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A Multi-Scale Evaluation of Landscape Relief Using Digital Elevation Data

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A MULTI-SCALE EVALUATION OF LANDSCAPE RELIEF USING DIGITAL
ELEVATION DATA

A Thesis

Presented to

The Faculty of the Department of Geology

San José State University

In Partial Fulfillment

of the Requirements for the Degree

Master of Science

by

Tania Michelle Treis

May 2011

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The Designated Thesis Committee Approves the Thesis Titled

A MULTI-SCALE EVALUATION OF LANDSCAPE RELIEF USING DIGITAL
ELEVATION DATA

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ABSTRACT

A MULTI-SCALE EVALUATION OF LANDSCAPE RELIEF USING DIGITAL ELEVATION DATA

by Tania Michelle Treis

Geomorphologists commonly use the term “relief” to define the difference in elevation of a landscape over a fixed horizontal distance or window size.

Relief is typically measured using a fixed-size sliding window analysis where the window size is arbitrarily chosen. This method, however, does not provide insight on how relief changes with or depends on scale.

A multi-scale analysis of digital elevation data (DEDs) was undertaken in this study. A sliding window analysis was employed, but the size of the window was varied from small to large for each landscape. A relief curve was generated and fitted to a saturating exponential function for each landscape based on the maximum relief measured at each window size. The resultant curves showed that most landscapes scale in the vertical dimension over a fixed horizontal distance. While relief is highly scale-dependent, the relief at each window size is generally proportional to the overall relief of the landscape.

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INTRODUCTION

Definitions and Uses of Relief

Relief is defined as the difference in maximum and minimum elevations across a width, or window, of interest (Fig. 1). Relief is an important concept most frequently studied not for its own intrinsic value, but as a morphometric parameter to explore other processes, such as erosion, slope stability, climate change, and crustal processes. A review of the relevant literature indicates that the definition and method of measuring relief vary depending upon the scale and also the purpose of the research. An understanding of how relief changes with scale can provide insight into the appropriate use of relief as a comparative tool.

Processes that Form Relief

Studies of relief have been performed to understand the underlying processes that shape the landscape. For example, Fielding et al. (1994) evaluated the topography of the Tibetan plateau to identify the geologic controls on the production and destruction of relief. Tibet has a high but relatively constant elevation, and it is characterized as having low relief in spite of its location in the middle of an active continental collision zone. Fielding et al. (1994) devised a

Figure 1. Relief as a function of window size. Slope angle stays constant, but total relief increases with an increase in the window size.

method of quantifying relief to show that it is generally low over large areas of the northern and central parts of the Tibetan plateau and is high over short horizontal distances along the edges of the plateau. Crustal versus surface processes were then evaluated to explain the large-scale low relief through the northern and central parts of Tibet.

Each of three scales of relief was defined and calculated by Fielding et al. (1994). Short-wavelength slopes were represented by hillslope gradients calculated across windows of 280 m in the north-south direction and 235 m in the east-west direction. The short-wavelength relief is, therefore, at an appropriate scale of 250 m. Swath profiles were used to calculate an intermediate scale of relief. Topographic profiles were taken across the greatest north-south and east-west directions in Tibet. Points were projected along 100-km-wide swaths of the profiles. The difference in elevation was calculated along each of the 100-km-wide segments, providing values of relief at the 100 km scale. Hypsometric analysis was considered to characterize large-scale relief at the 1000 km scale. A histogram of the internally drained area of Tibet was evaluated using Lambert Conformal Conic Projection Digital Elevation Models (DEMs) with a resolution of 1 km (Fielding et al., 1994).

Several factors were evaluated against the relief as calculated at each scale, including lithologic variations, erosional processes, and crustal fluid processes. Fielding et al. (1994) found that lithology seemed to have a relatively minor effect

on the spatial pattern of topographic relief. Low slopes were continuous along all lithologies in northern Tibet. Because the Tibetan plateau has a high desert climate with low temperatures and little precipitation, erosional processes level topography slowly. The area of low relief, however, is extremely large, and Fielding et al. (1994) postulated that erosion alone cannot account for the extent of low relief across the plateau when considering the active tectonic environment. They concluded that crustal processes may play a role in the large-scale flatness of the landscape. Viscous flow in the lower crust could be a cause of the low relief seen at the 1000 km scale in Tibet, where an originally low and flat surface was warped upward.

A differing hypothesis on the causes of the relief seen in the Tibetan plateau was presented by Liu-Zeng et al. (2008). They concluded that basin filling due to inefficient drainage smoothed out tectonically generated relief on the plateau. Liu-Zeng et al. (2008) calculated relief for their analysis using Shuttle Radar Topography Mission digital elevation data sets with 3 arc-second resolution. Three geomorphic indices at three scales were used, as in Fielding et al. (1994). Hypsometry was used to represent the approximate relief at a large scale. Local relief was calculated as the difference in elevation of 10 km by 10 km windows, which was defined as the intermediate scale. Local slope characterized relief at the smallest scale and was defined as the average gradient in 3° by 3° cell grids converted to meters and then averaged over a 200 m scale.

Three different regions of Tibet were then evaluated: the northern Tibetan plateau, the Tibet interior, and the eastern and southeastern Tibetan plateau.

Liu-Zeng et al. (2008) concluded that erosion and basin filling can account for the large-scale flat topography, a very different conclusion than that of Fielding et al. (1994). This leads to the question of whether the different conclusions could be attributed to the magnitude of order difference in the calculation of the intermediate scale between the studies.

Erosion

Relief has sometimes been used as a proxy for erosion rate. Abbot et al. (1997) used two measures of relief: ordinary relief and geophysical relief. Ordinary relief is simply the elevation difference between valley bottoms and adjacent hilltops. Ordinary relief, therefore, is considered only at the scale of a valley-hilltop pair. Geophysical relief is the mean elevation difference between two surfaces: “a smooth surface connecting the highest points in the current landscape and the current topography itself” (Abbot et al., 1997).

Using the definition of geophysical relief presented by Abbot et al. (1997), Small and Anderson (1998) calculated and compared the erosion rates of summits and valley floors in the Laramide ranges in the western U.S. Along gently sloped summits and ridges (summit flats), erosion rates were found to be about 10 m/my that were calculated on the basis of the concentrations of

cosmogenic radionuclides. In contrast, the erosion rates in the valleys between the summit flats were an order of magnitude faster. High rates of valley incision, despite slow rates of summit erosion, could result in peak uplift in response to removal of mass in the valleys (known as isostatic response). Relief production then increases as the summits rise.

Small and Anderson (1998) calculated the geophysical relief using 30 m DEMs of four summit flats in the Laramide mountain ranges, in areas that have not been influenced by glaciers. They found that the summits have stayed at the same elevation, even though several hundreds of meters of relief have been created. Peak elevations remain constant because isostatic response from valley incision equals summit erosion (Small and Anderson, 1998).

Montgomery and Brandon (2002) studied erosion rates and local relief through the evaluation of 10 m grid DEMs in the Olympic Mountains, Washington. They also performed a coarser global analysis (using 10 km grids) of the spatial distribution of local relief within 10-km-diameter circular areas for the major land masses in Asia, Europe, and North and South America. Their main goal was to evaluate Ahnert's (1970) linear relationship between erosion rates and mean local relief, where mean local relief is defined as the difference in elevation measured over a specified horizontal distance.

Montgomery and Brandon (2002) evaluated mean slope for 10 m DEMs over 10-km-diameter windows. Results for the Olympic Mountains showed a

linear relationship between mean slope and long term erosion rates at low slopes. However, there was a weak linear relationship (or a strongly non-linear relationship) between mean slope and erosion rates for steep slopes. The shift from a linear relationship between erosion rate and slope angle corresponded to a hillslope gradient of approximately 25° , which is the gradient at which landslides typically occur. They calibrated the erosion rates to the mean local relief for the Olympic Mountains. Windows with a 10 km width were also used to calculate mean local relief. Mean local relief appeared to be a control on erosion rate in the Olympic Mountains (Montgomery and Brandon, 2002).

Additional analyses of mean local relief and erosion rates in other tectonically active areas had similar results. Montgomery and Brandon (2002) evaluated mean local relief against erosion rate in several tectonically active areas that included the Ganges and Bramaputra River systems, India, Taiwan, the British Columbia Coast Range; the Denali portion of the Alaska Range; and the New Zealand and European Alps. They found that erosion rates did not support Anheert's (1970) linear correlation to relief. Erosion rates in Montgomery and Brandon's (2002) studies were based on published rates derived from sediment yields, bedrock river incision rates, and low-temperature thermochronometry. In tectonically active areas, erosion rates plot higher (i.e., the erosion rate is greater) than Anheert's relation.

Slope Stability, Threshold Hillslopes, and Rock Strength

Relief has also been used to understand slope stability and threshold hillslopes. Relief was defined in terms of uplift and erosion without consideration of rock strength in early evaluations (e.g., [Davis, 1899]). Large-scale assessments of the relief of particular landforms (e.g., [Fielding et al., 1994]) did not focus on the role of rock strength in generating topographic relief. Carson and Petley (1970) were among the first teams to empirically evaluate how tectonics, erosion, and lithology contribute to slope profiles.

While Carson and Petley (1970) did not relate slope directly to relief, the concept that lithology affects slope, and thus relief over distances (or windows) is a natural corollary addressed by Schmidt and Montgomery (1995). Schmidt and Montgomery (1995) extrapolated the idea of lithologic control on relief to mountain-scale material strength by generating a model for bedrock landsliding in order to predict the maximum size of stable hillslopes or mountain fronts. Their model incorporates bedrock material strength, where hillslope height is used as a proxy for relief. However, relief is defined as the change in elevation along features, such as valley walls and stream banks and is, therefore, examined on the scale of tens of meters.

Climate and Relief

Relief has been used to study how climate influences topography. Gabet et al. (2004) calculated relief as the maximum elevation minus the minimum elevation over 5-km-diameter circular windows. The average rainfall for various elevations was compared to relief. By looking at annual rainfall and hillslope angles in the Nepalese Himalayas, they proposed a mechanistic relationship between climate and hillslope relief. Generally, with increased rainfall, hillslope angle decreases. Drier conditions allow slopes to steepen and lengthen, while increased rainfall decreases slope stability and results in the removal of topography. Gabet et al. (2004) found no consistent relationship between lithology and mean hillslope angle, although lithologic differences may be responsible for the general distribution of hillslope angles.

Roe et al. (2003) also investigated how climate influences the form and evolution of mountain ranges. They defined relief as the depth of river valleys within mountains. Relief increases as valleys erode. The pattern and amount of precipitation distributed over a given river basin are important in setting the relief of the main river channel within that basin (Roe et al., 2003).

Relief has also been used to study the influence of climate on tectonics. Whipple et al. (1999) investigated whether climate change can significantly increase topographic relief. Relief is defined by Whipple et al. (1999) as a proxy for the volume of missing mass between summits and ridges; therefore, relief can

be increased only by a decrease in drainage density, assuming a condition of threshold hillslopes. Erosion is concentrated in the upper parts of the drainage basins as channels cut down farther into the valleys. Relief, therefore, increases when upstream channel reaches steepen and trunk channels incise (Whipple et al., 1999).

While studies of specific areas seem to indicate that climate influences relief, the global spatial distribution analysis of mean local relief performed by Montgomery and Brandon (2002) showed that tectonically active mountain ranges have a mean local relief of 1000-2000 m, independent of the specific tectonic context or climate zone. Neither a relation nor a structural/geomorphic control, however, was hypothesized to explain the observation of a relatively consistent mean local relief across diverse topographies.

