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Determination of Demagnetizing Factors in Nanomagnets by First-Order Reversal Curves and Ferromagnetic Resonance

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DETERMINATION OF DEMAGNETIZING FACTORS IN NANOMAGNETS BY
FIRST-ORDER REVERSAL CURVES AND FERROMAGNETIC RESONANCE

A Thesis

Presented to

The Faculty of the Department of Physics and Astronomy

San José State University

In Partial Fulfillment

of the Requirements for the Degree

Master of Science

by

Karl Muster

August 2019

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The Designated Thesis Committee Approves the Thesis Titled

DETERMINATION OF DEMAGNETIZING FACTORS IN NANOMAGNETS BY
FIRST-ORDER REVERSAL CURVES AND FERROMAGNETIC RESONANCE

by

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APPROVED FOR THE DEPARTMENT OF PHYSICS AND ASTRONOMY

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August 2019

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ABSTRACT
DETERMINATION OF DEMAGNETIZING FACTORS IN NANOMAGNETS BY
FIRST-ORDER REVERSAL CURVES AND FERROMAGNETIC RESONANCE

by Karl Muster

Magnetic reversal in nickel nanowire arrays due to an axial, external magnetic field was studied through first-order reversal curve (FORC) measurement. A general function modeling the observed Preisach distribution was developed using the moving Preisach model. Fitting allowed determination of the sample's demagnetizing field at saturation and modeling magnetic reversal with respect to the internal magnetic field. Magnetization dynamics in the sample were studied through ferromagnetic resonance (FMR) measurement to aid in interpreting FORC fit results. Combining FORC and FMR results yielded the saturation magnetization and effective demagnetizing factors of the sample. Similar analysis was performed for a thin film sample, indicating an effective magnetizing interaction. The calculated axial demagnetizing factor of the array was significantly larger than the expected value for an isolated nanowire by the ellipsoid approximation. Therefore, this study demonstrates successful quantitative analysis of the demagnetizing interaction between nanowires in the array.

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LIST OF ABBREVIATIONS

#D	virtually #-dimensional
AC	alternating (current)
CFD	observed coercive field distribution
CGS	centimeter-gram-seconds system of units
DC	direct (current)
FMR	ferromagnetic resonance
FORC	first-order reversal curve
FWHM	full width at half-maximum magnitude
GPIO	General Purpose Interface Bus (or IEEE-488)
IFD	observed interaction field distribution
LLG	Landau-Lifshitz-Gilbert equation
MRAM	magneto-resistive random-access memory
$\mathcal{O}$	order of magnitude
QCM	quartz crystal microbalance
RF	radio-frequency electromagnetic wave
SFD	switching field distribution of particles
SI	International System of Units
SMA	SubMiniature version A connector
VNA	vector network analyzer
VSM	vibrating sample magnetometer

I. INTRODUCTION AND THEORY

The magnetic properties of nanomagnets were measured using two types of measurement, one static and one dynamic. Analysis procedures were formulated to characterize each sample's magnetic properties. This study focused on two types of nickel nanomagnets, the thin film and the nanowire array, drawn in FIG. 1.1. This chapter outlines the theory behind the static and dynamic measurements and the information gained. In each measurement regime, the theory accounted for typical features when measuring thin films and nanowire arrays.

FIG. 1.1. (a) A thin film is a 2D nanomagnet with two relevant axes: “in-plane” and “perpendicular.” (b) A nanowire array is a parallel assembly of 1D nanomagnets with two relevant axes: “axial” and “transverse.”

1.1 Introduction

This section introduces the research in the context of past studies. Measurement of static magnetism in nanomagnets was motivated by their unique physical properties and technological applications. Measurement techniques and interpretations gradually developed along with advancements in magnetic memory device technology. The developed analysis procedure in this study was inspired by the recent interpretations.

1.1.1 *Nanomagnetism*

The magnetic behavior of a sample can depend on its size, shape, and material. Nanomagnets are magnetic objects with at least one geometric dimension comparable in size to the magnetic feature being studied [1]. A magnetic feature relevant to this study is

a magnetic domain, a region of material $\mathcal{O}(10\text{nm})$ to $\mathcal{O}(10\mu\text{m})$ wide that is magnetized in the same direction [1]. Small enough isolated particles, 60 nm for nickel [1], have only enough space for one domain, allowing a simpler model to be used. On the other hand, layered nanomagnets $\mathcal{O}(1\text{nm})$ thick have tunable properties related to the exchange interaction [2]. The behavior of nanomagnets is typically geometry-dependent and fundamentally different from that of bulk magnets.

The thin film in FIG. 1.1a is a 2D nanomagnet. Magnetization is constrained to be in-plane due to the small perpendicular thickness, $\mathcal{O}(100\text{nm})$. The nanowire is a 1D nanomagnet. Magnetization is constrained to be axial by the small transverse diameter, $\mathcal{O}(10\text{nm})$. Nanowires in an array illustrated in FIG. 1.1b are transversely packed close enough, $\mathcal{O}(100\text{nm})$, to experience the magnetic field of neighboring nanowires. In this thesis, interactions between nanowires in an array were analyzed quantitatively.

Emergent magnetic properties of nanomagnets have both inhibited and enabled engineering applications. Small magnetic cells in magneto-resistive random-access memory (MRAM) use a magnetic domain as a memory storage bit [3]. Miniaturization has introduced layered nanomagnets with improved qualities and has reduced the energy needed to write a bit [3]. On the other hand, bits have become more prone to environmental thermal energy and neighboring bit interactions that cause an unplanned change in the bit's state [3]. Therefore, control and measurement of magnetization have been essential in developing effective magnetic memory devices.

1.1.2 *Development of First-order Reversal Curve Measurement*

The Preisach model, discussed in Section 1.2, was developed by Ferenc Preisach in 1935 [4] and preceded its technological applications. The Preisach model was generalized into its modern, mathematical description in the late 1980s by Isaak Mayergoyz [5] and others [6, 7, 8]. The model was initially applied to describe the static magnetic properties of particles in naturally-occurring magnetic materials [9] measured using a different type

of measurement [10]. In 2000, the first-order reversal curve (FORC) measurement technique, discussed in Section 2.2, was popularized as an experimental extension of the Preisach model for geomagnetism [11].

Static nanomagnetic measurement has co-evolved with technological progress. Moore's Law, the reduction of technology feature size over time, has directed research toward man-made nanomagnets with controlled material properties. The Preisach model and modified models were used to characterize grains in early magnetic recording media [12]. Motivated by recent development of spintronics and nano-scale layered structures, groups have focused on nanowire arrays with non-uniform composition [2] or geometry [13]. There have also been studies on multi-layered films [14] to study the magnetic behavior of interfaces and hetero-structures.

This thesis focused on the magnetic properties of simple arrays of magnetic nanowires. Nanowire arrays were first fabricated and studied for their geometry-dependent magnetic properties controlled during fabrication [15] and interactions between neighboring nanowires [16]. Industry had gained interest in nanowire arrays as a possible magnetic recording medium by the late 1990s [17]. By 2004, various research groups had developed a cheap and simple method of making arrays of magnetic nanowires using anodized aluminum-oxide nano-pore templates [18, 19]. The nanowire array samples could be made using only wet chemistry instead of more expensive techniques like photolithography. These nanowire arrays were a common subject of early FORC measurement studies [20].

Measurement of man-made nanomagnets has resulted in new interpretations of the Preisach model. The moving Preisach model expands upon the moving Preisach model and allows more accurate interpretations of experimental data [12]. Pike et al. performed simulations of the moving Preisach model with other modifications to account for experimental features [21, 22]. Another variation is high-speed FORC measurement,

which characterizes transient magnetic reversal [23]. New models allow more accurate and detailed characterization of nanomagnets.

1.1.3

Thesis Contribution

This study obtained quantitative results by fitting experimental FORC data. This differed from other studies that treated FORC data as a “fingerprint,” or a qualitative chart to identify physical interactions in measured samples [2, 20]. The fit functions discussed in Section 2.2 implemented the quantitative interpretations by Gilbert et al. to model typical FORC results for arrayed nanomagnets [22]. The fit parameters were the intrinsic magnetic properties of the elements in the array and the strength of the interaction between elements.

A dynamic measurement called ferromagnetic resonance (FMR) has been a standard measurement to characterize nanomagnets quantitatively by fitting. FMR results provide additional physical quantities to characterize magnetic interactions in the samples. Interactions within thin films and nanowire arrays were compared quantitatively. Results from FORC and FMR were also combined to approximate other physical parameters according to theory.

The analysis was performed on nickel samples with various nanowire length. However, this thesis only presents quantitative results from a single sample. Most of the determined physical quantities in this thesis only pertain to an array of particular geometry and material.

Thin nickel films represented in FIG. 1.1a were also measured. These results provided a benchmark for verifying the sample preparation process and the derived equations with comparison to published results [24]. Thin film electrodeposition allowed direct measurement of deposition rate, visual observation, and faster preparation time. These advantages allowed the production of many nickel films to test. The procedure for making

nanowires, outlined in Section 2.1 and Appendix B, was informed by nickel film deposition tests.

