Efficient and effective large-scale vaccine distribution

John A. Muckstadt
Cornell University College of Engineering

Michael G. Klein
San Jose State University, michael.klein@sjsu.edu

Peter L. Jackson
Singapore University of Technology and Design

Robert M. Gougelet
Cornell University

Nathaniel Hupert
Cornell University

Follow this and additional works at: https://scholarworks.sjsu.edu/faculty_rasca

Recommended Citation

This Article is brought to you for free and open access by SJSU ScholarWorks. It has been accepted for inclusion in Faculty Research, Scholarly, and Creative Activity by an authorized administrator of SJSU ScholarWorks. For more information, please contact scholarworks@sjsu.edu.
Efficient and effective large-scale vaccine distribution

John A. Muckstadt, Michael G. Klein, Peter L. Jackson, Robert M. Gougelet, Nathaniel Hupert

A B S T R A C T

The goal of pandemic response is to provide the greatest protection, for the most people, in the least amount of time. Short response times minimize both current and future health impacts for evolving pathogens that pose global threats. To achieve this goal, efficient and effective systems are needed for distributing and administering vaccines, a cornerstone of pandemic response. COVID-19 vaccines were developed in record time in the U.S. and abroad, but U.S. data shows that they were not distributed efficiently and effectively once available. In an effort to “put vaccines on every corner,” pharmacies and other small venues were a primary means for vaccinating individuals, but daily throughput rates at these locations were very low. This contributed to extended times from manufacture to administration. An important contributing factor to slow administration rates for COVID-19 was vaccine transport and storage box size. In this paper, we establish a general system objective and provide a computationally tractable approach for allocating vaccines in a rolling horizon manner optimally. We illustrate the consequences of both box size and the number and capacity of dispensing locations on achieving system objectives. Using U.S. CDC data, we demonstrate that if vaccines are allocated and distributed according to our proposed strategy, more people would have been vaccinated sooner in the U.S. Many additional days of protection would have occurred, meaning there would have been fewer infections, less demand for healthcare resources, lower overall mortality, and fewer opportunities for the evolution of vaccine-evading strains of the disease.

1. Introduction

By definition, pandemics pose an acute health threat to the global population. Pandemic response therefore strives to provide the most protection to the greatest number of people in the least amount of time. To accomplish this, an efficient and effective system is needed for allocating, distributing and administering vaccines. Short response times are essential to minimizing both immediate health effects and future risks, for example through the evolution of new variants of the pandemic pathogen. The coronavirus disease 2019 (COVID-19) experience in the United States (U.S.) was marked by both dramatic success in the ultra-rapid development and licensing of effective vaccines in 2020 and, as we demonstrate in Sections 5 and 6, equally dramatic lapses in the efficient and effective distribution and administration of those vaccines in 2021 and 2022. Our goals in this paper are to: (1) present a quantitative performance measure for evaluating the consequences of a vaccine distribution system, (2) establish the necessity of designing any pandemic vaccine distribution system in accordance with established supply chain principles, (3) present a model and a computationally tractable algorithm for allocating supplies optimally each day for a properly managed system, (4) illustrate the importance of pooling and vaccine packaging (i.e., box size) on performance measures, and (5) show, using U.S. Centers for Disease Control and Prevention (CDC) data, the consequences of using policies employed in 2021 and 2022 rather than our proposed models.

Supply chain principles (see Muckstadt et al. (2001)) are foundational for the efficient and effective allocation, distribution, and administration of vaccines during a pandemic (Muckstadt et al., 2023). Modern industrial supply chains use a variety of metrics to assess performance across multiple domains (fulfillment, timeliness, wastage).
Pandemic vaccination systems lack a comparable standardized metric that focuses not on intermediary steps (e.g., transport time from factory to distribution warehouse) but on the ultimate outcome of the vaccination process, namely protected lives. Here we propose that the objective of such pandemic response efforts should be to maximize Vaccinated Person-Days (VPDs). VPDs measure the number of days from the time of an individual’s receipt of vaccine to the end of a vaccination campaign for all people. This will provide substantial protection from clinical disease and at least partial protection from onward transmission of the virus. Maximizing VPDs requires minimizing the time from a manufacturer’s production of vaccines to the time persons receive shots in their arms. Doing so provides the greatest protection for the most people in the least amount of time.

The need for a pandemic response strategy based on operations research has been stated by hospital administrators, for example, in testimony to the Pennsylvania State Senate (Murphy, 2021). Our objective is to describe an approach that fulfills this need. Our proposed approach provides the framework for vaccinating a population as soon as possible during a pandemic within a distribution system supply chain. Such a system for vaccines consists of information and business processes along with rules for allocating stocks optimally each day from manufacturing locations to distribution centers and from distribution centers to dispensing locations. This is particularly important at the beginning of an outbreak when a new vaccine is manufactured. For example, when a new vaccine was developed and approved for COVID-19, vaccine demand far exceeded the amount of supply available from initial manufacturing efforts. In this setting, vaccinating people as quickly as possible is essential, but this was not done in the U.S. (see infra). The consequence of this inefficiency was that more people became ill, more people were hospitalized and more people died than if the approach we describe had been followed. Speed is also an important consideration because a virus that becomes widely infective in a population mutates into variants. If modifications to initial vaccines are required to protect against a new variant, the surge of new infections will result in the need to provide new shots promptly, again with limited initial supply, and this cycle will repeat so long as there are new mutations. The importance of preventing the evolution of variants is therefore perhaps the single most important consequence of more rapid and effective roll out of vaccines, because of all the future lives saved from minimizing the potential for vaccine-escaping new virus variants to evolve.

Our paper is organized as follows. In the next Section 2, we provide background information on recent pandemics, review literature related to the design and operation of a vaccine distribution system and note existing mathematical approaches for allocating stocks in multi-echelon environments. In Section 3, we further develop the concept of maximizing VPDs, demonstrate that this concept can be stated as a linear program, and provide methods for finding the optimal strategy for administering vaccines. In Section 4, we propose a model and an algorithm for allocating daily supplies of vaccines throughout the distribution system, consistent with supply chain principles and the objective of maximizing VPDs established in Section 3. Specifically, we present a modeling approach for distributing vaccines in a three-echelon system consisting of a manufacturer’s warehouse, a collection of distribution warehouses supplied from a manufacturer’s warehouse, and a set of dispensing locations supplied from the distribution warehouses. In this Section, we also define algorithms for daily vaccine allocation in a rolling horizon manner. In Section 5, we illustrate these approaches with a case study that measures the consequences of both vaccine packaging box size and the number and capacity of dispensing locations on achieving system objectives, which shows the importance of following our proposed strategy. In Section 6, we present data obtained from the U.S. CDC pertaining to the effectiveness of the distribution and administration processes employed in the U.S. during 2021 and early 2022. We compare the effectiveness resulting from the use of these policies with our proposed approach. Our concluding remarks follow in Section 7. Note that the mathematical models presented in Sections 3 and 4 are not our main contribution to the evolving literature on pandemic response; rather, our contributions are (1) the introduction of a novel metric by which all such response systems may be measured (i.e., VPDs) and (2) our prescription for the design of efficient and effective systems that can maximize that metric in any response setting.