Models and Measures of Relief

Existing models often use relief as a proxy for other parameters, such as slope or height. Wood and Snell (1960) define the “grain” of the topography as the relief. They measured the topographic local relief of land areas within concentric circles, then plotted the relief with the circle diameter, and found the diameter beyond which there was little or no relief change. The diameter was used to symbolically represent the relief.

Relief has also been evaluated in terms of a hypsometric curve. The hypsometric curve approach examines elevations in drainage basins and is plotted with a drainage basin height on the y-axis and the area above the corresponding basin height on the x-axis. As elevation increases, the total area of higher elevation found within the drainage basin decreases. The hypsometric curve is typically used to assess the similarity of drainage basins (Strahler, 1952). However, some researchers have lingering doubts about the interpretation of the hypsometric curve (Carson and Kirkby, 1972). Issues with the landform classification schemes proposed by Strahler (1952, 1964) were noted by Willgoose and Hancock (1998) and included the scale dependence of the hypsometric curve, where the same landform viewed at different scales under Strahler's schemes yield different hypsometric curves.

More complex three-dimensional models have also been used to describe relief. Fractal Brownian surfaces have been compared to theoretical topography (Mandelbrot, 1977; Goodchild, 1980; Goodchild, 1982; Fournier et al., 1982; Mandelbrot, 1982). Fractal Brownian surfaces may resemble topographic landscapes, but they are not related to geomorphic processes. A model by Mark and Aronson (1984) compared fractal Brownian surfaces to real topography in 17 sample landscapes. The maximum circular area that could fit within a USGS DEM of 30 m scale was defined. Over 32,000 pairs of independent, randomly located points were chosen within the circle, and the distances and elevation

differences of the topography between each pair was calculated. Distances ranged from zero to the circle's diameter. The distances were then divided into equal classes and the variance of the elevation differences was calculated for each class. A log-log plot of the variance against the distance was generated.

For 16 of 17 plots, the topography had statistical properties similar to fractal Brownian surfaces, but with a change in dimensions of the surface at different scale ranges. The majority of plots showed straight segments divided by distinct breaks. The breaks were located at approximately 5 to 6 km distances, which were noted by Mark and Aronson (1984), but no geologic or geomorphic explanation for the distance between breaks was provided.

Limitations of Relief

A major limitation of relief is its dependence on the scale at which it is calculated. Shreve (1979) recognized that studies of geomorphic processes are largely related to scales of interest. While over 30 years have elapsed since Shreve's insight was published, the absence of the link between scales and relief remains a deficiency in the field of geomorphology.

A second limitation of the current use of the term "relief" is that it cannot intrinsically differentiate the role of landscape form (i.e., a smooth hill versus a jagged peak) in overall relief (Fig. 2), nor can it differentiate how small-scale

Figure 2. Representation of three generalized landscapes. Landscape B could have the greatest relief if evaluating relief strictly as the difference in elevation. Landscape A could have the greatest relief if considering the hill slope angle and width. Landscape C could have the greatest relief if considering the amount of material eroded from an initially planar surface.

relief is related to landscape-level relief. Whipple et al. (1999) recognized that there are several components of relief, including hillslope relief, trunk channel relief, and tributary relief; however, a definition of relief that unites and considers each of these components does not exist. Some attempts at a landscape-level analysis can be found in the literature (e.g., Fielding et al. 1994). The methods, while scientifically reasonable, were defined as convenient and appropriate for the analysis of the Tibetan plateau but not necessarily for other landscapes. The current practice of arbitrarily choosing a measurement and definition of relief may not be representative of the actual geomorphic processes and dynamics of the landscape that act on varying scales.

Fractal comparisons that quantify the shape of landforms (Mark and Aronson, 1984; Pal, 2008; Mandelbrot, 1977) are complex. Fractal models are primarily concerned with the scales at which landscapes exhibit roughness of a certain magnitude and the signals that these scales represent (e.g., tectonic patterns and climatic patterns). While fractal analysis approaches the questions surrounding scalability of landscape features, it has not been tied back to definitions of relief. Understanding the role of relief at different scales is important to allow for more meaningful comparisons for any research topic in which relief is a parameter (e.g., erosion, climate change, and crustal processes). A scale-comprehensive approach to the analysis of relief would therefore allow for a broader understanding of the factors that affect relief (or vice versa), such as

rock properties, watershed properties, erosion, and tectonics. It also provides insight into the scalability of relief within and across landscape features.

As described in this paper, I performed a scale-comprehensive evaluation of relief through evaluation of digital raster data sets (DEDs) over multiple window sizes. I then derived a function that describes the change in relief over the landscape as the window size increased, in order to understand how small-scale relief may relate to larger scale relief. The analysis, therefore, captured the components of the landscape that manifest at different scales, such as a stream channel bed at a small window size, the difference between mountain peaks and valleys at a moderate window size, and whole mountain ranges at a larger window size. These features when presented as a single function are like a simplified fractal analysis. The function was then compared across different landscapes of varying relief. The relief measured at individual window sizes was evaluated to determine if it could accurately portray the relief of a landscape or allow an accurate comparison across landscapes. The inherent similarities in all landscapes were also examined by evaluating the change in relief as the window size changed.

METHODS

Data Sets

DEDs were obtained primarily from the U.S. Geological Survey (USGS) Seamless server. I chose one to four landscape samples, defined by rectangular areas ranging from 210 km² to over 2500 km² from each of the 24 terrestrial geophysical provinces across the continental U.S. and for a total of 38 landscapes. The average landscape area was 963 km². Areas with high topography were chosen where possible, as well as landscapes with unique geomorphic features, such as Mt. Whitney, Mt. Washington, Zion Canyon, Mt. St. Helens, and Yosemite Valley. The selected DEDs cover a single topographic unit in most instances, such as one mountain range or one river valley. Additionally, six Hawaiian Islands and a section in southeastern Puerto Rico were sampled for a total of 45 sample sites. The 45 DEDs (also called “landscapes”) are presented by geographic province in Figures 3 and 4.

Focal Statistics

The DED analysis was based on a commonly used window method (Anhert, 1970; Fielding, et al., 1994; Sobel et al., 2003; Gabet et al., 2004; Clark, et

Figure 3. DEDs sampled across the 24 geomorphic provinces of the continental U.S. DED sizes vary from 210 km² to over 2500 km². Actual DEDs are represented in the figure.

Figure 4. DEDs sampled across the Hawaiian Islands and Puerto Rico.

al., 2006;). The window method defines relief by calculating the difference between maximum and minimum elevations over constant, equal areas (i.e., a fixed sliding window). ArcGIS's focal statistics are used to subtract the "focalmin" from the "focalmax" for a given window size. Focalmax gives the maximum value, and focalmin gives the minimum value from a set of pixels within a specified range. The range can be specified as either a circle of radius X or a rectangle of sides Y, Z. A new raster data set is generated when focal statistics are calculated. Each pixel of the new data set, within the defined window size, is assigned a value calculated as the difference between the maximum value and the minimum value of all pixels within that window size. An example of how focal statistics are applied is presented in Figure 5.

The DED analysis for this study employed the window method; however, the window method was repeated several times for each landscape using varying sized windows. Window sizes were increased from 2 pixels by 2 pixels up to a window size large enough to capture most of the relief across the entire DED or a maximum window size of 300 pixels by 300 pixels. Window sizes larger than 300 pixels by 300 pixels required too much data processing time to complete. Each pixel represented 30 m for most of the data sets used in this study (i.e., a window of 2 pixels by 2 pixels represented a 60 m by 60 m square area). The maximum difference in elevation for each window size was plotted against the window size for each landscape, resulting in a curve that represented

Figure 5. Schematic demonstrating the focalmax neighborhood function. Focal statistics is a function within ArcGIS. The diameter of a circle or two sides of a rectangle is defined. Focalmax evaluates all cells within the circle or rectangle and assigns the central pixel that value. Focalmin assigns the central pixel the minimum value.

the relief of the landscape at multiple scales.

Additional methodologies were used to calculate the focal statistics in order to assess the effects of the methods on the results. Lower resolution DEDs, circular versus square windows, a fixed instead of a moving window, and varying landscape extents over the same location were all used to test the methodologies. A subset of landscapes was used for each methodology, including Blue Ridge, Pacific Coast-Mt. Diablo, Ozarks, Central Plains-Iowa, and Cascade Sierra-Mt. Whitney. These landscapes were chosen because they spanned the range of topography seen in the overall data set.

The effect of the resolution of the DED on the curve was evaluated by resampling the raster data sets from a 30 m to a 90 m resolution for the five representative landscapes. The window method is typically performed using circular windows; however, rectangular windows were used for this analysis for faster processing time. Rectangular windows provide slightly different results than circular windows of the same radius due to inclusion of edges not found within a circle of the same radius. A fixed versus a sliding window was also analyzed where the relief was calculated for one window centered on a chosen point. The window size was increased, and the relief was calculated. This method showed how the elevation changed from one particular point as the area surrounding the point was enlarged. Finally, the analysis of the DEDs of variable extents was performed in order to understand the dependency of the

results on the size of the DED chosen. Various sized 30 m DEDs surrounding Mt. Diablo were analyzed using rectangular windows.

RESULTS

Graphical Analysis and Curve Fitting

Several functions were considered to fit the window size versus maximum relief curves for each landscape, including exponential, log, and split linear functions. The exponential function did not allow for the leveling off of maximum relief as was seen in the data. Log functions fit the data well with R^2 values mostly between 0.95 and 0.99 for the curves; however, the best-fit log function predicted negative relief for small window sizes that has no physical meaning. Linear functions fit to portions of the curve required a subjective selection of endpoints that inhibited reproducibility.

The saturating exponential function best fit the curve. The saturating exponential function predicted the relief at a window size of zero to be zero and allowed the maximum relief to level off as window sizes were enlarged. In the saturating exponential function

$$y = R_{\max} (1 - e^{-kw}) \quad (1)$$

based on the derivative

$$\frac{\delta R}{\delta w} = k(R_{\max} - R)$$

where $R_{\max}$ is the maximum relief of the maximum window size, R (or y in equation 1) is the maximum relief at any given window size, k is a constant scaling function, and w is window size. The resolution of the DED must be factored in to the window size (i.e., the factor w for a DED with 90 m resolution must be multiplied by 3 to be appropriately compared to the analysis of a DED with 30 m resolution). A goodness-of-fit calculation was made by calculating the difference between the maximum relief of the actual and the modeled curve for each window size, and then by calculating the average of the difference for all windows for a landscape. The error is expressed as a percent by dividing the average error by the maximum relief of the largest window. This calculation gave a general measure of how well the saturating exponential function fits the calculated curve. Appendix I includes the measured curve, the saturating exponential model curve, and the goodness-of-fit calculations for each landscape.

k Values

The k value was calculated for each curve by minimizing the percent error between the modeled curve and the actual curve in the goodness-of-fit calculation, as previously described. The k values were then plotted against the maximum relief for each landscape (Fig. 6).

Figure 6. The k values for each of 44 sampled DEDs (landscapes). The St. Lawrence River Valley is not shown as it has a k value of 0.2 and skews the presentation of the results. Numbers match the ID presented in Table 3.

The k values calculated using the alternative methodologies are summarized in Table 1, and the graphs and raw data are provided in Appendix II. Table 2 summarizes the k values for the varying sizes of DED covering the same area. The k values were relatively consistent for all methods except for the fixed-window method.

