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Developing an Empirical Correlation for the Thermal Spreading Resistance of a Heat Sink

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DEVELOPING AN EMPIRICAL CORRELATION FOR THE THERMAL
SPREADING RESISTANCE OF A HEAT SINK

A Thesis

Presented to

The Faculty of the Department of Mechanical Engineering

San José State University

In Partial Fulfillment

of the Requirements for the Degree

Masters of Science

by

Andrew R. Werdowatz

May 2016

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The Designated Thesis Committee Approves the Thesis Titled

DEVELOPING AN EMPIRICAL CORRELATION FOR THE THERMAL
SPREADING RESISTANCE OF A HEAT SINK

by

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APPROVED FOR THE DEPARTMENT OF MECHANICAL ENGINEERING

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May 2016

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ABSTRACT

DEVELOPING AN EMPIRICAL CORRELATION FOR THE THERMAL SPREADING RESISTANCE OF A HEAT SINK

by Andrew R. Werdowatz

Heat sinks are a critical component in numerous thermal management strategies, ranging from consumer electronics to data centers. The ability to perform an accurate thermal analysis of a heat sink is a crucial step in the design process. In situations where the heat sink is larger in area than the component it is being used to cool, thermal spreading resistance takes effect. Multiple analytical solutions have been proposed and published that can be used to calculate thermal spreading resistance. The difficulty lies in the fact that most of these solutions contain a very complex set of equations and are not very practical for use by the industry. As a result, the present study was aimed at developing a simple set of simple empirical equations that can be used to calculate the thermal spreading resistance of a heat sink. Using a CFD (computational fluid dynamics) software package, a model was developed that was used to calculate the spreading resistance of a heat sink. Using this model, a set of parametric studies was conducted that varied a number of geometric and thermal characteristics of a heat sink. Using the data collected from the studies, an extensive curve fitting analysis was conducted which resulted in the development of a set of simple empirical equations that can be used to calculate the thermal spreading resistance of the heat sink. The equations were shown to be in excellent agreement with previous analytical solutions, in most cases within $\pm 5\%$. As a result, it was concluded that the developed equations can be used to accurately calculate the spreading resistance of a heat sink over the stated range of valid parameters.

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NOMENCLATURE

A_s	heat source area [m^2]
A_b	heat sink base area [m^2]
w	half of the total width of the heat sink [m]
w_1	half of the total width of the heat source [m]
l	half of the total length of the heat sink [m]
l_1	half of the total length of the heat source [m]
t	thickness of heat sink base plate [m]
β	w/l , aspect ratio of heat spreader
ε	l_1/l , non-dimensional side length ratio
τ	t/l , non-dimensional base thickness
γ	w_1/w , non-dimensional side length ratio
k	thermal conductivity [W/m-K]
h	heat transfer coefficient [W/m ² -K]
Bi	Biot number (ht/k)
Bi_*	modified Biot number used for developed empirical equation
R_t	total thermal resistance [$^{\circ}C/W$]
R_s	thermal spreading resistance [$^{\circ}C/W$]
R_m	material thermal resistance [$^{\circ}C/W$]
R_f	external thermal resistance [$^{\circ}C/W$]
T_i	source plane temperature [$^{\circ}C$]
T_s	surface temperature of the heat spreader [$^{\circ}C$]
T_f	ambient temperature [$^{\circ}C$]
Q	heat power [W]
q	heat flux [W/m ²]
T	temperature
Ψ	non-dimensional thermal spreading resistance

$R_{t,A_b=A_s}$ Total thermal resistance of a heat sink when the heat source area and heat sink base area are equal to one another

$R_{t,A_b \neq A_s}$ Total thermal resistance of a heat sink when the heat source area and heat sink base area are not equal to one another

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1.0 INTRODUCTION

As the electronics industry continues to develop smaller and smaller electronic components while at the same time increasing their power densities, the ability to efficiently cool these components in space-saving environments is of increasing concern. The continued trend of miniaturization has resulted in an increasing number of scenarios where an electronics package (e.g. integrated chip (IC) package) is far smaller in surface area than the heat sink that is used to dissipate the heat generated by the chip package during operation. Figure 1 illustrates such a scenario where the red hatched square in the figure represents the chip package (i.e., heat source). When this situation occurs, a phenomenon known as thermal spreading resistance occurs and increases the overall thermal resistance of the heat sink, decreasing its thermal efficiency. Thermal spreading resistance can be defined as the resistance that a flow of heat experiences as it travels from a region of a smaller surface area to that of a larger surface area (e.g. IC package to heat sink baseplate) [1]. The heat flows from the heat source into the heat sink baseplate via one-dimensional conduction and, as it begins to flow through the heat sink baseplate, it changes to three-dimensional conduction. Spreading resistance acts to resist this change in heat flow direction. Thermal spreading resistance causes a non-uniform temperature distribution within the heat sink and, therefore, increases its overall thermal resistance [1].

Figure 1. Situation where the heat sink is larger in surface area than the chip package

The ability to efficiently remove heat from electronic packages plays a vital role in the performance and reliability of electronic components. Improper design of a thermal management system can lead to electronic failure and/or performance issues. If the effects of thermal spreading resistance are neglected during the design process, a heat sink will be calculated as having a lower overall thermal resistance. As a result, when the design is implemented, the heat sink will not dissipate the heat as efficiently as was calculated, which could lead to electronic failure or decreased performance due to power throttling. Therefore, in such cases where thermal spreading resistance exists, a way to easily analyze and compute the effect that certain geometric and/or environmental factors have on thermal spreading resistance would be very beneficial when designing thermal management systems for electronic packaging applications.

1.1. Literature Review

Several studies have developed analytical solutions for calculating thermal resistance for certain geometric configurations. However, most of the derived solutions are very complex, involving multiple infinite series, and therefore are not very practical for use by the electronics industry. The first solution was presented by Kennedy [2] in 1960 in his article titled *Spreading Resistance in Cylindrical Semiconductor Devices*. Kennedy's solution was based on the Laplace equation in cylindrical coordinates for a solid cylinder with a circular heat source of smaller area applied to one of the ends of the cylinder. Kennedy solved the Laplace equation for three separate cases involving different combinations of adiabatic and isothermal boundary conditions. For each case, Kennedy solved the Laplace equation in order to obtain a solution for the temperature distribution within the cylinder. Kennedy stated that, for all cases, the thermal spreading resistance could be calculated as a function of the derived temperature distribution, the thermal conductivity of the material, the heat flux from the heat source, and the area of the heat source. However, for each case, Kennedy's solution assumes at least one isothermal boundary condition, which for many real world applications is not an accurate assumption. As a result, Kennedy's solution can result in values that underestimate thermal spreading resistance by, in some cases, orders of magnitude [3].

Due to the limitations of Kennedy's solution, Nelson and Sayers [4] were the first to extend Kennedy's solution by adding a finite resistance to the base of the cylinder. Nelson and Sayers stated that external resistance plays a major role in the thermal spreading resistance and therefore must be taken into account when modeling spreading

resistance. They also expanded the research to include a solution for the thermal spreading resistance for a two-dimensional planar case and compared the results of the axisymmetric and planar cases to a three-dimensional solution, obtained via a computer program, in order to see when a simplified model would be acceptable. Nelson and Sayers used a control volume analysis based on the finite element method to solve the axisymmetric and planar models. The total thermal resistance of the heat sink was modeled as the sum of the thermal spreading resistance and the external resistance of the heat sink, where the thermal spreading resistance was stated to be a function of the non-dimensional temperature rise of the heat sink and geometric parameters. They then solved both scenarios for the non-dimensional temperature rise and compared the results to those of the three-dimensional solution. Based on these comparisons, Nelson and Sayers concluded that the axisymmetric model could be used to represent the spreading resistance in a three-dimensional case for small aspect ratios, and the planar model could be used when the heat sink baseplate is very thin and has a low external thermal resistance. For any other situation, the researchers concluded that the best results would be obtained using the three-dimensional solution [4].

Similar to Nelson and Sayers, Lee, Song, and Moran [3] developed an analytical solution for thermal spreading resistance using the two-dimensional Laplace equation in cylindrical coordinates. The problem setup was identical to that of Kennedy [2]. However, Lee et al. applied adiabatic boundary conditions to all sides of the heat sink, including the source plane. A Newtonian cooling boundary condition was also applied to the top surface of the heat spreader plate. The heat source was modeled as a uniform heat

flux which was applied to a small concentric circle on the top side of the heat sink which was modeled as a cylinder. Lee et al. stated that the total thermal resistance could be calculated as the sum of the thermal spreading resistance, the external resistance, and material resistance. Using the specified boundary conditions, Lee et al. derived a solution for the temperature distribution in the heat spreader and then used this solution to calculate the average and maximum thermal spreading resistance in the heat sink [3]. It was shown that the obtained non-dimensional, infinite series solution provided excellent convergence within a 100 terms with previous numerical data found from Nelson and Sayers [3].

Ellison [5] developed a solution for the maximum thermal spreading resistance for geometries with non-unity aspect ratios (i.e., the heat source is not placed at the centroid of the heat sink). Ellison set up the problem using the steady-state three-dimensional heat conduction equation and assumed a uniform heat flux from the heat source as well as adiabatic boundary conditions for the sides and source plane (excluding the heat source) of the heat sink/spreader. A Newtonian cooling boundary condition was also applied to the top surface of the heat spreader plate. Functions for the heat source and temperature distribution were then derived that satisfied the imposed boundary conditions. These equations were then used to solve the three-dimensional heat conduction equation. Ellison stated that by equating like coefficients, the three-dimensional heat equation could be simplified into a one-dimensional differential equation in the z direction. This simplified equation was then solved and its solution was used to fully define the heat source and temperature distribution equation previously derived. Noting that the total

thermal resistance is the sum of the thermal spreading resistance, the external resistance, and material resistances, an equation for the thermal spreading resistance was found by using the temperature distribution and the heat source equations. Using this solution method, Ellison derived an analytical solution for the thermal spreading resistance that took the form of an infinite series as a function of the heat source and heat spreader geometry. Ellison's solution provides a more general equation for the spreading resistance due to the fact that the heat source is not constrained to the centroid of the heat sink.

