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CONTROL OF HYDROSTATIC TRANSMISSION WIND TURBINE

A Thesis

Presented to

The Faculty of the Department of Electrical Engineering

San José State University

In Partial Fulfillment

of the Requirement for the Degree

Master of Science

by

Danop Rajabhandharaks

December 2014

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The Designated Thesis Committee Approves the Thesis Titled

CONTROL OF HYDROSTATIC TRANSMISSION WIND TURBINE

by

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APPROVED FOR THE DEPARTMENT OF ELECTRICAL ENGINEERING

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December 2014

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ABSTRACT

CONTROL OF HYDROSTATIC TRANSMISSION WIND TURBINE

by Danop Rajabhandharaks

In this study, we proposed a control strategy for a wind turbine that employed a hydrostatic transmission system for transmitting power from the wind turbine rotor via a hydraulic transmission line to a ground level generator. Wind turbine power curve tracking was achieved by controlling the hydraulic pump displacement and, at the other end of the hydraulic line, the hydraulic motor displacement was controlled so that the overall transmission loss was minimized. Steady state response, dynamic response, and system stability were assessed. The maximum transmission efficiency obtained ranged from 79% to 84% at steady state when the proposed control strategy was implemented. The leakage and friction losses of the hydraulic components were the main factors that compromised the efficiency. The simulation results showed that the system was stable and had fast and well-damped transient response. Double wind turbine system sharing hydraulic pipes, a hydraulic motor, and a generator were also studied. The hydraulic pipe diameter used in the double-turbine system increased by 27% compared to the single-turbine system in order to make the transmission coefficient comparable between both systems. The simulation results suggested that the leakage losses were so significant that the efficiency of the system was worsened compared with the single-turbine system. Future studies of other behavioral aspects and practical issues such as fluid dynamics, structure strength, materials, and costs are needed.

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NOMENCLATURE

Symbol	Definition	Unit
ρ_{air}	Air Density	kg/m ³
R	Wind Blade Radius	m
A	Wind Blade Swept Area	m ²
U	Wind Speed	m/s
P_{wind}	Aerodynamic Power	W
C_p	Power Coefficient	dimensionless
λ	Tip-Speed Ratio	dimensionless
β	Pitch Angle	rad
P_{rotor}	Rotor Rotational Power	W
ω_r	Rotor/Pump Rotational Speed	rad/s
τ_{rotor}	Rotor Torque	N-m
C_q	Torque Coefficient	dimensionless
τ_{pump}	Hydraulic Pump Torque	N-m
J_r	Pump/Rotor Inertia	kg-m ²
V_p	Pump Displacement	m ³ /rad
Δp_p	Differential Pressure at Pump Side	MPa
$\eta_{mech,p}$	Pump Mechanical Efficiency	dimensionless
Q_p	Pump Flow Rate	m ³ /s
$k_{leak,p}$	Pump Leakage Coefficient	m ³ /(s·Pa)
$k_{HP,p}$	Pump Hagen-Poiseuille Coefficient	(m/s) ² ·kg/Pa
$V_{p,max}$	Maximum Pump Volumetric Displacement	m ³ /rad
$\omega_{p,nom}$	Nominal Pump Speed	rad/s
$\eta_{v,pump}$	Pump Volumetric Efficiency	dimensionless
$\nu_{p,nom}$	Nominal Fluid Kinematic Viscosity at Pump Side	m ² /s
$\rho_{p,nom}$	Nominal Pump Fluid Density	kg/m ³
$\Delta p_{p,nom}$	Nominal Differential Pressure at Pump Side	Pa
Δp_x	Differential Pressure at Center of Straight Pipe	Pa
β_{fluid}	Fluid Bulk Modulus	Pa
V_{fluid}	Pipe Volume	m ³
Q_m	Motor Flow Rate	m ³ /s
δp	Pressure Loss in Pipe	Pa
f	Friction Factor	dimensionless
L_{pipe}	Pipe Length	m
ρ_{fluid}	Fluid Density	kg/m ³
D_{pipe}	Pipe Diameter	m
A_{pipe}	Pipe Cross-sectional Area	m ²
Re	Reynolds Number	dimensionless
r_{pipe}	Roughness of the Pipe	m

Symbol	Definition	Unit
ν_{fluid}	Fluid Kinematic Viscosity	m^2/s
Δp_m	Differential Pressure at Motor Side	MPa
$\eta_{mech,m}$	Motor Mechanical Efficiency	dimensionless
V_m	Motor Displacement	m^3/rad
J_m	Motor Inertia	$kg \cdot m^2$
ω_m	Motor Rotational Speed	rad/s
τ_{load}	Generator Load Torque	N-m
$k_{leak,m}$	Motor Leakage Coefficient	$m^3/(s \cdot Pa)$
$k_{HP,m}$	Motor Hagen-Poiseuille Coefficient	$(m/s)^2 \cdot kg/Pa$
$V_{m,max}$	Maximum Motor Volumetric Displacement	m^3/rad
$\omega_{m,nom}$	Nominal Motor Speed	rad/s
$\eta_{v,motor}$	Motor Volumetric Efficiency	dimensionless
$\nu_{m,nom}$	Nominal Fluid Kinematic Viscosity at Motor Side	m^2/s
$\rho_{m,nom}$	Nominal Motor Fluid Density	kg/m^3
$\Delta p_{m,nom}$	Nominal Differential Pressure at Motor Side	Pa
ε_{angle}	Generator Phase Angle Difference	rad
θ_{grid}	Phase of Grid Voltage	rad
θ_m	Synchronous Generator's Electrical Angle	rad
ω_{sync}	Generator Synchronous Speed	rad/s
K_s	Synchronizing Torque Coefficient	N-m/rad
K_d	Damping Torque Coefficient	N-m/(rad/s)
P_{out}	Output Power Generated by Synchronous Generator	W
$C_{p,max}$	Maximum Power Coefficient	dimensionless
λ_{opt}	Optimal Tip-Speed Ratio	dimensionless
C_T	Transmission Coefficient	dimensionless
$Loss_{mech,pump}$	Pump Mechanical Loss	dimensionless
$Loss_{leak,pump}$	Pump Leakage Loss	dimensionless
$Loss_{mech,motor}$	Motor Mechanical Loss	dimensionless
$Loss_{leak,motor}$	Motor Leakage Loss	dimensionless
$Loss_{fric}$	Friction Loss	dimensionless
$Loss_{total}$	Total Hydrostatic Transmission Loss	dimensionless

I. INTRODUCTION

A. Wind Energy

The energy from wind has been harnessed for serving humans for a long time. In early history, it was widely used for sailing ships, grinding grain, and pumping water in different locations in the world [1]. The use of wind energy for generating electricity became apparent in 1940s when the largest wind turbine (1.25 MW) of that time was developed and operated to generate electricity [1]. The increase in the price of oil and the concern over limited fossil fuel resources triggered a number of government funded wind energy projects worldwide [1]. By far, wind energy is the fastest growing energy source [2] compared with the other renewable energy sources. In the United States, the net wind energy generation increases at a rate approximately 22,000 thousand MWh per year as shown in Figure 1 from 2007 to 2013.

Figure 1: Net electricity generated by wind energy in the U.S. from 2004 to 2013 [3]

B. Conventional Wind Turbine Basics

A wind turbine is a device that transforms the kinetic energy of moving air (i.e., wind) into electrical energy. Figure 2 shows the structure of a typical wind turbine. It is composed of turbine blades, a tower, and a nacelle that houses a gearbox, a generator, a power converter, and a controller. Air flow over the blades causes the turbine shaft to rotate. The gearbox is employed to step up the rotational speed to a higher speed at which the generator can produce electrical power more efficiently. The power converter and transformer convert the frequency and the voltage level so that the power can be fed into the power grid [4]. The controller ensures that the wind turbine is operated safely and effectively.

Figure 2: Structure of a typical wind turbine

C. Problem Statement

The weight of a wind turbine significantly affects the costs of production, transportation, installation, and maintenance [5]. The heavier the weight, the higher the cost. For instance, an asynchronous generator with a squirrel cage rotor (ABB, Finland; rated power=2.3MW) is reported by Jeppsson et al to weigh 6580 kg [6]. The gearbox in a Siemens 2.3 MW Mk II wind turbine (Winergy; ratio=1:91), reported by Jeppsson et al, weighs 23 tons [6]. A power converter (ABB, Finland; up to 9 MVA) is reported by ABB to weigh between 5000 to 6000 kg [7]. A 2.5 MVA vacuum cast coil transformer (ABB, Finland) weighs 6200 kg [8]. The nacelle of Siemens 2.3 MW Mk II wind turbine weighs 82 tons in total [6]. The cost of construction and structure materials for supporting such a massive nacelle at about 100 meter height on top of the tower is substantial. If the weight of the nacelle could be lowered, these costs could be reduced.

Reliability is one of the most important qualities for a wind turbine. Poor reliability can lead to expensive maintenance and unnecessary operational loss due to downtime, which can significantly increase the cost of electrical energy production. In a conventional megawatt wind turbine, the gearbox by far is one of the most expensive components [9] and the weakest link of the entire structure [10]. Misalignment of the gearbox due to wind gusts and turbulence is a common cause for failure [10]. It is not uncommon for a gearbox to fail on average every 5 years while the designed lifetime of a wind turbine is typically about 20 years [10]. The gearbox replacement is usually a part of preventive maintenance every 5 years; therefore, it contributes significantly to the operating cost [10].

D. Literature Review

Many solutions have been proposed to solve the gearbox reliability issue. One of the most direct solutions is to find a way to entirely remove the gearbox without using any other means for replacing its function. This idea leads to a direct-drive or gearless wind turbine, pioneered in 1993 by the Enercon company of Germany and the ScanWind company of Norway [10]. To remove the gearbox, the generator inside the nacelle needs modifications. Equation (1) [10] shows the relationship between the generator rotational speed, the number of magnetic pole pairs n , and the frequency f in Hz.

$$f = n \frac{rpm}{120} \quad (1)$$

For a 4 pole-pair machine, in order to generate 60 Hz electrical power,

$$60 = 4 \times \frac{rpm}{120}$$

$$rpm = 1800$$

the generator has to operate at 1800 rpm. Assuming the rotor of the wind turbine is rotated at 18 rpm, the gear ratio required to step up the rotational speed would be 100. The concept of the direct-drive wind turbine is to increase the number of magnetic pole pairs. Using Equation (1) again to obtain the same 60 Hz operating frequency with the generator rotating at 18 rpm, the number of magnetic pole pairs has to be increased to 400 pairs. This idea seems reasonable and straightforward to solve the gearbox issue. However, the direct-drive generator requires a large amount of an expensive rare earth element magnet, which leads to high cost, large size, and heavy weight nacelle [11]. Another problem with the direct-drive generator is that, in order to obtain high flux

density, the small air gap between the rotor and the stator must be kept within a few millimeters. Achieving such a high precision can be a manufacturing challenge [11].

Another proposed solution is to employ hydrostatic transmission to step up rotational speed and transmit power in place of the gearbox. A hydrostatic transmission system contains a hydraulic pump, a hydraulic motor, and a hydraulic pipe/hose system. Hydrostatic transmission has been utilized in automobile application [12, 13] and recently introduced to wind turbine application. Hydrostatic transmission can offer many positive benefits. For instance, it allows the use of continuously variable transmission (CVT) that is capable of varying the input-output speed ratio continuously and smoothly [11, 14, 15], unlike the conventional gearboxes that offer a limited number of gear ratios. Because the low-speed shaft and high-speed shaft are decoupled, synchronous generators can potentially be used instead of induction generators because the fluctuations are more tolerable with hydrostatic transmission and the need of the power electronic converters can be eliminated [11, 13].

In addition to the aforementioned benefits, the weight of the nacelle can be significantly lowered with the hydrostatic transmission configuration. A mechanical gearbox is about 3 to 4 times heavier than the hydrostatic transmission system (a gearbox reported by Jeppsson et al weighs 23 tons [6] and that reported by NewScientist weighs 15 tons [16] versus a hydrostatic transmission reported by NewScientist weighs 6 tons [16]). The generator can be situated on the ground [13] (an asynchronous generator reported by Jeppsson et al weighs 6580 kg [6]). Thus, the weight of the generator does not contribute to the total weight of the nacelle. Moreover, the need for the power

electronic converter (one reported by ABB weighs 5000 to 6000 kg [7]) can be eliminated because a synchronous generator can be used. The transformer (one reported by ABB weighs 6200 kg [8]) can also be moved to the ground (a transformer is usually installed inside the nacelle for offshore wind farms to avoid I^2R losses [5]). In this example, the total nacelle weight reduction can be reduced by removing the gearbox (15 tons [16]), adding hydrostatic transmission (6 tons [16]), removing the generator (6.58 tons [6]), removing the power electronic converter (5 tons [7]), and removing the transformer (6.2 tons [8]). The total nacelle weight reduction is 26.78 tons. Compared with the total nacelle weight (reported by ABB [8]) of 82 tons for the conventional wind turbine, the reduction can be $26.78/82=0.33$ or 33%. Figure 3 illustrates hydrostatic transmission wind turbine and its components.

