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Dynamics and Control of A Suspended ATN Vehicle

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DYNAMICS AND CONTROL OF A SUSPENDED ATN VEHICLE

A Thesis

Presented to

The Faculty of the Department of Mechanical Engineering

San José State University

In Partial Fulfillment

of the Requirements for the Degree

Master of Science

by

Wade P. Brown

August 2016

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The Designated Thesis Committee Approves the Thesis Titled

DYNAMICS AND CONTROL OF A SUSPENDED ATN VEHICLE

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ABSTRACT

DYNAMICS AND CONTROL OF A SUSPENDED ATN VEHICLE

by Wade P. Brown

In order to enhance the research into Automation Transit Network (ATN) systems performed by the Spartan Superway team, a simulation environment and controller is developed for a suspended ATN vehicle. The equations of motion are derived for a suspended ATN vehicle constrained to move on a circular guideway (referred to as the Planar System) and for an ATN vehicle constrained to move on a guideway of arbitrary geometry (referred to as the General System). Additionally, a controller using feedback linearization is designed that tracks the position of the ATN for a conditioned displacement profile and minimizes the lateral acceleration experienced by the passenger. A literature review is first performed that covers the background of ATN systems and other important concepts important to vehicle control. The equations of motion for both the Planar System and General System are derived by the use of hand calculations and the MotionGenesis application. MotionGenesis is also used for the geometric calculations to allow for the interpolation of the arbitrary geometry guideway, the formation of the displacement profile, and the formation of the wind profile. SIMULINK is used to form the simulation environment built from the theoretical work and run validation simulations. Using a circular guideway with a radius of 100 m , it is found that there is negligible deviation between simulations of the Planar System and simulations of the General System. A controller is successfully implemented on the Planar System using feedback linearization with dynamics derived from both the Planar System and General System. Finally, there is a proof-of-concept test of the dynamics of an ATN vehicle on an Euler spiral guideway.

DEDICATION

This thesis is dedicated in loving memory of my grandmother Virginia Patterson,
whose love and support in my youth made my future possible.

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CHAPTER 1

INTRODUCTION

1.1 Introduction to the Automated Transit Network Concept

During the 20th century, the reliance on the use of automobiles expanded with the creation of new highway systems throughout the rural and urban centers of the country. This increased reliance on automobiles allowed the average citizen a previously unparalleled degree of convenience in personal transportation. However, as the 20th century came to a close, it became apparent that cities depending only highway networks were victims of unforeseen negative externalities. Chief among these was the problem of traffic congestion, which could turn a nominally short commute into hours, as well as the long term problem of air pollution. While an obvious solution would be investment in public transportation, traditional public transportation is nominally slower and less convenient than automobiles. An attractive middle ground solution, which was originally developed in the mid 20th century, is an Automated Transit Network or ATN. An ATN is a network of exclusive guideways, typically elevated, on which a fleet of small vehicles are allowed to move. The vehicles can move on the guideway with a captive bogie, where the vehicle can only move tangent to the guideway (similar to conventional rail vehicles), or be vehicles on an open paved surface. The vehicles are all controlled autonomously and have no set time-table. Thus, the vehicles will travel directly from the point of origin to the desired destination of the passengers. This system of networked vehicles retains the advantages of traditional train systems, such as reducing congestion and being powered by the electric grid (which can utilize renewable energy) while vastly decreasing the amount of time required for an individual commuter to reach his or her destination. In addition, the exclusive guideway network, which is typically above grade, enhances safety by

ensuring that the vehicle remains segregated from pedestrian and bicycle traffic. There are currently five public transportation systems around the world that qualify as ATN systems which are in service and carry passengers (Furman, Fabian, Ellis, Muller, & Swenson, 2014). These five systems are in the following locations:

- The Morgantown PRT at West Virginia University
- The Parkshuttle Rivium metro-feeder outside Rotterdam (1999)
- The Masdar City PRT in Abu Dhabi (2010)
- The Terminal 5 Shuttle at London Heathrow Airport (2011)
- The Nature Park Shuttle in Suncheon Bay, South Korea (2014)

Of these five systems, the technology behind the ATN at Suncheon Bay in South Korea (which was developed by the company Vectus PRT) is investigated in the Literature Review of this thesis.

However, this novel form of public transportation has unique engineering challenges. These challenges include the structural design of the vehicle and guideway, the energy source and power consumption of the ATN vehicles, the control of the individual ATN vehicles, and the routing method for the entire network. This thesis will focus primarily on the control of the individual vehicle. This thesis will specifically present a controller for a suspended ATN vehicle using a captive bogie design that tracks position and minimizes centripetal acceleration. The general vehicle and system design is being referenced from work done by Beamways AB (B. Gustafsson, 2014) and the INIST sponsored Spartan Superway research (Furman, 2016). Beamways AB, a design firm in Sweden, is designing an ATN system using a suspended vehicle. See Figure 1 for an example of the Beamways AB vehicle.

Figure 1. An artistic representation of what the Beamways ATN system will look like. Property of Beamways AB. Reprinted with permission.

Spartan Superways, an engineering team comprised of undergraduate and graduate students at San José State University, is developing the technology using concepts original to Beamways. As this thesis is being developed under the guidance of the INIST sponsored Spartan Superway project, the vehicle parameters are similar to the designs developed by members of the Spartan Superway team. The research in this thesis will investigate a novel method for the vehicle to simultaneously track position and maintain rider comfort using state control. To simplify the vehicle dynamics, the bogie is considered to be a point mass which can track its position on a straight or curved guideway and the suspended vehicle is considered to be a point mass which is suspended from the bogie mass by means of a rigid rod. The rod is allowed to tilt normal to the direction of the guideway. This simplification ignores the complicated internal dynamics of the vehicle suspension which is still under development. Additionally, a simplified model is easier to validate.

The following chapter describes a review of the literature focusing in three areas. The first is a background of the existing ATN design literature and includes a case study of a full scale model system. This information is obtained from relevant textbooks and

whitepapers. Next a review of historical research in ATN control is conducted which gives a background in developing classical and state control algorithms. Finally a review of the modern control techniques used for conventional rail vehicles is presented. While these vehicles are much heavier and larger than the ATN vehicles being analyzed and the research can be specific to a certain rail line or particular rail vehicle, there are similarities in the dynamics and control between conventional train vehicles and ATN vehicles. These papers are referenced for controller design strategies as the field of rail vehicle control is mature compared to ATN control. These three sources will allow for the development of a thesis that attempts to solve a historically defined problem with modern tools.

1.2 Literature Review

1.2.1 Introduction to the Literature Review

The first section of the literature review gives an introduction to the terminology and concepts of ATN systems. Special emphasis is placed on how each of the concepts relates to controller research. The second section reviews research specific to ATN systems. This includes a case study of the commercially developed Vectus ATN system, an open guideway vehicle tested by GM, and a study researching full state control of an idealized ATN vehicle. As discussed earlier, the dynamical behavior of an ATN vehicle is similar to the dynamical behavior of a conventional railway vehicle. Therefore, a useful resource for ATN vehicle control to investigate research modern control of conventional railway vehicles. Three different research projects are reviewed, one in the position and velocity control of a high speed train with multiple cars, one in the position and velocity control of a freight train, and one paper in the implementation of a passenger train that uses tilting for active suspension.

1.2.2 ATN System Background with Respect to Control Research Applications

There is a desire to understand existing research performed in the past on the control of ATN vehicles. The body of research in ATN typically uses a standardized performance criteria for system analysis and vehicle control. Additionally, some of the terminology the literature uses is in the context of urban planning or system design rather than control engineering. Therefore, there is a need to understand the physical meaning of the terminology used in ATN literature.

Passenger comfort and transit time are conflicting ATN vehicle controller performance criteria. At a system level, the two variables that govern both of these criteria are the acceleration and jerk of the pod car. The acceleration (a) is the second time derivative of the position of the vehicle. The jerk (J) is defined as the time derivative of acceleration or the third time derivative of vehicle's position. The acceleration and jerk of the vehicle may not exceed given values at any point in time during motion. The exact value depends the regulations governing the ATN system as well as other factors (such as if the passengers are standing or sitting) (Anderson, 1978). At a system level, the acceleration and jerk are defined from an acceleration profile. Figure 2 shows an example of an acceleration profile.

Figure 2. A graph showing an acceleration profile. Property of John Edward Anderson, PhD, P.E. Reprinted with permission.

The acceleration profile of the vehicle is defined first as the acceleration of the vehicle over a given transit time. The acceleration is assumed to increase at a linear rate to a maximum acceleration value, hold the maximum acceleration value until close to the cruising velocity, and then decrease at a linear rate to zero. There are three importation performance parameters from the acceleration profile: the maximum acceleration a_M of the vehicle and the positive and negative jerk of the vehicle, J_1 and J_2 . From the acceleration profile, it is possible through basic calculus to determine other useful information about the ATN such the velocity profile of the vehicle and the stopping distance of the vehicle. Different acceleration profiles are given for different vehicle events such as a vehicle merging with other vehicles or leaving a station. Controller validation studies will usually use a particular acceleration profile from a certain event to test the controller.

While this thesis concerns vehicle control rather than guideway design, some discussion is necessary to better understand the control research. The length of a straight section of guideway is governed by the acceleration requirements of a vehicle discussed above. For the purposes of this thesis, this distance will be assumed to be a constant and the acceleration profile used as an input for the controller. Curved sections of track can

either be circular arcs or spiral segments. Additionally, curved segments of track can exploit the super elevation principle. Super elevation (also referred in conventional rail literature as “canting”) is the concept of tilting the ATN vehicle during travel. This allows for a reduction in the centrifugal force on the passengers in the vehicle. This allows for either a higher speed around a given radius in the guideway or for the radius to be reduced. Either the guideway can be tilted or the vehicle body can be tilted to achieve this effect. Equation 1 (Anderson, 1978) estimates the reduction in centrifugal force.

$$a_n = g \tan \phi + e \approx g(\phi + e) \quad (1)$$

e is the elevated angle of the guideway and ϕ is the angle between the acceleration due to gravity (g) and the normal to the floor of the vehicle. The centripetal acceleration a_n is the acceleration vector from the vehicle body pointing to the center of the guideway arc. This concept is important to understanding the difference between supported and suspended captive bogie ATN vehicles.

The core of an ATN system is the vehicle which carries passengers between destinations. The two main types of vehicles are captive bogie ATN vehicle, such as the Vectus ATN system, and ATN vehicles for open guideways, such as the ULTra PRT ATN system and the experimental GM ATN system (Muller, 2009). The open guideway vehicle is where the vehicle is allowed to move freely on a guideway and must control for its longitudinal and lateral position on the guideway. The captive bogie ATN vehicle is where the vehicle has a bogie fixed to the track and can only move tangentially to the track. Additionally a captive bogie ATN vehicle may be Support-Vehicle-System type vehicle (otherwise known as a supported vehicle) or a Hanging-Vehicle-System type vehicle (otherwise known as a suspended vehicle) (Anderson, 2013). Figure 3 shows a comparison between each vehicle type. On the left is a supported vehicle and on the right

is a suspended vehicle. d represents the chord length on a curved section of guideway. The symbols F_c and w represent the centrifugal force and weight respectively. h is the distance of the center of mass on the supported vehicle to the guideway. h_1 is the distance from the suspended vehicle center of mass to the revolute joint of the suspended vehicle and h_2 is the distance from the revolute joint to the guideway.

Figure 3. A comparison of a suspended vehicle (right) and a simply supported vehicle (left). Property of John Edward Anderson, PhD, P.E. Reprinted with permission.

The supported vehicles have simpler construction than the suspended vehicles, however, supported vehicles must have a tilted guideway in order to take advantage of the super-elevation principle. Suspended vehicles can exploit the super-elevation principle by means of the vehicle body tilting inward. Lastly, for any type of vehicle, there can be point-following and car-following vehicle control schemes (Kornhauser, Lion, McEvaddy, & Garrard, 1974). Point following vehicles are given a position reference to a point moving along the guideway with a given acceleration profile. The reference for car following vehicles is based on the position of adjacent vehicles (Kornhauser et al., 1974). For both types of control, the reference value is decided from either a central computer (referred to as synchronous control) or by each vehicle (referred to as asynchronous control).

An important concept for rating an ATN system, as well as measuring performance of a position tracking algorithm, is minimum headway. Headway is the time it takes a trailing vehicle to be in the same position as the leading vehicle (Anderson, 1978) and has

units in seconds. Figure 4 shows a diagram explaining the concept of minimum headway. Let the front vehicle be f and the rear vehicle be e . Each vehicle has a length of L_v and is moving forward along x . If the speed of f is V_L and the speed of e is $V_L + \Delta V$, then H_0 describes the distance between e and f .

Figure 4. A diagram illustrating the concept of headway. Property of John Edward Anderson, PhD, P.E. Modified with permission.

$$T_{min} = \frac{L + H}{V_L} \quad (2)$$

Equation 2 provides a calculation for minimum headway time (Anderson, 1978) where L is the length of the vehicle, H is the minimum spacing between adjacent vehicles, and V_L is the velocity of the vehicle on a given segment of track. The headway is an important system design parameter as it describes the density of vehicles on the guideway. Headway is used as a performance criterion in control applications as the vehicle position overshoot or lag must not exceed H .

Station design is important to review as it can influence the acceleration profile. There are two different types of passenger boarding and disembarking stations referenced in ATN literature. The first type is an on-line (also referred to as an in-line station) in which are stations located on the main guideway (Muller, 2009). These types of stations have an inherent design flaw in that the passengers entering a stopped vehicle may interrupt the motion of other vehicles. The other type of station is the off-line station

where the vehicle leaves the main guideway to stop and allow passengers to enter and leave the vehicle. The station layout in Figure 5 (Kerr, 2013) shows different sections of an off-line station.

Figure 5. An example of an offline station which shows different sections of straight track. Property of Anthony Kerr. Reprinted with permission.

The track sections in Figure 5 are as follows: section a) is a turn-out section that leads to the passenger loading area. Section b) is a length of the track that allows for the vehicle to decelerate to the passenger loading area in section c). Section d) is a length of track allowing for the vehicle to accelerate up to speed to the merging on-ramp in Section e).

1.2.3 Vectus Review

Vectus Ltd is a private firm developing both research and commercial ATN systems. The firm has commercial operations in South Korea and Sweden (J. Gustafsson, 2009) and a private ATN test track in Sweden for system validation. The Vectus ATN system uses a captive bogie on a supported vehicle. As is the case for captive bogie vehicles, the stations are all offline stations. The vehicles are configured to have a dynamic headway and use asynchronous control. The unique feature of the Vectus ATN system vehicle is that the motive force comes from linear induction motor elements embedded in the guideway. The

linear motors embedded in the guideway have the advantage of giving constant propulsive force to the vehicle as well as allowing for variation in the amount of propulsive force by varying the distance between the linear inductor elements (Gustafsson, 2009). An image of the test track is shown in Figure 6. The test track used for the experiments is 400m long and has an elevation gradient of 1:50 over the entire length of the track.

Figure 6. The Vectus Test Track. Copyright IEEE 2009. Reprinted with permission.

The control system for the Vectus ATN vehicle must maintain the performance requirements dictated by Table 1. The performance of the ATN system and the vehicle is sourced from the whitepaper “Vectus PRT Concept and Test Track Experience” (J. Gustafsson, 2009) and the whitepaper “Automated Transit Networks (ATN): A Review of the State of the Industry and Prospects for the Future” (Furman et al., 2014). In addition to these stated performance values, there are two documented tests of the vehicle. The first is a test where all of the linear induction elements in the track were operational at a constant voltage and frequency. However, the force on the vehicle is not constant, as the vehicle is subjected to gravity as it moves around the test track and the force from the linear induction elements varies based on the distance between the elements. Figure 7

Table 1. Table of VECTUS Performance Data

Performance Parameter	Value
Minimum Headway (s)	3
Maximum Velocity ($\frac{m}{s}$)	12.5
Maximum Acceleration ($\frac{m}{s^2}$)	2
Maximum Deceleration ($\frac{m}{s^2}$)	5
Maximum Jerk ($\frac{m}{s^3}$)	2.5

shows the velocity of the vehicle as a function of time for this experiment. The blue line represents the velocity of the vehicle in $\frac{m}{s}$. All of the linear induction elements in the track are operational at a constant voltage and frequency.

Figure 7. Vectus PRT Constant Voltage/Frequency Test. Copyright IEEE 2009. Reprinted with permission.

Additionally, another test was performed on the control of the vehicle on the test track to determine the required voltage and frequency inputs to the linear induction elements.

Figure 8 shows the aforementioned test. The red line represents the required voltage by the motor controller and the green line represents the frequency. The blue line represents the velocity of the vehicle.

Figure 8. Vectus Dynamic Response Test. Copyright IEEE 2009. Reprinted with permission.

1.2.4 GM Open Guideway Vehicle Research

An investigation into the historical research of ATN systems is conducted to give context to the research. While the construction of the vehicles and control technology are not similar to the current research, the objectives are similar, namely to design a controller to track position on a guide way and maintain passenger comfort. The paper "Automated Control of Guideway Systems" (Smisek & Harder, 1979) published from research performed by the General Motors Corporation is an example of historical research that has a similar objective to the research presented in this thesis. The ATN system used for this research was a modified camper van on an open guideway. The objectives of this paper are to develop a controller for position tracking (henceforth referred to as the longitudinal controller) and to develop a controller to keep the vehicle on the center of the guideway (henceforth referred to as the lateral controller). Figure 9 shows the vehicle chassis used for the experiments.

Figure 9. The Transmode chassis used for the GM ATN research. Property of General Motors. Reprinted with permission.

Only a review of longitudinal controller is provided in this section. The longitudinal controller was designed by first determining a simplified model for the vehicle, which in the case of the paper is first order system. The proposed controller for the longitudinal system is divided into a lead component to provide pole zero cancellation of the resulting plant. A lag controller with a general gain (three total gains) is also placed in series to provide additional control over the position. The gains in the lag controller are set such that the natural frequency and damping coefficient are as high as possible to ensure rider comfort. The designed controller is then used to control the test vehicle on the guideway. From the experiments with the vehicle, it was determined that minimizing position error is inversely related to passenger comfort. Alternatively, values for the three gains that optimize passenger comfort cause variations in the position error. The ATN research from General Motors, although dated, provides excellent context to the objective of this thesis. This thesis investigates if using a state-controller algorithm allows for a optimal solution to position tracking and passenger comfort for longitudinal control (as well as centripetal acceleration).