Similar to Ellison, Muzychka, Culham, and Yovanovich [6] developed an analytical solution for the thermal spreading resistance for eccentric heat sources (i.e., heat sources not placed at the centroid of the heat spreader) on rectangular heat flux channels (heat spreader). Muzychka et al. [6] also derived an equation for the thermal spreading resistance for a case where the heat flux channel was composed of two different materials (i.e., an interface material and a heat sink/spreader), as well as cases that involved multiple heat sources. The governing equation used by Muzychka et al. [6] was the three-dimensional Laplace equation and a uniform heat flux was used to model the heat source. The governing equation was then solved using the imposed boundary conditions (very similar to Lee et al. [3] and Ellison [5]) in order to obtain a solution for the mean temperature excess. From this equation, an analytical solution for the thermal spreading resistance as a function of non-dimensional geometric and heat source parameters, was derived. Muzychka et al. [6] went on to analyze the spreading resistance for three separate cases using the derived solution in order to show its real world application.

The models developed by Lee et al. [3] as well as Nelson and Sayers [4] relied on axisymmetric assumptions and, in the case of non-circular heat sources and heat spreaders, equivalent radii would have to be used in order to transform the rectangular geometry into equivalent circular geometry. As a result, Feng and Xu [7] stated that due to this change in the coordinate system, error would be introduced when using the above model for square geometries. Based on this error, Feng and Xu developed an analytical model for thermal spreading resistance for square heat spreaders using the steady-state three-dimensional heat conduction equation in Cartesian coordinates. The model assumes a uniform heat flux from the heat source and, similar to Lee et al. [3], adiabatic boundary conditions for all edges of the heat spreader/heatsink, as well as a Newtonian cooling boundary condition at the top surface of the heat spreader plate [7]. The three-dimensional heat conduction equation was then solved and an equation for the local non-dimensional excess temperature and the non-dimensional heat flux distribution was derived. The total thermal resistance (defined in the same manner as Song et al. [3] and Ellison [5]) was then found by dividing the maximum temperature of the projected heating area by the total heating power produced by the heat source. From this equation a solution for the thermal spreading resistance was derived that took the form of a non-dimensionalized, infinite series solution as a function of the Biot number, as well as heat source and heat spreader geometric parameters. The derived equation is quite complex and requires that multiple infinite and double infinite series solution be solved.

Other studies have focused on deriving solutions for thermal spreading resistance. Sadeghi, Bahrami, and Djilali [8] noted that out of the extensive literature regarding

thermal spreading resistance, a general model had not yet been proposed that can be applied to arbitrarily shaped heat sources (as the previous models only considered circular or cubic geometry). As a result, Sadeghi, et al. [8] stated that the thermal spreading resistance for various shaped heat sources could be estimated using a solution for a hyperelliptical source as long as the area and aspect ratio of the arbitrary shape is identical to that of the hyperelliptical shape. Sadeghi, et al. [8] studied multiple different shaped heat sources including triangular, rhombic, rectangular with round ends, polygonal, and a circular segment and found that the developed model agreed well with previous data. Karmalkar, Mohan, Nair, and Yeluri [9] developed closed form models for thermal spreading resistance for common heat source and heat sink geometries encountered in electronic devices and integrated circuits.

1.2. Objectives of the Present Study

The goal of the present study was to analyze the effects of certain geometric parameters of a heat sink have on thermal spreading resistance in order to derive a set of empirical equations that could be used to calculate the thermal spreading resistance as a function of several geometric and thermal characteristics of the heat sink. As was mentioned in the previous section, the solution obtained by Feng and Xu is a very cumbersome equation and, therefore, not a very applicable solution for use by the electronic cooling industry. As a result, a set of much simpler empirical equations that show similar results to the solution proposed by Feng and Xu, would be very beneficial for the industry.

The problem setup provided by Feng and Xu [7] was chosen as the basis of the current research due to the fact that the derived solution is set up specifically for cubic heatsinks and heat sources, which is a very common geometry found in the electronics industry. The heat sink parameters that were varied in order to quantify the effect they have on thermal spreading resistance include the heat sink base plate thickness, heat source/base plate side length ratio, heat sink material (i.e. a change in thermal conductivity), external thermal resistance, and heat sink base plate aspect ratio.

2.0 METHODOLOGY

2.1 Overview of Methodology

The present study developed a computational fluid dynamics (CFD) model using ANSYS IcePak, which was used to model a heat sink similar to the solution setup stated by Feng and Xu (outlined in Section 2.2.). Using this CFD model, multiple parametric studies were conducted that varied the different heat sink parameters stated in the previous section which were then used to calculate the effects that the various parameters have on the thermal spreading resistance. A CFD model was used as opposed to solely using the analytical equation derived by Feng and Xu in order to simplify the gathering of data for the parametric studies as well as to gain experience using ANSYS Icepak. Based on the results obtained from the parametric studies, an equation was developed using curve fitting analysis that calculates the thermal spreading resistance as a function of certain geometric and thermal characteristics. The developed equation can be used during the design process to analyze how changing certain heat sink parameters will affect the thermal spreading resistance.

The following sections outline in detail the model setup and solution proposed by Feng and Xu, the development and benchmarking of the CFD model, the development of the parametric study (i.e. how each parameter was varied), how thermal spreading resistance was calculated using the CFD model, and, lastly, how the empirical equation was developed as well as a comparison to the analytical solution.

2.2 Model Setup and Solution Method for Spreading Resistance (Feng and Xu)

As previously stated, the present study developed a CFD model using the model setup used by Feng and Xu. Figure 2 shows a general chip package (represented by the red square) and heat sink setup that was used as the basis of the research proposed by Feng and Xu. Figure 3 shows the detailed geometry setup that was used by Feng and Xu in order to derive an equation for the thermal spreading resistance.

Figure 2. Chip package and heat sink setup (figure adapted from Feng and Xu [7])

Figure 3. Model setup (figure adapted from Feng and Xu [7])

As can be seen, the heat sink baseplate as well as the heat source were modeled as simple rectangles centered in the x-y plane. The heat dissipated by the heat source was modeled as a constant heat flux (q) [7]. Feng and Xu stated that, due to the symmetrical nature of the problem setup, it would only be necessary to consider one quarter of the

heat sink as the remaining quarters would be identical [7]. The governing equation used for the model was the steady state three-dimensional heat conduction equation assuming constant thermal conductivity of the heat sink [7]. The governing equation, along with the imposed boundary conditions, were stated as follows [7]:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

$$\frac{\partial T}{\partial x_{x=0}} = \frac{\partial T}{\partial x_{x=l}} = \frac{\partial T}{\partial y_{y=0}} = \frac{\partial T}{\partial y_{y=w}} = 0 \quad (1)$$

$$k \frac{\partial T}{\partial z_{z=0}} = hT \quad (2)$$

$$k \frac{\partial T}{\partial z_{z=t}} = \begin{cases} q & 0 < x \leq l_1 \quad 0 \leq y \leq w_1 \\ 0 & l_1 < x \leq l \quad w_1 \leq y \leq w \end{cases} \quad (3)$$

Where T was said to be the local excess temperature relative to the ambient temperature [7]. The above boundary conditions impose adiabatic edge conditions for the heat sink baseplate (1) as well as the portion of the bottom surface of the heat sink that does not contain the heat source (3) and a Newtonian cooling boundary (2) for the top surface of the heat sink. In order to simplify the derivation, Feng and Xu [7] non-dimensionalized the above equation and boundary conditions. Using the imposed boundary conditions, the non-dimensional steady state three-dimensional heat conduction equation was solved via Fourier expansion. Based on this derivation, an equation for the local non-dimensional excess temperature as well as the non-dimensional heat flux distribution were derived.

In order to develop an equation for the thermal spreading resistance, it was stated that the total thermal resistance of a heat sink could be calculated as the sum of the material thermal resistance, external thermal resistance, and the thermal spreading

resistance [7]. Feng and Xu proposed the following thermal resistance network, shown in Figure 4, to model the total thermal resistance of the heat sink (again Figure 4 is a recreation of a figure from Feng and Xu's original work).

Figure 4. Thermal resistance network proposed by Feng and Xu [7]

Where

$$R = R_{sm} + R_f = R_s + R_m + R_f$$

Where R is the total thermal resistance, R_m is the material thermal resistance, R_s is the thermal spreading resistance, and R_f is the external thermal resistance. Equations were given to calculate the material thermal resistance (R_m) as well as the external thermal resistance (R_f) [7]. They are as follows:

$$R_m = \frac{t}{kA_b} \quad R_f = \frac{1}{hA_b} = \frac{t}{Bi k A_b} \quad \text{where: } Bi = \frac{ht}{k}$$

Where t is the thickness of the heat sink baseplate, k is the thermal conductivity of the heat sink material, h is the heat transfer coefficient, A_b is the area of the heat sink baseplate, and Bi is the Biot number. As can be seen, the external thermal resistance equation does not take into account the overall surface efficiency of the heat sink (which is a function of fin efficiency and area ratio of the fin surface area to that of the base area) and therefore will result in a lower external thermal resistance than would be observed in a real world scenario [10]. In order to compensate for this, using an effective heat transfer coefficient value (h_{eff}) is recommended, that is,

$$h_{eff} = \frac{1}{\eta_0 A_t R_{t,0}} \quad 2.2.1$$

Where η is the overall surface efficiency of the heat sink, A_t is the total surface area of the heat sink (surface area of the fins and the exposed surface area of the base), and $R_{t,0}$ is the effective thermal resistance of the heat sink [10].