Figure 3: Hydrostatic transmission wind turbine

SeaAngel[®] [17] is a living example of an offshore hydrostatic transmission wind turbine, developed since 2011 by Mitsubishi Power Systems Europe (MPSE). It contains no gearbox. It uses Digital Displacement[®] Transmission (DDT) [18], developed by Artemis Intelligent Power (Edinburgh, Scotland), for its drive system. DDT consists of a variable Digital Displacement[®] Pump (DDP) and a variable Digital Displacement[®] Motor (DDM) in place of a conventional gearbox. A brushless synchronous generator is also employed instead of a popular asynchronous generator for a conventional wind turbine and no power electronic converter is used. An advantage of DDT compared with other variable displacement machines, such as Bent Axis Pump and Axial Piston Pump is that its efficiency is fairly uniform at different operating point as shown in Figure 1.1 by Yigen [19] and, thus, this technology allows the SeaAngel[®] to be efficient and competitive compared with the conventional gearbox [16]. Currently, the SeaAngel[®] is still under testing, according the milestones reported by Mitsubishi Power System [17].

Modeling and control aspects of the hydrostatic transmission wind turbine have been reported in literature. For instance, Pusha et al [20, 21], Hamzehlouia et al [22-28], Deldar et al [29], Izadian et al [30] developed a hydraulic transmission model and control strategy technique for wind turbine using a fixed displacement pump and a fixed displacement motor. Varpe [31] and Dolan et al [13] developed controllers to control variable displacement motor for maximum energy capture objective. Both of them used fixed displacement pump and a variable displacement motor. A difference between Varpe [31] and Dolan et al [13] was that Dolan et al employed synchronous generator but Varpe employed asynchronous generator. Dutta et al [32] presented a method to use hydraulic

accumulators for energy storage to improve energy capture efficiency. Wang et al [33] proposed the model predictive control (MPC) to increase the response time of the rotor speed to reach the optimal tip-speed ratio. Laguna et al [34] studied overall efficiency and dynamics of the model having the fixed displacement hydraulic pump in the nacelle and the variable displacement hydraulic motor and the synchronous generator on the ground. Despite the abovementioned potential advantages of the hydrostatic transmission wind turbine, thorough studies on control aspects using a variable displacement hydraulic pump and a variable displacement hydraulic motor in a model and having the motor and a synchronous generator on the ground are still lacking.

E. Study Objective and Report Structure

The objective of this study is to develop a control strategy for a wind turbine with hydrostatic transmission that transmits the wind power to a ground level generator using a variable displacement hydraulic pump and a variable displacement hydraulic motor. The control objective is to maximize aerodynamic energy capture and overall efficiency. We study two system configurations: a single turbine with a dedicated hydrostatic transmission system and a double turbine configuration sharing a hydrostatic transmission system and a generator. Modeling and simulation are conducted in MATLAB/Simulink. The mathematical models of the wind turbine with hydraulic components are defined in Chapter 2. The strategies to maximize aerodynamic energy capture for the wind turbine are proposed in Chapter 3. MATLAB/Simulink simulation models and results are discussed in Chapter 4. Two Turbine configuration analysis is discussed in Chapter 5. Conclusion is presented in Chapter 6.

II. HYDROSTATIC TRANSMISSION WIND TURBINE MODEL

A. Introduction

Figure 4 shows the overall hydrostatic transmission wind turbine model used in this study. The system is composed of turbine blades, a variable displacement hydraulic pump, hydraulic transmission pipes, a variable displacement hydraulic motor, and a synchronous generator on the ground level. In an actual system, there are other auxiliary components for hydrostatic transmission. For instance, an auxiliary hydraulic pump and two check valves are required to ensure that the pressure value at any location of the hydraulic transmission pipe is never lower than, typically, 2 MPa [35] to prevent cavitation in the system. There are also pressure relief valves in the system to prevent the pressure value exceeding the system limit. For simplicity, those auxiliary components are excluded in this study. Only the components in Figure 4 are considered.

Figure 4: Hydrostatic transmission wind turbine model

B. Aerodynamic Power and Torque Model

Power of the wind blowing over the circular area swept by the blades at a speed of U , P_{wind} , can be characterized by Equation (2) [1].

$$P_{wind} = \frac{1}{2} \rho_{air} A U^3 \quad (2)$$

ρ_{air} is the density of air and A is the blade swept area. Since wind power conversion is not 100% efficient, the available rotational power at the rotor (P_{rotor}) can be characterized using Equation (3) [1].

$$P_{rotor} = P_{wind} C_p(\lambda, \beta) \quad (3)$$

P_{rotor} is the power available as mechanical rotational power at the low-speed shaft. $C_p(\lambda, \beta)$ is the characteristic power coefficient of the wind turbine, whose value is dependent on the tip-speed ratio λ and blade pitch angle β . The tip-speed ratio is defined in Equation (4) [1].

$$\lambda = \frac{\omega_r R}{U} \quad (4)$$

ω_r is the rotor rotational speed and R is the blade length. The aerodynamic torque at the low-speed shaft (τ_{rotor}) can be expressed as [1]

$$\tau_{rotor} = \frac{P_{rotor}}{\omega_r} = \frac{1}{2} \rho_{air} A R C_q(\lambda, \beta) U^2. \quad (5)$$

C_q is the characteristic torque coefficient and is defined as [1]

$$C_q(\lambda, \beta) = \frac{C_p(\lambda, \beta)}{\lambda}. \quad (6)$$

For this study, C_p model is obtained from SimPowerSystems (The MathWorks, Natick, MA) and is modeled as

$$C_p(\lambda, \beta) = c_1 \left(\frac{c_2}{\lambda_i} - c_3 \beta - c_4 \right) e^{-\frac{c_5}{\lambda_i}} + c_6 \lambda \quad (7)$$

$$\frac{1}{\lambda_i} = \frac{1}{\lambda + 0.08\beta} - \frac{0.035}{\beta^3 + 1}$$

where $c_1=0.5176$, $c_2=116$, $c_3=0.4$, $c_4=5$, $c_5=21$, and $c_6=0.0068$.

C. Variable Displacement Hydraulic Pump

The variable displacement hydraulic pump converts the turbine rotor rotational motion to fluid flow. Using Newton's 2nd law, the first-order dynamic equation can be expressed as Equation (8) [13].

$$\tau_{rotor} - \tau_{pump} = J_r \dot{\omega}_r \quad (8)$$

J_r is the moment of inertia of the rotor and τ_{pump} is the resistant torque imposed by the hydraulic pump on the turbine rotor and can be expressed as [13]

$$\tau_{pump} = \frac{V_p \Delta p_p}{\eta_{mech,p}}.$$

Δp_p is the differential pressure between high and low side of the pump. $\eta_{mech,p}$ is the pump mechanical efficiency. V_p is the pump displacement which can be controlled directly according to a per-defined control strategy. Equation (8) can be expressed as [13]

$$\dot{\omega}_r = \frac{1}{J_r} \left(\tau_{rotor} - \frac{V_p \Delta p_p}{\eta_{mech,p}} \right).$$

Fluid flow rate at the pump side can be characterized as [28]

$$Q_p = V_p \omega_r - k_{leak,p} \Delta p_p. \quad (9)$$

Q_p is the fluid flow rate at the pump side. $k_{leak,p}$ is the leakage coefficient at the pump side, which can be expressed as the following equations [28].

$$\begin{aligned}
k_{leak,p} &= \frac{k_{HP,p}}{v_{fluid} \rho_{fluid}} \\
k_{HP,p} &= \frac{V_{p,max} \omega_{p,nom} (1 - \eta_{v,pump}) v_{p,nom} \rho_{p,nom}}{\Delta p_{p,nom}}
\end{aligned} \tag{10}$$

v_{fluid} is the fluid kinematic viscosity. ρ_{fluid} is the fluid density. $k_{HP,p}$ is the Hagen-Poiseuille coefficient for the pump, which depends on maximum pump volumetric displacement $V_{p,max}$, nominal pump angular speed $\omega_{p,nom}$, pump volumetric efficiency $\eta_{v,pump}$, nominal pump fluid kinematic viscosity $v_{p,nom}$, nominal pump fluid density $\rho_{p,nom}$, and nominal pump differential pressure $\Delta p_{p,nom}$.

D. Hydraulic Transmission Line

In this study a model of a straight, cylindrical pipe system is used. Differential pressure and flow rate dynamics can be expressed as [28]

$$\Delta \dot{p}_x = \frac{\beta_{fluid}}{V_{fluid}} (Q_p - Q_m) \tag{11}$$

Δp_x is the pressure difference between the high and the low pressure side at the center of the straight pipe system. β_{fluid} is the hydraulic fluid bulk modulus. V_{fluid} is the volume of the fluid in the pipe or hose. Q_p and Q_m are the flow rate at the pump and motor side, respectively. Pressure loss in the pipe system due to friction can also be significant and should be considered. The Darcy equation and Haaland approximation suggest that [36]

$$\delta p = f \frac{L_{pipe}}{D_{pipe}} \frac{\rho_{fluid}}{2 A_{pipe}^2} Q^2. \tag{12}$$

δp is the pressure loss along the pipe due to friction. L_{pipe} is the length of the pipe. D_{pipe} is the cross-sectional diameter of the pipe. A_{pipe} is the cross-sectional area of the pipe. Q is the flow rate in the pipe. f is the friction factor, which is modeled as in Equation (13) [36].

$$f \equiv f(Re) = \begin{cases} 64 / Re & \text{for } Re \leq 2000 \\ f(2000) + \frac{f(4000) - f_L(2000)}{4000 - 2000} (Re - 2000) & \text{for } 2000 < Re < 4000 \\ \frac{1}{\left(-1.8 \log_{10} \left(\frac{6.9}{Re} + \left(\frac{r_{pipe} / D_{pipe}}{3.7} \right)^{1.11} \right) \right)^2} & \text{for } Re \geq 4000 \end{cases} \quad (13)$$

r_{pipe} is the height roughness of the pipe internal surface and Re is the Reynolds number which can be expressed as [36]

$$Re = \frac{QD_{pipe}}{A_{pipe}V_{fluid}}. \quad (14)$$

Pump differential pressure (Δp_p) and motor differential pressure (Δp_m) can now be expressed as the following equations.

$$\Delta p_p = \Delta p_x + \frac{\delta p}{2} \quad (15)$$

$$\Delta p_m = \Delta p_x - \frac{\delta p}{2} \quad (16)$$

E. Variable Displacement Hydraulic Motor

A variable displacement hydraulic motor, at the receiving end of the line, converts fluid flow back to rotational motion. The motor's dynamics can be expressed by a first-order dynamic equation as shown in Equation (17) [28].

$$\begin{aligned}\eta_{mech,m} V_m \Delta p_m &= J_m \dot{\omega}_m + \tau_{load} \\ \dot{\omega}_m &= \frac{1}{J_m} (\eta_{mech,m} V_m \Delta p_m - \tau_{load})\end{aligned}\quad (17)$$

$\eta_{mech,m}$ is the motor mechanical efficiency. V_m is the motor volumetric displacement. Δp_m is the difference in pressure between the high and low side at the motor side. J_m is the moment of inertia of the high-speed shaft and that of the generator rotor. ω_m is the motor rotational speed. τ_{load} is the torque produced by the load (i.e., synchronous generator). Fluid flow rate at the motor side can be modeled as Equation (18) [28].

$$Q_m = V_m \omega_m + k_{leak,m} \Delta p_m \quad (18)$$

Q_m is the fluid flow rate at the motor side. $k_{leak,m}$ is the leakage coefficient at the motor side, which can be expressed as the following equations [28].

$$\begin{aligned}k_{leak,m} &= \frac{k_{HP,m}}{v_{fluid} \rho_{fluid}} \\ k_{HP,m} &= \frac{V_{m,max} \omega_{m,nom} (1 - \eta_{v,motor}) v_{m,nom} \rho_{m,nom}}{\Delta p_{m,nom}}\end{aligned}\quad (19)$$

$k_{HP,m}$ is Hagen-Poiseuille coefficient for the motor, which can be calculated from maximum motor displacement $V_{m,max}$, nominal motor angular speed $\omega_{m,nom}$, motor

volumetric efficiency $\eta_{v,motor}$, nominal motor fluid kinematic viscosity $\nu_{m,nom}$, nominal motor fluid density $\rho_{m,nom}$, and nominal motor differential pressure $\Delta p_{m,nom}$.

