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Dodge-Romig sampling plans: Misuse, frivolous use, and expansion for usefulness.

Avik Ganguly
San Jose State University

DODGE-ROMIG SAMPLING PLANS:
MISUSE, FRIVOLOUS USE, AND EXPANSION FOR USEFULNESS

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by
Avik Ganguly

APPROVED FOR THE DEPARTMENT OF INDUSTRIAL AND SYSTEMS ENGINEERING

Nov. 6, 2009

Dr. H.S.Jacob Tsao, Department of Industrial and Systems Engineering   Date

M Patel   Nov. 6, 2009

Dr. Minnie Patel, Department of Industrial and Systems Engineering   Date

Maryam Talakoob   Nov 6, 2009

Maryam Talakoob, Application Systems Engineer, Wells Fargo, CA 94105   Date

APPROVED FOR THE UNIVERSITY

2/5/10

Associate Dean Office of Graduate Studies and Research   Date

ABSTRACT

DODGE-ROMIG SAMPLING PLANS: MISUSE, FRIVOLOUS USE, AND EXPANSION FOR USEFULNESS

by Avik Ganguly

The Dodge-Romig plans and the Military Standard 105 Plans are the most popular published sampling plans covered in detail in many standard textbooks on statistical quality assurance such as *Introduction to Statistical Quality Control* by Douglas C. Montgomery. In this thesis, we focus on Dodge-Romig AOQL plans. The Dodge-Romig AOQL plans are designed to minimize the average total inspection (ATI) for a given AOQL and a specified process average. It is argued that the Dodge-Romig plans as currently tabulated (particularly in the standard publications or textbooks on acceptance sampling), are not useful. However, they can be made useful if the plans focus on ranges of the (incoming) process-average values that are greater than the target AOQL. Whether one knows the exact value of process average p or just the exact range of the process averages containing an uncertain p, one can use the pair of (n, c) listed under the corresponding range to ensure the target AOQL with the minimum ATI. The thesis provides example expansions of some of the published Dodge-Romig AOQL plans. A new concept of certainty line is also developed which is a measure of process-average (p) associated with gamma risk (γ) whose numerical value is 0.000033. For any given lot size if the calculated or knowledge base process-average of a manufacturer falls below or equal to the value of the certainty line, then one would not be required to consider any sampling plan to achieve a worst case outgoing quality.

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LIST OF SYMBOLS

c	Acceptance number
γ	Gamma risk
d	Number of nonconforming items
n	Sample size
N	Lot size
p	Process-average
P_a	Probability of acceptance
Z	Standard Normal Random Variable

LIST OF ABBREVIATIONS

AOQ	Average Outgoing Quality
AOQL	Average Outgoing Quality Limit
ATI	Average Total Inspection
LTPD	Lot Tolerance Percent Defective

CHAPTER 1

INTRODUCTION

Although statistical process control has been the centerpiece for modern statistical quality assurance, acceptance sampling remains a useful tool for a company to control the quality of raw materials or parts shipped from the suppliers, particularly when no company representatives are present at the suppliers' manufacturing facilities. Among the categories of single, multiple, and continuous sampling plans, single sampling plans are effective and most popular. In addition to the fundamental single sampling where the sample size n and acceptance number c are to be determined by user-specified the AQL and LTPD and the corresponding producer's and buyer's risk levels, the Dodge-Romig plans and the Military Standard 105 Plans are the most popular published sampling plans and are covered in detail in many standard textbooks on statistical quality assurance. In this thesis, after pointing out a common misuse of the Dodge-Romig plans as well as a common frivolous use, I propose an expansion of any of the published Dodge-Romig plans and address the resulting usefulness.

The thesis focuses on Dodge-Romig AOQL plans. Each of the published plan tables corresponds to a target AOQL and contains several columns corresponding to a set of non-overlapping ranges of process-average p (i.e., incoming quality or incoming percentage of non-conforming) and a number of rows corresponding to the same number of non-overlapping lot-size ranges. Note that the non-overlapping ranges of process-average p constitute the entire interval between 0 and the target AOQL. In particular, all the process-average (p) values of all the listed ranges are smaller than the target AOQL.

For illustration purposes, we focus on the single sampling plan published for AOQL = 3.0% and focus on the lot size 8500. A single sampling is specified by a pair of n and c, where n denotes the sample size and c denotes the acceptance number or the critical number. A lot is accepted if the number of defective items among the n sampled items is no greater than c. Each of the pairs of (n, c) listed for lot size 8500 guarantees an AOQL of 3.0%, but the (n, c) pair listed under the process-average range of (0.00% - 0.06%), for example, should be selected if the process-average p falls within this range. This particular pair produces the minimum ATI approximately, among all the (n, c) pairs listed for lot size 8500. A common misuse of this plan or any other Dodge-Romig AOQL plan can be illustrated as follows. "Furthermore, to use the plans, we must know the process average [...]." (Montgomery 682). This is a misuse, and one does *not* have to know exactly what the (incoming) process-average p is. Had one known the exact value of the process-average p and had one known that this process-average p is less than 3.0%, there is no need to conduct the sampling to begin with. This is because the 100% rectifying inspection of the Dodge-Romig plans guarantees that the outgoing quality will be strictly better than the (incoming) process-average. Note again that all the process-average values of all the process-average ranges of a Dodge-Romig plan are smaller than the target AOQL. If the process-average p is indeed 0.0006 (i.e., 0.06%), why would want to bother with any sampling plan at all to achieve a 0.03 (i.e., 3%) worst-case outgoing quality? (3.0% is 50 times worse than 0.06 %.) This misconception results from the fact that ATI is a function of the (incoming) process-average p and calculation of ATI for range selection requires the value of p. I will provide numerical examples to illustrate how minute the probability is for any particular lot of 8500 to have 3.0% rate of nonconforming (i.e., 255 defective items) if the process-average p is indeed 0.0006 (i.e., 0.06%).