Feng and Xu stated that the total thermal spreading resistance from the projected heating area to the ambient could be calculated as the difference between the maximum temperature at the projected heating area and that of the ambient (i.e. excess temperature) divided by the total heating power dissipated by the heat source [7]. Due to the symmetrical nature of the problem setup, it was stated that this maximum temperature would be located at the centroid of the heat sink [7]. Therefore,

$$R_t = \frac{T_{max}}{Q} = \frac{T|_{x=0,z=t}}{Q} = \frac{T^*|_{X=0,Y=0,Z=t}}{kA_s} = R_s + R_m + R_f$$

where the asterisk represents the non-dimensional version of the local excess temperature equation. Using the equations for the material and external thermal resistances, along with the derived equation for the local non-dimensional excess temperature, Feng and Xu developed a closed form equation for the non-dimensional thermal spreading resistance as a function of geometric and thermal parameters (thermal conductivity, heat transfer coefficient etc.). The derived expression is as follows:

$$\Psi = k\sqrt{A_s R_s} = \sqrt{\frac{\varepsilon\gamma}{\beta}} \left[\sum_{m=1}^{\infty} C_{m0} + \sum_{n=1}^{\infty} C_{0n} + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn} \right]$$

where

$$C_{m0} = \frac{\gamma[(\tau m\pi + Bi) \exp(\tau m\pi) + (\tau m\pi - Bi) \exp(-\tau m\pi)] \sin(\varepsilon m\pi)}{(m\pi)^2 [(\tau m\pi + Bi) \exp(\tau m\pi) - (\tau m\pi - Bi) \exp(-\tau m\pi)]} \quad [7]$$

$$C_{0n} = \frac{\beta \varepsilon [(\tau n\pi + Bi) \exp(\tau n\pi) + (\tau n\pi - Bi) \exp(-\tau n\pi)] \sin(\gamma n\pi)}{(n\pi)^2 [(\tau n\pi + \beta Bi) \exp(\frac{\tau n\pi}{\beta}) - (\tau n\pi - \beta Bi) \exp(\frac{-\tau n\pi}{\beta})]} \quad [7]$$

$$C_{mn} = \frac{2[(\tau \xi \pi + Bi) \exp(\tau \xi \pi) + (\tau \xi \pi - Bi) \exp(-\tau \xi \pi)] \sin(\varepsilon m\pi) \sin(\gamma n\pi)}{mn \xi \pi^3 [(\tau \xi \pi + Bi) \exp(\tau \xi \pi) - (\tau \xi \pi - Bi) \exp(-\tau \xi \pi)]} \quad [7]$$

where

$$\xi = \sqrt{m^2 + \left(\frac{n}{\beta}\right)^2} \quad \beta = \frac{W}{L} \quad \varepsilon = \frac{L_1}{L} \quad \gamma = \frac{W_1}{W} \quad \tau = \frac{t}{L} \quad [7]$$

where L, W, L₁, W₁, and t represent the dimensions of the heat spreader outlined in Figure 3.

It was shown that the solution developed by Feng and Xu converged within 40 terms of the infinite series (i.e. $m=n=40$) and showed similar results to those of Kennedy and Lee, et al. [7]. However, Feng and Xu stated that their solution would provide a more accurate result for cubic heat spreaders, as those of Kennedy and Lee, et al. resort to using equivalent radii when calculating thermal spreading resistance for cubic geometries which can introduce error into the model [7]. The above solution model was the basis for the CFD model that was developed and used to analyze the effects that heat sink and environmental parameters have on the thermal spreading resistance.

2.3 Development of the CFD Model

2.3.1 Why ANSYS Icepak?

ANSYS Icepak is a CFD software package that utilizes the same background code employed by the well-known ANSYS Fluent, but it has been specifically designed to be used by the electronics cooling industry. It offers numerous built-in features catered to the electronics industry such as heat sinks, heat sources, real world chip packages, PCB boards, and fans as well as the ability to bring in PCB boards and chip packages designed in other electronic design software. The most useful feature for the present study is the built-in output of the total thermal resistance of a heat sink, which will be very useful in finding the portion of the total thermal resistance that can be attributed to thermal spreading resistance of the heat sink (a more detailed explanation of how this output will be used can be found in Section 2.3.2.4). Icepak also offers the ability to perform parametric studies, which will be used to vary the parameters of the heat sink in order to

see the effects they have on the thermal spreading resistance (a more detailed explanation of the parametric study that will be conducted can be found in Section 2.4).

2.3.2 Development of the CFD Model

2.3.2.1 Physical Setup of the Model

Recall from Figure 2, the solution setup proposed by Feng and Xu, consisted of a heat sink with a heat source placed centrally on its bottom surface. Similarly, the CFD model was developed using a simple block object and a two-dimensional heat source object taken from the ANSYS Icepack component library. Due to the simplicity of the model, the cabinet of the model (i.e., the model boundary) was set equal to the size of the block object. Figures 5 and 6 show the end result of the developed model, where the red hatched square represents the heat source.

Figure 5. Isometric view of the mesh

Figure 6. Bottom surface of the CFD model

As can be seen, the model is very simple and resembles the model proposed by Feng and Xu [7] in Figure 3. As for the boundary conditions of the model, a newton cooling boundary (i.e., external thermal resistance) was applied to the top surface of the block object (opposite the surface of the heat source) while adiabatic edge conditions (i.e. heat flux is equal to zero) were applied to the remaining sides of the block. The external thermal resistance applied to the top surface of the plate can be thought of as the effective resistance that would be present if there was finned heat sink connected to the top surface of the plate. The ambient temperature and the thermal dissipation power (TDP) of the heat source were set equal to 0 C and 15 W, respectively. Recalling that thermal resistance is calculated as the difference in the maximum temperature and that of the ambient divided by the TDP, the ambient temperature as well as the TDP have no effect on the thermal resistance of thermal resistance of the heat sink as the value of the maximum temperature will simply fluctuate if these values are varied.

2.3.2.2 Mesh Setup and Analysis

Due to the simple nature of the geometry of the developed model, a constant set of mesh controls was for the entirety of the model. Figure 7 shows the mesh control parameters that were used for the model.

Figure 7: Mesh controls used for the model

Figures 8-10 on the following pages show the resulting mesh that was generated using the meshing controls outlined in Figure 7. Figure 8 shows an isometric view of the mesh of the model. As can be seen, it is a very fine mesh that will result in a detailed temperature profile for the block element (heat spreader). Figures 9 and 10 show a z-x plane at the top surface of the model and a y-z cut plane through the model, respectively.

Figure 8. Isometric view of the mesh

Figure 9. z-x plane view of the mesh

Figure 10. y-z cut plane view of the mesh

In order to determine the appropriate element size that was chosen for the mesh controls (i.e. the maximum element size for the x, y, and z direction from Figure 7), a mesh analysis was conducted on the model. The model was run using different maximum element sizes in order to obtain the effect that a decrease in element size has on the desired output parameters. Table 1 shows the different element sizes that were chosen for the mesh analysis, as well as the resulting values for the number of elements, maximum temperature, and thermal resistance of the model.

Table 1. Mesh analysis

Max Element Size w/in Assembly (mm)	Number of Elements	Max Temperature °C	Thermal Resistance °C/W
0.1	320000	23.68	1.579
0.15	88445	23.75	1.583
0.2	40000	23.79	1.586
0.25	19200	23.82	1.588
0.35	9747	23.71	1.581

Based on the data in Table 1, a max element size of 0.2 mm was chosen for the model as it significantly reduced the number elements as well as kept the minimum volume of the mesh elements at an acceptable value, with negligible loss in accuracy (ANSYS recommends a minimum element volume of $1\text{e-}13\text{ m}^3$; using a maximum element size of 0.2 mm resulted in a minimum volume of $8\text{e-}12\text{ m}^3$ [11]). Table 2 shows the percent difference in the calculated values for the maximum temperature and thermal resistance for a 0.2 mm maximum element size vs a 0.1 mm maximum element size, that is

$$\% \text{ Difference in Maximum Temperature} = \frac{T_{0.2\text{ mm}} - T_{0.1\text{ mm}}}{T_{0.2\text{ mm}}}$$

$$\% \text{ Difference in Thermal Resistance} = \frac{R_{0.2\text{ mm}} - R_{0.1\text{ mm}}}{R_{0.2\text{ mm}}}$$

Table 2. Solution results comparison of 0.2 mm vs. 0.1 mm max element size

Using Max Element Size of 0.2 mm vs 0.1 mm	
Reduction in the Number of Elements	88%
% Difference in Max Temperature	0.46%
% Difference in Thermal Resistance	0.46%

As can be seen from Table 2, the difference between using a 0.1 mm max element and 0.2 mm max element size is negligible (less than 0.5%). However, the reduction in the number of elements is significant, thereby reducing the runtime for each solution run. As a result of this analysis, it was concluded that using 0.2 mm as the maximum element size will provide sufficient accuracy for the developed CFD model.

In order to check the quality of the generated mesh, Icepak offers two quality check features: face alignment and skewness of the mesh. Face alignment is measured as the deviation from the perfect face alignment of 90 degrees of two adjacent planes [11]. Skewness determines how close to an ideal shape (equilateral or equiangular), a face or cell is [11]. In order to obtain convergence and accurate results, ANSYS recommends that the face alignment value be greater than 0.15 (a value of 1.0 is perfect) and the skewness value should be greater than 0.2 (a value of 1.0 is perfect) [11]. Figure 11 shows the face alignment (left) and skewness (right) quality check for the generated mesh (the y-axis represents the number elements and the x-axis represents the face alignment value). As can be seen, due to the geometry of the model both the face alignment and skewness values are exactly 1.0 for all elements within the model, which is an excellent mesh and, therefore, will provide accurate results.