F. Synchronous Generator

A synchronous generator is used in this study. Connected to the hydraulic motor, the synchronous generator generates electricity at the constant synchronous speed ω_{sync} . For simplicity, the synchronous generator is modeled by a second-order system as shown in Figure 5 and Equation (20). ε_{angle} is the phase angle difference between the phase of the grid voltage and the synchronous generator's electrical angle. Synchronizing torque coefficient (K_s) and damping torque coefficient (K_d) are chosen so that the generator model has a fast and stable response.

$$\begin{aligned}\varepsilon_{angle} &= \theta_{grid} - \theta_m \\ \tau_{load} &= K_s \varepsilon_{angle} + K_d \dot{\varepsilon}_{angle}\end{aligned}\tag{20}$$

Figure 5: Simplified synchronous generator modeling [39]

Output power (P_{out}) generated by the synchronous generator is given by Equation (21) neglecting generator losses.

$$P_{out} = \tau_{load} \omega_m \quad (21)$$

III. CONTROL STRATEGY

A. Introduction

Typically, there are three main operational regions for variable-speed wind turbines [37]. Figure 6 shows the output power vs. wind speed curve (i.e., the power curve) and the operational regions of a variable-speed wind turbine. Region 1 refers to a condition when the turbine is not producing output power due to low wind speed under its threshold value or during start-up. The rotor rotates freely until the rotor speed reaches the threshold value, and the turbine moves into the region 2 operating region. The main control objective when operating at region 2 is to maximize wind energy capture. To achieve this, the rotor speed of the wind turbine is controlled such that the characteristic power coefficient (C_p) is maximized. Detailed techniques for maximizing the wind energy capture for conventional variable-speed wind turbines are discussed by Johnson et al [37]. Region 3 refers to a condition when the turbine operates at high wind speed and the output power is constant at the maximum regardless of wind speed. A typical control objective in this region is to prevent the output power exceeding the rated value in order to protect the system from power overloading, which can cause damage to the wind turbine. This control objective can be realized by adjusting the blade pitch angle to lower the power coefficient. If the wind turbine continues to experience high wind speed beyond the cutout speed, the wind turbine will be shutdown. In this study, region 1 operation is not discussed.

Figure 6: Wind turbine system operational regions

B. Region 2 Control Strategy

In this section the variable-speed hydrostatic transmission wind turbine control strategies are developed with the goal of maximizing wind power capture. All variables and their units are defined in Nomenclature at the beginning of this thesis.

1) Maximization of Wind Turbine Power Coefficient Strategy

Power coefficient (C_p) is the efficiency of a wind turbine converting wind power (P_{wind}) to rotor rotational power (P_{rotor}), i.e., $C_p = P_{rotor}/P_{wind}$. The control law [37] for region 2 can be applied so that C_p is optimized at steady state by setting the last term in Equation (8) to

$$\begin{aligned} \frac{V_p \Delta p_p}{\eta_{mech,p}} &= K \omega_r^2 \\ K &= \frac{1}{2} \rho_{air} A R^3 \frac{C_{p,max}}{\lambda_{opt}^3} \end{aligned} \quad (22)$$

where $C_{p,max}$ is the maximum power coefficient and λ_{opt} is the optimal tip-speed ratio that renders the maximum power coefficient. A detailed analysis of this region 2 control law is discussed by Johnson et al [37]. Rearranging Equation (22) so that

$$V_p = \frac{K\omega_r^2\eta_{mech,p}}{\Delta p_p}. \quad (23)$$

It can be seen that this control strategy can be realized by controlling V_p according to Equation (23). As a result, at steady state, the rotor speed converges to the speed that gives optimal tip-speed ratio and maximum energy conversion is achieved (i.e., $C_p=C_{p,max}$).

2) *Maximization of Transmission Coefficient Strategy*

The Transmission Coefficient (C_T) is defined as the efficiency of power transfer in hydrostatic transmission system (i.e., $C_T=P_{out}/P_{rotor}$). By identifying all losses associated with the hydrostatic transmission and minimizing the combined loss, C_T can be maximized. When both C_p and C_T are maximized, the overall system efficiency ($C_p C_T$) is maximized. There are five main losses in a hydrostatic transmission system as discussed below.

Pump Mechanical Loss ($Loss_{mech,pump}$): From Equation (8), at steady-state,

$$\eta_{mech,p}\tau_{rotor} = V_p\Delta p_p. \quad (24)$$

From Equation (24), it can be shown that the pump mechanical loss ($Loss_{mech,pump}$) at steady state can be characterized as

$$Loss_{mech,pump} = (1 - \eta_{mech,p}) K \omega_r^3. \quad (25)$$

Pump Leakage Loss ($Loss_{leak,pump}$): From Equation (9), it can be shown that the pump leakage loss ($Loss_{leak,pump}$) at steady state can be modeled as

$$Loss_{leak,pump} = k_{leak,p} \Delta p_p^2. \quad (26)$$

Motor Mechanical Loss ($Loss_{mech,motor}$): Recall Equation (17). At steady state,

$$\eta_{mech,m} V_m \Delta p_m = \tau_{load}. \quad (27)$$

From Equation (27), it can be shown that the motor mechanical loss ($Loss_{mech,motor}$) at steady state can be characterized as

$$Loss_{mech,motor} = (1 - \eta_{mech,m}) V_m \Delta p_m \omega_m. \quad (28)$$

Motor Leakage Loss ($Loss_{leak,motor}$): Recall Equation (18). It can be shown that the motor leakage loss ($Loss_{leak,motor}$) at steady state can be characterized as

$$Loss_{leak,motor} = k_{leak,m} \Delta p_m^2. \quad (29)$$

Friction Loss ($Loss_{fric}$): From Equation (12) and (13), it can be shown that the friction loss ($Loss_{fric}$) at steady state can be modeled as

$$Loss_{fric} = f \frac{L_{pipe}}{D_{pipe}} \frac{\rho_{fluid}}{2A_{pipe}^2} Q^3. \quad (30)$$

Total Hydrostatic Transmission Loss ($Loss_{total}$): Total hydrostatic transmission loss ($Loss_{total}$) can be summarized as shown in Equation (31).

$$\begin{aligned}
 Loss_{total} = & Loss_{mech,pump} + Loss_{leak,pump} + Loss_{mech,motor} + \dots \\
 & \dots Loss_{leak,motor} + Loss_{fric}
 \end{aligned} \tag{31}$$

Transmission coefficient (C_T) can be characterized as

$$C_T = \frac{P_{out}}{P_{rotor}} = 1 - \frac{Loss_{total}}{P_{rotor}}. \tag{32}$$

Figure 7 shows C_T as a function of V_m at three different wind speeds ($U=4, 8$, and 12m/s), assuming rotor speed is related to the wind speed by the optimal tip-speed ratio (Parameters from Table 1 are used in generating these three curves). It can be seen that C_T peaks at different V_m and therefore C_T can be maximized by a properly chosen V_m .

Figure 7: Transmission coefficient (C_T) as a function of motor displacement (V_m). Note, U is the wind speed.

C_T Optimization Process: The maximum C_T can be found by taking the partial derivative of Equation (32) with respect to V_m and setting it to zero as shown in Equation (33).

$$\frac{\partial}{\partial V_m} C_T = \frac{\partial}{\partial V_m} \left(\frac{Loss_{total}}{P_{rotor}} \right) = 0 \quad (33)$$

Since P_{rotor} is independent of V_m , Equation (33) can be simplified as

$$\frac{\partial}{\partial V_m} Loss_{total} = 0. \quad (34)$$

By solving Equation (34) for V_m , optimal V_m ($V_{m,opt}$) can be obtained (i.e., $V_m = V_{m,opt}$).

Figure 8 shows $V_{m,opt}$ for each wind speed U . A linear approximation equation can be obtained from the graph as

$$V_{m,opt} = 0.0002 \frac{\omega_r R}{\lambda_{opt}} - 0.0003. \quad (35)$$

C_T can now be maximized by controlling $V_m = V_{m,opt}$ according to Equation (35). Figure 9 is the overall system-level block diagram for the proposed system.

Figure 8: Optimal motor displacement ($V_{m,opt}$) as a function of wind speed (U) and its linear approximation equation. Note that $U = \omega_r R / \lambda_{opt}$.

Figure 9: Overall system level block diagram including control strategy to maximize C_p and C_T in round-dotted boxes

IV. SIMULATION RESULTS IN MATLAB/SIMULINK

In this section, steady state and transient responses of the hydrostatic transmission wind turbine are studied by computer simulation according to the model and control strategies proposed in Chapter 2 and Chapter 3. Table 1 shows all variables and their values used in all simulations in this study.

A. Steady State Response

Steady state responses as a function of U are shown in Figure 10 to Figure 15. ω_m is constant at 125.66 rad/s at steady state regardless of U because the 6-pole synchronous generator is used in this study. ω_r increases linearly as a function of U because the control law guarantees that ω_r is proportional to U at steady state. Δp_p and Δp_m increase as a function of rotor speed and, therefore, as a function of U . The difference between Δp_p and Δp_m is due to loss from friction δp . V_p and V_m also increase as a function of U according to control strategy given by Equation (23) and (35). τ_{load} from the synchronous generator increases as a function of U . P_{wind} , P_{rotor} , and P_{out} increase as a function of U . Note that $P_{rotor}/P_{wind}=C_p$, $P_{out}/P_{rotor}=C_T$, and $P_{out}/P_{wind}=C_p C_T$. As shown in Figure 15, C_p is constant regardless of U because of the control strategy proposed in Equation (23). C_T , optimized by Equation (35), decreases as a function of U (5% decrease from end to end). Table 2 shows all losses associated with C_T , suggesting that $Loss_{leak,pump}$ and $Loss_{fric}$ are higher as U is higher and, thus, worsen C_T . The overall system efficiency ($C_p C_T$) decreases as function of U , following C_T .

Table 1: Wind turbine parameter values used in this study

Symbol	Definition	Unit	Value
ρ_{air}	Air Density	kg/m ³	1.225
R	Wind Blade Radius	m	63
A	Wind Blade Swept Area	m ²	πR^2
β	Pitch angle	Deg	0
$C_{p,opt}$	Optimal Power Coefficient	dimensionless	0.48
λ_{opt}	Optimal Tip-Speed Ratio	dimensionless	8.1
β_{fluid}	Fluid Bulk Modulus	Pa	1.40403e9
D_{pipe}	Pipe Diameter	m	0.15
L_{pipe}	Pipe Length	m	100
r_{pipe}	Roughness of the Pipe	m	1.5e-5
ρ_{fluid}	Fluid Density	kg/m ³	852.8
ν_{fluid}	Fluid Kinematic Viscosity	m ² /s	18.786e-6
J_r	Pump/Rotor Inertia	kg-m ²	38759228
$\eta_{mech,p}$	Pump Mechanical Efficiency	dimensionless	0.95
$\eta_{v,pump}$	Pump Volumetric Efficiency	dimensionless	0.95
$V_{p,max}$	Maximum Pump Volumetric Displacement	m ³ /rad	0.3
$\omega_{p,nom}$	Nominal Pump Speed	rad/s	1
$\nu_{p,nom}$	Nominal Fluid Kinematic Viscosity at Pump Side	m ² /s	18.876e-6
$\Delta p_{p,nom}$	Nominal Pressure Different at Pump Side	Pa	2e7
J_g	Motor Inertia	kg-m ²	534.116
$\eta_{mech,m}$	Motor Mechanical Efficiency	dimensionless	0.95
$\eta_{v,motor}$	Motor Volumetric Efficiency	dimensionless	0.95
$V_{m,max}$	Maximum Motor Volumetric Displacement	m ³ /rad	8e-4
$\omega_{m,nom}$	Nominal Motor Speed	rad/s	125.6637
$\nu_{m,nom}$	Nominal Fluid Kinematic Viscosity at Motor Side	m ² /s	18.786e-6
$\Delta p_{m,nom}$	Nominal Pressure Different at Motor Side	Pa	2e7
ω_{sync}	Synchronous Speed	rad/s	125.6637
K_s	Synchronizing Torque Coefficient	N-m/rad	2.67e5
K_d	Damping Torque Coefficient	N-m/(rad/s)	1.07e4

Figure 10: Rotor speed (ω_r) and motor speed (ω_m) as functions of wind speed (U)

Figure 11: Pump differential pressure (Δp_p) and motor differential pressure (Δp_m) as functions of wind speed (U)

Figure 12: Pump displacement (V_p) and motor displacement (V_m) as functions of wind speed (U)

Figure 13: Generator torque load (τ_{load}) as a function of wind speed (U)

Figure 14: Wind power (P_{wind}), rotor rotational power (P_{rotor}), and output power (P_{out}) as functions of wind speed (U)

Figure 15: Transmission coefficient (C_T), power coefficient (C_p), and overall system efficiency ($C_T C_p$) as functions of wind speed (U)

Table 2. Loss analysis computed from Equation (24) to (31). The values are calculated in percentage compared with rotor rotational power.

Loss Type	Wind Speed (U)		
	4 m/s	8 m/s	12 m/s
$Loss_{mech,pump}$	5.0%	5.0%	5.0%
$Loss_{leak,pump}$	3.6%	4.2%	5.3%
$Loss_{mech,motor}$	4.4%	4.3%	4.2%
$Loss_{leak,motor}$	1.2%	1.3%	1.6%
$Loss_{fric}$	2.4%	4.3%	4.9%
$Loss_{total}$	16.5%	19.0%	20.9%

B. System Response

The system responses are shown in Figure 16 to Figure 22. The input U is a unit step function with an initial condition of 4m/s at $t < 50$ s and the magnitude of 6, 10, and 12 at time=50, 100, and 150s, respectively. As U changes, ω_m remains constant at 125.663 rad/s and ω_r tries to keep up with U so that it operates at $\omega_r = \lambda_{opt}U/R$ where $\lambda_{opt}=8.1$. Δp_p , Δp_m , and τ_{load} also change according to U . V_p and V_m are controlled according to the control law so that the system reaches the optimal efficiency at steady state. Figure 22 shows that C_T , C_p , and C_TC_p track their optimal points as U varies.