Some theoreticians or practitioners suggest the following frivolous use of Dodge-Romig plans. Although one does not have to know the exact value of the (incoming) process-average p, he/she only needs to know exactly which of the ranges listed for the given lot size contains p and should use the (n, c) pair of the corresponding range (i.e., column). In this use, although the exact knowledge of the (incoming) process-average p is not needed, one must have the exact knowledge of the range. This use is frivolous because, once again, had one known exactly that the process-average is within a given listed range, there is no need for the sampling. Once again, this is because the rectifying inspection guarantees that the outgoing quality is strictly better than the incoming process-average. Note again that all the process-average values of all the process-average ranges of a Dodge-Romig plan are smaller than the target AOQL.

CHAPTER 2

FUNDAMENTAL CONCEPTS

Single-Sampling Plan

A single-sampling is defined by the three entities namely lot size N, sample size n and acceptance number c. Thus for a lot size of N a random sample of n units is inspected and the number of nonconforming items d is observed. If the number of nonconforming items d is less than or equal to acceptance number c, the lot will be accepted. On the other hand, if $d > c$ then the lot will be rejected.

Probability of Acceptance

The probability of acceptance P_a is the probability that $d \leq c$. It is a probabilistic measure and the result can vary from 0 to 1. It is also used in the calculation of AOQ and ATI. P_a is large (in the order of 0.9 and more) when the incoming process-average is low.

Percent Defective

Percent defective is a measure of quality in terms of percentage. It can vary from 0% to 100%. It is also termed as fraction defective. It represents the number of defective items per 100 items present in a lot of size N. In my thesis, it is mainly referred as incoming process-average p. This is a quality measure which defines the incoming lot from a supplier.

Concept of Rectifying Inspection

The phenomenon of 100% screening or inspection of rejected lots forms the basis of rectifying inspection technique. In case of a lot being sentenced, all the discovered defective items are either removed for subsequent work or they are returned to the supplier or replaced with a stock of good items. For example if the incoming lots have a fraction defective of p_o then some of these lots will be accepted and some of them will be rejected. The outgoing quality of the accepted lots will have a fraction defective of $p_{o.}$ However, the rejected lots will be screened and their final fraction defective will be zero which is definitely less than p_o since p_o is a positive real number. The overall outgoing lots thereby are combination of lots with fraction defective p_o and fraction defective zero. Therefore, the outgoing lots from the inspection activity will have a fraction defective measure of say p_1 which is less than $p_{o.}$ In short, rectifying inspection is a technique which guarantees an overall outgoing quality of lots which is strictly greater than the overall quality of the incoming ones. The Dodge-Romig sampling plans hold good only if rectifying inspection is adopted.

Average Outgoing Quality (AOQ)

AOQ is the measure of quality in the lot that results from the application of rectifying inspection.

AOQ can be illustrated as follows: "It is simple to develop a formula for average outgoing quality (AOQ). Assume that the lot size is N and that all discovered defectives are replaced with good units. Then in lots of size N, we have

1. *n* items in the sample that, after inspection, contain no defectives, because all discovered defectives are replaced.

2. $N - n$ items that, if the lot is rejected, also contain no defectives

3. $N - n$ items that, if the lot is accepted, contain $p(N - n)$ defectives

Thus, lots in the outgoing stage of inspection have an expected number of defective units equal to $P_a p\ (N - n)$, which we may express as an *average fraction defective,* called the **average outgoing quality** or AOQ = $\frac{P_a p(N-n)}{N}$," (Montgomery 659).

The AOQ varies as the fraction defective of the incoming lot varies.

Average Outgoing Quality Limit (AOQL)

The maximum ordinate on the AOQ curve (while varying *p)* is the AOQL. It represents the worst possible average quality that would result from the rectifying inspection program. For the purpose of this thesis, I have always used the target AOQL to be 3.0%. There are also sampling plans for different AOQL values such as 0.1%, 0.25%, 0.5%, 0.75%, 1%, 1.5%, 2%, 2.5%, 4%, 5%, 7%, and 10%.