The resulting k values for the five circular windows were nearly the same as for the square windows, suggesting that the shape of the window does not have a significant influence on the results. A circular window weights the results across a given radius (r) versus a square with a side $2r$. The square includes an area approximately $(4 - \pi) * r^2$ greater than the circle. Nevertheless, because the window slides across the entire landscape, this difference in weighting did not seem to make any significant difference in the overall analysis. Where the resolution of the DEDs was varied and the k values compared, the k values were the same for 30 m and 90 m DEDs of the same landscape. A 90 m DED has the same maximum elevation at a window size of 1 that the 30 m DED has at a window size of 3. Results showed that the analysis is not sensitive to the resolution of the DEDs for 30 m and 90 m resolutions. The majority of the curve forms over window sizes up to 3000 m in the horizontal plane (the ground surface). The error in the curve is similar, even though fewer data points are available in the 90 m resolution DEDs. Lower resolution DEDs provide an advantage over the higher resolution DEDs in terms of processing speed using

Table 1. The k values for the alternative methodologies tested.

	k Value				
	Pacific Border – Mt. Diablo	Blue Ridge	Central Lowlands - Iowa	Cascade-Sierra – Mt. Whitney	Central Lowlands – Iowa
Rectangular window (principal study)	0.025	0.013	0.027	0.015	0.027
Circular Window	0.022	0.013	0.026	0.015	0.026
90 m DED	0.022	0.014	0.026	0.015	0.026

Table 2. Summary of k values for various sized landscapes around Mt. Diablo, Pacific Coast Border.

Mt. Diablo Landscape	k value
544 km ² (23.3 km x 23.3 km)	0.012
210 km ² (14.5 km x 14.5 km)	0.025
19.6 km ² (4.43 km x 4.43 km)	0.012
3.73 km ² (1.9 km x 1.9 km)	0.012

ArcGIS.

The results of the fixed versus sliding window analysis were somewhat surprising. The saturating exponential curve generally did not fit the data for the fixed-window analysis. Only one window is generated per window size when using a fixed-window approach. The entire landscape within the defined area is analyzed at every window size by using the sliding-window approach. Effectively, a much smaller area is analyzed using the fixed-window approach. The results of the sliding-window analysis suggest that it is not only the changes in maximum elevation over a given window size that matters, but also the overall area or “landscape” that is analyzed. In one given location, the maximum elevation change is not likely to be realized for a single window size. It is only by looking at many windows of the same size over a larger feature or “landscape” that the maximum change can be found.

The results suggest that, within any given landscape, even an expanding window analyzing a single location is not necessarily a good indicator of the overall relief that characterizes that landscape. These results seemed to raise questions about the size of the landscape that should be analyzed and whether or not results differ dramatically if different sized landscapes are assessed.

Landscape sizes in the main analysis varied from 210 km² to over 2500 km² with an average of 963 km²; however, each landscape evaluated was unique. The analysis of overlapping extents in the same area was performed to

understand the effect of landscape size on the resultant curve. The Mt. Diablo area within the Pacific Coast geomorphic province was reassessed with different sizes of overlapping DEDs ranging in area from 3.73 km² to 544 km². The results all fit a saturating exponential function with fitting errors of 2 to 3 percent. The principal landscape chosen to represent Mt. Diablo was 210 km² and had a k value of 0.022 with a fitting error of 4 percent. Both of the smaller landscape extents (3.73 km² areas and 19.58 km²) had k values of 0.012. The largest landscape extent with an area of 544.5 km² had a k value of 0.009. While the k values varied depending upon the extent of the DED, directionality of the variation was not seen (i.e., k values did not vary based on size of the DED sampled). The 210 km² area had the largest k value of 0.022. The largest extent at 544.4 km² had the smallest k value. The smallest extents, which covered the peak of Mt. Diablo, had intermediate k values of 0.012. Since all higher relief landscapes of the principal analysis had k values that ranged from 0.005 to 0.033, a variation of 0.013 is fairly large; however, the variation seen in the differing landscape extents for Mt. Diablo still falls within the range of all other higher relief landscapes.

The resultant k values for the principal study of the 45 landscapes are presented in Figure 6 and summarized in Table 3. The k values ranged from 0.005 to 0.033 for most landscapes and for all landscapes with a maximum relief above 300 m with two exceptions. Zion Canyon and Yosemite Valley each had k

values of 0.075. The k value varied from 0.02 to 0.2 for landscapes with maximum relief below 300 m. The median k value was 0.024, and the mode was 0.025. Statistical values are summarized in Table 4.

DISCUSSION

The Saturating Exponential Function

The saturating exponential function was originally developed to calculate diffusion coefficients. After an initial adjustment period, the concentration of the diffusing substance in the surrounding medium increases according to the function

$$y = y_{\infty} (1 - e^{-k(x+\delta)}) \quad (2) \text{ (Dibdin, 1988)}$$

where δ is an unknown interval, x is the time, and the concentration y rises to an asymptote y_{∞} as $x \rightarrow \infty$. The exponential constant k describes a function where, as the time x increases, $y_{\infty} - y$ decreases linearly. The landscape relief analysis fits the saturating exponential function for every landscape tested, where the variable x , time, is replaced by w , window size. As window size increases, the curve approaches an asymptote, or the maximum relief seen in the sample landscape extent, and relief at a zero window size is thus zero. The odds of finding a value

Table 3. Summary data for the 45 landscape DEDs used in the analysis.

ID	Geomorphic Province	Landscape County, State or Feature Name	Size of DED (km ²)	Max Relief for DED (m)	<i>k</i> value	Fitting Error (%)
1	Adirondacks	Essex County, NY	163	1057	0.021	3
2	Appalachian Plateau	Venango County, PA	380	240	0.009 ₅	5
3	Basin and Range	Clark County, NV Spring Mountains	291	1984	0.015	6
4		Lander County, NV	238	2953	0.016	5
5	Blue Ridge	Jackson, Swain, Hayward Counties, NC	256	1286	0.017	6
6	Cascade-Sierra Mountains	Yakima, Skamania, WA Mt. St. Helens	465	2717	0.009 ₅	3
7		Alpine, El Dorado Counties, CA Kirkwood	229	1890	0.015	7
8		Mariposa County, CA Yosemite Valley	127	1519	0.075	7
9		Fresno, Tulare, Inyo, Counties CA Mt. Whitney	1432	2583	0.015	7
10	Central Lowlands	Alcona County, MI	167	142	0.032	9
11		Ida County, IA	526	114	0.027	9
12	Coastal Plains	Oceola County, FL	349	37	0.09	9
13		Lincoln and Union Counties, LA	312	64	0.1	6
14	Colorado Plateaus	Mesa, Garfield Counties, CO	729	865	0.025	9
15		Navajo County, CO	463	1869	0.015	8
16		Coconino County, AZ Grand Canyon	332	1650	0.025	8
17		Washington County, UT Zion Canyon	148	1234	0.075	6
18	Columbia Plateau	Wallowa, Union, Baker Counties, OR	686	1611	0.025	4

ID	Geomorphic Province	Landscape County, State or Feature Name	Size of DED (km ²)	Max Relief for DED (m)	<i>k</i> value	Fitting Error (%)
19	Great Plains	Edwards County, KS	1820	65	0.02	9
20		Custer, Pennington Counties, SD Black Hills	238	871	0.017	8
21	Interior Low Plateau	Breckinridge, Grayson Counties, KY	311	134	0.075	7
22	Lower California	San Diego County, CA	259	990	0.022	5
23	Middle Rocky Mountains	Park County, WY	811	1746	0.015	7
24	New England	Cheshire County, NH	438	438	0.025	12
25		Coos County, VT Mt. Washington	264	1439	0.016	4
26	Northern Rocky Mountains	Idaho County, ID and Ravalli County, MT	827	1745	0.02	6
27	Ouachita	Le Flore County, OK	174	490	0.03	5
28	Ozark Plateaus	Madison County, MO	187	298	0.065	3
29	Pacific Border	Josephine, Curry Counties, OR	288	1475	0.025	3
30		Contra Costa County, CA	130	1040	0.025	3
31	Piedmont	Cherokee County, SC	274	182	0.05	6
32	Southern Rocky Mountains	Pitkin, Chaffee, Gunnison, Lake Counties, CO Aspen	909	1593	0.023	3
33		San Juan County, CO	393	1656	0.02	6
34	St. Lawrence Valley	St. Lawrence County, NY	357	72	0.2	6
35	Superior Upland	Ontonagon County, MI	224	230	0.07	7
36	Valley and Ridge	Huntington, Mifflin, Centre Counties, PA	291	529	0.03	4
37		Tazwell, Smythe Counties, VA	414	761	0.03	4
38	Wyoming Basin	Carbon County, WY	323	802	0.018	1
39	OTR	Puerto Rico	230	1145	0.023	5

ID	Geomorphic Province	Landscape County, State or Feature Name	Size of DED (km ²)	Max Relief for DED (m)	<i>k</i> value	Fitting Error (%)
40	OTR	Maui	1865	3050	0.01	4
41	OTR	Hawaii	6482	4198	0.005	9
42	OTR	Oahu	966	1224	0.018	8
43	OTR	Kauai	1001	1598	0.018	8
44	OTR	Molokai	418	1506	0.033	6
45	OTR	Lanai	227	1024	0.022	5

Table 4. Statistics for *k* values

	Total Data Set	Less than 300 m relief	Greater than 300 m relief
Count	45	10	33
Mean <i>k</i> Value	0.036	0.062	0.020
Std Deviation	0.035	0.029	0.006
Median <i>k</i> Value	0.024	0.068	0.020
Mode <i>k</i> Value	0.025	--	0.025

of greater relief decreases linearly as the window size increases and the maximum relief is approached, defined by k . The saturating exponential function has two sources of error that may impact the calculation of k values (Dibdin, 1988). The curve fit is sensitive to the choice of the maximum value of y_{∞} for a diffusing substance (equation 2). However, for the multi-scale analysis y_{∞} is replaced by $R_{\max}$ (equation 1), which has a definitive value for each landscape based on the DED extent. The second source of error for the saturating exponential function is in the transformation from the saturating curve to the logarithmic linear regression from which k is calculated. The scale is compressed where most of the change in the dependent variable occurs (i.e., at low x values for equation 2), while it is expanded where y approaches y_{∞} and the fractional error is large (i.e., at high x values for equation 2) (Dibdin, 1988). This condition, however, was overcome by visually fitting the curve to the point data and by obtaining more point data within the lower range than the upper range of the saturating exponential function.

The k value for all but two landscapes with maximum relief greater than 300 m was between 0.005 and 0.033, and it was between 0.005 and 0.2 for all landscapes. The approximation of the landscape relief curves to the saturating exponential function had fitting errors between 2 and 9 percent in all 45 samples. The narrow range in k values for all landscapes suggests that most landscapes

across a fixed horizontal plane (the x , y -dimension) are scales of each other in the vertical plane (z -dimension) (Fig. 7).

The “ z -scaling” hypothesis was tested by analyzing two landscapes, Mt. Whitney, California and Mt. Washington, New Hampshire, which have the same k value but different maximum relief. The relief of Mt. Washington was scaled to the relief of Mt. Whitney (i.e., multiplying the z values of the DED with the scaling factor); then the multi-scale window analysis was run again. The resultant saturating exponential curves are similar, indicating the landscapes are indeed scales of each other in the vertical plane (Fig. 8).

Range of k Values

Geophysical provinces across the U.S. exhibiting the lowest maximum relief include the St. Lawrence River Valley, the Coastal Plains, the Appalachian Plateau, the Interior Low Plateau, the Superior Uplands, the Ozarks, the Piedmont, the Central Lowlands, and the Great Plains. The k values in these areas, however, were found to have greater variability than landscapes with maximum relief greater than 300 m. One possible explanation for the greater variation in k values over low relief landscapes is the dependence of k on the proportionate change in relief as window size increases. With limited maximum relief, small changes in relief over relatively short distances can represent a large

Figure 7. Schematic of landscape scaling in z-dimension (height).