1.2.5 Optimal Sample-Data Control of PRT Vehicles

The research paper titled "Optimal State Control of PRT Vehicles" (Kornhauser et al., 1974) demonstrates how to control a highly idealized ATN vehicle using state control techniques. The state control is formulated in discrete time and subjected to random noise to better provide for more realistic simulations. This paper treats the ATN as a point moving on a straight section of guideway. The equation of motion of the ATN are not derived, but given as

$$M\dot{v} = -F_D + F - F_M - Mg \sin \theta \quad (3)$$

The ATN system in Equation 3 is idealized as a particle of mass M that moves along a straight guideway at velocity v . The forces on the vehicle are the propulsive force F , the rolling resistance F_M , the weight from the vehicle due to a sloped guideway and a drag force F_D . The definition of F_D in Equation 3 is

$$F_D = C_D(v + v_W)^2 \quad (4)$$

The drag force is a function of the drag coefficient C_D and sum of v and the wind velocity v_W . The unique development used in order to provide jerk control is to model F as a first order system which has the form of

$$\dot{F} = \frac{-1}{\tau}F + Ki \quad (5)$$

Where τ is a time constant, i is the control input to the motor, and K is the gain to the input. Given expressions for the motion of the ATN vehicle and the definitions of F_D and F , Equation 3 is reformulated using the error, e , between the ATN vehicle position and the

desired position, x_c . The expression for e is

$$x = e + x_c \quad (6)$$

Substituting from Equation 6 into Equation 3 produces

$$M\ddot{e} = -C_D(\dot{e} + \dot{x}_c + v_w)^2 + F - Mg \sin \theta - M\ddot{x}_c - F_M \quad (7)$$

The next step is to create a non-dimensional form of the equations of motion. This is performed by first choosing a non-dimensional form for each of the terms in the equations of motion.

$$y = \frac{e}{H} \quad (8)$$

The position error e is divided by minimum headway spacing between each ATN vehicle.

$$\sigma = \frac{t}{T} \quad (9)$$

The elapsed time, t , is divided by the minimum headway time T (Refer to Equation 2).

$$T = \frac{H}{v_N} \quad (10)$$

In turn, the minimum headway time T is defined as the minimum headway spacing divided by the nominal cruising velocity, v_N .

$$w = \frac{v_W}{v_N} \quad (11)$$

The non-dimensional wind velocity is v_W divided by v_N .

$$f = \frac{TF}{Mv_N} \quad (12)$$

The non-dimensional force is a function of T , the force on the ATN vehicle F , M , and v_N .

$$v^* = \frac{T^2 Ki}{Mv_n} \quad (13)$$

The non-dimensional input is defined as a function of the square of T , the motor input signal i and its gain K (which has units of $kg \frac{m}{s^3}$), M , and v_N .

$$\frac{d}{dt} = T \frac{d}{d\sigma} \quad (14)$$

Finally, a non-dimensional form of the time derivative is needed, which is found from the derivative of Equation 9. Using the non-dimension forms in Equations 8 through 14 with Equation 7 and then solving for y yields

$$\ddot{y} = \frac{-C_D}{M}(\dot{y} + \dot{y}_c + w)^2 + f - \frac{Tg \sin \theta}{v_N} - \ddot{y}_c - \frac{F_M T}{Mv_N} \quad (15)$$

Using the non-dimension forms in Equations 8 through 14 with Equation 5 yields

$$\dot{f} = \frac{-T}{\tau} f + v^* \quad (16)$$

Linearization of Equation 15 is performed by finding the derivative of Equation 15 with respect to $\dot{y}$ and evaluating at v_N . This operation yields

$$\ddot{y} = \frac{-2C_D H}{M}(w+1)\dot{y} + f - d$$

$$d = \frac{Tg \sin \theta}{v_n} + \frac{F_M T}{v_N} + \frac{C_D H}{M}(w+1)^2 \quad (17)$$

The continuous state space model is formed from Equation 17 as

$$\mathbf{x}(t) = \begin{pmatrix} y \\ \dot{y} \\ \ddot{y} \\ \dot{f} \end{pmatrix} \quad A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{-2C_D H}{M} & 1 \\ 0 & 0 & 0 & \frac{-T}{\tau} \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad u = \dot{v} \quad (18)$$

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + Bu$$

The state space model is then converted to discrete time

$$\mathbf{x}(k+1) = \Phi \mathbf{x}(k) + Du(k) \quad (19)$$

The discrete plant matrix, Φ , is defined as

$$\Phi = e^{AT} \quad (20)$$

Likewise, the control matrix is defined as

$$D = \int_0^T e^{AT} B dt \quad (21)$$

The resulting discrete time state space model is used with the LQR algorithm which has the form

$$J = \lim_{N \rightarrow \infty} \int_0^{NT} [x'(\tau)Qx(\tau) + u'(\tau)Ru(\tau)] d\tau \quad (22)$$

The LQR algorithm uses the state vector x and control input u from Equation 19. The weights for the error and control energy are Q and R respectively.

The resulting state model was experimented with in simulations for varying acceleration profiles and events that can occur during normal operation. The purpose of the simulations is to determine the effects of varying the sample time, testing the effects of random wind gusts, and the effect of noise on the performance of the controller.

Additionally, three different vehicle acceleration profiles were tested. The first is the vehicle running at nominal mainline velocity (an acceleration profile of zero), the second is an acceleration profile for an slot-advance event, and the third is an acceleration profile for a vehicle leaving an offline station.

The vehicle parameters and performance parameters for the simulation are in Table 2. This table gives the constant values from the derivation of the state space model. Table 2 includes some of the industry specific terminology discussed in the previous section of this thesis especially with regards to the jerk of the system.

Table 2. Performance Parameters of Optimally Controlled PRT Vehicle

System Parameter	Value
Nominal Mainline Velocity	$v_N = 50 \frac{ft}{s}$
Nominal Headway Time	$T = 1.0 s$
Vehicle and Passenger Weight	3200 lbs
Vehicle Length	10 ft
Maximum Acceleration in Mainline Operation	$4 \frac{ft}{s^2}$
Maximum Acceleration for Merging and Maneuvering	$6 \frac{ft}{s^2}$
Maximum Emergency Deceleration	$25 \frac{ft}{s^2}$
Maximum Jerk in Mainline Operation	$4 \frac{ft}{s^3}$
Maximum Headway Error	10 ft
Propulsion System Time Constant	0.1
Drag Coefficient C_D	0.025

Figure 10 shows the acceleration profile of an ATN vehicle performing a slot-advance event. The acceleration of the ATN vehicle ramps up from zero to a maximum value, holds this acceleration, and then ramps down to a minimum value until the ATN vehicle returns to its nominal velocity. This acceleration profile is used as a setpoint for state space model in Equation 19.

Figure 10. The acceleration profile for an ATN vehicle advancing its location relative to other cars, ©1974 by the Regents of the University of Minnesota. Reprinted with permission of the University of Minnesota's Center for Urban and Regional Affairs (CURA).

Figure 11 shows a simulation that tests an ATN vehicle moving at constant velocity

subjected to a step change in wind velocity. The maximum jerk, acceleration, position error, and velocity error are related to the sampling time of the controller. This figure demonstrates the performance of the controller when the ATN vehicle is traveling at a constant velocity and subjected to disturbances.

Figure 3

Figure 11. Performance criteria of an ATN vehicle traveling at constant velocity, ©1974 by the Regents of the University of Minnesota. Reprinted with permission of the University of Minnesota's Center for Urban and Regional Affairs (CURA).

Figure 12 shows the acceleration response of an ATN vehicle given a reference input of the slot advance acceleration profile (see Figure 10). The resulting acceleration for a sampling time of $T = 0.2s$ and $T = 0.05s$ is shown. This shows the ability of the controller to track this particular acceleration profile.

Figure 12. Response to slot advance acceleration profile, ©1974 by the Regents of the University of Minnesota. Reprinted with permission of the University of Minnesota's Center for Urban and Regional Affairs (CURA).

The graph in Figure 13 shows the performance criteria as functions of sampling time for the slot advance simulation. As the sampling time of the controller is modified, the change in the maximum jerk, maximum acceleration, maximum position error, and maximum velocity error can be seen.

Figure 13. Performance criteria as functions of sampling time for the slot advance acceleration profile, ©1974 by the Regents of the University of Minnesota. Reprinted with permission of the University of Minnesota's Center for Urban and Regional Affairs (CURA).

This paper is a useful reference as it gives an example of state space control of an ATN vehicle. Concepts taken from this paper are the use of the vehicle acceleration and jerk as performance criteria as well as making the vehicle setpoint a function of time.

1.2.6 Optimal Cruise Control of Heavy-Haul Trains Equipped with Electronically Controlled Pneumatic Brake Systems

In the operation of conventional freight rail, the three main factors in the cost of operation are the energy consumption, the transit time, and maintenance. Optimized control of a train can be used to design a controller for these competing requirements. The controller proposed in “Optimal Cruise Control of Heavy-Haul Trains Equipped with Electronically Controlled Pneumatic Brake Systems” (Chou & Xia, 2007) models the train as a series of n unique masses connected by springs and dampers. The position of each car is x_i and the mass of each car is m_i . The forces that couple the individual masses are the spring and damping forces between adjacent cars. The spring constants between each adjacent car are k_i and k_{i-1} . Likewise, the damping constants between each adjacent car are d_i and d_{i-1} . Note for the locomotive and the last car, there is only one set of spring and damping forces, as these cars only have one adjacent car. Each individual car is also subject to rolling resistance which is modeled as a constant force ($m_i c_0$) summed with a linear resistive force ($m_i c_v \dot{x}_i$). Each car is assumed to have the same rolling resistance constants. The locomotive also is subjected to air resistance which is based on the mass of the entire train multiplied by an air resistance coefficient and the velocity of the locomotive. Additionally, each individual car is subjected to a force from gravity due to traveling on a sloped track of θ and a centrifugal force of $0.004 D_i m_i$. With these declarations in hand, the motion equation for the first car, the locomotive, in the train

($i = 1$) is

$$\begin{aligned}
m_1\ddot{x}_1 = & u_1 - k_1(x_1 - x_2) - d_1(\dot{x}_1 - \dot{x}_2) \\
& - (c_0 + c_v\dot{x}_1)m_1 - c_a\dot{x}_1^2 \left(\sum_{i=1}^n m_i \right) - 9.98\sin\theta_1 m_1 - 0.004D_1m_1
\end{aligned} \tag{23}$$

The motion equation for the last car ($i = n$) is

$$\begin{aligned}
m_n\ddot{x}_n = & u_n - k_{n-1}(x_n - x_{n-1}) - d_{n-1}(\dot{x}_n - \dot{x}_{n-1}) \\
& - (c_0 + c_v\dot{x}_n)m_n - 9.98\sin\theta_n m_n - 0.004D_n m_n
\end{aligned} \tag{24}$$

The system of equations for all other cars are

$$\begin{aligned}
m_i\ddot{x}_i = & u_i - k_i(x_i - x_{i+1}) - k_{i-1}(x_i - x_{i-1}) - d_i(\dot{x}_i - \dot{x}_{i+1}) - d_{i-1}(\dot{x}_i - \dot{x}_{i-1}) \\
& - (c_0 + c_v\dot{x}_i)m_i - 9.98\sin\theta_i m_i - 0.004D_i m_i
\end{aligned} \tag{25}$$

The input force of u_1 in Equation 23 can be positive or negative. The input forces u_i and u_n in Equations 25 and 24 can only be negative as individual cars are not powered.

A state space model formed from Equations 23, 25 and 24 allows for each individual car to have its own control signal. In practice, this is not realizable as the signal bandwidth does not support individual control of up to 200 individual train cars over the speeds at which the train travels. The current industry standard is to have one control signal for the locomotive group and one for all of the other cars. This is potentially inefficient as it is possible on certain grades that in the case of decelerating the train system, the brakes would be applied to a car going uphill. The paper introduces the concept of “fencing”, which is to have several control inputs based on the expected forces on the train. The number of control groups is calculated every sampling period based on the expected forces from the variations in slope and curvature of the track. If the change in

slope or curvature exceeds a threshold, the number of control groups changes. Figure 14 shows an example of a train controlled without fencing. In this example, there are only two control inputs, one for the locomotive and one for the entire series of cars. This control method inefficient as car 6 is braking even though the car is traveling uphill.

Figure 14. The control inputs for the entire train grouped together. Reprinted from Control Engineering Practice, Vol. 15, Chou, M. & Xia, X., Optimal cruise control of heavy-haul trains equipped with electronically controlled pneumatic brake systems, pages 511-519, Copyright 2007, with permission from Elsevier.

Figure 15 shows an example of fencing with the control inputs to the individual cars based on the expected forces on each car. In this example cars 3 through 5 are on a downhill slope and have a braking input while car 6 is on a uphill slope and is allowed to coast.

Figure 15. How the control inputs are grouped when fencing is applied. Reprinted from Control Engineering Practice, Vol. 15, Chou, M. & Xia, X., Optimal cruise control of heavy-haul trains equipped with electronically controlled pneumatic brake systems, pages 511-519, Copyright 2007, with permission from Elsevier.

Two controllers are tested in this paper. The first is an open loop controller that assumes a constant velocity of v_d . The open loop controller is not expected to perform well in simulations, but is included to provide comparisons. A second closed-loop controller is also tested with the poles selected by use of the LQR algorithm. The LQR are

decided based on the in-train force, the fuel consumption, and the traveling time to the control input.

The plots in Figure 16 show the response of the heavy haul train controlled with the open-loop controller. The 1st plot shows velocity tracking for a given reference velocity. The 2nd plot shows the forces within the train over the same time period. The 3rd plot shows the control signal generated by the controller. The 4th plot shows the variation in elevation for the simulation. These plots are used as a benchmark for the closed loop tests.

Figure 16. The simulation results of a heavy haul train controlled with the open-loop controller. Reprinted from Control Engineering Practice, Vol. 15, Chou, M. & Xia, X., Optimal cruise control of heavy-haul trains equipped with electronically controlled pneumatic brake systems, pages 511-519, Copyright 2007, with permission from Elsevier.

Figure 17 shows the heavy-haul train controlled with the closed loop controller. This control was tuned using weights that equally favored velocity tracking and minimization of control force. The 1st plot shows velocity tracking for a given reference velocity. The 2nd plot shows the forces within the train over the same time period. The 3rd plot shows

the control signal generated by the controller.

Figure 17. The simulation results of a heavy haul train controlled with the closed-loop controller. Reprinted from Control Engineering Practice, Vol. 15, Chou, M. & Xia, X., Optimal cruise control of heavy-haul trains equipped with electronically controlled pneumatic brake systems, pages 511-519, Copyright 2007, with permission from Elsevier.

It can be seen that the train controlled with the closed loop controller has superior tracking to the velocity reference than the open loop controller. The control input and simulated forces within the train are similar between the open loop controller and closed loop controller.

This paper describes a different system than the ATN vehicle analyzed in this thesis. The train system in this paper consists of a simpler vehicle model, incorporates a series of interconnected train cars as opposed to an independent vehicle, and is much heavier than the ATN vehicle. However, this paper still helps in guiding the design of the ATN controller. The first guideline is that the vehicle model should be simple to avoid requiring a specific vehicle type but still retains some of the internal dynamics. The paper also gives guidance on how to use the LQR method for multiple competing design criteria. Lastly, the simulation results give some insight into how to present the data, such as displaying the different design parameters as a function of distance.

1.2.7 Robust Sampled-Data Cruise Control Scheduling of High Speed Train

Another paper that discusses control of conventional trains is “Robust Sampled-Data Cruise Control Scheduling of High Speed Train” (Li, Yang, Li, & Gao, 2014). In this paper, the train system is treated as a series of n masses connected by a spring. In addition, there are also force terms for rolling resistance, air resistance, and forces due to wind tangent to the path of the train. Figure 18 show the idealization of the passenger train, where the mass of each car is m_i , the velocity of each car is v_i , the position between each car is x_i .

Figure 18. This diagram shows how the high speed passenger train is idealized. Reprinted from Transportation Research Part C: Emerging Technologies, Vol. 46, Li, Shukai, Yang, Lixing, Li, Kepin & Gao, Ziyu, Robust sampled-data cruise control scheduling of high speed train, pages 274-283, Copyright 2014, with permission from Elsevier.

The change in position of car i in the train is

$$\dot{x}_i = v_i - v_{i+1} \quad (26)$$

There is a spring force between each car and the adjacent cars with spring constant k . Additionally, all individual cars are subjected to a rolling resistance force which is the sum of a constant force ($m_i c_0$) and a linear damping force ($m_i c_1 v_i$). The lead car is also subjected to two additional forces. The first is a force from air resistance defined by the product of the mass of the entire train, the square of the lead car velocity (v_1^2) and an air resistance constant (c_2). The second is a disturbance force from wind gusts tangent to the track designated by w . Finally, each car has an input force expressed by u_i . The system of

equations for the lead car (the locomotive) is

$$\begin{aligned} m_1 \dot{v}_1(t) = & u_1 - kx_1 - (c_0 + c_1 v_1) m_1 \\ & - c_2 \left(\sum_{i=1}^n m_i \right) v_1^2 + w \end{aligned} \quad (27)$$

The system of equations for the end car is

$$\begin{aligned} m_n \dot{v}_n(t) = & u_n + kx_{n-1} \\ & - (c_0 + c_1 v_n) m_n \end{aligned} \quad (28)$$

The system of equations for all other cars is

$$\begin{aligned} m_i \dot{v}_i(t) = & u_i + kx_{i-1} - kx_i \\ & - (c_0 + c_1 v_i) m_i \end{aligned} \quad (29)$$

A continuous state space model is formed from Equations 26, 27, 29, and 28. The proposed form of the controller is

$$\mathbf{u}(t) = K_1 \dot{\mathbf{x}}(t) + K_2 \dot{\mathbf{v}}(t) \quad (30)$$

The continuous controller in Equation 30 is transformed into discrete form using a zero order hold. The discrete controller is

$$\mathbf{u}(t) = \mathbf{u}(t_k) = K_1 \dot{\mathbf{x}}(t_k) + K_2 \dot{\mathbf{v}}(t_k) \quad t_k - 1 \leq t < t_k \quad (31)$$

Lyapunov stability theory is used to choose the gains of K_1 and K_2 .

Figure 19 shows the velocity setpoint for the robust controller. There is an

acceleration period to the first cruising velocity followed by the short deceleration period to the second cruising velocity.

Figure 19. The velocity setpoint for the lead car in a high speed passenger train. Reprinted from Transportation Research Part C: Emerging Technologies, Vol. 46, Li, Shukai, Yang, Lixing, Li, Kepin & Gao, Ziyu, Robust sampled-data cruise control scheduling of high speed train, pages 274-283, Copyright 2014, with permission from Elsevier.

Figure 20 shows the relative displacement between trains for a system simulated with the robust controller designed in this paper. The relative displacements for each individual train are superimposed on each other. When the train is at cruising velocity, all of the relative displacements between the individual cars go to zero.

Figure 20. The relative displacement between the train cars in a high speed train. Reprinted from Transportation Research Part C: Emerging Technologies, Vol. 46, Li, Shukai, Yang, Lixing, Li, Kepin & Gao, Ziyu, Robust sampled-data cruise control scheduling of high speed train, pages 274-283, Copyright 2014, with permission from Elsevier.

Additionally, Figure 21 shows the velocity curves for each car when compensated by the sampled data controller. The top plot (a) shows the evolution of the velocity curves from $t = 280$ s to $t = 400$ s and the bottom plot shows (b) the evolution of the velocity curves from $t = 680$ s to $t = 800$ s. After the acceleration and deceleration period, there is short period that velocity of each car diverges.

Figure 21. The velocity of each individual train car for a system simulated with the robust controller designed in this paper. Reprinted from Transportation Research Part C: Emerging Technologies, Vol. 46, Li, Shukai, Yang, Lixing, Li, Kepin & Gao, Ziyu, Robust sampled-data cruise control scheduling of high speed train, pages 274-283, Copyright 2014, with permission from Elsevier.

This paper discusses a method of designing a sampled data controller for a high speed passenger train. The research in this paper is useful as it shows a way on modeling forces on a vehicle fixed to the track and provides an example for another method of control.

1.2.8 Suspension Control for Conventional Trains

Research into actively controlled railway suspensions using modern control strategies was also investigated to aid in modeling the tilting of the ATN vehicle discussed in this paper. The paper titled “Integrated Tilt with Active Lateral Secondary Suspension Control for High Speed Railway Vehicles” (Zhou, Zolotas, & Goodall, 2011) discusses the combination of a tilting and lateral actuator on a high speed railway vehicle to compensate for the centrifugal forces felt by the passenger when the train moves around a curve.

Figure 22 shows a diagram of the tilting railcar. The tilting train car has a stiff primary suspension between the wheelset and the vehicle bogie coupled with a tilting mechanism on which the vehicle body sits. The vehicle body is attached to the tilting mechanism via a set of air springs. The tilting mechanism is designed such that the rail car can tilt up to 10° with a tilting center above the floor of the railcar.