Average Total Inspection (ATI)

ATI can be illustrated as follows: "Another important measure relative to rectifying inspection is the total amount of inspection required by the sampling program. If the lots contain no defective items, no lots will be rejected, and the amount of inspection per lot will be the sample size *n*. If the items are all defective, every lot will be submitted to

100% inspection, and the amount of inspection per lot will be the lot size *N*. If the lot quality is $0 < p < 1$, the average amount of inspection per lot will vary between the sample size *n* and the lot size *N*. If the lot is of quality *p* and the probability of lot acceptance is P_a, then the **average total inspection** per lot will be:

$ATI = n + (1 - P_a)(N - n)$" (Montgomery 661).

CHAPTER 3

THE DODGE-ROMIG SAMPLING PLAN (AOQL): MISUSE

The Dodge-Romig single sampling tables obtained from the book *Sampling Inspection Tables, Single and Double Sampling* by H. F. Dodge and H. G. Romig give AOQL sampling plans for various AOQL values ranging from 0.1% to 10%. The tables are provided for both single and double sampling. For illustration purpose, I have focused on the plan published for AOQL = 3% and for the lot sizes 8,500 and 75,000.

Table 3.1: Single Sampling Plan for AOQL = 3.0% (Dodge and Romig 202)

Lot Size	Process Average 0 to 0.06%		Process Average 0.07 to 0.60%		Process Average 0.61 to 1.20%		Process Average 1.21 to 1.80%		Process Average 1.81 to 2.40%		Process Average 2.41 to 3.00%	
	n	*c*	*n*	*c*	*n*	*c*	*n*	*c*	*n*	*c*	*n*	*c*
7001-10,000	28	1	46	2	65	3	105	5	170	8	280	13
50,001-100,000	28	1	65	3	125	6	215	10	385	17	690	29

I have arbitrarily chosen the row 16 and 19 of the published table. For calculation purpose I have chosen 8,500 and 75,000 which represent the median value of the interval 7,001-10,000 and 50,001-100,000 respectively. Each of the above pairs of (n, c) for lot sizes 8,500 and 75,000 guarantees an AOQL of 3.0%. Also, these particular pairs of (n, c) produce the minimum ATI approximately, among all the (n, c) pairs listed for lot sizes 8,500 and 75,000 respectively. However, the (n, c) pair listed under the process-average range of (0.00% - 0.06%), for example in our case (28, 1), should be selected if the process-average p falls within this range. In order to use the plans, the text books (Montgomery) state the necessity of having the knowledge of the process-average which can also be termed as average fraction nonconforming of the incoming product. This is a misuse of the knowledge of incoming process average, and I argue that one does not have to know the exact incoming process-average p. If one knows the incoming process-average p and if $p < 3.0\%$, then there will not be any necessity to adopt any sampling plan. This is because the Dodge-Romig plan is only applicable when the rejected lots are subjected to 100% inspection. This rectifying inspection ensures the outgoing quality of the lot will always be better than the incoming process-average.

I will use the following numerical example to defend my argument. Let us consider the initial range of the incoming process-average p (0 to 0.06%).

Problem statement: Calculate the probability of a lot with lot size 8,500 to have 3% or more defective items given that the incoming process-average is 0.06%.

Solution: Given lot size $N = 8500$

Number of defective items = 3% of 8500 = 255

Incoming process-average $p = 0.06\% = 0.0006$

Hence for a given lot size of 8,500 let us calculate the probability of having 255 or more number of defective items when the incoming process-average is 0.0006

To calculate,

$P\ (X \geq 255)$

Where X is a binomial random variable with parameters N and p.

By the principle of normal approximation to the binomial distribution (Montgomery and Runger 132)

$P\ (X \geq 255) = 1 - P\ (X < 255) = 1 - P\ (X \leq 254)$

$$1 - P\left[\frac{X - Np}{\sqrt{Np(1-p)}}\right] \leq \frac{[254 - (8500 \times 0.0006)]}{\sqrt{[8500 \times 0.0006 \times (1 - 0.0006)]}}$$

$$1 - P\left(Z \leq \frac{248.9}{2.2576}\right)$$

$1 - P\ (Z \leq 110.2498228)$

We know that, from the cumulative standard normal distribution table (Montgomery and Runger 713)

$P\ (Z \leq 3.99) = 0.999967$

$P\ (Z \leq 110.2498228) \approx 1$

Hence $P\ (X \geq 255) \approx 0$

Thus the example demonstrates that the probability of having 3.0% or more defective items within a lot with lot size of 8500 when the incoming process-average is 0.06% is approximately equal to zero. It is to be noted that all the process-average in Table 3.1 is less than or equal to the target AOQL. If one knows the incoming process-average p to be 0.06%, then one would not require consideration of any sampling plan.