Figure 8. Saturating exponential function analysis of a scaled landscape. The elevations for the Mt. Washington DED were multiplied by 1.8, such that the maximum relief for Mt. Washington equaled that of Mt. Whitney. The focal statistics were run on the new DED for each window size, and the results plotted. The k value for both Mt. Whitney and the scaled Mt. Washington is 0.015. The analysis shows that the landscapes are scales of each other in the vertical plane (z-dimension) over the same horizontal plane (x,y-dimension).

proportion of the overall relief. For example, the maximum relief in the sample landscape within Florida (Coastal Plains province) is 37 m. Florida is a large plateau that is minimally above sea level. Peninsular Florida, which contains extensive swampland, has a relatively elevated central spine of rolling landscape, dotted with lakes and springs (Gall and Gall, 2004). At a window size of 2 pixels and on a grid with 30 m resolution (60 m across a horizontal plane), the maximum relief was found to be 15 m or approximately 41 percent of the total relief seen in the landscape. For a landscape with a maximum relief of 300 m, the equivalent proportion would be an elevation change of 123 m over 60 m across a horizontal plane. A 15 m change in elevation over 60 m would represent a rolling topography (as seen in the central spine of the state) with a 14 degree angle of incline. Due to the minimally varying topography, a large proportion of the total relief can be reached over a limited horizontal distance, potentially skewing the k value upward. In contrast, for a landscape with a maximum relief of 300 m to have 41 percent of the relief realized within a 60 m horizontal plane (a two-pixel window), the slope of the landscape would have to be 64 degrees, requiring a cliff face or series of cliff faces totaling 123 m. It is probably not geomorphically possible for a landscape with 300 m of total relief to have 123 m of relief realized within 60 m of horizontal distance (Fig. 9). The k value for the Central Florida DED is 0.09, and no landscapes with greater than 300 m of relief have k values that high, not even Zion Canyon with its vertical sandstone cliffs (k

Figure 9. Topographic noise within low relief landscapes. Topographic noise can explain the occurrence of a larger range of k values. Landscape a) is a realistic scenario representing the topography within Central Florida, while the topography in b) has the same k value as Central Florida but is not likely realistic given the high angle of slope or large cliff faces required. No landscapes over 300 m had k values higher than 0.075 (Zion Canyon and Yosemite Valley).

= 0.075). The k values for low relief landscapes have greater topographic noise, which may explain why a larger range in values is observed.

Landscapes with maximum relief greater than 300 m have k values with a narrower range of variability that ranged between 0.005 and 0.033. As with the low relief landscapes, the change in maximum elevation per window is proportional to the overall maximum relief. For example, the Black Hills (South Dakota), and Mt. Whitney (California) have similar k values at 0.017 and 0.015, and maximum relief of 871 m and 2583 m, respectively. Within a two-pixel window, the Black Hills exhibits a maximum elevation change of 100 m or 11 percent of the total elevation change, while Mt. Whitney exhibits a 319 m elevation change or 12 percent of the total elevation change (maximum relief). Due to the higher average total elevation, the data have less topographic noise for high relief landscapes than for low relief landscapes, which can explain the narrower range of k values.

Correlation Between k Values and Landscape Features

Landscape evolution is a result of tectonic uplift, countered by down-cutting by erosion and mass movement processes (Willgoose and Hancock, 1998). The basic landform processes acting on low relief landscapes are different from those of the high relief landscapes. Low relief landscapes are described as

transport limited where erosion cannot act as quickly to remove soils, and the landscape is therefore mantled by soil. Transport-limited landscapes are typically found in areas with high weathering rates (e.g., hot and wet climates) and with low erosion rates (e.g., flat to rolling topography). For transport-limited landscapes, landform modification can occur through fluvial transport but is dominated by mass-wasting mechanisms, such as soil creep and rainsplash (Willgoose and Hancock, 1998).

The profile of a typical transport-limited landscape was initially explained by Gilbert (1909). Smith and Bretherton (1972) quantified Gilbert's model through a linear stability analysis. Valleys form where runoff converges and erosion outpaces infilling. Transport-limited mass-wasting processes can result in only minor modification to the topography over large areas. The existence of a fluvial feature within the landscape can have an effect on the k value of a low relief landscape. The change in relief from the fluvial feature occurs over a relatively short distance but represents a large proportion of the overall change in relief, skewing the k value upward. Where only diffusive mechanisms, such as soil creep occur, the change in relief over a small window is gradual, and the k value is small. Other low relief landscapes may have low k values where no fluvial feature is found within the sampled DED, such as the Central Lowlands landscape of Michigan or Iowa (Fig. 10).

High relief landscapes are a result of tectonic activity and erosion.

Landscape	Window Size [†]	y (units)	R_{max} (units)	k
L_1	20	10	60	0.009
L_2	20	50	60	0.090

[†] Vertical not to same scale as horizontal - approximately 3:1 vertical to horizontal

Figure 10. A potential explanation for the variation in k values for low relief landscapes. L_1 has no fluvial feature within the landscape extent. A change in relief over a small window size is a small proportion of the overall change in relief of the landscape, resulting in a small k value. L_2 has a fluvial feature; therefore, a large proportion of the overall change in relief is seen over the small window, resulting in a higher k value.

Features with greater than 300 m of relief may be weathering limited (also known as source limited) since weathering-limited landscapes are more common where slopes are steeper.

Modeling of weathering-limited transport has been completed by Howard (1994), including the modeling of fluvial processes of steep sloped terrain and of mass movement including slope failure. The variation in k seen for landscapes with a maximum relief greater than 300 m is related to the incision of the landscape. The smallest k values represented volcanic landscapes that have experienced limited incision, potentially due to their relatively young age, such as the islands of Maui and Hawaii. The landscapes with relatively higher k values appear to be associated with the downcutting of uplifted plateaus, such as the Colorado Plateau (Colorado, Utah, and Arizona) and the Columbia Plateau (Washington). Where k values are larger, the maximum relief over the smallest window sizes is greater in proportion to the total relief; that is the landscape has “cliff-like” features most likely formed by river or glacial erosion or even large-scale landslides.

Extreme examples that demonstrate the relationship between landscapes with cliff-like features and relatively high k values include Zion Canyon (Utah) and Yosemite Valley (California). The k value for both of these landscapes is 0.075. Zion Canyon is part of the Markagnut Plateau. The canyon was carved by the North Fork Virgin River and is predominantly comprised of Navajo

sandstone, a 400- to 600-m-thick sequence of Middle Jurassic age (Prothero and Schwab, 2004). The walls of the canyon are as much as 900 m high. Likewise, Yosemite Valley is an impressive steep-walled, U-shaped, relatively narrow canyon through which the Merced River flows. The Yosemite Valley is as deep as 1500 m near Half Dome and approximately 900 m deep in other areas. The granite of Yosemite Valley also forms steep cliff-faced walls, such as those of El Capitan. The valley is believed to have been formed, at least in part, by glacial erosion (Beatty, 1943). The vertical cliffs of both Zion Canyon and Yosemite Valley are characterized by a large change in relief over a very short distance that gives rise to larger k values. The downcutting of a relatively planar initial surface can result in disproportionately steep slopes compared to the overall maximum elevation change of the landscape (Fig. 11).

Newly created surfaces, such as volcanoes, can have a high landscape-level maximum relief or elevation, but they are not yet highly incised by erosional forces; therefore, they have less-steep micro-topography within the overall landscape feature. These landscapes have slightly lower k values. The lowest k values were represented by Hawaii, Maui, and Mt. St. Helens (Washington), all volcanic features. The relatively narrow range in k values across all landscapes with greater than 300 m of relief, however, suggests that the overall maximum elevation and the micro-elevation changes are in some way correlated.

a)

b)

Figure 11. High k value relief demonstrated at Zion Canyon. Zion Canyon has a high k value that can be related to the cliff-like features and the deep incision of the valley, as shown in the (a) photograph and (b) cross section through the valley. Photo by Tobias Alt, Creative Commons Free License.

The Hawaiian Islands allow for a direct comparison of relief at each window size, overall relief, and k value (Fig. 12). All of the Hawaiian Islands are composed of similar rock types, but depending upon age, they have undergone different amounts of erosion. Uplift rate is lower farther away from the volcanic hotspot (Kious and Tilling, 1996). Hawaii and Maui, the two youngest islands, have the lowest k values at 0.005 and 0.01, respectively, and the greatest overall maximum relief of any of the islands; further, Hawaii has over 4250 m of elevation change over the whole island. Kauai and Oahu, the oldest islands, have experienced the most erosion. These two islands both have higher k values ($k=0.018$) than Hawaii and Maui, but they also have a considerably lower maximum overall relief. As the islands erode, the k value increases while the overall relief decreases. Erosion acts to increase relief at the smaller or micro-scale, consistent with the hypothesis of Small and Anderson (1998). On the larger landscape-scale, however, Hawaii has greater relief.

Evaluation of Landscape Evolution using the Multi-Scale Model of Relief

The dependence on an arbitrarily chosen scale for most studies where relief is used as a parameter raises concerns regarding the validity of those comparisons. The multi-scale approach to defining relief provides insight into the relationship of the scale-related components of relief. This section revisits the

Figure 12. The k values versus maximum relief for the four major Hawaiian Islands. Ages of islands are indicated in millions of years. Hawaii has the smallest k value and has experienced the least erosion. Kauai and Oahu have the largest k values and have experienced the most erosion.

existing studies and discusses how relief at the multi-scale level influences different comparative parameters, such as erosional processes, rock strength, and climatic effects on relief.

Influence of Erosional Processes on Landscape Form

The rate of change of elevation of a point on the landscape is equal to the difference between the local divergence of sediment flux and the uplift rate (U)

$$\frac{\Delta z}{\Delta t} = U - \left(\frac{q_{s \text{ in}} - q_{s \text{ out}}}{\Delta x} \right)$$

where $q_{s \text{ in}}$ is the flux of sediment in, $q_{s \text{ out}}$ is the flux of sediment out, Δx is an interval of distance along the hillslope, and $\Delta z/\Delta t$ is the change in elevation.

Erosive forces generate a feedback loop of isostatic rebound due to mass removal, steepening slopes, and more erosion.

The studies performed by Small and Anderson (1998) compared relief to erosion rates at summit peaks and valley bottoms that can be thought of as analogous to the evaluation of relief at a single window size around a single point (versus a sliding window). The multi-scale analysis suggests that such an assessment may not accurately portray the elements of the overall landscape that define relief via the k value. One valley may not be representative of the forces that influence the overall landscape in the region as demonstrated by the fixed-point analysis of Mt. Diablo. The assessment using a single window size may,

however, provide insight into the dynamics of the processes that form particular elements within the landscape (i.e., components of the saturating exponential function for the multi-scale window analysis). The purpose of Small and Anderson's (1998) study is to discern the erosion rate differences between peaks and valleys, which is a valid study of erosion versus relief. It is, however, important to understand the overall landscape dynamics that may not be represented at a single scale. Similarly, for the case of the Hawaiian Islands, the small-scale relief on the island of Kauai is greater than the small-scale relief on Hawaii. The maximum relief at a two-pixel window on Kauai is 465 m but only 333 m on Hawaii. Hawaii, however, has greater overall relief.

The effect of erosion on relief for a volcanic area, such as Hawaii, is overall landscape-level relief decreases as small-scale relief (i.e., between ridges and valleys) increases. As the overall relief of the landscape decreases, the k value increases (Fig. 13). This circumstance may be unique to the Hawaiian volcanic hotspot (Kious and Tilling, 1996). Erosion and isostatic rebound are acting to form the landscape, and erosion rates exceed uplift rates as the islands move farther away from the hotspot. The k values may, therefore, increase for landscapes where uplift rates are less than the erosive rates. However, at some time, the micro-relief should smooth as valleys widen, which should cause a decrease in k value. Alternatively, as the landscape decreases in overall relief, the variability in the k value increases.