### 2. Literature review

In 2009, an influenza-A virus subtype H1N1 emerged that had never been seen before in humans. The World Health Organization (WHO) declared a global pandemic on June 11, 2009 (WHO, 2009). The H1N1 response included sending 78 million vaccine doses to 77 countries, with limited global production capacity and limited local capacity to transport and store the vaccines (Fineberg, 2014). In the U.S., vaccines were distributed in proportion to each state’s population. Vaccines and other pharmaceuticals were sent to regions which were not in need and those in need were left without a vaccine when they needed it most (Larson and Teytelman, 2012). Furthermore, vaccines needed to be stored and transported within a temperature range of 2°C to 8°C. The vaccines became unusable in freezing temperatures or with prolonged exposure to heat (CDC, 2010).

COVID-19, the illness caused by infection with the SARS-CoV-2 coronavirus, was declared a global pandemic on March 11, 2020 (AJMC, 2020). Critical supply chains were strained, personal protective equipment (PPE) was scarce, hospital Emergency Departments (EDs) were overwhelmed, and symptomatically unstable patients required transfer between hospitals (Ramachandran et al., 2020; Ranney et al., 2020; Livingston et al., 2020; CDC, 2020). The U.S. began vaccination after Emergency Use Authorizations for two-dose vaccines manufactured by Pfizer-BioNTech and Moderna, followed by a single dose vaccine, manufactured by Janssen/Johnson & Johnson. Despite the extensive Federal emphasis over the last two decades on influenza pandemic planning (CDC, 2017), the COVID-19 pandemic had a far greater impact than the H1N1 pandemic (Peters et al., 2020). According to Alam et al. (2021), significant challenges for the COVID-19 vaccine supply chain included “Lack of vaccine monitoring bodies” and “Inappropriate coordination with local organizations”.

Two types of questions must be addressed for pandemic planning: who should be given priority for receiving vaccines and how should the distribution system be structured including what approach should be employed to best allocate available supplies every day. The first question is answered by health system experts and policy makers. Throughout the COVID-19 pandemic, this question was answered differently by authorities in different states. We do not address this question in this paper. The answer to the second question must be based on several factors including how convenient local access to dispensing locations will be ensured for diverse populations. We also do not address the siting of dispensing locations. While Bertsimas et al. (2022) study where to locate mass vaccination facilities for COVID-19, we focus on the effects of the number and capacity of dispensing locations on system performance. Furthermore, we do not address how to determine the optimal number of distribution warehouses or the optimal number of dispensing locations having various throughput rates. We do show in Section 5, that having higher throughput locations is preferable. Dispensing locations should be supplied daily to ensure supplies are used expeditiously with short, predictable and repeatable lead times. This is an essential requirement to ensure the maximum number of persons are vaccinated in the shortest amount of time.

Duijzer et al. (2018) reviewed the vaccine supply chain literature in four sections: product, production, allocation and distribution. Product studies help identify what to include in the seasonal influenza (flu) vaccine each year (Wu et al., 2005; Ozaltn et al., 2011), the optimal number of doses per vial (Dhamodharan and Proano, 2012), when to commit or defer a seasonal flu vaccine (Kornish and Keeney, 2008; Ozaltn et al., 2011) or update to a new one (Cho, 2010). For pediatric vaccination programs, selection problems include combining vaccines
to immunize against several diseases with one injection (Hall et al., 2008), pricing decisions (Robbins and Lunday, 2016) and vaccination scheduling (Engineer et al., 2009; Smalley et al., 2011).

Studies on vaccine production consider the uncertainty in supply and price for required ingredients (Kazaz et al., 2016), leading to supplier selection challenges (Federguen and Yang, 2008), supplier competition (Federguen and Yang, 2009), and advance selling (Cho and Tang, 2013). To manage production uncertainty, Chick et al. (2005) propose a cost-sharing contract between the government public health service and the vaccine manufacturer. However, when considering both demand-side and supply-side characteristics, Arifoglu et al. (2012) found that the equilibrium seasonal flu vaccine demand may be greater than the socially optimal demand. To address the need for seasonal flu vaccine market coordination, new contracts are proposed by Dai et al. (2016), Chick et al. (2017) and Arifoglu and Tang (2020).

Motivated by the H1N1 pandemic, Teytelman and Larson (2013) develop heuristics for the vaccine allocation problem with the objective to reduce the sum of total infections using real-time data on infection rates over time. Tanner and Naimo (2010) develop a branch-and-cut algorithm to improve the efficiency for solving a mixed-integer programming formulation of a vaccine allocation problem with a minimum cost objective. Yarmand et al. (2014) formulate the vaccine allocation problem for a single decision maker as a two-stage stochastic linear program to minimize the expected number of vaccine doses administered for an infectious disease outbreak. Enayati and Ozaltin (2020) proposed a nonlinear optimization formulation to minimize the amount of vaccine used with equity constraints by age group. Recently, Thul and Powell (2023) proposed a reinforcement learning approach to allocate COVID-19 vaccines and test kits from stockpiles to population zones, considering the uncertainty in the level of infection in each zone.

The two-dose vaccines developed for COVID-19 led to the need for new vaccination plans. Tuite et al. (2021) note that during early distribution of COVID-19 vaccines, reserving supply for second doses delays the receipt of first doses, and that it is better to provide more first doses in order to vaccinate more people as soon as possible. Mak et al. (2022) compare policies for two-dose vaccines including holding back second doses and stretching the time between doses. Combined with a Susceptible, Infected, Recovered (SIR) Epidemic Model, they suggest that, rather than two-dose vaccines, a single-dose vaccine with lower efficacy can be more effective in slowing down infections. In Section 3, we consider the relationship between production capacity and vaccination policies for two-dose vaccines. We provide methods for finding the optimal vaccination plan, use examples to demonstrate the use of the methods and emphasize that the system’s objective should be to maximize VPDs.