Figure 11. Face alignment (left) and skewness of the mesh (right)

Lastly, Icepak offers a built-in macro function called the automatic case check tool which searches the model for any possible errors that have the potential to cause

inaccuracies when solving the model. The automatic case check tool was run on the above model; and no errors were found.

2.3.2.3 Solution Setup

Figure 12 shows the solution setup parameters used for the model. The solution was run using a steady state analysis with the only solution variable being temperature, as solving for velocity was unnecessary as there was no fluid flow within the model. As a result of the cabinet being identical in size to the model, it was also unnecessary to solve for both natural convection and radiation.

Figure 12. Solution controls used for the model

Figure 13 shows the convergence criteria used for the model. Convergence criteria are used to determine acceptable solution residuals, which are an indication of the numerical error within a model, the lower the residual value the less error that exists within the model [12]. The only criteria that is of concern for the current model is energy (due to there being no flow within the model), which was set to $1e-7$. It is generally accepted that solution residuals of $1e-4$ are considered loosely converged, while solution residuals greater than $1e-6$ are considered tightly converged [12]. Also, the maximum number of iteration was set to 100 for the model.

Figure 13. Convergence criteria used for the model

Based on the parameters outlined in Figures 12 and 13, Figure 14 shows the solution residuals for an individual solution run of the developed CFD model. Due to their being no fluid flow within the model the only solution residual present is energy which is represented by the red line. It can be seen from Figure 14 that using the

convergence criteria outlined in Figure 13 the model converged within six iterations out of an allowable 100.

Figure 14. Solution residuals

2.3.2.4 Calculation of Thermal Spreading Resistance using the CFD Model

Recall from Section 2.2 that Feng and Xu stated that the total thermal resistance of the heat sink can be calculated as

$$R_t = \frac{T_{max}}{Q} = R_s + R_m + R_f$$

Where $T_{\max}$ is the difference between the maximum temperature of the heat sink and the ambient temperature, Q is the total heating power from the heat source, R_m is the material thermal resistance, R_s is the thermal spreading resistance, and R_f is the external thermal resistance.

For each run of the CFD model, the thermal resistance was calculated as

$$R_t = \frac{T_{\max, \text{heatsink}}}{Q} \quad (2.3.2.4.1)$$

Where the ambient temperature is set equal to zero. The ambient temperature has no effect on the resistance of heat sink as long as the external heat transfer coefficient remains constant. As a result the ambient temperature was set equal to zero simply out of convenience. Using Equation 2.3.2.4.1, the thermal spreading resistance was calculated by running two separate solutions for each desired combination of geometry. The first trial was used to calculate the resistance of the heat sink with the area of the heat source set equal to the heat sink base area (Figure 15, right on the following page), and the second trial was used to calculate the thermal resistance with the area of the heat source set equal to the desired area for the given trial (Figure 15, left on the following page).

Figure 15. Heat source not equal to baseplate area (left), heat source equal to baseplate area (right)

Recalling that spreading resistance only occurs when the heat source is of dissimilar area than that of the heat sink base plate, this technique can be used to calculate the thermal spreading resistance for a given geometric setup by calculating the difference between the two trials, that is

$$R_s = R_{t, A_s \neq A_b} - R_{t, A_s = A_b} \quad (2.3.2.4.2)$$

Where A_b is the area of the base plate and A_s is the area of the heat source. Since the only difference between the two trials is the area of the heat source, the increase in total thermal resistance between the two values can be attributed to spreading resistance. In order to generalize the results, R_s was then be non-dimensionalized using the equation presented by Feng and Xu, that is

$$\Psi = k\sqrt{A_s}R_s \quad (2.3.2.4.3)$$

Where k is the thermal conductivity of the material and A_s is the area of the heat source.

The above method eliminates the need to calculate individual values for the material and external thermal resistances as they will remain constant for a given parametric trial regardless of the area ratio of the heat source and baseplate. The method outlined in this section was used to calculate spreading resistance of all parametric trials.

2.3.2.5 Benchmarking the CFD Model

In order to assess the validity of the developed CFD model it was necessary to benchmark it against the results obtained by Feng and Xu [7]. In order to accomplish this, an algorithm was developed, using Python programming (Python is a programming language that is commonly used for mathematical operations), that solves the analytical solution outlined in Section 2.2 (Refer to Appendix A for the developed Python script.). Using the developed algorithm, a few benchmark cases were conducted, and the results were compared to those obtained via the CFD model.

Figure 16 shows the results of both the developed CFD model and the solution derived by Feng and Xu (obtained via the Python algorithm) for a parametric analysis varying the heat source/heatsink side length ratio for multiple external resistance values (legend for Figure 16-18: CFD represents the results found using the CFD model; Feng represents the results obtained via Feng and Xu analytical solution). As can be seen, the results agree extremely well with the analytical closed form solution; in most cases the difference between the two values is within $\pm 1\%$.

Figure 16. Analytical Results vs CFD results for varying side length ratios and Biot numbers

Similar to Figure 16, Figure 17 shows the results of a parametric analysis varying the non-dimensional base thickness for both the CFD model and the results obtained using the Python algorithm that models the analytical solution derived by Feng and Xu. Again, it can be seen that the results agree very well with the analytical model, in most cases the error is within $\pm 2\%$.

Figure 17. Analytical Results vs CFD results for varying base thicknesses and Biot numbers

Similar to the previous two Figures, Figure 18 shows the results of a parametric analysis varying the aspect ratio of the heat sink base plate for both the CFD model and the analytical solution derived by Feng and Xu. Once again, it can be seen that the results are in excellent agreement with the analytical model; in most cases the error is within $\pm 0.5\%$.

Figure 18. Analytical Results vs CFD results for varying aspect ratios and Biot numbers

Based on the results from Figures 16-18, it was concluded that the developed CFD model accurately models the solution setup presented by Feng and Xu, as the results are shown to be in excellent agreement with the analytical solution. Therefore, the developed CFD model was used for the present study.

2.4 Development of the Parametric Analysis

In order to analyze the effect that certain heat sink parameters have on the thermal spreading resistance of the heat sink multiple parametric studies were conducted that

varied the heat sink base plate thickness, heat source/base plate side length ratio, heat sink material (i.e., thermal conductivity), external thermal resistance (Biot number), and heat sink base plate aspect ratio. Each parametric study varied one of the previously stated parameters, while all remaining parameters were held constant using a baseline set of parameters (refer to Table 3 for these baseline values). Also, each parametric study was conducted multiple times with varying external thermal resistance values in order to obtain a set of curves that show the effect that an individual parameter, as well as the external thermal resistance, has on the spreading resistance within a heat sink. The external thermal resistance was varied by changing the Biot number of the heat sink, namely the external heat transfer coefficient at the top surface of the plate (Refer back to Section 2.2 for the definition of the Biot number).

Table 3: Baseline set of parameters

Parameter	Value
Non-Dimensional Base Thickness (τ)	0.08
Heat Source/Base Plate Side Length Ratio (γ)	0.2
Aspect Ratio (α)	1.0
Thermal Dissipation Power (W)	15
Heat Sink Material (conductivity W/m-k)	Al-2024-T6 (177)
Ambient Temperature ($^{\circ}\text{C}$)	0

Referring to Figure 19 on the following page, the non-dimensional base thickness, τ , can be defined as twice the base thickness divided by the side length of the heat sink base plate or

$$\tau = \frac{2t}{X}$$

Figure 19. non-dimensional base thickness

Referring to Figure 20, the aspect ratio can be defined as the ratio of the two side lengths of the heat sink,

$$\text{aspect ratio} = \alpha = \frac{X}{Y}$$

Figure 20. Aspect ratio of the heat spreader

Referring again to Figure 20, the heat source/base plate side length ratio can be defined as the side length of the heat source divided by the side length of the base plate,

$$\varepsilon = \frac{X_1}{X} \text{ and } \gamma = \frac{Y_1}{Y}$$

For all the parametric studies, ε will equal γ , that is

$$\frac{X_1}{X} = \frac{Y_1}{Y}$$

However, using the data gathered from the parametric studies where ε and γ are equal, a correlation will be developed that will allow for ε and γ to differ from one another. Also, for the parametric study varying the aspect ratio, only one side of the heat sink, X , will be varied in order to create a change in the aspect ratio of the heatsink while the remaining side, Y , is held at a constant value.

A constant thermal dissipation power (TDP) was used to represent the heat source as opposed to a constant heat flux due to the fact that when conducting the parametric study for the heat source/heat sink area ratio, a heat flux value would need to have been varied along with the area ratio in order to keep a constant total power being supplied by the heat source. The ambient temperature was set as 0 °C so that the thermal resistance can be easily calculated as the maximum temperature in the source plan divided by the TDP. It was observed that any change in the TDP or the ambient temperature had no effect on the thermal spreading resistance or the overall heat sink resistance and, therefore, these were set as constants.

Table 4 on the following page shows the variation range that was developed for each of the parameters stated above that will be used to conduct the parametric trials using the CFD model.