CHAPTER 4

THE DODGE-ROMIG SAMPLING PLAN (AOQL): FRIVOLOUS USE

The frivolous use of the Dodge-Romig sampling plan (AOQL) is quite similar to that of its misuse. Looking back to Table 3.1, for any given range of lot size (e.g., 7,001-10,000) there are six different non-overlapping ranges of incoming process-average *p*. Some theoreticians or practitioners suggest the following frivolous use of Dodge-Romig plans. Although for a given lot size one does not have to know the exact value of the incoming process-average *p*, one must know the exact range (in our case one among the six non-overlapping ranges listed in Table 3.1), which contains the incoming process-average *p*. In this use, one must use the (*n*, *c*) pair of the corresponding range (i.e., column). For example, if one knows that the incoming process-average *p* lies within the range of 1.21 to 1.80% and the lot size is 9,000, then one would choose (105, 5) as the (*n*, *c*) pair for the purpose of sampling. This is a frivolous use. There is no need for sampling if one knows that the incoming process-average *p* lies exactly within a given listed range. Once again, this is because rectifying inspection guarantees that the outgoing quality is always better than the incoming process-average. Note that all the process-average values of all the process-average ranges of a Dodge-Romig plan are smaller than the target AOQL.

CHAPTER 5

THE DODGE-ROMIG SAMPLING PLAN (AOQL):

EXPANSION FOR USEFULNESS – CERTAINTY LINE

Having stated the misuse and frivolous use of Dodge-Romig sampling plan, I argue that the currently published tables can be made more useful. In this part, I am introducing a concept of certainty line which is a measure of process-average p associated with a gamma risk (γ) whose numerical value is 0.000033. For any given lot size if the calculated or knowledge base process-average of a manufacturer falls below or equal to value of the certainty line then one would not require to consider any sampling plan to achieve a worst case outgoing quality. The following numerical example will show how we can calculate the certainty line for any given lot size.

Problem Statement: Find the process-average p such that a lot of size N will have number of defective items being greater than or equal to N times lot percentage (in our case AOQL of 3.0%). The probability of such measure should be smaller than or equal to the γ risk which is equal to 0.000033.

Solution: Let us consider the lot size N to be 100

Hence, the problem can be redefined as follows.

Find the process average p such that a lot size of 100 will have number of defective items being greater than or equal to 3 (i.e. 100×0.03).

Given:

N=100

Lot Percentage or AOQL = 3.0%

Risk (γ) = 0.000033 (Smallest number in Normal Distribution Table) (Montgomery and Runger 713)

To calculate:

$$P(c \geq 3) \leq 0.000033$$

Where c is binomial random variable with parameters N and p.

By the principle of normal approximation to the binomial distribution (Montgomery and Runger 132)

$$1 - P(c < 3) \leq 0.000033$$

$$1 - P\left(\left(\frac{X-Np}{\sqrt{Np \times (1-p)}}\right) < \left(\frac{3-100p}{\sqrt{100p \times (1-p)}}\right)\right) \leq 0.000033$$

Multiplying both sides by -1,

$$-1 \times \left[1 - P\left(\left(\frac{X-Np}{\sqrt{Np \times (1-p)}}\right) < \left(\frac{3-100p}{\sqrt{100p \times (1-p)}}\right)\right)\right] \geq -1 \times 0.000033$$

$$-1 + P\left(\left(\frac{X-Np}{\sqrt{Np \times (1-p)}}\right) < \left(\frac{3-100p}{\sqrt{100p \times (1-p)}}\right)\right) \geq -0.000033$$

Adding 1 on both sides,

$$P\left(\left(\frac{X-Np}{\sqrt{Np \times (1-p)}}\right) < \left(\frac{3-100p}{\sqrt{100p \times (1-p)}}\right)\right) \geq 1 - 0.000033$$

$$P\left(\left(\frac{X-Np}{\sqrt{Np \times (1-p)}}\right) < \left(\frac{3-100p}{\sqrt{100p \times (1-p)}}\right)\right) \geq 0.9999667$$

We know that, $P(Z \leq x) \geq 0.9999667 \Rightarrow x \geq 3.99$(approximately) (Montgomery and Runger 713)

$$\frac{3-100p}{\sqrt{100p\times(1-p)}} \geq 3.99 \tag{1}$$

$$(3-100p)^2 \geq [3.99 \times \{100p \times (1-p)\}]^2$$

$$9 + 10000p^2 - 600p \geq 1592.01p - 1592.01p^2$$

$$11592.01p^2 - 2192.01p + 9 \geq 0$$

$$p = \frac{2192.01 \pm \sqrt{2192.01^2 - (4 \times 11592.01 \times 9)}}{2 \times 11592.01}$$

Therefore $p = 0.184896$ or $p = 0.004199$

Case 1:

Substituting $p = 0.184896$ in the equation (1)

$$\frac{3 - 100\text{x}0.184896}{\sqrt{100 \times 0.184896 \times (1 - 0.184896)}} = -3.989972$$

Since -3.989972 < 3.99;

We can disregard the value $p = 0.184896$

Case 2:

Substituting $p = 0.004199$ in the equation (1)

$$\frac{3-100\text{x}0.004199}{\sqrt{100 \times 0.004199 \times (1-0.004199)}} = 3.990040$$

Since 3.990040>3.99;

We choose the value $p = 0.004199$

Hence for a given lot size of 100 if one knows the incoming process-average p to be less than or equal to 0.004199 or 0.4199% then one would not require to consider any sampling plan. The Table 5.1 represents some examples of various lots with different lot size and their corresponding certainty line. Similarly, from the above numerical example we can draw certainty line for any given lot size. In this use, one can utilize his/her knowledge of incoming process average in order to decide whether or not he/she must consider any sampling plan. This will improve the efficiency of sampling using Dodge-Romig sampling plan and reduce cost by considering sampling plan only when it is necessary in order to ensure the target AOQL.