Supply chain design, inventory and dispensing locations have specific characteristics that impact vaccine distribution. Supply chain design considerations include vaccine perishability (Masoumi et al., 2012; Chung and Kwon, 2016) and the need to keep vaccines in a temperature controlled environment (Saif and Elhedhili, 2016; Lin et al., 2020; Lim et al., 2022). Inventory considerations include stockpile management, studied by Jacobson et al. (2006) for pediatric vaccines, and vaccine inventory control policies to reserve inventory for high priority groups such as healthcare professionals and the elderly (Samil et al., 2012; Lee et al., 2015). Vaccine dispensing facility locations also need to be determined with capacity needs managed accordingly (Lee et al., 2009; Ekici et al., 2014; McCoy and Johnson, 2014; Ramirez-Nafarrate et al., 2015). Another important issue is the sequence in which individuals are vaccinated. Recent studies on equitable vaccine distribution for COVID-19 include in-country vaccine allocation (Balcik et al., 2022; Dastgoshade et al., 2022) and between countries vaccine allocation (Katz et al., 2021; Ye et al., 2022). Although health equity is not studied in this paper, our proposed measure, VPDs, could be useful to compare the extent of vaccination coverage achieved by one group to another. Considering that the public health system determines the sequence of vaccination for subgroups, it would be possible to measure inequities that occur in VPDs within a country. VPDs could also be used to measure inequities transparently between countries and identify the extent of progress made in reducing the gap in vaccine access for people in low- and middle-income countries (LMICs) compared to high income countries (HICs).

There is also considerable literature on managing inventories in multi-echelon systems. In the seminal paper on this topic, Clark and Scarf (1960) demonstrate the importance of managing inventories on an echelon basis rather than on an installation basis. Eppen and Schrage (1981) discuss the advantages of pooling inventories in two echelon systems. The possibility of inventory imbalance in the lower echelons is discussed in both papers. Imbalance occurs when inventories needed by one location are on hand at a different location causing increased expected costs. Due to this imbalance, finding optimal policies becomes extremely difficult and usually computationally intractable. In the context of vaccines, ensuring that imbalance cannot occur is essential due to the limited supplies and refrigeration requirements. In Section 5, we present an optimization model in which imbalance is not possible. Our proposed model employs news-vendor-like cost functions, as described in Muckstadt and Supatra (2010). When the Principles of Supply Chain Management are in place as described in Muckstadt et al. (2023), imbalance will not occur. This is a key characteristic of our modeling effort.

3. Long-range tactical planning: the relationship between production capacity and vaccination policies

Suppose a production plan exists for two-dose vaccines over a planning horizon consisting of $P$ periods, where $P$ is the last period in which vaccine supply limits vaccinations. Thereafter, vaccinations may be limited by other factors such as vaccinator capacity or reduced vaccine demand. An important question is, how many persons should receive a first dose in period $p$ given a supply plan of $S_k$, units, $p=1,...,P$? The answer to this question is not immediately obvious. Some suggest that only one half of a period’s capacity should be allocated for first doses.

Define $V_p$ as the number of persons receiving their first dose in period $p$. Let $L$ be the number of periods following the first dose in which the second dose will be administered. Thus if the first dose is given in period $p$, then the second dose will be administered in period $p+L$.

The relationship between the production plan and a vaccination plan may be expressed as follows. For periods $p \leq L$,

$$V_p \leq \sum_{j=p}^{p-1} S_j - \sum_{j=p+1}^{p-1} V_j.$$  

For periods $p > L$,

$$V_{p+L} \leq \sum_{k=p-L}^{p-1} S_k - \sum_{j=p-L}^{p-1} V_j - \sum_{k=p-L}^{p-1} V_k.$$  

Thus the number of persons vaccinated in period $L+j$ cannot exceed the total production available through that period minus the number of persons receiving their first dose through the previous period, $L+j-1$, minus those who require second doses through period $L+j$.

Suppose the production quantity will be $S$ units each week and the number of persons receiving the first dose is the same each week. Then the total supply over the $P$ week planning horizon is $SP$ doses. The total number of vaccine doses administered is $VP + V(P - L) = VP(2P-L)$. The number of persons receiving first doses each week occurs when total supply equals total demand. Then $V = SP(2P-L)$. When $P$ is large relative to $L$, say $P = 26$ weeks and $L = 3$ weeks, then $V \approx S/2$. Therefore initially only one half of the first week’s production is used to vaccinate people and the other half is held back for future use. This is not a surprising observation since for every first dose there must be
a corresponding second dose $L$ periods later. Is this the best way to use the production capacity? This requires us to define what we mean by best.

By best we maximize the total number of periods that persons have received first doses over the planning horizon. Persons receiving their first dose in period 1 have $P$ periods of protection. Persons receiving their first dose in period 2 have $P - 1$ periods of protection, and, ultimately, persons receiving their first dose in period $P$ will have 1 period of protection. If a person receives a first dose in period $p$, then the person will receive a second dose in period $p + L$ and hence, the person will be doubly-vaccinated through period $P + L$ and beyond. In the U.S., for example, in California, Michigan, New York and Pennsylvania, the COVID-19 vaccination policy included scheduling both first and second doses so that second doses would be administered $L$ periods after the first dose at dispensing locations.

The goal of maximizing the number of vaccinated persons in the shortest amount of time is therefore to maximize the total number of vaccinated person-periods, that is, to solve the following linear program: maximize $\sum_{p=1}^{P} V_p(P - p + 1)$ subject to constraints given in (1). However, there is another way to express the constraints for this linear program. Suppose we rewrite the constraints (1) as material balance constraints, so that our optimization problem becomes:

$$\text{maximize } \sum_{p=1}^{P} V_p(P - p + 1)$$

subject to $I_p = I_{p-1} + S_L - V_p - V_{p-L}, \quad p = 1, \ldots, P,$

where $I_p$ is the period $p$ ending inventory and the other variables and parameters are defined as before. Our desire is to provide a vaccine distribution system that provides the most protection, for the most patients, in the least amount of time. The objective is, therefore, to maximize what we term Vaccinated Person-Days (VPDs). Note that by maximizing this expression, we also maximize the number of second dose VPDs.

Let us return to the example where the production quantity is $S$ units each week and $V_p = S/2$ for all periods. The value of this policy is $(S/2) \sum_{p=1}^{P} (P - p + 1) = (S/2) \left( \frac{P(P + 1)}{2} \right)$.

Another possible policy is as follows. For periods 1 through $L$, let $V_p = S$, and for periods $L + 1$ through period $2L$, $V_p = 0$. During these latter periods only second doses are administered. What is the number of vaccinated persons in periods of protection with this policy? Consider the first 2$L$ periods. The value is $S(P + P - 1 + P - 2 + \cdots + P - (L - 1)) = S L(P - (L - 1)/2).

Compare this to the value associated with the previous policy for these same periods. The value of the first policy is $(S/2)(P + P - 1 + P - 2 + \cdots + P - 2L + 1)$. But $(S/2)(P + P - 1 + P - 2 + \cdots + P - 2L + 1) < S L(P - (L - 1)/2).$ When, for example, $P = 12$ and $L = 3$, the first policy has a value of $S/2$ whereas the second policy has a value of $33S$. The difference occurs because the value of the first dose administered to persons in periods $L + 1$ through $2L$ in the first policy has fewer vaccinated person-periods than the same persons being vaccinated in one of the periods 1 through $L$ in the second policy.