Table 4. Variation of parameters

Base Thickness	Heat Source/Base Plate	Material	Aspect Ratio	Biot Number
τ	Side length Ratio ε	(conductivity)	α	ht/k
0.04	0.05	Al-Diecast (100)	1	0.113
0.06	0.1	Al-6061-T4 (154)	1.5	0.0565
0.08	0.15	Al-6061-T6 (167)	2	0.0377
0.1	0.2	Al-2024-T6 (177)	2.5	0.0282
0.12	0.25	Al-Extruded (205)	3	0.0226
0.15	0.3	Al-6063-T5 (209)	4	0.0188
0.2	0.35	Al 1100 (218)	5	0.0161
0.25	0.4	Al-Pure (240)	6	0.0141
0.3	0.45	Cu-Pure (387.6)	7	0.0113
0.35	0.5	Cu-Pure-ref (401)	8	0.0075
0.4	0.55	-	9	0.00565
0.5	0.6	-	10	0.00452
0.6	0.7	-	-	0.00377
0.7	0.8	-	-	0.00283
0.8	0.9	-	-	0.00226
0.9	1	-	-	0.00161
1.0	-	-	-	0.00113

The materials that were used for the parametric study were chosen based on two factors. One, all of the materials used for the parametric study can be found in the ANSYS Icepak material library, thereby, eliminating the need to create new material profiles; and secondly,, the materials offer a wide range of thermal conductivity (from 100-401 W/m²-K), which is necessary to see the effects that thermal conductivity has on the thermal spreading resistance.

2.5 Development of an Empirical Correlation for Spreading Resistance

For each parametric study the output values for the total thermal resistance were inserted into Excel and, using the method outlined in Section 2.3.2.4, the non-

dimensional thermal spreading resistance was calculated for all solution runs. The results were then graphed using Excel, and then, via extensive data analysis, a set of equations were developed that calculates the non-dimensional thermal spreading resistance of a heat sink as a function of the geometric and thermal characteristics of the heat sink. The results of all the parametric studies are presented in the following section

3.0 RESULTS AND DISCUSSION

3.1. CFD Results

Figures 21-23 show temperature contour plots for an isometric view, the z-x plane at the bottom surface of the heat sink base plate (red hatched square represents the heat source), and a z-x cut plane, respectively, of the CFD model results for the base case set of parameters outlined in Table 3. As would be expected, it can be seen from these figures that the temperature decreases as the distance from centroid of the plate increases. Recalling the definition of spreading resistance from Section 1.1, this non-uniform temperature distribution is the result of spreading resistance. Due to this non-uniform temperature profile, the overall thermal resistance of the plate increases.

Figure 21. Isometric view of temperature contour plot ($\epsilon = 0.2$)

Figure 22. z-x-plane view of temperature contour plot ($\epsilon = 0.2$)

Figure 23. x-y cut plane of temperature contour plot ($\epsilon = 0.2$)

To demonstrate this increase in overall thermal resistance, the overall resistance as well as the spreading resistance was calculated for the base case scenario shown in Figures 21-23, using the method outlined in Section 2.3.2.4. Recalling from Section 2.3.2.4, in order to calculate the spreading resistance, the CFD model needs to also be run with the heat source area equal to the base plate area. Figure 24 shows an isometric view of the temperature contour plot for such a case, using the baseline set of parameters. As would be expected, this situation produces a uniform temperature profile within the heat sink base plate, which results in a lower overall thermal resistance.

Figure 24. Isometric view of temperature contour plot ($\epsilon = 1.0$)

Recalling Section 2.3.2.4, the thermal resistance of both cases can be calculated as

$$R_t = \frac{T_{max,heatsink} - T_{ambient}}{Q}$$

Where the ambient temperature in the above equation was set equal to zero $^{\circ}\text{C}$ and the maximum temperature was located at the centroid of the heat source in the source plane.

Therefore,

$$R_{t@ \epsilon=0.2} = \frac{23.82^{\circ}\text{C} - 0^{\circ}\text{C}}{15\text{ W}} = 1.59 \frac{^{\circ}\text{C}}{\text{W}}$$

And

$$R_{t@ \epsilon=1.0} = \frac{4.67^{\circ}\text{C} - 0^{\circ}\text{C}}{15\text{ W}} = 0.31 \frac{^{\circ}\text{C}}{\text{W}}$$

Recalling that the spreading resistance can be calculated as the difference of the two previously calculated values, the thermal spreading resistance for the base case scenario can be calculated as

$$R_s = R_{t@ \varepsilon=1.0} - R_{t@ \varepsilon=0.2} = 1.59 - 0.31 = 1.28 \frac{^{\circ}\text{C}}{\text{W}}$$

In order to make these results more universal, the values for thermal spreading resistance can be non-dimensionalized, again using the method outlined in Section 2.3.2.4, that is

$$\Psi = k\sqrt{A_s}R_s = \left(177 \frac{\text{W}}{\text{m}^{\circ}\text{C}}\right) \left(\sqrt{.004^2 \text{ m}}\right) 1.28 \frac{^{\circ}\text{C}}{\text{W}} = 0.906$$

The same calculation was performed for all solution runs of the CFD model, and the results in the following section have all been non-dimensionalized using this method.

3.2. Parametric Analysis Results

3.2.1. Base Thickness Variation

Figure 24 shows the results of the parametric analysis varying the non-dimensional base thickness of the heat sink base plate at varying Biot numbers (i.e. external thermal resistance at the top surface of the plate). For each individual curve in Figure 25, the heat transfer coefficient as well as the thermal conductivity of the material were held constant while the base thickness was varied. However, since the Biot number is also a function of the base thickness, the Biot number will vary as the non-dimensional base thickness changes (refer to Section 2.2. for the definition of the Biot number). As a

result, the values of the Biot number stated in the legend of Figure 24, were calculated with

$$t = 0.04 * \frac{X}{2}$$

Figure 25. Parametric analysis results for the base thickness variation

The most intriguing result of Figure 25 is that as the non-dimensional base thickness increases beyond approximately 0.4, the non-dimensional spreading resistance becomes independent of both the Biot number and the non-dimensional base thickness and solely becomes a function of the side length ratio of the heat source and heat sink as

well as the aspect ratio. This result will be helpful in correlating the data into a useful equation for the thermal spreading resistance. It can also be seen that as the Biot number decreases (i.e. external resistance increases), the non-dimensional spreading resistance increases.

3.2.2. Side Length Ratio Variation

Figure 26 shows the results of the parametric analysis varying the side length ratio of the heat source and the heat sink at varying Biot numbers.

Figure 26. Parametric analysis results for the side length ratio variation

As can be seen, for all values of the Biot number the non-dimensional spreading resistance collapses to zero when the side length ratio is equal to one. This result follows the definition for spreading resistance, as spreading resistance should only be present when the heat source and heat sink are of differing areas. Also, similar to Figure 25, the non-dimensional spreading resistance increases as the Biot number decreases. Figure 26 also shows that, depending on the Biot number, the non-dimensional spreading resistance peaks between $\varepsilon = 0.2$ and $\varepsilon = 0.35$. However, due to the dependence on the square root of the heat source area in the conversion from the non-dimensional spreading resistance to the actual spreading resistance (recall equation 2.3.2.4.3), the shape of the curves in Figure 26 change significantly when graphed for the dimensional values. Figure 27 shows the dimensional spreading resistance as a function of the side length ratio of a specific solution run of the CFD model.

Figure 27. Dimensional spreading resistance vs. side length ratio

As is evident, Figure 27 differs significantly from Figure 26 in that the dimensional spreading resistance continually decreases as the side length ratio increases, ultimately trending to zero as the side length ratio goes to 1.0. This is as expected due to the fact that as the difference in area between the heat source and heat sink base plate decreases, the change in heat flow direction within the heat sink base plate is less severe therefore, reducing the thermal spreading resistance of the heat sink.

3.2.3. Aspect Ratio Variation

Figure 28 shows the results of the parametric analysis varying the aspect ratio of the heat sink at varying Biot numbers.

Figure 28. Parametric analysis results for the aspect ratio variation

As Figure 28 shows, the shape of the curve for the non-dimensional spreading resistance changes as a function of the Biot number. At higher Biot number values the non-dimensional spreading resistance decreases with increasing aspect ratio. However, at lower Biot number values the non-dimensional spreading resistance increases with increasing aspect ratio. Due to the nature of how this particular parametric study was conducted, the surface area of the top surface of the heat spreading increased as the aspect ratio was increased, which in turn decreased the overall thermal resistance of the heat spreader. At higher Biot values, this decrease in heat sink resistance significantly increased the amount of heat the top surface of the heat sink could dissipate, therefore decreasing its spreading resistance. However, at lower Biot number values the external resistance is significantly higher, therefore reducing the amount of heat that can be dissipated to the environment which increases the amount of heat that must flow within the heat sink, as a result increasing its spreading resistance. To illustrate this Figure 29 on the following page shows the dimensional spreading resistance for a high and low Biot number.

Figure 29. Dimensional spreading resistance vs. aspect ratio

As can be seen, at the higher Biot number value the spreading resistance continually decreases with increasing aspect ratio. However, at the lower Biot number value the spreading resistance tends to increase as the aspect ratio increases, peaking at around $\alpha = 8$.

3.2.4. External Thermal Resistance (Biot Number) Variation

Figure 30 shows the results of the parametric analysis varying the external thermal resistance (Biot number variation) of the heat sink. The Biot number was varied by changing the external heat transfer coefficient for the top surface of the plate. For the specific geometry of the developed CFD model the Biot numbers that were used corresponded to external resistance values ranging from $0.1 - 10.0 \text{ } ^\circ\text{C/W}$.

Figure 30. Parametric analysis result for the Biot number variation

As can be seen, the spreading resistance increases as the Biot number decreases (i.e. external thermal resistance increases). This result is somewhat intuitive in that a smaller Biot number corresponds to a higher external thermal resistance. Due to this higher resistance, the heat is not as easily dissipated by the top surface of plate and, therefore more of the heat must flow through the plate increasing its spreading resistance.

3.2.5. Material Variation

Recalling equation 2.3.2.4.3, the conversion from non-dimensional spreading resistance values to actual dimensional values is dependent upon the thermal conductivity of the material. As a result, it was unnecessary to conduct multiple parametric studies at various Biot numbers as the correlations that will be developed will calculate the non-dimensional value and then convert it to the true dimensional value using equation

2.3.2.4.3. However, a single parametric study was conducted in order to show the effect that thermal conductivity has on spreading resistance. Figure 31 shows the dimensional spreading resistance vs. the thermal conductivity of the material.