Figure 22. An diagram showing the concept of the tilting train. Reprinted from *Mechatronics*, Vol. 21, Zhou, Ronghui, Zolotas, Argyrios & Goodall, Roger, Integrated tilt with active lateral secondary suspension control for high speed railway vehicles, pages 1108-1122, Copyright 2011, with permission from Elsevier.

The rail vehicles are assumed to be traveling on a canted (curved inward) track. The normal component of the centripetal acceleration (a_c) felt by the passenger is a key parameter in evaluating passenger comfort. The centripetal acceleration can be estimated in terms of the train car velocity (v), the radius of the curve (R) and gravity (g). The angle of the canted track is θ_0 and the angle tilting railcar is θ_v . The expression for a_c is

$$a_c = \frac{v^2}{R} \cos(\theta_0 - |\theta_v|) - g \sin(\theta_0 - |\theta_v|) \quad (32)$$

However, it is possible to control the angle of θ_v . The normal component of the

centrifugal acceleration for a train with controlled tilting can be estimated by

$$a_c = \frac{v^2}{R} \cos(\theta_0 - |\theta_v|) - g \sin(\theta_0 + |\theta_v|) \quad (33)$$

The angle θ_v in Equation 33 can be controlled to have the component from gravity cancel the component from the centripetal acceleration of the railcar to minimize the acceleration felt by the passengers.

The simplest control method is referred to as “nulling” and measures the lateral acceleration and uses Equation 33 to determine to the setpoint angle of the tilting actuator. However, this control method was found unsuitable for commercial usage as the coupling between the roll dynamics and lateral dynamics of the rail car caused motion sickness in the passengers. An alternative control method of “precedence control” which determines a tilt command from the bogie acceleration. This control method avoids negative effects of the coupled lateral dynamics. This controller is too complicated for the research this thesis as the proposed model does not incorporate lateral dynamics of the bogie or the vehicle. However, while the paper dismissed the use of “nulling” to control the tilting actuator as a suitable strategy, the concept is ideal for the control of the ATN vehicle in the thesis. The vehicle being analyzed in this thesis does not have sufficient complexity in the idealization to have lateral dynamics, so the “nulling” control method is a useful initial approach.

CHAPTER 2

OBJECTIVE

The primary objective of this research is to deliver a SIMULINK model for a controlled idealized ATN vehicle with an active suspension element. While there is existing research being performed on multiple ATN vehicles in the Spartan Superway team, there has been a lack of a dedicated research into a control algorithm for the vehicle. In addition, most of the literature on controller design investigates supported ATN vehicles as opposed to the suspended design being researched by the Spartan Superway team. The controller in the SIMULINK model is a point-following controller and the controller's performance is judged by the maximum acceleration and jerk experienced by the vehicle, as well as the error between the vehicle and its desired position. The desired end-users for this simulation environment and controller are future engineering design teams on the Spartan Superway project. It is intended that the design teams are able to integrate the controller in a SIMULINK application interfaced with a microprocessor (e.g. Arduino) to control a test vehicle as well as use the SIMULINK model to test their own design. The research is split into three different overlapping sections: the formulation of the equations of motion for the ATN vehicle, the design of the SIMULINK model and controller, and the analysis of the results of simulations to validate the research objectives.

The formulation of the equations of motion for the ATN vehicle is performed first as these equations govern the controller design and the validation simulations. Because of the complexity of real world systems coupled with the fact that there is no final design for the suspended ATN vehicle, a simple idealization is utilized. For the first idealization (henceforth referred to as the Planar System), the ATN vehicle is simplified to two point masses, one representing the bogie of the ATN vehicle and one representing the center of mass of the vehicle body. Between the point mass representing the bogie and the vehicle

body, there is an arm that is allowed to rotate perpendicular to the guideway. In the Planar System, it is assumed that the bogie point mass is constrained to move on a circular arc. The equations of motion for the Planar System are derived by hand and by MotionGenesis. The second idealization (henceforth referred to as the General System) is the same as the Planar System with the exception that the bogie point mass is constrained to move on a numerically defined arbitrary curve in space. The equations of motion of the General System are based on numerically interpolated unit vectors calculated from the guideway geometry. Both systems are subjected to a propulsive force from the bogie, a stabilizing force on the vehicle body, friction forces and air resistance, as well as a disturbance force from wind gusts. Approximate values for all of the parameters are obtained by a combination of communication with Bengt Gustafsson, the director of Beamways AB, referencing existing designs, and using information from the literature review.

The resulting equations of motion are used to design a state-space controller using feedback linearization and the pole placement method to determine both the propulsive and stabilizing force mentioned above. The goal of the controller is to track the position of the ATN vehicle on the guideway with minimal error at the highest possible velocity while simultaneously ensuring that the acceleration and jerk in motion tangent and normal to the guideway remain below the targeted levels. There is a controller for both the Planar System and the General System.

In order to validate both the equations of motion and the resulting controller, there are multiple simulations on a virtual test track with a 100 meter radius. The first set of simulations compares the hand derived and computer derived equations of motion for the Planar System for a vehicle with constant velocity subjected to a disturbance force. The next set of simulations validates the numerical method of determining the numerical unit vectors of the General System. Next, the equations of motions for the general system are

compared to the hand derived equations under the same initial conditions and subjected to the same forces as the Planar System comparison study.

A set of simulations is performed to analyze the state space controller derived from the dynamics of the Planar System. This set of simulations uses three different sets of closed loop poles for the bogie and one set of closed loop poles for the angle position. Then a controller defined from the dynamics of the General System is used to control the output of a plant defined by the dynamics of the Planar System. This set of simulations shows the viability of using numerically defined guideway geometry to control the output of a suspended ATN vehicle.

The last set of simulations demonstrates the dynamics of an ATN vehicle on a guideway shaped like an Euler spiral. An Euler spiral is a curve in which the curvature is linearly related to the displacement along the curve. This geometry is used in transit engineering and demonstrates the ability of the General System to simulate complex guideway geometry.

CHAPTER 3

METHODOLOGY

3.1 Dynamical System Idealization

In order to construct the equations of motion for the ATN vehicle, a suitable idealization must be made and numerical values must be assigned to the associated parameters. The process for assigning these values is a combination of communications with consultants from Beamway, referencing selected resources from the literature review, and using engineering data from other sectors of the Spartan Superway team. The relevant parameters for the rigid idealization are the radius of the curve, the mass of the bogie point mass, the mass of the vehicle body point mass, the length of the pendulum arm between the bogie point mass and vehicle body point mass, the coefficient of viscous damping on the rotating ATN vehicle body, the coefficient for rolling resistance, and the coefficient for air resistance. The mass values have been discussed in a cited communication with a consultant from Beamways AB, the length of the pendulum arm is found by using the maximum angle deflection in the "The Tradeoff Between Supported vs. Hanging Vehicles" (Anderson, 2013) and from the geometry from a CAD model of a proposed cabin created by visiting Swedish students to the Spartan Superway design center during the summer of 2015. The rolling resistance is based on the weight and wheel diameter from the Beamways AB design. The air resistance is based on the drag coefficient of a half sphere and the profile area from a CAD model designed by Swedish students interning for a summer with the Spartan Superway team. The exact equations and process for determining these values are listed in the theoretical work section and the exact values used in the simulations are detailed in the Results section.

3.2 Forming the Equations of Motion and Validation Simulations

For the Planar System, the equations of motion are formed by hand and by using the computer algebra software MotionGenesis (Mitiguy, 2015). This software is also used for other complex symbolic calculations. Once the equations of motion have been determined, MATLAB code for the equations is used to create SIMULINK blocks. These SIMULINK blocks are then used for validation simulations in order to determine that the simulated dynamical system behaves as expected. The General System follows the same steps; however, the equations of motion are derived only with MotionGenesis.

3.3 Numerically Defined Unit Vectors

A dataset containing the guideway geometry that has path displacement as an independent variable and the X, Y, and Z positions in an inertial frame as the dependent variables is generated. This dataset is used in conjunction with the MATLAB Curve Fitting toolbox to fit 5th order splines for each function of X, Y, and Z to the path displacement. The resulting splines are used to generate a dataset of the geometric information of the General System as a function of path displacement. This dataset of geometric information is used in interpolation blocks in the SIMULINK models.

3.4 Designing the Controller

Once the equations of motion have been validated, a controller is designed for both the Planar and General Systems. The controller is formed by creating a state space model around the desired setpoints and then using the pole placement algorithm in the MATLAB Control System toolbox to find the gains of the system. The controller is implemented in SIMULINK along with the SIMULINK model of the dynamical system for validation purposes.

CHAPTER 4

ATN VEHICLE DYNAMICS

As this research involves designing a control algorithm for the suspended ATN vehicle, the research must begin by a formulation of the equations of motion. Two ATN systems are studied, the first system being an idealized ATN vehicle constrained to move on an arc (the “Planar System”) in which the equations of motion are manually derived using Kane’s method and derived again using MotionGenesis. Both of these sections are explicit in detail in order to provide a tutorial. The second system is the same ATN suspended vehicle on an arbitrary curve in space that is defined through numerical values (the “General System”). The equations of motion for the General System are derived only with MotionGenesis, but the mathematical theory behind the concepts of the model is discussed.

4.1 Planar System - Manual Derivation

While the final goal of this research is a controller that is based on the dynamics of the General System, it is advantageous to first solve for the equations of motion for the Planar System to provide a hand calculated comparison to the MotionGenesis derived equations. The kinematics and the expressions for the forces of the Planar System are formulated and the equations of motion are then derived using Kane’s Method in terms of two generalized speeds.

In Figure 23, let $\hat{N}_x$, $\hat{N}_y$, and $\hat{N}_z$ form a right-handed set of mutually perpendicular unit vectors fixed in an inertial frame N . The guideway geometry is assumed to be an arc of radius r_0 with center N_0 that is fixed in the plane formed by $\hat{N}_x$ and $\hat{N}_y$. Let B be a particle of mass m_B which is allowed to freely move along the arc for

a distance s . Next, let $\hat{S}_x$, $\hat{S}_y$, and $\hat{S}_z$ form a right-handed set of mutually perpendicular unit vectors fixed in a reference frame S . $\hat{S}_x$, $\hat{S}_y$, and $\hat{S}_z$ have the following properties:

- (1) $\hat{S}_x$ is tangent to the guideway arc
- (2) $\hat{S}_y$ is normal to the guideway arc
- (3) $\hat{S}_z = \hat{S}_x \times \hat{S}_y$

Figure 23. Planar System - Kinematics Overview

Figure 24 shows the orientation of $\hat{S}_x$, $\hat{S}_y$, $\hat{S}_z$ relative to $\hat{N}_x$, $\hat{N}_y$, $\hat{N}_z$ in terms of s and r_0 .

Figure 24. Relationship between $\hat{S}_x, \hat{S}_y, \hat{S}_z$ and $\hat{N}_x, \hat{N}_y, \hat{N}_z$

Table 3 gives the mathematical relationship between $\hat{S}_x, \hat{S}_y, \hat{S}_z$ and $\hat{N}_x, \hat{N}_y, \hat{N}_z$.

Table 3. S Unit Vectors Relation - Planar System

	$\hat{N}_x$	$\hat{N}_y$	$\hat{N}_z$
$\hat{S}_x$	$\cos \frac{s}{r_0}$	$\sin \frac{s}{r_0}$	0
$\hat{S}_y$	$-\sin \frac{s}{r_0}$	$\cos \frac{s}{r_0}$	0
$\hat{S}_z$	0	0	1

As S has a simple angular velocity in N , Figure 24 and Table 3 permit one to write the angular velocity of S in N , $\omega^{S/N}$, by inspection. However, for tutorial purposes, a formal derivation of $\omega^{S/N}$ is performed using the definition (Kane & Levinson, 2005)

$$\omega^{S/N} = \hat{S}_x \left(\frac{{}^N d\hat{S}_y}{dt} \cdot \hat{S}_z \right) + \hat{S}_y \left(\frac{{}^N d\hat{S}_z}{dt} \cdot \hat{S}_x \right) + \hat{S}_z \left(\frac{{}^N d\hat{S}_x}{dt} \cdot \hat{S}_y \right) \quad (34)$$

Substituting the expressions for $\hat{S}_x, \hat{S}_y, \hat{S}_z$ in terms of $\hat{N}_x, \hat{N}_y, \hat{N}_z$ from Table 3 into

Equation 34 and evaluating the derivatives produces

$$\boldsymbol{\omega}^{S/N} = \hat{\mathbf{S}}_z \left(\frac{\dot{s}}{r_0} \left(-\sin \frac{s}{r_0} \hat{\mathbf{N}}_x + \cos \frac{s}{r_0} \hat{\mathbf{N}}_y \right) \cdot \left(-\sin \frac{s}{r_0} \hat{\mathbf{N}}_x + \cos \frac{s}{r_0} \hat{\mathbf{N}}_y \right) \right) \quad (35)$$

The dot product in Equation 35 is evaluated to give $\boldsymbol{\omega}^{S/N}$ as

$$\boldsymbol{\omega}^{S/N} = \frac{\dot{s}}{r_0} \hat{\mathbf{S}}_z \quad (36)$$

Moving on to the kinematics of B , the position vector from $r^B N_0$ to B , is found by inspection of Figure 23 as

$$\mathbf{r}_B = r_0 \hat{\mathbf{S}}_y \quad (37)$$

The velocity of B in N , $\mathbf{v}^{B/N}$, is given by:

$$\mathbf{v}^{B/N} = \frac{d}{dt} \mathbf{r}^B + \boldsymbol{\omega}^{S/N} \times \mathbf{r}^B \quad (38)$$

Substituting from Equations 36 and 37 into Equation 38 gives

$$\mathbf{v}^{B/N} = 0 + \frac{\dot{s}}{r_0} \hat{\mathbf{S}}_z \times r_0 \hat{\mathbf{S}}_y \quad (39)$$

Evaluating the cross product in Equation 39 yields

$$\mathbf{v}^{B/N} = \dot{s}(-\hat{\mathbf{S}}_x) \quad (40)$$

The acceleration of B in N , $\mathbf{a}^{B/N}$, is given by

$$\mathbf{a}^{B/N} = \frac{d}{dt} \mathbf{v}^{B/N} + \boldsymbol{\omega}^{B/N} \times \mathbf{v}^{B/N} \quad (41)$$

Substituting from Equations 36 and 40 into Equation 41 yields

$$\mathbf{a}^{B/N} = \ddot{s}(-\hat{\mathbf{S}}_x) + \frac{\dot{s}}{r_0}(-\hat{\mathbf{S}}_x) \times \frac{\dot{s}}{r_0}\hat{\mathbf{S}}_z \quad (42)$$

Evaluating the cross product in Equation 42 produces

$$\mathbf{a}^{B/N} = -\ddot{s}\hat{\mathbf{S}}_x - \frac{\dot{s}^2}{r_0}\hat{\mathbf{S}}_y \quad (43)$$

Now that an expression for $\mathbf{a}^{B/N}$ is in hand, analysis of the kinematics of the suspended vehicle can begin. Additional hardware components must be taken into account to represent a suspended ATN vehicle. Referring to Figure 23, let C be a particle of mass m_C that represents the vehicle cabin. Let C be constrained to B by means of massless rod BC of length l_c . The axis of rotation of BC is parallel to $\hat{\mathbf{S}}_x$ at B . BC makes an angle of q_B with $\hat{\mathbf{S}}_z$. Next, let $\hat{\mathbf{A}}_x$, $\hat{\mathbf{A}}_y$, and $\hat{\mathbf{A}}_z$ form a right-handed set of mutually perpendicular unit vectors fixed in reference frame A . $\hat{\mathbf{A}}_x$, $\hat{\mathbf{A}}_y$, and $\hat{\mathbf{A}}_z$ have the following properties:

- (1) $\hat{\mathbf{A}}_x$ is equal to $\hat{\mathbf{S}}_x$
- (2) $\hat{\mathbf{A}}_z$ points from C to B
- (3) $\hat{\mathbf{A}}_y = \hat{\mathbf{A}}_z \times \hat{\mathbf{A}}_x$

Figure 25 shows the orientation of $\hat{\mathbf{A}}_x$, $\hat{\mathbf{A}}_y$, $\hat{\mathbf{A}}_z$ relative to $\hat{\mathbf{S}}_x$, $\hat{\mathbf{S}}_y$, $\hat{\mathbf{S}}_z$.

Figure 25. Planar System - Frame A

Table 4 gives the mathematical relationship between $\hat{A}_x$, $\hat{A}_y$, $\hat{A}_z$ and $\hat{S}_x$, $\hat{S}_y$, $\hat{S}_z$ in terms of q_B .

Table 4. A Unit Vectors Relation - Planar System

	$\hat{S}_x$	$\hat{S}_y$	$\hat{S}_z$
$\hat{A}_x$	1	0	0
$\hat{A}_y$	0	$\cos q_B$	$\sin q_B$
$\hat{A}_z$	0	$-\sin q_B$	$\cos q_B$

As A has a simple angular velocity in S, refer to Figure 25 and Table 4 to obtain $\omega^{A/S}$

$$\omega^{A/S} = \dot{q}_B \hat{A}_x \quad (44)$$

To find the the angular velocity of A in N, the addition theorem for angular velocities

(Kane & Levinson, 2005) can be utilized which yields

$$\boldsymbol{\omega}^{A/N} = \boldsymbol{\omega}^{A/S} + \boldsymbol{\omega}^{S/N} \quad (45)$$

Substituting from Equations 44 and 36 into Equation 45 produces

$$\boldsymbol{\omega}^{A/N} = \dot{q}_B \hat{\mathbf{A}}_x + \frac{\dot{s}}{r_0} \hat{\mathbf{S}}_z \quad (46)$$

Refer to Figure 23 to obtain the position vector $\mathbf{r}'_C$ from B to C by inspection

$$\mathbf{r}'_C = -l_C \hat{\mathbf{A}}_z \quad (47)$$

The position vector from N_0 to C is found by adding Equations 47 and 37

$$\mathbf{r}_C = r_0 \hat{\mathbf{S}}_y - l_C \hat{\mathbf{A}}_z \quad (48)$$

The velocity of C in N , $\mathbf{v}^{C/N}$, is

$$\mathbf{v}^{C/N} = \frac{N d}{dt} \mathbf{r}_B - \frac{N d}{dt} l_C \hat{\mathbf{A}}_z \quad (49)$$

Substituting from Equation 37 into Equation 49 and evaluating the derivatives yields

$$\mathbf{v}^{C/N} = -\dot{s} \left(1 + \frac{l_c}{r_0} \sin q_B \right) \hat{\mathbf{S}}_x + \dot{q}_B l_c \hat{\mathbf{A}}_y \quad (50)$$

The acceleration of C in N , $\mathbf{a}^{C/N}$, is defined as

$$\mathbf{a}^{C/N} = \frac{N d}{dt} \mathbf{v}^{C/N} \quad (51)$$

Substituting from Equation 50 into Equation 51 and evaluating the derivative yields

$$\begin{aligned}
\mathbf{a}^{C/N} = & \hat{\mathbf{S}}_x \left[-\ddot{s} \left(1 + \frac{l_c}{r_0} \sin q_B \right) - 2\dot{s}\dot{q}_B \frac{l_c}{r_0} \cos q_B \right] \\
& + \hat{\mathbf{S}}_y \left[-\frac{\dot{s}^2}{r_0} \left(1 + \frac{l_c}{r_0} \sin q_B \right) \right] \\
& + \hat{\mathbf{A}}_y [\dot{q}_B l_c] \\
& + \hat{\mathbf{A}}_z [\dot{q}_B^2 l_c]
\end{aligned} \tag{52}$$