Calculation of k value of hypothetical landscape

Time	Window Size (units)	y (units)	$R_{\max}$ (units)	k
t_1	0.5	0.9	5.6	0.199
t_2	0.5	0.7	7.4	0.350
t_3	0.5	1.5	2.5	1.833

b)

Figure 13. Erosional effects on k values for a model Hawaiian landscape. a) Schematic of erosional effects on k value for a generic topography analogous to the Hawaiian Islands. Greater amounts of total erosion lowers the overall relief from time 1 (t_1) to time 3 (t_3) but increases the k value through increased micro-relief. b) Calculation of the k value of the hypothetical landscape. The landscape is not to scale; therefore, unrealistic k values are calculated; however, the calculation demonstrates the point that the k value increases from t_1 to t_3 .

Montgomery and Brandon (2002) performed a large-scale analysis of relief compared with erosion rates to evaluate the relationship between mean local relief and erosion rates. They used 10-km windows, which are equivalent to the approximately 33-pixel window size in my study. The multi-scale analysis shows that a single window size can be representative of the entire relief curve and can be compared to other landscapes provided a consistent k value. This comparison deteriorates in deeply incised landscapes, such as Zion Canyon or Yosemite Valley where higher k values are seen. For example, with a k value of 0.075, Yosemite Valley has a maximum relief of 1431 m at the 30 m sliding window, while the entire landscape has a maximum relief of 1519 m. Mt. Whitney has a maximum relief of 1133 m at a 30 m sliding window but has a maximum landscape relief much greater than Yosemite Valley at 2554 m. It is difficult to determine whether the relief over 30-m windows represents the “mean local relief” where k values are different. The vast majority of landscapes, however, seem to have sufficiently similar k values that a comparison of a single window size is probably meaningful and appropriate.

Slope Stability, Threshold Hillslope, and Rock Strength

The study by Schmidt and Montgomery (1995) evaluated lithologic effects on relief. Relief was defined as the change in elevation along features, such as valley walls and stream banks and was considered on a scale of tens of meters.

As in the erosion studies of Small and Anderson (1998), a definition of relief at the scale of a landscape feature only captures a small portion of the landscape processes that form relief.

The multi-scale analysis of relief suggests that there is some relationship between the maximum relief of an overall landscape and the micro-relief of the landscape where rock strength may play a role, not just at the relief between single ridges and valleys but also at the scale of the overall topographic unit. The multi-scale analysis of relief generally supports this hypothesis when extrapolated to the landscape scale upon which uplift acts where micro-scale relief is proportional to the overall relief of the landscape.

The mechanism that relates small- and large-scale relief may be described in terms of rock failure. At failure, the strength of a rock unit is equal to the lithostatic pressure (P), defined as

$$P = \rho gh$$

where ρ is the density of the rock, g is gravity, and h is the height of the rock unit. The strength is further diminished by spacing of fractures and joints within the rock.

Fractures and joints in the rock reduce the maximum height of a rock unit before failure. With increased rock strength, steeper cliffs can form in a landscape. A substrate like granite is competent and can form sheer rock cliffs. Formation of a series of steep cliff faces as elevation increases could result in an

overall greater maximum elevation change in a landscape. Therefore, the relationship between the steepness of the landscape at a small scale could reflect the steepness of the overall topographic unit. Schmidt and Montgomery (1995) back-calculated the limit to topographic development based on the weakest members in a rock mass and found that large-scale rock strength may control the regional limit to topographic development. For example, more friable sedimentary rocks that cover the Black Hills have less competence (Martin et al., 2004). Steep slopes form, but their maximum heights are dictated by the rock strength.

An evaluation of rock strength and slope stability seems to be appropriate particularly for landscapes with higher k values (e.g., Yosemite Valley, Zion Canyon) as k values appear to be influenced by rock properties. A straight comparison of valleys and ridge peaks, however, does not capture the anomalous nature of landscapes, such as Zion Canyon where the overall landscape elevation is proportionally less than the increase in relief over the smallest window sizes compared with most other landscapes.

Climate and Relief

Studies of climate and relief (Whipple et al., 1999; Roe et al., 2003; Gabet et al., 2004) are primarily concerned with comparing changes in topographic relief to average rainfall. Gabet et al. (2004) used the familiar fixed-size moving

window to define the landscape relief, while others looked to the individual ridge/valley scale. Limitations similar to those discussed for the studies of erosion rates and rock strength constrain these comparisons. The comparisons are useful for describing the physical processes and mechanisms at small scales, but they may not describe the overall relief and incision of the larger topographic unit.

The role of climate versus tectonic uplift can also be examined using the multi-scale approach. A comparison of climate could be addressed in terms of k value and maximum overall relief. Most landscapes appear to be proportional to each other in the vertical dimension having similar k values (Fig. 7), regardless of climate, for landscapes with maximum relief over 300 m. Further studies that evaluate the patterns of overall relief, k value, and rainfall amounts could provide useful insights into the role of weather on overall relief. Climate may also be a more significant factor where landscape-level maximum relief is low (below 300 m), supporting existing hypotheses of climate's role on hillslope evolution (Roe et al., 2003; Gabet et al., 2004) and providing further insight into how landscapes may change due to climate change.

CONCLUSIONS

The term relief has been used broadly in the geologic literature. Relief is most often defined in geomorphic analysis over an arbitrarily chosen window size. This study sought to understand the concept of relief over more than one window size by performing a landscape-scale analysis that incorporates the changes in relief as window size increases. The results showed that most landscapes scale over a fixed horizontal distance.

There is generally a directly proportional relationship between the overall maximum relief of a landscape and the maximum relief at each window size. Exceptions were found in unusual topography, such as Zion Canyon or Yosemite Valley or in landscapes that have a relatively flat topography over large distances.

Relief follows a saturating exponential function and maximum relief increases with window size. The k value may vary with the nature of incision, the rock strength, and the rate of uplift. The k values in the lower range for landscapes with maximum relief over 300 m likely represent smoother slopes that have experienced less incision (e.g., volcanoes or newly uplifted ranges). Larger k values may suggest a more highly incised landscape, more competent bedrock, and potentially, a decrease in tectonic uplift. Both maximum relief and k value should be considered and are components of the concept of relief.

A single window size represents just one point on the landscape relief curve, but if used consistently, it can be used to compare multiple landscapes, provided the landscapes have the same or similar k value. Landscape-scale relief can be used as a comparative tool by considering a greater area than just the point of interest. The area of interest should be considered within the overall topographic unit in which it lies.

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1: ADIRONDACKS - ESSEX COUNTY, NY

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
	(m)	(m)	(m)
2	117	43.47	73.53
5	219	105.36	113.64
10	334	200.21	133.79
20	491	362.50	128.50
30	561	494.05	66.95
40	619	600.68	18.32
50	672	687.12	15.12
60	751	757.18	6.18
70	817	813.97	3.03
80	889	860.00	29.00
90	892	897.32	5.32
100	904	927.56	23.56
110	906	952.08	46.08
120	925	971.95	46.95
130	939	988.06	49.06
140	998	1001.12	3.12
150	1018	1011.71	6.29
160	1020	1020.28	0.28
170	1020	1027.24	7.24
180	1021	1032.88	11.88
190	1025	1037.45	12.45
200	1026	1041.15	15.15
300	1057	1055.06	1.94
		Average	35.54
		Error	3%
		k value	0.021

1: Adirondacks - Essex County, NY

2: APPALACHIAN PLATEAU - VENANGO COUNTY, PA

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
	(m)	(m)	(m)
2	58	41.53	16.47
5	138	90.75	47.25
10	178	147.18	30.82
20	193	204.10	11.10
30	200	226.12	26.12
40	201	234.63	33.63
50	221	237.92	16.92
60	223	239.20	16.20
70	230	239.69	9.69
80	232	239.88	7.88
90	232	239.95	7.95
100	232	239.98	7.98
110	239	239.99	0.99
120	239	240.00	1.00
130	239	240.00	1.00
140	240	240.00	0.00
150	240	240.00	0.00
200	240	240.00	0.00

Average 13.06

Error 5%

k value 0.095

2: Appalachian Plateau - Venango County, PA

3: BASIN AND RANGE – SPRING MOUNTAINS - CLARK COUNTY, NV

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	199	58.64	140.36
5	319	143.36	175.64
10	446	276.36	169.64
20	647	514.22	132.78
30	842	718.95	123.05
40	942	895.16	46.84
50	1046	1046.82	0.82
60	1084	1177.37	93.37
70	1111	1289.72	178.72
80	1210	1386.43	176.43
90	1303	1469.67	166.67
100	1390	1541.31	151.31
110	1477	1602.97	125.97
120	1532	1656.05	124.05
130	1553	1701.73	148.73
140	1590	1741.05	151.05
150	1611	1774.89	163.89
300	1984	1961.96	22.04
Average			127.30
Error			6%
k value			0.015

3: Basin and Range - Clark County, NV

4: BASIN AND RANGE – LANDER COUNTY, NV

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
	(m)	(m)	(m)
2	90	38.48496	51.52
5	185	93.95182	91.05
10	264	180.6803	83.32
20	445	334.6459	110.35
30	579	465.8467	113.15
40	673	577.6487	95.35
50	730	672.92	57.08
60	768	754.1049	13.90
70	810	823.2861	13.29
80	858	882.2384	24.24
90	918	932.4743	14.47
100	932	975.2825	43.28
150	1055	1111.143	56.14
200	1118	1172.189	54.19
300	1222	1211.943	58.67

Average 59.18
Error 5.00
k value 0.02

4: Basin and Range - Lander County, NV

5: BLUE RIDGE – JACKSON, SWAIN, HAYWARD COUNTIES, NC

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	124	33.01	90.99
5	188	80.93	107.07
10	326	156.77	169.23
20	498	294.43	203.57
30	604	415.30	188.70
40	662	521.45	140.55
50	730	614.65	115.35
60	791	696.49	94.51
70	827	768.35	58.65
80	872	831.46	40.54
90	908	886.87	21.13
100	953	935.52	17.48
110	989	978.25	10.75
120	1022	1015.77	6.23
130	1051	1048.71	2.29
140	1082	1077.63	4.37
150	1122	1103.04	18.96
200	1198	1190.48	7.52
300	1286	1259.97	26.03
Average			69.68
Error			5%
k value			0.013

5: Blue Ridge - Jackson, Swain, Hayward Counties, NC
Window Size Versus Maximum Elevation Change Using 30m DED

**6: CASCADE-SIERRA – MT. ST. HELENS - YAKIMA, SKAMANIA
COUNTIES, WA**

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	116	51.14	64.86
5	254	126.04	127.96
10	415	246.23	168.77
20	651	470.15	180.85
30	860	673.78	186.22
40	1036	858.95	177.05
50	1164	1027.34	136.66
60	1271	1180.47	90.53
70	1362	1319.72	42.28
80	1461	1446.35	14.65
90	1548	1561.51	13.51
100	1638	1666.22	28.22
110	1709	1761.45	52.45
120	1782	1848.05	66.05
130	1848	1926.80	78.80
140	1930	1998.42	68.42
150	2006	2063.54	57.54
200	2382	2310.62	71.38
300	2717	2559.84	157.16
Average			93.86
Error			3%
k value			0.0095