Figure 31. Parametric analysis result for the material variation

As would be expected, the spreading resistance decreases as the conductivity of the material increases. Again, this result is fairly intuitive in that as the thermal conductivity increases the resistance of the material decreases, ultimately trending to zero as the thermal conductivity tends to infinity. Due to the lower material resistance the heat can more easily flow within the plate and, as a result, it's spreading resistance decreases.

3.3. Developed Empirical Equation for Spreading Resistance Calculation

Using the data collected from the parametric studies along, with the developed algorithm solving the analytical solution developed by Feng and Xu, a set of empirical

equations were developed that can be used to more easily calculate the thermal spreading resistance within a heat sink. The following equations were developed using extensive curve fitting analysis in Excel. Equation 3.3.1, along with its coefficients in equations 3.3.2 – 3.3.4, can be used to calculate the non-dimensional spreading resistance for the stated ranges of the side length ratio (ϵ), non-dimensional base thickness (τ), and the Biot number (Bi) stated under the various coefficients.

$$\Psi = kR_s\sqrt{A_s} = \Theta * (1 - E) * T \quad (3.3.1)$$

Where

$$\Theta = 0.7723 - 129.4Bi_*^3 + 29.956Bi_*^2 - 2.996Bi_* \quad (3.3.2)$$

$$with \ 0.032 \leq Bi_* \leq 0.00113$$

and

$$E = 4.9006\epsilon^4 - 14.482\epsilon^3 + 16.385\epsilon^2 - 6.015\epsilon + 0.2575 \quad (3.3.3)$$

$$with \ 0.05 \leq \epsilon \leq 1.0$$

and

$$T = 0.1536\tau^{-.728}((5.6119e^{-8.129\epsilon})\tau + 1.1214\epsilon^{.1377}) \quad (3.3.4)$$

$$with \ 0.1 \leq \tau < 0.4$$

Due to the nature of how equations 3.3.3 and 3.3.4 were developed, the variable Bi_* must be calculated as,

$$Bi_* = \frac{h * .04 * X}{k} \quad (3.3.5)$$

Where h is the external heat transfer coefficient of the heat sink, X is the side length of the heat sink, and k is the thermal conductivity of the heat sink (Note, the actual Biot number may vary from Equation 3.3.5).

In order to better understand the purpose of the different coefficients in Equations 3.3.2 – 3.3.4 it would be beneficial to graph each coefficient as a function of its parameter (i.e. Θ is a function of the Biot number, E is a function of the side length ratio, and T is a function of the non-dimensional base thickness). The Purpose of Equation 3.3.2 is to account for the effects that the Biot number has on the thermal spreading resistance. Figure 32 shows the result of graphing Equation 3.3.2 over the valid range of the Biot Number.

Figure 32. Equation 3.3.2 vs. Biot Number

It can be clearly seen that the resulting graph is very similar in shape to Figure 30 which showed the results of the parametric study varying the Biot number. This similarity between the two graphs is the result of Equation 3.3.2 being developed in order to compensate for the effects that the Biot number has on the spreading resistance. Figure 30 shows that as the Biot number increases the non-dimensional spreading resistance decreases. Similarly, Figure 32 shows that as the Biot number increases the resulting value of Equation 3.3.2 decreases and therefore, via Equation 3.3.1, the non-dimensional spreading resistance decreases.

Similar to Figure 32, Figure 33 shows the results of graphing Equation 3.3.4 over the valid range of the side length ratio.

Figure 33. Equation 3.3.2 vs. Side Length Ratio

Once again, Equation 3.3.3 follows the same trend as the results of the parametric trial varying the side length ratio (Figure 26). As one might expect, the similarity between Figures 26 and 33 is a result of Equation 3.3.3 being developed in order to compensate for the effects that the side length ratio has on the spreading resistance.

Lastly, Equation 3.3.4 is a function of the non-dimensional base thickness and the side length ratio and its main function is to account for the effects that the non-dimensional base thickness has on the thermal spreading resistance. Figure 34 shows the results of graphing Equation 3.3.4 over the valid range of the non-dimensional base thickness (Note: although Equation 3.3.4 depends on the side length ratio as well, a constant value for the side length ratio was used in order to illustrate what Equation 3.3.4 accomplishes).

Figure 34. Equation 3.3.2 vs. Non-dimensional Base Thickness

As can be seen, Equation 3.3.4 follows the same trend as the results of the parametric trial varying the non-dimensional base thickness (Figure 25). Again, this similarity arises out of the fact that Equation 3.3.4 is used to compensate for the effects that the non-dimensional base thickness has on the thermal spreading resistance.

Recalling Figure 25, it was shown that as the non-dimensional base thickness increases beyond about 0.4, the non-dimensional spreading resistance became independent of both the Biot number and the non-dimensional base thickness. As a result, the spreading resistance becomes only a function of the side length ratio of the heat source and heat sink, as well as the aspect ratio. Using this insight, Equation 3.3.6 was developed which calculates the non-dimensional spreading resistance solely as a function of the side length ratio for non-dimensional base thickness values greater than or equal to 0.4 and with an aspect ratio of 1.0.

$$\Psi = kR_s\sqrt{A_s} = 0.0362\varepsilon^2 - 0.6045\varepsilon + 0.5655 \quad \tau \geq 0.4 \quad (3.3.6)$$

Noting from Equations 3.3.3 - 3.3.4 as well as 3.3.6, the side length ratio is held constant for both side of the heat source (i.e. $\varepsilon = \gamma$ from Section 2.4). However, this situation may not always be the case and, as a result, it is necessary to be able to calculate the spreading resistance when these two variables differ from one another. It was found that the spreading resistance of a heat sink with differing ε and γ values can be easily calculated using the following relationship

$$\Psi_{\varepsilon,\gamma} = \frac{\Psi_{\varepsilon} + \Psi_{\gamma}}{2} \quad (3.3.7)$$

Where Ψ_ε is the non-dimensional spreading resistance calculated via either Equation 3.3.1 or 3.3.6 (depending on the geometric characteristics of the heat sink) with the ε variable in those equations set to the desired ε value. Similarly, Ψ_γ is the non-dimensional spreading resistance calculated via either Equation 3.3.1 or 3.3.6 with the ε variable in those equations changed to the desired γ value.

Equations 3.3.1 and 3.3.6 present a significant reduction in complexity when compared to the analytical solution developed by Feng and Xu. The current solution employs only polynomial, power, and exponential functions in order to calculate the spreading resistance whereas the solution proposed by Feng and Xu contains infinite and double infinite sums with very complex coefficients. The current solution can be easily solved by anyone with a working knowledge of Excel, whereas the solution by Feng and Xu takes significantly more effort to solve.

As can be seen, equations 3.3.1 – 3.3.6 do not take into account the aspect ratio of the heat sink (i.e. β). Unfortunately, a discernable correlation for the aspect ratio was unable to be found. Further analysis of the data is suggested in order to find this correlation and further increases the usability of the developed equations. The following Section shows a comparison between the developed empirical equation and the analytical solution proposed by Feng and Xu.

3.4. Comparison of Developed Empirical Equation with the Analytical Solution

In order to verify that Equations 3.3.1 – 3.3.6 will accurately calculate the non-dimensional spreading resistance, it is necessary to compare the developed empirical

equations with the analytical solution proposed by Feng and Xu. As a result, Figures 35-41 on the following pages show multiple comparisons of the analytical solution and Equation 3.3.1 at various combination of the Biot number, non-dimensional base thickness, and the side length ratio (the analytical solution was again solved using the Python script outlined in Appendix A). Figures 35-39 show a variation in the side length ratio of the heat source and heat sink for various combinations of the non-dimensional base thickness and the Biot number, while Figures 40 and 41 show a variation in the non-dimensional base thickness at different side length ratios and Biot numbers. For each Figure below, the stated Biot number is calculated via Equation 3.3.5 (the actual Biot number of the heat sink may vary when using the equations).

Figure 35. Comparison of Equation 3.3.1 for $\tau = 0.1$ and $Bi = 0.025$

Figure 36. Comparison of Equation 3.3.1 for $\tau = 0.15$ and $Bi = 0.0158$

Figure 37. Comparison of Equation 3.3.1 for $\tau = 0.2$ and $Bi = 0.009$

Figure 38. Comparison of Equation 3.3.1 for $\tau = 0.3$ and $Bi = 0.0036$

Figure 39. Comparison of Equation 3.3.1 for $\tau = 0.35$ and $Bi = 0.00113$

Figure 40. Comparison of Equation 3.3.1 for $\epsilon = 0.2$ and $Bi = 0.025$

Figure 41. Comparison of Equation 3.3.1 for $\epsilon = 0.7$ and $Bi = 0.009$

As can be seen from the previous figures, Equation 3.3.1 agrees extremely well with the stated analytical solution over the range of acceptable values. In most cases, the error between Equation 3.3.1 and the analytical solution are within $\pm 5\%$, with a maximum error of around 10 – 12% when the values for the non-dimensional spreading resistance approach zero. However, as the non-dimensional spreading resistance approaches zero, its effect on the overall thermal performance of a heat sink are greatly reduced and, therefore, these larger errors will not have a significant effect on the analysis of a heat sink as the spreading resistance will become negligible at these lower values. As a result of the above analysis, it was concluded that Equation 3.3.1, along with its coefficients in Equations 3.3.2 – 3.3.4, accurately calculates the non-dimensional spreading resistance over the stated range of parameters.