Custom Python programs performed calculations and fitting of FORC and FMR data according to Section 2.2 and Section 2.3, respectively. The programs and analysis methods developed for this study could be suitable for future studies of magnetic interactions in nanomagnets.

1.2 Static Magnetometry

Magnetometry describes the magnetization of a ferromagnetic sample, $\vec{M}$, in the presence of an applied external magnetic field, $\vec{H}$. These fields are components of the magnetic flux density, $\vec{B}$,

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) . \quad (1.1)$$

Separating the $\vec{B}$ field into $\vec{H}$ and $\vec{M}$ distinguishes the controlled external magnetic field from the measured magnetic behavior of the ferromagnetic material itself. The fields referenced are illustrated in FIG. 1.2, along with the demagnetizing field introduced in section Section 1.2.4.

FIG. 1.2. (a) A magnetic object is placed in an applied external field, $\vec{H}$ (b) causing the material to have a magnetization, $\vec{M}$. The dashed box represents a neighboring domain. (c) The neighbor creates a dipole field. Note the direction is against the magnetization, so it is known as the demagnetizing field, $\vec{H}_d$. (d) The total magnetic flux density, $\vec{B}$, has a continuous normal component across interfaces.

Static magnetometry is applicable when the external field has a small enough rate of change, $d\vec{H}/dt$, such that the measured $\vec{M}$ is practically independent of this rate. This allows assuming quasi-static equilibrium, in which $\vec{M}$ is in a local minimum energy state at all times.

1.2.1

Units

The explicit SI unit for the magnetic field is Amperes per meter (A/m). The magnetic field is more commonly presented implicitly as a magnetic flux density, $\mu_0\vec{H}$ and $\mu_0\vec{M}$, using the unit Tesla (T) to match that of $\vec{B}$. The CGS units are equally common, Oersted (Oe) for magnetic field and Gauss (G) for the magnetic flux density.

In this thesis, the magnetic field is presented explicitly as A/mm. The magnetic field conversion is $1 \text{ A/mm} = 4\pi \text{ Oe}$ and corresponds to the magnetic flux density $4\pi \times 10^{-4} \text{ T} = 1.257 \text{ mT} = 4\pi \text{ G}$. The chosen SI unit for the magnetic moment was $\text{A} \cdot \text{mm}^2$, which is equivalent to the CGS unit memu (milli-electro-magnetic unit). The shared use of milli-meters in units of the field, the moment, and the sample width was also done for convenience, especially for the unit of the Preisach density, mm^4/A .

1.2.2

Magnetic Reversal

Uniform external magnetic field with magnitude H was applied to a sample. The magnetic dipole moment, m , was measured. The magnetic dipole moment is a scalar that represents the sample's magnetization as an equivalent single magnetic dipole by

$$m = \iiint \vec{M} \cdot \hat{z} dV , \quad (1.2)$$

where $\hat{z}$ is the direction of positive applied field. While $\vec{M}$ is a field inside the sample, m is found by the instrument via the magnetic dipole field outside the sample.

On the microscopic level, gradually changing the magnitude of the external field, H , causes the equilibrium magnetic domain state to evolve. This can be observed by a change in the magnetic moment, m . Measurements during this process form the magnetization curve, $m(H)$.

Ferromagnetic materials exhibit magnetic memory, or hysteresis. Magnetic memory is experimentally observed by the dependence of the magnetization curve, $m(H)$, on the magnetic history, $H(t)$, illustrated in FIG. 1.3. The state of magnetic domains depends on the previous domain configuration as well as the present applied field. A domain state in quasi-static equilibrium is a state of local minimum total energy. Creating or re-shaping domains from a previous state also comes at an energy cost in real magnets. In static magnetometry, the magnetic interactions are described by analyzing magnetic memory in the sample.

FIG. 1.3. (a) The sample's domains evolve from “A” to “C” by aligning with (b) the applied field, $\vec{H}$. (c) The magnetic history, $H(t)$, is a linearly increasing external field, (d) resulting in an increasing minor magnetization curve, $m(H)$. The major curve is also shown for comparison.

Saturation is the process of erasing magnetic memory in the sample by applying a large enough external field, $H > H_s$, illustrated on the microscopic scale in FIG. 1.3a. In this case the lowest energy state is a magnetization aligned with the external field, resulting in the magnetic moment $m = m_s = M_s \cdot V$. Measuring saturation moment allowed estimation of the total volume of magnetic material using the known saturation magnetization. The saturation magnetization strength in bulk nickel is $M_s = 521.7 \pm 0.3 \text{ A/mm}$ [25].

Saturation is an ordered state that can be reached by any previous domain state when applying a large magnetic field. Simply, the magnetic memory of the original domain state is erased. A major curve, also called “the zero-order reversal curve,” is the magnetization curve when $H(t)$ is a linear sweep from one saturated state to the opposite. A major curve is virtually a repeatable measurement since it starts and ends without magnetic memory.

Below the saturation field, the demagnetizing field begins to favor domain configurations with zero net magnetic moment as the global minimum energy state.

However, anisotropy and pinning energy barriers resist change in magnetization. In the Stoner-Wohlfarth model, anisotropy is the energy barrier of rotating magnetization through an axis of higher energy [1]. Pinning is the local anisotropy near a defect that prevents the growth of domains. The anisotropy and pinning energy barrier results in a meta-stable equilibrium state with non-zero magnetic moment called the remnance. When the applied field reaches zero, the sample remains magnetized with $m = m_r$.

Anisotropy also results in a negative applied field required to reach zero moment, known as the coercivity, H_c . In major curve measurements, coercivity and remnance characterize the sample's magnetic domain behavior. In magnetic samples composed of particles, for example, a correlation has been shown between particle/grain size and coercivity [10]. Coercivity and remnance have also been figures of merit for engineered permanent magnets [26]. Domain growth continues until negative saturation. The magnetization aligns with the applied field at negative saturation, which is simply $-H_s$ and $-m_s$ for a typical sample.

A minor curve is the magnetization curve starting from a non-uniform domain state rather than saturation. FIG. 1.3d compares a typical minor and major curve. The minor curve also does not necessarily end in saturation. These are more arbitrary curves that depend on the sample's magnetic memory, realized in its initial domain state. Domain formation and growth are random processes of evolving domain state. Several domain states at a particular applied field H but different $H(t)$ may have equal m . Therefore, minor curves are not generally repeatable.

In practice the initial state of a minor curve is usually one along the major curve in the opposite direction, which are virtually repeatable. This initial field is the reversal field, H_a . A minor loop is a minor curve from H_a to a particular field and back to H_a . A FORC is a minor curve from H_a to saturation. FORCs were measured and analyzed in this study, as outlined in Section 2.2.

FORCs provide details about the initial state at the reversal field. The curves tend to be initially constant in m with changing H . This illustrates the integrity of the initial state from the anisotropy and pinning energy barrier. The steep change in M along a minor curve also occurs at a lower H_c compared to the major curve, which illustrates the lower energy barrier of a mixed-domain state compared to that of a saturated state. This is because some energy barriers had been overcome in creating the mixed initial state from saturation. Therefore, FORC measurements systematically use magnetic history to identify hysteretic reversal events.

1.2.3

The Classic Preisach Model

The classic Preisach model is a general and complete representation of a magnetic sample's response to magnetic history. The model is “general” in that it can describe hysteresis in any ferromagnetic sample. The model is “complete” in that assuming the model provides a prediction of the magnetic moment given any magnetic history, $H(t)$, in the quasi-static equilibrium regime [5].

The magnetic sample is generalized to be a collection of abstract elements called hysterons. A hysteron can be in one of two discrete magnetic states, “+” or “-.” The hysteron's descending and ascending switching fields are h_a and h_b , respectively, as labeled in FIG. 1.4. By nature of hysteresis, $h_b > h_a$.

FIG. 1.4. (a) A hysteron is characterized by its switching fields, (h_b, h_a) , (b) corresponding to a coordinate on the Preisach plane.

Consider a hysteron with switching fields $h_a < 0$ and $h_b > 0$ in the positive-saturated state, “+.” At zero external field, the hysteron is still “+.” The hysteron transitions to state “-” when $H(t) < h_a$. The state is still “-” when the applied field returns to $H(t) = 0$.

Therefore, a hysteron is a binary memory unit.

A magnetic sample can be modeled as a collection of hysterons. The Preisach plane is the space of possible hysteron switching fields, (h_b, h_a) . Since $h_b > h_a$, the Preisach plane is a triangular half-plane below the diagonal line, $h_b = h_a$. The presence of hysterons in a particular system is represented by a Preisach distribution, $\rho(h_b, h_a)$ [5]. The Preisach distribution can be visualized as a smooth density plot, $\rho(h_b, h_a)$, with h_b as the horizontal axis and h_a as vertical. Hysteron states at a particular time are represented by regions of state, $\gamma(h_b, h_a) = \{-1, +1\}$ [5]. The boundary between “+” and “-” regions changes in response to the magnetic history, as hysterons undergo reversal.