### 3.1. Determining an optimal vaccination plan

To begin, let us assume that $S_p \geq S_{p-L}$. That is, the supply in period $p$ is at least as large as the supply $L$ periods previously. In practice, this is most likely to be the case, for example, as it was in the COVID-19 pandemic. If the supplies are monotonically non-decreasing, then this condition holds. However, the condition that $S_p \geq S_{p-L}$ is more general. We now state the following theorem:

**Theorem 1.** When $S_p \geq S_{p-L}$, the optimal vaccination plan is $V_p = S_p - V_{p-L}$.

*Proof.* See Appendix.

Under the assumptions of Theorem 1, when $S_p \geq S_{p-L}$, then $V_p = S_p$ for $p = 1, \ldots, L$. When this is not the case, we can use the simplex method to obtain the optimal solution to the following linear program:

$$\text{maximize } \sum_{p=1}^{P} V_p(P - p + 1)$$

subject to $I_p = I_{p-1} + S_p - V_p - V_{p-L}, \quad I_p \geq 0.$

(2)

Let us consider a more interesting example. Suppose two manufacturing warehouses each supply a vaccine requiring second doses administered $L = 3$ weeks following first dose administration. Suppose that the planned production quantities arriving at these warehouses each week are as displayed in Table 1. Observe that the supply plan for manufacturer warehouse 1 satisfies the condition $S_1 \geq S_{p-L}$ whereas the supply plan for manufacturer warehouse 2 does not.

The maximum vaccinated person-weeks are $923,700$ and $675,900$ for warehouses 1 and 2, respectively, which can be obtained using the proposed linear program (2). The resulting 26-week optimal vaccination plans are provided in the plots shown in Fig. 1.

For manufacturer warehouse 1, we see that the total number of vaccinations increases throughout weeks 1 through 26 as expected. However, the first plot shows that the number of first dose vaccinations drops every three weeks due to the requirement for second doses. For manufacturer warehouse 2, the total number of vaccinations is not monotonically increasing because we need to have a supply for second doses. In week 24, manufacturer warehouse 2 would not have a sufficient supply to meet second dose requirements if any first doses were administered. The second dose requirements are satisfied from that week’s production plus stock carried into that week. Rather than reducing first doses administered in week 24 – $3 = 21$, the required reduction in first doses occurs in week 23. This ensures that the number of vaccinated person-weeks is maximized.

Our goal in the example provided in Table 1 and Fig. 1 is to show the implications of a dynamic supply environment in which the assumptions of Theorem 1 do not hold. The graphs in Fig. 1 show how the vaccination plan depends on the dynamics of vaccine production. Therefore, to maximize VPDs, it is best for vaccine production to ensure that $S_p \geq S_{p-L}$.

### 4. Distributing vaccines in a dynamic pandemic environment: A myopic allocation model

In Section 3, we established that the objective of a vaccine distribution system should be to maximize the number of VPDs. To implement this policy, we develop a modeling environment and an algorithm for allocating available supplies daily, in a rolling horizon manner, throughout the distribution system supply chain.

Consider a distribution system supply chain for managing the manufacture and distribution of vaccines with three echelons. This vaccine distribution supply chain consists of a manufacturing facility at which vaccines are being produced, packaged, and stored; a collection of regional distribution warehouses that receive shipments from the manufacturer’s warehouse daily; and a collection of dispensing locations that receive daily vaccine supplies from a regional distribution warehouse. During the early phase of distributing vaccines to the public, production rates and cumulative production to date are well below overall demand for immunizing the entire U.S. population. Once supply exceeds demand, the need to allocate supplies optimally is no longer necessary. In Section 3, we denoted the time that this occurs as period $P$. When the supply chain is managed in a manner consistent with the Principles set forth in Muckstadt et al. (2023), all lead times will be short, predictable and repeatable, and box sizes and dispensing location capabilities will be consistent. Finally, considering that the supply of vaccines is limited, inventories must be pooled to ensure
that the maximum number of people in high-risk populations receive vaccines in a timely manner.

Daily usage quantities at a dispensing location are related to the demand for vaccines, the capacity to administer the vaccines and any losses of supply (breakage, lack of proper handling or temperature control, etc.). When daily forecasts for usage are made for each dispensing location, forecast errors are noted. Based on both the forecasts and forecast errors, we assume we can construct probability distributions for usage over appropriate lead times. The daily usage at dispensing location, forecast errors are noted. Based on both the forecasts and

### 4.1. Nomenclature

Let us now state basic nomenclature that will be used in our model. The model is a periodic review model and hence time is measured in periods, or days. Let

- \( t \) refer to a day in the planning horizon of length \( T \) days,
- \( k \in K \) refer to a location: \( k = 0 \) is the manufacturer’s warehouse, \( k = i \in I \) is a regional distribution warehouse, \( k = j \in J \) is a dispensing location, so \( K = \{0\} \cup I \cup J \).
- \( H(i) \subseteq J \) is the set of locations receiving vaccine supply from regional distribution warehouse \( i \in I \).
- \( T_I \) is the lead time to ship vaccines to distribution warehouse \( i \) from the manufacturer’s warehouse plus the time needed for this shipment to be available for distribution.

We assume each dispensing location is in only one set \( H(i) \) and that shortage and holding costs are charged each day at each dispensing location. Let \( D_{ij} \) represent the usage random variable at dispensing location \( j \) on day \( t \). We assume that \( D_{ij} \) is integer valued and assumes values in the interval \( [\alpha_j, \beta_j] \). Let \( \gamma_j = \beta_j - \alpha_j \). We assume that \( \gamma_j < \alpha_{j+1} \) for all \( t \). By making this assumption, we are guaranteeing that the net inventory at a dispensing location at the end of a day prior to an allocation decision is always less than the minimum usage in the following day. Hence any positive amount of inventory on-hand at a dispensing location at the end of a day will be used on the following day with certainty. While globally optimal solutions may differ from locally optimal decisions, because of our assumptions about the usage random variable and the cost function, the optimal policy for managing the allocation of vaccine supplies is found by acting locally, that is, we should employ a myopic policy. That means we act optimally through time by acting optimally each day. The models we will develop are based on this observation. Therefore, if the distribution warehouse has an adequate supply, the vaccine supply at a dispensing location should be in the interval \( [\alpha_j, \beta_j] \). The exact value will depend on the shortage and holding costs and on the amount of supply available for allocation at the distribution warehouse.

Let us introduce some additional nomenclature. Let

- \( S_0 \) be the vaccine supply available for allocation from the manufacturer’s warehouse to the distribution warehouses on day \( t \),
- \( S_i \) be the vaccine supply available for allocation from distribution warehouse \( i \) at the beginning of day \( t \),
- \( y_{0i} \) be the amount allocated from the manufacturer’s warehouse to the distribution warehouse \( i \) on day \( t \),
- \( y_{ij} \) be the amount allocated from distribution warehouse \( i \) to dispensing location \( j \) on day \( t \).