Figure 16: Wind speed (U) as a function of time

Figure 17: Rotor speed (ω_r) and motor speed (ω_m) as functions of time

Figure 18: Differential pump pressure (Δp_p) and differential motor pressure (Δp_m) as functions of time

Figure 19: Pump displacement (V_p) and motor displacement (V_m) as functions of time

Figure 20: Generator torque load (τ_{load}) as a function of time

Figure 21: Wind power (P_{wind}), rotor rotational power (P_{rotor}), and output power (P_{out}) as functions of time

Figure 22: Transmission coefficient (C_T), power coefficient (C_p), and overall system efficiency ($C_T C_p$) as functions of time

C. System Stability Analysis

In this section the dynamic equation of the turbine system is linearized and its stability is assessed [38].

Let $\vec{X}(t)$ be systemstatespace variable defined as

$$\vec{X}(t) = \begin{bmatrix} \theta_m(t) \\ \omega_r(t) \\ \Delta p_x(t) \\ \omega_m(t) \end{bmatrix}. \quad (36)$$

$\vec{X}(t)$ can be written in terms of an operating system trajectory $\vec{X}_n$ and a small change $\Delta \vec{X}(t)$ as

$$\begin{aligned}\vec{X}(t) &= \vec{X}_n + \Delta\vec{X}(t) \\ &= \begin{bmatrix} \theta_{m,n} \\ \omega_{r,n} \\ \Delta p_{x,n} \\ \omega_{m,n} \end{bmatrix} + \begin{bmatrix} \Delta\theta_m(t) \\ \Delta\omega_r(t) \\ \Delta(\Delta p_x(t)) \\ \Delta\omega_m(t) \end{bmatrix}.\end{aligned}\quad (37)$$

Similarly, let $\vec{I}(t)$ be system input and defined as the following equation.

$$\vec{I}(t) = [U(t) \quad \omega_{sync}(t) \quad \theta_{grid}(t)] \quad (38)$$

$\vec{I}(t)$ can be expressed as the sum of the unperturbed value $\vec{I}_n$ and the variation $\Delta\vec{I}(t)$ as

$$\begin{aligned}\vec{I}(t) &= \vec{I}_n + \Delta\vec{I}(t) \\ &= [U_n \quad \omega_{sync,n} \quad \theta_{grid,n}] + [\Delta U(t) \quad \Delta\omega_{sync}(t) \quad \Delta\theta_{grid}(t)]\end{aligned}\quad (39)$$

The nonlinear system state space representation of the system can be written as

$$\frac{d}{dt} \vec{X}(t) = F(\vec{X}(t), \vec{I}(t)) \quad (40)$$

where F is the 4x4 dimensional vector function. If small changes are considered, the nonlinear system can be linearized at an operating point as

$$\begin{aligned}\frac{d}{dt} \Delta\vec{X}(t) &= \left. \frac{\partial F}{\partial \vec{X}} \right|_{\vec{X}_n, \vec{I}_n} \Delta\vec{X}(t) + \left. \frac{\partial F}{\partial \vec{I}} \right|_{\vec{X}_n, \vec{I}_n} \Delta\vec{I}(t) \\ &= A\Delta\vec{X}(t) + B\Delta\vec{I}(t)\end{aligned}\quad (41)$$

where A and B represent the Jacobian matrices and are given by the following equations.

$$\left. \frac{\partial F}{\partial X} \right|_{\bar{X}_n, \bar{I}_n} = A^{4 \times 4} = \begin{bmatrix} \frac{\partial F_1}{\partial \theta_m} & \frac{\partial F_1}{\partial \omega_r} & \frac{\partial F_1}{\partial \Delta p_x} & \frac{\partial F_1}{\partial \omega_m} \\ \frac{\partial F_2}{\partial \theta_m} & \frac{\partial F_2}{\partial \omega_r} & \frac{\partial F_2}{\partial \Delta p_x} & \frac{\partial F_2}{\partial \omega_m} \\ \frac{\partial F_3}{\partial \theta_m} & \frac{\partial F_3}{\partial \omega_r} & \frac{\partial F_3}{\partial \Delta p_x} & \frac{\partial F_3}{\partial \omega_m} \\ \frac{\partial F_4}{\partial \theta_m} & \frac{\partial F_4}{\partial \omega_r} & \frac{\partial F_4}{\partial \Delta p_x} & \frac{\partial F_4}{\partial \omega_m} \end{bmatrix}_{\bar{X}_n, \bar{I}_n} \quad (42)$$

$$\left. \frac{\partial F}{\partial I} \right|_{\bar{X}_n, \bar{I}_n} = B^{4 \times 3} = \begin{bmatrix} \frac{\partial F_1}{\partial U} & \frac{\partial F_1}{\partial \omega_{sync}} & \frac{\partial F_1}{\partial \theta_{grid}} \\ \frac{\partial F_2}{\partial U} & \frac{\partial F_2}{\partial \omega_{sync}} & \frac{\partial F_2}{\partial \theta_{grid}} \\ \frac{\partial F_3}{\partial U} & \frac{\partial F_3}{\partial \omega_{sync}} & \frac{\partial F_3}{\partial \theta_{grid}} \\ \frac{\partial F_4}{\partial U} & \frac{\partial F_4}{\partial \omega_{sync}} & \frac{\partial F_4}{\partial \theta_{grid}} \end{bmatrix}_{\bar{X}_n, \bar{I}_n} \quad (43)$$

MATLAB command “linmod” is employed to evaluate these. Jacobian matrices and the eigenvalues of the system are calculated at wind speed $U=4, 8$, and 12 m/s as shown in Table 3. As shown, all the eigenvalues are on the left-half plane. Therefore, the system is stable at these operating points.

Table 3. Jacobian matrices and eigenvalues of the system at specified operating point

Operating Point	Jacobian Matrices and Eigenvalues
$U = 4m/s,$ $\bar{I}_n = \begin{bmatrix} 4 \\ 125.6637 \\ 2.5132e5 \end{bmatrix}^T,$ $\bar{X}_n = \begin{bmatrix} 2.5132e5 \\ 0.5143 \\ 3.3242e6 \\ 125.6637 \end{bmatrix}$	$A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & -0.0687 & 0 & 0 \\ 0 & 1.4407e8 & -16.0180 & -3.9714e5 \\ -500 & 8.8658 & 8.8906e-7 & -20.0005 \end{bmatrix},$ $B = \begin{bmatrix} 0 & 0 & 0 \\ 0.0088 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 20 & 500 \end{bmatrix}, Eigenvalue = \begin{bmatrix} -0.0687 \\ -10.0067 + 20.0067i \\ -10.0067 - 20.0067i \\ -16.0051 \end{bmatrix}$
$U = 8m/s,$ $\bar{I}_n = \begin{bmatrix} 8 \\ 125.6637 \\ 2.5132e5 \end{bmatrix}^T,$ $\bar{X}_n = \begin{bmatrix} 2.5132e5 \\ 1.0286 \\ 1.0035e7 \\ 125.6637 \end{bmatrix}$	$A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & -0.1373 & 0 & 0 \\ 0 & 2.2917e8 & -13.5946 & -1.0322e6 \\ -500 & 25.9145 & 2.3107e-6 & -20.0079 \end{bmatrix},$ $B = \begin{bmatrix} 0 & 0 & 0 \\ 0.0177 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 20 & 500 \end{bmatrix}, Eigenvalue = \begin{bmatrix} -0.1373 \\ -10.0430 + 20.0507i \\ -10.0430 - 20.0507i \\ -13.5164 \end{bmatrix}$
$U = 12m/s,$ $\bar{I}_n = \begin{bmatrix} 12 \\ 125.6637 \\ 2.5132e5 \end{bmatrix}^T,$ $\bar{X}_n = \begin{bmatrix} 2.5132e5 \\ 1.5429 \\ 2.0555e7 \\ 125.6637 \end{bmatrix}$	$A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & -0.2060 & 0 & 0 \\ 0 & 2.6158e8 & -10.9054 & -1.6669e6 \\ -500 & 52.4178 & 3.7315e-6 & -20.0307 \end{bmatrix},$ $B = \begin{bmatrix} 0 & 0 & 0 \\ 0.0265 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 20 & 500 \end{bmatrix}, Eigenvalue = \begin{bmatrix} -0.2060 \\ -10.0988 + 20.1441i \\ -10.0988 - 20.1441i \\ -10.7385 \end{bmatrix}$

V. DOUBLE TURBINE CONFIGURATION

A. Introduction

The cost of a multi-turbine system could be reduced if some components can be combined. In this section, a double-turbine system with combined hydraulic pipe, hydraulic motor, and generator, as shown in Figure 23, is analyzed.

Figure 23: Hydrostatic transmission wind turbine with shared hydraulic pipes, hydraulic motor, and generator

B. Hydrostatic Transmission Modeling for Double Turbines

In this section, the hydrostatic transmission wind turbine model discussed in Chapter 2, Equation (2) to (21), is reviewed and applied for a double-turbine configuration. In the following analysis, the subscript (1 or 2) of a variable denotes the turbine (#1 or #2) that the variable is associated with.

1) Aerodynamic Power and Torque Model

From Equation (2), aerodynamic wind powers (P_{wind1} and P_{wind2}) for a double-turbine configuration can be characterized by the following equations.

$$P_{wind1} = \frac{1}{2} \rho_{air} A_1 U_1^3 \quad (44)$$

$$P_{wind2} = \frac{1}{2} \rho_{air} A_2 U_2^3 \quad (45)$$

From Equation (3), rotor rotational powers (P_{rotor1} and P_{rotor2}) can be characterized by the following equations.

$$P_{rotor1} = P_{wind1} C_{p1}(\lambda_1, \beta_1) \quad (46)$$

$$P_{rotor2} = P_{wind2} C_{p2}(\lambda_2, \beta_2) \quad (47)$$

From Equation (5), aerodynamic torques (τ_{rotor1} and τ_{rotor2}) can be expressed as the following equations.

$$\tau_{rotor1} = \frac{P_{rotor1}}{\omega_{r1}} = \frac{1}{2} \rho_{air} A_1 R_1 C_{q1}(\lambda_1, \beta_1) U_1^2 \quad (48)$$

$$\tau_{rotor2} = \frac{P_{rotor2}}{\omega_{r2}} = \frac{1}{2} \rho_{air} A_2 R_2 C_{q2} (\lambda_2, \beta_2) U_2^2 \quad (49)$$

2) Variable Displacement Hydraulic Pump

From Equation (8), the first-order dynamic equation for double turbine configuration can be expressed as the following equations.

$$\dot{\omega}_{r1} = \frac{1}{J_{r1}} \left(\tau_{rotor1} - \frac{V_{p1} \Delta p_{p1}}{\eta_{mech,p1}} \right) \quad (50)$$

$$\dot{\omega}_{r2} = \frac{1}{J_{r2}} \left(\tau_{rotor2} - \frac{V_{p2} \Delta p_{p2}}{\eta_{mech,p2}} \right) \quad (51)$$

From Equation (9), fluid flow rates at the pump side (Q_{p1} and Q_{p2}) can be characterized as the following equations.

$$Q_{p1} = V_{p1} \omega_{r1} - k_{leak,p1} \Delta p_{p1} \quad (52)$$

$$Q_{p2} = V_{p2} \omega_{r2} - k_{leak,p2} \Delta p_{p2} \quad (53)$$

From Equation (10), leakage coefficients at the pump side ($k_{leak,p1}$ and $k_{leak,p2}$) can be expressed as the following equations.

$$k_{leak,p1} = \frac{k_{HP,p1}}{v_{fluid} \rho_{fluid}} \quad (54)$$

$$k_{HP,p1} = \frac{V_{p,max1} \omega_{p,noml} (1 - \eta_{v,pumpl}) v_{p,noml} \rho_{p,noml}}{\Delta p_{p,noml}}$$

$$\begin{aligned}
k_{leak,p2} &= \frac{k_{HP,p2}}{v_{fluid} \rho_{fluid}} \\
k_{HP,p2} &= \frac{V_{p,max2} \omega_{p,nom2} (1 - \eta_{v,pump2}) v_{p,nom2} \rho_{p,nom2}}{\Delta p_{p,nom2}}
\end{aligned} \tag{55}$$

3) Hydraulic Transmission Line

Since the fluid flow from the hydraulic pump of wind turbine #1 and #2 are combined to a single pipe system, the model of this part is different from that in the single-turbine configuration. Differential pressure and flow rate dynamics can be expressed as

$$\Delta \dot{p}_x = \frac{\beta_{fluid}}{V_{fluid}} (Q_{p1} + Q_{p2} - Q_m) \tag{56}$$

where Q_{p1} and Q_{p2} are the flow rate at the pump of turbine #1 and #2 and Q_m is the flow rate at the motor side. All other symbols have the same definition as those defined in Chapter 2.

Pressure loss equation and friction factor for the double-turbine configuration can be characterized using Equation (12) and (13), respectively, from the single-turbine configuration.

4) Variable Displacement Hydraulic Motor and Synchronous Generator

Equation (17) to (21) from Chapter 2 can be applied to the double-turbine configuration.