The classic Preisach model is based on the claim: if a sample’s Preisach distribution and magnetic history since saturation are known, the sample’s magnetic moment can be predicted [5]. The classic Preisach model asserts the total magnetic moment of the sample

can be found by the Preisach integral,

$$m = m_s \left[\iint \rho(h_b, h_a) \gamma(h_b, h_a) dh_b dh_a \right]. \quad (1.3)$$

The sample's magnetic moment is the sum of states of all hysterons in the system. The regions of hysteron state can be found from the magnetic history. It is clear from Equation (1.3) that the volume under the entire Preisach distribution is the saturation magnetic moment.

The Preisach integral applies to major curves, minor curves, or any general magnetic history. The Preisach model relates the magnetic memory to the measured state of a sample. In the case of this study, the Preisach distribution of each ferromagnetic sample was plotted from FORCs using a procedure outlined in Section 2.2.

One caveat is that a typical sample also has a reversible component. This is evident from a major loop measurement that is not constant outside the hysteretic region, or at fields greater than the field at which the ascending and descending major curves join. This may be due to paramagnetic or diamagnetic contributions from the sample or the hardware. The reversible component can be modeled by hysterons with $h_a = h_b$, which cannot be represented as a density in the Preisach plane. To account for this, the Preisach distribution describes only irreversible changes to a sample's magnetic moment. Therefore, m_s in Equation (1.3) is slightly less than the total saturation moment measured from major curves.

The Preisach distribution is traditionally shown and interpreted in a modified basis [27],

$$h_u = \frac{h_b + h_a}{2} \quad (1.4a)$$

$$h_c = \frac{h_b - h_a}{2}. \quad (1.4b)$$

The coercive field, h_c , is the hysteron width, related to the coercivity mentioned in Section 1.2.2. The interaction field, h_u , is the offset of the hysteron from the center. These parameters are illustrated in FIG. 1.5.

FIG. 1.5. (a) A hysteron can be characterized in the (h_c, h_u) basis by describing its width and center. (b) This can be mapped to the Preisach plane, rotated and scaled from FIG. 1.4.

The major loop of a typical isolated magnetic particle is square-shaped and inversion symmetric at the origin. Therefore, a component of a nanomagnet, like a single isolated nanowire, can be represented as a single hysteron with $h_u = 0$ and a particular h_c . A sample that is a collection of non-interacting particles with a range of h_c can, therefore, be treated as a distribution in h_c , known commonly as a switching field distribution (SFD). The Preisach distribution of a sample without strong interactions is interpreted physically as the SFD of that sample [28, 5].

1.2.4 *The Demagnetizing Field*

A region of magnetized material produces a magnetic field around it called the demagnetizing field, H_d , illustrated in FIG. 1.2. There is a resulting magnetostatic energy of this field interacting with the magnetization in other regions [1],

$$E_{ms} = \frac{\mu_0}{2} \int \vec{H}_d \cdot \vec{M} dV . \quad (1.5)$$

In quasi-static equilibrium, the magnetization field locally minimizes total energy. The resulting demagnetizing field is parallel to the magnetization if the applied field is along a principal axis of the sample, like those labeled in FIG. 1.1. Defining this direction as $\hat{x}$,

$$H_{d,x} = -N_x M_x, \quad (1.6)$$

where N_x is the demagnetizing factor in the $\hat{x}$ direction [1]. The demagnetizing factors depend on sample geometry and are known for simple geometries like the sphere and infinite plane [1]. The internal field, $\tilde{H}$, is the local magnetic field within the material. Since measurements were taken along principal axes, the internal, external, and demagnetizing field magnitudes are added as scalars,

$$\tilde{H} = H - H_d. \quad (1.7)$$

Magnetic reversal of particles in the sample depends on the magnetic field of that particle's local environment, the internal field. However, the axes of the observed Preisach distribution are the controlled external fields. The demagnetizing factor at a particular time depends on the sample's magnetic moment, which depends on the particular magnetic history. Therefore, an observed Preisach distribution would not model a sample's magnetic reversal for any magnetic history, as the classic Preisach model claims. Systems with strong demagnetizing field require a modified model, known as “the moving Preisach model” [8, 5].

A hysteron can instead be intrinsically modeled by its internal switching fields, $(\tilde{h}_b, \tilde{h}_a)$, described by Equation (1.7). A hysteron would be observed to reverse at a field that is displaced from its intrinsic position by the demagnetizing field [8]. The collection of hysterons in a sample form an intrinsic Preisach distribution, $\tilde{\rho}(\tilde{h}_b, \tilde{h}_a)$. The observed Preisach distribution $\rho(h_a, h_b)$ with respect to the applied field could be modeled as

$$\rho(h_a, h_b) = \tilde{\rho}(\tilde{h}_a, \tilde{h}_b). \quad (1.8)$$

The observed Preisach density at a point could be mapped from the intrinsic distribution with the transformation, $(h_b, h_a) \rightarrow (\tilde{h}_b, \tilde{h}_a)$ if the demagnetizing field were known. This general transformation is formulated in Derivation A.1.

FIG. 1.6 illustrates the hysteron displacements in the cases of a strong magnetizing field ($H_d < 0$) and a strong demagnetizing field ($H_d > 0$). The transformation from the intrinsic Preisach distribution to the observed Preisach distribution is described in Derivation A.2.1 and Derivation A.2.2 for these two cases.

FIG. 1.6. In the moving Preisach model, the observed Preisach distribution [shaded] can be described as an intrinsic distribution [dotted] that is distorted by the (de)magnetizing field [arrows]. (a) An intrinsic distribution with magnetizing interactions contracts to a “boomerang” shape when measured with the FORC method. (b) An intrinsic distribution with demagnetizing interactions expands to a “wishbone” shape.

1.2.5

Reversal in Thin Films

Thin films behave like a 2D nanomagnet with magnetization confined in-plane. Hysteresis was observed when the external field was applied in-plane with the thin film. A region in a domain of perpendicular magnetization would be demagnetized by neighboring regions along two in-plane dimensions. A region in a domain of in-plane magnetization would be demagnetized by neighboring regions in one in-plane dimension, perpendicular to $\vec{M}$. The perpendicular magnetization case involves a generally larger opposing demagnetizing field than in the in-plane magnetization case and, therefore, larger magnetostatic energy. In-plane magnetization is energetically preferred in a thin

film. Approximating the thin film as an infinite plane [1],

$$N_{\parallel} = 0 ; N_{\perp} = 1 . \quad (1.9)$$

An ellipsoid with aspect ratio equal to that of the thin film in this study, $w/t = 5 \times 10^4$, has a small in-plane demagnetizing factor, $N_{\parallel} = 2 \times 10^{-5}$ [1]. The small $N_{\parallel}$ supports treating the sample as an infinite plane with no area dependence.

Observed Preisach distributions from thin films were typically a single peak, similar to a 2D Dirac delta. The peak was located near $(h_c, h_u) = (H_c, 0)$, where H_c was the coercivity of the sample's major curve.

Research involving thin films used the Preisach distribution to describe particles or grains in magnetic recording media [12]. The switching field is one component of the Day plot, a traditional plot for analysis of major curves to identify grain size and magnetic behavior [10]. A distribution in grain sizes resulted in a switching field distribution (SFD). The analogue to the SFD in the classic Preisach model was the distribution in h_c .

Simulation studies had shown that particles treated as hysterons underestimated the width of the SFD [28]. Simulation research had been done to correct this using the moving Preisach model and other modifications [21]. Upon close inspection, the observed Preisach distribution is skewed with regions of low hysteron density, or tails. The faint tails formed a “boomerang” shape, as illustrated in FIG. 1.6(a). Recent studies recognized asymmetric Preisach distributions as evidence of strong interactions within the sample [29, 22].

Domain growth is the main mechanism of reversal in thin films. The applied field interacts with the $\mathcal{O}(100\text{nm})$ [1] region of in-homogeneous magnetization between domains, the domain wall. An interplay of torques and local field gives the domain wall velocity, causing the field-aligned domain to grow [1]. The local field involved in domain growth was accounted for by an effective magnetizing field, $N_{\parallel} < 0$. Recall that the

macroscopic magnetization resulted in zero in-plane (de)magnetizing field, $N_{\parallel} = 0$ (Equation (1.9)). Domain pinning and anisotropy might also systematically affect the observed switching field and contribute to the effective demagnetizing factor.

The Preisach distribution's boomerang shape was accounted for by the moving Preisach model with an effective magnetizing field. The observed Preisach distribution was smaller than the intrinsic distribution. The hysteron movement is illustrated in FIG. 1.6 and explained in Derivation A.2.1.

1.2.6 *Reversal in Nanowire Arrays*

A nanowire is easily magnetized in the axial direction since the nanowire behaves as a 1D nanomagnet. Single nanowires were assumed to have axial and transverse demagnetizing factors equal to that of an infinite wire [1],

$$N_{ax} = 0; N_{tr} = 1/2. \quad (1.10)$$

The large transverse demagnetizing factor introduces anisotropy and results in large coercivity in the axial major curve. Ellipsoids with aspect ratio equal to those of nanowires in this study, $l/d \approx 60$, have a small axial demagnetizing factor, $N_{ax} = 0.0011$ [1].