Now that expressions for $\mathbf{a}^{B/N}$ and $\mathbf{a}^{C/N}$ are in hand, the next step is to establish expressions for the external forces and torques on the system. Figure 26 is a free body diagram for B . F_P is the propulsion force on the bogie from the wheels on the guideway and acts in the $\hat{\mathbf{S}}_x$ direction. F_R is a resistive force that acts in the $-\hat{\mathbf{S}}_x$ direction and is defined as

$$F_R = c_0 + c_1 \dot{s} + c_2 \dot{s}^2 \tag{53}$$

Equation 53 is referred to as the Davis equation and is used in research performed in conventional train simulations (Li et al., 2014). The value of F_R is the sum of a constant force, a linear damping force (to model rolling resistance), and a damping force proportional to the square of the speed (to model air resistance). $\tilde{F}_W$ is the force on the ATN Vehicle from wind gusts and is considered a disturbance on the system. Because the wind gusts are possible in any direction, $\tilde{F}_W$ acts in the direction of the independent unit vector $\hat{\mathbf{W}}$. Also present is the force due to the weight of the bogie, $m_B g$ which acts in the $\hat{\mathbf{S}}_z$ direction. Additionally, the weight and motion of C is expressed by R_v which acts in the $-\hat{\mathbf{A}}_z$ direction. Lastly, there are the reaction forces R_y and N which act in the $\hat{\mathbf{S}}_y$ and

the $\hat{S}_z$ directions respectively. The sum of the external forces on B is

$$\Sigma \mathbf{F}^B = (F_P - F_R) \hat{\mathbf{S}}_x + (\tilde{F}_W) \hat{\mathbf{W}} + (N - m_B g) \hat{\mathbf{S}}_z - (R_v) \hat{\mathbf{A}}_z + (R_y) \hat{\mathbf{S}}_y \quad (54)$$

Figure 26. Planar System - Bogie Free Body Diagram

Figure 27 is the free body diagram of BC . T_A is the control torque from a motor about the revolute joint. There is also damping torque T_R about the revolute joint defined as

$$T_R = c_c \dot{q}_B \quad (55)$$

Lastly, f_C represents a non-linear friction torque about revolute joint which is explained in further depth in Chapter 5.4. The sum of the external torques on BC is given by

$$\Sigma\tau = (T_A - T_R - f_C)\hat{S}_x \quad (56)$$

$$\Sigma \mathbf{F}^C = (F_A - F_{R,2} - f_C) \hat{\mathbf{A}}_y + (-m_C g) \hat{\mathbf{S}}_z + (\tilde{F}_W) \hat{\mathbf{W}} + (R_v) \hat{\mathbf{A}}_z \quad (57)$$

Figure 28. Planar System - Vehicle Body Free Body Diagram

With the kinematics and the external forces of the system in hand, the equations of motion are found using Kane's Method. The first step is to select appropriate generalized speeds. As this system is relatively simple, it is sufficient to select the time derivatives of s and q_B respectively, that is,

$$u_1 = \dot{s} \quad u_2 = \dot{q}_B \quad (58)$$

From this point onward in the section the following substitutions are used for the particles

and generalized speeds for the purpose of compact notation

$$\begin{aligned}
Q_1 &= B & Q_2 &= C \\
F_1 &= S & F_2 &= A \\
u_1 &= \dot{s} & u_2 &= \dot{q}_B
\end{aligned} \tag{59}$$

Due to this system having only holonomic constraints, the partial velocities for a particle with respect to generalized speed u_r are given by

$$\hat{\mathbf{v}}_{u_r}^{Q_i/N} = \frac{\partial \mathbf{v}^{Q_i/N}}{\partial u_r} \tag{60}$$

Likewise, the partial angular velocities for a reference frame with respect to generalized speed u_r are given by

$$\hat{\boldsymbol{\omega}}_{u_r}^{F_i/N} = \frac{\partial \boldsymbol{\omega}^{F_i/N}}{\partial u_r} \tag{61}$$

Using the definition of the partial velocities in Equations 60 and 61 and substituting $\mathbf{v}^{B/N}$ from Equation 40 as well as $\boldsymbol{\omega}^{C/N}$ from Equation 50 allows for calculation of each partial velocity by evaluating the partial derivative.

Table 5. Planar System - Partial Velocities and Partial Angular Velocities

	u_1	u_2
Q_1	$-\hat{\mathbf{S}}_x$	0
Q_2	$-\left(1 + \frac{l_c}{r_0} \sin q_B\right) \hat{\mathbf{S}}_x$	$(l_c) \hat{\mathbf{A}}_y$
F_1	$\frac{1}{r_0} \hat{\mathbf{S}}_z$	0
F_2	$\frac{1}{r_0} \hat{\mathbf{S}}_z$	$\hat{\mathbf{A}}_x$

Table 5 shows all of the partial velocities and partial angular velocities for the system.

Each entry in Table 5 refers to the partial velocity of particle or reference frame on the left most column with respect to the generalized speed on the top most row. Given expressions

for all partial velocities and partial angular velocities in the system, the next step is to use this information to form the generalized active forces and generalized inertia forces. The generalized active force contribution $\mathcal{F}_{u_r}^{Q_i}$ for a particle Q_i w.r.t to a given generalized speed, u_r , is

$$\mathcal{F}_{u_r}^{Q_i} = \frac{\partial \mathbf{v}^{Q_i/N}}{\partial u_r} \cdot \Sigma \mathbf{F}^{Q_i} \quad (62)$$

Similarly, the generalized active force contribution $\mathcal{F}_{u_r}^{F_i}$ for a reference frame or rigid body F_i w.r.t to a generalized speed, u_r , is

$$\mathcal{F}_{u_r}^{F_i} = \frac{\partial \boldsymbol{\omega}^{F_i/N}}{\partial u_r} \cdot \Sigma \boldsymbol{\tau}^{F_i} \quad (63)$$

Evaluating Equations 62 and 63 using the expressions in Equation 59 and the entries in Table 5, one can find each generalized active force contribution for the system. Note, as there are no external torques that act on F_1 , there are no generalized active forces of F_1 . The generalized active force contribution for Q_1 with respect to u_1 is

$$\mathcal{F}_{u_1}^{Q_1} = -F_P + F_R - \tilde{F}_W (\hat{\mathbf{W}} \cdot \hat{\mathbf{S}}_x) \quad (64)$$

The generalized active force contribution for Q_1 with respect to u_2 is

$$\mathcal{F}_{u_2}^{Q_1} = 0 \quad (65)$$

The generalized active force contribution for Q_2 with respect to u_1 is

$$\mathcal{F}_{u_1}^{Q_2} = -\tilde{F}_W \left(1 + \frac{l_c}{r_0} \sin q_B \right) (\hat{\mathbf{W}} \cdot \hat{\mathbf{S}}_x) \quad (66)$$

The generalized active force contribution for Q_2 with respect to u_2 is

$$\mathcal{F}_{u_2}^{Q_2} = (\tilde{F}_W(\hat{\mathbf{W}} \cdot \hat{\mathbf{A}}_y))l_c - mgl_c \sin q_B \quad (67)$$

The generalized active force contribution for F_2 with respect to u_1 is

$$\mathcal{F}_{u_1}^{F_2} = 0 \quad (68)$$

The generalized active force contribution for F_2 with respect to u_2 is

$$\mathcal{F}_{u_2}^{F_2} = T_A - T_R - f_C \quad (69)$$

With the generalized active forces in hand, the next step is find the generalized inertia forces of the system. The generalized inertia force contribution $\mathcal{F}_{u_r}^{*Q_i}$ for particle Q_i with respect to a generalized speed, u_r , is

$$\mathcal{F}_{u_r}^{*Q_i} = \frac{\partial \mathbf{v}^{Q_i/N}}{\partial u_r} \cdot m_{Q_i} \mathbf{a}^{Q_i/N} \quad (70)$$

As the only rotating components in this system are inertia-less reference frames, there are no generalized inertia forces of the rotating components of the system. The generalized inertia force contribution for Q_1 with respect to u_1 is

$$\mathcal{F}_{u_1}^{*Q_1} = m_B \ddot{s} \quad (71)$$

The generalized inertia force contribution for Q_1 with respect to u_2 is

$$\mathcal{F}_{u_2}^{*Q_1} = 0 \quad (72)$$

The generalized inertia force contribution for Q_2 with respect to u_1 is

$$\mathcal{F}_{u_1}^* Q_2 = m_C \left[\left(1 + \frac{l_c}{r_0} \sin q_B \right) \left(\ddot{s} \left(1 + \frac{l_c}{r_0} \sin q_B \right) - 2\dot{s}\dot{q}_B \frac{l_c}{r_0} \cos q_B \right) \right] \quad (73)$$

The generalized inertia force contribution for Q_2 with respect to u_2 are

$$\mathcal{F}_{u_1}^* Q_2 = m_C \left[l_c^2 \ddot{q}_B + \cos q_B \left(-\frac{\dot{s}^2}{r_0} \left(1 + \frac{l_c}{r_0} \sin q_B \right) \right) \right] \quad (74)$$

The next step is to express Kane's Equations for each generalized speed in terms of the generalized active and generalized inertia forces. Kane's Equations with respect to $\dot{s}$ (u_1) are

$$\sum_{i=1}^2 \mathcal{F}_{u_1}^{Q_i} = \sum_{i=1}^2 \mathcal{F}_{u_1}^{*Q_i} \quad (75)$$

Likewise, Kane's Equations with respect to $\dot{q}_B$ (u_2) are

$$\sum_{i=1}^2 \mathcal{F}_{u_2}^{Q_i} = \sum_{i=1}^2 \mathcal{F}_{u_2}^{*Q_i} \quad (76)$$

To find the equations of motion for $\ddot{s}$, substitute the generalized active forces in Equations 64 and 66 and the generalized inertia forces in Equations 71 and 73 into Equation 75 which gives

$$\begin{aligned} & -F_P + F_R - \tilde{F}_W \left[1 + \left(1 + \frac{l_c}{r_0} \sin q_B \right) \right] (\hat{\mathbf{W}} \cdot \hat{\mathbf{S}}_x) = \\ & m_B \ddot{s} + m_C \left[\left(1 + \frac{l_c}{r_0} \sin q_B \right) \left(\ddot{s} \left(1 + \frac{l_c}{r_0} \sin q_B \right) - 2\dot{s}\dot{q}_B \frac{l_c}{r_0} \cos q_B \right) \right] \end{aligned} \quad (77)$$

Solving for $\ddot{s}$ yields

$$\ddot{s} = \frac{-\left(-F_P + F_{R_1} - Z_0 + f_B\right) + 2l_C m_C \cos q_B \left(1 + \frac{l_C}{r_0} \sin q_B\right) \frac{\dot{q}_B \dot{s}}{r_0}}{m_B + m_C \left(1 + \frac{l_C}{r_0} \sin q_B\right)^2} \quad (78)$$

Where Z_0 is

$$Z_0 = \tilde{F}_W \left[1 + \left(1 + \frac{l_C}{r_0} \sin q_B \right) \right] (\hat{\mathbf{W}} \cdot \hat{\mathbf{S}}_x) \quad (79)$$

To find the equations of motion for $\ddot{q}_B$, substitute the generalized active forces in Equations 65, 67, and 69 as well as the generalized inertia forces in Equations 72 and 74 into Equation 76 which gives

$$\begin{aligned} & T_A - T_R - f_C + (\tilde{F}_W (\hat{\mathbf{W}} \cdot \hat{\mathbf{A}}_y)) l_C - m g l_C \sin q_B \\ & = m_C \left[l_C^2 \ddot{q}_B + \cos q_B \left(-\frac{\dot{s}^2}{r_0} \left(1 + \frac{l_C}{r_0} \sin q_B \right) \right) \right] \end{aligned} \quad (80)$$

Solving for $\ddot{q}_B$ yields

$$\ddot{q}_B = \frac{T_A - T_R - f_C}{m_C^2 l_C} + \frac{\tilde{F}_W (\hat{\mathbf{W}} \cdot \hat{\mathbf{A}}_y)}{m_C l_C} + \cos q_B \left(\frac{r_0}{l_C} + \sin q_B \right) \frac{\dot{s}^2}{r_0^2} - \frac{g}{l_C} \sin q_B \quad (81)$$

4.2 General System - Theoretical Overview

In the General System, the ATN vehicle is constrained to move on an arbitrary curve in space. The advantage of such a constraint is that an ATN vehicle can be simulated over any possible guideway geometry. Only MotionGenesis is used to formulate the equations of motion for the General System. Additionally, an overview of the system and mathematical operation are provided rather than an explicit derivation of the exact equations.

Figure 29. General System - Kinematics Overview

Referring to Figure 29, let $\hat{N}_x$, $\hat{N}_y$, and $\hat{N}_z$ form a right-handed set of mutually perpendicular unit vectors fixed in the inertial frame N . The guideway geometry is an arbitrary curve in space that is defined by its distance from the point N_0 . Let B be a particle of mass m_B is allowed to freely move along the guideway for distance s . The the position vector r_B from N_0 to B is defined as

$$r_B(s) = f_x(s)\hat{N}_x + f_y(s)\hat{N}_y + f_z(s)\hat{N}_z \quad (82)$$

The definitions of $f_x(s)$, $f_y(s)$, and $f_z(s)$ in Equation 82 are known arbitrary functions of s which are defined numerically. Next, let $\hat{S}_x$, $\hat{S}_y$, and $\hat{S}_z$ be unit vectors fixed in S . As was the case with the Planar System, $\hat{S}_x$, $\hat{S}_y$, and $\hat{S}_z$ have the following properties:

- (1) $\hat{S}_x$ is tangent to the guideway
- (2) $\hat{S}_y$ is normal to the guideway

$$(3) \hat{S}_z = \hat{S}_x \times \hat{S}_y$$

Table 6. S Unit Vectors Relation - General System

	$\hat{N}_x$	$\hat{N}_y$	$\hat{N}_z$
$\hat{S}_x$	R_{11}	R_{12}	R_{13}
$\hat{S}_y$	R_{21}	R_{22}	R_{23}
$\hat{S}_z$	R_{31}	R_{32}	R_{33}

Table 6 shows the relationship between $\hat{N}_x$, $\hat{N}_y$, $\hat{N}_z$ and $\hat{S}_x$, $\hat{S}_y$, and $\hat{S}_z$. R_{11} through R_{33} in Table 6 are each numerical functions of s . It is important to note that the values of R_{11} through R_{33} cannot be arbitrary and must form a rotation matrix. To demonstrate, allow $\hat{S}_x$, $\hat{S}_y$, $\hat{S}_z$ to be defined as

$$\begin{pmatrix} \hat{S}_x \\ \hat{S}_y \\ \hat{S}_z \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} \hat{N}_x \\ \hat{N}_y \\ \hat{N}_z \end{pmatrix} \quad (83)$$

Before attempting to use equations of motion developed through the method in this chapter, it must be proven that the 3X3 matrix in Equation 83 (henceforth referred to as $R(s)$) has the properties of a rotation matrix, namely orthogonality. This is proven by determining if $R(s)$ has a determinant equal to one over the range of s , that is, if

$$\det(R(s)) = 1 \quad (84)$$

Additionally, $R(s)$ is multiplied with its transpose and the result is compared element by element to an identity matrix, that is, a test is performed to see if

$$R(s)R^T(s) = I \quad (85)$$

This test is conducted on each different guideway geometry before carrying out the simulations (an example of the results for this validation test are Figures 41 and 42 in Chapter 6.3).

An explanation of the kinematic terms of the General System is now provided. Due to the length of the equations, neither an explicit derivation nor explicit record of the equations is provided here. First, the velocity of B in N is defined as

$$\mathbf{v}^{B/N} = -\dot{s} \left[R_{11} \hat{\mathbf{N}}_x + R_{12} \hat{\mathbf{N}}_y + R_{13} \hat{\mathbf{N}}_z \right] \quad (86)$$

The reason that the definition in Equation 86 is used is that it defines the velocity vector as the speed of B along the guideway multiplied by the instantaneous tangent vector. With definitions of $\mathbf{v}^{B/N}$, $\hat{\mathbf{S}}_x$, $\hat{\mathbf{S}}_y$, and $\hat{\mathbf{S}}_z$ in hand, the acceleration of B in N , $\mathbf{a}^{B/N}$, can be expressed as unspecified function of defined terms. This expression is

$$\mathbf{a}^{B/N} = f(\dot{s}, \ddot{s}, R_{11}, \dots, R_{33}, \dot{R}_{11}, \dots, \dot{R}_{33}) \quad (87)$$

For a similar expression of the acceleration of C in N , $\mathbf{a}^{C/N}$, it first must be stated that the rotation of BC and the expressions of $\hat{\mathbf{A}}_x$, $\hat{\mathbf{A}}_y$, and $\hat{\mathbf{A}}_z$ relative to $\hat{\mathbf{S}}_x$, $\hat{\mathbf{S}}_y$, and $\hat{\mathbf{S}}_z$ are exactly the same between the General System and the Planar System. With this in hand, $\mathbf{a}^{C/N}$ can be expressed as

$$\mathbf{a}^{C/N} = f(\dot{s}, q_B, \dot{q}_B, R_{11}, \dots, R_{33}, \dot{R}_{11}, \dots, \dot{R}_{33}, \ddot{R}_{11}, \dots, \ddot{R}_{33}) \quad (88)$$

Note how the expressions of $\mathbf{a}^{B/N}$ and $\mathbf{a}^{C/N}$ in Equations 87 and 87 respectively are dependent on the time directives of R_{11} through R_{33} . It is desired to transform the expressions from Equations 87 and 88 into a form that is only dependent on geometric

properties and state variables of the system. Let each individual function of R_{11} through R_{33} also be referred to as R_{ij} with i and j as index values from 1 to 3. An expression for $\dot{R}_{ij}$ in terms of the $\dot{s}$ and $\frac{dR_{ij}}{ds}$ is given as

$$\dot{R}_{ij} = \frac{dR_{ij}}{ds} \dot{s} \quad (89)$$

Equation 89 equates the change in R_{ij} with respect to time to the change in R_{ij} with respect to space multiplied by $\dot{s}$. This changes $\dot{R}_{ij}$ to an expression of geometric properties and state variables. Likewise, using the chain rule and Equation 89, each expression of $\ddot{R}_{ij}$ is turned into an expression in terms of $\dot{s}$, $\ddot{s}$, $\frac{dR_{ij}}{ds}$, and $\frac{d^2R_{ij}}{ds^2}$

$$\ddot{R}_{ij} = \frac{d^2R_{ij}}{ds^2} \dot{s}^2 + \frac{dR_{ij}}{ds} \ddot{s} \quad (90)$$

Substituting Equation 89 into Equation 87 yields

$$\mathbf{a}^{B/N} = f(\dot{s}, \ddot{s}, R_{11}, \dots, R_{33}, \frac{dR_{11}}{ds}, \dots, \frac{dR_{33}}{ds}) \quad (91)$$

Likewise, substituting Equations 89 and 90 into Equation 88 yields

$$\mathbf{a}^{C/N} = f(\dot{s}, \ddot{s}, q_B, \dot{q}_B, R_{11}, \dots, R_{33}, \frac{dR_{11}}{ds}, \dots, \frac{dR_{33}}{ds}, \frac{d^2R_{11}}{ds^2}, \dots, \frac{d^2R_{33}}{ds^2}) \quad (92)$$

Now, $\mathbf{a}^{B/N}$ and $\mathbf{a}^{C/N}$ are only functions of the geometric properties of the guideway and state variables. The advantage of this is that the spatial derivatives of R_{ij} discussed above do not change in time and therefore can be solved prior to the simulation of the system.