Table 5.1: Certainty Line for Lots with Different Lot Size.

Lot Size	**Certainty Line**	**Gamma Risk**
100	0.41990%	0.000033
200	0.68050%	0.000033
300	0.86581%	0.000033
400	1.00757%	0.000033
500	1.11212%	0.000033
600	1.21525%	0.000033
800	1.36383%	0.000033
1000	1.47762%	0.000033
2000	1.81044%	0.000033
3000	1.98410%	0.000033
4000	2.09620%	0.000033
5000	2.17660%	0.000033
7000	2.28700%	0.000033
10000	2.39050%	0.000033
20000	2.55483%	0.000033
50000	2.71029%	0.000033
100000	2.79213%	0.000033

For instance, if the lot size is 100,000 then one would require considering sampling using Dodge-Romig sampling plan only if one knows that the incoming process-average p is greater than 2.79%. In a way, one does not have to even consider sampling for the first five columns from Table 3.1 listing five different non-overlapping ranges of incoming process average up to 2.40 %.

CHAPTER 6

THE DODGE-ROMIG SAMPLING PLAN (AOQL): EXPANSION OF TABLES

Considering Table 5.1, it is seen that the certainty line tends to grow larger with larger lot sizes, leaving the initial columns of ranges of incoming process average not of any use. I believe that the current plans can be made useful by focusing on ranges of incoming process-average values that are greater than the target AOQL which is 3.0% in our case. In this chapter, I will provide example expansions of some of the published Dodge-Romig AOQL plans. Before doing that, it is necessary to select the (n, c) pairs under different non-overlapping listing of ranges greater than the target AOQL of 3.0%. In this thesis, I have maintained equal width (0.6%) of non-overlapping ranges as I moved from the target AOQL to higher values. The (n, c) pairs must be chosen in such a way that each pair ensures target AOQL with minimum ATI respectively. For illustration purpose, I have considered the lot size of 8500(Median value of the range of 7001-10,000 range). The table below shows different (n, c) pairs which assures the target AOQL of 3.0% with minimum ATI respectively.

Table 6.1: Example Expansion of Dodge-Romig AOQL Plans

Lot Size	Process Average 3.01 to 3.60%		Process Average 3.61 to 4.20%		Process Average 4.21 to 4.80%		Process Average 4.81 to 5.40%		Process Average 5.41 to 6.00%		Process Average 6.01 to 6.60%	
	n	*c*	*n*	*c*	*n*	*c*	*n*	*c*	*n*	*c*	*n*	*c*
7001-10,000	441	20	147	7	64	3	45	2	45	2	28	1

A program in C (Appendix A) was written to identify (*n*, *c*) pairs which establish the target AOQL of 3.0%. It is to be noted that for every value of acceptance number c, there exists a unique sample size *n* whose AOQL will be approximately equal to 3.0%. Appendix B lists some of the (*n*, *c*) pairs obtained for a given lot size of 8500, all of which meeting the targeted AOQL of 3.0%. Note that there are only 39 pairs of (*n*, *c*) obtained. The 39^{th} pair of (*n*, *c*) is (848, 38). For *c* being 39 and onwards the corresponding unique sample size which meets the target AOQL of 3.0% are found out to be greater than 850. Considering those pairs for calculation and comparison of ATI adds complexity to the process. This is because, for a lot size of 8500 the calculation of both AOQL and ATI can rely upon binomial distribution only until $n/N \leq 0.10$. This is more of a hypergeometric phenomenon after we exceed the sample size of 850 for a lot size 8500. Appendix C shows the sample calculation for ATI for the range of 3.01 to 3.60%. ATI is a function of incoming process average and hence for the column 3.01 to 3.60% the incoming process average *p* is considered as 3.305% which is the mean of the range 3.01 to 3.60%. For all the (*n*, *c*) pairs listed in Appendix B, ATI is calculated for column 1 (3.01 to 3.60%) and each pair is compared against all the other pairs for the calculation of minimum ATI. In this way, I provide the example expansion of already published Dodge-Romig sampling plans.

CHAPTER 7

CONCLUSION AND FUTURE SCOPE OF RESEARCH

Conclusion

With the help of my numerical argument, it is evident that one does not have to know the exact incoming process-average p in order to adopt a sampling plan. Not only that, one does not even have to know the exact range which contains the incoming process-average p in order to consider a sampling plan. However, for any given lot size N, if the incoming process-average p is known, then one can compare the knowledge base about the incoming process-average p against the certainty line for that given lot size and decide whether or not he/she must consider any sampling plan. This will not only save cost and time, but also improve the overall efficiency of a manufacturing environment. In the case of the supplier being remotely located, such practice will help improve the relationship of the supplier with the customer company. Dodge-Romig plans as currently tabulated can also be made more useful by focusing upon the ranges of the (incoming) process-average values that are greater than the target AOQL.

