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Hydraulic Sheet Resistance In Paper-Based Porous Media

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HYDRAULIC SHEET RESISTANCE IN PAPER-BASED POROUS MEDIA

A Thesis

Presented to

The Faculty of the Department of Mechanical Engineering

San José State University

In Partial Fulfillment

of the Requirements for the Degree

Master of Science

by

Stefan M. Doser

May 2018

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The Designated Thesis Committee Approves the Thesis Titled

HYDRAULIC SHEET RESISTANCE IN PAPER-BASED POROUS MEDIA

by

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May 2018

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ABSTRACT

HYDRAULIC SHEET RESISTANCE IN PAPER-BASED POROUS MEDIA

by Stefan M. Doser

This work investigates the special case of in-plane fluid flow of a Newtonian incompressible fluid across a thin paper-based porous medium at very low Reynolds numbers ($Re \ll 1$). Fluid transport with these characteristics is used in emerging devices such as microscale paper-based analytical devices (μ PADs). The mathematical similarity between Darcy's law and Ohm's law is considered, and hydraulic equivalents of current, voltage and resistance are determined to propose hydraulic sheet resistance. Darcy flow is predicted under these conditions and tested by experiment at two flow rates of 5 $\mu\text{L}/\text{min}$ and 10 $\mu\text{L}/\text{min}$. A device was designed and fabricated to ensure a deterministic 310 μm gap that directs prescribed flow, unidirectionally across Grade 50 Whatman filter paper. Pressure was measured along the direction of flow over a 125 mm distance at six pressure ports placed at uniform increments of 25 mm. Measurements were recorded over a time period up to 48 hours at discrete intervals with at least four replicates. Measurements of the pressure profile showed a linear relationship as predicted by Darcy's law, which allow hydraulic permeability, hydraulic bulk resistivity and hydraulic sheet resistivity to be calculated as 324 mm^2 , 2995 s^{-1} and 9433 $(\text{mm}\cdot\text{s})^{-1}$ respectively. Among replicates measured under the same set of controllable experimental conditions, the data also show a nonlinear relationship, suggesting transition into a nonlinear flow regime dependent upon inlet pressure and media tortuosity.

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TABLE OF CONTENTS

LIST OF TABLES	ix
LIST OF FIGURES	x
LIST OF SYMBOLS AND ABBREVIATIONS	xi
1. INTRODUCTION	1
1.1 Background	1
1.1.1 Consideration and Selection of Fluid	2
1.1.2 Consideration and Selection of Porous Media	3
1.2 Hypothesis	4
1.3 Significance	5
2. THEORY	6
2.1 Darcy's Law	6
2.2 Review of the Four-Point Probe Method	10
2.3 Defining Electrical Sheet Resistance	12
2.4 Applying Darcy's Law to Sheet Resistance	15
3. RELATED WORK.....	21
3.1 Flow Regimes within Porous Media	21
3.1.1 Flow Regime Identification Criteria	21
3.1.2 The Weak Inertia Regime	23
3.1.3 The Forchheimer Regime	24
3.1.4 The Turbulent Regime	26
3.2 Explanations for Non-Darcy Flow in Porous Media	27
3.2.1 Inertial Effects as a Contributor to Non-Darcy Flow	27
3.2.2 Porous Media Qualities as a Contributor to Non-Darcy Flow	27
3.3 Boundary Condition Effects in Porous Media Flow	28
3.4 Paper-Based Porous Media Flow Device Research	29
3.4.1 Current Application of Paper-Based Porous Media	29
3.4.2 Time-Delay Research in Paper-Based Porous Media	30
3.4.3 Flow Control Research in Paper-Based Porous Media	31
4. METHODOLOGY.....	33
4.1 Selection of Paper Media and Sample Preparation	33
4.2 Design of the Hydraulic Resistance Tester	35
4.2.1 Functional Requirements	35
4.2.2 Kinematic Design and Construction	36

4.3 Fluid Flow Rate Device and Tube Selection	37
4.4 Pressure Measurement Methodology.....	39
4.4.1 Determining Pressure in Porous Media	39
4.4.2 Pressure Measurement Selection	39
4.5 Experimentation.....	42
4.5.1 Experimental Preparation	42
4.5.2 Experimental Conditions	42
4.6 Sources of Uncertainty	44
4.6.1 Relative and Combined Error	44
4.6.2 Unquantified Uncertainty	46
5. RESULTS AND DISCUSSION	49
5.1 Collected Data	49
5.1.1 Best Data Representation	49
5.1.2 Linear and Nonlinear Determination	51
5.2 Analysis	53
5.2.1 Curve Fit Equations	53
5.2.2 Special Observations and Explanations	55
5.2.3 Calculated Material Properties	58
6. CONCLUSIONS	61
6.1 Addressing the Hypothesis.....	61
6.2 Discoveries from Experimentation	61
6.3 Recommendations for Future Work	62
6.3.1 Recommendations for Methodology.....	62
6.3.2 Recommendations for Testing	63
REFERENCES CITED	65
APPENDIX A: APPARATUS TECHNICAL DOCUMENTS.....	74
Bill of Material.....	74
Top Plate 2D.....	75
Bottom Plate 2D	76
APPENDIX B: COORDINATE MEASUREMENT MACHINE FLATNESS.....	77
Coordinate Measurement Machine Bottom Bracket	77
Coordinate Measurement Machine Top Bracket Main	78
Coordinate Measurement Machine Top Bracket Edges	79
APPENDIX C: EXPERIMENTAL RAW DATA CHARTS.....	80
Pressure Readings Separated by Location Over Time.....	80

Pressure Readings Separated by Time over Location	82
Curve Fitted Pressure Readings Over Location at One Instance	84

LIST OF TABLES

Table 1.	Summary of uncertainty as relative error for permeability k calculation.....	46
Table 2.	Maximum pressure, linear regression fit values for flow at 10 $\mu\text{L}/\text{min}$ and 5 $\mu\text{L}/\text{min}$	53
Table 3.	Flow rate, coefficients and regression fit values for data sets fitted by equation (54).....	54
Table 4.	Flow rate, coefficients and regression fit values for data sets fitted by equation (54).....	55
Table 5.	Flow rate and calculated material permeability k (mm^2) from equation (51) of linear data sets.....	59

LIST OF FIGURES

Figure 1.	Diagram of the four-point probe method on a surface with the outer probes.....	11
Figure 2.	A representation of the hypothetical current density field assumed in electrical sheet resistance definition.....	13
Figure 3.	A representation of the hypothetical flow density field from inlet to outlet with.....	18
Figure 4.	Scanning electron microscope (SEM) image of grade 50 Whatman filter paper.....	34
Figure 5.	Data set 5 pressure readings separated by measurement location as a function of time.....	49
Figure 6.	Data set 5 pressure readings as a function of distance from inlet to outlet, measured at.....	50
Figure 7.	Data set 5 at hour 24.75 showing individual pressure readings and a curve.....	51
Figure 8.	Data set 2 at hour 25 showing individual pressure readings and a curve fitting the data as.....	52
Figure 9.	Data set 9 at hour 23 showing individual pressure readings and a curve fitting the data as.....	56
Figure 10.	Data set 10 at hour 25 showing individual pressure readings and a curve fitting the data as.....	56
Figure 11.	Data set 6 at hour 46.50 showing individual pressure readings and a curve fitting the data as.....	57
Figure 12.	Maximum recorded pressure drop for linear pressure profile data sets (1, 2, 6, 7 & 8).....	58
Figure 13.	Maximum recorded pressure drop for linear pressure profile data sets (3, 4, 5, 9 & 10).....	59

LIST OF SYMBOLS

Symbol	Description
--------	-------------

a	Placeholder constant
A	Area (m ²)
A	Regression fit constant
b	Placeholder constant
B	Regression fit constant
C	Regression fit constant
d	Average particle diameter (m)
d_p	Pore diameter (m)
d_{part}	Particle diameter (m)
D	Diameter of porous media bed (m)
D	Regression fit constant
E	Vector of electromagnetic field force (volt/m)
F	Vector of flow field force (m/s ²)
g	Scaler force of gravity (m/s ²)
G	Vector of gravitational force (m/s ²)
h	Height above elevation datum (m)
h_1	First manometer height (m)
h_2	Last manometer height (m)
i	Current density (A/m ²)
i	Vector of current density field (ampere/m ²)
I	Current (ampere)
k	Darcy permeability coefficient (m ²)
k_f	Forchheimer permeability coefficient (m ²)
K	Darcy's hydraulic conductivity
K'	Remainder of Darcy's hydraulic conductivity
l	Length (m)
L	Length of flow path (m)
n	A finite number of terms in a series
N	Dimensionless factor of proportionality
P	Pressure (Pa)
q	Flow density (m/s)
q	Vector of flow density field (m/s)
Q	Volume flow rate (m ³ /s)
r	Radius (m)
R	Electrical resistance (Ω)
R_h	Hydraulic resistance (m/s)

Re	Reynolds number
Re_m	Modified Reynolds number
Re_p	Pore Reynolds number
s	Probe spacing (m)
S_{part}	Particle surface area (m ²)
t	Thickness (m)
u	Range of uncertainty
v_p	Pore velocity (m/s)
V	Voltage (V)
V_{part}	Particle volume (m ³)
w	Width (m)
x	Linear distance (m)
X	Dimensionless inertial contribution ratio
y	Source of uncertainty
z	Height direction parallel to gravity (m)
β	Inertial resistance coefficient
γ	Weak inertia coefficient
ϵ	Dimensionless voidage in porous media
μ	Dynamic fluid viscosity (Pa·s)
ρ	Fluid density (kg/m ³)
ρ_a	Particle apparent density (kg/m ³)
ρ_b	Electrical bulk resistivity ($\Omega \cdot m$)
ρ_{bulk}	Particle bulk density (kg/m ³)
ρ_s	Electrical sheet resistivity (resistance) ($\Omega/\square$)
σ	Hydraulic conductivity (s)
σ_e	Electromagnetic conductivity ($\Omega^{-1} \cdot m^{-1}$)
τ_b	Hydraulic bulk resistivity (s ⁻¹)
τ_s	Hydraulic sheet resistivity (resistance) (m ⁻¹ ·s ⁻¹)
Φ	Energy per unit mass (m ² /s ²)

1. INTRODUCTION

1.1 Background

The concept of hydraulic resistance as it applies to the hydraulic-electrical analogy has been established in research going back since the industrial revolution. The theoretical framework for the hydraulic-electrical analogy can be seen in continuum mechanics equations such as the Hagen-Poiseuille relation for cylindrical ducts, or alternatively by manipulation of the Navier-Stokes equations for a direct albeit cumbersome solution. However, the hydraulic-electrical analogy can be attributed in part to Henry Darcy and a relationship he discovered known as Darcy's law. Simply put, closed hydraulic systems function similarly to electrical circuits with hydraulic equivalents of voltage, current and resistance which behave linearly with respect to impetus and flow.