C. Region 2 Control Strategy

1) Maximization of Wind Turbine Power Coefficient Strategy

Similar to Equation (23), this control strategy can be realized by independently controlling V_{p1} and V_{p2} for the double-turbine configuration as shown below.

$$V_{p1} = \frac{K_1 \omega_{r1}^2 \eta_{mech,p1}}{\Delta p_{p1}} \quad (57)$$

$$V_{p2} = \frac{K_2 \omega_{r2}^2 \eta_{mech,p2}}{\Delta p_{p2}} \quad (58)$$

At steady state, the rotor speeds (ω_{r1} and ω_{r2}) can independently converge to the speed that gives optimal tip-speed ratio for maximum energy conversion efficiency (i.e., $C_{p1}=C_{p,max1}$ and $C_{p2}=C_{p,max2}$).

2) Maximization of Transmission Coefficient Strategy

Similar to the definition given in Chapter 3, the transmission coefficient (C_T) is defined as the efficiency of power transfer in hydrostatic transmission system (i.e., $C_T=P_{out}/(P_{rotor1}+P_{rotor2})$). To optimize C_T , five main losses are identified for the double-turbine configuration as shown below.

Pump Mechanical Loss ($Loss_{mech,pump1}$ and $Loss_{mech,pump2}$)

$$Loss_{mech,pump1} = (1 - \eta_{mech,p1}) K_1 \omega_{r1}^3 \quad (59)$$

$$Loss_{mech,pump2} = (1 - \eta_{mech,p2}) K_2 \omega_{r2}^3 \quad (60)$$

Pump Leakage Loss ($Loss_{leak,pump1}$ and $Loss_{leak,pump2}$)

$$Loss_{leak,pump1} = k_{leak,p1} \Delta p_{p1}^2 \quad (61)$$

$$Loss_{leak,pump2} = k_{leak,p2} \Delta p_{p2}^2 \quad (62)$$

Motor Mechanical Loss ($Loss_{mech,motor}$)

$$Loss_{mech,motor} = (1 - \eta_{mech,m}) V_m \Delta p_m \omega_m \quad (63)$$

Motor Leakage Loss ($Loss_{leak,motor}$)

$$Loss_{leak,motor} = k_{leak,m} \Delta p_m^2 \quad (64)$$

Friction Loss ($Loss_{fric}$)

$$Loss_{fric} = f \frac{L_{pipe}}{D_{pipe}} \frac{\rho_{fluid}}{2A_{pipe}^2} Q^3 \quad (65)$$

Total Hydrostatic Transmission Loss ($Loss_{total}$)

$$\begin{aligned} Loss_{total} = & Loss_{mech,pump1} + Loss_{mech,pump2} + Loss_{leak,pump1} + \dots \\ & \dots Loss_{leak,pump2} + Loss_{mech,motor} + Loss_{leak,motor} + Loss_{fric} \end{aligned} \quad (66)$$

Similar to the single-turbine configuration, the transmission coefficient (C_T) can be characterized as

$$C_T = \frac{P_{out}}{P_{rotor1} + P_{rotor2}} = 1 - \frac{Loss_{total}}{P_{rotor1} + P_{rotor2}}. \quad (67)$$

C_T Optimization Process: The maximum C_T can be found by taking the partial derivative of Equation (67) with respect to V_m and setting it to zero as shown in Equation (68).

$$\frac{\partial}{\partial V_m} C_T = \frac{\partial}{\partial V_m} \left(\frac{Loss_{total}}{P_{rotor1} + P_{rotor2}} \right) = 0 \quad (68)$$

Since P_{rotor1} and P_{rotor2} are independent of V_m , Equation (68) can be simplified as

$$\frac{\partial}{\partial V_m} Loss_{total} = 0. \quad (69)$$

By solving Equation (69) for V_m , optimal V_m ($V_{m,opt}$) can be obtained (i.e., $V_m = V_{m,opt}$).

Figure 24 shows $V_{m,opt}$ as a function of wind speed 1 and 2 (U_1 and U_2) in a contour plot at steady state. It can be shown that $V_{m,opt}$ depends on the rotor speed of both turbines (ω_{r1} and ω_{r2}), as shown in Equation (70).

$$V_{m,opt} = f(\omega_{r1}, \omega_{r2}) \quad (70)$$

$V_{m,opt}$ for the double-turbine configuration can be implemented using a lookup table.

Figure 25 is the overall system-level block diagram for the proposed double-turbine system.

Figure 24: Motor displacement (V_m) as a function of wind speed 1 (U_1) and wind speed 2 (U_2). Note, when compared with the single-turbine system, Figure 8, V_m is no longer linear.

Figure 25: Overall system level block diagram including control strategy for maximizing C_{p1} , C_{p2} , and C_T (in round-dotted boxes)

D. Simulation Results in MATLAB/Simulink

In this section, steady state and transient responses of the hydrostatic transmission wind turbine in the double-turbine configuration are simulated according to the model and control strategies proposed above. Parameters in Table 1 are used for both turbines except $D_{pipe}=0.19$ m (27% increase from 0.15 m used in the single turbine), which is selected so that C_T for the single- and double-turbine are comparable when $U_1=U_2$.

1) Steady State Response

Steady state of the system as a function of wind speed U_1 and U_2 are shown in Figure 26 to Figure 42. The motor speed ω_m is constant at 125.66 rad/s at steady state regardless of U_1 and U_2 , similar to the case of single turbine configuration. ω_{r1} is a function of U_1 and not dependent on U_2 . ω_{r2} is a function of U_2 and not dependent on U_1 . Δp_p ($\Delta p_{p1} = \Delta p_{p2} = \Delta p_p$) and Δp_m increase as a function of U_1 and U_2 . V_{p1} , V_{p2} , and V_m increase as a function of U_1 and U_2 according to control strategy given by Equation (57), (58), and (70). τ_{load} from the synchronous generator increases as a function of U_1 and U_2 . P_{wind1} and P_{rotor1} are functions of U_1 and independent of U_2 . P_{wind2} and P_{rotor2} are functions of U_2 and independent of U_1 . P_{out} increase as a function of U_1 and U_2 . Note that $P_{rotor1}/P_{wind1} = C_{p1}$, $P_{rotor2}/P_{wind2} = C_{p2}$, and $P_{out}/(P_{rotor1} + P_{rotor2}) = C_T$. As shown in Figure 40 and Figure 41, C_{p1} and C_{p2} are constant ($C_{p1} = C_{p2} = 0.48$) regardless of U_1 and U_2 because of the control strategy proposed in Equation (57) and (58). C_T , optimized by Equation (70), decreases as a function of U_1 and U_2 .

Figure 26: Rotor speed 1 (ω_{r1}) in rad/s as a function of wind speed 1 (U_1) and wind speed 2 (U_2)

Figure 27: Rotor speed 2 (ω_{r2}) in rad/s as a function of wind speed 1 (U_1) and wind speed 2 (U_2)

Figure 28: Motor speed (ω_m) in rad/s as a function of wind speed 1 (U_1) and wind speed 2 (U_2)

Figure 29: Differential pump pressure (Δp_p) in MPa as a function of wind speed 1 (U_1) and wind speed 2 (U_2)

Figure 30: Differential motor pressure (Δp_m) in MPa as a function of wind speed 1 (U_1) and wind speed 2 (U_2)

Figure 31: Pump displacement 1 (V_{p1}) in m^3/rad as a function of wind speed 1 (U_1) and wind speed 2 (U_2)

Figure 32: Pump displacement 2 (V_{p2}) in m^3/rad as a function of wind speed 1 (U_1) and wind speed 2 (U_2)

Figure 33: Motor displacement (V_m) in m^3/rad as a function of wind speed 1 (U_1) and wind speed 2 (U_2)

Figure 34: Generator torque load (τ_{load}) in kN-m as a function of wind speed 1 (U_1) and wind speed 2 (U_2)

Figure 35: Wind power 1 (P_{wind1}) in MW as a function of wind speed 1 (U_1) and wind speed 2 (U_2)

Figure 36: Wind power 2 (P_{wind2}) in MW as a function of wind speed 1 (U_1) and wind speed 2 (U_2)

Figure 37: Rotor rotational power 1 (P_{rotor1}) in MW as a function of wind speed 1 (U_1) and wind speed 2 (U_2)

Figure 38: Rotor rotational power 2 (P_{rotor2}) in MW as a function of wind speed 1 (U_1) and wind speed 2 (U_2)

Figure 39: Output power (P_{out}) in MW as a function of wind speed 1 (U_1) and wind speed 2 (U_2)

Figure 40: Power coefficient 1 (C_{p1}) as a function of wind speed 1 (U_1) and wind speed 2 (U_2)

Figure 41: Power coefficient 2 (C_{p2}) as a function of wind speed 1 (U_1) and wind speed 2 (U_2)

Figure 42: Transmission coefficient (C_T) as a function of wind speed 1 (U_1) and wind speed 2 (U_2)

2) System Response

The system responses of the double turbine configuration are shown in Figure 43 to Figure 51. Two inputs, wind speed U_1 and U_2 , are unit step functions as shown in Figure 43. As U_1 and U_2 change, ω_m remains at 125.66 rad/s and ω_{r1} and ω_{r2} try to keep up with U_1 and U_2 so that both wind turbines operate at optimal tip-speed ratio, similar to the single turbine configuration. Δp_p , Δp_m , and τ_{load} also change according to U_1 and U_2 . V_{p1} , V_{p2} , and V_m are controlled according to the control law so that the system reaches the optimal efficiency at steady state. Figure 51 shows that C_T , C_{p1} , and C_{p2} track their optimal points as U_1 and U_2 vary.

Figure 43: Wind speeds (U_1 and U_2) as functions of time

Figure 44: Rotor speeds (ω_{r1} and ω_{r2}) and motor speed (ω_m) as functions of time

Figure 45: Differential pump pressure ($\Delta p_{p1} = \Delta p_{p2} = \Delta p_p$) and differential motor pressure (Δp_m) as functions of time

Figure 46: Pump displacements (V_{p1} and V_{p2}) and motor displacement (V_m) as functions of time

Figure 47: Generator torque load (τ_{load}) as a function of time

Figure 48: Wind power 1 (P_{wind1}) and rotor rotational power 1 (P_{rotor1}) as functions of time

Figure 49: Wind power 2 (P_{wind2}) and rotor rotational power 2 (P_{rotor2}) as functions of time

Figure 50: Total wind power ($P_{wind1}+P_{wind2}$), total rotor rotational power ($P_{rotor1}+P_{rotor2}$), and output power (P_{out}) as functions of time

Figure 51: Transmission coefficient (C_T) and power coefficients (C_{p1} and C_{p2}) as functions of time

It is worth noting that when one turbine is operating at a much higher speed than the other one, the system pressure will stay at a high value (following the higher wind speed turbine) because V_m is adjusted according to Equation (70) to maximize C_T . As a result, the leakage flow rate of the hydraulic pump at the lower speed wind turbine ($k_{leak,p}\Delta p_p$) will be significant comparing to the pumped flow rate due to the rotor speed ($V_p\omega_r$). In fact, if $V_p\omega_r$ cannot overcome $k_{leak,p}\Delta p_p$, the aerodynamic power from the low speed turbine does not contribute to the power generation. In this case, two individual wind turbines operating at $U_1=12\text{m/s}$ and $U_2=4\text{m/s}$ will produce more power than what the double-turbine configuration wind turbine can produce.

3) System Stability Analysis

In this section, the dynamic equation of the double-turbine configuration is linearized and its stability is assessed [38].

