $N_{mn}(E)$ goes as $E^{-1/2}$ as expected for one dimensional case.

We should note that for a thick wire Eq. (3.41b) reduces to the density of states for a bulk 3-dimensional system. It has been shown in Appendix I that for larger wire radius a , the energy separation between various sub-band decrease as a^{-2} . Hence, m and n summation in Eq. (3.41b) can be changed to an integration. Then, $N(E) \approx E^{1/2}$ for 3-dimensions.

Using Eq. (3.39) and (3.40), we have

$$\Sigma_{Im} = g \left(\frac{\delta}{a} \right)^2 \left(\frac{1}{4\pi} \right) \sum_{m, n} \frac{\alpha_{mn}^2}{k_{fmn}} \quad (3.42)$$

We can now calculate the relaxation time τ_{mn} from Eq. (3.31)

3.3. Electrical Conductivity

Now, we can derive expression for electrical conductivity of the thin wire. As discussed earlier, electron can move only along the z -direction, and its motion perpendicular to the z -direction is quantized. Since Fermi energy crosses different sub-bands, (see Fig. 2) all electrons with energies close to the Fermi energy take part in the conduction. In the previous section, we calculated the relaxation time for the electrons in sub-bands m and n . We can now define the elastic mean free path ℓ_{mn} as

$$\ell_{mn} = v_{f,mn} \tau_{mn} \quad (3.43)$$

where $v_{f,mn} = \hbar k_{f,mn} / M$ is the Fermi velocity in sub-band defined by m and n . The diffusion coefficient in the corresponding sub-band can be written as

$$D_{mn} = v_{f,mn} \ell_{mn} . \quad (3.44)$$

If $N_{mn}(E_f)$ is the density of states of m,n sub-band, then following the discussions of Chapter 2, we can write the conductivity :

$$\sigma = \sum_{m,n} N_{mn}(E_f) e^2 D_{mn} . \quad (3.45)$$

Note that $N_{mn}(E_f)$ is the one dimensional density of states given by Eq. (3.38). Using (3.38) and (3.44), we finally obtain

$$\sigma = \frac{e^2}{\hbar} \sum_{m,n} \ell_{mn} \quad (3.46)$$

where we have used $v_{fmn} = \hbar k_{fmn} / M$ and ℓ_{mn} given by Eq. (3.43). We should note that Eq. (3.46) corresponds to a quantum mechanical contribution to the conductivity. For example, in the classical limit, if $\hbar \rightarrow 0$, $\sigma \rightarrow \infty$, i.e., there is no resistance.

In next chapter we give the results of σ calculated from Eq. (3.46) for various values of roughness parameters and the wire radius.

4. RESULTS AND CONCLUSIONS

Equation (3.46) derived in the previous chapter shows that the total conductivity σ is due to the conduction of all electrons in different sub-bands. Writing explicit expression for ℓ_{mn} from Eq. (3.42) and (3.31), we have

$$\sigma = \frac{e^2}{h} \sum_{m,n} \frac{k_{fmn}}{x_{mn} \Sigma_{Im}} a^2 \quad (4.1)$$

where k_{fmn} and Σ_{Im} are given by Eq. (3.41a) and (3.42), respectively. Furthermore, x_{mn} are the zeros of the Bessel function. We have calculated σ from Eq. (4.1) for various values of wire radius a . In these calculations, we have taken $E_f = 0.1$ eV, roughness parameters $\delta = 5$ Å and $g = k_f^{-1}$. The summation in Eq. (4.1) is carried out numerically.

In Fig. 4, we have plotted σ as a function of a^2 . For large values of a , ($a \gg \lambda_f$) we have a straight line indicating that the conductivity σ is proportional to the area of the wire. This is an expected result and agrees with the geometrical considerations. However, for smaller values of a , between 50 Å and 250 Å, we find $\sigma \approx a^{5/2}$ dependence (see Fig. 4b). For a between 10 Å and 50 Å ($a < \lambda_f$), Fig. 4c shows higher dependence of σ on a ($\approx a^3$). In addition, we find also oscillations in σ for small a (Fig. 4c). These correspond to the oscillations in the density of states discussed in Fig. 3.

The experimental results of Ref. 12 for σ also indicate a^2 dependence for $a > 1000$ Å. This agrees with our results discussed in Fig. 4a. However, we do not know any experiments for very thin wire which show different a dependence. There is,

however, recent experimental interest¹¹ for semiconductor GaAs wires. Because of the small effective mass in GaAs and large Fermi wave length, we predict that even for thick wires, one should be able to observe $a^{5/2}$ or a^3 dependence for the conductivity.

APPENDIX I

From Eq. (3.40) - (3.41b), we write the density of state as

$$N(\epsilon_f) = \frac{Ma}{2\pi \hbar^2} \sum_{m,n} \frac{1}{[x_f^2 - x_{mn}^2]^{1/2}} \quad (A.1)$$

where

$$x_f^2 = \frac{2M}{\hbar^2} a^2 E_f, \quad (A.2)$$

and x_{mn} are the zeros of the Bessel function. For thick wire, $a \rightarrow \infty$, contribution in (A.1) comes only from very large values of m and n . Then, we can use the asymptotic limit for x_{mn} :

$$x_{mn} = n\pi + \frac{m\pi}{2} + \frac{\pi}{4} \approx n\pi + \frac{m\pi}{2} \quad (A.3)$$

The separation between the energy levels decreases as the wire's radius increases. At the limit of bulk metal, $a \rightarrow \infty$, the summation of the density of state becomes an integration

$$\sum_{m,n} \rightarrow \int_0^\infty dm \int_0^\infty dn \quad (A.4)$$

By setting $x = n\pi$ and $y = m\pi/2$ we have

$$N(E_f) = \frac{Ma}{\pi^3 \hbar^2} \int_0^\infty \int_0^\infty \frac{dx dy}{\sqrt{x_f^2 - (x+y)^2}} \quad (A.5)$$