6: Cascade-Sierra Mtns - Mt. St. Helens - Yakima, Skamania WA

Window Size Versus Maximum Elevation Change Using 30m DED

7: CASCADE-SIERRA – KIRKWOOD - ALPINE, DORADO COUNTIES, CA

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	97	55.86	41.14
5	204	136.56	67.44
10	331	263.26	67.74
20	591	489.85	101.15
30	818	684.88	133.12
40	982	852.75	129.25
50	1208	997.23	210.77
60	1380	1121.58	258.42
70	1464	1228.62	235.38
80	1518	1320.74	197.26
90	1561	1400.04	160.96
100	1666	1468.28	197.72
110	1728	1527.03	200.97
120	1762	1577.59	184.41
130	1780	1621.10	158.90
140	1793	1658.56	134.44
150	1798	1690.80	107.20
200	1848	1795.90	52.10
300	1890	1869.00	21.00
Average			139.97
Error			7%
k value			0.015

7: Cascade-Sierra Mtns - Kirkwood - Alpine, El Dorado, CA
Window Size Versus Maximum Elevation Change Using 30m DED

8: CASCADE-SIERRA – YOSEMITE VALLEY – MARIPOSA COUNTY, CA

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	609	211.58	397.42
5	847	475.01	371.99
10	925	801.48	123.52
20	1149	1180.07	31.07
30	1431	1358.90	72.10
40	1431	1443.37	12.37
50	1434	1483.28	49.28
60	1463	1502.13	39.13
70	1466	1511.03	45.03
80	1468	1515.23	47.23
90	1469	1517.22	48.22
100	1519	1518.16	0.84
Average			103.18
Error			7%
k			0.075

8: Cascade Sierra Mtns: Yosemite, Mariposa, CA

Window Size Versus Maximum Elevation Change Using 30m DED

**9: CASCADE-SIERRA – MT. WHITNEY – FRESNO, TULARE, INYO
COUNTIES, CA**

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	319	76.33918684	242.6608
5	454	186.6385748	267.3614
10	634	359.7912969	274.2087
20	956	669.466536	286.5335
30	1133	936.0064844	196.9935
40	1304	1165.419544	138.5805
50	1435	1362.877194	72.12281
80	1667	1805.015351	138.0154
100	1820	2006.654796	186.6548
150	2087	2310.753803	223.7538
200	2358	2454.400002	96.4
300	2583	2554.305462	28.69454

Average 179.3316

Error 0.069428

k value 0.015

9: Cascade-Sierra Mtns - Mt. Whitney - Fresno, Tulare, Inyo, CA

Window Size Versus Maximum Elevation Change Using 30m DED

10: CENTRAL LOWLANDS – ALCONA COUNTY, MI

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	24	8.80	15.20
5	48	21.00	27.00
10	65	38.89	26.11
20	85	67.12	17.88
30	93	87.63	5.37
40	99	102.52	3.52
50	100	113.33	13.33
60	105	121.18	16.18
70	111	126.88	15.88
80	117	131.02	14.02
90	120	134.03	14.03
100	123	136.21	13.21
150	134	140.83	6.83
200	142	141.76	0.24

Average
Error
k value

13.49
9%
0.032

10: Central Lowlands - Alcona, MI

Window Size Versus Maximum Elevation Change Using 30m DED

11: CENTRAL LOWLANDS – IDA COUNTY, IA

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	23	5.99	17.01
5	34	14.40	19.60
10	48	26.97	21.03
20	60	47.57	12.43
30	65	63.29	1.71
40	70	75.29	5.29
50	75	84.45	9.45
60	78	91.44	13.44
70	84	96.78	12.78
80	89	100.85	11.85
90	98	103.96	5.96
100	98	106.34	8.34
150	108	112.01	4.01
200	114	113.49	0.51

Average 10.24

Error 0.09

k value 0.027

11: Central Lowlands - Ida County, IA

Window Size Versus Maximum Elevation Change Using 30m DED

12: COASTAL PLAINS – OCEOLA COUNTY, FL

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	16	6.10	9.90
5	23	13.41	9.59
10	27	21.96	5.04
20	28	30.88	2.88
30	32	34.51	2.51
40	33	35.99	2.99
50	34	36.59	2.59
60	35	36.83	1.83
70	35	36.93	1.93
80	35	36.97	1.97
90	35	36.99	1.99
100	36	37.00	1.00
150	37	37.00	0.00
Average Error			3.40
			9%

12: Coastal Plains - Ocala County, FL

Window Size Versus Maximum Elevation Change Using 30m DED

13: COASTAL PLAINS – LINCOLN, UNION COUNTIES, LA

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	23	11.60	11.40
5	45	25.18	19.82
10	47	40.46	6.54
20	55	55.34	0.34
30	58	60.81	2.81
40	60	62.83	2.83
50	63	63.57	0.57
60	63	63.84	0.84
70	63	63.94	0.94
80	63	63.98	0.98
90	63	63.99	0.99
100	63	64.00	1.00
150	64	64.00	0.00

Average 3.77
Error 0.06
k value 0.1

13: Coastal Plains - Lincoln and Union Counties, LA

Window Size Versus Maximum Elevation Change Using 30m DED

14. COLORADO PLATEAUS – MESA, GARFIELD COUNTIES, CO

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	218	42.19	175.81
5	334	101.64	232.36
10	412	191.34	220.66
20	521	340.35	180.65
30	568	456.40	111.60
40	609	546.78	62.22
50	639	617.17	21.83
60	666	671.99	5.99
70	679	714.69	35.69
80	696	747.93	51.93
90	729	773.83	44.83
100	737	794.00	57.00
110	763	809.70	46.70
120	763	821.93	58.93
130	775	831.46	56.46
140	779	838.88	59.88
150	790	844.66	54.66
200	832	859.17	27.17
300	865	864.52	0.48
Average			79.20
Error			9%
k value			0.025

14: Colorado Plateau 2 - Mesa, Garfield, CO

15. COLORADO PLATEAUS – NAVAJO COUNTY, CO

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	154	49.92	104.08
5	267	122.04	144.96
10	412	235.26	176.74
20	631	437.76	193.24
30	806	612.05	193.95
40	906	762.06	143.94
50	970	891.17	78.83
60	1026	1002.30	23.70
70	1052	1097.96	45.96
80	1067	1180.28	113.28
90	1089	1251.14	162.14
100	1103	1312.13	209.13
150	1284	1510.98	226.98
200	1449	1604.91	155.91
300	1689	1670.24	18.76

Average 132.77

Error 8%

k values 0.015

15: Colorado Plateau - Navajo, CO

Window Size Versus Maximum Elevation Change Using 30m DED

16. COLORADO PLATEAUS – GRAND CANYON – COCONINO COUNTY,

AZ

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	317	80.47	236.53
5	437	193.88	243.12
10	643	364.98	278.02
20	987	649.22	337.78
30	1110	870.60	239.40
40	1161	1043.00	118.00
50	1163	1177.27	14.27
60	1188	1281.84	93.84
70	1275	1363.27	88.27
80	1293	1426.70	133.70
90	1430	1476.09	46.09
100	1431	1514.56	83.56
150	1595	1611.20	16.20
200	1650	1638.88	11.12

Average 138.56

Error 8%

k value 0.025

16: Colorado Plateaus - Grand Canyon, Coconino, AZ

Window Size Versus Maximum Elevation Change Using 30m DED

17. COLORADO PLATEAUS – ZION CANYON, WASHINGTON COUNTY, UT

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	453	171.89	281.11
5	651	385.89	265.11
10	775	651.10	123.90
20	1016	958.66	57.34
30	1044	1103.94	59.94
40	1119	1172.56	53.56
50	1143	1204.98	61.98
60	1163	1220.29	57.29
70	1192	1227.52	35.52
80	1211	1230.94	19.94
90	1215	1232.56	17.56
100	1217	1233.32	16.32
150	1230	1233.98	3.98
200	1234	1234.00	0.00
Average			75.25
Error			6%
k value			0.075

18: COLUMBIA PLATEAU – WALLOWA, UNION, BAKER COUNTIES, OR

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	167	81.50	85.50
5	358	196.35	161.65
10	507	369.62	137.38
20	689	657.49	31.51
30	882	881.68	0.32
40	980	1056.27	76.27
50	1106	1192.25	86.25
60	1202	1298.15	96.15
70	1251	1380.62	129.62
80	1324	1444.85	120.85
90	1372	1494.88	122.88
100	1404	1533.84	129.84
110	1462	1564.18	102.18
120	1523	1587.81	64.81
130	1599	1606.21	7.21
140	1644	1620.54	23.46
150	1648	1631.70	16.30
200	1648	1659.74	11.74
300	1671	1670.08	0.92
Average			73.94
Error			4%
k value			0.025

18: Columbia Plateau - Wallowa, Union, Baker Counties, OR
Window Size Versus Maximum Elevation Change Using 30m DED

19: GREAT PLAINS – EDWARDS COUNTY, KS

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	16	2.55	13.45
5	19	6.19	12.81
10	25	11.78	13.22
20	29	21.43	7.57
30	33	29.33	3.67
40	37	35.79	1.21
50	40	41.09	1.09
80	49	51.88	2.88
100	51	56.20	5.20
150	56	61.76	5.76
200	60	63.81	3.81
300	65	64.84	0.16
Average			5.90
Error			9%
k value			0.02

19: Great Plains - Edwards, KS

Window Size Versus Maximum Elevation Change Using 30m DED

**20: GREAT PLAINS – BLACK HILLS – CUSTER, PENNINGTON COUNTIES,
SD**

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	100	29.12	70.88
5	165	70.98	94.02
10	309	136.17	172.83
20	400	251.05	148.95
30	467	347.97	119.03
40	524	429.74	94.26
50	565	498.72	66.28
60	589	556.92	32.08
70	610	606.02	3.98
80	629	647.45	18.45
90	640	682.40	42.40
100	660	711.88	51.88
150	728	802.99	74.99
200	799	841.93	42.93
300	871	865.69	5.31
Average			69.22
Error			0.08
k values			0.017

20: Great Plains - Black Hills, Custer, Pennington Counties, SD
Window Size Versus Maximum Elevation Change Using 30m DED

**21: INTERIOR LOW PLATEAU – BRECKINRIDGE, GRAYSON COUNTIES,
KY**

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	49	18.67	30.33
5	72	41.90	30.10
10	90	70.70	19.30
20	108	104.10	3.90
30	117	119.88	2.88
40	122	127.33	5.33
50	122	130.85	8.85
60	123	132.51	9.51
70	128	133.30	5.30
80	131	133.67	2.67
90	134	133.84	0.16
100	134	133.93	0.07

Average 9.87
Error 7%
k value 0.075

22: LOWER CALIFORNIA – SAN DIEGO COUNTY, CA

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	83	42.62	40.38
5	183	103.12	79.88
10	317	195.51	121.49
20	478	352.40	125.60
30	540	478.32	61.68
40	635	579.36	55.64
50	686	660.46	25.54
60	714	725.54	11.54
70	754	777.76	23.76
80	775	819.68	44.68
90	784	853.31	69.31
100	803	880.30	77.30
150	935	953.49	18.49
200	938	977.85	39.85
300	990	988.65	1.35

Average 53.10

Error 5%

k value 0.022

22: Lower California - San Diego County, CA

23: MIDDLE ROCKY MOUNTAINS – PARK COUNTY, WY

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	250	51.60	198.40
5	296	126.16	169.84
10	430	243.20	186.80
20	656	452.53	203.47
30	913	632.70	280.30
40	1068	787.77	280.23
50	1103	921.25	181.75
60	1126	1036.13	89.87
70	1187	1135.01	51.99
80	1225	1220.11	4.89
90	1289	1293.37	4.37
100	1308	1356.41	48.41
150	1441	1561.97	120.97
200	1566	1659.07	93.07
300	1746	1726.60	19.40