This work explores the hydraulic electrical analogy and applies it to the concept of electrical sheet resistance which is widely used in the semiconductor manufacturing industry. A notable departure from the classical hydraulic-electrical analogy is the inclusion of porous media, which introduces additional considerations to the flow dynamics. The inclusion of porous media requires observance to specific theoretical frameworks but within this work remains the subject of incompressible low Reynolds number flow of a Newtonian fluid.

An important distinction between the microfluidic experimental model tested in this work and the industrial application of electrical sheet resistance is the aspect ratio of sheet length and width relative to the thickness. In this work, empirical data are obtained with aspect ratios of approximately 500:1 where electrical sheet resistance is often measured at aspect ratios of 10000:1 or greater. Herein, an expanded discussion is presented as to other similarities and differences between the microfluidic model and electrical sheet resistance to explore how the behavior of hydraulic sheet resistance may behave as understood by Darcy's law.

1.1.1 Consideration and Selection of Fluid

Although gases and liquids are both of interest for porous media flow, the scope of this study is limited to liquids. An important advantage of liquids is that they are nearly completely incompressible. Gas, by contrast, is a compressible fluid which in a closed system can exhibit non-steady state behavior with respect to flow rate and pressure. Choosing an incompressible fluid ensures that any kind of harmonic behavior can be ignored as a source of error, provided that the liquid delivery is steady without pressure transients.

The chosen liquid is water. In addition to being isotropic, water is a Newtonian fluid meaning that the viscous stress and strain rate are not dependent on the stress state of velocity of flow but solely on temperature [1]. This is an important consideration because viscosity directly contributes to hydraulic resistance as predicted by Darcy's law.

For these reasons, water is an ideal candidate as a fluid for study. In addition, choosing distilled or deionized water is most preferred because it ensures a minimum number of particles within the water. These particulates can cause contamination of the porous medium that will affect the hydraulic sheet resistance throughout experimentation.

1.1.2 Consideration and Selection of Porous Media

For this study, only a few media were considered for use. One option is the same porous medium of the related work involving reverse-emulsion electrophoretic displays or (REED) technologies which uses spray-deposited powder titanium dioxide [2]. However, this can be cost prohibitive and introduces other unwanted factors such as the particle size variation, nonuniform agglomeration and repeatability of the deposited material.

Another consideration is an unbound porous medium such as sand, similar to the original medium studied by Darcy. However, sand is not as applicable within the field of microelectromechanical devices because of the large pore size relative to the channel and gap widths typically found in these devices. In addition, preference is given to fixed matrix porous media over a media composed of free particles which can exhibit unwanted behavior such as rearrangement and nonuniform channeling of fluid.

These considerations leave fibrous materials such as cellulose paper-based media as preferred subjects for study. Not only are these porous media constrained within a matrix, but these materials can be easily manufactured with tight dimensional thickness

tolerances of approximately $\pm 15 \mu\text{m}$, and consistent porosity as in the case of the porous media studied in previous work [3]. In addition, because paper is easy to manufacture, such sheets are also inexpensive making paper-based media ideal for disposable microdevices [4].

1.2 Hypothesis

This work investigates the special case of lateral fluid flow of a Newtonian incompressible fluid across thin paper-based porous medium in a confined conduit. For this case, the hypothesis is that pressure drop will behave linearly (i.e., as in Darcy flow) with distance along the porous medium, for Reynolds numbers that are much less than one ($\text{Re} \ll 1$).

Furthermore, an accurate statement describing hydraulic sheet resistance can be derived based on Darcy's law, similar to the statement of electrical sheet resistance based on Ohm's law. The statement proposed would have a form similar to the electrical sheet resistance equation (1) and would be supported by experimental data gathered.

$$\Delta V = -\left(\frac{I\rho_s}{2\pi}\right)\ln r \quad (1)$$

In equation (1), ΔV is the voltage drop from the current source to a point located at a distance r away, I is current, and ρ_s is electrical sheet resistivity [5]. For the proposed statement of hydraulic sheet resistance, the hydraulic equivalent terms of equation (1) will be determined based on the parallel between Ohm's law and Darcy's law.

1.3 Significance

There is still much to be discovered in the field of porous media flow and the mechanisms that contribute to flow regime condition. Understanding this behavior by exploring alternative theoretical frameworks can lead to advancement in fields that use this technology. It is becoming more common for thin paper-based porous media to be used in biomedical fluidic devices and thus it is important to understand if the special case of thin porous media flow requires a correction to the pressure and flow rates predicted by Darcy's law [4]. One immediate application of this work is to predict permeability of a common fluidic device materials. This is useful in the display technology field which uses manufactured and deposited materials. Hydraulic sheet resistance can be used as a quality control criterion to determine a medium's permeability in mass production. Alternatively, it can be used as a design criterion to determine the time for an image to resolve as well as the stability of an image [2].

2. THEORY

2.1 Darcy's Law

The study of porous media goes back to the mid 1800s with the first studies focused on the fluid mechanics of flow through bulk porous material determined directly from experimentation. In the years since, many have attempted to connect these observations to a theoretical framework. This section will focus on these first observations by Darcy and a theoretical framework that can be applied.

The study of flow through porous media began with Darcy's law published by Henry Philibert Gaspard Darcy in his book *Les Fontaines Publiques de la Ville de Dijon* in 1856 [6]. Darcy's law was developed based on the study of hydrology as it applied to groundwater behavior and the design of aquifers. Darcy's law describes volume flow rate Q as a function of the cross-sectional area of porous media normal to flow A , the "hydraulic conductivity" K , the change in manometer height reading h where subscripts 1 and 2 denote inlet and outlet flow respectively, and the length of the flow path L as shown in equation (2) below [7].

$$Q = AK \frac{(h_1 - h_2)}{L} \quad (2)$$

This relationship in (2) was found based on a series of experiments through a porous medium with a known flow rate and the pressure drop from inlet to outlet recorded, similar to the experiments done in this thesis to determine hydraulic sheet resistance.

Because Darcy’s law is based solely on empirical evidence, it lacks a satisfactory connection to a greater physical theory of continuity as some may describe as a “mass-point” or “microscopic” approach [8]. As such, many investigators have attempted to relate Darcy’s law to one of these theories with varying degrees of success. Because Darcy’s law is essential to this paper’s topic, some focus will be placed on the most notable of these derivations to give an adequate background on this theory for proper context.

First we must change the form of Darcy’s law so that we can relate physical and dynamical parameters to field forces. This transformation is first done by experiment to confirm that Darcy’s law is invariant of earth’s gravitational field with respect to flow direction. We can visualize a field in three-dimensional space where at any point there exists a scalar value h defined by the height above an elevation datum with respect to which the flow moves perpendicularly. We can breakdown Darcy’s law into flow rate Q divided by the cross-sectional area A , as a ratio called flow density q .

$$q = \frac{Q}{A} \tag{3}$$

The flow density represents an impetus in the direction of flow, while still being dependent on the manometer head (∇h), and a generalized impedance term K which remains a lumped parameter capturing both physical and geometric conditions [7].

$$\mathbf{q} = -K \nabla(h) \tag{4}$$

From the K term we can extract fluid density ρ and dynamic viscosity μ , as well as an average particle diameter d and dimensionless factor of proportionality N which captures the geometry of the paths through the porous media [7]. The K' accounts for any missed terms from the lumped parameter K .

$$\mathbf{q} = -K'(Nd^2)\left(\frac{\rho}{\mu}\right) \nabla(h) \quad (5)$$

We can now address the impetus term by relating the manometer height to the hydrostatic equation for pressure P ,

$$P = \rho g(h - z) \quad (6)$$

thus allowing us to express the gradient of h in terms of pressure and a height z above a reference datum. Gravitational acceleration is represented by the customary variable g .

$$-\nabla(h) = -\left(\frac{1}{\rho g}\right) \nabla(P) - \nabla(z) \quad (7)$$

By multiplying both sides by g we can rewrite the expression as below.

$$-g \nabla(h) = -\left(\frac{1}{\rho}\right) \nabla(P) - g \nabla(z) \quad (8)$$

The last term is the force of gravity pointing downward towards the earth and can be re-expressed as vector $\mathbf{G}$.

$$-g \nabla(z) = \mathbf{G} \quad (9)$$

Now we can change the expression for the impetus of flow to the below.

$$-g \nabla(h) = \mathbf{G} - \left(\frac{1}{\rho}\right) \nabla(P) \quad (10)$$

The terms on the right-hand side of the equation express direction and magnitude of the force applied upon a unit mass of fluid by both gravity and the gradient of fluid pressure in the direction of flow. Another insight is that the gravitational term g is the final concealed term of K' [7]. Now that we can equate the manometer height to physical forces, we will continue to use the left side of equation (10) and move the scalar g term inside the gradient operation as below.

$$-g \nabla(h) = -\nabla(gh) \quad (11)$$

In this expression the (gh) term represents the work required to elevate a unit mass of matter from the elevation datum to the height h in the manometer. It follows that,

$$gh = \Phi \quad (12)$$

where the expression can be viewed as the energy per unit mass or potential Φ . A more complete expression for the energy per unit mass is,

$$\Phi = gz + \int \frac{dP}{\rho} \quad (13)$$

which shows the additional contribution from the pressure field [7]. Now the expression,

$$\mathbf{F} = -\nabla(\Phi) \quad (14)$$

is the intensity of the force field $\mathbf{F}$ acting upon the fluid. Finally, we can combine equations (5), (11), (12) and (14) to re-express Darcy's law within the context of physical properties and field-forces.

$$\mathbf{q} = -(Nd^2)\left(\frac{\rho}{\mu}\right) \nabla(\Phi) = \sigma \mathbf{F} \quad (15)$$

Where the hydraulic bulk conductivity σ of the system is given by,

$$\sigma = [-(Nd^2)(\frac{\rho}{\mu})] \quad (16)$$

Hydraulic bulk conductivity has the units of seconds. When we compare this version of Darcy's law in (15) to Ohm's law, we can see the parallel between them.

$$\text{Darcy's law:} \quad \mathbf{q} = \sigma \mathbf{F} = -\sigma \nabla(\Phi) \quad (17)$$

$$\text{Ohm's law:} \quad \mathbf{i} = \sigma_e \mathbf{E} = -\sigma_e \nabla(V) \quad (18)$$

In Ohm's law $\mathbf{i}$ is the current density, σ_e is the electrical conductivity, $\mathbf{E}$ is the electromagnetic field force, and V is the electrical potential [7].