To determine how to find the functions of R_{ij} from the guideway geometry, $\hat{S}_x$, $\hat{S}_y$, and $\hat{S}_z$ must be related to $\mathbf{r}_B$. Mathematical expressions for $\hat{S}_x$, $\hat{S}_y$, and $\hat{S}_z$ based on $\mathbf{r}_B$

are(Kane & Levinson, 2005)

$$\hat{\mathbf{S}}_x = \frac{{}^N d\mathbf{r}^B}{ds} \quad (93)$$

$$\hat{\mathbf{S}}_y = \frac{\frac{{}^N d}{ds} \left(\frac{{}^N d\mathbf{r}^B}{ds} \right)}{\left| \frac{{}^N d}{ds} \left(\frac{{}^N d\mathbf{r}^B}{ds} \right) \right|} \quad (94)$$

$$\hat{\mathbf{S}}_z = \hat{\mathbf{S}}_x \times \hat{\mathbf{S}}_y \quad (95)$$

The unit vector definitions of $\hat{\mathbf{S}}_x$, $\hat{\mathbf{S}}_y$, and $\hat{\mathbf{S}}_z$ in Equations 93, 94, and 95 are related to each of function of R_{ij} by:

$$\begin{aligned} R_{ij} &= \hat{\mathbf{S}}_i \cdot \hat{\mathbf{N}}_j \\ \hat{\mathbf{S}}_1 &= \hat{\mathbf{S}}_x & \hat{\mathbf{S}}_2 &= \hat{\mathbf{S}}_y & \hat{\mathbf{S}}_3 &= \hat{\mathbf{S}}_z \\ \hat{\mathbf{N}}_1 &= \hat{\mathbf{N}}_x & \hat{\mathbf{N}}_2 &= \hat{\mathbf{N}}_y & \hat{\mathbf{N}}_3 &= \hat{\mathbf{N}}_z \end{aligned} \quad (96)$$

Additionally, it can be seen from Equation 96 and Equations 93, 94, and 95 that R_{ij} are functions only of s . As an analytical expression for R_{ij} now exists, the next step is to interpolate the guideway geometry to obtain an accurate estimation of R_{ij} . While the guideway geometry may be defined from a GIS map file or a CAD model, it must eventually have a form as a numerical table that relates s to f_x , f_y , and f_z from 82. Table 7 shows an example of the type of guideway geometry that is needed to interpolate functions for f_x , f_y , and f_z . Table 7 is for a guideway that is a circular track with a radius of $100m$, a geometry that is used for the simulations in this thesis.

Table 7. Guideway Geometry - 100m Circle Example

$s \text{ (m)}$	$f_x(s) \text{ (m)}$	$f_y(s) \text{ (m)}$	$f_z(s) \text{ (m)}$
0	100.0000	0	0
6.2832	99.8027	6.2791	0
12.5664	99.2115	12.5333	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$
609.4690	98.2287	-18.7381	0
615.7522	99.2115	-12.5333	0
622.0353	99.8027	-6.2791	0

The interpolation method used must ensure a smooth function of f_x , f_y , and f_z up to the 4th spatial derivative. To explain this requirement, refer to Equation 92 and note how an expression for $\mathbf{a}^{C/N}$ would require an expression for $\frac{d^2 R_{ij}}{ds^2}$. Next, refer to Equation 96 and note that the $\frac{d^2 R_{ij}}{ds^2}$ require an expression $\frac{d^N \hat{\mathbf{S}}_x}{ds}$, $\frac{d^N \hat{\mathbf{S}}_y}{ds}$, and $\frac{d^N \hat{\mathbf{S}}_z}{ds}$. Finally, referring to Equation 94, it is seen that $\hat{\mathbf{S}}_y$ requires $\frac{d^2 f_x}{ds^2}$, $\frac{d^2 f_y}{ds^2}$, and $\frac{d^2 f_z}{ds^2}$. Therefore, the function of the 4th spatial derivative of f_x , f_y , and f_z is required for accurate simulation of the system. As a consequence of this, the interpolation function must be smooth up to the 4th spatial derivative of s . An appropriate interpolation function is a 5th order spline constructed from the guideway geometry. The MATLAB Curve Fitting toolbox is used to construct a spline interpolation for f_x , f_y , and f_z . Once an interpolation exists, MotionGenesis is used to produce a MATLAB function that calculates R_{ij} and the needed spatial derivatives from the interpolated functions of f_x , f_y , and f_z (See Appendix A.3 for the MotionGenesis script, note that the resulting MATLAB function needed to be modified to perform operations on arrays).

4.3 MotionGenesis Overview

MotionGenesis is a computer algebra system (CAS) software tool specialized for dynamics applications (Mitiguy, 2015). MotionGenesis is used in this thesis to derive equations of motion for the Planar System and the General System, calculate the

expressions for R_{ij} , $\frac{dR_{ij}}{ds}$, and $\frac{d^2R_{ij}}{ds^2}$ in terms of the guideway geometry functions f_x , f_y , and f_z , solve for the setpoint displacement function (see 5.1), and solve for the wind function (see 5.3). In order to provide a tutorial for the MotionGenesis used in this thesis, a detailed derivation of the Planar System is performed and specific sections of the General System relating to using spatial derivatives is covered.

4.3.1 Planar System - Derivation with MotionGenesis

The equations of motion for the Planar System are also formulated using the computer algebra software MotionGenesis. The script commands used to generate the equations of motion are discussed in depth in order to function as a tutorial. The derivation in MotionGenesis follows a similar procedure to the manual derivation: the kinematics of the system are derived, force contributions are taken into account, and then the equations of motion are formulated. For the complete set of commands used in the script, refer to Appendix A.

In order for MotionGenesis to form the equations of motion, the parameters of the system need to be declared. The parameters of the system are the constants, specified functions, and variables used in the equations. For this system, the `Constant` command is used to assign a value for the radius of the curve, the acceleration due to gravity, the mass of the bogie and cabin, the length of the arm between the bogie and the cabin, the rolling resistance and air resistance coefficients of the system, and the damping coefficient of the arm. Additionally, variables for path displacement and cabin angle are declared with the `Variable` command. Lastly, each external force and torque is declared with the `Specified` command. The difference between variables and specified functions in MotionGenesis is that specified functions are user inputs and variables are outputs. The MotionGenesis syntax is shown below

```

(19) Constant r0, g, m_B, m_C, l_c, c0, c1, c2, cc
(20) Variable s'', q_B''
(21) Specified F_P % Bogie Propulsion Force
(22) Specified T_A % Car Tilting Control Torque
(23) Specified F_W % Wind Gust Force
(24) Specified WX %Wind Gust Direction X Component
(25) Specified F_R1 = ( c0 + (c1*s'') + (c2*s''^2) )
-> (26) F_R1 = c0 + c1*s'' + c2*s''^2

(27) Specified T_R2 = cc*q_B'
-> (28) T_R2 = cc*q_B'

```

For this MotionGenesis script, the desire is to produce MATLAB code of the equations of motion for a SIMULINK model. Most of these forces and constants are defined in the SIMULINK model. Therefore, except in the case of the damping force on the bogie and the damping torque on the vehicle arm, the declared constants and functions are left as arbitrary.

The next step is to declare the reference frames and bodies in the mechanical system. Additionally, the masses and geometric properties (if needed) are declared here. The first declaration is an inertial reference frame (equivalent to frame N in the Planar System derivation in Chapter 4.1) which is declared with the `NewtonianFrame` command. Subsequent reference frames (frames S and A in the Planar System derivation in Chapter 4.1) are declared with `RigidFrame`. Point mass particles (particles B and C in the Planar System derivation in Chapter 4.1) are assigned with the `Particle` command. Rigid bodies can be declared with the `RigidBody` command, however, as the derivation in this thesis is for a system of point masses, this command is not used. The `.SetMass` method assigns a mass value to the particle or rigid body object. The commands used are

```
NewtonianFrame N
```

```
RigidFrame S0, ARM
```

```
Particle B, C
```

```
B.SetMass(m_B)
```

```
C.SetMass(m_C)
```

After the declarations of the system parameters and objects, the kinematics of the declared objects can be found. The `Rotate` command calculates the angular velocity and angular acceleration of a reference frame or rigid body with respect to another reference frame or rigid body. The `Translate` command finds the velocity and acceleration of a particle given a position vector from another point. The commands and responses are

```
(34) S0o.Translate(N0,0>)
-> (35) p_No_S0o> = 0>
-> (36) v_S0o_N> = 0>
-> (37) a_S0o_N> = 0>

(38) S0.RotateZ(N,(s/r0))
-> (39) S0_N = [cos(s/r0), sin(s/r0), 0; -sin(s/r0), cos(s/r0), 0; 0,
0, 1]
-> (40) w_S0_N> = s'/r0*S0z>
-> (41) alf_S0_N> = s''/r0*S0z>

(42) B.Translate(N,S0o,r0*S0y>)
-> (43) p_S0o_B> = r0*S0y>
-> (44) v_B_N> = -s'*S0x>
-> (45) a_B_N> = -s''*S0x> - s'^2/r0*S0y>

(46) ARM.RotateX(S0,q_B)
-> (47) ARM_S0 = [1, 0, 0; 0, cos(q_B), sin(q_B); 0, -sin(q_B), cos(
q_B)]
```

```

-> (48) w_ARM_S0> = q_B'*ARMx>
-> (49) w_ARM_N> = q_B'*ARMx> + s'/r0*S0z>
-> (50) alf_ARM_S0> = q_B''*ARMx>
-> (51) alf_ARM_N> = q_B''*ARMx> + q_B'*s'/r0*S0y> + s''/r0*S0z>

(52) ARMo.Translate(N,B,0>)
-> (53) p_B_ARMo> = 0>
-> (54) v_ARMo_N> = -s'*S0x>
-> (55) a_ARMo_N> = -s''*S0x> - s'^2/r0*S0y>

(56) C.Translate(N,ARMo,-l_c*ARMz>)
-> (57) p_ARMo_C> = -l_c*ARMz>
-> (58) v_C_N> = l_c*q_B'*ARMy> - (1+l_c*sin(q_B)/r0)*s'*S0x>
-> (59) a_C_N> = l_c*q_B''*ARMy> + l_c*q_B'^2*ARMz> + (-s''-l_c*(2*cos(q_B)*q_B'*s'+
sin(q_B)*s'')/r0)*S0x> - (r0+l_c*sin(q_B))*s'^2/r0^2*S0y>

```

When the kinematic commands are used, MotionGenesis generates output that corresponds to the results of the kinematic calculation. In order for the equations of motion to be correctly generated, it is important that each particle be assigned an acceleration relative to the inertial frame.

The external forces on a particle or point are defined by the .AddForce method.

This method requires a vector of the external forces. The method used in context is

```

(64) W> = WX*Nx> + sqrt(1-(WX^2))*Ny>
-> (65) W> = WX*Nx> + sqrt(1-WX^2)*Ny>

(66) B.AddForce((-m_B*g*Nz>) - (F_R1*S0x>) - (F_CF_B*S0x>) + (F_P*
S0x>) + (F_W*W>))
-> (67) Force_B> = F_W*WX*Nx> + F_W*sqrt(1-WX^2)*Ny> - g*m_B*Nz> + (F_P-
F_CF_B-F_R1)*S0x>

```

```
(68) C.AddForce( (-m_C*g*Nz>) + (F_W*W>))
-> (69) Force_C> = F_W*WX*Nx> + F_W*sqrt(1-WX^2)*Ny> - g*m_C*Nz>
```

Note, that the vector $W>$ representing the wind direction needs to be defined here explicitly.

Similarly, external torques on a reference frame or rigid body are defined by the `.AddTorque` as follows

```
(62) ARM.AddTorque( (T_A - T_R2 - T_CF_C)*ARMx> )
-> (63) Torque_ARM> = (T_A-T_CF_C-T_R2)*ARMx>
```

After the external forces and torques have been defined, one can form the equations of motion for the system. This script uses three commands to implement Kane's method to determine the equations. The first is `SetGeneralizedSpeeds`, which defines what the generalized speeds are. Next, the `.GetDynamicsKane()` command forms the equations of motion. The time derivatives of the generalized speeds, s'' and q_B'' , are solved for with `Solve` command.

```
(66) SetGeneralizedSpeeds(s',q_B')
(67) DynEqn = System.GetDynamicsKane()
-> (68) DynEqn[1] = F_P + F_W*WX*cos(s/r0) + F_W*sqrt(1-WX^2)*sin(s/r0)
+ F_W*(1+l_c
*sin(q_B)/r0)*(WX*cos(s/r0)+sqrt(1-WX^2)*sin(s/r0)) + 2*l_c*m_C*cos(q_B)
*(1+l_c*sin(
q_B)/r0)*q_B'*s'/r0 + (m_B+m_C*(1+l_c*sin(q_B)/r0)^2)*s'' - F_R1

-> (69) DynEqn[2] = -l_c*(F_A+F_W*sqrt(1-WX^2)*cos(q_B)*cos(s/r0)+m_C*
cos(q_B)*(r0+l
_c*sin(q_B))*s'^2/r0^2-F_R2-g*m_C*sin(q_B)-F_W*WX*cos(q_B)*sin(s/r0)-l_c
*m_C*q_B'')
```

```

(70) Solve(DynEqn,s'',q_B'')
-> (71) s'' = (F_R1-F_P-F_W*WX*cos(s/r0)-F_W*sqrt(1-WX^2)*sin(s/r0)-F_W
*(1+l_c*sin(q
_B)/r0)*(WX*cos(s/r0)+sqrt(1-WX^2)*sin(s/r0))-2*l_c*m_C*cos(q_B)*(1+l_c*
sin(q_B)/r0)
*q_B'*s'/r0)/(m_B+m_C*(1+l_c*sin(q_B)/r0)^2)

-> (72) q_B'' = -(F_R2+g*m_C*sin(q_B)+F_W*WX*cos(q_B)*sin(s/r0)-F_A-F_W*
sqrt(1-WX^2)
*cos(q_B)*cos(s/r0)-m_C*cos(q_B)*(r0+l_c*sin(q_B))*s'^2/r0^2)/(l_c*m_C)

```

The final commands are used to generate the MATLAB code, which is used in the SIMULINK model. The command `ClearOutputEncode` is used to instruct MotionGenesis to disregard any previous variables or values assigned to the code generation engine. The command `OutputEncode` is used to list which variables one wishes numerical output for. Finally, the command `Code Algebraic()` allows for the specification of a MATLAB function with inputs specified by the user. The commands used in context are:

```

(75) ClearOutputEncode
(75) OutputEncode s',s'',q_B',q_B''
(76) Code Algebraic() Sys_2D.m(s,s',q_B,q_B',F_P,F_A,m_b, m_c, l_c,g,
r0,c0,c1,c2,cc,F_W,WX,F_CF_B,F_CF_C)
The file Sys_2D.m exists. Overwrite it? (Y/N): y

At the MATLAB prompt, type Sys_2D
To change input values, edit the file Sys_2D.m

```

This concludes the overview and brief tutorial for the Planar System derived with MotionGenesis. The `Help` command should be used for further information for any MotionGenesis command employed in this section.

4.3.2 General System - Working with Spatial Derivatives

The derivation of the General System in MotionGenesis is similar to the derivation of the Planar System. However, one key difference is that the equations of motion for the General System must be defined in terms of a spatial derivative of a function.

MotionGenesis does not have a command to form implicit spatial derivatives of a function, so an alternative method is used. Initially, the `Specified` command is employed to declare a specified function for R_{ij} , $\dot{R}_{ij}$, and $\ddot{R}_{ij}$. The time derivatives are declared by adding two apostrophes after each specified function. The full command used is

```
(2) Specified R11'',R12'',R13'',R21'',R22'',R23'',R31'',R32'',R33''
```

Before the time derivatives of R_{ij} can be equated to functions of the spatial derivatives, specified functions must be created for each spatial derivative of R_{ij} . Because there is no command for an implicit spatial derivative, each spatial derivative must be declared as its own unique function. Therefore, $\frac{dR_{11}}{ds}$ is specified as `R11s`, $\frac{dR_{12}}{ds}$ is specified as `R12s`, and so on. Likewise $\frac{d^2R_{11}}{ds^2}$ is specified as `R11s2`, $\frac{d^2R_{12}}{ds^2}$ is specified as `R12s2`, and so on. The spatial derivatives are declared below

```
(15) Specified R11s,R12s,R13s,R21s,R22s,R23s,R31s,R32s,R33s
```

```
(16) Specified R11s2,R12s2,R13s2,R21s2,R22s2,R23s2,R31s2,R32s2,R33s2
```

Once there specified functions for each spatial derivative of R_{ij} , it is possible to relate the specified time derivatives of R_{ij} to it spatial derivatives. The `SetNoDt` command is used to specify a definition for a specified function or it time derivative with the added caveat that no time derivatives are taken of the definition. An expression relating each time derivative of R_{ij} to its spatial derivative using Equations 89 and 90 and the `SetNoDt` command. The commands and responses used in context are

```
(17) SetNoDt(R11' = s'*R11s)
```

```
-> (18) R11' = R11s*s'
```

$$\begin{aligned}
(19) \quad & \text{SetNoDt}(R12' = s' * R12s) \\
\rightarrow (20) \quad & R12' = R12s * s' \\
\\
(21) \quad & \text{SetNoDt}(R13' = s' * R13s) \\
\rightarrow (22) \quad & R13' = R13s * s' \\
\\
(23) \quad & \text{SetNoDt}(R21' = s' * R21s) \\
\rightarrow (24) \quad & R21' = R21s * s' \\
\\
(25) \quad & \text{SetNoDt}(R22' = s' * R22s) \\
\rightarrow (26) \quad & R22' = R22s * s' \\
\\
(27) \quad & \text{SetNoDt}(R23' = s' * R23s) \\
\rightarrow (28) \quad & R23' = R23s * s' \\
\\
(29) \quad & \text{SetNoDt}(R31' = s' * R31s) \\
\rightarrow (30) \quad & R31' = R31s * s' \\
\\
(31) \quad & \text{SetNoDt}(R32' = s' * R32s) \\
\rightarrow (32) \quad & R32' = R32s * s' \\
\\
(33) \quad & \text{SetNoDt}(R33' = s' * R33s) \\
\rightarrow (34) \quad & R33' = R33s * s' \\
\\
(35) \quad & \text{SetNoDt}(R11'' = s'' * R11s + s'^2 * R11s2) \\
\rightarrow (36) \quad & R11'' = R11s2 * s'^2 + R11s * s'' \\
\\
(37) \quad & \text{SetNoDt}(R12'' = s'' * R12s + s'^2 * R12s2) \\
\rightarrow (38) \quad & R12'' = R12s2 * s'^2 + R12s * s''
\end{aligned}$$

```

(39) SetNoDt(R13'' = s''*R13s + s'^2 * R13s2)
-> (40) R13'' = R13s2*s'^2 + R13s*s''

(41) SetNoDt(R21'' = s''*R21s + s'^2 * R21s2)
-> (42) R21'' = R21s2*s'^2 + R21s*s''

(43) SetNoDt(R22'' = s''*R22s + s'^2 * R22s2)
-> (44) R22'' = R22s2*s'^2 + R22s*s''

(45) SetNoDt(R23'' = s''*R23s + s'^2 * R23s2)
-> (46) R23'' = R23s2*s'^2 + R23s*s''

(47) SetNoDt(R31'' = s''*R31s + s'^2 * R31s2)
-> (48) R31'' = R31s2*s'^2 + R31s*s''

(49) SetNoDt(R32'' = s''*R32s + s'^2 * R32s2)
-> (50) R32'' = R32s2*s'^2 + R32s*s''

(51) SetNoDt(R33'' = s''*R33s + s'^2 * R33s2)
-> (52) R33'' = R33s2*s'^2 + R33s*s''

```

The final step to ensure that MotionGenesis uses the spatial derivatives in formulating the equations of motion is to instruct MotionGenesis to use the explicit form (which has been defined above) of the time derivatives of R_{ij} when calculating the partial velocities of the system. As the partial velocities of the system are calculated from the kinematic terms, each kinematic term is redefined from an expression based on the time derivatives of R_{ij} to an expression based on the spatial derivatives of R_{ij} . The `Explicit` command redefines an expression such that all explicitly defined terms are expressed as their definition. Using

the `Explicit` command ensures all kinematic terms are expressed in the spatial derivatives of R_{ij} . This step is critical as `MotionGenesis` does not natively perform this step when finding partial velocities. The commands used in context are:

```
w_S0_N> := Explicit(w_S0_N>)
alf_S0_N> := Explicit(alf_S0_N>)

v_S0o_N> := Explicit(v_S0o_N>)
a_S0o_N> := Explicit(a_S0o_N>)

v_B_N> := Explicit(v_B_N>)
a_B_N> := Explicit(a_B_N>)

w_ARM_N> := Explicit(w_ARM_N>)
alf_ARM_N> := Explicit(alf_ARM_N>)

v_ARMo_N> := Explicit(v_ARMo_N>)
a_ARMo_N> := Explicit(a_ARMo_N>)

v_C_N> := Explicit(v_C_N>)
a_C_N> := Explicit(a_C_N>)
```

Once the kinematic terms are redefined in terms of the spatial derivatives of R_{ij} , the `SetGeneralizedSpeeds` command and `System.GetDynamicsKane()` can be used without concern.