Average
Error
k value

128.92
7%
0.015

23: Middle Rocky Mountains - Park County, WY

Window Size Versus Maximum Elevation Change Using 30m DED

24: NEW ENGLAND – CHESHIRE COUNTY, NH

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	49	21.36	27.64
5	117	51.47	65.53
10	206	96.89	109.11
20	275	172.34	102.66
30	307	231.10	75.90
40	310	276.87	33.13
50	311	312.51	1.51
60	318	340.27	22.27
70	318	361.89	43.89
80	318	378.72	60.72
90	331	391.84	60.84
100	338	402.05	64.05
110	339	410.00	71.00
120	344	416.19	72.19
130	353	421.02	68.02
140	353	424.77	71.77
150	353	427.70	74.70
200	391	435.05	44.05
300	438	437.76	0.24
Average			56.27
Error			0.13
k values			0.025

25: NEW ENGLAND – MT WASHINGTON, COOS COUNTY, NH

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	78	45.32	32.68
5	187	110.64	76.36
10	328	212.77	115.23
20	479	394.07	84.93
30	560	548.57	11.43
40	716	680.23	35.77
50	868	792.42	75.58
60	948	888.02	59.98
70	1014	969.48	44.52
80	1045	1038.90	6.10
90	1086	1098.06	12.06
100	1117	1148.47	31.47
110	1144	1191.43	47.43
120	1192	1228.03	36.03
130	1291	1259.23	31.77
140	1362	1285.81	76.19
150	1415	1308.46	106.54
160	1422	1327.76	94.24
170	1429	1344.21	84.79
180	1430	1358.22	71.78
190	1433	1370.17	62.83
200	1439	1380.34	58.66
		Average	57.11
		Error	4%
		k value	0.016

**26: NORTHERN ROCKY MOUNTAINS – IDAHO COUNTY, ID; RAVALLI
COUNTY, MT**

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	182	68.42	113.58
5	340	166.06	173.94
10	534	316.31	217.69
20	720	575.29	144.71
30	956	787.32	168.68
40	1155	960.92	194.08
50	1314	1103.05	210.95
60	1364	1219.42	144.58
70	1396	1314.69	81.31
80	1413	1392.69	20.31
90	1434	1456.55	22.55
100	1458	1508.84	50.84
110	1476	1551.65	75.65
120	1496	1586.70	90.70
130	1516	1615.39	99.39
140	1563	1638.89	75.89
150	1601	1658.12	57.12
200	1665	1713.04	48.04
300	1745	1740.67	4.33
Average			104.96
Error			6%
k value			0.02

26: Northern Rocky Mountains - Idaho, ID and Ravalli, MT
Window Size Versus Maximum Elevation Change Using 30m DED

27: OUACHITA – LE FLORE COUNTY, OK

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	51	28.54	22.46
5	105	68.25	36.75
10	183	127.00	56.00
20	280	221.08	58.92
30	338	290.78	47.22
40	366	342.41	23.59
50	397	380.67	16.33
60	417	409.00	8.00
70	434	430.00	4.00
80	442	445.55	3.55
90	450	457.07	7.07
100	456	465.60	9.60
200	490	488.79	1.21

Average 22.67

Error 5%

k value 0.03

27: Ouachita - Le Flore County, OK

28: OZARK PLATEAUS – MADISON COUNTY, MO

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	49	36.33	12.67
5	105	82.69	22.31
10	145	142.43	2.57
20	202	216.79	14.79
30	260	255.60	4.40
40	293	275.87	17.13
50	296	286.45	9.55
60	296	291.97	4.03
70	296	294.85	1.15
80	297	296.36	0.64
90	297	297.14	0.14
100	298	297.55	0.45
Average			7.49
Error			3%
k value			0.065

28: Ozark Plateaus - Madison County, MO

29: PACIFIC BORDER – JOSEPHINE, CURRY COUNTIES, OR

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	97	71.94	25.06
5	218	173.32	44.68
10	373	326.27	46.73
20	534	580.37	46.37
30	723	778.26	55.26
40	929	932.38	3.38
50	1037	1052.41	15.41
60	1103	1145.88	42.88
70	1143	1218.68	75.68
80	1202	1275.38	73.38
90	1234	1319.54	85.54
100	1279	1353.92	74.92
110	1316	1380.71	64.71
120	1364	1401.56	37.56
130	1368	1417.81	49.81
140	1374	1430.46	56.46
150	1379	1440.31	61.31
200	1416	1465.06	49.06
300	1475	1474.18	0.82
Average			47.84
Error			3%
k value			0.025

29: Pacific Border - Josephine, Curry Counties, OR

Window Size Versus Maximum Elevation Change Using 30m DED

30: PACIFIC BORDER – MT. DIABLO – CONTRA COSTA COUNTY, CA

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	61	44.77	16.23
5	151	108.33	42.67
10	288	205.38	82.62
20	464	370.20	93.80
30	613	502.47	110.53
40	728	608.63	119.37
50	773	693.81	79.19
60	813	762.18	50.82
70	857	817.04	39.96
80	878	861.07	16.93
90	898	896.41	1.59
100	903	924.76	21.76
110	944	947.52	3.52
120	962	965.78	3.78
130	976	980.44	4.44
140	985	992.20	7.20
150	996	1001.64	5.64
200	1029	1027.23	1.77
300	1040	1038.59	1.41
Average			37.01
Error			4%
k value			0.022

30: Pacific Border - Mt. Diablo, Contra Costa County, CA
Window Size Versus Maximum Elevation Change Using 30m DED

31: PIEDMONT – CHEROKEE COUNTY, SC

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	37	17.32	19.68
5	73	40.26	32.74
10	86	71.61	14.39
20	113	115.05	2.05
30	132	141.39	9.39
40	142	157.37	15.37
50	158	167.06	9.06
60	165	172.94	7.94
70	171	176.50	5.50
80	174	178.67	4.67
90	177	179.98	2.98
100	182	180.77	1.23

Average
Error
k values

10.42
6%
0.05

31: Piedmont - Cherokee County, SC

32: SOUTHERN ROCKY MOUNTAINS – ASPEN, COUNTY, CO

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	157	71.62	85.38
5	256	173.05	82.95
10	412	327.31	84.69
20	641	587.37	53.63
30	853	793.99	59.01
40	1043	958.16	84.84
50	1081	1088.60	7.60
60	1191	1192.24	1.24
70	1232	1274.58	42.58
80	1323	1340.00	17.00
90	1333	1391.99	58.99
100	1368	1433.29	65.29
110	1430	1466.10	36.10
120	1494	1492.18	1.82
130	1499	1512.89	13.89
140	1506	1529.35	23.35
150	1584	1542.43	41.57
160	1593	1552.82	40.18
Average			44.45
Error			0.03
k value			0.023

32: So. Rocky Mountains - Aspen: Pitkin, Chaffee, Gunnison, Lake, CO

33: SOUTHERN ROCKY MOUNTAINS – SAN JUAN COUNTY, CO

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	182	64.93	117.07
5	430	157.59	272.41
10	569	300.18	268.82
20	741	545.95	195.05
30	962	747.17	214.83
40	1133	911.91	221.09
50	1272	1046.79	225.21
60	1331	1157.22	173.78
70	1342	1247.64	94.36
80	1356	1321.66	34.34
90	1366	1382.27	16.27
100	1420	1431.88	11.88
110	1457	1472.51	15.51
120	1471	1505.77	34.77
130	1520	1533.00	13.00
140	1524	1555.30	31.30
150	1567	1573.55	6.55
200	1612	1625.67	13.67
300	1656	1651.90	4.10
Average			103.37
Error			6%
k values			0.02

34: ST. LAWRENCE RIVER VALLEY – ST. LAWRENCE COUNTY, NY

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	34	23.74	10.26
5	50	45.51	4.49
10	55	62.26	7.26
20	70	70.68	0.68
30	72	71.82	0.18

Average 4.57
Error 6%
k value 0.2

34: St. Lawrence Valley - St. Lawrence County, NY

Window Size Versus Maximum Elevation Change Using 30m DED

35: SUPERIOR UPLAND – ONTONAGON COUNTY, MI

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	65	30.05	34.95
5	131	67.92	63.08
10	159	115.79	43.21
20	179	173.28	5.72
30	182	201.84	19.84
40	200	216.01	16.01
50	210	223.05	13.05
60	217	226.55	9.55
70	219	228.29	9.29
80	224	229.15	5.15
90	229	229.58	0.58
100	230	229.79	0.21
Average			18.39
Error			7%
k value			0.07

**36: VALLEY AND RIDGE – HUNTINGTON, MIFLIN, CENTRE COUNTIES,
PA**

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	56	30.81	25.19
5	115	73.69	41.31
10	193	137.11	55.89
20	282	238.68	43.32
30	348	313.92	34.08
40	404	369.67	34.33
50	438	410.96	27.04
60	456	441.56	14.44
70	468	464.22	3.78
80	475	481.01	6.01
90	479	493.45	14.45
100	484	502.66	18.66
150	512	523.12	11.12
200	529	527.69	1.31

Average 23.64
Error 4%
k value 0.03

36: Valley and Ridge - Huntington, Miflin, Centre Counties, PA
Window Size Versus Maximum Elevation Change Using 30m DED

37: VALLEY AND RIDGE – TAZWELL, SMYTHE COUNTIES, VA

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	67	44.32	22.68
5	171	106.00	65.00
10	254	197.24	56.76
20	352	343.35	8.65
30	440	451.60	11.60
40	498	531.79	33.79
50	577	591.20	14.20
60	637	635.21	1.79
70	639	667.81	28.81
80	652	691.96	39.96
90	654	709.86	55.86
100	663	723.11	60.11
150	722	752.55	30.55
200	761	759.11	1.89

Average 30.83
Error 0.04
k value 0.03

37: Valley and Ridge - Tazwell, Smythe Counties, VA

Window Size Versus Maximum Elevation Change Using 30m DED

38: WYOMING BASIN – CARBON COUNTY, WY

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	127	28.36	98.64
5	240	69.03	170.97
10	352	132.11	219.89
20	432	242.46	189.54
30	443	334.64	108.36
40	458	411.62	46.38
50	489	475.93	13.07
60	514	529.64	15.64
70	533	574.51	41.51
80	573	611.98	38.98
90	581	643.29	62.29
100	604	669.43	65.43
200	705	780.09	75.09
300	802	798.38	3.62

Average 82.10
Error 10%
k value 0.018

39: PUERTO RICO

Window Size (pixels)	Maximum Relief (m)	Max Relief Saturating Exponential Function (m)	Difference (m)
2	90	51.47	38.52
5	241	124.38	116.61
10	378	235.25	142.74
20	534	422.18	111.81
30	634	570.69	63.30
40	729	688.69	40.30
50	810	782.45	27.54
60	875	856.94	18.05
70	897	916.12	19.12
80	914	963.15	49.15
90	929	1000.51	71.51
100	943	1030.20	87.20
200	1145	1133.49	11.50
average			61.34
error			5%
k			0.023

39: OTR - Puerto Rico

40: MAUI

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
	(m)	(m)	(m)
2	463.7	177.64	286.06
5	813.9	424.90	389.00
10	1087.1	790.61	296.49
20	1341.1	1376.30	35.20
30	1788.7	1810.20	21.50
40	2115.3	2131.64	16.34
50	2364.8	2369.76	4.96
60	2555.6	2546.17	9.43
70	2751.3	2676.86	74.44
80	2918.2	2773.67	144.53
90	3023.3	2845.40	177.90
100	3050.4	2898.53	151.87