Let $\vec{X}(t)$ be system statespace variable defined as

$$\vec{X}(t) = \begin{bmatrix} \theta_m(t) \\ \omega_{r1}(t) \\ \omega_{r2}(t) \\ \Delta p_x(t) \\ \omega_m(t) \end{bmatrix}. \quad (71)$$

$\vec{X}(t)$ can be written in terms of an operating system trajectory $\vec{X}_n$ and a small change $\Delta\vec{X}(t)$ as

$$\begin{aligned} \vec{X}(t) &= \vec{X}_n + \Delta\vec{X}(t) \\ &= \begin{bmatrix} \theta_{m,n} \\ \omega_{r1,n} \\ \omega_{r2,n} \\ \Delta p_{x,n} \\ \omega_{m,n} \end{bmatrix} + \begin{bmatrix} \Delta\theta_m(t) \\ \Delta\omega_{r1}(t) \\ \Delta\omega_{r2}(t) \\ \Delta(\Delta p_x(t)) \\ \Delta\omega_m(t) \end{bmatrix}. \end{aligned} \quad (72)$$

Similarly, let $\vec{I}(t)$ be system input and defined as the following equation.

$$\vec{I}(t) = \begin{bmatrix} U_1(t) & U_2(t) & \omega_{sync}(t) & \theta_{grid}(t) \end{bmatrix} \quad (73)$$

$\vec{I}(t)$ can be expressed as the sum of the un-perturbed value $\vec{I}_n$ and the variation $\Delta\vec{I}(t)$ as

$$\begin{aligned}\vec{I}(t) &= \vec{I}_n + \Delta\vec{I}(t) \\ &= \begin{bmatrix} U_{1,n} & U_{2,n} & \omega_{sync,n} & \theta_{grid,n} \end{bmatrix} + \begin{bmatrix} \Delta U_1(t) & \Delta U_2(t) & \Delta\omega_{sync}(t) & \Delta\theta_{grid}(t) \end{bmatrix} \quad (74)\end{aligned}$$

The nonlinear system state space representation of the system can be written as

$$\frac{d}{dt} \vec{X}(t) = F(\vec{X}(t), \vec{I}(t)) \quad (75)$$

where F is the 5x5 dimensional vector function. If small changes are considered, the nonlinear system can be linearized at an operating point as

$$\begin{aligned}\frac{d}{dt} \Delta\vec{X}(t) &= \left. \frac{\partial F}{\partial X} \right|_{\vec{X}_n, \vec{I}_n} \Delta\vec{X}(t) + \left. \frac{\partial F}{\partial I} \right|_{\vec{X}_n, \vec{I}_n} \Delta\vec{I}(t) \\ &= A\Delta\vec{X}(t) + B\Delta\vec{I}(t)\end{aligned} \quad (76)$$

where A and B represent the Jacobian matrices and are given by the following equations.

$$\left. \frac{\partial F}{\partial X} \right|_{\vec{X}_n, \vec{I}_n} = A^{5 \times 5} = \begin{bmatrix} \frac{\partial F_1}{\partial \theta_m} & \frac{\partial F_1}{\partial \omega_{r1}} & \frac{\partial F_1}{\partial \omega_{r2}} & \frac{\partial F_1}{\partial \Delta p_x} & \frac{\partial F_1}{\partial \omega_m} \\ \frac{\partial F_2}{\partial \theta_m} & \frac{\partial F_2}{\partial \omega_{r1}} & \frac{\partial F_2}{\partial \omega_{r2}} & \frac{\partial F_2}{\partial \Delta p_x} & \frac{\partial F_2}{\partial \omega_m} \\ \frac{\partial F_3}{\partial \theta_m} & \frac{\partial F_3}{\partial \omega_{r1}} & \frac{\partial F_3}{\partial \omega_{r2}} & \frac{\partial F_3}{\partial \Delta p_x} & \frac{\partial F_3}{\partial \omega_m} \\ \frac{\partial F_4}{\partial \theta_m} & \frac{\partial F_4}{\partial \omega_{r1}} & \frac{\partial F_4}{\partial \omega_{r2}} & \frac{\partial F_4}{\partial \Delta p_x} & \frac{\partial F_4}{\partial \omega_m} \\ \frac{\partial F_5}{\partial \theta_m} & \frac{\partial F_5}{\partial \omega_{r1}} & \frac{\partial F_5}{\partial \omega_{r2}} & \frac{\partial F_5}{\partial \Delta p_x} & \frac{\partial F_5}{\partial \omega_m} \end{bmatrix}_{\vec{X}_n, \vec{I}_n} \quad (77)$$

$$\left. \frac{\partial F}{\partial I} \right|_{\bar{X}_n, \bar{I}_n} = B^{5 \times 4} = \begin{bmatrix} \frac{\partial F_1}{\partial U_1} & \frac{\partial F_1}{\partial U_2} & \frac{\partial F_1}{\partial \omega_{sync}} & \frac{\partial F_1}{\partial \theta_{grid}} \\ \frac{\partial F_2}{\partial U_1} & \frac{\partial F_2}{\partial U_2} & \frac{\partial F_2}{\partial \omega_{sync}} & \frac{\partial F_2}{\partial \theta_{grid}} \\ \frac{\partial F_3}{\partial U_1} & \frac{\partial F_3}{\partial U_2} & \frac{\partial F_3}{\partial \omega_{sync}} & \frac{\partial F_3}{\partial \theta_{grid}} \\ \frac{\partial F_4}{\partial U_1} & \frac{\partial F_4}{\partial U_2} & \frac{\partial F_4}{\partial \omega_{sync}} & \frac{\partial F_4}{\partial \theta_{grid}} \\ \frac{\partial F_5}{\partial U_1} & \frac{\partial F_5}{\partial U_2} & \frac{\partial F_5}{\partial \omega_{sync}} & \frac{\partial F_5}{\partial \theta_{grid}} \end{bmatrix}_{\bar{X}_n, \bar{I}_n} \quad (78)$$

MATLAB command “linmod” is employed to evaluate these. Jacobian matrices and the eigenvalues of the system are calculated at wind speed $(U_1=4, U_2=4)$; $(U_1=4, U_2=8)$; $(U_1=4, U_2=12)$; $(U_1=8, U_2=8)$; $(U_1=8, U_2=12)$; and $(U_1=12, U_2=12)$ as shown in Table 4. As shown, all the eigenvalues are on the left-half plane. Therefore, the system is stable at these operating points.

Table 4. Jacobian matrices and eigenvalues of the system at specified operating point

Operating Point	Jacobian Matrices and Eigenvalues
$U_1 = 4m/s$ $U_2 = 4m/s$ $\bar{I}_n = \begin{bmatrix} 4 \\ 4 \\ 125.6637 \\ 2.5132e5 \end{bmatrix}^T$, $\bar{X}_n = \begin{bmatrix} 2.5132e5 \\ 0.5143 \\ 0.5143 \\ 3.3409e6 \\ 125.6637 \end{bmatrix}$	$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & -0.0687 & 0 & 0 & 0 \\ 0 & 0 & -0.0687 & 0 & 0 \\ 0 & 8.8226e7 & 8.8226e7 & -19.5675 & -4.9512e5 \\ -500 & 8.8783 & 8.8783 & 1.7784e-6 & -20.0012 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.0088 & 0 & 0 & 0 \\ 0 & 0.0088 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 20 & 500 \end{bmatrix}$, $Eigenvalue = \begin{bmatrix} -0.0687 \\ -0.0687 \\ -10.0181+20.0134i \\ -10.0181-20.0134i \\ -19.5325 \end{bmatrix}$
$U_1 = 4m/s$ $U_2 = 8m/s$ $\bar{I}_n = \begin{bmatrix} 4 \\ 8 \\ 125.6637 \\ 2.5132e5 \end{bmatrix}^T$, $\bar{X}_n = \begin{bmatrix} 2.5132e5 \\ 0.5143 \\ 1.0286 \\ 7.7908e6 \\ 125.6637 \end{bmatrix}$	$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & -0.0687 & 0 & 0 & 0 \\ 0 & 0 & -0.1373 & 0 & 0 \\ 0 & 2.9750e7 & 1.6737e8 & -16.0195 & -9.4062e5 \\ -500 & 10.1954 & 30.5879 & 3.3785e-6 & -20.0073 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.0088 & 0 & 0 & 0 \\ 0 & 0.0177 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 20 & 500 \end{bmatrix}$, $Eigenvalue = \begin{bmatrix} -0.0687 \\ -0.1373 \\ -10.0618+20.0603i \\ -10.0618-20.0603i \\ -15.9033 \end{bmatrix}$
$U_1 = 4m/s$ $U_2 = 12m/s$ $\bar{I}_n = \begin{bmatrix} 4 \\ 12 \\ 125.6637 \\ 2.5132e5 \end{bmatrix}^T$, $\bar{X}_n = \begin{bmatrix} 2.5132e5 \\ 0.5143 \\ 1.5429 \\ 1.5106e7 \\ 125.6637 \end{bmatrix}$	$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & -0.0687 & 0 & 0 & 0 \\ 0 & 0 & -0.2060 & 0 & 0 \\ 0 & 3.9851e7 & 2.1348e8 & -13.2124 & -1.4850e6 \\ -500 & -0.0019 & 58.6726 & 5.3338e-6 & -20.0264 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.0088 & 0 & 0 & 0 \\ 0 & 0.0265 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 20 & 500 \end{bmatrix}$, $Eigenvalue = \begin{bmatrix} -0.0687 \\ -0.2060 \\ -10.1389+20.1717i \\ -10.1389-20.1717i \\ -12.9610 \end{bmatrix}$

Operating Point	Jacobian Matrices and Eigenvalues
$U_1 = 8m/s$ $U_2 = 8m/s$ $\bar{I}_n = \begin{bmatrix} 8 \\ 8 \\ 125.6637 \\ 2.5132e5 \end{bmatrix}^T$, $\bar{X}_n = \begin{bmatrix} 2.5132e5 \\ 1.0286 \\ 1.0286 \\ 1.0882e7 \\ 125.6637 \end{bmatrix}$	$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & -0.1373 & 0 & 0 & 0 \\ 0 & 0 & -0.1373 & 0 & 0 \\ 0 & 1.2573e8 & 1.2573e8 & -14.5680 & -1.1881e6 \\ -500 & 28.3258 & 28.3258 & 4.2673e-6 & -20.0140 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.0177 & 0 & 0 & 0 \\ 0 & 0.0177 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 20 & 500 \end{bmatrix}$, $Eigenvalue = \begin{bmatrix} -0.1373 \\ -0.1373 \\ -10.0941 + 20.1037i \\ -10.0941 - 20.1037i \\ -14.3939 \end{bmatrix}$
$U_1 = 8m/s$ $U_2 = 12m/s$ $\bar{I}_n = \begin{bmatrix} 8 \\ 12 \\ 125.6637 \\ 2.5132e5 \end{bmatrix}^T$, $\bar{X}_n = \begin{bmatrix} 2.5132e5 \\ 1.0286 \\ 1.5429 \\ 1.7112e7 \\ 125.6637 \end{bmatrix}$	$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & -0.1373 & 0 & 0 & 0 \\ 0 & 0 & -0.2060 & 0 & 0 \\ 0 & 9.1835e7 & 1.7034e8 & -12.8347 & -1.6334e6 \\ -500 & 22.0740 & 66.2262 & 5.8669e-6 & -20.0347 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.0177 & 0 & 0 & 0 \\ 0 & 0.0265 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 20 & 500 \end{bmatrix}$, $Eigenvalue = \begin{bmatrix} -0.1373 \\ -0.2060 \\ -10.1653 + 20.2106i \\ -10.1653 - 20.2106i \\ -12.5387 \end{bmatrix}$
$U_1 = 12m/s$ $U_2 = 12m/s$ $\bar{I}_n = \begin{bmatrix} 12 \\ 12 \\ 125.6637 \\ 2.5132e5 \end{bmatrix}^T$, $\bar{X}_n = \begin{bmatrix} 2.5132e5 \\ 1.5429 \\ 1.5429 \\ 2.2167e7 \\ 125.6637 \end{bmatrix}$	$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & -0.2060 & 0 & 0 & 0 \\ 0 & 0 & -0.2060 & 0 & 0 \\ 0 & 1.4581e8 & 1.4581e8 & -11.8088 & -1.9303e6 \\ -500 & 56.9838 & 56.9838 & 6.9332e-6 & -20.0558 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.0265 & 0 & 0 & 0 \\ 0 & 0.0265 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 20 & 500 \end{bmatrix}$, $Eigenvalue = \begin{bmatrix} -0.2060 \\ -0.2060 \\ -10.2184 + 20.3040i \\ -10.2184 - 20.3040i \\ -11.4278 \end{bmatrix}$

VI. CONCLUSION

The dynamics and control of a wind turbine system with a hydrostatic transmission were studied in this work. In this system, the nacelle contained a variable displacement hydraulic pump. A hydraulic transmission pipe connected the pump to a variable displacement motor on the ground level. The hydraulic motor was mechanically connected to a synchronous generator that converted the rotational power to electrical power. A double-turbine system was also included in the study.

The objective of this research was to develop a control strategy that maximized the power conversion efficiency from captured aerodynamic power to the production of the electrical power. The pump displacement was controlled to achieve maximum wind power capture and the motor displacement was controlled to minimize the losses in the hydraulic transmission system.

The stability of the proposed control strategy was established by linearization analysis and was validated by computer simulation using MATLAB/Simulink. The results showed that the system was stable and had fast and well-damped response.

This thesis provides a theoretical basis for the control of a wind turbine system using hydraulic transmission. While the study includes all the basic elements in the system, there are many aspects of the system behavior (e.g., fluid dynamics) and practical issues such as structure strength, materials, and costs that need further study. Considering the benefits of such a wind turbine system, the realization and implementation of such a wind turbine system will be a fertile research area for years to come.