With a more complete theoretical analogy between hydraulic and electrical behavior based on physical properties and forces, we can examine the theory of electrical sheet resistance and attempt to apply Darcy's law to propose a theory of hydraulic sheet resistance.

2.2 Review of the Four-Point Probe Method

The discussion of the four-point probe method and electrical sheet resistance are nearly inseparable as the theory of electrical sheet resistance comes from this testing method. Originally, the four-point probe method was used to test the resistivity of the earth as described in a paper published in 1915 by Frank Wenner. In this paper Wenner describes submerging four probes at regular distances in-line in the ground where the outer two probes are an anode and cathode that conduct current past the inner two probes that measure the electrical potential [9]. With this setup Wenner was able to

experimentally determine the resistivity of the soil in much the same way Darcy was able to measure hydraulic resistivity sixty years earlier.

Later in 1954, L.B. Valdes wrote a paper describing the application of the four-point probe method on the surface of germanium crystals of various geometries and positions relative to boundary conditions. Valdes' work covers experimental test scenarios along with the mathematical expressions for predicting electrical resistivity for each case. One such case is measuring the surface of a germanium crystal slab with a thickness approximately equal to or greater than the spacing of the probes [10]. Figure 1 shows this four-point probe setup on a conductive surface.

Figure 1. Diagram of the four-point probe method on a surface with the outer probes conducting current and the inner probes reading the potential difference.

Uhlir Jr. adds to this research with the insight that testing progressively thinner slabs requires a larger correction divisor in order to calculate the electrical resistivity. Uhlir concludes that this is more desirable in the statement,

“In general, a large correction divisor is desirable because it indicates that a relatively large potential difference is to be measured. For this reason, the use of the four-point probe on thin slices should be and, in fact, is found in practice to be quite satisfactory [11].”

In 1957 the the four-point probe method was applied to thin sheets in a paper authored by F.M. Smits. Smits expands the work of Valdes and Uhlir of four-point probe testing to an infinite sheet with a relatively small thickness compared to probe spacing. From this, Smits makes reference to sheet resistivity as a function of distance of dipoles and formally defines sheet resistance [5].

2.3 Defining Electrical Sheet Resistance

Although not explicitly derived in Smits’ work, a complete description can be shown by application of Ohm’s law and visualizing the current density field approximated as an expanding outer surface of a cylinder.

Figure 2. A representation of the hypothetical current density field assumed in electrical sheet resistance definition.

Starting with Ohm's law from equation (18) we can make substitutions for each term starting with current density. Knowing that current density i is current I divided by area A , and that the area of the outer surface of a cylinder, we can make the substitution into Ohm's law.

$$\text{cylinder wall area } A = 2\pi r t \quad (19)$$

$$i = \frac{I}{A} = \frac{I}{2\pi r t} \quad (20)$$

For the right-hand side, we will substitute electrical conductivity σ_e with electrical bulk resistivity ρ_b and re-express electric field-force intensity E_e as the rate of change of the potential (voltage) per unit length which is the radius of the expanding ring of the current density field shown in figure 2.

$$\sigma_e = \frac{1}{\rho_b} \quad (21)$$

$$-E = -\frac{dV}{dr} \quad (22)$$

With these substitutions into Ohm's law we get equation (23) below. With some manipulation we have equation (24) with the definite integrals to describe the voltage V measured at a point r distance from the current source [12].

$$\frac{I}{2\pi r t} = -\left(\frac{1}{\rho_b}\right)\left(\frac{dV}{dr}\right) \quad (23)$$

$$\int_0^V dV = -\frac{\rho_b I}{2\pi t} \int_0^r \frac{dr}{r} \quad (24)$$

Smits introduces the sheet resistivity term ρ_s shown in equation (25) asserting that from bulk resistivity one can determine sheet resistivity, where t is sheet thickness [5].

$$\rho_s = \frac{\rho_b}{t} \quad (25)$$

Dimensional analysis of electrical sheet resistivity shows it has units of ohms (Ω) which can explain why the terms “sheet resistivity” and “sheet resistance” are often used interchangeably. However, this is more commonly and perhaps accurately expressed as ohms per square ($\Omega/\square$) to distinguish it from resistance and reinforce its appropriate application in 2-D resistivity which is independent of sheet dimensions perpendicular to the thickness direction [12].

Equation (25) can be substituted into equation (24) and the definite integrals are resolved to give equation (26) below,

$$\Delta V = -\left(\frac{I\rho_s}{2\pi}\right)\ln r \quad (26)$$

where ΔV is the voltage drop from the current source to a point at a radial distance r [5].

From equation (26) we can apply the scenario of a four point probe test where voltage is measured between the current source and sink and get the expression below,

$$\Delta V = V_2 - V_3 = \frac{I\rho_s}{2\pi}\ln\left(\frac{s_2s_3}{s_1s_4}\right) \quad (27)$$

where V_2 and V_3 is the voltage reading of the inner two probes and s_1, s_2, s_3, s_4 is the spacing between the respective probes as referenced in figure 1 [12]. For the case with equal probe spacing and being relatively large compared to thickness ($s \gg t$), we can re-express equation (28) as below [5].

$$\Delta V = \frac{I\rho_s}{\pi}\ln 2 \quad (28)$$

Other variations of the expression above are given for four-point probe measurements of rectangular and circular samples of finite dimensions and alternative test configurations [5], although we will ignore these variations for the scope of this thesis.

2.4 Applying Darcy's Law to Sheet Resistance

To begin, we will re-define certain terms and demonstrate the analogy between Ohm's law and electrical sheet resistance to Darcy's law and hydraulic sheet resistance.

We will define a new term for hydraulic bulk resistivity τ_b . Similar to electrical theory, we will also define hydraulic bulk resistivity as the inverse of hydraulic conductivity shown in equation (29) in which the units of τ_b are an inverse second (s^{-1}).

$$\sigma = \frac{1}{\tau_b} \quad (29)$$

By extension, hydraulic sheet resistivity is defined as below,

$$\tau_s = \frac{\tau_b}{t} \quad (30)$$

where τ_s is in units of an inverse meter second ($m^{-1} \cdot s^{-1}$). We can apply these terms to Darcy's law from equation (17) and convert to a simplified form.

$$\mathbf{q} = \frac{1}{\tau_b} \mathbf{F} \quad (31)$$

$$\mathbf{q}(A) = \frac{l}{\tau_b} \mathbf{F}(l) \quad (32)$$

In equation (32) cross-sectional area A is multiplied by the flow density on the left-hand side and the area is split into two length terms on the right-hand side. We can re-express the hydraulic resistivity term as hydraulic resistance as in equation (33), similar to the equivalent electrical expression in equation (34).

$$\frac{\tau_b}{l} = R_h \quad (33)$$

$$\frac{\rho_b l}{A} = \frac{\rho_b}{l} = R \quad (34)$$

These expressions above can now be substituted into equation (17) which can be rewritten as,

$$Q = \frac{\Phi}{R_h} \quad (35)$$

where the flow rate Q is expressed in units of (m^3/s) and is analogous to current I , hydraulic resistance R_h is expressed in units of ($\text{m}^{-1} \cdot \text{s}^{-1}$) and is analogous to electrical resistance R , and energy per unit mass Φ is expressed in units of (m^2/s^2) and is the analogous term for voltage V . In addition, the energy per unit mass term Φ defined in equation (13) shows that the energy contribution is from both the work to move against the gravity field and the pressure difference within a field. The gradient of Φ has dimensions of a meter per second squared (m/s^2). This is dimensionally consistent with equation (17) with the units of flow density q are meters per second (m/s) and the units of hydraulic conductivity are seconds. Equation (35) does not contain a unit of mass because it is cancelled out of Φ by the pressure and density terms as shown in equation (13).

With the above hydraulic equivalents established we can now hypothesize a statement for hydraulic sheet resistance. We can substitute equations (3) and (29) into the form of Darcy's law from equation (17). In addition, we will express the magnitude of the hydraulic field force F as below.

$$F = \frac{d\Phi}{dx} \quad (36)$$

Unlike electrical sheet resistance, saturated flow must move down the sheet to the sink or outlet. Figure 4 shows the flow density field from inlet to outlet of the test setup as a comparison to electrical sheet resistance shown in figure 3.

Figure 3. A representation of the hypothetical flow density field from inlet to outlet with boundary conditions tested in this work.

With these substitutions we can re-express Darcy's law as below.

$$d\Phi = -\frac{Q\tau_b}{A} dx \quad (37)$$

Then we separate the area into width w and thickness t of the flow inlet and apply equation (30) to include τ_s . We now integrate to get equation (39) below which is the hydraulic analogy to equation (1).

$$\int d\Phi = -\frac{Q\tau_s}{w} \int dx \quad (38)$$

$$\Delta\Phi = -\left(\frac{Q\tau_s}{w}\right)\Delta x \quad (39)$$

Equation (39) addresses the hypothesis as an expression of hydraulic sheet resistance.

For convenience, equation (39) and equation (13) can be combined and expressed in a way that presents a more general form of Darcy's law.

$$gz + \frac{\Delta P}{\rho} = -\left(\frac{Q\tau_s}{w}\right)\Delta x \quad (40)$$

Here we see how the electrical and hydraulic analogy diverge as the potential drop is linear for the above equation but logarithmic for electrical sheet resistance from equation (1). This can be explained by the initial setup shown in figure 3 that shows how fluids move in-plane. It is uncertain if saturated fluid flow can move radially in practical devices as surface current does in figure 2. For the case of electrical sheet resistance, the expanding current density field emanates radially from a point source explaining the logarithmic voltage decrease. Although there is a theoretical line source (perpendicular to planar radial flow) well-known in fluid mechanics theory, in practical hydraulic flow the conservation of mass must be observed so the flow source has finite area. In addition, a radial movement in hydraulics requires an outlet around the source as compared to electrical sheet conductance that may assume an infinite sheet. For hydraulics there cannot be an infinite saturated porous medium sheet as it would require an infinite hydraulic potential Φ to allow flow.