Recall that this amount will be available for use at the dispensing location on that day.

Thus any allocation on day 1, the current day, must satisfy \( S_{0i} \geq \sum_{j \in H(i)} y_{0j} \) and \( S_{0i} \geq \sum_{j \in H(i)} y_{ij} \geq 0 \).

### 4.2. A model for allocating vaccines to dispensing locations

We now construct a model and describe an algorithm for allocating distribution warehouse supplies to dispensing locations. Our approach implements the myopic policy for managing supplies.

We assume that events occur as follows each day for distribution warehouse \( i \) and the dispensing locations in \( H(i) \). First, for a given day \( t \), we observe the net inventory at dispensing location \( j \), which we denote by \( s_j \) and the on-hand supply at distribution warehouse \( i \). Second, knowing these inventory levels, we construct probability distributions

---

Table 1: Planned production quantity by week.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturer Warehouse 1</td>
<td>2000</td>
<td>2100</td>
<td>2200</td>
<td>2500</td>
<td>3700</td>
<td>4400</td>
<td>4600</td>
<td>4800</td>
<td>5000</td>
<td>5200</td>
<td>5400</td>
<td>5600</td>
<td>5800</td>
</tr>
<tr>
<td>Manufacturer Warehouse 2</td>
<td>2000</td>
<td>2100</td>
<td>2200</td>
<td>2500</td>
<td>3700</td>
<td>3800</td>
<td>3900</td>
<td>4000</td>
<td>4100</td>
<td>4200</td>
<td>4000</td>
<td>3800</td>
<td>3600</td>
</tr>
</tbody>
</table>

---

Fig. 1. 26-week vaccination plans for manufacturer warehouses 1 and 2.
of the day’s usage and cost functions for all dispensing locations, and make allocations to them. Third, usage occurs and, fourth, shortage and holding costs are incurred at the dispensing locations.

Define $G_j(S_j)$ to be the expected cost associated with having a supply of $S_j$ vaccines available to satisfy usage requests at dispensing location $j$. Let $b$ denote the per unit shortage cost and $r$ the per unit holding cost. Then $G_j(S_j) = b \sum_{d \leq S_j} (d - S_j) P(D_j = d) + r \sum_{d \leq S_j} (S_j - d) P(D_j = d)$, where $P(D_j = d)$ is the probability that usage at dispensing location on the current day is $d$, which is a newsvendor cost expression. Next, define

$$\Delta G_j(S_j) = G_j(S_j + 1) - G_j(S_j) = -b + (r + b) \sum_{d \leq S_j} P(D_j = d). \quad (3)$$

To find the optimal allocation of distribution warehouse $i$’s supply to all $j \in H(i)$, solve

$$\min_{j \in H(i)} \left\{ \sum_{j \in H(i)} G_j(y_{ij} + x_j) \right\} \text{ s.t. } S_j \geq \sum_{j \in H(i)} y_{ij}, \text{ and } y_{ij} \geq 0. \quad (4)$$

Since supplies are not shared between distribution warehouses in our model, every distribution warehouse would allocate supplies using model (4).

Let $S_j = y_{ij} + x_j$. Observe that dispensing location $j$’s minimum expected cost occurs when $S_j$ is the smallest value for which $\Delta G_j(S_j) \geq 0$, that is, the smallest value for which

$$\sum_{x \leq S_j} P[D_j = x] \geq \frac{b}{r + b}. \quad (5)$$

Let $S_j^*$ denote this optimal value for $S_j$. If $S_j^* \geq \sum_{j \in H(i)} (S_j^* - x_j)$, then each dispensing location will incur the minimum expected cost on day 1 with an allocation $y_{ij} = S_j^* - x_j$. Consequently, in this case, $\sum_{j \in H(i)} G_j(S_j^*)$ is the minimum total expected dispensing location cost on day 1.

Suppose, however, that $S_j^* < \sum_{j \in H(i)} (S_j^* - x_j)$. Then we determine the optimal allocations $y_{ij}^*$ using Algorithm 1.

**Algorithm 1: Algorithm for Finding $y_{ij}^*$ when $S_j^* < \sum_{j \in H(i)} (S_j^* - x_j)$**

**Step 0.** For all $j \in H(i)$, set $U_j = x_j$, $V \leftarrow 0$.

**Step 1.** Compute $G_j(U_j)$, for all $j \in H(i)$.

**Step 2.** while $V \leq S_j$ do

a: Select $j^*$ such that $\Delta G_j(U_j) = \arg\min_j \Delta G_j(U_j) > 0$. Note that $\Delta G_j(U_j) > 0$.

b: Set $U_{j^*} = U_{j^*} + 1$.

c: Set $U_j = U_j + 1$.

d: Set $V \leftarrow V + 1$.

e: Recompute $G_j(U_j)$.

**Repeat.*

**Step 3.** Compute $G_j(U_j)$ and $\sum_{j \in H(i)} G_j(U_j)$, the minimum total expected dispensing location cost on day 1 given $S_j < \sum_{j \in H(i)} (S_j^* - x_j)$. Set $y_{ij}^* = U_j - x_j$.

Observe that recomputing $\Delta G_j(U_j)$ in Step 2 (e) can be quickly accomplished using expression (3) and noting that $\sum_{x \leq S_j} P[D_j = x] = \sum_{x < S_j} P[D_j = x] + P[D_j = S_j]$.

Algorithm 1 provides an efficient way to determine the optimal allocations to the dispensing locations when $S_j^* < \sum_{j \in H(i)} (S_j^* - x_j)$.

There are several reasons why we constructed the model as we did. While our mathematical formulation permits fair and transparent allocation decisions to be made, it also is easy to implement. However, these reasons alone do not justify the use of our models.

To this point we have said nothing about the probability distributions that are key elements in our model and provide the basis for determining the allocations from a distribution warehouse to dispensing locations. In practice, these probability distributions are represented by triangular distributions. Users are asked to provide minimum, maximum and most likely usage for a day at each dispensing location. This approach is easily understood by health system officials and is implemented easily (Hupert et al., 2002). Perhaps of greatest importance, the recommended allocation decisions make sense to public health officials.

Another important question pertains to the way we formulated our optimization model. The formulation employs a newsvendor cost function to determine the allocation of vaccines. To run the model, users are asked to specify the desired probability of meeting all demand over the appropriate lead time. By answering this question, we obtain an estimate of $b/(r + b)$. Since each unit short corresponds to a missed VPD, we set $b = 1$ and then solve for $r$. The resulting values are then used to obtain the corresponding allocations.