Future Scope of Research

A further study can be conducted to rethink the expansion of Dodge-Romig plans for those ranges of the incoming process-average (listed in Table 6.1) which is greater than the target AOQL. It is noteworthy that the pair of n and c which gives the minimum ATI for the range of p from 4.81 to 5.40% is (45, 2). The similar pair provides us with the minimum ATI for the ranges that are smaller than the target AOQL (Table 3.1: under the range of 0.07 to 0.60%). Further research can be done to determine whether one must consider sampling at all in cases where, for example, the required AOQL is 3% and incoming process-average p is known to be in the range between 4.81% and 5.40%. In such cases, the rejection probability may be so close to one that virtually every lot is rejected, all items of a lot are inspected, and all non-conforming items are replaced by conforming items. Moreover, in this era of building good quality products, as opposed to the earlier era of screening out the bad quality ones, the supplier should improve the quality through statistical process control or be replaced by a better supplier.

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APPENDIX A

PROGRAM TO OUTPUT (n, c) PAIRS FOR ANY GIVEN LOT SIZE WHICH MEETS TARGET AOQL OF 3.0%.

```
#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include <math.h>
#define LINE_MAX  100
int fileFound = 0;
int printDetails = 0;
double Factorial(int n){
        double fctrl= 1;
        for (int i = n; i > 1; i--)
        {
                fctrl = fctrl * i;
        }
        return fctrl;
}
double SemiFactorial(int n, int j){
        double sFctrl= 1;
        for (int i = n; j > 0; j--, i--)
        {
```

```
        sFctrl = sFctrl * i;
    }
    return sFctrl;
}
double MathematicalCombination(int sample, int acptNbr){
    double fctrlCmb;
    double fctrlNC, fctrlC;
    switch (acptNbr)    {
    case 0:
        fctrlCmb = 1;
        break;
    case 1:
        fctrlCmb = sample;
        break;
    default:
        if (acptNbr == sample) {
            fctrlCmb = 1;
        }
        else {
            fctrlNC = SemiFactorial(sample, acptNbr);
            fctrlC = Factorial(acptNbr);
            fctrlCmb = fctrlNC / fctrlC;
```

```
                }
                break;
        }
        return fctrlCmb;
}
double AOQ(int lotSize, double pctD, int sample, int acptNbr){
        int sampleLessAcptNbr;
        double fctrlCmb;
        double pPower, onePPower, probabilityAcpt, totalProbabilityAcpt, lotSizeCalc,
calculatedAOQ;
        /* don't use power function when pctD is equal to 1.00 due to significant digit
error */
        /* when pctD is equal to 1, AOQ is zero; (1 - pctD) raised to a power is zero when
pctD is 1 */
        if (1 == (int) pctD) {
                calculatedAOQ = 0;
        }
        else {
                totalProbabilityAcpt = 0;
                for (int i = 0; i <= acptNbr; i++) {
                        fctrlCmb = MathematicalCombination(sample, i);
                        sampleLessAcptNbr = sample - i;
```

```
                pPower = pow(pctD, i);
                onePPower = pow((1 - pctD), sampleLessAcptNbr);
                probabilityAcpt = fctrlCmb * pPower * onePPower;
                totalProbabilityAcpt += probabilityAcpt;
            }
            lotSizeCalc = (double) (lotSize - sample) / lotSize;
            calculatedAOQ = totalProbabilityAcpt * pctD * lotSizeCalc;
        }
        return calculatedAOQ;
}
double CalculateAOQL(int lotSize, double pctD, int sample, int acptNbr, FILE *fDtl,
FILE *fC, FILE *fn){
        double aoq, aoqMax[3];
        int a, s, aMax[3], sMax[3];
        double p, pMax[3];
        char buffer[200];

        aoqMax[2] = pMax[2] = 0;
        aMax[2] = sMax[2] = 0;
        for (a = 0; a <= acptNbr; a++) {
            aoqMax[1] = pMax[1] = 0;
            aMax[1] = sMax[1] = 0;
```

```
for (s = (a + 1); s <= sample; s++) {
    aoqMax[0] = pMax[0] = 0;
    aMax[0] = sMax[0] = 0;
    for (p = pctD; p < (1 + pctD); p += pctD) {
        aoq = AOQ(lotSize, p, s, a);
        if (printDetails) {
            sprintf(buffer, "%5d  %5d  %5d  %10.6lf
%+15.10E\n", lotSize, a, s, p, aoq);
            fputs(buffer, fDtl);
        }
        if (aoq > aoqMax[0]) {
            aoqMax[0] = aoq;
            aMax[0] = a;
            sMax[0] = s;
            pMax[0] = p;
        }
    }
    sprintf(buffer, "%5d  %5d  %5d  %10.6lf  %+15.10E
MaxPerSample(Max per C=%d n=%d)\n",
        lotSize, aMax[0], sMax[0], pMax[0], aoqMax[0], a, s);
    fputs(buffer, fn);
    if (aoqMax[0] > aoqMax[1]) {
```

```