For completeness, we can substitute the right-hand side of equation (39) with equation (30), (29), and (16) and get a more familiar expression of Darcy's law shown in equation (41).

$$gz + \frac{\Delta P}{\rho} = -\left(\frac{Q\mu}{Nd^2\rho tw}\right) \Delta x \quad (41)$$

From here, Nd^2 can also be combined as a single term k which is the Darcy permeability coefficient which is different from K (Darcy's hydraulic conductivity term). Further simplification can be done by multiplying all terms by ρ to get another recognizable form of Darcy's law as equation (42) [13].

$$\rho gz + \Delta P = -\left(\frac{Q\mu}{ktw}\right) \Delta x \quad (42)$$

3. RELATED WORK

3.1 Flow Regimes within Porous Media

3.1.1 Flow Regime Identification Criteria

Porous media flow follows Darcy's law for sufficiently low velocities in most soil based porous media, but when one considers different fluids and the alternative porous media, it is better to distinguish flow behavior by Reynolds number instead of flow velocity. It is generally agreed that Darcy flow occurs at $Re \ll 1$ but there is little consensus on the exact transition conditions. Recent research shows the transition is at a Reynolds number between one and ten [14] [15] [16]. Specifically, research done by [17] suggests the upper limit of Darcy's law is at $Re = 1$, whereas [16] suggests this occurs at a Reynolds number of 2 or 3. [18] suggests non-Darcy flow begins at a Reynolds number of 5, while other research suggests the transition as high as $Re = 10$ [15] [16] [13]. With research suggesting such a wide range of Reynolds numbers as the upper limit of the Darcy regime, one could speculate that the conditions for Darcy flow are not entirely understood or that Reynolds number is not an adequate indicator of flow regime.

Because of the uncertainty in using Reynolds number to predict flow behavior, some researches have employed a modified Reynolds number which takes into account the voidage of the porous media [19]. The modified Reynolds number requires definition of the voidage term ϵ , and an equivalent particle diameter d_{part} as defined below,

$$\epsilon = 1 - \frac{\rho_{bulk}}{\rho_a} \quad (43)$$

$$d_{part} = \frac{6V_{part}}{S_{part}} \quad (44)$$

where ρ_{bulk} is bulk density, ρ_a is apparent density, V_{part} is particle volume and S_{part} is particle surface area [20]. From this we can define a modified Reynolds number as,

$$Re_m = \frac{Re}{1 - \epsilon} = \frac{\rho q d_{part}}{\mu(1 - \epsilon)} \quad (45)$$

where ρ and μ are the fluid density and dynamic viscosity, respectively [20]. The modified Reynolds number is used to capture particle irregularity and voidage which Forchheimer-type equations do not directly take into account [21]. This is important because research suggests voidage and cell volume have a significant effect on flow regime [20] [19] [22].

Other such examples of modified Reynolds numbers include the particle Reynolds number, interstitial Reynolds number, column Reynolds number and the pore Reynolds number, of which all are attempts to normalize Reynolds numbers to a feature of porous media structure [23]. For example, the pore Reynolds number is based on the capillary representation of porous media and is defined as,

$$Re_p = \frac{v_p d_p \rho}{\mu} \quad (46)$$

where v_p is the pore velocity and d_p is the pore diameter. Also, suggested by the data is that the stable laminar regime (linear) transitions at about $Re_p \approx 180$ [23].

Other researchers have developed alternative criteria to classify flow regime, such as flow visualization [24] and the current fluctuation measurement method [23]. However, both of these methods require direct measurement or observation of a flow. An indirect method has been developed that attributes the contribution of inertia to overall pressure as the criterion for the linear regime, inertial regime and turbulent regime. This inertial contribution ratio is defined as,

$$X = 1 - \frac{\mu q L}{k \Delta P} \quad (47)$$

where L is flow distance and k is the permeability coefficient. Here, an X value below 0.70 corresponds to the linear regime (Darcy), 0.70 to 0.91 is the inertial regime (as in equation 3), and values above 0.91 represents the turbulent regime [23].

3.1.2 The Weak Inertia Regime

Forchheimer was the first to describe flow behavior beyond the Darcy regime in 1901 but recent research by [25] suggests that there are sub-regimes which characterize all non-Darcy flow. These regimes past the Darcy regime are first the weak inertia regime, followed by the Forchheimer regime (or strong inertia), a transitional regime from Forchheimer regime to turbulence, and finally turbulence [25]. The transitioning between these regimes is partially explained by growing contribution of inertial forces over viscous forces as Reynolds number is increased [26]. The weak inertia regime is one in which the viscous and inertial forces are on the same order of magnitude of flow resistance. Here we see the contribution from Darcy's law as the first expression, added

by a second expression with the weak inertia coefficient (γ) and proportional to the flow velocity cubed [26], where q is the magnitude of the corresponding vector in equation (31).

$$-\frac{dp}{dx} = \frac{\mu}{k}q + \frac{\gamma}{\mu}\rho^2 q^3 \quad (48)$$

The numerical deviation of the weak inertia equation from Darcy's law was first shown by [27] and again by [28] and analytical derivation for homogeneous isotropic and spatially periodic porous media was done by [29]. The weak inertia regime was confirmed numerically by [30] and [25] and experimentally confirmed by [31].

3.1.3 The Forchheimer Regime

Beyond the weak inertia regime is the Forchheimer regime originally described as the below expression,

$$-\frac{dp}{dx} = aq + bq^2 \quad (49)$$

where coefficients a and b are constants [32]. In the Forchheimer regime, it is important to note that the inertia term is squared instead of cubed as in the weak inertia regime.

Recent work has made progress in providing some insight into the constants by relating them to flow properties. Because of these efforts we can express the Forchheimer regime as,

$$-\frac{dp}{dx} = \frac{\mu}{k_f}q + \beta\mu q^2 \quad (50)$$

where k_f is the Forchheimer permeability constant (in units of m^2) and β is an inertial resistance coefficient [33] [34] [35]. The second form of Forchheimer's equation was defined by [36] [14] [37]. It is important to note that the Forchheimer permeability constant k_f is not hydraulic permeability constant used in Darcy's law because of the transition of the weak inertia regime, or $k \neq k_f$. It is also generally assumed that the reason for the transition between the weak inertia regime and the Forchheimer regime is due to viscous dissipation or the “conversion of kinetic energy into internal energy by work done against the viscous stress [26].”

There have been many sources that can be credited with corroborating the Forchheimer equation such as [33] [38] [39] [40] [41] [42] [43] [44] [45]. Irmay [46] derived the Forchheimer equation from the Navier-Stokes equation using a model of packed spheres of equal diameter for a homogeneous isotropic medium. The Forchheimer equation for high velocity isothermal flow through porous media was derived based on fundamental laws of continuum mechanics [47] [42] [16]. In addition, the Forchheimer equation was derived by the averaging theorem for idealized porous media [48] and by a matched asymptotic expression for a rigid porous media [49]. The Forchheimer equation was also derived for some special cases such as non-idealized media with an ideal Newtonian fluid [50], and compressible fluids [51].

Simulation has been conducted in recent research, providing some insights into generalized flow. Despite the existence of the weak inertia regime, research involving

Navier-Stokes simulations and laboratory experiments suggests the Forchheimer equation can accurately predict pressure drops over the entire range of Reynolds numbers because the Forchheimer equation effectively simplifies to Darcy's law at low Reynolds numbers [52]. This is supported by numerical simulations of steady-state flow of incompressible Newtonian fluids at a range of Reynolds numbers through both two-dimensional and three-dimensional porous media. This research shows a more acute transition of the weak inertia regime in three-dimensional porous media as compared to two-dimensional porous media [53]. Other recent research suggests that a shortened weak inertia regime correlates to a tighter distribution of grain size in the porous media [54].

3.1.4 The Turbulent Regime

There is limited research on the turbulent regime but an important distinction within the literature is if the fluid is compressible or incompressible. Research investigating water shows that beyond the Forchheimer regime a sudden decrease in hydrostatic pressure is observed. Researchers attribute this sudden pressure drop to a flow transition to the turbulent regime [21] [55].

Interestingly, research involving gas does not show this sharp pressure drop effect but rather an increase in the rate of change of the pressure gradient for an incremental increase of flow rate [20] [56]. For these studies, turbulent transition is determined by inertial contribution to pressure drop by a factor X , as shown in (47). However, an observation for beds of particle-based porous media is that irregularity in particle shape

as measured by its sphericity seems to be the largest contributor to how readily porous media flow may transition to the turbulent regime [20].

3.2 Explanations for Non-Darcy Flow in Porous Media

3.2.1 Inertial Effects as a Contributor to Non-Darcy Flow

The earliest research related nonlinear (non-Darcy) porous media flow to turbulence [33], but later research rejects this suggesting nonlinearity starts before turbulence because of the presence of a large linear term and lack of a sharp transition which usually marks turbulent flow [39] [14] [43]. Generally, most researchers attribute the nonlinear behavior to inertial forces in laminar flow [57] [58]. This is supported by [59] [28] that also suggest nonlinear behavior is caused only by the presence of inertial forces. [60] proposes nonlinearity is due solely to inertia and an inertia-viscous cross effect. Other research attributes nonlinearity between flow velocity and pressure drop specifically to streamline pattern deformation by inertial forces [40].

3.2.2 Porous Media Qualities as a Contributor to Non-Darcy Flow

Another body of research focuses on the qualities of the porous media itself and its effect on the flow path as an explanation for nonlinear (non-Darcy) behavior. One example is research attributing nonlinearity to macro-roughness of pores [61]. This is supported by recent experimental research that calculates Forchheimer coefficients in unconsolidated porous media and suggests that the inertial term β is smaller for spherical glass beads than for cubic natural sand grains [54]. Another example is research that

attributes nonlinearity to kinetic energy losses due to constrictions in porous media [62]. Similarly, [43] attributes nonlinearity to convective acceleration and deceleration of fluid particles in porous media flow, [63] relates nonlinearity to microscopic inertial force manifested as the interfacial drag force, and [64] to channel curvature. It is also possible that all the examples above are contributing to the inertial forces which cause nonlinear behavior. If so, a significant quality of porous media that contributes to nonlinearity is tortuosity [65]. Thus, an increase in tortuosity corresponds to an increase of the Forchheimer coefficient [26].

3.3 Boundary Condition Effects in Porous Media Flow

Limited research has been published on the wall effects of closed porous media flow. Most research investigates a cylindrical containment vessel holding the porous media. Within this context, research suggests that particle geometry and packing arrangement have a significant effect on how the vessel wall contributes to the internal Reynolds number [23]. Assuming spherical particles, early research suggests that both wall effects diminish and porosity profiles become established at a distance of approximately 3.5 particle diameters away from the wall [66]. More recent work measures the bed-to-particle diameter ratio as an indicator of the wall effects. This work suggests that wall effects should be considered when the diameter of the bed (D) is less than ten times the particle diameter (d_{part}), or $D < 10 d_{part}$ [67].