This integration in cylindrical coordinates can be written as

$$N(E_f) = \frac{Ma}{\pi^3 \hbar^2} \int_0^{\pi/2} d\theta \int_0^{\rho'} \frac{\rho d\rho}{\sqrt{x_f^2 - \rho^2 (\cos \theta + \sin \theta)^2}} \quad (A.6)$$

where $x = \rho \cos \theta$, $y = \rho \sin \theta$, $dx dy = \rho d\rho d\theta$ and $\rho' = x_f / (1 + \sin \theta)^{1/2}$. Doing the straight forward integration, we obtain

$$N(E_f) = \frac{1}{2\pi^3 a E_o} \left(\frac{E_f}{E_o} \right)^{1/2} \quad (A.7)$$

where

$$E_o = \frac{\hbar^2}{2Ma^2} \quad (A.8)$$

REFERENCES

1. C. Kittel, Introduction to Solid State Physics, 6th ed. (Wiley, New York, 1986).
2. J. M. Ziman, Principles of the Theory of Solids, 2nd ed. (Cambridge, London, 1972).
3. J. M. Ziman, Electrons and Phonons, 1st ed. (Oxford Press, London, 1960).
4. N. W. Ashcroft, N. D. Mermin, Solid State Physics, (Holt, New York, 1976).
5. C. Kittel, H. Kroemer, Thermal Physics, 2nd ed. (Freeman, San Francisco, 1980).
6. K. L. Chopra, Thin Film phenomena, (McGraw Hill, New York, 1969).
7. R. B. Dingle, Proc. Roy. Soc. A 201, 545 (1950) .
8. K. Fuchs, Proc. Camb. Phil. Soc. 34 , 100 (1938) .
9. E. H. Sondheimer, Adv. in Phys. 1, 1 (1952) .
10. H. Sakaki, T. Noda, K. Hirakawa, M. Tanka, and T. Matsusue, Appl. Phys. Lett. 51, 1934 (1987).
11. E. Kapon, D. M. Hwang, and R. Bhat, Phys. Rev. Lett. 63, 430 (1989).
12. N. Giordano, W. Gilson, and D.E. Prober, Phys. Rev. Lett. 43, 725 (1979).
13. L. I. Schiff, Quantum Mechanics, 3rd ed. (McGraw Hill, New York, 1968).
14. A. Maradudin, A. McGurn, Phys. Rev. B 30, 3136 (1984).
15. Z. Tesanovic, M. V. Jaric and S. Maekawa, Phys. Rev. Lett. 57, 2760 (1986).
16. N. Trivedi, N. Ashcroft, Phys. Rev. B 35, 298 (1988).
17. K. Arya, Y. Chang, M. Raj, J. P. Lu, and J. L. Birman, Bull. of Am. Phys. Soc. 35, 644, (1990).
18. P. A. Badoz, A. Briggs, E. Rosencher, F. Arnaud d'Avitaya, and C. d'Anterrosches, Appl. Phys. Lett. 51, 169 (1987).

19. J. M. Phillips, J. L. Batstone, J. C. Hensel, and M. Cerullo, Appl. Phys. Lett. 57,1895 (1987).
20. J. C. Hensel, R. T. Tung, J. M. Poate, and F. C. Unterwald, Phys. Rev. Lett. 54,1840 (1985).
21. G. Arfken, Mathematical Methods for Physicists, 3rd ed. (Academic Press, Orlando, 1985).
22. J. D. Jackson, Classical Electrodynamics, 2nd ed. (Wiley, New York, 1975).
23. H. Margenau, G. M. Murphy, The Mathematics of Physics and Chemistry, 2nd ed. (Nostrand, New York, 1956).
24. M. Abramowitz, I. A. Stegun, Handbook of Mathematical Functions, (Dover, New York, 1972).

FIGURE CAPTIONS

Figure 1. (a) Schematic diagram of an infinite smooth wire : a is the radius of the wire.

(b) Schematic diagram of an infinite rough wire : a is the average radius of the wire, and $\xi(\phi, z)$ is the rough surface profile.

Figure 2. Quantized energies of an electron in a cylindrical wire calculated from Eq. (3.14) : The eigen values are split into sub-bands and are parabolic within a band. The vertical axis is scaled by $E_0 = \hbar^2 / 2Ma^2$. The various sub-bands correspond to the zeros of the Bessel function. The dotted horizontal line is the Fermi level.

Figure 3. Density of states $N(E)$ in units of $2\pi a E_0$ calculated from Eq. (3.41b) : E_0 is defined by $E_0 = \hbar^2 / 2Ma^2$. The various peaks correspond to different energy bands.

Figure 4. Plots of conductivity σ as function of a^2 for various scales of the wire radius : Each vertical axis is scaled by e^2 / π . The roughness parameters are $\delta = 5 \text{ \AA}$ and $g = k_F^{-1}$. The Fermi level is $E_F = 0.1 \text{ eV}$, and the Fermi wave length is $\lambda_F = 40 \text{ \AA}$.

(a) The straight line shows that the conductivity is proportional to the cross sectional area of the wire when $a \approx 1000 \text{ \AA}$.

(b) The wire radius a is between $50 \text{ \AA}$ and $150 \text{ \AA}$.

(c) The conductivity oscillates when a is between $10 \text{ \AA}$ and $70 \text{ \AA}$.

Figure 1 (a)

1 (b)

Figure 2

Figure 3

Figure 4 (a)

Figure 4 (b)

Figure 4 (c)