The magnetostatic interaction between nanowires can be understood by considering a point-dipole, a magnet of negligible size or shape. The field produced around a point-dipole with magnetic moment $\vec{m}$ is [27]

$$\vec{H}_{dip}(\vec{r}) = \frac{1}{4\pi} \left[3 \frac{(\vec{m} \cdot \vec{r})\vec{r}}{r^5} - \frac{\vec{m}}{r^3} \right]. \quad (1.11)$$

The treatment of nanowires as magnetic dipoles is qualitatively illustrative but not quantitatively accurate because of close nanowire spacing.

Considering $\vec{m}$ in the axial direction and the field only evaluated for $\vec{r}$ in the sample plane, the first term becomes zero. The second term represents the magnetic field, H_{dip} ,

due to the magnetic moment of a single nanowire, m_{nw} ,

$$\vec{H}_{dip}(r) = -\frac{m_{nw}}{4\pi r^3} \hat{z}. \quad (1.12)$$

This demagnetizing dipole field is roughly illustrated in FIG. 1.2.

The negative sign in Equation (1.12) illustrates a preferred anti-parallel alignment of neighboring nanowires at remnance, or in the absence of an external magnetic field. As mentioned earlier, this is counter-acted by the nanowire's shape anisotropy.

The array used in this study could be idealized as an infinite 2D triangular lattice of nanowires. Clime et al. developed equations for the demagnetizing field when varying nanowire and array geometries in simulation [30]. In the “dipolar regime”, the demagnetizing field in the array agreed with that of an array of point dipoles with field described by Equation (1.12) [30]. The sample used was not in this regime due to close nanowire packing.

The demagnetizing field was modeled with a mean-field approach, in which the demagnetizing field strength was assumed to be homogeneous and proportional to the sample's magnetic moment. A mean-field approach maintains the accuracy of farther nanowire contributions [30, 22]. Since the real sample's nanowires were not a perfect triangular lattice, the demagnetizing field may have been in-homogeneous across the sample.

The Preisach distribution for arrayed structures had been described as a “wishbone” shape, like in FIG. 1.6b. The wishbone shape can be described as an interaction field distribution (IFD) in the $\pm h_u$ direction and a coercive field distribution (CFD) along the h_c axis. These distributions can be accounted for by the dipole interactions between nanowires in the array [22, 21]. Derivation A.2.2 describes how the IFD and CFD arise from the demagnetizing field via the moving Preisach model.

The theoretical model outlined above guided analysis of experimental Preisach distributions, described in Section 2.2. A multi-variable fit based on the description in Derivation A.2.2 provided estimates for the magnitude of the dipole field strength and the intrinsic hysteron distribution. Measurement and fit results are reported in Section 3.2.

1.3 Magnetization Dynamics

In addition to studying the magnetization in quasi-static equilibrium, the magnetization's frequency response, or magnetization dynamics, was studied. A static external field was applied to saturate the sample, avoiding the effects of domains and reversal. The static field also resulted in a torque causing precession of magnetization. The theory of this precession is outlined in this section. Measurements of magnetization dynamics indicated the precession driven by microwaves, discussed in Section 2.3.

1.3.1 *Ferromagnetic Resonance*

The magnetic moment, m , is fundamentally related to quantum mechanical angular momentum. The magnetization can be expressed in terms of the angular momentum per unit volume,

$$\vec{M} = -|\gamma|\vec{J}, \quad (1.13)$$

where γ is the gyromagnetic ratio. The gyromagnetic ratio can be expressed in terms of known physical constants,

$$\gamma = -g \frac{\mu_B}{\hbar}, \quad (1.14)$$

where μ_B is the Bohr magneton. The effective g-factor, g , is approximately 2.00 for an isolated electron (with spin $m_s = \hbar/2$) and is between 2.0 and 3.0 in a typical ferromagnetic material [31, 32]. The negative sign is due to the electron's charge.

Like classical angular momentum, the magnetization undergoes precession in the presence of a torque. In this case the applied magnetic flux, $\vec{B}$, provides a torque,

$$\vec{\tau} = \frac{d\vec{J}}{dt} = \vec{M} \times \vec{B}. \quad (1.15)$$

The magnetic flux includes the internal field but not magnetization, so $\vec{B} = \mu_0 \vec{H}_{int}$.

Substituting this field and Equation (1.13) yields the equation of motion for $\vec{M}$,

$$\frac{d\vec{M}}{dt} = -|\gamma| \vec{M} \times \mu_0 \vec{H}_{int} . \quad (1.16)$$

This general precession effect for a magnetic moment is known as Larmour precession.

FIG. 1.7 illustrates the directions of fields in this effect. Note that precession is

“left-handed” in this case. The frequency of precession is the Larmour frequency,

$\omega_L = |\gamma| \mu_0 H$. Determining the resonant frequency allows analysis of a sample’s internal magnetic field.

FIG. 1.7. In FMR measurements, the magnetization undergoes precession about a static, external magnetic field, $\vec{H}$, while driven by a dynamic external field supplied by microwaves on a waveguide.

Real magnetic systems also experience damping due to the properties of the material, illustrated by the Landau-Lifshitz-Gilbert (LLG) equation [33],

$$\frac{d\vec{M}}{dt} = -|\gamma| \vec{M} \times \mu_0 \vec{H}_{int} + \frac{\alpha}{M_s} \vec{M} \times \frac{d\vec{M}}{dt} . \quad (1.17)$$

FMR is a behavior of ferromagnets observed by absorption of microwave power, or radio-frequency (RF) electromagnetic waves. The magnetization vector in the material undergoes precession in the presence of an external microwave magnetic field. The magnetization and the RF field are treated as phasors and are related by the tensor,

$$M_i(t) = \chi_{ij}H_j(t) , \quad (1.18)$$

known as the AC susceptibility. FMR can be compared to a driven oscillator in classical mechanics. A driven harmonic oscillator has a large amplitude and is in-phase when the driving frequency is close to the oscillator's natural frequency. In the case of FMR, the natural frequency is roughly the Larmour frequency. FIG. 1.7 shows the layout for this measurement.

The curve for complex magnetic susceptibility theoretically depends on microwave frequency, f ; the sample's saturation magnetization strength, M_s ; the effective g-factor, g ; and the damping factor, α . Fitting the complex susceptibility curve yielded these physical constants for a particular magnetic sample.

1.3.2 *Magnetization Dynamics in Thin Films*

Consider FMR measurement with in-plane static field, $H \hat{x}$, and in-plane RF field, $\vec{h}(t) \perp \hat{x}$. Note that the static field and magnetization directions are different from FIG. 1.7. The static equilibrium for $\vec{M}$ was assumed to be aligned with the applied field, which is theoretically true for a principal axis of the material. An additional RF magnetization, $\vec{M}(t) \perp \hat{x}$, was introduced.

The susceptibility is derived in Derivation A.3. A derivation with angle dependence was derived in the dissertation by Lei Lu [34], which this derivation was adapted from. A simpler derivation was also given in the commonly-cited article by Kittel [24]. The derived susceptibility equation was

$$\chi_{yy}(H, \omega) = \frac{M_s(M_s + H) + iM_s\alpha H_L}{H(M_s + H) - (1 + \alpha^2)H_L^2 + i\alpha H_L(M_s + 2H)} , \quad (1.19)$$

where $H_L = \omega/(|\gamma|\mu_0)$ is the resonance condition for Larmour precession. The derived trend of resonant field versus frequency used for fitting was written in terms of physical parameters,

$$H_{res} = \frac{-M_s + \sqrt{M_s^2 + 4H_L^2(1 + \alpha^2)}}{2}. \quad (1.20)$$

Equation (1.20) is derived in Derivation A.3. The resonant field, or field of maximum susceptibility magnitude, was verified numerically from the theoretical susceptibility, Equation (1.19), within 10% accuracy (over-estimated values).

This equation predicted $H_{res}(\omega = 0) = 0$. Experimental results could contradict this if an additional field, H_k , were added to the external field, H_{ext} . In the case of crystalline anisotropy, the lattice introduces an energetically preferred direction that can be treated as an effective magnetic field [24]. The applied field was replaced with $H_{res} = H_{ext} + H_k$ when fitting to estimate any field offset that may have been present.

The susceptibility curve's width in field, the full-width at half-maximum magnitude (FWHM), was expected to have a linear trend versus frequency. The FWHM was evaluated numerically and compared to the result derived in the dissertation by Lei Lu [34]. A correction factor was introduced, resulting in the linear relationship between resonance FWHM and microwave frequency,

$$\Delta_H = 2\sqrt{3}\alpha H_L + \Delta_{H0}, \quad (1.21)$$

where Δ_{H0} accounted for in-homogeneity in the sample.

1.3.3 *Magnetization Dynamics in Nanowire Arrays*

A similar derivation to that of the thin film was performed to find an expression for susceptibility of a nanowire array. For consistency with the thin film case, $\hat{z}$ was defined as being perpendicular to the sample plane, the axial direction in this case. The static field and static magnetization were along $\hat{z}$, and the RF field was along $\hat{y}$ like before. This layout with labeled axes is shown in FIG. 1.7.