There is little research published on the inlet and outlet pressure drop effects for porous media flow. One work presupposes flows with both restricted and unrestricted paths, described as a fluid surface fraction. The data suggest that a porous media length, in which the pressure drop contributed by the inlet and outlet dominate over the porous media pressure drop, is on the order of the average pore diameter of the porous media [68]. The same research also assert that most porous media are so restrictive that the inlet and outlet pressure drops have an insignificant contribution to the total pressure drop [68].

3.4 Paper-Based Porous Media Flow Device Research

3.4.1 Current Application of Paper-Based Porous Media

The most widely researched application of paper-based porous media flow devices is in the biotechnology fields. This technology has enabled autonomous and multiplexing capability in drug screening and *in vitro* devices [4]. A great appeal for the technology is its use of capillary forces for fluid manipulation allowing low cost and disposability of the devices [69]. These advantages are particularly well suited in the subject of genomics. After the completion of the human genome project, companies such as Illumina Inc. and Fluidigm Corp. have focused on developing increasingly automated genetic testing for purposes of drug discovery and biomarker identification as some applications. This technology includes the use of paper-based devices as well as next-generation sequencing and picoliter volume droplet systems [4].

Another biotechnology field of high applicability is point-of-care diagnostics, in which many companies like Abbott Laboratories have expanded diagnostic products to handheld devices that use microfluidics and electrochemical detection. One such device, the i-STAT system, analyzes blood chemistry to detect electrolytes, metabolites, and gases as well as perform lateral flow immunoassays [4]. Other companies such as Daktari Diagnostics Inc. and Diagnostics for All, focuses on building low-cost high-performance devices for developing world markets to address everything from HIV detection to veterinary tests to environmental monitoring tests [4]. Based on these applications, porous media research in device design focuses on both time-delay mechanisms and flow control mechanisms for microfluidic mixing, multiplexing and signaling.

3.4.2 Time-Delay Research in Paper-Based Porous Media

Techniques in time-delay vary depending upon the desired function. For devices that seek to indicate the completion of a test or assay, deposited paraffin wax can be used as an inhibitor of flow. One device stacks layers of porous media with some layers coated in wax and non-permeable material with cutouts for through-plane flow paths [70]. These devices use a single sample entry path that splits in two paths for the assay conduit and the fluidic timer conduit. This design benefits by being self-calibrating as environmental effects such as humidity have equal impact to the rate of testing and the time tracking [71]. In addition, properties of the fluid such as viscosity and density, as

well as properties of the porous media such as permeability and tortuosity, are also self-calibrating within the microfluidic device [71]. These types of wax-deposited devices show an accuracy of 97% and precision of 90% when compared to the assay completion time in experiments with untrained operators [71].

Other devices use time-delay to control automated sequential delivery of multiple fluids to a testing or detection zone or zones. These techniques use dissolvable sugar (sucrose) applied to paper to delay all flow until the sucrose is dissolved and flow resumes [72]. One team of researchers test this by dipping strips of paper in various concentrations of sucrose solutions from 10% to 70% and then allowed the test strips to dry before testing. An asymptotic relationship is observed between time delay and concentration with an arrival time of 44 seconds for a 0% sucrose samples to 53 minutes for a 70% sucrose sample [72]. An approximate 12% error in arrival time between a control strip without sucrose and sucrose samples was observed across experiments which takes into account self-calibrating factors such as the fluid, paper and environment [72].

3.4.3 Flow Control Research in Paper-Based Porous Media

Other research involving paper-based porous media devices explore flow control for mixing and pathing. Similar to some time-delay mechanisms, these mechanisms often use inkjet deposition of wax (or another hydrophobic material) onto paper-based porous media and a secondary process such as heating to let the hydrophobic material permeate

the porous media [73] [74] [70]. One study uses commercially available printers to deposit prescribed concentrations of wax onto paper for two-fluid mixing. This study used synthetic food dye to give a colorimetric response to correlate fluidic mixing ratio and the brightness intensity setting on printers. This study showed an approximately 4% difference across all ratios from the expected mixed ratio to measured mixed ratio [74]. Interestingly, this methodology can create complex 2-D fluidic circuits as the highest concentrations of wax deposition could completely block flow in some regions while leaving selective degrees of permeability in other regions [74].

4. METHODOLOGY

4.1 Selection of Paper Media and Sample Preparation

The medium selected for study is the Whatman® quantitative filter paper, hardened low-ash, grade 50 Whatman 1450-916, from GE Healthcare Bio-Sciences, Pittsburgh, Pennsylvania, USA. This filter has an ash content of $\leq 0.015\%$ where ash content is determined by ignition of the cellulose filter at 900°C in air [75]. This ash content is indicative of the material purity which is a desired property for the chosen medium as impurities can affect testing results.

This filter paper has a liquid flow rate of 2685 s/100 mL as measured by the Herzberg method where pre-filtered deaerated water is applied to a filter with a 10 cm^2 area at a constant hydrostatic pressure head of 10 cm of water [75]. This measurement along with an average pore size of $2.7\text{ }\mu\text{m}$ is indicative of the overall fluid permeability of the filter. This filter is fairly restrictive to flow, allowing pressure in the short transverse direction to equally distribute as fluid flows in the long transverse direction.

Figure 4. Scanning electron microscope (SEM) image of grade 50 Whatman filter paper.

This paper is hardened with a highly glazed surface finish which helps to strengthen the paper while wet, allowing extended use without the filter breakdown. In addition, the glazed surface helps to keep the surface free of loose surface fibers. The manufacturer reports an average pore size of $2.7\ \mu\text{m}$ for the medium which, at the maximum flow rate used in experiments, results in a Reynolds number of 1.7×10^{-14} [3]. An image of this filter is shown in figure 4 at 300 times magnification, allowing the matrix of cellulose fibers to be seen.

The overall rectangular dimensions of the filter paper of 150 mm by 230 mm allow it to be cut and placed in the 88 mm x 147 mm test area. The thickness of the filter paper is 121 μm , but when wetted swells to approximately 150 μm based on sample testing [3].

4.2 Design of the Hydraulic Resistance Tester

4.2.1 Functional Requirements

To test the application of Darcy's law in the special case of sheet resistance, a device was built to direct fluid into a porous medium of a fixed cross section. The device has one port for fluid flow of a known rate to enter the device and a reservoir for the fluid to gather and push evenly into the thin porous medium. Along the length of the flow direction, several ports were placed to test the local pressure in order to track pressure drop as a function of the distance traveled. Additional pressure ports, transverse to the flow direction, are included to confirm an even pressure field as the fluid moves through the porous medium. The device has one exit open atmospheric pressure at the end of the porous medium, with the same cross sectional area of the porous medium. To test Darcy's law, the total volume of fluid into the device must travel to the exit, necessitating proper sealing at each pressure port and along the outer perimeter of the device. Finally, a high degree of flatness for the internal surfaces as well as deterministic spacing is ensured to allow a fixed cross section throughout the device.

4.2.2 Kinematic Design and Construction

The design of the Hydraulic Resistance Tester relies on kinematic principles with the goal to both accurately achieve a known desired cavity thickness in the Z direction and reliably achieve this thickness with repeated testing trials. In addition, the Hydraulic Resistance Tester is designed such that feature tolerances in the X-Y directions are not critical to function, and as a result, the testing apparatus has the fewest tolerance capability requirements possible for the manufacturing process.

To achieve these design goals, two 6061-T6 aluminum plates were machined from the flattest available stock material to the design shown in appendix A. The top plate features a groove along three sides of the perimeter for a rubber gasket, as well as a series of tapped holes for pressure port readings and another series of tapped holes within the groove for fastening to the bottom plate and proper sealing. The most critical dimensional control for both of these plates is the flatness of the primary datum surfaces. To ensure the highest degree of flatness possible, both plates were lapped on the primary datum surface. Measurement by a coordinate measurement machine confirmed the flatness for both plates and will be discussed in greater detail in Section 4.6 below.

The top plate of the Hydraulic Resistance Tester is designed to fit a 0.79 mm thick nitrile rubber gasket within the machined groove, with additional space along the outer edge for the displacement of the gasket once compressed. The gasket also includes cut center holes along its length allowing screws between the top and bottom plates when

fastened together. The depth of the groove is chosen such that proper sealing is accomplished once compressed to the desired thickness of a nominal 315 μm .

To create the desired thickness, polyester spacers are used between the plates. Originally, particle standards were considered because of their tight diameter tolerance ensuring a high confidence of cavity thickness within the Hydraulic Resistance Tester. However, particle standards had many drawbacks such as the potential for Hertz contact stress from the lime glass particle standards and inconsistent particle placement. Alternatively, BoPET (polyester) shim stock has a tolerance of approximately 15 μm , zero risk of contact stress and higher repeatability in placement. After these considerations, rectangular cut polyester shim stock strips were chosen as the ideal spacers between the plates. In all experiments they were placed in three areas: two evenly apart at both sides of the flow exit and one at the back center nearest to the inlet. This triangular placement ensures repeatability once the Hydraulic Resistance Tester is assembled and can be seen on the bill of materials in appendix A.

4.3 Fluid Flow Rate Device and Tube Selection

To control liquid volume flow rate into the kinematic device, a syringe pump (Harvard Apparatus Model 11, Holliston, Massachusetts, USA) is used. This is a low cost syringe pump with microliter (μL) resolution that operates by a spiral gear, controlled by high resolution stepper motors which depresses a fixtured syringe. The syringe diameter is programmed into the syringe pump and the internal microprocessor

drives the stepper motor to produce the prescribed fluid flow. The syringe pump has a rated accuracy of $\pm 0.5\%$ of the displayed flow rate and a “pusher advance” resolution of $0.8\text{ }\mu\text{m}$ travel per motor step.

The typical syringe used has a capacity of 35 mL and the displaced fluid runs through a sealed polyethylene tube of 1/16th inch (1.59 mm) inner diameter by 1/8th inch (3.18 mm) outer diameter which is the same tubing used throughout the experimental setup wherever tubing is used for transport fluid. A 10-32 UNF by 1/16th inch (1.59 mm) inner diameter polypropylene adaptor is used to connect the tubing to the pressure ports of the kinematic device to create a watertight seal.

4.4 Pressure Measurement Methodology

4.4.1 Determining Pressure in Porous Media

The task of selecting the ideal pressure measurement methodology is complicated by the challenge of initially predicting the expected pressure within the testing device. To predict pressure, Darcy's law requires a value of permeability k to be quantified, dependent on the porous medium. If Forchheimer flow is present, inertial coefficient β is also required which is dependent on the geometry and pathing of the porous medium. The task of determining media permeability and inertial coefficient is not trivial and is more often determined experimentally or avoided entirely in other works. As shown in previous sections, there is little theoretical work available to estimate permeability in anything other than very idealized media such as packed spheres.