A similar analysis was done in order to compare Equation 3.3.6 with the solution proposed by Feng and Xu. Figure 42 shows the comparison of analytical solution with Equation 3.3.6 for non-dimensional base thickness values at opposite ends of the acceptable range (i.e. $\tau = 0.4$ and $\tau = 0.9$).

Figure 42. Comparison of Equation 3.3.6 for $\tau = 0.4$ and 0.9

As can be seen, the change in values from $\tau = 0.4$ and $\tau = 0.9$ using the analytical solution is negligible, and Equation 3.3.6 is sufficiently accurate in calculating the non-dimensional spreading resistance within the given range. Once again, the errors between Equation 3.3.6 and the analytical solution are, in most cases, $\pm 5\%$ with a maximum error of about $\pm 10\%$ when the values for the non-dimensional spreading resistance approach zero. As was with Equation 3.3.1, this larger error near the low end values for the non-dimensional spreading resistance will not significantly affect the analysis of heat sink. Due to the results in Figure 42, it was concluded that Equation 3.3.6 accurately calculates the non-dimensional spreading resistance over the stated range of parameters.

Lastly, in order to verify the relationship stated in Equation 3.3.7, an analysis was conducted that compared Equation 3.3.7 with the analytical solution at various combinations of ε , γ , τ , and Biot number (both Equations 3.3.1 and 3.3.6 were used in this analysis). Table 5 shows the results of this analysis.

Table 5. Empirical Equation vs. Feng and Xu analytical solution for various combinations of when $\varepsilon \neq \gamma$

ε	γ	τ	Bi	ψ	Feng	% Difference
0.1	0.9	0.1	0.0294	0.4419	0.4444	-1%
0.15	0.85	0.12	0.0225	0.4629	0.4651	0%
0.2	0.8	0.15	0.0181	0.4459	0.4431	1%
0.25	0.75	0.18	0.0158	0.4226	0.4219	0%
0.3	0.7	0.21	0.011	0.4027	0.4000	1%
0.35	0.65	0.25	0.0081	0.3705	0.3688	0%
0.4	0.6	0.28	0.00542	0.3465	0.3508	-1%
0.45	0.55	0.32	0.00407	0.3185	0.3289	-3%
0.5	0.5	0.35	0.00316	0.3195	0.3156	1%
0.2	0.8	0.6	0.0181	0.276	0.2581	6%
0.3	0.7	0.9	0.009	0.274	0.2576	6%

As can be seen, the relationship in Equation 3.3.7 in conjunction with either Equation 3.3.1 or 3.3.6 (depending on the chosen parameters) agrees extremely well with the analytical solution with a maximum error for the tested values of about 6%. Based on this analysis, it was concluded that the relationship in Equation 3.3.7 can accurately calculate the non-dimensional spreading resistance for heat sources with dissimilar side length ratios, ε and γ .

Based on the above analysis it was concluded that Equations 3.3.1 – 3.3.7 can be used to accurately calculate the non-dimensional spreading resistance over the stated range of parameters. The developed equations provide a significant reduction in

complexity when compared to the analytical solution, and can be easily used to calculate the spreading resistance in a heat sink when conducting a thermal analysis. Similar to the code developed to solve the analytical solution, a Python script was written that will take a set of input parameters (i.e., ε , γ , τ , h , k , and the side length of the heat sink) and calculate the non-dimensional spreading resistance based on Equations 3.1.3 – 3.3.7 (see Appendix B for the developed Python script).

4.0 CONCLUSION

A CFD model was developed that was shown to be in excellent agreement with the analytical solution proposed by Feng and Xu (with an error of $\pm 2\%$ for most cases). Using this model, a set of parametric studies was conducted that varied the base thickness, side length ratio, aspect ratio, thermal conductivity of the material, and the external thermal resistance (i.e., Biot number variation) in order to obtain the effect they have on the thermal spreading resistance. Based on the data collected from these parametric studies, a set of empirical equations was developed that can be used to accurately calculate the non-dimensional spreading resistance within a heat sink. The equations were shown to be accurate to within $\pm 5\%$ in most cases and provide a significant reduction in complexity over previously derived analytical solutions. However, further study is suggested in order to develop a correlation for the aspect ratio in order to make the developed solution applicable for heat sinks aspect ratios other than one.

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APPENDIX A. Python Script for Feng and Xu Analytical Solution

```
def Feng(paramValues, parameter):
    results_nd = []
    results_d = []
    print('\n\nEnter values for the constant parameters: ')
    while True:
        try:
            print('\n!!Enter negative one (-1) for %s!!'
%parameter)
            t = float(input('\t Please enter a value for
Tau: '))
            g = float(input('\t Please enter a value for
gamma: '))
            e = float(input('\t Please enter a value for
epsilon: '))
            bi = float(input('\t Please enter a value for
the Biot Number: '))
            As = float(input('\t Please enter the surface
area of the heat source (m^2): '))
            k = float(input('\t Please enter the thermal
conductivity of the heat sink material (W/m-k): '))
        except ValueError:
            print('\n')
            print("Last entry was invalid")
            continue
        else:
            break

    for i in range(0,15):
        if parameter.lower() == '1':
            t = paramValues[i]
        elif parameter.lower() == '2':
            b = paramValues[i]
        elif parameter.lower() == '3':
            g = paramValues[i]
        elif parameter.lower() == '8':
            e = paramValues[i]
        elif parameter.lower() == '4':
            bi = paramValues[i]
        elif parameter.lower() == '5':
            As = paramValues[i]
        elif parameter.lower() == '6':
            k = paramValues[i]
```

```

b = 1
Cm0 = 0
for m in range(1,101):
    import math
    w = (t * m * math.pi + bi) * math.exp(t * m *
math.pi)
    x = (t * m * math.pi - bi) * math.exp(-t * m *
math.pi)
    y = math.sin(g * m * math.pi)
    z =(m * math.pi)**2
    Cm = (g * (w + x) * y) / (z * (w - x))
    Cm0 += Cm

C0n = 0
for n in range(1,101):
    import math
    w = (t * n * math.pi + b * bi) * math.exp((t *
n * math.pi) / b)
    x = (t * n * math.pi - b * bi) * math.exp((-t *
n * math.pi) / b)
    y = math.sin(g * n * math.pi)
    z = (n * math.pi)**2
    Cn = (b * g * (w + x) * y)/(z * (w - x))
    C0n += Cn

Cmna = 0
for m in range(1,101):
    for n in range (1,101):
        import math
        p = math.sqrt(m**2 +(n/b)**2)
        w = (t * p * math.pi + bi) * math.exp(t * p
* math.pi)
        x = (t * p * math.pi - bi) * math.exp(-t *
p * math.pi)
        y = math.sin(g * m * math.pi) * math.sin(g
* n * math.pi)
        z = m * n * p * (math.pi)**3
        Cmn = (2 * (w + x) * y) / (z * (w - x))
        Cmna += Cmn

import math
nd_s_resistance = math.sqrt((g * g) / b) * (Cm0 +
C0n + Cmna)
s_resistance = (((nd_s_resistance / k)**2) / As)
results_nd.append(nd_s_resistance)

```



```

        results_d.append(s_resistance)
    return results_nd, results_d

print('\n')
print('This program can be used as a spreading resistance
calculator \nor it can be used to conduct a parametric
study \n\nSolutions are based on Feng and Xu')
choice = input('Run prgram as: \n  a) A Spreading Reistance
Calculator \n  b) Spreading Resistance Parametric Analysis
\n \nChoose a or b: ')

if choice == 'a':
    while True:
        try:
            t = float(input('\tPlease enter a value
for Tau: '))
            b = float(input('\tPlease enter a value
for Beta: '))
            g = float(input('\tPlease enter a value
for gamma: '))
            e = float(input('\tPlease enter a value
for epsilon: '))
            bi = float(input('\tPlease enter a value
for the Biot Number: '))
            As = float(input('\tPlease enter the
surface area of the heat source (m^2): '))
            k = float(input('\tPlease enter the
thermal conductivity of the heat sink material (W/m-k): '))
        except ValueError:
            print('\n')
            print("Last entry was invalid")
            continue
        else:
            break

    Cm0 = 0
    for m in range(1,101):
        import math
        w = (t * m * math.pi + bi) * math.exp(t * m *
math.pi)
        x = (t * m * math.pi - bi) * math.exp(-t * m *
math.pi)
        y = math.sin(e * m * math.pi)
        z =(m * math.pi)**2
        Cm = (g * (w + x) * y) / (z * (w - x))

```

```

        Cm0 += Cm

    C0n = 0
    for n in range(1,101):
        import math
        w = (t * n * math.pi + b * bi) * math.exp((t *
n * math.pi) / b)
        x = (t * n * math.pi - b * bi) * math.exp((-t *
n * math.pi) / b)
        y = math.sin(g * n * math.pi)
        z = (n * math.pi)**2
        Cn = (b * e * (w + x) * y)/(z * (w - x))
        C0n += Cn

    Cmna = 0
    for m in range(1,101):
        for n in range (1,101):
            import math
            p = math.sqrt(m**2 +(n/b)**2)
            w = (t * p * math.pi + bi) * math.exp(t * p
* math.pi)
            x = (t * p * math.pi - bi) * math.exp(-t *
p * math.pi)
            y = math.sin(e * m * math.pi) * math.sin(g
* n * math.pi)
            z = m * n * p * (math.pi)**3
            Cmn = (2 * (w + x) * y) / (z * (w - x))
            Cmna += Cmn

    import math
    nd_s_resistance = math.sqrt((e * g) / b) * (Cm0 +
C0n + Cmna)
    s_resistance = nd_s_resistance / (k *
math.sqrt(As))

    print('\n')
    A = 'Non-dimensional Rs = ' +
str('{:.2e}'.format(float(nd_s_resistance)))
    B = 'Dimensional Rs = ' +
str('{:.2e}'.format(float(s_resistance))) + ' C/W'
    print(A)
    print('\n')
    print(B)

if choice == 'b':