Because vaccine supplies are initially limited, achieving the desired probability of meeting all demand is not likely. Knowing only this desired probability, we would have no way to allocate supplies other than to use simple rules such as population-based allocations. Hence, having an approach that considers the factors included in our model is essential. Our algorithms yield allocations that are consistent with the primary goal of maximizing VPDs while also ensuring decision makers can prioritize those who receive the vaccines each day. That is, depending on population demographics, availability of dispensing capability, and other factors, the desired probability of satisfying demand each day may differ by dispensing location and over time. By implementing the model in a rolling horizon manner at each allocation entity ensures the process for allocating vaccines is transparent and consistent across the country.

4.3. Manufacturer warehouse to distribution warehouse allocations

We will now see how to optimally allocate vaccines from the manufacturer’s warehouse to the distribution warehouses. As was the case in our previous model, allocations in each period are limited by the available supply at a location. In particular, we know that $S_0 \geq \sum_{j \in H(i)} y_{0j}$.

For simplicity, let us assume that $T_0 = 1$ for all distribution warehouses. Consistent with adherence to the Principles as described in Muckstadt et al. (2023), we assume that events on a day occur in the following manner.

(1) The net inventories at all dispensing locations, the on-hand inventory at each distribution warehouse and in-transit to them from the manufacturer’s warehouse, and the on-hand vaccine stock at the manufacturer’s warehouse are known at the beginning of day $t$ prior to any vaccine usage at dispensing locations or making allocation decisions on day $t$.

(2) Allocation decisions, based on net inventories and usage probabilities, are made at each distribution warehouse using only stock on-hand at the beginning of the day.

(3) Allocations from distribution warehouses arrive at dispensing locations in time for use on day $t$.

(4) Shipments from the manufacturer’s warehouse made on day $t - 1$ arrive at the distribution warehouses.

(5) Allocation decisions are made at the manufacturer’s warehouse for vaccines that will arrive at distribution warehouses on day $t - 1$.

(6) Vaccine dispensing occurs at all dispensing locations.

(7) Inventory arrives at the manufacturer’s warehouse from the production facility.

Based on our understanding of the sequence of daily events, we can establish the uncertainty that must be accounted for when making allocation decisions at the manufacturer’s warehouse. We must first determine the earliest time today’s shipment from the manufacturer’s warehouse could arrive and be used at a dispensing location. Since $T_0 = 1$, today’s shipment will arrive at distribution warehouses on day $t + 1$. Note that they are available to be shipped to dispensing locations only on day $t + 2$ or beyond. The vaccine supplies at a distribution warehouse and its dispensing location partners needed to meet usage requirements over days $t$, $t + 1$ and $t + 2$ consist of the net inventories at all dispensing locations at the beginning of day $t$, the on-hand inventory at each distribution warehouse on day $t$, the in-transit supply from the manufacturer’s warehouse that will arrive on day $t$, and allocation from the manufacturer’s warehouse on day $t$. This stock must be adequate to satisfy dispensing requirements on days $t$ through $t + 2$. 
Let $Y_i$ denote the sum of on hand inventories at all dispensing locations $j \in H(i)$ at the beginning of day $t$, the on-hand inventory at distribution warehouse $i$ at the beginning of day $t$, and the in-transit amount from the manufacturer’s warehouse that will arrive at distribution warehouse $i$ on day $t$. Let $X_i$ be the random variable for total usage at the dispensing locations $j \in H(i)$ over days $t$ through $t+2$, and let $P[X_i = x]$ be the probability that $X_i = x$. The supply available to meet this demand is $Y_i + y_{01}$. For simplicity, let us assume that $t = 1$.

Let $G_i(Y_i + y_{01} - x_i)$ be the cost incurred if the supply is $Y_i + y_{01}$ and the usage over days 1 through 3 is $x_i$. Since $X_i$ is a random variable, the unconditional imputed expected cost is $\sum_{x_i} G_i(Y_i + y_{01} - x_i) P[X_i = x_i]$.

We have used the term *imputed expected cost* in the preceding discussion. The cost expression does not represent an incurred expected cost at distribution warehouse $i$ but rather the implied expected costs that will be incurred at the dispensing locations supplied by that distribution warehouse. To determine the optimal allocation of the manufacturer’s warehouse’s stock of $s_{01}$, we solve

$$\min \sum_{x_i} G_i(Y_i + y_{01} - x_i) P[X_i = x_i] \text{ s.t. } s_{01} \geq \sum_{x_i} y_{01}, \text{ and } y_{01} \geq 0.$$

We propose approximating $P[X_i = x_i]$ by a normal distribution. This is likely to be appropriate since $X_i$ is the sum of the usage random variables of the dispensing locations $j \in H(i)$ over a multi-day horizon. When $T_0 > 1$, as is likely to be the case, then the normal approximation will certainly be appropriate. Explicit expressions exist for $G_i()$ and the optimal allocation will then be a fractile one.

5. Case study

In this section, we illustrate the importance of managing a vaccine distribution system that employs the optimization methods described in the previous sections. We demonstrate the consequences of both box size and the number and capacity of dispensing locations on achieving system objectives. In our illustration, three potential dispensing system designs are compared for New Hampshire (NH), a state with a population of approximately 1,330,000. The first is one that has been used in NH consisting of four large vaccination locations and 9 smaller ones. The second system consists of four large vaccination sites and 43 smaller sites, with varying dispensing capacities among the smaller locations. Some of these smaller locations may be mobile, moving from one physical location to another on a predetermined schedule. In the third system, the general population would be vaccinated in 181 of the 200 pharmacies in the state.

In all scenarios, individuals receive two doses of the vaccine. We also consider box size restrictions. For the Moderna vaccine, a box contains 100 doses. We explore the effect of box size on our performance measures. Specifically, we let the box size be either 1 or 100 doses. For each scenario, we simulate system performance over a 180 day horizon following a warm-up period. By administering 9800 doses per day for 180 days, it is possible to vaccinate approximately 70% of the NH population, which is the number of persons expected to desire vaccination.

Data pertaining to the three scenarios are displayed in Table 2. For each dispensing location type we provide the minimum, maximum and most likely vaccine usage rates per day. We are assuming supply and vaccination capabilities are known and unchanging over the 180 day horizon, and therefore these numbers remain constant. The maximum usage on a day represents the most optimistic case. If every aspect of the dispensing system’s process works perfectly and all persons requesting vaccinations present themselves, then the maximum could be observed. When things go poorly, the usage could be as low as the minimum number. However, the most likely daily usage is estimated for each site. We ask decision makers to make these estimates daily for each dispensing location. For simplicity, we assume usage has a triangular distribution with these parameter values. For these experiments, we set the desired probability of not running out of stock to 0.90 for every dispensing location. Our results are not dependent on the choice of these probabilities. The values displayed in Table 2 were selected so that the coefficient of variation of the system’s usage process for all three scenarios would be approximately equal to 0.1. Thus, from a system perspective, daily usage should most often range from a low of 9700 to a high of 9900 doses, which is a predictable and repeatable system environment. Dispensing locations do experience more variability individually each day which, in turn, impacts system performance. As stated earlier, we do not prescribe who should receive vaccinations on a day. Priorities must be established by officials and persons should be scheduled accordingly for the administration of the vaccine.