Also challenging is determining what flow regime will be present for a given experimental setup. Research and theory show that pressure will increase cubically or quadratically depending on the inertial regime state, leading to very different expected pressure drops in the porous medium. Research shows there is an inherent difficulty in predicting this, and most methods often requires observation of the flow before this determination is made.

4.4.2 Pressure Measurement Selection

Resistance to flow is measured by pressure difference, and common methods for pressure measurement include manometers and silicon membrane transducers. The

selection of manometers for these experiments is driven by the parameters of the experiment, considerations of off-the-shelf devices, and merits of manometers.

The experimental setup benefits from a pressure device that will allow the air that initially occupies the porous medium to be completely displaced by the water as it is pumped into the device. If the chosen pressure measurement device does not allow this, air would become trapped in the tapped holes and tubes that are connected to the pressure sensor, potentially exhibiting compressible fluid behavior. In this scenario it is also uncertain if the pressure sensor is exposed to air, liquid or both, as the testing apparatus is handled and the trapped air between the testing device and sensor are minimized. Also, an important parameter is that the experiment would require six pressure measurement devices making a low cost method more ideal.

To compound these challenges, the parameters of the experiment were not completely defined at the time of pressure measurement device selection. Before the device is built the smallest achievable gap in experimentation cannot easily be predicted. Because Darcy's law predicts pressure drop is linearly proportional to the gap or thickness, the difference between the design target of 111 μm and the achieved thickness of 315 μm , means a pressure change of nearly a factor of three.

Off-the-shelf devices such as silicon membrane transducers do not well support these experimental parameters. Pressure sensors need to be sourced with a high degree of certainty on the expected pressure ranges needed. Similarly, the accompanying data

acquisition technology requires a high degree of certainty on pressure measurement resolution. Pressure measurement systems for wet environments with wide ranges, high accuracy and resolution are commercially available such as omega engineering's PX209 Series but are cost prohibitive when six sensors are required [76].

However, there are distinct advantages with silicon membrane transducer solutions. Plate thickness can be optimized to prevent trapped air within device and data acquisition technology can operate continuously capturing more data and can provide greater insight into the time dependent behavior of pressure profiles. If approximate pressure ranges can be predicted or referenced from earlier work, transducer cost can be reduced by choosing narrower ranges. Finally, if there is little concern for compressible flow behavior, dry air transducers are ideal because of a greatly reduced cost compared to wet pressure transducers. The manufacturer Honeywell offers a good selection of such pressure transducers that fit these functional requirements [77].

A reliable, low-cost method of measuring pressure is to fabricate manometers from polyurethane tubing and vertical hard plastic tubes to measure water column height visually. This solution is ideal when internal pressure cannot be easily estimated as the time to build taller manometers is minimal with little impact to costs. Manometer height can be adjusted by selection of shorter or longer tubes, depending on the observed pressures between experiments. For example, a range of column heights between 10 mm and 1200 mm corresponds to hydrostatic pressure between 980 Pa and 11770 Pa.

Because the greatest flow rate is 10 $\mu\text{L}/\text{min}$ and the relatively large section of porous medium flow compared to pores, the inlet pressure losses are negligible and gauge pressure can be reasonably determined by the recorded manometer height above the plane of flow [68]. Another benefit of manometers is that air can be completely displaced by the inlet water removing compressible fluid behavior. Coincidentally, manometers were used in Darcy's original experiment [6].

4.5 Experimentation

4.5.1 Experimental Preparation

Before data collection, the device was pre-wetted and tested to ensure even pressure distribution. This is done by placing manometers on the three pressure ports that run transverse to the flow direction closest to the inlet and closing all other pressure ports. Fluid is ran through the device and manometer height is recorded across all three ports once at steady state to confirm an even pressure distribution in the transverse direction of flow. This is repeated for the three pressure ports running transverse to flow nearest to the outlet.

4.5.2 Experimental Conditions

Experimentation is performed with the Hydraulic Resistance Tester set level so gravitational effects can be ignored. The Hydraulic Resistance Tester is assembled with two stacked Whatman filter paper samples and the 0.0125 inch (315 μm) polyester shims around the perimeter. The Hydraulic Resistance Tester is closed with the socket head

screws tightened in an alternating star pattern in two rounds of torquing. A torque screwdriver set to 4 N·m is used to ensure consistent tightening and to prevent over torque. A 35 mL syringe with distilled water is prepared and then placed in the Harvard Apparatus model 11 syringe pump. Pneumatic tubes are connected from the syringe to the Hydraulic Resistance Tester. Pneumatic tubes are also connected to the pressure ports of the Tester as manometers as seen in the bill of material in appendix A.

Experiments are done at flow rates of 10 $\mu\text{L}/\text{min}$ and 5 $\mu\text{L}/\text{min}$. The syringe pump is set to the appropriate flow rate, the pump is turned on and the time is recorded for the beginning of the experiment. After 21 to 24 hours, first measurements of the pressures are taken to ensure a steady state and that capillary effects can be ignored. Although pressure readings start at different times across experiments, all overlap the 24 to 26 hour interval to ensure comparable data sets. Some experiments are tracked into the 46 to 48 hour time interval to better understand the transitory behavior of the test setup.

Within intervals of data collection, pressure readings are taken by ruler from the top surface of the Hydraulic Resistance Tester to the top of each water column's meniscus in the manometers. These reading are taken every 15 minutes in approximately a two hour duration between hour 24 and hour 26.

4.6 Sources of Uncertainty

4.6.1 Relative and Combined Error

For this work uncertainty is calculated as a relative error as it contributes to the calculation of porous media permeability. Permeability k , is defined by Darcy's law and shown in equation (51).

$$k = -\left(\frac{Q\mu}{tw}\right)\frac{\Delta x}{\Delta P} \quad (51)$$

The uncertainty of any given source is applied to equation (51) giving an absolute uncertainty and then divided by the original k value to give a relative error as a percentage of k as shown in equation (52). Thus, relative error shows how each source of uncertainty affects the final uncertainty of k . Also, because not all sources of uncertainty are symmetric about a center value (such as error due to material compression) or because a symmetric uncertainty source corresponds to an asymmetric error in k , relative error is shown as either a positive or negative away from an ideal k value where all inputs have zero uncertainty.

$$Relative\ error = \frac{\Delta k}{k}(100) \quad (52)$$

The propagation of uncertainty in permeability can be traced through the terms in equation (51). These factors include the flow accuracy of the pump which is taken from the product specification previously discussed. Pressure measurements were taken by observation, thus an uncertainty of ± 0.5 mm is assigned to manometer height. This

corresponds to a ± 5 Pa pressure difference which has a variable impact on the relative error across experiments. However, calculation shows this relative error has a range of $\pm 0.2\%$ to $\pm 0.5\%$ across all data sets. Fluid viscosity is dependent on ambient temperature during the experiments and was recorded between 21°C and 22.7°C with an average of 22°C throughout all experiments. These temperatures correspond to an average dynamic viscosity by table of 0.9544 mPa·s with a tolerance of ± 0.2270 mPa·s which in turn correlate to an uncertainty in k of $\pm 2.38\%$. Another small contributor are the machining tolerances of the tapped holes expressed in equation (51) as ΔX . A reasonable positional tolerance for each hole is ± 0.075 mm which when doubled (to account the position of two holes) correspond to an uncertainty of $\pm 1.09\%$.

The largest contributors to uncertainty are factors that affect the thickness such as the material tolerance of the spacer shim. Per the information provided by the distributor (not available by manufacturer directly) the material tolerance for the shim is ± 0.00063 inches (± 0.016 mm) which correspond to a $+4.8\%$ and -5.3% uncertainty of k [78]. More difficult to determine is the compression of the material once in assembly. A reasonable estimation of compression is approximately 10% of the original thickness which corresponds to an 11.1% uncertainty of k .

Table 1. Summary of uncertainty as relative error for permeability k calculation.

Source of uncertainty	Tolerance	Relative error of k (-%)	Relative error of k (+%)
Pump accuracy	+/-5%	-5%	+5%
Spacer compression	+0 / -10%	-11.1%	+0
Spacer material tolerance	+/-0.016 mm	-5.3%	+4.8%
Pressure measurement	+/-5 Pa	-0.5%	+0.5%
Fluid viscosity	+/-0.0227 mPa-s	-2.38%	+2.38%
Machining tolerance	+/-0.075 mm	-0.11%	+0.11%

Applying law of propagation of uncertainty, a square root of a sum of squares is used to combine uncertainties and provide an estimate of total uncertainty [79]. Combined error u is defined as,

$$u = \left(\left(\frac{\partial f}{\partial y_1} u_1 \right)^2 + \left(\frac{\partial f}{\partial y_2} u_2 \right)^2 + \left(\frac{\partial f}{\partial y_3} u_3 \right)^2 + \dots \left(\frac{\partial f}{\partial y_n} u_n \right)^2 \right)^{1/2} \quad (53)$$

where f is the function under analysis containing the sources of error, y_i is the first source of uncertainty and u_i is the range of uncertainty of y_i . The second source of uncertainty is y_2 and so forth to n uncertainties. When the six sources of uncertainty shown above are evaluated by equation (53) we yield a combined error of $u = 74.5 \text{ mm}^2$ for a $10 \text{ }\mu\text{L/min}$ flow assuming a 2400 Pa , and a $u = 74.6 \text{ mm}^2$ for a $5 \text{ }\mu\text{L/min}$ flow assuming 1200 Pa .

4.6.2 Unquantified Uncertainty

There are other sources of uncertainty that cannot be quantified as an uncertainty of k for various reasons. This may be because the uncertainties themselves cannot be easily quantified, or because the sources have a complex interaction with multiple variables within equation (53), or possibly because they affect permeability in indirect ways not

accounted for by equation (51). An example is the surface roughness of the plates that were not measured and may have had an effect on the transition of flow regime. This is a difficult uncertainty to resolve because it was not quantified, there is no method to correlate the roughness of the boundary condition of closed porous media flow, and because the mechanisms that affect flow regime are complex.

The width of the flow cross section is an example of a source of uncertainty with a complex effect on calculated permeability. As stated previously, paper samples were cut into the sample widths by precision craft knife (X-ACTO #1, Elmer's Products, Inc., High Point, NC) and a finely graduated ruler. A reasonable estimation for the width tolerance is a tenth of a millimeter as this is approximately the width of the blade. However, the tolerance effect is not limited to the sensitivity to equation (51) but there is also the compounding effect of thin unobstructed flow paths which would have a significant impact to calculated permeability. It is also difficult to determine if the variation in width actually translates to creating these channels because the complaint rubber gasket has a high likelihood of constricting flow through the porous medium. However, there is reason to believe this type of event is rare as there was only one instance observed with leakage through the sides. In this single occurrence, zero pressure readings for all manometers were recorded from a 10 $\mu\text{L}/\text{min}$ to 1000 $\mu\text{L}/\text{min}$ flow rate.