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APPENDIX A: MATLAB/SIMULINK FOR SINGLE TURBINE

A. Simulink Base Model

Figure 52 (Appendix A): “Wind Turbine” symbol and detailed model

Figure 53 (Appendix A): “Pump System” symbol and detailed model

Figure 54 (Appendix A): “Motor System” symbol and detailed model

Figure 55 (Appendix A): “Pressure System” symbol and detailed model

Figure 56 (Appendix A): “Synchronous Generator” symbol and detailed model

B. MATLAB Code for Assigning System Parameters

```

1  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
2  %Define constant variable value
3
4  %Wind and Turbine Parameter
5  rho_air=1.225;           %Air Density in kg/m^3
6  R=63;                   %Wind blade Radius in m
7  area=pi()*R^2;          %Blade swept area in m^2
8  pitch=0;                %Pitch angle in degree
9  c1=0.5176;              %Parameter for power coefficient
10 c2=116;                  %Parameter for power coefficient
11 c3=0.4;                  %Parameter for power coefficient
12 c4=5;                    %Parameter for power coefficient
13 c5=21;                   %Parameter for power coefficient
14 c6=0.0068;               %Parameter for power coefficient
15 Cp_opt=0.48;             %Optimal power coefficient
16 lamda_opt=8.1;           %Optimal Tip-speed ratio
17
18 %Hydrostatic Transmission Line Parameter
19 bulk_mod=1.40403e9;      %Fluid bulk modulus in Pa
20 Dp=0.15;                 %Pipe Diameter in m
21 Lp=100;                  %Pipe Length in m
22 Ch=pi()* (Dp/2)^2*Lp;    %Chamber or pipe volume in m^3
23 rho_fluid=852.8;         %Fluid density in kg/m^3
24 v=18.786e-6;             %Kinematic viscosity in m^2/s
25 r=1.5e-5;                %Pipe roughness internal surface
26
27 %Pump/Rotor Parameters
28 Jr=38759228;             %Pump/Rotor inertia in kg-m^2
29 n_mech_pump=0.95;        %Pump/Rotor mechanical efficiency
30 n_vol_pump=0.95;         %Pump volumetric efficiency
31 Vp_max=0.3;              %Maximum pump displacement in m^3/rad
32 w_pump_nom=1;            %Nominal pump speed in rad/s
33 v_pump_nom=18.786e-6;    %Nominal pump kinematic viscosity in m^2/s
34 p_pump_nom=2e7;          %Nominal pump pressure in Pa
35 kHP_pump=Vp_max*w_pump_nom*(1-n_vol_pump)*v_pump_nom*rho_fluid/p_pump_nom;
36 %Pump Hagen-Poiseuille coefficient
37 kleak_pump=kHP_pump/(v*rho_fluid);
38 %Pump leakage Coefficient
39
40 %Motor/Generator Parameters
41 Jg=534.116;              %Motor/Generator inertia in kg-m^2
42 n_mech_motor=0.95;       %Motor/Generator mechanical efficiency
43 n_vol_motor=0.95;        %Motor volumetric efficiency
44 Vm=0.8e-3;               %Motor displacement in m^3/rad
45 w_motor_nom=125.6637;    %Nominal motor speed in rad/s
46 v_motor_nom=18.786e-6;  %Nominal motor kinematic viscosity in m^2/s
47 p_motor_nom=2e7;         %Nominal motor pressure in Pa
48 kHP_motor=Vm*w_motor_nom*(1-n_vol_motor)*v_motor_nom*rho_fluid/p_motor_nom;
49 %Motor Hagen-Poiseuille coefficient
50 kleak_motor=kHP_motor/(v*rho_fluid);
51 %Motor leakage coefficient
52 w_motor_sync=125.6637;  %Motor synchronous speed
53
54 Kp=500*Jg;               %Motor Angle Controller
55 Kd=20*Jg;                %Motor Speed Controller

```

Figure 57 (Appendix A): MATLAB code for assigning system parameters

C. MATLAB Code and Simulink for Steady State Response

```

58 %Initialize State Variables and Input Parameters
59 %These values are chosen so that Simulink can solve the devised model
60 xInitial(1,1)=1.2566e5; %Grid Angle Initialization
61 xInitial(1,2)=1.2566e5; %Motor Angle Initialization
62 xInitial(1,3)=125.6637; %Motor Speed Initialization
63 xInitial(1,4)=1.5429; %Rotor Speed Initialization
64 xInitial(1,5)=4.2055e7; %System Pressure Initialization
65
66 %Set Wind Speed and Motor Displacement to Simulate
67 set_Wind_Speed=4:1:12; %Matrix containing Wind speed
68 set_Vm=1e-4:1e-4:60e-4; %Matrix containing Motor Displacement
69
70 %Main Loop
71 dws=size(set_Wind_Speed);
72 dvm=size(set_Vm);
73 ws=1;
74 vm=1;
75 results=zeros(dvm(1,2)+1,dws(1,2)+1,21); %Matrix containing all the output
76
77 for ws=1:dws(1,2)
78     Wind_Speed=set_Wind_Speed(ws);
79     display(Wind_Speed)
80
81     for vm=1:dvm(1,2)
82         Vm=set_Vm(vm);
83
84         sim('OneTurbine_SteadyState')
85
86         %Copy all the outputs to results matrix
87         results(vm+1,1,:)=Vm;
88         results(1,ws+1,:)=Wind_Speed;
89         d=size(wp.signals.values);
90         results(vm+1,ws+1,1)=wp.signals.values(d(1,1)); %Rotor Speed
91         d=size(wg.signals.values);
92         results(vm+1,ws+1,2)=wg.signals.values(d(1,1)); %Motor Speed
93         d=size(Pp.signals.values);
94         results(vm+1,ws+1,3)=Pp.signals.values(d(1,1)); %Pump Pressure
95         d=size(Pm.signals.values);
96         results(vm+1,ws+1,4)=Pm.signals.values(d(1,1)); %Motor Pressure
97         d=size(Pwind.signals.values);
98         results(vm+1,ws+1,5)=Pwind.signals.values(d(1,1)); %Wind Power
99         d=size(Protor.signals.values);
100        results(vm+1,ws+1,6)=Protor.signals.values(d(1,1)); %Rotor Power
101        d=size(Pout.signals.values);
102        results(vm+1,ws+1,7)=Pout.signals.values(d(1,1)); %Output Power
103        d=size(Vp.signals.values);
104        results(vm+1,ws+1,8)=Vp.signals.values(d(1,1)); %Pump Displacement
105        d=size(cp.signals.values);
106        results(vm+1,ws+1,9)=cp.signals.values(d(1,1)); %Power Coefficient
107        d=size(Mg.signals.values);
108        results(vm+1,ws+1,10)=Mg.signals.values(d(1,1)); %Generator Torque
109        results(vm+1,ws+1,11)=results(vm+1,ws+1,7)/results(vm+1,ws+1,6);
110                                     %Transmission Coefficient
111     end
112 end

```

Figure 58 (Appendix A): MATLAB code for steady state response

Figure 59 (Appendix A): Workspace (part 1) for “OneTurbine_SteadyState.mdl”

Figure 60 (Appendix A): Workspace (part 2) for “OneTurbine_SteadyState.mdl”

D. MATLAB Code and Simulink for System Response

```

57 %Initialize State Variables and Input Parameters
58 %These values are chosen so that Simulink can solve the devised model
59 xInitial(1,1)=1.2566e5; %Grid Angle Initialization
60 xInitial(1,2)=1.2566e5; %Motor Angle Initialization
61 xInitial(1,3)=125.6637; %Motor Speed Initialization
62 xInitial(1,4)=1.5429; %Rotor Speed Initialization
63 xInitial(1,5)=4.2055e7; %System Pressure Initialization
64
65 %Run Simulink Model
66 sim('OneTurbine_Dynamics')
67
68 %Construct a matrix that contains all results
69 dtout_total=size(tout); %tout is exported from the simulation
70 results=zeros(dtout_total(1,1),2,13);
71 for a=1:13
72     results(:,1,a)=tout;
73 end
74 results(:,2,1)=wp.signals.values; %Pump Speed
75 results(:,2,2)=wg.signals.values; %Motor Speed
76 results(:,2,3)=Pp.signals.values; %Pump Pressure
77 results(:,2,4)=Pm.signals.values; %Motor Pressure
78 results(:,2,5)=Pwind.signals.values; %Wind Power
79 results(:,2,6)=Protor.signals.values; %Rotor Rotational Power
80 results(:,2,7)=Pout.signals.values; %Output Power
81 results(:,2,8)=Vp.signals.values; %Pump Displacement
82 results(:,2,9)=cp.signals.values; %Power Coefficient
83 results(:,2,10)=Mg.signals.values; %Generator Torque
84 results(:,2,11)=results(:,2,7)./results(:,2,6); %Transmission Coefficient
85 results(:,2,12)=Vm.signals.values; %Motor Displacement
86 results(:,2,13)=U.signals.values; %Wind Speed
87
88 %Export to Excel
89 xlswrite('wp.xls',results(:,1))
90 xlswrite('wg.xls',results(:,2))
91 xlswrite('Pp.xls',results(:,3))
92 xlswrite('Pm.xls',results(:,4))
93 xlswrite('Pwind.xls',results(:,5))
94 xlswrite('Protor.xls',results(:,6))
95 xlswrite('Pout.xls',results(:,7))
96 xlswrite('Vp.xls',results(:,8))
97 xlswrite('cp.xls',results(:,9))
98 xlswrite('Mg.xls',results(:,10))
99 xlswrite('ct.xls',results(:,11))
100 xlswrite('Vm.xls',results(:,12))
101 xlswrite('U.xls',results(:,13))

```

Figure 61 (Appendix A): MATLAB code for system response

Figure 62 (Appendix A): Workspace (part 1) for “OneTurbine_Dynamics.mdl”

Figure 63 (Appendix A): Workspace (part 2) for “OneTurbine_Dynamics.mdl”

E. MATLAB Code and Simulink for Linearization

```

72 %Initialize State Variables and Input Parameters
73 %These values are chosen so that Simulink can solve the devised model
74 - xInitial(1,1)=1.5429; %Rotor Speed Initialization
75 - xInitial(1,2)=125.6637; %Motor Speed Initialization
76 - xInitial(1,3)=4.2055e7; %System Pressure Initialization
77 - xInitial(1,4)=1.2566e5; %Grid Angle Initialization
78 - xInitial(1,5)=1.2566e5; %Motor Angle Initialization
79
80 %Set Wind Speed
81 - Wind_Speed=12;
82 - Vm=0.0002*Wind_Speed-0.0003; %Satisfy optimal Vm at steady state
83
84 %Acquire State Variable Value at Steady State
85 - sim('OneTurbine_SteadyState')
86 - [a b] =size(yout);
87 - v_equil = Wind_Speed;
88 - wg_equil = yout(a,2);
89 - wr_equil = yout(a,3);
90 - P_equil = yout(a,4);
91 - Angle_grid_equil = yout(a,5);
92 - Angle_motor_equil = yout(a,6);
93
94 %Linearization
95 - [A B C D] = linmod('OneTurbine_Linearization',[wr_equil; wg_equil;...
96 - P_equil; Angle_motor_equil],[v_equil 125.6637 Angle_grid_equil]);

```

Figure 64 (Appendix A): MATLAB code for linearization

Figure 65 (Appendix A): Workspace (part 1) for “OneTurbine_Linearization.mdl”

Figure 66 (Appendix A): Workspace (part 2) for “OneTurbine_Linearization.mdl”

APPENDIX B: MATLAB/SIMULINK FOR DOUBLE TURBINE

A. Simulink Base Model

Figure 67 (Appendix B): “Pressure System” symbol and detailed model (part 1)

B. MATLAB Code for Assigning System Parameters

```

1  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
2  %Define constant variable value
3
4  %Wind and Turbine Parameter
5  rho_air=1.225;           %Air Density in kg/m^3
6  R=63;                   %Wind blade Radius in m
7  area=pi()*R^2;          %Blade swept area in m^2
8  pitch1=0;               %pitch angle in degree
9  pitch2=0;
10 c1=0.5176;              %Parameter for power coefficient
11 c2=116;                 %Parameter for power coefficient
12 c3=0.4;                 %Parameter for power coefficient
13 c4=5;                   %Parameter for power coefficient
14 c5=21;                  %Parameter for power coefficient
15 c6=0.0068;              %Parameter for power coefficient
16 Cp_opt=0.48;            %Optimal power coefficient
17 lamda_opt=8.1;          %Optimal Tip-speed ratio
18
19 %Hydrostatic Transmission Line Parameter
20 bulk_mod=1.40403e9;      %Fluid bulk modulus in Pa
21 Dp=0.19;                %Pipe Diameter in m
22 Lp=100;                 %Pipe Length in m
23 Ch=pi()*(Dp/2)^2*Lp;    %Chamber or pipe volume in m^3
24 rho_fluid=852.8;         %Fluid density in kg/m^3
25 v=18.786e-6;            %Kinematic viscosity in m^2/s
26 r=1.5e-5;               %Roughness on the pipe internal surface
27
28 %Pump/Rotor Parameters
29 Jr=38759228;            %Pump/Rotor inertia in kg-m^2
30 n_mech_pump1=0.95;      %Pump/Rotor mechanical efficiency
31 n_mech_pump2=0.95;
32 n_vol_pump=0.95;        %Pump volumetric efficiency
33 Vp_max=0.3;             %Maximum pump displacement in m^3/rad
34 w_pump_nom=1;           %Nominal pump speed in rad/s
35 v_pump_nom=18.786e-6;   %Nominal pump kinematic viscosity in m^2/s
36 p_pump_nom=2e7;         %Nominal pump pressure in Pa
37 kHP_pump=Vp_max*w_pump_nom*(1-n_vol_pump)*v_pump_nom*rho_fluid/p_pump_nom;
38                          %Pump Hagen-Poiseuille coefficient
39 kleak_pump1=kHP_pump/(v*rho_fluid);
40 kleak_pump2=kHP_pump/(v*rho_fluid);
41                          %Pump leakage Coefficient
42
43 %Motor/Generator Parameters
44 Jg=534.116;             %Motor/Generator inertia in kg-m^2
45 n_mech_motor=0.95;      %Motor/Generator mechanical efficiency
46 n_vol_motor=0.95;       %Motor volumetric efficiency
47 Vm=0.8e-3;             %Motor displacement in m^3/rad
48 w_motor_nom=125.6637;   %Nominal motor speed in rad/s
49 v_motor_nom=18.786e-6; %Nominal motor kinematic viscosity in m^2/s
50 p_motor_nom=2e7;        %Nominal motor pressure in Pa
51 kHP_motor=Vm*w_motor_nom*(1-n_vol_motor)*v_motor_nom*rho_fluid/p_motor_nom;
52                          %Motor Hagen-Poiseuille coefficient
53 kleak_motor=kHP_motor/(v*rho_fluid);
54                          %Motor leakage coefficient
55 w_motor_sync=125.6637;  %Motor synchronous speed
56
57 Kp=500*Jg;              %Motor Angle Controller
58 Kd=20*Jg;               %Motor Speed Controller