                    aoqMax[1] = aoqMax[0];
                    aMax[1] = aMax[0];
                    sMax[1] = sMax[0];
                    pMax[1] = pMax[0];
                }
            }
            sprintf(buffer, "%5d  %5d  %5d  %10.6lf  %+15.10E
MaxPerAcceptanceNbr(Max per C=%d)\n",
                lotSize, aMax[1], sMax[1], pMax[1], aoqMax[1], a);
            fputs(buffer, fC);
            if (aoqMax[1] > aoqMax[2]) {
                aoqMax[2] = aoqMax[1];
                aMax[2] = aMax[1];
                sMax[2] = sMax[1];
                pMax[2] = pMax[1];
            }
        }
        sprintf(buffer, "%5d  %5d  %5d  %10.6lf  %+15.10E
MaxForAllAcceptanceNbrs(Max for all C)\n",
            lotSize, aMax[2], sMax[2], pMax[2], aoqMax[2]);
        fputs(buffer, fC);
        return aoqMax[2];
```

```
}
void Usage(char *programName){
        fprintf(stderr,"Usage : %s L P S A\n",programName);
        fprintf(stderr,"        L is LotSize, P is PercentDefective, S is Sample and A is
AcceptanceNumber\n");
        fprintf(stderr,"   or : %s -f filename\n",programName);
        fprintf(stderr,"        filename contains LotSize, PercentDefective, Sample and
AcceptanceNumber\n");
        fprintf(stderr,"   or : %s -p L P S A\n",programName);
        fprintf(stderr,"   or : %s -p -f filename\n",programName);
        fprintf(stderr,"        -p causes each aoq result to be logged to outDtl.txt\n");
        fprintf(stderr,"Output: outDtl.txt - aoq details if -p option is specified\n");
        fprintf(stderr,"        outn.txt   - summary for sample\n");
        fprintf(stderr,"        outC.txt   - summary for acceptance number\n");
}
/* returns the index of the first argument that is not an option; i.e.
does not start with a dash or a slash
*/
int HandleOptions(int argc,char *argv[]){
        int i,firstnonoption=0;

        for (i=1; i< argc;i++) {
```

```
if (argv[i][0] == '/' || argv[i][0] == '-') {
        switch (argv[i][1]) {
                /* An argument -? means help is requested */
        case '?':
                Usage(argv[0]);
                break;
        case 'h':
        case 'H':
                if (!stricmp(argv[i]+1,"help")) {
                        Usage(argv[0]);
                        break;
                }
                /* An argument -f means the input data is in a file */
        case 'f':
        case 'F':
                fileFound = 1;
                break;
                /* An argument -p means the details are written to an
output file */
        case 'p':
        case 'P':
                printDetails = 1;
```

```
                break;
            default:
                fprintf(stderr,"unknown option %s\n",argv[i]);
                break;
            }
        }
        else {
            if (firstnonoption == 0) {
                firstnonoption = i;
            }
        }
    }
    return firstnonoption;
}
int main(int argc,char *argv[]){
    int argIndex, argCnt, lotSize, sample, acptNbr;
    char line[100];
    FILE *f = NULL;
    FILE *fn = NULL;
    FILE *fDtl = NULL;
    FILE *fC = NULL;
    double pctD, aoqL;
```

```
// Minimum 3 arguments - input file option
if (argc < 3) {
        Usage(argv[0]);
        return 1;
}
/* handle the program options */
argIndex = HandleOptions(argc,argv);
argCnt = argc - fileFound - printDetails;
if (!fileFound && argCnt < 5) {
        Usage(argv[0]);
        return 1;
}
fn = fopen("outn.txt", "w");
if (fn == NULL) {
        fprintf(stderr, "Unable to open sample output file: outn.txt\n");
}
fDtl = fopen("outDtl.txt", "w");
if (fDtl == NULL) {
        fprintf(stderr, "Unable to open detail output file: outDtl.txt\n");
}
fC = fopen("outC.txt", "w");
if (fC == NULL) {
```

```
			fprintf(stderr, "Unable to open acceptance number output file:
outC.txt\n");
		}
		if (fn && fDtl && fC) {
			if (fileFound) {
				f = fopen(argv[argIndex], "r");
				if (f == NULL) {
					fprintf(stderr, "File not found: %s\n", argv[argIndex]);
				}
				else {
					while (fgets(line, LINE_MAX, f) != NULL) {
						sscanf(line, "%d %lf %d %d", &lotSize, &pctD,
&sample, &acptNbr);
						aoqL = CalculateAOQL(lotSize, pctD, sample,
acptNbr, fDtl, fC, fn);
					}
					fclose(f);
				}
			}
			else {
				lotSize = atoi(argv[argIndex]);
				pctD = atof(argv[++argIndex]);
```

```
                    sample = atoi(argv[++argIndex]);

                    acptNbr = atoi(argv[++argIndex]);

                    aoqL = CalculateAOQL(lotSize, pctD, sample, acptNbr, fDtl, fC,
fn);

                }

                fclose(fDtl);

                fclose(fC);

                fclose(fn);

        }

        return 0;

}
```