CHAPTER 5

ATN VEHICLE SIMULATION

For the ATN vehicle simulation and control research, a SIMULINK model is constructed using the equations of motion derived in the previous chapter as the plant and a controller designed using the Control Systems Toolbox in MATLAB. The plant dynamics use a MATLAB function block in SIMULINK that uses code generated by the MotionGenesis script in Appendix A.1. The controller is implemented in SIMULINK blocks outside of the plant. The controller uses feedback linearization to negate the nonlinear dynamics of the plant and decouple the dynamics of the bogie and the vehicle angle. Additionally, proportional control is applied to the error of the bogie position and to the error of the angular position of the vehicle body. This controller tracks the position of the ATN vehicle and minimizes the centrifugal force on the passenger. The setpoints for both the displacement and the angular position are calculated in SIMULINK blocks separate from the plant and the controller. Additionally, there are two blocks that calculate system dynamics outside of the MATLAB function block: the force due to wind gusts and the non-linear friction force.

5.1 Displacement Profile Calculation

The controller for the ATN vehicle requires that the vehicle be able to track a position on a guideway with restrictions on the maximum velocity, acceleration, and jerk. If a constant position setpoint is used with a P controller, then the velocity, acceleration, and jerk are governed completely by the location of the closed loop poles. A simple method of ensuring that the controller output meets the design criteria is to have the setpoint of the ATN vehicle be a displacement profile that meets the required conditions. The displacement profile chosen for this is the piecewise function $s_0(t)$ comprised of the

functions $f_1(t)$ (a waiting period for passenger ingress), $f_2(t)$ (an acceleration segment), $f_3(t)$ (a segment where is ATN is at cruising velocity), $f_4(t)$ (a deceleration segment), and $f_5(t)$ (a waiting period for passenger egress). The breakpoints of the piecewise function are the time t_a , t_b , t_c , and t_d . The only other parameter of $s_0(t)$ is the cruising velocity v_0 . The form $s_0(t)$ is

$$s_0(t) = \begin{cases} f_1(t) & 0 \leq t < t_a \\ f_2(t) & t_a \leq t < t_b \\ f_3(t) & t_b \leq t < t_c \\ f_4(t) & t_c \leq t \leq t_d \\ f_5(t) & t \geq t_d \end{cases} \quad (97)$$

$$f_1(t) = 0$$

$$f_2(t) = y_a + \frac{1}{2} \frac{v_0}{t_b - t_a} - \frac{v_0}{t_b - t_a} t_a + \frac{(v_0)(t_b - t_a)}{4\pi^2} \cos 2\pi \frac{t - t_a}{t_b - t_a}$$

$$f_3(t) = v_0(t - t_b) + f_1(t_b)$$

$$f_4(t) = y_b + \frac{1}{2} \frac{-v_0}{t_d - t_c} + \frac{v_0}{t_d - t_c} t_d + \frac{(v_0)(t_d - t_c)}{4\pi^2} \cos 2\pi \frac{t - t_c}{t_d - t_c}$$

$$f_5(t) = x_0$$

The profile defined in Equation 97 is the integral of the “Transition” function from the MotionGenesis library (Mitiguy, 2015). The “Transition” function is a function that combines a linear function with a trigonometric term to produce a function that increases from a defined minimum value to a maximum value with no local extrema while simultaneously ensuring a zero 1st and 2nd derivative at the terminal points. This second property is advantageous as it ensures an optimal response in acceleration and jerk from the controlled plant. In order to find the values for t_a , t_b , t_c , and t_d , the piecewise profile

functions form a system of non-linear equations that can be solved for the individual time steps using the constraints that each adjacent function must have the same position at their shared time step and that the maximum absolute value of the 2nd derivative of the acceleration and deceleration segments must equal the maximum allowed acceleration. The symbolic algebra capabilities of MotionGenesis were used to generate a MATLAB function that solves for the time steps. From the time steps, a SIMULINK block can be made that dynamically changes the setpoint as a function of simulation time. Figure 30 shows an example of the displacement function over 100 m using the piecewise functions from Equation 97.

Figure 30. Displacement Profile Example, $a_m = 5 \frac{m}{s^2}$, $v_0 = 15 \frac{m}{s}$, $x = 100m$

5.2 Angle Setpoint Calculation

The setpoint for the angular position q_B of the suspended vehicle is not determined by an externally defined setpoint, but rather from velocity of the ATN bogie. Using the mechanics of suspended ATN vehicles (Anderson, 1978) and control of tilting high speed trains (Zhou et al., 2011), a simple method of determining the angular value to minimize the reaction forces on the passenger is developed. This method is based on the derivation of the equations of motion for the Planar System and is used without modification with the General System. Refer to Chapter 4.1 for the definition of the terms used for the derivation of the angle setpoint formula.

The development of the angle setpoint formula is based on the reaction forces felt by the passenger relative to A .

- A force that acts in the $\hat{A}_x$ direction pushes the passenger against the rear face of the seat or throws the passenger forward from the seat.
- A force that acts in the $\hat{A}_y$ direction pushes the passenger to either side of the seat.
- A force that acts in the $\hat{A}_z$ direction pushes the passenger against the cushion of the seat or lifts the passenger from the seat.

Forces on the passenger in the direction of $\hat{A}_x$ cannot be compensated by rotation of the vehicle body and therefore warrant no further discussion or analysis. It is shown that there is a trade between forces in the directions of $\hat{A}_y$ and $\hat{A}_z$. Figure 31 shows a free body for an ATN passenger at steady cruising velocity and constant vehicle angle.

Figure 31. Free Body Diagram for ATN passenger P with mass m_P at steady cruising velocity and constant vehicle angle. Centrifugal force is included.

The reaction force R_H acts in the $\hat{A}_y$ direction and R_V is the force that acts in the $\hat{A}_z$ direction. The weight ($m_P g$) of the passenger acts in the $-\hat{S}_z$ direction. The centrifugal force, F_C , acts in the $\hat{S}_y$ direction. The expression for F_C is defined with terms from Equation 52

$$F_C = \frac{\dot{s}^2}{r_o} \left(1 + \frac{l_c}{r_o} \sin q_B \right) \quad (98)$$

Referring to the forces in Figure 31, the definition for F_C in Equation 98, and Table 4, R_H

at steady cruising velocity and constant vehicle angle is

$$R_H = -m_P \frac{\dot{s}^2}{r_0} \left(1 + \frac{l_c}{r_0} \sin q_B \right) \cos q_B + m_P g \sin q_B \quad (99)$$

Likewise, the expression for R_V at steady cruising velocity and constant vehicle angle is

$$R_V = m_P \frac{\dot{s}}{r_0} \left(1 + \frac{l_c}{r_0} \sin q_B \right) \sin q_B + m_P g \cos q_B \quad (100)$$

As the length of the vehicle body arm is usually much less than the radius of the curve, it safe to neglect the $\frac{l_c}{r_0}$ term in Equation 99, which simplifies R_H to

$$R_H = -m_P \frac{\dot{s}^2}{r_0} \cos q_B + m_P g \sin q_B \quad (101)$$

From an ergonomics standpoint, forces that act in the $\hat{A}_z$ direction are preferable over forces that act in the $\hat{A}_y$ direction. Therefore, the vehicle angle setpoint is defined by the value q_B that results when R_H is zero. Setting R_H to 0 in Equation 101 gives

$$m_P \frac{\dot{s}}{r_0} \cos q_B = m_P g \sin q_B \quad (102)$$

Solving for q_B in Equation 102 gives

$$q_B = \text{atan} \frac{\dot{s}^2}{gr} \quad (103)$$

Although the angle setpoint formula is defined for a guideway that is an arc of constant radius, the same setpoint for q_B is used for any possible guideway geometries. Therefore,

a useful alternative expression for q_B , which is in terms of a varying curvature, κ , is

$$q_B = \text{atan} \frac{\dot{s}^2 \kappa}{g} \quad (104)$$

With κ defined as the inverse of the radius (Weisstein, 2016b) at a given path length.

$$\kappa = \frac{1}{r(s)} \quad (105)$$

As this thesis investigates the control of vehicles over a numerically defined guideway, it is convenient to express the angle setpoint formula as the form in Equation 104. This is because the numerical guideway may have straight segments which have an effective infinite radius.

5.3 Wind Force Calculation

The magnitude and direction of the wind force are controlled by MATLAB functions embedded in the SIMULINK program. The wind force needs to be an intermittent force that ramps up to a maximum force value and then ramps back down to zero. The 1st and 2nd derivatives of function governing wind force must be zero. The piecewise function F_M is created to model the wind force as a zero force segment, a ramp up period, a ramp down period, and another zero force segment. The maximum force of F_M is F_W and F_M acts in the $\hat{W}$ direction. The ramp up and ramp down sections of F_M are modeled by the "Transition" function from MotionGenesis. The two time steps are t_a

and t_b which are the beginning and end of the wind force respectively. The form of F_M is

$$F_M = \begin{cases} 0 & 0 \leq t < t_a \\ \frac{1}{2\pi} F_{W,0} \left(4\pi \frac{t_a-t}{t_a-t_b} - \sin 4\pi \frac{t_a-t}{t_a-t_b} \right) \hat{W} & t_a \leq t < t_a + \frac{t_b-t_a}{2} \\ \frac{1}{2\pi} F_{W,0} \left(-2\pi + 2\pi \frac{t_a+t_b-2t}{t_a-t_b} - \sin 2\pi \frac{t_a+t_b-2t}{t_a-t_b} \right) \hat{W} & t_a + \frac{t_b-t_a}{2} \leq t < t_b \\ 0 & t \geq t_b \end{cases} \quad (106)$$

An example output for the wind force described by Equation 106 is in Figure 32. While Figure 32 is for a specific maximum force, start time, and stop time, all simulations in this thesis have the same profile.

Figure 32. Wind Profile Example, $t_a = 7s$, $t_b = 13s$, $F_{W,0} = 230.15N$.

5.4 Non-Linear Friction Force

The nonlinear friction force refers to combined effect of static friction and the Stribeck effect(Makkar, Dixon, Sawyer, & Hu, 2005). The Stribeck effect is when the coefficient of friction decreases from the static coefficient to the kinetic coefficient. These frictional forces involve discontinuities and thus can be difficult to model in a simulation as the discontinuities represent situations where the time derivative goes to infinity. A solution is to use a mathematical model comprised of hyperbolic tangent functions to replicate the discontinuous behavior. The coefficient of friction modeled with hyperbolic tangent functions (Makkar et al., 2005) is

$$f(\dot{x}) = \gamma_1 (\tanh \gamma_2 \dot{x} - \tanh \gamma_3 \dot{x}) + \gamma_4 \tanh \gamma_5 \dot{x} + \gamma_6 \dot{x} \quad (107)$$

The mathematical model in Equation 107 is dependent on the velocity, $\dot{x}$, and the constants γ_1 , γ_2 , γ_3 , γ_4 , γ_5 , and γ_6 . It should be noted that γ_6 represents viscous friction and is set to zero in the simulations as viscous friction is defined by other forces in the equations of motion. Table 8 shows the constants used for the coefficient of friction model for the suspended vehicle body. The constants for the coefficient of friction of the bogie are not included due the discontinuous nature of the non-linear friction causing discontinuities in the acceleration of the bogie. These acceleration discontinuities in the motion of the bogie result in unrealistic jerk results of the bogie. The velocity of the bogie does not cross zero and the effect of the nonlinear friction forces is only prevalent in the beginning of the simulation. Therefore, it is acceptable to discount the nonlinear friction effects of the bogie. The values of γ_1 , γ_2 , γ_3 , γ_4 , and γ_5 are defined from the design guidelines in the paper "A New Continuously Differentiable Friction Model for Control Systems Design" (Makkar et al., 2005) and accepted static and kinetic friction coefficients

(Avallone & III, 1987) of lubricated steel on steel. The coefficients are defined here as these values are used in all simulations where the nonlinear friction effects are present.

Table 8. Non-Linear Friction Model Coefficients

	Vehicle Arm Angle (q_B)
γ_1	0.025
γ_2	100
γ_3	10
γ_4	0.025
γ_5	100
γ_6	0

The plot of the non-linear friction model for the vehicle arm angle is in Figure 33.

Figure 33. Plot of the Non-Linear friction model for the vehicle arm angle. Uses the coefficients in Table 8

5.5 Feedback Linearization

As the dynamics of the ATN vehicle are nonlinear and coupled, the system must be linearized in order to use pole placement control techniques. The linearization method chosen is feedback linearization which uses the expressions of the equations of motion to cancel out the nonlinear terms (Levine, 1996). The system is first expressed in state space as

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}) = M^{-1}(\mathbf{x}) (f(\mathbf{x}, \mathbf{u})) \quad (108)$$

where $M(\mathbf{x})$ represents the total mass of the ATN vehicle as well as the rotational inertia of the suspended vehicle. Also note how the wind and nonlinear friction forces are not included in this expression. This is because the wind force cannot be predicted and the non-friction forces are too difficult to model.

In the case of the Planar System it is possible to rewrite Equation 108 as

$$\dot{\mathbf{x}} = M^{-1}(\mathbf{x}) (f(\mathbf{x}) + \mathbf{u}) \quad (109)$$

The form of the state equations in Equation 109 allows for each input to individually cancel out the nonlinear terms of each component of the system. In the case of the Planar System, the propulsion force can be set to cancel the nonlinear terms of $\ddot{s}$ and the stabilizing torque to cancel out the nonlinear terms of $\ddot{q}_B$. In order to perform the cancellation, the following expression for each input is

$$u_i = (-f_i(\mathbf{x}) + M_i^{-1}(\mathbf{x})\ddot{u}_i) \quad (110)$$

Using this expression for u cancels the nonlinear terms, decouples the dynamics of the bogie from the vehicle arm angle, and allows for P control of the system.

The full non-linear Planar System in state space form using the terms introduced in Chapter 4 is

$$\begin{aligned}
 x_1 &= s & x_2 &= \dot{s} & x_3 &= q_B & x_4 &= \dot{q}_B \\
 u_2 &= F_P & u_4 &= T_A \\
 f_2(\mathbf{x}) &= \ddot{s} & f_4(\mathbf{x}) &= \ddot{q}_B
 \end{aligned}$$

$$\mathbf{x} = \begin{pmatrix} x_2 \\ M_2^{-1}(\mathbf{x})f_2(\mathbf{x}) \\ x_4 \\ M_4^{-1}(\mathbf{x})f_4(\mathbf{x}) \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ M_2^{-1}(\mathbf{x}) & 0 \\ 0 & 0 \\ 0 & M_4^{-1}(\mathbf{x}) \end{pmatrix} \begin{pmatrix} u_2 \\ u_4 \end{pmatrix} \quad (111)$$

Inserting the expression in Equation 110 for u_2 and u_4 gives the following system

$$\dot{\mathbf{x}} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{u}_2 \\ \tilde{u}_4 \end{pmatrix} \quad (112)$$

This 4th order linear system can be decoupled into two 2nd order linear systems, which allows for the application of 2nd order system design rules.

The strategy for controlling the Planar System is extended to controlling the General System. However, this approach presents some additional challenges, chief among which is that the General System technically introduces 27 additional states in the form of the functions of R_{ij} , $\frac{dR_{ij}}{ds}$, and $\frac{d^2R_{ij}}{ds^2}$. This means that the state space form of the General System has a total of 31 states. Due to length of the equations, the separation of the input forces from the state variables is no longer possible. The state space system is

now expressed as

$$\dot{\mathbf{x}} = \begin{pmatrix} x_2 \\ f_2(\mathbf{x}, \mathbf{u}) \\ x_4 \\ f_4(\mathbf{x}, \mathbf{u}) \end{pmatrix} \quad (113)$$

The linearization is performed by first creating a new vector comprised of the nonlinear function f_2 and f_4 . Then the input $\mathbf{u}^*$ that causes both f_2 and f_4 to equal zero is found.

$$\begin{pmatrix} f_2(\mathbf{x}, \mathbf{u}^*) \\ f_4(\mathbf{x}, \mathbf{u}^*) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (114)$$

Where $\mathbf{u}^*$ is a function of all 31 states and the output is a vector of F_P and T_A . The resulting system has form seen in Equation 112.

5.6 P Control - Pole Placement

Pole placement control (Levine, 1996) is used to artificially set the poles of the states to values that ensures the desired response. Although the feedback linearization was able to decompose the 4th order system into two 2nd order system, there is still some challenge as to how to select the best choice in poles for the decoupled system governing the motion of the bogie. Because a fast response time is desired without any overshoot, the poles are chosen such that the system is critically damped. The value of the poles is based on the acceleration and jerk felt by the passengers rather than a specific settling time. The dynamics of the linearized decoupled bogie system is given by the expression:

$$x_1(t) = p(t) \left(1 - e^{-\lambda t} (c_1 + c_2 t) \right) \quad (115)$$

From Equation 115, it can be seen that poles further along the left hand plane cause the displacement to better adhere to the displacement setpoint profile. However, poles further on the left hand plane have a higher initial acceleration and jerk that may exceed comfort levels. While there are methods of optimally choosing the poles (such as the LQR), the use of that technique is beyond the scope of this thesis. Therefore, multiple selections of the values of the poles were selected for the bogie system and multiple simulations were run for each selection. The same pole placement strategy is used to control the vehicle body angle based on the setpoint defined in Chapter 5.2. However, only a single set of poles for this controller are tested.

CHAPTER 6

RESULTS AND DISCUSSION

6.1 Results Introduction

The results of this thesis are from SIMULINK simulations of the system described in this thesis. The following simulations are conducted to validate the theoretical work in this thesis:

- A simulation that compares the MotionGenesis derived equations for a Planar System with hand derived equations for the Planar System.
- A geometry analysis on the General System for circular guideway with a 100 m radius
- A simulation that compares the MotionGenesis derived equations for a General System with hand derived equations for the Planar System.
- A simulation that experiments with a controller derived from the Planar System dynamics to control a plant derived from the Planar System Dynamics.
- A simulation that experiments with a controller derived from the General System dynamics to control a plant derived from the Planar System Dynamics.
- A geometry analysis and dynamics simulation of a spiral guideway using the General System

The purpose of these simulations is to provide a proof-of-concept for the ideas discussed in this thesis. The primary goal is to prove the suitability of the General System for dynamical simulation and for control applications. Additional conclusions drawn from these studies are discussed but not explored further.

A variable step differential equation solver (ode45) is used for the simulations. In order to allow for the replication of these simulation, the SIMULINK solver parameters are provided below:

- Absolute Tolerance = $1E - 4$
- Relative Tolerance = $1E - 4$
- Minimum Step Size = $1E - 4$
- Maximum Step Size = $1E - 3$
- Initial Step Size = $1E - 4$
- Maximum Consecutive Minimum Steps = 10

The solver parameters are deliberately kept coarse to reduce the simulation time and to provide a worst-case example of the performance of the dynamical models presented here. Experimentation with additional solver parameters is outside the scope of this research.