In the static magnetometry, only the second term in Equation (1.11) was considered. However, the dynamic magnetization is not necessarily perpendicular to $\hat{r}$. The first term accounts for the transverse component of the dipole field during precession. Unlike in Equation (1.12), this factor represented a magnetizing interaction, negative effective N_d . This meant the magnetic dipole interaction energetically prefers in-phase precession between neighboring moments, which are typically anti-parallel in the axial component.

Variables for static and dynamic demagnetizing factors were included when deriving the expected FMR equations for nanowire arrays. The expected axial susceptibility of a nanowire array was,

$$\chi(H, \omega) = \frac{M_s(H + N_\Delta M_s) + i\alpha M_s H_L}{(H + N_\Delta M_s)^2 - (1 + \alpha^2)H_L^2 - i2\alpha H_L(H + N_\Delta M_s)} , \quad (1.22)$$

derived in Derivation A.4. The resonant field had an expected linear dependence on frequency,

$$H_{res} = H_L \sqrt{1 + \alpha^2} - N_\Delta M_s , \quad (1.23)$$

where $N_\Delta = N_{tr} - N_{ax}$. Note that this equation did not allow fitting for M_s , N_{ax} or N_{tr} independently. The effective demagnetizing factors of the nanowire array were estimated with additional information from other measurements. The linear trend also prevents an independent estimate of uniaxial anisotropy. The FWHM was fit using the same equation as for the thin film, Equation (1.21).

II. EXPERIMENTAL METHODS

The experiment consisted of fabricating a magnetic thin film and a nanowire array. The static and dynamic magnetic properties of the samples were measured. Calculations and fitting provided estimates for magnetic quantities describing the samples. This section outlines the experimental procedure and methods.

2.1 Electrodeposition

Nickel metal was electrodeposited into alumina (Al_2O_3) templates to form nanowire arrays. The templates were fabricated by and purchased from InRedox. Each template was a circular alumina film with an 8 mm diameter and gold film electrical contact applied to the back side. The alumina had nano-pores with 80 ± 9 nm diameter that extended 50 ± 2 μm from the front surface to the gold contact. The pores were parallel and close-packed into an array across the entire front surface.

During the electrodeposition process, nickel metal deposited on the exposed gold inside the nano-pore. Nickel continued to deposit on existing nickel to gradually fill the nano-pore from back to front. Nanowire length was controlled by stopping the electrodeposition at the desired time.

The resulting sample was an array of nano-pores in alumina that were partially filled with nickel. The only magnetic contribution when measuring was from the nickel nanowires, so there was no need to remove the alumina or gold. The electrodeposition setup, recipe, and procedure are outlined in Appendix B.

2.2 First-Order Reversal Curve Measurement

A vibrating sample magnetometer (VSM) measures a sample's magnetic moment in a constant external magnetic field. The sample's magnetic properties are found by measuring the magnetic moment while sweeping the magnetic field. The resulting trend of magnetic moment versus applied field are used to characterize the samples with regard to magnetic memory and reversal at quasi-static equilibrium.

2.2.1

Hardware

The VSM hardware and control components are outlined in FIG. 2.1. External field was produced by a Helmholtz coil with the sample at its center. The applied field strength and direction was controlled by the current supplied to the coils with a ratio of 16 A/mm field per 1 A current. A gaussmeter measured the recorded applied magnetic field, H . A built-in feedback system adjusted the current based on measured H [35].

FIG. 2.1. The layout for vibrating-sample magnetometry (VSM) measurements is shown above. The sample experienced a static external magnetic field provided by a Helmholtz coil. The sample's magnetic moment was measured. This measurement was controlled by an instrument unit with commands sent from a LabVIEW program using GPIB.

Magnetic moment, m , was measured by the VSM's inductor coil. Vibrating the magnetic sample induced an alternating current at the vibration frequency. The instrument unit contained a lock-in amplifier that determined the induced EMF amplitude, which was proportional to m . This procedure allowed the instrument to distinguish the sample magnetic moment from the external magnetic field. The conversion from the induced signal to m value was made possible by the vibrating motor's design, ensuring vibrating amplitude and frequency were consistent within 1% [35]. The proportion in this conversion was set by the user during calibration.

The sample was adhered to a plastic holder vertically suspended from the vibrating motor by a long metal rod. The sample was positioned at the center of the Helmholtz coil. The center corresponded to the saddle-point of measured m sensitivity. The saddle point was found by measuring m_s while adjusting the sample's position. Screws allowed fine adjustment of the sample-holding rod's angle and vertical displacement. This calibration was performed whenever a new sample was mounted.

The VSM instrument unit controlled the applied current and read the induced signal amplitude in the inductor coil and the applied field from the gaussmeter. The instrument unit converted signal amplitude to sample magnetic moment, m . The instrument unit also controlled the current supply to approach a desired applied field, H_{des} . The front panel included controls to perform an automated hysteresis sweep. Instead, commands and queries were sent between the computer and the instrument unit via GPIB. The custom program that ran the measurements is shown in Appendix C.

2.2.2 *Measurement*

FORC measurements illustrate the magnetic memory of a sample. While a major curve characterizes magnetic reversal from saturation, FORC measurements show reversal from intermediate states. This is done by sweeping minor curves upward from the major descending curve. The FORCs filled the major loop's interior, or the hysteretic region.

The procedure for a single FORC is

- Apply a large positive magnetic field to reach magnetic saturation.
- Reduce the applied field gradually to a particular reversal field, H_a .
- Incrementally sweep the applied field, H_b , upward toward positive saturation.

This traces out a single FORC.

- Record measurements, $m(H_a, H_b)$.

An example FORC is illustrated in FIG. 2.2. A custom LabVIEW program discussed in Appendix C performed a sweep procedure using a list of field values.

FIG. 2.2. An example FORC (a) applied fields and (b) resulting curve are shown. The measurement involved (dotted) an initializing major curve and (solid) a measured minor curve. An arbitrary single measurement, $m(H_b, H_a)$, is also labeled.

The step sizes for H_a and H_b were chosen to be equal for simpler numerical analysis. Typical FORC measurements contained 50 to 200 FORCs.

2.2.3 Preisach Distribution Calculation

Static magnetization was analyzed by calculating and describing the Preisach distribution. A FORC distribution emphasizes and simplifies the details of magnetic reversal when performing minor curve measurements. A switching field distribution is shown using a Preisach distribution, as described in Section 1.2.

The Preisach distribution was directly calculated from FORC measurements by the mixed derivative,

$$\rho(h_a, h_b) = -\frac{1}{2} \frac{d^2 m}{dH_a dH_b}. \quad (2.1)$$

Notice that this is essentially the inverse of the Preisach integral, Equation (1.3). Then the distribution is rotated to h_c and h_u axes, as mentioned in Section 1.2.2. This plot was the focus of the FORC measurement analysis.

2.2.4 Moving Preisach Model Fitting

After obtaining a clean Preisach distribution from FORC measurements, the Preisach distribution was fit according to theoretical descriptions of the moving Preisach model, described in Section 1.2. The fit equations formed the typical “boomerang” shape for thin

films and the “wishbone” shape for nanowire arrays. The fit variables were the (de)magnetizing field magnitude at saturation, $H_{ds} = |N|M_s$, and variables describing the intrinsic hysteron distribution, $\tilde{\rho}(h_b, h_a)$.

The classic Preisach model describes a hysteron distribution based on two applied magnetic field values, or $\rho(h_b, h_a)$. The moving Preisach model’s distribution additionally depends on the sample’s present magnetic moment to calculate the internal magnetic field, or $\rho(h_b, h_a, M)$ [8]. A simple fit of the Preisach distribution is a function of three independent variables to solve for one dependent variable, forming a distribution in 3D space.

A 3D fit using the moving Preisach model would have been simple to implement but practically limited and mathematically redundant. FORC measurements are magnetic moment values depending on the applied fields, or $m(h_a, h_b)$. The variable m is not strictly an independent variable. The physical parameters being fit define a surface within the 3D variable space, (h_b, h_a, m) . After fitting using the 3D method, the resulting Preisach distribution cannot be visualized on the Preisach plane without the raw FORC data. Furthermore, the dependent variable values being fit, ρ , are directly calculated from m .

Instead, the fit equation used in this thesis predicted the FORC magnetization, $m(h_a, h_b)$, within the fit function. The prediction relied on a self-consistent relationship between m and ρ in the Preisach integral,

$$m = m_s \left[\iint \rho(h_b, h_a) \gamma(h_b, h_a) dh_b dh_a \right] \quad (1.3)$$

where γ is a hysteron’s present state. This reduced the number of independent variables from three to two. The function could be evaluated and plotted on the Preisach plane using particular fit variable inputs.

The sample’s magnetic moment in FORC measurements, $m(h_a, h_b)$, can be predicted using Equation (1.3). The Preisach distribution, ρ , is integrated over regions of “+” and

“-” state, γ . The regions of state were predictable during FORC measurements, as illustrated in FIG. 2.3, since the magnetic history was procedural. The fit functions used in this study started by approximating a general effective Preisach distribution with a function that was easily integrated. The effective distribution's position, size, and amplitude were evaluated from an analytic solution that was self-consistent in the moving Preisach model.