```

Figure 69 (Appendix B): MATLAB code for assigning system parameters

C. MATLAB Code and Simulink for Steady State Response

```

61 %Initialize State Variables and Input Parameters
62 %These values are chosen so that Simulink can solve the devised model
63 xInitial(1,1)=1.2566e5; %Grid Angle Initialization
64 xInitial(1,2)=1.2566e5; %Motor Angle Initialization
65 xInitial(1,3)=125.6637; %Motor Speed Initialization
66 xInitial(1,4)=1.5429; %Rotor Speed 1 Initialization
67 xInitial(1,5)=1.5429; %Rotor Speed 2 Initialization
68 xInitial(1,6)=5.8991e7; %System Pressure Initialization
69
70 %Set Wind Speed and Motor Displacement to Simulate
71 set_Wind_Speed1=4:1:12; %Matrix containing Wind speed 1
72 set_Wind_Speed2=4:1:12; %Matrix containing Wind speed 2
73 set_Vm=1e-4:1e-4:60e-4; %Matrix containing Motor Displacement
74
75 %Main Loop
76 dws=size(set_Wind_Speed1);
77 dws2=size(set_Wind_Speed2);
78 dvm=size(set_Vm);
79 ws=1;
80 ws2=1;
81 vm=1;
82 results=zeros(dvm(1,2)+1,dws(1,2)+1,32,dws2(1,2));
83 %Matrix containing all the output
84 for ws2=1:dws2(1,2)
85
86 for ws=1:dws(1,2)
87 Wind_Speed1=set_Wind_Speed1(ws);
88 Wind_Speed2=set_Wind_Speed2(ws2);
89
90 for vm=1:dvm(1,2)
91 Vm=set_Vm(vm);
92
93 sim('TwoTurbine_SteadyState')
94
95 %Copy all the outputs to results matrix
96 results(vm+1,1,:)= Vm;
97 results(1,ws+1,:)=Wind_Speed1;
98 d=size(wp1.signals.values);
99 results(vm+1,ws+1,1,ws2)=wp1.signals.values(d(1,1)); %Rotor Speed1
100 d=size(wg.signals.values);
101 results(vm+1,ws+1,2,ws2)=wg.signals.values(d(1,1)); %Motor Speed
102 d=size(Pp.signals.values);
103 results(vm+1,ws+1,3,ws2)=Pp.signals.values(d(1,1)); %Pump Pressure
104 d=size(Pm.signals.values);
105 results(vm+1,ws+1,4,ws2)=Pm.signals.values(d(1,1)); %Motor Pressure
106 d=size(Pwind1.signals.values);
107 results(vm+1,ws+1,5,ws2)=Pwind1.signals.values(d(1,1)); %Wind Power1
108 d=size(Protor1.signals.values);
109 results(vm+1,ws+1,6,ws2)=Protor1.signals.values(d(1,1)); %Rotor Power1
110 d=size(Pout.signals.values);
111 results(vm+1,ws+1,7,ws2)=Pout.signals.values(d(1,1)); %Output Power
112 d=size(Vp1.signals.values);
113 results(vm+1,ws+1,8,ws2)=Vp1.signals.values(d(1,1));
114 %Pump Displacement1
115 d=size(cp1.signals.values);
116 results(vm+1,ws+1,9,ws2)=cp1.signals.values(d(1,1));
117 %Power Coefficient1

```

Figure 70 (Appendix B): MATLAB code for steady state response (part 1)

```

118 -     d=size(Mg.signals.values);
119 -     results(vm+1,ws+1,10,ws2)=Mg.signals.values(d(1,1)); %Generator Torque
120 -     d=size(wp2.signals.values);
121 -     results(vm+1,ws+1,11,ws2)=wp2.signals.values(d(1,1)); %Rotor Speed2
122 -     d=size(Pwind2.signals.values);
123 -     results(vm+1,ws+1,12,ws2)=Pwind2.signals.values(d(1,1)); %Wind Power2
124 -     d=size(Protor2.signals.values);
125 -     results(vm+1,ws+1,13,ws2)=Protor2.signals.values(d(1,1)); %Rotor Power2
126 -     d=size(Vp2.signals.values);
127 -     results(vm+1,ws+1,14,ws2)=Vp2.signals.values(d(1,1));
128 -                                     %Pump Displacement2
129 -     d=size(cp2.signals.values);
130 -     results(vm+1,ws+1,15,ws2)=cp2.signals.values(d(1,1));
131 -                                     %Power Coefficient2
132 -     results(vm+1,ws+1,16,ws2)=results(vm+1,ws+1,7,ws2)/...
133 -         (results(vm+1,ws+1,6,ws2)+results(vm+1,ws+1,13,ws2));
134 -                                     %Transmission Coefficient
135 -     end
136 - end
137
138 %Construct Optimal Vm Table for each results variable
139 [row col]=size(results(:, :, 16, ws2));
140 [C I]=max(results(2:row, 2:col, 16, ws2));
141 [a b]=size(I);
142
143 if ws2==1
144     Opt_Vm_Table=zeros(dws2(1,2)+1,b+1,18);
145     j=1;
146     for j=1:18
147         Opt_Vm_Table(1,2:dws2(1,2)+1,j)=set_Wind_Speed1;
148     end
149     end
150
151     for i=1:b
152         Opt_Vm_Table(ws2+1,1,:)=Wind_Speed2;
153         j=1;
154         for j=1:16
155             Opt_Vm_Table(ws2+1,i+1,j)=results(I(i)+1,i+1,j,ws2);
156         end
157         Opt_Vm_Table(ws2+1,i+1,17)=C(i);
158         Opt_Vm_Table(ws2+1,i+1,18)=results(I(i)+1,1,j,ws2);
159     end
160 end

```

Figure 71 (Appendix B): MATLAB code for steady state response (part 2)

Figure 72 (Appendix B): Workspace for “TwoTurbine_SteadyState.mdl”

D. MATLAB Code and Simulink for System Response

```

213 %Initialize State Variables and Input Parameters
214 %These values are chosen so that Simulink can solve the devised model
215 - xInitial(1,1)=1.5429; %Rotor Speed 1 Initialization
216 - xInitial(1,2)=1.5429; %Rotor Speed 2 Initialization
217 - xInitial(1,3)=5.8991e7; %System Pressure Initialization
218 - xInitial(1,4)=125.6637; %Motor Speed Initialization
219 - xInitial(1,5)=1.2566e5; %Grid Angle Initialization
220 - xInitial(1,6)=1.2566e5; %Motor Angle Initialization
221
222 %Look up table for optimal Vm
223 - Opt_Vm=[0.001 0.0012 0.0014 0.0016 0.0019 0.0022 0.0024 0.0027 0.0030;...
224           0.0012 0.0013 0.0015 0.0017 0.0020 0.0022 0.0025 0.0028 0.0030;...
225           0.0014 0.0015 0.0017 0.0019 0.0021 0.0023 0.0026 0.0028 0.0031;...
226           0.0016 0.0017 0.0019 0.0020 0.0022 0.0024 0.0027 0.0029 0.0032;...
227           0.0019 0.0020 0.0021 0.0022 0.0024 0.0026 0.0028 0.0030 0.0033;...
228           0.0022 0.0022 0.0023 0.0024 0.0026 0.0028 0.0030 0.0032 0.0034;...
229           0.0024 0.0025 0.0026 0.0027 0.0028 0.0030 0.0031 0.0033 0.0036;...
230           0.0027 0.0028 0.0028 0.0029 0.0030 0.0032 0.0033 0.0035 0.0037;...
231           0.0030 0.0030 0.0031 0.0032 0.0033 0.0034 0.0036 0.0037 0.0039];
232
233 %Run Simulink Model
234 - sim('TwoTurbine_Dynamics')
235
236 %Construct a matrix that contains all results
237 - dtout_total=size(dtout);
238 - results2=zeros(dtout_total(1,1),2,19);
239 - for a=1:19
240 -     results2(:,1,a)=tout;
241 - end
242 - results2(:,2,1)=wp1.signals.values; %Rotor Speed1
243 - results2(:,2,2)=wp2.signals.values; %Rotor Speed2
244 - results2(:,2,3)=wg.signals.values; %Motor Speed
245 - results2(:,2,4)=Pp.signals.values; %Pump Pressure
246 - results2(:,2,5)=Pm.signals.values; %Motor Pressure
247 - results2(:,2,6)=Pwind1.signals.values; %Wind Power1
248 - results2(:,2,7)=Pwind2.signals.values; %Wind Power2
249 - results2(:,2,8)=Protor1.signals.values; %Rotor Rotational Power1
250 - results2(:,2,9)=Protor2.signals.values; %Rotor Rotational Power2
251 - results2(:,2,10)=Pout.signals.values; %Output Power
252 - results2(:,2,11)=Vp1.signals.values; %Pump Displacement1
253 - results2(:,2,12)=Vp2.signals.values; %Pump Displacement2
254 - results2(:,2,13)=cp1.signals.values; %Power Coefficient1
255 - results2(:,2,14)=cp2.signals.values; %Power Coefficient2
256 - results2(:,2,15)=Mg.signals.values; %Generator Torque
257 - results2(:,2,16)=results2(:,2,11)./(results2(:,2,10)+results2(:,2,27));
258 - %Transmission Coefficient
259 - results2(:,2,17)=Vm.signals.values; %Motor Displacement
260 - results2(:,2,18)=U1.signals.values; %Wind Speed1
261 - results2(:,2,19)=U2.signals.values; %Wind Speed2

```

Figure 73 (Appendix B): MATLAB code for system response

Figure 74 (Appendix B): Workspace for “TwoTurbine_Dynamics.mdl”

E. MATLAB Code and Simulink for Linearization

```

212 %Initialize State Variables and Input Parameters
213 %These values are chosen so that Simulink can solve the devised model
214 - xInitial(1,1)=1.5429; %Rotor Speed 1 Initialization
215 - xInitial(1,2)=1.5429; %Rotor Speed 2 Initialization
216 - xInitial(1,3)=5.8991e7; %System Pressure Initialization
217 - xInitial(1,4)=125.6637; %Motor Speed Initialization
218 - xInitial(1,5)=1.2566e5; %Grid Angle Initialization
219 - xInitial(1,6)=1.2566e5; %Motor Angle Initialization
220
221 %Look up table for optimal Vm
222 - Opt_Vm=[0.001 0.0012 0.0014 0.0016 0.0019 0.0022 0.0024 0.0027 0.0030;...
223           0.0012 0.0013 0.0015 0.0017 0.0020 0.0022 0.0025 0.0028 0.0030;...
224           0.0014 0.0015 0.0017 0.0019 0.0021 0.0023 0.0026 0.0028 0.0031;...
225           0.0016 0.0017 0.0019 0.0020 0.0022 0.0024 0.0027 0.0029 0.0032;...
226           0.0019 0.0020 0.0021 0.0022 0.0024 0.0026 0.0028 0.0030 0.0033;...
227           0.0022 0.0022 0.0023 0.0024 0.0026 0.0028 0.0030 0.0032 0.0034;...
228           0.0024 0.0025 0.0026 0.0027 0.0028 0.0030 0.0031 0.0033 0.0036;...
229           0.0027 0.0028 0.0028 0.0029 0.0030 0.0032 0.0033 0.0035 0.0037;...
230           0.0030 0.0030 0.0031 0.0032 0.0033 0.0034 0.0036 0.0037 0.0039];
231
232 %Set Wind Speed
233 - Wind_Speed1=12;
234 - Wind_Speed2=12;
235 - Vm=Opt_Vm(Wind_Speed1-3, Wind_Speed2-3); %Satisfy optimal Vm at steady state
236
237 %Acquire State Variable Value at Steady State
238 - sim('TwoTurbine_SteadyState')
239 - [a b] =size(yout);
240 - v_equil1 = Wind_Speed1;
241 - v_equil2 = Wind_Speed2;
242 - wr_equil1 = yout(a,3);
243 - wr_equil2 = yout(a,4);
244 - P_equil = yout(a,5);
245 - wg_equil = yout(a,6);
246 - Angle_grid_equil = yout(a,7);
247 - Angle_motor_equil = yout(a,8);
248 - w_grid_equil = w_motor_sync;
249
250 %Linearization
251 - [A B C D] = linmod('TwoTurbine_Linearization',[wr_equil1;...
252           wr_equil2; P_equil; wg_equil; Angle_motor_equil],[v_equil1...
253           v_equil2 w_grid_equil Angle_grid_equil]);

```

Figure 75 (Appendix B): MATLAB code for linearization

Figure 76 (Appendix B): Workspace for “TwoTurbine_Linearization.mdl”