6.2 Planar System - Comparison Study

The first simulation tests the generated MATLAB code from MotionGenesis against the manually derived Equation 78 and Equation 81. A set of simulations is run for each system in which an ATN vehicle with no damping or external forces given an initial bogie velocity of $50 \frac{m}{s}$ and be subjected to a $500 N$ wind force from $100 s$ to $200 s$ using the wind profile in Equation 106. Each simulation has a running time of $300 s$.

Table 9. Comparison Study - Parameters

Parameter	Value
g	$9.8 \frac{m}{s^2}$
r_0	100 m
c_0	0 N
c_1	$0 \frac{Ns}{m}$
c_2	$0 \frac{Ns^2}{m^2}$
c_c	$0 \text{ Nm}(s)$
l_c	0.75 m
m_B	1150 m
m_C	100 m

The physical parameters of the system for the comparison study with a wind disturbance are shown in Table 9. Despite having no damping, the ATN vehicle in this study is given the mass and size of a realistic ATN vehicle.

Table 10. Planar System - Comparison Study - Initial Conditions

Initial Condition	Value
s_0	0 m
$\dot{s}_0$	$50 \frac{m}{s}$
$q_{B,0}$	0 rad
$\dot{q}_{B,0}$	$0 \frac{\text{rad}}{s}$

The initial conditions for the first study are shown in Table 10. The initial condition for the bogie velocity is much higher than what would be expected in realistic operation. In order to provide a reference of what the simulated motion of the particle is under the initial conditions specified by Table 9, the state information of the hand derived equations is in Figure 34 and Figure 35. Figure 34 shows the position and the velocity of the bogie traveling on a circular track of 100 m as defined by Equation 78. The plot shows that with the initial conditions from Table 10, the bogie travels 15 km in 300 s .

Figure 34. Bogie position and bogie velocity from simulations based on the hand derived equations.

Figure 35 shows the angular displacement and angular velocity of the suspended vehicle under the initial conditions shown in Table 10. Because of the high velocity of the bogie, the angular displacement exceeds that which exists during normal operation of the ATN vehicle.

Figure 35. Vehicle angle and vehicle angular velocity from simulations of the hand derived equations.

Figure 36 shows the error of the bogie position and velocity between simulations using the hand derived equations and simulations with MotionGenesis derived equations. The error over the course of the simulation is orders of magnitude smaller than the total displacement.

Figure 36. Bogie position error and bogie velocity error between simulations of the MotionGenesis equations and the hand derived equations.

Figure 37 shows the error of the vehicle angular position and velocity between simulations using the hand derived equations and simulations with MotionGenesis derived equations. The error for these states is higher as the angular position and angular velocity of the vehicle body are dynamically coupled to the bogie velocity.

Figure 37. Vehicle angular position error and vehicle angular velocity error between simulations of the MotionGenesis equations and the hand derived equations.

Table 11 shows the maximum error for each state variable. From the values in this table, it can be safely concluded that MotionGenesis plant for the Planar System has errors in position tracking of less than $1E - 9$ for simulations that run for less time and with a lower velocity.

Table 11. Comparison Study - Constant Bogie Force

	Error
s	$9.293E - 06 \text{ m}$
$\dot{s}$	$9.625E - 05 \frac{\text{m}}{\text{s}}$
q_B	$8.713E - 03^\circ$
$\dot{q}_B$	$7.873E - 04 \frac{\text{rad}}{\text{s}}$

As the distance and simulation time for this set of simulations vastly exceed the

distances and simulation time that are used for the simulations with control input. Therefore, it is reasonable to expect that the MotionGenesis derived equations will not show deviation in excess of the error shown in these simulations.

6.3 General System - Numerical Unit Vector Analysis

The geometry data for the General System is a circular track with a radius of 100 *m* (replicating the r_0 value from the planar system analysis) that has been divided into 100 equally spaced points. A 5th order spline has been fit to this geometry data with 5000 points to be used to calculate the R_{ij} functions mentioned in Chapter 4.2. The validation methods from Equations 84 and 84 in Chapter 4.2 are performed to confirm that the numerical unit vectors are acceptable. Figure 38 shows the determinant of the estimated rotation matrix (refer to Equation 84) compared with unity. A result of zero or close to zero implies that the R matrix is an orthogonal matrix and thus has the properties of a rotation matrix. The highest error is at the beginning and the end of the path. This is to be expected as it is at those two points where the spline interpolation is the least accurate.

Figure 38. The determinant of the numerical rotation matrix subtracted by 1 for every position on the path.

Figure 39 shows each element of the matrix result of Equation 85 compared with an identity matrix. As with the previous figure, a result of zero or close to zero implies that the R matrix is orthogonal.

Figure 39. Each individual element of the numerical rotation matrix multiplied by its transpose and subtracted by an identity matrix for every position on the path.

Figures 38 and 39 show that the numerical rotation matrix produced by the MATLAB code has the properties that define a rotation matrix.

With regards to the error, the maximum determinant error is on the order of $10E - 6$ and the maximum error of the product of the numerical rotation matrix multiplied by its transpose is on the order of $10E - 4$. With regards to the order of magnitude of the error, it is difficult to determine the threshold for acceptability. A sensitivity analysis of the error of the numerical rotation matrix with respect to the error of the General System simulation in SIMULINK is outside the scope of this thesis. However, the determinant error seen in Figure 38 can be compared to the simulation error in the General System comparison study in the next section to establish rough estimate of the sensitivity. An additional test is

to determine if values of $\hat{S}_x$, $\hat{S}_y$, and $\hat{S}_z$ for a circular guideway determined by the method prescribed in Chapter 4.2 match $\hat{S}_x$, $\hat{S}_y$, and $\hat{S}_z$ as specified in Table 3. This test is additional to the validation tests described in Chapter 4.2. To test this, first let $\hat{S}_x$, $\hat{S}_y$, and $\hat{S}_z$ determined by the method prescribed in Chapter 4.2 be expressed as $\hat{\hat{S}}_x$, $\hat{\hat{S}}_y$, and $\hat{\hat{S}}_z$. Then calculate one minus the dot product of $\hat{\hat{S}}_x$ and $\hat{S}_x$

$$\hat{S}_x \text{ Error} = 1 - |(R11, R12, R13) \cdot (\cos s/r_0, \sin s/r_0, 0)| \quad (116)$$

Then, calculate one minus the dot product of $\hat{\hat{S}}_y$ and $\hat{S}_y$ for all values of s

$$\hat{S}_y \text{ Error} = 1 - |(R21, R22, R23) \cdot (-\sin s/r_0, \cos s/r_0, 0)| \quad (117)$$

Finally, calculate one minus the dot product of $\hat{\hat{S}}_z$ and $\hat{S}_z$ for all values of s

$$\hat{S}_z \text{ Error} = 1 - |(R31, R32, R33) \cdot (0, 0, 1)| \quad (118)$$

If the results of Equations 116, 117, and 118 are equal or close to zero for all values of s , then $\hat{\hat{S}}_x$, $\hat{\hat{S}}_y$, and $\hat{\hat{S}}_z$ can be assumed to be parallel to $\hat{S}_x$, $\hat{S}_y$, and $\hat{S}_z$. Figure 40 shows the results of Equations 116, 117, and 118 for s for a vehicle traveling from 0 m to 15072 m (24 laps) for a circular guideway with a radius of 100 m . This distance traveled is higher than the circumference of the circular guideway to test if consecutive laps traveled affect the unit vector value. From Figure 40, it is seen that the dot product error is on the order of $10E - 6$ over 24 laps. Of note is the observation that the maximum error occurs at the beginning and the end of each lap which corresponds to the beginning and the end of each dataset, where the spline interpolation is the least accurate.

Figure 40. The results for Equations 116, 117, and 118 over 15072 m .

6.4 General System - Comparison Study

A comparison between the hand-derived equations of motion for the Planar System and the computer derived equations of motion for the General System is conducted to determine the open loop accuracy of using numerically defined unit vectors. The guideway geometry profile is a circle of 100 m . The General System uses a 100 datapoints of this geometry data to create an interpolated function of 5000 datapoints. The simulation is an ATN vehicle moving at a constant velocity of 50 $\frac{m}{s}$ over a time of 300 s subjected to a wind force of 500 N using the wind profile in Equation 106. Refer to Table 9 and Table 10 for the parameters and initial conditions of the simulation. Refer to Figure 34 and Figure 35 for the open loop response of the hand derived equations. Figure 41 shows the

error in the bogie position and bogie velocity, while still acceptable, are much greater than the error seen in the Planar System comparison study. The cause of this greater error is that in addition to any error from simulation timing, there is also the numerical error from the estimated unit vectors from the guideway geometry. Of note is that the velocity error (which in turn propagates error for the other states) is the greatest during the period when the wind gust is active.

Figure 41. Difference in simulation values of the states s and $\dot{s}$ of the manually derived equations versus the General System plant.

Figure 42 shows the error in the angular position and velocity of the vehicle body is much more pronounced. As the angular position and velocity is coupled to the bogie velocity, the error is higher in these. Like bogie velocity error, the highest error is during the time

of the wind gust.

Figure 42. Difference in simulation values of the states q_B and $\dot{q}_B$ of the manually derived equations versus the General System plant.

The maximum error for each state for this comparison study is in Table 12. As an additional check on the accuracy of the simulation, the curvature at each instance can be calculated from the values of R_{ij} generated at each simulation step. The calculated curvature is then compared to the known curvature of the circular guideway, which is $0.01 \frac{1}{m}$.

Table 12. General System - Comparison Study - Constant Bogie Force

Max Error in s	$4.552E-02 \text{ m}$
Max Error in $\dot{s}$	$6.870E-04 \frac{m}{s}$
Max Error in q_B	$3.940E-02^\circ$
Max Error in $\dot{q}_B$	$3.592E-03 \frac{rad}{s}$

Figure 43 shows the error between the known curvature and the interpolated curvature. The maximum error is on the order of $10E - 7$ at the beginning and end of each lap around the guideway. From this figure, it can be assumed that the simulation of the General System is accurately estimating the curvature of the guideway.

Figure 43. Curvature error of the General System.

6.5 Planar System - Controlled Plant

A controller that tracks a position on the guideway and maintains acceleration and jerk under prescribed maximum values is presented for the Planar System. To test for robustness, the mass of the vehicle used by the plant is made greater than the mass of the vehicle used by the feedback linearization controller and for the gain calculation. The

results from the simulation are the state values over time, the position error, a measure of the performance criteria over time, and an estimate of the energy used to control the vehicle. The plant is subjected to nonlinear friction on the vehicle arm angle and a wind profile defined by Equation 106 in Chapter 5.3 with a magnitude of 518N (equivalent to a 60 mph wind gust) from 20 s to 60 s. The vehicle parameters for this simulation are shown in Table 13.

Table 13. Planar System - Controlled Plant - Parameters

Parameter	Value
g	$9.8 \frac{m}{s^2}$
r_0	$100 m$
c_0	$0 N$
c_1	$5 \frac{Ns}{m}$
c_2	$0.7 \frac{Ns^2}{m^2}$
c_c	$0.5 Nms$
l_c	$0.75 m$
m_B (Plant)	$1400 m$
m_B (Controller)	$1150 m$
m_C	$100m$
Bogie Poles	$[-0.4, -0.4], [-4, -4], [-12.0, -12.0]$
Vehicle Angle Poles	$[-10, -10]$

The initial conditions for the ATN vehicle are shown in Table 14

Table 14. Planar System - Controlled Plant - Initial Conditions

Initial Condition	Value
s_0	$0 \frac{m}{s^2}$
$\dot{s}_0$	$0 \frac{m}{s}$
$q_{B,0}$	0 rad
$\dot{q}_{B,0}$	$0 \frac{\text{rad}}{s}$

The displacement profile conditions are shown in Table 15. It is critical to note that the displacement profile condition in Table 15 must be applied by means of the piecewise function specified by Equation 97 in Chapter 5.1 in order to achieve low acceleration and jerk values throughout the motion of the ATN vehicle.

Table 15. Planar System - Controlled Plant - Displacement Profile

Performance Parameter	Value
Max Acceleration	$5 \frac{m}{s^2}$
Cruise Velocity	$15 \frac{m}{s}$
Total Displacement	$1000 m$

The bogie position and velocity is shown in Figure 44. It can be seen that faster poles decrease the error between the specified displacement and the bogie position. Additionally, only the poles of $[-4, -4]$ and $[-12, -12]$ were able to have the ATN vehicle meet the specified cruising velocity of $15 \frac{m}{s}$.

Figure 44. Bogie position and bogie velocity over time for three sets of poles for a given displacement profile.

The vehicle arm angular position and angular velocity are shown in Figure 45. The angle setpoint is defined by Equation 105.

Figure 45. Vehicle angular position and vehicle angular velocity over time for three sets of poles for a given displacement profile.

Figure 46 shows the error in the bogie position and vehicle angular position tracking. It must be stated that the setpoints for both the bogie position and the vehicle angular position are dynamic quantities. As was noticed in 44, the faster poles decrease the error for the bogie position. However, faster bogie poles cause a higher initial angular position error and the bogie poles selection have mixed effect on the performance of the controller during the period of the wind gust. Therefore, it can be concluded that fast position tracking of the bogie is inversely related to the attenuation of centrifugal force.

Figure 46. Tracking error for the bogie position and vehicle angular position.

Figure 47 shows the acceleration and jerk of the ATN vehicle tangent to the guideway. As expected, the faster the poles, the higher acceleration and jerk values. However, due to the nature of the transition function described in Equation 97 in Chapter 5.1, the acceleration does not exceed the specified value of $5 \frac{m}{s^2}$ and there are no large fluctuations in the jerk. However, the absence of large jerk values must also be attributed to the fact that the friction and disturbance forces on the ATN vehicle are continuous functions for the entire simulation period.

Figure 47. Performance criteria over time for the bogie controller.

Figure 48 shows the acceleration and jerk of the ATN vehicle normal to the guideway.

Due to the relatively slow motion of the vehicle body, neither the acceleration nor jerk of the vehicle body have large values relative to the acceleration and jerk of the ATN vehicle tangent to the guideway.

Figure 48. Performance criteria over time for the vehicle angle controller.

Figure 49 shows an estimate for the horizontal and vertical reaction forces per unit mass (expressions for which are in Equations 99 and 100) felt by the passenger. It is seen that the horizontal reaction acceleration is close to zero during the phase when the ATN vehicle is at cruising velocity, which is the desired effect of the vehicle angle control. The vertical reaction force increases to slightly above the acceleration due to gravity; however, this is an acceptable tradeoff to minimizing the horizontal reaction force.

Figure 49. Reaction forces felt by the passenger over time.

The estimated power consumption for both the bogie position actuator and the vehicle arm angle actuator is shown in Figure 50. As this research is working purely with simulations, the estimated power gives an indication if a controller is practical to implement in a prototype. For example, the maximum peak power (for bogie poles of $[-12, -12]$) demanded by the bogie is 55 kW , which is roughly equivalent to the engine of a small city car, which, given the size and application of the ATN vehicle, is appropriate.

Figure 50. Power consumption over time.

A summary of the maximum values from Figure 44 through 50 for each set of poles are in Table 16.

Table 16. Planar System - Controlled Plant - Performance

	Bogie Poles: [-0.4, -0.4]	Bogie Poles: [-4, -4]	Bogie Poles: [-12, -12]
Max Bogie Acceleration	2.045 $\frac{m}{s^2}$	4.814 $\frac{m}{s^2}$	4.977 $\frac{m}{s^2}$
Max Bogie Jerk	0.711 $\frac{m}{s^3}$	2.429 $\frac{m}{s^3}$	2.594 $\frac{m}{s^3}$
Max Angle Acceleration	0.014 $\frac{m}{s^2}$	0.044 $\frac{m}{s^2}$	0.048 $\frac{m}{s^2}$
Max Angle Jerk	0.011 $\frac{m}{s^3}$	0.063 $\frac{m}{s^3}$	0.068 $\frac{m}{s^3}$
Max Bogie Power	24.564 kW	47.164 kW	48.773 kW
Max Vehicle Angle Power	0.013 kW	0.024 kW	0.024 kW
Max Path Tracking Error	76.145 m	7.535 m	2.504 m
Max Horizontal Reaction Force	0.025 $\frac{m}{s^2}$	0.071 $\frac{m}{s^2}$	0.075 $\frac{m}{s^2}$
Max Vertical Reaction Force	10.088 $\frac{m}{s^2}$	10.056 $\frac{m}{s^2}$	10.056 $\frac{m}{s^2}$

6.6 Planar System - No Vehicle Angle Control

In order to better understand the ATN vehicle dynamics and to provide an analysis on the usability of the controller, a set of simulations is run where the bogie is controlled via a feedback linearization force and pole placement exactly the same as in the previous section, but no control input is provided to the vehicle angle. The purpose of this test is to determine if feedback linearization is an efficient choice to minimize the centrifugal force felt by the passenger. Refer to Tables 13, 14, and 15 for the vehicle parameters (with the exception of the vehicle angle poles, which do not apply for this set of simulations), initial conditions, and the properties of the displacement profile. Figure 51 shows the states of the bogie vehicle for the simulation. The results are virtually the same for the controlled plant with both bogie control and vehicle angle control. It can be concluded that neglecting angular control does not affect the performance of the bogie controller.

Figure 51. Bogie position and bogie velocity over time for three sets of poles for a given displacement profile.

The angular position and angular velocity of the vehicle body are shown in Figure 52. From these results, it is shown that the steady state value of the vehicle angle is close to vehicle angle for the controlled angular position in Figure 45. However, the slight variation in the angular position due to the wind gust shows that neglecting vehicle angle control leaves the vehicle susceptible to disturbances. Additionally, while the steady state angle value is the optimal angle for negated centrifugal force felt by the passenger, there is a lag in achieving this angle compared to the fully controlled ATN vehicle.

Figure 52. Vehicle angular position and vehicle angular velocity over time for three sets of poles for a given displacement profile.

Figure 53 shows the reaction forces felt by the passenger during motion of the vehicle. The horizontal reaction force does eventually come close to zero; however, it takes additional time to do so compared to the fully controlled vehicle. Additionally, without vehicle control, there is no way to compensate against the wind gust. Therefore, the simple conclusion is that the fully controlled ATN vehicle has superior performance to the fully controlled ATN vehicle. However, this set of simulations does demonstrate that the feedback linearization method cancels useful nonlinear behavior of the system.

Figure 53. Reaction forces felt by the passenger over time.

6.7 Planar System - Controlled Plant w/ Feedback Linearization from General System Dynamics

A set of simulations of a plant that is defined by the equations of motion of the Planar System and controlled with a feedback linearization controller defined by the equations of motion of the General System is performed. Refer to Tables 13, 14, and 15 for the system parameters, initial conditions, and displacement profile. The same wind force of 518 N from 20 s to 60 s is applied and the vehicle arm angle is subjected to nonlinear friction. This simulation is of considerable interest as it represents the goal of using a guideway defined by geometry to control an ATN vehicle. The bogie position and velocity is shown in Figure 54. As with the previous study, it can be seen that faster poles

decrease the error between the specified displacement and the bogie position. As noted before, only the poles of $[-4, -4]$ and $[-12, -12]$ were able to have the ATN vehicle meet the specified cruising velocity of $15 \frac{m}{s}$.

Figure 54. Bogie position and bogie velocity over time for three sets of poles for a given displacement profile.

Figure 55 shows the vehicle angular position and angular velocity. The response for these states is similar to the Planar System simulation, which implies that is possible to control angular position using a controller derived from the General System.

Figure 55. Vehicle angular position and vehicle angular velocity over time for three sets of poles for a given displacement profile.