FIG. 2.3. During FORC measurement, (shaded) the regions of hysteron states, $\gamma(h_b, h_a)$, are predictable from (star) the applied field, (H_b, H_a) . The magnetic moment can be predicted by integrating (oval) the Preisach distribution according to Equation (1.3).

The effective distributions were chosen to be based on Gaussian functions,

$$G(x) = \frac{A}{\Delta_x \sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{x - x_0}{\Delta_x} \right)^2 \right] \quad (2.2)$$

where A is the area under the curve ($A = 1$ for the normal distribution), Δ_x is the width parameter (standard deviation), and x_0 is the peak location (mean). The indefinite integral of the Gaussian function is an error function,

$$y(x) = \int G(x) dx = A \operatorname{erf} \left[\frac{x - x_0}{\Delta_x \sqrt{2}} \right] + y(x_0). \quad (2.3)$$

A suitable 2D distribution is the 2D Gaussian distribution,

$$G(x, y) = G(x) G(y) = \frac{V}{2\pi \Delta_x \Delta_y} \exp \left[-\frac{1}{2} \left(\frac{x - x_0}{\Delta_x} \right)^2 - \frac{1}{2} \left(\frac{y - y_0}{\Delta_y} \right)^2 \right] \quad (2.4)$$

where V is the volume under the surface, (x_0, y_0) is the peak coordinate, and Δ_x and Δ_y represent the horizontal and vertical widths.

Integrating the effective distribution approximated m , proportional to the (de)magnetizing field. The internal field seen by the hysteron was estimated by adding the (de)magnetizing field to the applied field. Lastly, the density of hysterons at the point on the observed Preisach plane had “moved” from the corresponding point on the intrinsic Preisach distribution. The hysteron displacement is illustrated in FIG. 1.6 and described simply in Derivation A.2.1 and Derivation A.2.2.

2.2.5 *Boomerang Distribution Fit*

This section presents the fit function used to fit a boomerang distribution as several equations. These equations are derived and explained in Derivation A.5. FIG. 2.4 shows an example of the fit function output using arbitrary inputs.

FIG. 2.4. Hystérons were modeled with respect to the internal field as a 2D Gaussian intrinsic distribution. In this arbitrary example, the intrinsic distribution ($V = 100$, $\tilde{\Delta}_{avg} = 12$) was centered at the origin ($\tilde{h}_{c0} = 0$, $\tilde{h}_{u0} = 0$) with 3:4 aspect ratio ($\tilde{\Delta}_r = 0.75$). The boomerang distribution fit function output the observed Preisach distribution of the hystérons in the presence of a large magnetizing field ($H_{ds} = 12$), which caused hystérons to shift inward. The shift was estimated from an effective distribution calculated from function inputs.

Observed boomerang distributions had a prominent peak at a particular coercivity, ($h_c = h_{c,0}, h_u \approx 0$). Less prominent tails sometimes extended in the h_a or h_b directions. A 2D Gaussian effective distribution like Equation (2.4) was chosen to approximate a general boomerang shape.

The effective distribution predicts the magnetizing field during the descending major curve and ascending FORC,

$$-H_{d\downarrow}(h_a) = H_{ds} \operatorname{erf} \left[\frac{h_a - h_{a,0}}{\Delta_a \sqrt{2}} \right] \quad (2.5a)$$

$$-H_{d\uparrow}(h_b, h_a) = \frac{H_{ds} - H_{d\downarrow}}{2} \operatorname{erf} \left(\frac{h_b - h_{b,0}}{\Delta_b \sqrt{2}} \right) + \frac{H_{ds} + H_{d\downarrow}}{2} \quad (2.5b)$$

where $H_{ds} = |N_{||}|M_s$, the magnetizing field strength at positive saturation. The effective distribution's parameters need to be self-consistent and expressed in terms of the fit variables. The magnetizing field translates hysterons from the intrinsic center to the effective center if the effective distribution is centered at

$$h_{a,0} = \tilde{h}_{a,0} \quad (2.5c)$$

$$h_{b,0} = \tilde{h}_{b,0} - \frac{H_{ds}}{2} \quad (2.5d)$$

and has widths

$$\Delta_b = \tilde{\Delta}_b - \frac{H_{ds}}{2} \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) \quad (2.5e)$$

$$\Delta_a = \tilde{\Delta}_a - H_{ds} \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right). \quad (2.5f)$$

The magnetizing field contracted and shifted the intrinsic distribution in the $-h_b$ direction.

The parameter solutions, Equation (2.5), support previous claims that the observed Preisach distribution of a sample with strong magnetic interactions inaccurately describes the sample's intrinsic magnetic reversal behavior [28]. For example, Equation (2.5)e and Equation (2.5)f predicted that the coercive field range found from FORC measurements, Δ_c , under-values the sample's intrinsic switching field distribution (SFD), $\tilde{\Delta}_c$.

The intrinsic distribution, $\tilde{\rho}(\tilde{h}_b, \tilde{h}_a)$, was chosen to be a 2D Gaussian (Equation (2.4)) in the intrinsic field basis, $(\tilde{h}_b, \tilde{h}_a)$. The observed distribution is the intrinsic distribution in terms of the applied fields and scaled by a correction factor for volume,

$$\begin{aligned} \rho(h_b, h_a) &= \frac{\tilde{\Delta}_b \tilde{\Delta}_a}{\Delta_b \Delta_a} \tilde{\rho}(h_b - H_{d\uparrow}, h_a - H_{d\downarrow}) \\ &= \frac{V}{2\pi\Delta_b\Delta_a} \exp\left[-\frac{1}{2}\left(\frac{h_b - H_{d\uparrow} - \tilde{h}_{b,0}}{\tilde{\Delta}_b}\right)^2 - \frac{1}{2}\left(\frac{h_a - H_{d\downarrow} - \tilde{h}_{a,0}}{\tilde{\Delta}_a}\right)^2\right]. \end{aligned} \quad (2.6)$$

The fit function inputs were the descending and ascending fields h_a and h_b , represented by a point on the Preisach plane. The output was the hysteron density ρ . The intrinsic peak location was $(\tilde{h}_{c,0}, \tilde{h}_{u,0})$ in the (h_c, h_u) basis. The intrinsic widths, $\tilde{\Delta}_a$ and $\tilde{\Delta}_b$, were

substituted with their average, $\tilde{\Delta}_{avg} = \sqrt{\tilde{\Delta}_a \tilde{\Delta}_b}$, and aspect ratio, $\tilde{\Delta}_r = \tilde{\Delta}_b / \tilde{\Delta}_a$. The volume under the distribution, V , was the total moment of all hysterons, which is theoretically equal to saturation moment according to Equation (1.3). The magnetizing field strength, $H_{ds} = M_s N_{||}$, is the effective internal field produced by a saturated sample.

2.2.6 *Wishbone Distribution Fit*

This section presents the fit function used to fit a wishbone distribution as several equations. These equations are derived and explained in Derivation A.6.1 and Derivation A.6.2.

Observed wishbone distributions had two long tails, the coercive field distribution (CFD) and the interaction field distribution. The CFD was angled in the $+h_c$ and $-h_u$ direction. The IFD was in the h_u direction and sometimes had a h_c component. The two distributions seemed to have the same range in h_a , which is qualitatively supported in Derivation A.2.2. The effective distribution was approximated as the sum of two 1D distributions with the same h_a range and different h_b range. FIG. 2.5 shows an example of the fit function output using arbitrary inputs.

FIG. 2.5. Hysterons were modeled with respect to the internal field as a 2D Gaussian intrinsic distribution. In the arbitrary example shown, the intrinsic distribution ($V = 100$, $\tilde{\Delta}_{avg} = 1$) was centered at the origin ($\tilde{h}_{c0} = 0$, $\tilde{h}_{u0} = 0$) with 2:1 aspect ratio ($\tilde{\Delta}_r = 2$). The wishbone distribution fit function output the observed Preisach distribution of the hysterons. A large demagnetizing field ($H_{ds} = 10$, $V_r = 0.6$), caused hysterons to shift outward. The shift was estimated from an effective distribution calculated from function inputs.