Another source of uncertainty that cannot be correlated to a known uncertainty of k is the flatness of the plates. Despite that flatness measurements were taken on both plates

by a coordinate measurement machine, it is unclear exactly how these surface topologies effect calculated permeability. Variations in both plate's flatness could lead to localized flow velocities and permeabilities as the distance between plates narrow choking flow and compressing the porous medium. Obviously, large variations in flatness could lead to serious doubt over all experimentation as the flow through the porous medium would not be homogenous in the transverse direction to flow (across the width of the paper). This is why the transverse pressure was taken at the experiment's onset to confirm a reasonable homogeneous flow. The measurements from a Zeiss O-INSPECT 543 coordinate measurement machine confirm both plate's flatness within two micrometers at the onset of experimentation and can be seen in appendix B.

5. RESULTS AND DISCUSSION

5.1 Collected Data

5.1.1 Best Data Representation

Data collected are shown in two forms; one as figure 5 which plots pressure for each individual port over time, and another shown in figure 6 which stacks the pressure readings of each pressure port at the location the pressure port is away from the inlet.

Figure 5. Data set 5 pressure readings separated by measurement location as a function of time, measured at intervals over a time interval from 22 hours to 26 hours.

Figure 5 of data set 5 is helpful to show the time dependent behavior to confirm the overall increasing trend of pressure over time. The graph shows that this increase is constant and roughly linear which is true for nearly all cases. Other graphs shown in Appendix C show pressure eventually holding constant, but this is rarer. Because of this observed behavior, some experiment's data collection was extended into the 48 hour range of which all suggest that this increase is not constant and may even reverse. What

can be said of the data is that the pressure profiles (pressure drop between ports) remains approximately constant. A possible explanation for this is that over time the part of the wetted filter paper exposed to air accumulates particles increasing hydraulic resistance. This could explain a slow increase on all pressure port readings but constant pressure differences between ports. Although this is an interesting phenomenon that deserves investigation, this work seeks to isolate or remove the transience from the data to investigate the steady-state behavior.

Figure 6. Data set 5 pressure readings as a function of distance from inlet to outlet, measured at intervals over a time interval from 22 hours to 26 hours.

Figure 6 collapses data set 5 onto the pressure measurement positions as measured as the distance from the inlet. Data shown in this way allow visualization of the pressure profile throughout the porous medium which is ideal to investigate Darcy's law. However, analyzing data as shown in figure 6 is not preferred because the transient behavior of the data prevents a deterministic pressure reading for a port. A solution is to

isolate only the pressure readings at the same instance across all data sets. This allows representative data sets of individual pressures as shown in figure 7. The instance chosen to isolate is hour 25 (or as close as possible) as this is in the middle of observation for all data sets and therefore establishes behavior before and after, confirming a relatively stable process. Figure 7 is the representative pressures of data set 5, showing a very useful form of the data which allows easy comparison of all data and as such will be the form in which data sets will be discussed.

Figure 7. Data set 5 at hour 24.75 showing individual pressure readings and a curve fitting the data as a function of distance from inlet to outlet.

5.1.2 Linear and Nonlinear Determination

Figure 7 shows a curve that is fitted to data set 5. The fitted curve shown in these graphs is done by first determining if the data set is linear or nonlinear in nature. For example, data set 5 is an experiment at the high flow rate (10 $\mu\text{L}/\text{min}$) but another data set with the same flow rate shows a different behavior as seen in figure 8. Interestingly,

even the seemingly linear behavior seen in figure 8 is not entirely linear as the sixth pressure port has a zero pressure reading that is omitted from graphs. The lowest pressure measured is approximately 100 Pa, so the sixth pressure port should be measurable if the data were completely linear.

Figure 8. Data set 2 at hour 25 showing individual pressure readings and a curve fitting the data as a function of distance from inlet to outlet.

To objectively determine if a data set is linear or nonlinear, R^2 values are calculated for the first four and five pressure readings in each data set. A reasonable criterion for linearity is a 4-point and 5-point R^2 value greater than 0.98. The reason that a 6-point R^2 value is not considered is because most data sets do not have a pressure reading for the sixth pressure port. A 4-point R^2 value is useful to see a data set's deviation from linearity as nearly all data sets could be considered linear based on the first four pressure ports. Table 2 shows an overview of all data sets gathered by experimentation with

relevant R^2 values calculated and the determination of each data set as “Linear” or “Nonlinear.”

Table 2. Maximum pressure, linear regression fit values for flow at 10 $\mu\text{L}/\text{min}$ and 5 $\mu\text{L}/\text{min}$.

Data Set	Flow Rate ($\mu\text{L}/\text{min}$)	Maximum Pressure Drop (Pa)	Linear R^2 (4 points)	Linear R^2 (5 points)	Best Fit Equation Type
1	10	2550	0.9934	0.9569	NA**
2	10	2256	0.9983	0.9991	Linear
3	10	10088	0.9002	NA*	Nonlinear
4	10	8656	0.9482	0.8465	Nonlinear
5	10	10088	0.9502	0.8318	Nonlinear
6	5	1442	0.9966	0.9971	Linear
7	5	990	0.9723	0.9857	Linear
8	5	1098	0.9922	0.9943	Linear
9	5	2501	0.9489	0.8590	Nonlinear
10	5	3217	0.9699	0.8158	Nonlinear

* Not available because local pressure at port 5 exceeded manometer height. ** Only partially meets “Linear” classification criteria.

5.2 Analysis

5.2.1 Curve Fit Equations

For the nonlinear data sets many curve types were applied and an R^2 value calculated in the attempt to find the best form that describes the data. In alignment with Forchheimer's equation, quadratic forms were initially investigated but these fits did not have a satisfactory R^2 value. Alternatively, logarithmic equations fit these nonlinear data sets quite well with the form shown in equation (54). Table 3 shows the logarithmic curve fitted equations and R^2 value for all non-zero pressure values within data sets classified as nonlinear. In table 3, A is a shape factor, B controls the location of the

asymptotic pressure drop, and C represents the maximum pressure within the porous medium.

$$A \ln\left(\frac{B - x}{B}\right) + C \quad (54)$$

Table 3. Flow rate, coefficients and regression fit values for data sets fitted by equation (54).

Data Set	Flow Rate ($\mu\text{L}/\text{min}$)	A	B	C	R ²
1	10	600	141.7	2630	0.9999
3	10	1575	139.8	10400	0.9996
4	10	1200	139.7	8650	0.9994
5	10	1635	139.8	11150	0.9991
9	5	570	121.0	2600	0.9999
10	5	470	124.0	3300	0.9993

Similarly, we can display the linear curve fit data sets that follow equation (55) with values shown in table 4. In Table 4, coefficient C has the same meaning as table 3 and represents the maximum pressure within the porous medium as an arithmetic shift above the independent axis, and D is the slope of linear pressure drop within the porous medium.

$$Dx + C \quad (55)$$

Table 4. Flow rate, coefficients and regression fit values for data sets fitted by equation (55).

Data Set	Flow Rate ($\mu\text{L}/\text{min}$)	D	C	Linear R^2 (5 points)
1	10	-8.5	2730.0	0.9569
2	10	-12.0	2400.7	0.9991
6	5	-13.1	1621.2	0.9971
7	5	-6.7	1127.5	0.9857
8	5	-7.6	1232.2	0.9943

5.2.2 Special Observations and Explanations

From table 2 we can see data set 1 does not meet the linear classification criteria because the 5-point R^2 value fails to meet the 0.98 criterion. However, it should be noted that data set 1 is unique in that the 4-point R^2 values does meet the 0.98 criterion and in all other cases, both the 4-point and 5-point R^2 values either both pass or fail. It is also important to note that chronologically data set 1 was performed first and over repeated experiments the surface condition of the plates became worse. This is because over the long exposure to water, the aluminum material slowly oxidized. Polishing abrasives where applied to remove the oxidation but surface condition was changed after this first lost test exposure. It is also reasonable to think that surface condition did not continue to change afterwards as water exposure was taken into consideration with regards to test duration. Finally, data set 1 is unique because it is the only data set that does not conform to the pattern that linear data sets have lower maximum pressure drop than nonlinear data sets at the same flow rate. Because of these reasons, data set 1 will be treated as a special case and shown in both tables 3 and 4 for comparison but treated as a linear data set.

Another important phenomenon is the unusual asymptotic pressure drop in data sets 9 and 10. In all other data sets, pressure profiles show this drop close to the outlet at approximately 140 mm but data sets 9 and 10 show this drop at approximately 125 mm which is shown as coefficient B in table 3 and in figures 9 and 10. Data sets 9 and 10 are also at the low flow rate and both have nonlinear behavior.

Figure 9. Data set 9 at hour 23 showing individual pressure readings and a curve fitting the data as a function of distance from inlet to outlet.

Figure 10. Data set 10 at hour 25 showing individual pressure readings and a curve fitting the data as a function of distance from inlet to outlet.

An explanation for this behavior is that this early pressure drop is symptomatic of porous media flow (under these conditions) in which inertia falls below a critical value. Acute local pressure drop at approximately 125 mm can also be observed in the linear data as well despite that initial pressure is much lower for linear profiles. Figure 11 shows this early pressure drop for the linear flow with a low maximum pressure, which supports the assertion that the early pressure drop is related to viscous forces dominating over inertial forces. Also, a zero pressure reading does not mean there is zero pressure but rather the pressure is too low to be detected by the manometer. It is interesting that data set 10 and data set 6 both show this behavior despite a difference of maximum pressure by a factor of two. Because of these reasons, it is the recommendation of this author that this phenomenon be studied further.

Figure 11. Data set 6 at hour 46.50 showing individual pressure readings and a curve fitting the data as a function of distance from inlet to outlet.

5.2.3 Calculated Material Properties

According to the accepted theory of porous media flow, material permeability can only be determined by Darcy (linear) flow because nonlinear flow includes the quadratic inertial term. The way to visualize this is to plot maximum pressure drop versus flow rate. For linear data sets maximum pressure drop for 10 $\mu\text{L}/\text{min}$ is approximately double the maximum pressure drop at 5 $\mu\text{L}/\text{min}$ as shown in figure 12. This behavior indicates a consistent material permeability as opposed to nonlinear pressure profiles as shown in figure 13. Data collection at different flow rates is required to know if nonlinear pressure profile data sets comply with Forchheimer's equation.