APPENDIX B

LIST OF (*n*, *c*) PAIRS WHICH MEET THE TARGET AOQL OF 3.0%

FOR A GIVEN LOT SIZE OF 8500

Lot Size	Percent defective	n	c	PA	AOQL
8500	0.0370	258	12	0.836504638	0.030011227
8500	0.0370	281	13	0.837849438	0.029975591
8500	0.0370	303	14	0.842323601	0.030054998
8500	0.0370	326	15	0.843988359	0.0300299
8500	0.0370	349	16	0.845805943	0.03000989
8500	0.0360	372	17	0.871706009	0.030008018
8500	0.0360	395	18	0.874156535	0.030007223
8500	0.0360	418	19	0.876625478	0.03000658
8500	0.0360	441	20	0.879097998	0.03000558
8500	0.0360	464	21	0.881562531	0.030003825
8500	0.0360	487	22	0.884010196	0.030001018
8500	0.0360	510	23	0.886433959	0.029996926
8500	0.0360	533	24	0.888828635	0.029991377
8500	0.0360	556	25	0.891189992	0.029984243
8500	0.0360	579	26	0.893514931	0.029975427
8500	0.0360	601	27	0.897286594	0.030018354
8500	0.0360	624	28	0.899481297	0.030004155
8500	0.0360	647	29	0.901636899	0.029988229
8500	0.0360	669	30	0.90508008	0.030018417
8500	0.0360	692	31	0.907111406	0.029997427
8500	0.0360	714	32	0.910335243	0.030019214
8500	0.0360	737	33	0.912249029	0.02999346
8500	0.0360	759	34	0.915269673	0.030007493
8500	0.0360	782	35	0.917072594	0.02997727
8500	0.0360	804	36	0.919905066	0.029984143
8500	0.0360	826	37	0.922627866	0.029986925
8500	0.0360	848	38	0.925246179	0.029985813

APPENDIX C

SAMPLE ATI CALCULATION FOR THE RANGE CONTAINING INCOMING PROCESS-AVERAGE FROM 3.01 TO 3.60%

LOTSIZE	n	c	AOQL	c/n	Pa when P = 0.03305	ATI	Minimum
8500	12	0	0.0294	0	0.66811037	2829.07918	950.2276
8500	28	1	0.02958	0.035714	0.763677409	2030.12499	
8500	45	2	0.03022	0.044444	0.814242857	1615.57664	
8500	64	3	0.03014	0.046875	0.838632762	1425.29402	
8500	84	4	0.03005	0.047619	0.854570432	1307.93524	
8500	105	5	0.0299	0.047619	0.864905104	1239.12166	
8500	126	6	0.02992	0.047619	0.875010948	1172.65832	
8500	147	7	0.03002	0.047619	0.884558785	1111.28047	
8500	169	8	0.02997	0.047337	0.890555158	1080.78498	
8500	191	9	0.02998	0.04712	0.896485981	1051.09798	
8500	213	10	0.03002	0.046948	0.902237868	1023.15479	
8500	235	11	0.03006	0.046809	0.907752671	997.42417	
8500	258	12	0.03001	0.046512	0.911013119	991.429874	
8500	281	13	0.02998	0.046263	0.914304547	985.33093	
8500	303	14	0.03005	0.046205	0.919303239	964.471349	
8500	326	15	0.03003	0.046012	0.92241393	960.18854	
8500	349	16	0.03001	0.045845	0.925463517	956.546875	
8500	372	17	0.03001	0.045699	0.928436448	953.668551	
8500	395	18	0.03001	0.04557	0.931322624	951.630133	
8500	418	19	0.03001	0.045455	0.934115815	950.475985	
8500	441	20	0.03001	0.045351	0.936812557	950.227599	Minimum
8500	464	21	0.03	0.045259	0.939411375	950.890188	
8500	487	22	0.03	0.045175	0.941912221	952.457375	
8500	510	23	0.03	0.045098	0.944316071	954.914592	
8500	533	24	0.02999	0.045028	0.946624633	958.241547	
8500	556	25	0.02998	0.044964	0.948840127	962.414031	
8500	579	26	0.02998	0.044905	0.950965123	967.405257	
8500	601	27	0.03002	0.044925	0.953785645	966.047187	
8500	624	28	0.03	0.044872	0.955697591	972.925775	
8500	647	29	0.02999	0.044822	0.957530335	980.514277	
8500	669	30	0.03002	0.044843	0.959947381	982.652063	
8500	692	31	0.03	0.044798	0.961597103	991.849823	
8500	714	32	0.03002	0.044818	0.963767159	996.1089	
8500	737	33	0.02999	0.044776	0.965253113	1006.74009	
8500	759	34	0.03001	0.044796	0.967203482	1012.87785	
8500	782	35	0.02998	0.044757	0.968542831	1024.78643	
8500	804	36	0.02998	0.044776	0.970297494	1032.59049	
8500	826	37	0.02999	0.044794	0.971948479	1041.26737	
8500	848	38	0.02999	0.044811	0.97350245	1050.75925	