The IFD distribution was found by saturating the CFD negatively during the ascending FORC. The demagnetizing field during the descending major curve and ascending FORC was predicted to be

$$H_{d\downarrow}^{\text{IFD}}(h_a) = H_{ds} \operatorname{erf}\left(\frac{h_a - h_{a,0}^{\text{IFD}}}{\Delta_a^{\text{IFD}} \sqrt{2}}\right) \quad (2.7a)$$

$$H_{d\uparrow}^{\text{IFD}}(h_b, h_a) = (1 - V_r) \max\left[H_{d\downarrow}, H_{ds} \operatorname{erf}\left(\frac{h_b - h_{b,0}^{\text{IFD}}}{\Delta_b^{\text{IFD}} \sqrt{2}}\right)\right] + V_r H_{d\downarrow} \quad (2.7b)$$

where $H_{ds} = |N_{ax}|M_s$ is the magnetizing field magnitude at positive saturation, and V_r is the proportion of total hysterons in the CFD (the ratio of CFD volume to the total

volume). The above equations are self-consistent under hysteron movement for

$$h_{a,0}^{\text{IFD}} = \tilde{h}_{a,0} \quad (2.7c)$$

$$h_{b,0}^{\text{IFD}} = \tilde{h}_{b,0} + H_{d\downarrow} \quad (2.7d)$$

$$\Delta_a^{\text{IFD}} = \tilde{\Delta}_u + \tilde{\Delta}_c + H_{ds} \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) \quad (2.7e)$$

$$\Delta_b^{\text{IFD}} = \tilde{\Delta}_u - \tilde{\Delta}_c + H_{ds} \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) . \quad (2.7f)$$

The intrinsic distribution is expanded primarily in the h_u direction since $\Delta_a^{\text{IFD}} \approx \Delta_b^{\text{IFD}}$ for large H_{ds} . A sufficiently large $\tilde{\Delta}_c$ introduces a counter-clockwise IFD tilt. Some experimental results did show a small IFD tilt.

The CFD distribution was defined by partially saturating the IFD positively during the ascending FORC. The demagnetizing fields during the descending major curve and ascending FORC were predicted to be

$$H_{d\downarrow}^{\text{CFD}}(h_a) = H_{d\downarrow}^{\text{IFD}}(h_a) \quad (2.8a)$$

$$H_{d\uparrow}^{\text{CFD}}(h_b, h_a) = (1 - V_r)H_{d\uparrow}^{\text{IFD}}(h_{a,0}^{\text{CFD}}, h_{b,0}^{\text{CFD}}) + V_r \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) \quad (2.8b)$$

The above equations are self-consistent for

$$h_{a,0}^{\text{CFD}} = \tilde{h}_{a,0} \quad (2.8c)$$

$$h_{b,0}^{\text{CFD}} = \tilde{h}_{b,0} + H_{ds} \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) . \quad (2.8d)$$

Self-consistent values for Δ_a^{CFD} and Δ_b^{CFD} were found to produce a distribution primarily in h_a . However, Δ_a^{CFD} and Δ_b^{CFD} were not used in the fit function.

The corresponding intrinsic coordinates were evaluated as

$$\tilde{h}_a = h_a - H_{d\downarrow} \quad (2.9a)$$

$$\tilde{h}_b^{\text{IFD}} = h_b - H_{d\uparrow}^{\text{IFD}} \quad (2.9b)$$

$$\tilde{h}_b^{\text{CFD}} = h_b - H_{d\uparrow}^{\text{CFD}} , \quad (2.9c)$$

and then converted to the $(\tilde{h}_c, \tilde{h}_u)$ basis. The function output was the weighted sum of the IFD and CFD distributions (with extra volume corrections),

$$\rho(h_c, h_u) = V_r^{\text{IFD}} \tilde{\rho}(\tilde{h}_c^{\text{IFD}}, \tilde{h}_u^{\text{IFD}}) + V_r^{\text{CFD}} \tilde{\rho}(\tilde{h}_c^{\text{CFD}}, \tilde{h}_u^{\text{CFD}}) \quad (2.10a)$$

$$V_r^{\text{IFD}} = (1 - V_r) \frac{\sqrt{2\tilde{\Delta}_u\tilde{\Delta}_c}}{\sqrt{\Delta_a^2 + \Delta_b^{\text{IFD}2}}} \quad (2.10b)$$

$$V_r^{\text{CFD}} = V_r \frac{\sqrt{2\tilde{\Delta}_u\tilde{\Delta}_c}}{\Delta_a} \quad (2.10c)$$

$$\tilde{\rho}(\tilde{h}_c, \tilde{h}_u) = \frac{V}{2\pi\tilde{\Delta}_u\tilde{\Delta}_c} \exp\left[-\frac{1}{2}\left(\frac{\tilde{h}_c - \tilde{h}_{c,0}}{\tilde{\Delta}_c}\right)^2 - \frac{1}{2}\left(\frac{\tilde{h}_u - \tilde{h}_{u,0}}{\tilde{\Delta}_u}\right)^2\right]. \quad (2.11)$$

The fit function inputs were the hysteron coercive and interaction fields, h_a and h_b ; represented by a point on the Preisach plane. The fit variables were the intrinsic peak location, $(\tilde{h}_{c,0}, \tilde{h}_{u,0})$; geometric mean, $\tilde{\Delta}_{avg}$ and aspect ratio $\tilde{\Delta}_r$ of intrinsic widths ($\tilde{\Delta}_c$ and $\tilde{\Delta}_u$); volume under the distribution, $V \approx m_s$; the magnetizing field strength, H_{ds} ; and the proportion of hysterons in the CFD, V_r . FIG. 2.5 shows an example of the function output. The values of V and V_r were refined after fitting by numerically integrating the fit result distribution.

Fit results were obtained for function inputs based on the experimental Preisach distributions from the nanomagnet samples: magnetic thin films and nanowire arrays. The fit results estimated the intrinsic hysterons representing magnetic memory in the nanomagnet and the (de)magnetizing field magnitude. The results are presented in Section 3.1 and Section 3.2.

2.3 Ferromagnetic Resonance Measurement

A Vector Network Analyzer (VNA) supplied and measured microwave power (≈ 10 GHz) reflected and transmitted across a magnetic sample. The VNA measured radio-frequency electromagnetic wave (RF) reflection and transmission, from which complex susceptibility was calculated as a function of frequency and applied static

magnetic field. The results were then compared to the FMR theory outlined in Section 1.3 to analyze the magnetization dynamics in the ferromagnetic sample.

2.3.1

Hardware

An external field was supplied by a large Helmholtz coil with current supplied by a computer-controlled power supply. Unlike the VSM, this setup did not use feedback to ensure a particular applied field. Instead the field was measured using a Hall probe near the sample. The current was determined by a user-defined proportion. The layout for this measurement is shown in FIG. 2.6.

FIG. 2.6. The layout for FMR measurements is shown above. The sample experienced a static external magnetic field provided by a Helmholtz coil. Microwave power was supplied and measured across the sample by a Vector Network Analyzer (VNA). This measurement was controlled by a LabVIEW program using GPIB.

The sample rested flat on a printed circuit board with a coplanar waveguide, which transmitted microwaves to the sample, as illustrated in FIG. 2.7. This holder was rotated so that the applied field could be oriented perpendicular or parallel to the sample plane. In the case of nanowire arrays, these orientations were axial and transverse, respectively. The RF power was supplied by and measured by a Vector Network Analyzer (VNA). The

VNA is a common instrument for characterizing a microwave network. RF power was always applied in the sample plane.

FIG. 2.7. The top-down layout for the sample holder is shown. The sample laid on top of a coplanar waveguide, which was connected to the microwave cables by SMA connectors.

The cables often affect VNA measurements. The setup used high quality, flexible, non-magnetic SMA cables. A cable calibration procedure was performed. The same correction was implemented during each measurement.

2.3.2 *Measurement*

The data acquisition program measured the scattering parameters while sweeping the applied magnetic field from positive saturation. The saturation field was 400 A/mm, decreased with step size typically 8 A/mm. The gaussmeter reported values in CGS units. A sweep was performed for each 1 or 2 GHz step in a 1-20 GHz range.

A portion of microwave power was absorbed when driving Larmour precession. Absorbed power was observed by measuring the proportion of incoming power a and outgoing power b described by the scattering matrix,

$$b_i = S_{ij} a_j, \quad (2.12)$$

with indices representing the left and right side as ports “1” and “2.” FIG. 2.8 illustrates the scattering parameters.

FIG. 2.8. A general 2-port network can be described by an S matrix, the complex reflection and transmission coefficients of power.

The VNA measured the four scattering parameters S_{ij} with respect to varying both the microwave frequency and the external magnetic field. The resulting data had two independent variables and 8 dependent variables. A full FMR measurement typically took 6 hours.

2.3.3 Waveguide Correction

Typically, a small length of cable or waveguide caused a small amount of attenuation and group delay to the S parameters. Attenuation and group delay were indicated by changes in the complex value's magnitude and phase, respectively. Standard cable calibration corrected the magnitude and phase of S parameters up to the sample holder ports. However, the desired S parameter values were those representing only the sample.

After cable calibration, waveguide correction was performed by measuring the sample holder with the sample, $[S^{Meas}]$, and without the sample, $[S^{Bkgnd}]$. From these two measurements and reasonable approximations, the S matrix describing the magnetic sample was evaluated, $[S^{Corr}]$. This procedure is generally referred to as “de-embedding.” The measurement with the sample was modeled as three two-port networks cascaded shown in FIG. 2.9, representing the left half of the sample holder and the left port, $[S^A]$, the desired sample network, $[S^{Corr}]$, and the right half and port, $[S^B]$

FIG. 2.9. (a) The waveguide was modeled as two general networks for each half, S^A and S^B , and the desired network for the sample placed in the center, S^{Corr} . Measurements were represented by the three networks in series, S^{Meas} . (b) Omitting S^{Corr} represented a blank waveguide measurement, S^{Bkgnd} .