```

```

print('\nWhich parameter would you like to vary?')
print('\n\tEnter (1) for Tau')
print('\n\tEnter (2) for Beta')
print('\n\tEnter (3) for gamma/epsilon')
print('\n\tEnter (4) for Biot Number')
print('\n\tEnter (5) for Heat Source Area (m^2)')
print('\n\tEnter (6) for Thermal Conductivity (W/m-k)')
parameter = input('Input one of the above numbers and
hit ENTER\n\n\t')
print('\nPlease enter 10 data points for the parametric
study: ')
i = 0
paramValues = []
for i in range(15):
    paramValues.append(float(input('\t')))

results_nd = []
results_d = []
results_nd, results_d = Feng(paramValues, parameter)
print('\nNon-Dimensional Spreading Resistance: \n')
print(results_nd)
print('\nSpreading Resistance: \n')
print(results_d)
print('\n\n')

import matplotlib.pyplot as plt
X = [paramValues]
Y = [results_nd]
plt.scatter(X,Y, s=100, marker = '+', color = 'red')
plt.ylabel('Non-Dimensional Spreading Resistacne')
plt.xlabel('%s' %parameter)
plt.grid(b = True, which = 'major')
plt.yscale('log')
plt.savefig('spreading_resistance_output.jpg', dpi=400)
plt.show()

file = open("spreading_resistance_output.csv", "w")
file.write("Non-Dimensional Values: \n")
for j in range (0,15):
    rnd = str(results_nd[j])
    file.write('%s \n' %rnd)
file.write("\n")
file.write("\n")
file.write("Dimensional Values: \n")
for j in range (0,15):

```

```
rd = str(results_d[j])  
file.write('%s \n' %rd)  
file.close()
```

Appendix B. Python Script for the Developed Emperical Equation

```
def empirical_equation(paramValues, parameter):
    results_nd = []
    results_d = []
    print('\n\nEnter values for the constant parameters: ')
    while True:
        try:
            print('\n!!Enter negative one (-1) for %s!!'
                %parameter)
            t = float(input('\tPlease enter a value for
Tau: '))
            e = float(input('\tPlease enter a value for
the side length ratio: '))
            sl = float(input('\tPlease enter a value for
the side length of the heat sink (m): '))
            h = float(input('\tPlease enter a value for
the heat transfer coefficient (W/m^2-K): '))
            k = float(input('\tPlease enter the thermal
conductivity of the heat sink material (W/m-K): '))
        except ValueError:
            print('\n')
            print("Last entry was invalid")
            continue
        else:
            break

    for i in range(0,10):
        if parameter.lower() == '1':
            t = paramValues[i]
        elif parameter.lower() == '2':
            e = paramValues[i]
        elif parameter.lower() == '3':
            h = paramValues[i]
        elif parameter.lower() == '4':
            As = paramValues[i]
        elif parameter.lower() == '5':
            k = paramValues[i]

        if t >= 0.4:
            import math
            nd_s_resistance = .0362 * e**2 - .6045 * e +
.5655
```

```

        s_resistance = nd_s_resistance / (k *
math.sqrt(As))

        print('\n')
        nd_value = 'Non-dimensional Rs = ' +
str('{:.2e}'.format(float(nd_s_resistance)))
        d_value = 'Dimensional Rs = ' +
str('{:.2e}'.format(float(s_resistance))) + ' C/W'
        print(nd_value)
        print('\n')
        print(d_value)

As = (e * sl) * (e * sl)
bi = (h * (.04 * sl) / 1000) / k

if bi >= .034:
    print("\nCalculated Biot number not in the
valid range, please try again with a different set of
parameters")

if bi <= .00113:
    print("\nCalculated Biot number not in the
valid range, please try again with a different set of
parameters")

if t < 0.4:
    if bi <= .034:
        if bi >= .00113:
            import math
            theta = .7723 - 129.4 * bi**3 + 29.956
* bi**2 - 2.996 * bi
            E = 4.9006 * e**4 - 14.482 * e**3 +
16.385 * e**2 - 6.015 * e + .2575
            T = (.1536 * t**(-.728)) * ((5.6119 *
math.exp(-8.129 * e)) * t + 1.1214 * e**(.1377))

            nd_s_resistance = theta * (1 - E) * T
            s_resistance = nd_s_resistance / (k *
math.sqrt(As))

            results_nd.append(nd_s_resistance)
            results_d.append(s_resistance)
        return results_nd, results_d

print('\n')

```

```

print('This program can be used as a spreading resistance
calculator \nor it can be used to conduct a parametric
study \n')
choice = input('Run program as: \n  a) A Spreading
Resistance Calculator \n  b) Spreading Resistance
Parametric Analysis \n \nChoose a or b: ')

if choice == 'a':
    while True:
        try:
            t = float(input('\tPlease enter a value
for Tau: '))
            g = float(input('\tPlease enter a value
for gamma: '))
            e = float(input('\tPlease enter a value
for epsilon: '))
            sl = float(input('\tPlease enter a value
for the side length of the heat sink (m): '))
            h = float(input('\tPlease enter a value
for the heat transfer coefficient (W/m^2-K): '))
            k = float(input('\tPlease enter the
thermal conductivity of the heat sink material (W/m-K): '))
        except ValueError:
            print('\n')
            print("Last entry was invalid")
            continue
        else:
            break

    if t >= 0.4:
        import math
        nd_s_resistance = .0362 * e**2 - .6045 * e +
.5655
        s_resistance = nd_s_resistance / (k *
math.sqrt(As))

        print('\n')
        A = 'Non-dimensional Rs = ' +
str('{:.2e}'.format(float(nd_s_resistance)))
        B = 'Dimensional Rs = ' +
str('{:.2e}'.format(float(s_resistance))) + ' C/W'
        print(A)
        print('\n')
        print(B)

```

```

As = (e * sl) * (g * sl)
bi = (h * ((.04 * sl) / 1000)) / k

if bi >= .034:
    print("\nCalculated Biot number not in the
valid range, please try again with a different set of
parameters")

if bi <= .00113:
    print("\nCalculated Biot number not in the
valid range, please try again with a different set of
parameters")

if t < 0.4:
    if bi <= .034:
        if bi >= .00113:
            if e == g:
                import math
                theta = .7723 - 129.4 * bi**3 +
29.956 * bi**2 - 2.996 * bi
                E = 4.9006 * e**4 - 14.482 * e**3 +
16.385 * e**2 - 6.015 * e + .2575
                T = (.1536 * t**(-.728)) * ((5.6119
* math.exp(-8.129 * e)) * t + 1.1214 * e**(.1377))
                nd_s_resistance = theta * (1 - E) *
T
                s_resistance = nd_s_resistance / (k
* math.sqrt(As))

                print('\n')
                A = 'Non-dimensional Rs = ' +
str('{:.2e}'.format(float(nd_s_resistance)))
                B = 'Dimensional Rs = ' +
str('{:.2e}'.format(float(s_resistance))) + ' C/W'
                print(A)
                print('\n')
                print(B)

            if e != g:
                import math
                theta_a = .7723 - 129.4 * bi**3 +
29.956 * bi**2 - 2.996 * bi
                E_a = 4.9006 * e**4 - 14.482 * e**3
+ 16.385 * e**2 - 6.015 * e + .2575

```



```

T_a = (.1536 * t**(-.728)) *
((5.6119 * math.exp(-8.129 * e)) * t + 1.1214 * e**(.1377))
nd_s_resistancea = theta_a * (1 -
E_a) * T_a

theta_b = .7723 - 129.4 * bi**3 +
29.956 * bi**2 - 2.996 * bi
E_b = 4.9006 * g**4 - 14.482 * g**3
+ 16.385 * g**2 - 6.015 * g + .2575
T_b = (.1536 * t**(-.728)) *
((5.6119 * math.exp(-8.129 * g)) * t + 1.1214 * g**(.1377))
nd_s_resistanceb = theta_b * (1 -
E_b) * T_b

nd_s_resistance =
(nd_s_resistance_a + nd_s_resistance_b) / 2
s_resistance = nd_s_resistance / (k
* math.sqrt(As))

print('\n')
nd_value = 'Non-dimensional Rs = '
+ str('{:.2e}'.format(float(nd_s_resistance)))
d_value = 'Dimensional Rs = ' +
str('{:.2e}'.format(float(s_resistance))) + ' C/W'
print(nd_value)
print('\n')
print(d_value)

if choice == 'b':
    print('\nWhich parameter would you like to vary?')
    print('\n\tEnter (1) for Tau')
    print('\n\tEnter (2) for gamma/epsilon')
    print('\n\tEnter (3) for Heat Transfer Coefficient')
    print('\n\tEnter (4) for Heat Source Area (m^2)')
    print('\n\tEnter (5) for Thermal Conductivity (W/m-k)')
    parameter = input('Input one of the above numbers and
hit ENTER\n\n\t')
    print('\nPlease enter 10 data points for the parametric
study: ')
    i = 0
    paramValues = []
    for i in range(10):
        paramValues.append(float(input('\t')))

    results_nd = []

```

```

results_d =[]
results_nd, results_d = emperical_equation(paramValues,
parameter)
print('\nNon-Dimensional Spreading Resistance: \n')
print(results_nd)
print('\nSpreading Resistance: \n')
print(results_d)
print('\n\n')

file = open("spreading_resistance_results.csv", "w")
file.write("Non-Dimensional Values: \n")
for j in range (0,10):
    rnd = str(results_nd[j])
    file.write('%s \n' %rnd)
file.write("\n")
file.write("\n")
file.write("Dimensional Values: \n")
for j in range (0,10):
    rd = str(results_d[j])
    file.write('%s \n' %rd)
file.close()

```