We report three principle performance measures over the 180 days: total number of vaccinations, VPDs and missed vaccination opportunities. The latter measure is of particular interest. Suppose on some day $j$ there is one unit of unused supply. Then a person who is planned to be vaccinated on day $j + 1$ could be vaccinated on day $j$. Furthermore, a person who would have been vaccinated on day $j + 2$ could be vaccinated one day earlier as well. In general, a person who would previously be vaccinated on day $j + k$ could be vaccinated on day $j + k − 1$. Thus this cascading would result in a possible increase of 180 − $j + 1$ VPDs. Thus for every unused dose on every day there is a corresponding number of missed vaccination person-days. Of course, if there is more than one unit of unused supply, the number of missed vaccination person-days increases accordingly. The design of the dispensing system has a very substantial impact on missed vaccination opportunities, as shown in Tables 3 and 4.

We simulated the system’s operations using Algorithm 1 to determine allocations to dispensing locations daily. Each replication is a 180 day simulation of the vaccine distribution system. We performed 500 replications of the 180 day system operation for each scenario and for box sizes 1 and 100. We show average values and 95% confidence intervals for our performance measures in Tables 3 and 4. In all scenarios, better performance is achieved with a smaller box size. For Scenario 3, the number of missed vaccination opportunities is three times more when the box size is 100. Independent of the box size, Table 3 shows better performance with fewer and larger dispensing locations. When the box size is 1, Scenario 1 outperforms the other scenarios with approximately four times fewer missed vaccination opportunities compared to Scenario 3. We also graphically show how on-hand day’s end vaccine supplies vary for an example replication. Notice the scale in the plots shown in Figs. 2 and 3. When the box size is 100, in comparison with Scenario 1, the daily number of missed vaccination opportunities is more than twice as large in Scenario 2 and more than eight times as large in Scenario 3.

---

**Table 2**

<table>
<thead>
<tr>
<th>Scenario 1</th>
<th>Vaccine usage</th>
<th>Scenario 2</th>
<th>Vaccine usage</th>
<th>Scenario 3</th>
<th>Vaccine Usage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Mode</td>
<td>Max</td>
<td>Min</td>
<td>Mode</td>
</tr>
<tr>
<td>9 Large</td>
<td>150</td>
<td>200</td>
<td>250</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>4 X-Large</td>
<td>1900</td>
<td>2000</td>
<td>2100</td>
<td>15</td>
<td>100</td>
</tr>
<tr>
<td>8 Large</td>
<td>110</td>
<td>150</td>
<td>190</td>
<td>8</td>
<td>100</td>
</tr>
<tr>
<td>4 X-Large</td>
<td>1525</td>
<td>1625</td>
<td>1725</td>
<td>4</td>
<td>100</td>
</tr>
</tbody>
</table>

---

J.A. Muckstadt et al.
The results of our experiments clearly demonstrate that performance measures, especially missed vaccination opportunities, vary based on the number and capacity of dispensing locations. The effect of box size on performance measures is also noteworthy. Thus we conclude that a smaller box size and having fewer and larger dispensing locations will provide the maximum benefit to both individuals and society.

We also observed that our proposed approach includes an efficient computationally tractable algorithm. In our experiments, completed on a 2.3 GHz Intel(R) Core(TM) i7-10610U CPU laptop with 16 gigabytes of RAM, Algorithm 1 was called over 100,000 times for each scenario. The average running time for Algorithm 1 was 5.5 ms with 13 dispensing locations and box size = 1, and 12.9 ms with 181 dispensing locations and box size = 1. In practice, we believe a well-designed system for a country such as the U.S. would contain approximately 200 distribution centers (DCs), each supporting up to 150 dispensing locations. Thus for each daily allocation decision, the computation time would be approximately 12.9 ms on a laptop computer.

6. COVID-19 vaccine distribution and administration in the U.S.

In the previous section, we demonstrated that the number and size of dispensing locations and box size impact the performance measures of a vaccine distribution system with poorly designed systems resulting in many missed VPDs. In practice, the vaccine distribution policy employed in the U.S. during the COVID-19 pandemic was even less effective than the policies evaluated in the experiments described in the previous section. In this section we present data pertaining to the effectiveness of the distribution and administration processes employed in the U.S. during 2021 and early 2022. These data allow us to estimate the missed VPDs resulting from the use of these policies. We also provide a lower bound on the amount of VPDs that would have occurred if the supply chain principles and optimization system we have proposed had been followed.

The data we use are publicly available from the CDC (CDC, 2023). They detail the daily vaccine distribution and administration amounts in the U.S. as a whole and in each state. In Fig. 4 we show the cumulative distribution and administration quantities from the end of 2020.
through the end of 2021. The point on the cumulative distributed graph corresponding to approximately 350 million doses occurred on May 20, 2021. The same point on the cumulative administered graph occurred on August 9, 2021. Assuming a first in first out administration plan, the doses distributed on May 20, 2021 did not get administered until 81 days after delivery. To compare the actual versus optimal values of VPDs, based on the data displayed in the graph, we assume supplies are no longer the constraint once the total distributed supply is 300 million doses. Thus we are looking at data from December 13, 2020 through April 30, 2021. Given these dates, \( P \), the length of the planning horizon is 139 days. The number of VPDs achieved during this period of time is approximately 11.4 billion, an impressive number. However, suppose a more efficient and effective system had been employed. In such a system, the time from distribution to administration would not exceed 6 days and for population centers it would be fewer days. Based on the CDC data and a 6-day delay between distribution and administration of vaccines, if an optimal strategy had been employed, approximately 13.5 billion VPDs of protection would have been achieved, an increase of approximately 18%. If the delay was less than 6 days, the number would be greater. These results suggest that substantial opportunity to decrease morbidity and mortality – and potentially to reduce the likelihood of new variants evolving – was missed. By the middle of October 2022, approximately 883 million doses had been distributed to states but only 633 million (71%) had been administered nationally; at the government purchase price of approximately $20 per dose, this means that $5 billion in vaccine went unused during this period. Given the quantity of supply that had not been administered, the question of the disposition of the unused supplies requires further investigation. Supplies may have expired that could have been used to vaccinate people elsewhere, for example via various multilateral mechanisms such as the COVID-19 Vaccines Global Access (COVAX) facility (WHO, 2023).