Average 133.98

Error 4%

k value 0.03

40: OTR - Maui

41: HAWAII

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
	(m)	(m)	(m)
2	333	124.07	208.93
5	798	303.33	494.67
10	987	584.75	402.25
20	1123	1088.05	34.95
30	1335	1521.24	186.24
40	1612	1894.09	282.09
50	1808	2215.01	407.01
60	1986	2491.22	505.22
70	2174	2728.96	554.96
80	2317	2933.59	616.59
90	2467	3109.71	642.71
100	2598	3261.30	663.30
110	2728	3391.77	663.77
120	2861	3504.08	643.08
130	2947	3600.73	653.73
140	3018	3683.93	665.93
150	3089	3755.53	666.53
160	3150	3817.17	667.17
170	3268	3870.21	602.21
180	3448	3915.87	467.87
190	3670	3955.17	285.17
200	3773	3988.99	215.99
210	3828	4018.11	190.11
220	3988	4043.16	55.16
230	4088	4064.73	23.27
240	4191	4083.30	107.70
250	4198	4099.27	98.73
260	4198	4113.02	84.98

Error 396.08

Average 9%

k value 0.015

41: OTR - Hawaii

42: OAHU

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
	(m)	(m)	(m)
2	314	127.50	186.50
5	580	294.28	285.72
10	765	517.81	247.19
20	924	816.57	107.43
30	993	988.93	4.07
40	1118	1088.38	29.62
50	1189	1145.75	43.25
60	1218	1178.86	39.14
70	1223	1197.95	25.05
80	1224	1208.97	15.03

Average
Error
k value

98.30
8%
0.055

42: OTR - Oahu

Window Size Versus Maximum Elevation Change Using 90m DED

43: KAUAI

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
	(m)	(m)	(m)
2	465	166.46	298.54
5	794	384.20	409.80
10	1018	676.03	341.97
20	1187	1066.07	120.93
30	1283	1291.10	8.10
40	1341	1420.94	79.94
50	1380	1495.84	115.84
60	1427	1539.06	112.06
70	1446	1563.99	117.99
80	1467	1578.38	111.38
90	1478	1586.68	108.68
100	1493	1591.47	98.47
110	1514	1594.23	80.23
120	1536	1595.83	59.83
130	1562	1596.75	34.75
140	1598	1597.28	0.72
Average			131.20
Error			8%
k value			0.055

43: OTR - Kauai

44: MOLOKAI

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
	(m)	(m)	(m)
2	361	272.99	88.01
5	820	592.56	227.44
10	1148	951.97	196.03
20	1312	1302.19	9.81
30	1345	1431.02	86.02
40	1385	1478.42	93.42
50	1486	1495.85	9.85
60	1506	1502.27	3.73
Average			89.29
Error			6%
k value			0.033

44: OTR - Molokai

45: LANAI

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
	(m)	(m)	(m)
2	248	124.83	123.17
5	396	284.13	111.87
10	475	489.43	14.43
20	631	744.93	113.93
30	821	878.31	57.31
40	959	947.94	11.06
50	1022	984.30	37.70
60	1023	1003.27	19.73
70	1024	1013.18	10.82

Average
Error
k value

55.56
5%
0.022

45: OTR - Lanai

Window Size Versus Maximum Elevation Change Using 90m DED

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km ² DED	131

90 M DEDS

5: Blue Ridge - Jackson, Swain, Hayward Counties, NC

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	151	102.73	48.27
5	401	241.51	159.49
10	562	437.27	124.73
15	669	595.95	73.05
20	783	724.57	58.43
25	845	828.83	16.17
30	895	913.34	18.34
35	962	981.85	19.85
40	1024	1037.37	13.37
45	1071	1082.38	11.38
50	1124	1118.87	5.13
100	1275	1255.88	19.12
Average			47.28
Error			4%
k value			0.014

5: Blue Ridge - Jackson, Swain, Hayward Counties, NC

28: Ozark Plateaus - Madison County, MO

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	90	90.39	0.39
5	161	177.44	16.44
10	260	249.58	10.42
15	293	278.91	14.09
20	294	290.83	3.17
25	295	295.68	0.68
30	295	297.65	2.65
35	298	298.45	0.45
40	299	298.78	6.04
45	299	298.91	6.74
50	299	298.96	5.53

Average
Error
k value

6.06
2%
0.06

11: Central Lowlands - Ida County, IO

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	25	15.02	9.98
5	50	33.59	16.41
10	63	56.33	6.67
15	71	71.72	0.72
20	78	82.15	4.15
25	87	89.20	2.20
30	94	93.98	0.02
35	98	97.22	0.78
40	104	99.41	5.12
Average			5.12
Error			5%
k value			0.026

11: Central Lowlands - Ida County, IA

Window Size versus Maximum Elevation Change using 90m DED

9: Cascade-Sierra - Mt. Whitney – Fresno, Tulare, Inyo Counties, CA

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	378	222.32	155.68
5	716	520.43	195.57
10	1096	936.01	159.99
15	1352	1267.85	84.15
20	1547	1532.83	14.17
25	1651	1744.42	93.42
30	1752	1913.38	161.38
35	1838	2048.30	210.30
40	2004	2156.03	152.03
45	2061	2242.06	136.30
50	2073	2310.75	134.15
100	2583	2554.31	127.32

Average 135.37
Error 5%
k value 0.015

9: Cascade-Sierra Mtns - Mt. Whitney - Fresno, Tulare, Inyo, CA

30: Pacific Border - Mt. Diablo – Contra Costa County, CA

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	123	129.59	6.59
5	332	294.57	37.43
10	599	506.34	92.66
15	751	658.59	92.41
20	812	768.04	43.96
25	864	846.73	17.27
30	901	903.30	2.30
35	938	943.97	5.97
40	962	973.21	11.21
45	986	994.23	34.42
50	1004	1009.35	37.52
100	1048	1046.57	37.53

Average
Error
k value

34.94
3%
0.022

30: Pacific Border - Mt. Diablo, Contra Costa County, CA

Window Size versus Maximum Elevation Change using 90m DED

CIRCULAR WINDOWS

5: Blue Ridge - Jackson, Swain, Hayward Counties, NC

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
4	164	65.26	98.74
10	285	157.01	127.99
20	453	294.89	158.11
40	593	522.26	70.74
60	699	697.57	1.43
80	841	832.75	8.25
100	921	936.98	15.98
120	1003	1017.34	14.34
140	1065	1079.31	14.31
160	1142	1127.09	14.91
180	1154	1163.93	9.93
200	1172	1192.34	20.34
300	1259	1261.93	2.93
400	1288	1280.89	7.11
Average			40.36
Error			3%
k			0.013

5: Blue Ridge - Jackson, Swain, Hayward Counties, NC

28: Ozark Plateaus - Madison County, MO

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
4	90	64.23	25.77
10	138	135.81	2.19
20	187	210.34	23.34
40	274	273.69	0.31
60	295	292.78	2.22
80	296	298.52	2.52
100	297	300.25	3.25
200	301	301.00	0.00
Average			7.45
Error			2%
k value			0.06

28: Ozark Plateaus - Madison County, MO

11: Central Lowlands - Ida County, IO

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
4	28	10.67	17.33
10	48	24.73	23.27
20	58	43.79	14.21
40	65	69.83	4.83
60	73	85.31	12.31
80	80	94.51	14.51
100	88	99.98	11.98
200	108	107.40	0.60
Average			12.38
Error			11%
k value			0.026

11: Central Lowlands - Ida County, IA

9: Cascade-Sierra - Mt. Whitney – Fresno, Tulare, Inyo Counties, CA

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
4	435	146.05	288.95
10	632	349.34	282.66
20	888	650.03	237.97
40	1263	1131.58	131.42
60	1463	1488.32	25.32
80	1594	1752.60	158.60
100	1758	1948.39	190.39
200	2299	2383.13	84.13
300	2508	2480.14	27.86
Average			158.59
Error			6%
k values			0.015

9: Cascade-Sierra Mtns - Mt. Whitney - Fresno, Tulare, Inyo, CA

30: Pacific Border - Mt. Diablo – Contra Costa County, CA

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
4	120	87.61	32.39
10	245	205.38	39.62
20	383	370.20	12.80
40	621	608.63	12.37
60	771	762.18	8.82
80	868	861.07	6.93
100	908	924.76	16.76
200	1029	1027.23	1.77
300	1040	1038.59	1.41
Average Error k			14.76425 1% 0.022

30: Pacific Border - Mt. Diablo, Contra Costa County, CA

FIXED POINT

5: Blue Ridge - Jackson, Swain, Hayward Counties, NC

5: Blue Ridge - Jackson, Swain, Hayward Counties, NC

28: Ozark Plateaus - Madison County, MO

28: Ozark Plateaus - Madison County, MO

11: Central Lowlands - Ida County, IO

Window Size (pixels)

9: Cascade-Sierra - Mt. Whitney, Fresno, Tulare, Inyo Counties, CA

9: Cascade-Sierra Mtns - Mt. Whitney - Fresno, Tulare, Inyo, CA

30: Pacific Border - Mt. Diablo – Contra Costa County, CA

30: Pacific Border - Mt. Diablo, Contra Costa County, CA

Window Size versus Max. Elevation Change, 30m DED, Windows Centered on Fixed Point

VARYING DED SIZE FOR MT DIABLO AREA

30: Pacific Border - Mt. Diablo – Contra Costa County, CA; 3.73 km² DED

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	47	42.52	4.48
5	122	100.81	21.19
10	213	185.02	27.98
20	335	314.11	20.89
30	400	404.17	4.17
40	457	467.00	10.00
50	508	510.84	2.84
60	559	541.42	17.58
70	612	562.76	49.24
80	612	577.65	34.35
90	612	588.03	23.97
100	612	595.28	16.72

Average 19.45
Error 3%
k value 0.012

30: Pacific Border - Mt. Diablo, Contra Costa County, CA

30: Pacific Border - Mt. Diablo – Contra Costa County, CA; 19.81 km² DED

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	64	66.41	2.41
5	160	157.48	2.52
10	263	289.02	26.02
20	467	490.66	23.66
30	613	631.35	18.35
40	728	729.50	1.50
50	773	797.97	24.97
60	813	845.75	32.75
70	860	879.08	19.08
80	865	902.34	37.34
90	885	918.56	33.56
100	910	929.88	19.88
200	956	955.29	0.71

Average
Error
k value

18.67
2%
0.012

30: Pacific Border - Mt. Diablo, Contra Costa County, CA

30: Pacific Border - Mt. Diablo – Contra Costa County, CA; 544.55 km² DED

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	62	54.46	7.54
5	160	130.83	29.17
10	281	245.14	35.86
20	467	432.27	34.73
30	613	575.13	37.87
40	728	684.18	43.82
50	773	767.43	5.57
60	813	830.98	17.98
70	860	879.49	19.49
80	876	916.52	40.52
90	895	944.79	49.79
100	914	966.38	52.38
200	1036	1031.32	4.68

Average
Error
k value

29.18
3%
0.009

30: Pacific Border - Mt. Diablo, Contra Costa County, CA

Window Size versus Maximum Elevation Change, 30m DED of 544.55 km²

30: Pacific Border - Mt. Diablo – Contra Costa County, CA; 209.79 km² DED

Window Size (pixels)	Maximum Relief	Max Relief Saturating Exponential Function	Difference
2	61	44.77	16.23
5	151	108.33	42.67
10	288	205.38	82.62
20	464	370.20	93.80
30	613	502.47	110.53
40	728	608.63	119.37
50	773	693.81	79.19
60	813	762.18	50.82
70	857	817.04	39.96
80	878	861.07	16.93
90	898	896.41	1.59
100	903	924.76	21.76
110	944	947.52	3.52
120	962	965.78	3.78
130	976	980.44	4.44
140	985	992.20	7.20
150	996	1001.64	5.64
200	1029	1027.23	1.77
300	1040	1038.59	1.41
Average			37.01
Error			4%
k value			0.022

30: Pacific Border - Mt. Diablo, Contra Costa County, CA