Figure 56 shows the error for the position and the angular position. The path tracking error is virtually the same as the Planar System simulation results shown in Figure 46, however, the the angular position error shows a similar profile but with higher error values. The reason for the higher error in the angular position is that the angular position of the vehicle body is more sensitive to the 2nd spatial derivatives of the numerically calculated unit vectors than the bogie position. Due to the realities of numerical differentiation, the 2nd spatial derivatives of the numerically calculated unit vectors have the highest error. Therefore it is reasonable to expect higher relative error for the vehicle angular position.

Figure 56. Tracking error for the bogie position and vehicle angular position.

The acceleration and jerk tangent to the guideway in Figure 57 shows minor variations from the previous study. Neither the acceleration nor the jerk exceed the limits prescribed by the displacement profile. However, a discontinuity in the acceleration causes minor spikes in the jerk values. This discontinuity is most likely a result of using the General System dynamics for the feedback linearization portion of the controller.

Figure 57. Performance criteria over time for the bogie controller.

The acceleration and jerk of the vehicle body normal to the guideway is shown in Figure 58. Due to discontinuities present in the numerical error of the General System, there is slightly higher jerk.

Figure 58. Performance criteria over time for the vehicle angle controller.

Figure 58 shows the reaction force for the ATN vehicle controlled by a controlled derived from the General System dynamics. Of note is the higher horizontal reaction force felt by the passenger during the time the ATN vehicle is subjected to the wind gust. This can be explained by the error in the predicted dynamics between the General System and the Planar System.

Figure 59. Reaction forces felt by the passenger over time.

Lastly, the estimated curvature from the General System controller is in Figure 60. This final measurement provides a check on the results shown beforehand to ensure that geometry predicted by the General System replicates the known geometry. The fact that the error is on the magnitude of $10E - 7$ implies that the General System is accurately predicting the geometry.

Figure 60. Estimated curvature over time as predicted by the General System controller.

Performance parameters and other simulation data for each set of poles are in Table 17.

Table 17. Planar System - General System Controller - Performance

	Bogie Poles [-0.4, -0.4]	Bogie Poles [-4, -4]	Bogie Poles [-12, -12]
Max Bogie Acceleration	2.047 $\frac{m}{s^2}$	4.815 $\frac{m}{s^2}$	4.978 $\frac{m}{s^2}$
Max Bogie Jerk	0.856 $\frac{m}{s^3}$	2.434 $\frac{m}{s^3}$	2.595 $\frac{m}{s^3}$
Max Angle Acceleration	0.020 $\frac{m}{s^2}$	0.064 $\frac{m}{s^2}$	0.071 $\frac{m}{s^2}$
Max Angle Jerk	0.023 $\frac{m}{s^3}$	0.178 $\frac{m}{s^3}$	0.214 $\frac{m}{s^3}$
Max Bogie Power	24.565 kW	47.192 kW	48.782 kW
Max Vehicle Angle Power	0.010 kW	0.015 kW	0.016 kW
Max Error	76.147 m	7.535 m	2.504 m
Max Horizontal Reaction Force	0.147 $\frac{m}{s^2}$	0.240 $\frac{m}{s^2}$	0.248 $\frac{m}{s^2}$
Max Vertical Reaction Force	10.088 $\frac{m}{s^2}$	10.056 $\frac{m}{s^2}$	10.056 $\frac{m}{s^2}$

6.8 General System - Euler Spiral Example

The validation of the General System involved simulations of a simple geometry. However, the appeal of the General System is that the dynamics constrained to any numerically defined geometry can be simulated. Therefore, an example of a different geometry is simulation without comparison to hand derived models. The purpose of this is to showcase the method developed in this thesis for a different, more complex guideway geometry. The guideway geometry chosen is that of an Euler Spiral in which a point of the guideway at position s in inertial frame N is defined as

$$\mathbf{r} = \cos A \frac{1}{2} \pi s^2 \hat{\mathbf{N}}_x + \sin A \frac{1}{2} \pi s^2 \hat{\mathbf{N}}_y + (0) \hat{\mathbf{N}}_z \quad (119)$$

Euler spirals are important concepts in ATN research as portions of such spirals are used to transition between straight and curved segments of the guideway. A unique property of the Euler spiral is that the curvature changes linearly with distance. Figure 61 shows an Euler spiral based on Equation 119 with a coefficient of $A = 2.5E - 5$ (Weisstein, 2016a). The vehicle begins travel at green circle on the plot where there the curvature is at a minimum and travels to the red X where the curvature is greatest.

Figure 61. Euler spiral Guideway Geometry - $A = 2.5E - 5$.

The first step is to confirm that a valid rotation matrix can be formed via the numerical calculated unit vectors. The method described in Chapter 4.2 used to analyze a circular guideway is also employed here. Figure 62 shows the determinant error for the rotation matrix over the path of the Euler spiral from Equation 84 in Chapter 4.2 compared with unity. The determinant error is close to zero providing evidence that the rotation matrix is valid across the entire guideway path.

Figure 62. The rotation matrix determinant error.

Figure 63 shows each element of the rotation matrix from Equation 85 compared with an identity matrix. The low error provides additional evidence that the method of determining unit vectors is valid for the methods described in Chapter 4.2. An important note to make is that Figures 62 and 63 validate both the interpolation method and the algebraic method of determining the unit vectors from the interpolated equations.

Figure 63. Individual element error of the rotation matrix multiplied by its determinant.

A dynamical study is conducted for an ATN vehicle moving from across the entire guideway geometry. The ATN vehicle is subject to damping from rolling and air resistance on the bogie as well as viscous damping on the vehicle body. The bogie is subjected to a constant force of 250N and the vehicle is allowed to move freely. The ATN vehicle is also subjected to a wind force of 500N from 30s to 80s. The results for this simulation are not compared with any other models. The physical parameters of the system for the ATN vehicle simulation on a Euler spiral with a wind disturbance are shown in Table 18.

Table 18. General System - Euler Spiral - Parameters

Parameter	Value
g	$9.8 \frac{m}{s^2}$
r_0	$100 m$
c_0	$0 N$
c_1	$5 \frac{Ns}{m}$
c_2	$0.7 \frac{Ns^2}{m^2}$
c_c	$1000 Nm(s)$
l_c	$0.75 m$
m_B	$1150 m$
m_C	$100 m$

The initial conditions for the ATN vehicle simulation on an Euler spiral are shown in Table 19. Of interest is the initial position of $1.1E - 6$. The reason for this infinitesimal initial condition is that the rotation matrix is poorly defined at $s = 0m$.

Table 19. General System - Euler Spiral - Initial Conditions

Initial Condition	Value
s_0	$1.1E - 6 m$
$\dot{s}_0$	$0 \frac{m}{s}$
$q_{B,0}$	0 rad
$\dot{q}_{B,0}$	$0 \frac{\text{rad}}{s}$

Figure 64 shows the path displacement and velocity of the ATN vehicle traveling over the Euler spiral guideway. As the force on the bogie is tangent to the guideway, the unique guideway geometry, the path displacement and velocity are similar to the simulations on the circular guideway.

Figure 64. Bogie position and bogie velocity over time for an ATN under open loop control travelling on a Euler spiral guideway.

Figure 65 shows the angular displacement and velocity of the ATN vehicle body over time. As the angle of the vehicle body is sensitive to the curvature of the guideway, these states show a different response than the ATN vehicle travelling over the circular guideway. As the curvature of the guideway increases, the vehicle angle increases as well. This is because the centrifugal force increases with the increasing curvature over the path length.

Figure 65. Vehicle angular position and vehicle angular velocity over time for an ATN under open loop control traveling on a Euler spiral guideway.

Figure 66 shows the curvature calculated from the numerically estimated unit vectors over the path length. As an important property of the Euler spiral is that the curvature is linearly related to the path length, a qualitative check on the accuracy of the General System is to check the curvature over the traveled distance.

Figure 66. The predicted curvature of the guideway over the course of the simulation.

CHAPTER 7

CONCLUSION AND RECOMMENDATIONS FOR FUTHER RESEARCH

From the simulation data in the comparison studies, it is seen that the General System is able to accurately simulate the dynamics of a suspended ATN vehicle. While there is some increased error in the simulation output between the General System dynamics and the Planar System dynamics, the error is orders of magnitude less than the maximum displacement. Additionally, another useful conclusion is that feedback linearization is possible using dynamics derived from the General System. This is an important conclusion as it means that a controller can be given numerical data from CAD or GIS to control a physical or prototype ATN vehicle. However, one consideration that the comparison studies and other simulation do not fully address is the limitations of the General System with regards to guideway geometry. An expected goal of the General System is to be able to simulate the dynamics on any numerically defined guideway. The two guideway geometries studied for this thesis are a circular guideway and an Euler spiral guideway. The geometry of these two guideways were interpolated with a 5th order spline. From the interpolated spline, a rotation matrix over the entire length of the guideway was formed. However, both of these guideways were continuous and did not contain any straight segments. Because some guideways of interest may have straight segments, these segments may have singularities due to an undefined unit normal vector. Therefore, the interpolation method of a 5th order spline may not be suitable for those particular guideway geometries. However, the tools to determine the rotation matrix over the guideway path can be used to to verify a particular interpolation method.

Future work for this research falls into three areas: increasing the complexity of the ATN idealization, increasing complexity of the features of the simulation environment, and researching the interpolation method of the numerical guideway data used for the

General System. Adding complexity to this idealization involves adding passive suspension components and determining if the vibrations cause the feedback linearization method to fail. Additional simulation environment features that increase the realism and provide a better engineering tool include discrete time sampling and measurement noise. Finally, new interpolation methods besides 5th order polynomial splines can be tried. This research should be performed in parallel with sensitivity studies to investigate the effect on granularity of the numerical guideway data and the accuracy of the simulation.

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Appendix A

Appendix A: MotionGenesis Scripts

A.1 Planar System Equations of Motion Derivation

```
Constant r0, g, m_B, m_C, l_c, c0, c1, c2, cc
Variable s'', q_B''
Specified F_P % Bogie Propulsion Force
Specified F_A % Car Tilting Control Force
Specified F_W % Wind Gust Force
Specified WX %Wind Gust Direction X Component
Specified F_R1 = ( c0 + (c1*s') + (c2*s'^2) ) % Davis Formula (kinda)
Specified F_R2 = cc*q_B'

NewtonianFrame N
RigidFrame S0, ARM
Particle B, C

B.SetMass(m_B)
C.SetMass(m_C)

S0o.Translate(No,0>)

S0.RotateZ(N,(s/r0))

B.Translate(N,S0o,r0*S0y>)

ARM.RotateX(S0,q_B)
ARMo.Translate(N,B,0>)

C.Translate(N,ARMo,-l_c*ARMz>)

W> = WX*Nx> + sqrt(1-(WX^2))*Ny>

B.AddForce((-m_B*g*Nz>) - (F_R1*S0x>) - (F_CF_B*S0x>) + (F_P*S0x>) + (
    F_W*W>))
C.AddForce((-m_C*g*Nz>) + ((F_A - F_R2 - F_CF_C)*ARMy>) + (F_W*W>))

SetGeneralizedSpeeds(s',q_B')

DynEqn = System.GetDynamicsKane()

Solve(DynEqn,s'',q_B'')

S_E = [s';s'';q_B';q_B''] %State Equations, x1 = s, x2 = s', x3 = q_B,
    x4 = q_B'

S_V = [s,s',q_B,q_B'] %State Variables
I_V = [F_P,F_A] %Input Signal
```

```

D_V = [F_W] %Disturbance Signal

B0 = D(S_E,I_V)

OutputEncode s',s'',q_B',q_B''
Code Algebraic() Sys_2D.m(s,s'',q_B,q_B',F_P,F_A,m_b, m_c, l_c ,g,r0,c0,c1
,c2,cc,F_W,WX)

ClearOutputEncode()

OutputEncode B0
Code Algebraic() B0_Lin.m(q_B,m_b, m_c, l_c ,g,r0)

NL_F1 = Solve(s'',F_P) %The non-linear bogie force %Opposite non-linear
force of the bogie
NL_F2 = Solve(q_B'',F_A) %The non-linear vehicle force %Opposite non-
linear force of the cabin

NL_F1 := NL_F1 - F_W*D(NL_F1,F_W)
NL_F2 := NL_F2 - F_W*D(NL_F2,F_W)

ClearOutputEncode
OutputEncode NL_F1
Code Algebraic() NL_FB_2D.m(s,s'',q_B,q_B',m_b, m_c, l_c ,g,r0,c0,c1,c2,cc
)

ClearOutputEncode
OutputEncode NL_F2
Code Algebraic() NL_FC_2D.m(s,s'',q_B,q_B',m_b, m_c, l_c ,g,r0,c0,c1,c2,cc
)

```

A.2 General System Equations of Motion Derivation

```

SetAutoZee(Off)

Constant g, m_B, m_C, l_c, c0, c1, c2, cc
Variable s'',q_B''
Specified R11'',R12'',R13'',R21'',R22'',R23'',R31'',R32'',R33''
Specified F_P % Bogie Propulsion Force
Specified F_A % Car Tilting Control Force
Specified F_W % Wind Gust Force
Specified WX %Wind Gust Direction X Component
Specified F_R1 = ( c0 + (c1*s') + (c2*s'^2) ) % Davis Formula (kinda)
Specified F_R2 = cc*q_B'
Specified F_CF_B % Coulomb friction of the bogie
Specified F_CF_C % Coulomb friction of the car angle

NewtonianFrame N
RigidFrame S0, ARM

```

```

Particle B, C

B.SetMass(m_b)
C.SetMass(m_c)

e_t> = R11*Nx>+R12*Ny>+R13*Nz>
R_B = [R11,R12,R13;R21,R22,R23;R31,R32,R33]

S0.Rotate(N,R_B)
S0o.SetVelocity(N,s'*e_t>)

B.Translate(N,S0o,0>)

ARM.RotateX(S0,q_B)
ARMo.Translate(N,B,0>)

C.Translate(N,ARMo,-l_c*ARMz>)

W> = WX*Nx> + sqrt(1-(WX^2))*Ny>

B.AddForce((-m_B*g*Nz>) - (F_R1*S0x>) - (F_CF_B*S0x>) + (F_P*S0x>) + (
    F_W*W>))
C.AddForce((-m_C*g*Nz>) + ((F_A - F_R2 - F_CF_C)*ARMy>) + (F_W*W>))

SetGeneralizedSpeeds(s',q_B')

DynEqn = System.GetDynamicsKane()

Solve(DynEqn,s'',q_B'')

S_E = [s';s'';q_B';q_B''] %State Equations, x1 = s, x2 = s', x3 = q_B,
    x4 = q_B'

S_V = [s,s',q_B,q_B'] %State Variables
I_V = [F_P,F_A] %Input Signal
D_V = [F_W] %Disturbance Signal

B0 = D(S_E,I_V)

OutputEncode s',s'',q_B',q_B''
Code Algebraic() Sys_3D1.m(R11,R12,R13,R21,R22,R23,R31,R32,R33,R11',R12
    ',R13',R21',R22',R23',R31',R32',R33',R11'',R12'',R13'',R21'',R22'',
    R23'',R31'',R32'',R33'',s,s',q_B,q_B',F_P,F_A,m_b,m_c,l_c,g,c0,c1,
    c2,cc,F_W,WX,F_CF_B,F_CF_C)

ClearOutputEncode()

OutputEncode B0
Code Algebraic() B0_Lin_3D.m(R11,R12,R13,R21,R22,R23,R31,R32,R33,R11',
    R12',R13',R21',R22',R23',R31',R32',R33',R11'',R12'',R13'',R21'',R22'

```



```

',R23'',R31'',R32'',R33'',s,s',q_B,q_B',F_P,F_A,m_b, m_c, l_c,g,c0,
c1,c2,cc,F_W,WX,F_CF_B,F_CF_C)

NL_F1 = Solve(s'',F_P) %The non-linear bogie force %Opposite non-linear
force of the bogie
NL_F2 = Solve(q_B'',F_A) %The non-linear vehicle force %Opposite non-
linear force of the cabin

NL_F1 := NL_F1 - F_W*D(NL_F1,F_W) - F_CF_B*D(NL_F1,F_CF_B) - F_CF_C*D(
NL_F1,F_CF_C) - F_A*D(NL_F1,F_A)
NL_F2 := NL_F2 - F_W*D(NL_F2,F_W) - F_CF_B*D(NL_F2,F_CF_B) - F_CF_C*D(
NL_F2,F_CF_C) - F_P*D(NL_F1,F_P)

ClearOutputEncode
OutputEncode NL_F1
Code Algebraic() NL_FB_3D.m(R11,R12,R13,R21,R22,R23,R31,R32,R33,R11',R12
',R13',R21',R22',R23',R31',R32',R33',R11'',R12'',R13'',R21'',R22'',
R23'',R31'',R32'',R33'',s,s',q_B,q_B',m_b, m_c, l_c,g,c0,c1,c2,cc)

ClearOutputEncode
OutputEncode NL_F2
Code Algebraic() NL_FC_3D.m(R11,R12,R13,R21,R22,R23,R31,R32,R33,R11',R12
',R13',R21',R22',R23',R31',R32',R33',R11'',R12'',R13'',R21'',R22'',
R23'',R31'',R32'',R33'',s,s',q_B,q_B',m_b, m_c, l_c,g,c0,c1,c2,cc)

```

A.3 General System - Unit Vector Coefficient Calculation

```

Variable x''',y''',z'''
Constant epi = 3E-16 NoUnits

NewtonianFrame N

r_s> = x*Nx> + y*Ny> + z*Nz>

e_t> = Dt(r_s>,N)
e_n> = Dt(e_t>,N)/(sqrt(Dot(Dt(e_t>,N),Dt(e_t>,N))))+epi)
e_b> = Cross(e_t>,e_n>)

de_t> = Dt(e_t>,N)
de_n> = Dt(e_n>,N)
de_b> = Dt(e_b>,N)

d2e_t> = Dt(de_t>,N)
d2e_n> = Dt(de_n>,N)
d2e_b> = Dt(de_b>,N)

R11 = Dot(e_t>,Nx>)
R12 = Dot(e_t>,Ny>)
R13 = Dot(e_t>,Nz>)

```

```

R21 = Dot(e_n>,Nx>)
R22 = Dot(e_n>,Ny>)
R23 = Dot(e_n>,Nz>)

R31 = Dot(e_b>,Nx>)
R32 = Dot(e_b>,Ny>)
R33 = Dot(e_b>,Nz>)

dR11 = Dot(de_t>,Nx>)
dR12 = Dot(de_t>,Ny>)
dR13 = Dot(de_t>,Nz>)

dR21 = Dot(de_n>,Nx>)
dR22 = Dot(de_n>,Ny>)
dR23 = Dot(de_n>,Nz>)

dR31 = Dot(de_b>,Nx>)
dR32 = Dot(de_b>,Ny>)
dR33 = Dot(de_b>,Nz>)

d2R11 = Dot(d2e_t>,Nx>)
d2R12 = Dot(d2e_t>,Ny>)
d2R13 = Dot(d2e_t>,Nz>)

d2R21 = Dot(d2e_n>,Nx>)
d2R22 = Dot(d2e_n>,Ny>)
d2R23 = Dot(d2e_n>,Nz>)

d2R31 = Dot(d2e_b>,Nx>)
d2R32 = Dot(d2e_b>,Ny>)
d2R33 = Dot(d2e_b>,Nz>)

ClearOutputEncode()
OutputEncode R11,R12,R13,R21,R22,R23,R31,R32,R33,dR11,dR12,dR13,dR21,
    dR22,dR23,dR31,dR32,dR33,d2R11,d2R12,d2R13,d2R21,d2R22,d2R23,d2R31,
    d2R32,d2R33
Code Algebraic Spline_Vect.m(x,y,z,x',y',z',x'',y'',z'',x''',y''',z''',x
    ''',y''',z''')

```

Appendix B: SIMULINK MODEL - PLANAR SYSTEM

Appendix C: SIMULINK MODEL - GENERAL SYSTEM

Figure 68. A SIMULINK Model for the continuous General System.