Let us now consider what happened in New York (NY) state using the CDC data (CDC, 2023). 40 million doses were distributed by January 6, 2022. The doses distributed on January 6, 2022 did not get fully administered until 153 days later on June 8, 2022. To estimate the number of missed VPDs we considered only these data for December 20, 2020 through November 19, 2021 (when approximately 35 million doses had been distributed to the state). The number of VPDs achieved using the state’s policy was approximately 5.4 billion; however, if an optimal distribution and administration strategy had been employed, an additional 872 million VPDs of protection (16%) would have been achieved. An estimated 17.2 million people were vaccinated in the state by November 19, 2021. Thus over 50 days of protection were missed by an average vaccinated person, which is approximately 1/3 of the total duration of the omicron variant surge in the U.S. This inefficiency was not limited to the early ramp up period or first year of COVID-19 vaccination campaigns: by the middle of October 2022, approximately 55.2 million doses had been distributed to NY state but only 42.6 million (77%) had been administered.

7. Conclusion

Vaccines form the backbone of public health defense against pandemics, and COVID-19 showed that these can be developed, tested, and licensed in less than a year for new pathogens. Efforts to shorten the vaccine development and manufacturing timeframe to 100 days or fewer now have powerful support among industry and non-governmental organizations (e.g., from the Coalition for Epidemic Preparedness Innovations (CEPI) (CEPI, 2023)). But to ensure the protection of the maximum number of people in the shortest amount of time using these vaccines, nations require efficient and effective distribution systems.

In this paper, we provide guidance for improving the operation of a vaccine distribution supply chain. We present a conceptual approach and algorithmic solution to this vaccine distribution problem. A major implication of our work is that such distribution system supply chains must have properly designed and operated information and business processes along with rules for allocating available stocks optimally each day to distribution centers and vaccine dispensing sites. Furthermore, policy makers should require manufacturers to deliver vaccines in box sizes consistent with dispensing location capabilities as specified by public health officials. In light of evidence indicating lower than optimal cumulative dispensing rates throughout in the U.S., we also note that each jurisdiction should design and size their intended dispensing locations to maximize population-wide throughput.

Our goal in this effort was not to develop new mathematical knowledge but rather to show how existing knowledge can be employed to improve system outcomes. Muckstadt et al. (2023) state several observations and guiding principles that should be the basis for the design and operation of supply chains required to meet the needs of the public during a pandemic or other large scale public health emergency. For vaccine supply chains, we have established that the system’s objective should be to maximize VPDs. We also observed that maximizing this statistic and minimizing missed vaccination opportunities requires using the smallest number of dispensing locations consistent with intentional efforts to target specific (e.g., historically underserved, geographically remote) populations, which may require
bespoke clinic locations and designs. Having fewer and higher through-
put dispensing locations achieves the public health goals that motivate
such campaigns: vaccinating the maximum number of people as soon
as possible. Using CDC data, we demonstrate the benefit of employing
this strategy and contrast it with outcomes that would have occurred if
the system actually used in the U.S. during the COVID-19 pandemic had
been operated optimally. Daily allocations throughout such a national
distribution system can be implemented in a rolling horizon manner
using the models presented here. Using the CDC data, we demonstrated
the effect of employing our strategy and contrasted it to the outcomes
that occur when optimally operating the system used in the U.S. during
the COVID-19 pandemic. In summary, if vaccines are allocated and
distribution system can be implemented in a rolling horizon manner
since all \( V_p \) variables are basic variables, all constraints in the follow-
ing dual problem are equality constraints. Let \( u_p \) represent the dual variable
subject to

\[
\begin{align*}
   u_1 + u_2 &= 8 \\
   u_2 + u_4 &= 7 \\
   u_3 + u_5 &= 6 \\
   u_4 + u_6 &= 5 \\
   u_5 + u_7 &= 4 \\
   u_6 + u_8 &= 3 \\
   u_7 &= 2 \\
   u_8 &= 1 \\
\end{align*}
\]

The solution to this problem is \( u_8 = 1, u_7 = 2, u_6 = 2, u_5 = 3, u_4 = 4, u_3 = 4, u_1 = 4 \). Note this is the solution independent of the values of the \( S_p \) parameters as long as \( V_p = S_p - V_{p-L} \geq 0 \).

We now show how the optimal dual variable values can be de-
termined in general. By construction we see that \( u_p = P - p + 1 \) for
\( p = P - L + 1, \ldots, P \). For \( p = P - 2L + 1, \ldots, P - L, u_p = u_{p-L+1} \).
For \( p = P - 3L + 1, \ldots, P - 2L, u_p = u_{p-1} + 1 \) and so on. Thus the
optimal dual variable values can be found easily for the vaccination
linear programming problem when \( V_p = S_p - V_{p-L} \geq 0 \). It is important
to observe that these dual variable values are optimal for any supply
plan that satisfies the condition that \( S_p \geq S_{p-L} \).

To illustrate, suppose \( S_1 = 5, S_2 = 10, S_3 = 6, S_4 = 11, S_5 = 7, S_6 = 12, S_7 = 7, S_8 = 12 \). Then we have:
\[
\begin{align*}
   V_1 &= 5, V_2 = 10, V_3 = 1, V_4 = 6, V_5 = 11, V_6 = 1, \quad \text{and} \quad V_7 = 6. \\
\end{align*}
\]

The objective function value is 181 for both the primal and dual
problems. Since the conditions of the duality theorem are satisfied, we
know that the proposed solution is optimal.  

## References

aged Care URL https://www.ajmc.com/view/a-timeline-of-covid-19-developments-

COVID-19 vaccine supply chain: Implications for sustainable development goals.
Int. J. Prod. Econ. 239, 108193.

in the influenza vaccine supply chain: Interventions in demand and supply sides.
Manage. Sci. 58 (6), 1072-1091.

Arifoglu, K., Tang, C.S., 2020. A two-sided incentive program for coordinating the

Balick, B., Yueessy, E., Akca, B., Karakaya, S., Gevsek, A.A., Baharmand, H., Sgar-
bossa, F., 2022. A mathematical model for equitable in-country COVID-19 vaccine


CDC, 2010. Influenza A (H1N1) 2009 Monovalent Vaccine Storage, Preparation,
Handling Q & A. Centers for Disease Control and Prevention, URL https://www.

CDC, 2017. National Pandemic Strategy, Centers for Disease Control and Preven-

Centers for Disease Control and Prevention, URL https://www.cdc.gov/coronavirus/2019-
nov-covid/cope-strategy/index.html.

CDC, 2023. COVID-19 Vaccinations in the United States, Jurisdiction. Centers for

100days.cepi.net/, (Accessed February 9, 2023).

Chick, S.E., Hanjia, S., Nasiry, J., 2017. Information elicitation and influenza vaccine
production. Oper. Res. 65 (1), 75-96.

vaccination. Oper. Res. 56 (6), 1493-1506.

Cho, S.-H., Tang, C.S., 2013. Advance selling in a supply chain under uncertain supply

Chung, S.H., Kwon, C., 2016. Integrated supply chain management for perishable

Manage. Sci. 6 (4), 475–490.