Figure 12. Maximum recorded pressure drop for linear pressure profile data sets (1, 2, 6, 7 & 8) with an “(H)” or “(L)” to denote high and low flow rate.

Figure 13. Maximum recorded pressure drop for linear pressure profile data sets (3, 4, 5, 9 & 10) with an “(H)” or “(L)” to denote high and low flow rate.

Replicate data is required for a strong consensus that these pressure trends hold true. However, assuming these trends are true we can calculate material permeability as shown in table 5.

Table 5. Flow rate and calculated material permeability k (mm^2) from equation (51) of linear data sets.

Data Set	Flow Rate ($\mu\text{L}/\text{min}$)	Material Permeability k (mm^2)	Hydraulic Bulk Resistivity ($1/\text{s}$)	Hydraulic Sheet Resistivity ($1/\text{mm-s}$)
1	10	306	3120	9828
2	10	346	2761	8695
6	5	257	3715	11700
7	5	374	2550	8033
8	5	337	2829	8909
Avg.		324	2995	9433

From table 5 we see an average calculated material permeability of 324 mm^2 which has a standard deviation of 44.6 mm^2 . It is important to note the calculated material permeabilities have a relatively high agreement considering only five data sets could be

used. By dividing fluid viscosity by k , we can show an expression for hydraulic bulk resistivity (or resistance) based on equation (56). By extension, we can express the proposed hydraulic sheet resistivity (or resistance) by dividing the hydraulic bulk resistivity by thickness as shown in equation (30), providing a proposed value for infinitesimally thin porous media.

$$\tau_b = \frac{\mu}{k} \quad (56)$$

6. CONCLUSIONS

6.1 Addressing the Hypothesis

The hypothesis predicts that the lateral fluid flow of a Newtonian incompressible fluid across thin paper-based porous media in a confined conduit, at Reynolds numbers much less than one ($Re \ll 1$), will be in the Darcy regime. Although some experimental trials do show Darcy flow, other experimental data contradict this by showing a nonlinear pressure profile and a nonlinear relationship between maximum pressure and flow rate. In addition, an expression of hydraulic sheet resistance was derived describing thin sheet paper-based porous media flow based on electrical sheet resistance. This was shown in equation (38) which has the same form as electrical sheet resistance shown in equation (1), except that impetus (as energy per unit mass) decreases linearly instead of logarithmically because of different flow density field propagation. In addition, the data do not contradict or preclude the validity of hydraulic sheet resistance.

6.2 Discoveries from Experimentation

It is important to emphasize that there are not enough data to draw any strong conclusions, but assuming these trends hold true we can make some observations. The most immediate observation from experimentation is the wide variation in maximum pressure drop across experiments with fixed conditions. More significant is the discovery that maximum pressure drop and pressure profile seem to be correlated with low pressures relating to linear pressure profiles, and high pressures to nonlinear (possibly

logarithmic) pressure profiles. Although more data are required to substantiate these correlations, the implication is that disordered and unconsolidated porous media such as paper-based porous media, can lead to both Darcy and Forchheimer regime flow with all other controllable experimental variables held constant. These correlations are likely due to a wide variance in overall tortuosity of the porous medium. The effect of which can cause Forchheimer flow (or weak inertia flow) when Reynolds numbers are much less than one, as in the case of this work where maximum Reynolds number is approximately 1.7×10^{-14} .

6.3 Recommendations for Future Work

6.3.1 Recommendations for Methodology

Different approaches are recommended with regards to experimental setup and execution. Based on this work, reasonable pressure ranges could be estimated to aid in the selection of low cost pressure transducer sensors that are usually built for dry environments. Sensors can be protected against fluid by using polyurethane tubing between port and sensor, and because there is little time delay in pressure readings compared to experimental time duration (due to low flow rate), there is little concern with capturing compressible fluid effects due to the trapped air. Data acquisition technology can provide more complete data, allowing better isolation of steady state conditions and better insight into the drivers of transient behavior.

In addition, more pressure sensors are recommended to capture pressure profiles. The acute pressure drop observed near the outlet has the greatest rate of change of nonlinear profiles and provides the greatest uncertainty in curve fitting. Also recommended is a pressure sensor as close to the inlet (but within the porous medium) as possible to determine if maximum theoretical pressure aligns with experimental pressure values.

It is also recommended that testing be done in a clean room or similarly controlled environment as free particle deposit on the exposed wetted porous media would have an effect on hydraulic resistance and is a likely contributor to the transient pressure data observed. It is also recommended for any replicate testing that flow rates are randomized to help control for possible changing conditions of the equipment.

6.3.2 Recommendations for Testing

To test hydraulic sheet resistance directly, more testing is required with different aspect ratios. This can be done by changing out gaskets and spacers and applying more layers of porous media so that the effects of compression are the same. Research suggests that testing progressively thinner aspect ratios will not support a consistent calculated hydraulic sheet resistance because wall effects will begin to dominate the effective hydraulic resistance [67], but for aspect ratios beyond a critical value, the concept of hydraulic sheet resistance may hold true.

Discoveries found from experimentation prompt other possible future work. Experimentation affirming the correlation of a maximum low pressure to linear pressure

profiles and high pressure to nonlinear pressure profiles by replicate testing is recommended. Although it is another possible avenue of future work, this testing would require conditions similar to those used here as it is unclear if these test conditions capture the transition of the Darcy regime and a nonlinear regime by coincidence, or if bimodal behavior can be observed at other flow rates with reduced instances of occurrence. Finally, testing with similar conditions at intermediate flow rates between 5 $\mu\text{L}/\text{min}$ and 10 $\mu\text{L}/\text{min}$, is recommended to confirm that linear pressure profiles correlate with Darcy's law and nonlinear pressure profiles correlate with the Forchheimer's equation or the weak inertia regime.

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APPENDIX A: APPARATUS TECHNICAL DOCUMENTS

* CAD MAINTAINED DRAWING* REV. CONTROLLED DOCUMENT MANUAL CHANGES UNAUTHORIZED PRINTED COPIES OF THIS DOCUMENT ARE CONSIDERED UNCONTROLLED.	BLOCK TOLERANCES APPLY TO METRIC UNITS ONLY. DUAL DIMENSIONS SHALL NOT BE FOR REFERENCE ONLY AND ARE DISPLAYED IN 3 PLACES FOR CONVERSION PURPOSES ONLY		UNITS: MM (INCH)		TOLERANCES: X = ± 0.2		3RD ANGLE PROJECTION		DESIGNED IN SOLIDWORKS		SCALE: 1:2		SHEET 1 OF 1		B	
	ANGULAR: X° = ± 1.0°		SIZE		B		THICKNESS:		8mm ± 0.01mm		FINISH:		-		-	
	MATERIAL:		6061-T6 ALUMINUM		PART DESCRIPTION:		CUSTOMER:		-		PROGRAM NAME:		-		CUSTOMER PART NUMBER:	
	LARGEST PART NUMBER:		TOP PLATE ALUMINIUM...		DATE:		APPROVED BY:		-		DATE:		-		REV:

APPENDIX B: COORDINATE MEASUREMENT MACHINE FLATNESS

ZEISS CALYPSO
6.2.20

Part name
Drawing number
Part ident
Time/Date
Operator

1
9/20/2017 6:18 PM
Mark Sullivan
Order number
Department
Text

— Positive peaks
○ Extreme point

inch	X	Y	Z	
Corner points	1	-0.9873	1.3384	0.0000
	2	-0.9873	7.0472	0.0000
	3	-4.3702	7.0472	0.0000
	4	-4.3702	1.3384	0.0000
Max	0.0000	-4.3702	1.3387	0.0000
Min	0.0000	-1.8333	2.6115	0.0000

0 inch
25000.1

Name	Value	Upper Limit	Points	Filter type	Lc	upr	Probe radius	Vmess [mm/sec]	Evaluation method
Flatness1	0.0011	0.0200	49	No Filter	-	-	1.5003		Minimum Feature

ZEISS CALYPSO
6.2.20

Part name
Drawing number
Part ident
Time/Date
Operator

1
9/20/2017 6:18 PM
Mark Sullivan
Order number
Department
Text

Positive peaks
Extreme point

inch		X	Y	Z
Corner points	1	-0.1233	0.1182	0.0000
	2	-0.1233	-0.9886	0.0000
	3	0.9886	-0.1233	0.0000
	4	0.9886	0.1182	0.0000
Max	0.0001	-2.9123	0.1250	0.0000
Min	0.0000	-0.1233	0.7472	0.0000

(6.0,04) PhotoClock

Name	Value	Upper Limit	Points	Filter type	Lc	upr	Probe radius	Vmsec [mm/sec]	Evaluation method
Flatness2	0.0014	0.0200	29	No Filter	-	-	1.5003		Minimum Feature

APPENDIX C: EXPERIMENTAL RAW DATA CHARTS

Pressure Readings Separated by Location Over Time:

Data set 1 pressure readings separated by measurement location over time.

Data set 2 pressure readings separated by measurement location over time.

Data set 3 pressure readings separated by measurement location over time.

Data set 4 pressure readings separated by measurement location over time.

Data set 5 pressure readings separated by measurement location over time.

Data set 6 pressure readings separated by measurement location over time.

Data set 7 pressure readings separated by measurement location over time.

Data set 8 pressure readings separated by measurement location over time.

Data set 9 pressure readings separated by measurement location over time.

Data set 10 pressure readings separated by measurement location over time.

Pressure Readings Separated by Time over Location:

Data set 1 pressure readings separated by time over measurement location.

Data set 2 pressure readings separated by time over measurement location.

Data set 3 pressure readings separated by time over measurement location.

Data set 4 pressure readings separated by time over measurement location.

Data set 5 pressure readings separated by time over measurement location.

Data set 6 pressure readings separated by time over measurement location.

Data set 7 pressure readings separated by time over measurement location.

Data set 8 pressure readings separated by time over measurement location.

Data set 9 pressure readings separated by time over measurement location.

Data set 10 pressure readings separated by time over measurement location.

Curve Fitted Pressure Readings Over Location at One Instance:

Data set 1 curve fitted pressure readings over location at one instance.

Data set 2 curve fitted pressure readings over location at one instance.

Data set 3 curve fitted pressure readings over location at one instance.

Data set 4 curve fitted pressure readings over location at one instance.

Data set 5 curve fitted pressure readings over location at one instance.

Data set 6 curve fitted pressure readings over location at one instance.

Data set 7 curve fitted pressure readings over location at one instance.

Data set 8 curve fitted pressure readings over location at one instance.

Data set 9 curve fitted pressure readings over location at one instance.

Data set 10 curve fitted pressure readings over location at one instance.