Starting with the equations for S parameters of two cascaded networks (in series), a formula for $[S^{Corr}]$ was approximated. The derivation is shown in Derivation A.7. The results conveyed that the scattering parameters related to reflection, S_{11} and S_{22} , nearly combined in series by addition. Transmission scattering parameters S_{12} and S_{21} combined in series by multiplication.

$$S_{11}^{Corr} = \frac{S_{11}^{Meas} - S_{11}^{Bkgnd}}{\sqrt{S_{12}^{Bkgnd} S_{21}^{Bkgnd}}} \quad (2.13a)$$

$$S_{12}^{Corr} = S_{12}^{Meas} / S_{12}^{Bkgnd} \quad (2.13b)$$

$$S_{21}^{Corr} = S_{21}^{Meas} / S_{21}^{Bkgnd} \quad (2.13c)$$

$$S_{22}^{Corr} = \frac{S_{22}^{Meas} - S_{22}^{Bkgnd}}{\sqrt{S_{12}^{Bkgnd} S_{21}^{Bkgnd}}} . \quad (2.13d)$$

The measurements with and without the sample were performed with the same procedure. The same background measurement data were used to correct multiple measurements if those measurements were taken within the same week. Correction was performed point-by-point to correct field-dependent signals from the blank waveguide that could interfere with fitting [36].

The amplitude of the resonance curve was much smaller than the constant offset. Therefore, background correction affected the offset amplitude and not the resonance amplitude. However, this correction did provide improvements to the phase of both the offset and the resonance curve. Raw measurements showed positive a monotonic relationship between phase and frequency, with phase increasing through multiple revolutions. This was expected for a transmission line of a particular length, which had a large group delay. Offset phase was nearly constant after waveguide correction, meaning group delay was reduced. The resonance phase was also more constant, which suggested the correction method provided a reference plane close to the resonance signal source.

2.3.4 *Susceptibility Calculation*

The observed absorption of microwave power was interpreted as a change in effective susceptibility of the sample. A transmission line model allowed relating scattering parameters to the model's inductance,

$$i\omega L = \frac{1 + S_{11} - S_{21}}{1 - S_{11}} Z_0 - R \quad (2.14)$$

$$L = L_0 + C\mu\mu_0, \quad (2.15)$$

where Z_0 , L_0 , and R described the transmission line, and the unknown constant C depended on the sample's geometry and had units of meters [34]. Since the relative permeability, μ , is typically very large in ferromagnets, the substitution $\chi = 1 + \mu \approx \mu$ was made. The predicted relationship between measured S parameters and the susceptibility was

$$S_\chi = i\omega \frac{C\mu_0}{Z_0} \chi + i\omega \bar{L}_0 + \bar{R} \quad (2.16a)$$

$$S_\chi \equiv \frac{1 + S_{11} - S_{21}}{1 - S_{11}}. \quad (2.16b)$$

The extracted Lorentzian signal from S_χ was theoretically proportional to susceptibility.

Pre-processing the data yielded two independent variables: applied field and RF frequency, and only two dependent variables: the real and imaginary S_χ . FMR fitting had typically been performed using the real and imaginary transmission S_{21} , but this relied on several assumptions for the sample holder design and the magnetic sample [34]. Calculated complex susceptibility was a more convincing representation of the theory. Buford, Dhagat, and Jander reported improved fitting after performing waveguide correction using all four S-matrix parameters [36]. In this study, each S-matrix parameter showed a resonance signal.

2.3.5 *Signal Extraction*

Proper fitting required segregating the desired resonance signal from the undesired components of measured S parameters. The real and imaginary S_χ were composed of

- large constant offset
- de-embedding errors
- desired resonance signal
- undesired background signal
- noise from measurement uncertainty.

The desired resonance curve was found after several fitting steps that were similar to those outlined by the National Institute of Standards and Technology [37]. The peak location and width were determined using a simple Lorentzian curve fit of the S_{21} magnitude versus applied field,

$$y = A \frac{\Delta_x^2}{(x - x_0)^2 + \Delta_x^2} + ax + b . \quad (2.17)$$

The transmission generally had less noise and larger resonance peaks. A linear background was added, which was noticeable for some curves. This involved a standard curve fit since complex phase was neglected, which otherwise would have contributed to an apparent shift and skew to the peak. A manually entered guess was occasionally

needed after the resonance peak was misidentified. The determined peak location and width was fixed in the next later correction to automatically identify the signal.

Next, all data were windowed to several times the resonance peak width. This ensured that any offset correction was performed for offset near the peak since curvature was sometimes observed.

A complex fit yielded the complex linear background of susceptibility. The signal was accounted for by a complex Lorentzian curve,

$$y_{signal}(x, A, \phi) = Ae^{i\phi} \frac{i\Delta_x x_0}{x^2 - x_0^2 + i\Delta_x x_0} , \quad (2.18)$$

to ensure the offset was accurately determined near the resonant field. The value of Equation (2.18) at $x = x_0$ is $y_{signal}(x = x_0) = Ae^{i\phi}$, corresponding to resonant susceptibility. Equation (2.18) accurately modeled the shape of the theoretical result, Equation (1.19), near the resonant field. FIG. 2.10 shows a comparison of fit and evaluated susceptibility curves for typical physical parameters and a frequency of 15 GHz.

FIG. 2.10. Evaluated susceptibility was plotted versus applied field at 15.0 GHz using the theoretical equation, Equation (1.19), with typical physical parameters for a nickel thin film. The complex Lorentzian curve, Equation (2.18), with the same resonance position, resonant amplitude, and FWHM as the theoretical curve (found numerically) was plotted for comparison. The curve shapes were not identical but were most similar near resonance, the main region of interest.

The resonance peak position and width, x_0 and Δx , were fixed to the previously found values and not fit during this step. The full equation for offset correction was

$$y(x) = (a_{re} + ia_{im})x + (b_{re} + ib_{im}) + y_{signal}(x, A, \phi) . \quad (2.19)$$

The offset position and resonance curve's phase were dependent variables. For example, zero phase yielded a real curve above the offset, while π phase yielded a real curve below the offset. Therefore, phase fitting was necessary to accurately determine the real and imaginary offsets.

The fit's linear part was subtracted, yielding only the Lorentzian part. According to Equation (2.16), the Lorentzian part was proportional to the sample's complex susceptibility.

2.3.6 *Susceptibility Fit*

The extracted Lorentzian signal from S_χ was divided by $i\omega\mu_0/Z_0$. This resulted in the complex susceptibility of the sample scaled by an unknown geometry-dependent factor, C , from Equation (2.16). A complex Lorentzian fit was performed again using Equation (2.18) to characterize FMR in the sample for each frequency.

The most relevant fit parameters were the linewidth, Δ_x , and resonance field, x_0 . The resonant field best represented the relationship between frequency and magnetic field in FMR. At higher magnetic fields, the magnetization experienced a larger torque. This caused resonant precession at a higher frequency. The equation relating resonant field and frequency for a sample is commonly known as the “Kittel equation” [34]. The linewidth arose from in-homogeneity and damping in the sample [34]. Together, the resonant field and linewidth described magnetization dynamics in the sample in resonance.

The linewidth parameter in Equation (2.18), Δ_x , was converted into the full width at half-maximum magnitude (FWHM) of the resonance curve to allow comparison with theory. For small linewidth, FWHM can be considered proportional to the linewidth parameter, $\text{FWHM} = \Delta_x \{ \sqrt{3} + \mathcal{O}[(\Delta_x/x_0)^2] \}$. The program used the exact expression,

$$\text{FWHM} = \sqrt{x_0} \left(\sqrt{x_0 + \Delta_x \sqrt{3}} - \sqrt{x_0 - \Delta_x \sqrt{3}} \right). \quad (2.20)$$

The two terms in Equation (2.20) were the low and high values such that

$|y_{\text{signal}}(x)| = |y_{\text{signal}}(x_0)|/2$. Equation (2.20) did not have a real solution for small resonance field and large linewidth. In these cases, $|y_{\text{signal}}(x)| > |y_{\text{signal}}(x_0)|/2$ for $x < x_0$, so the FWHM was not well-defined. Instead, the half-width at half-maximum was calculated and then doubled to find the effective FWHM.

After this step, the complex S-parameter measurements over a range of applied fields had yielded two real parameters: linewidth and resonant field. Other research groups used different methods of correction and measurement, but most of them arrived at values for linewidth and resonance. Measurements can now be easily compared between studies.

2.3.7

Kittel Fit

The theoretical trends of resonant field and linewidth versus frequency are presented in Section 1.3. Equation (1.20), Equation (1.21), and Equation (1.23) were fit from fit values of resonance field and FWHM. Resonant field and FWHM are functions of

- f , microwave frequency (the independent variable)
- g , sample g-factor
- M_s , magnitude of saturation magnetization
- α , Gilbert damping constant
- Δ_{H0} , in-homogeneous line broadening

To obtain consistent fit results, both trends were fit simultaneously using the same fitting program that the complex-valued fits used. The fitting was weighted by the resonant field and FWHM standard errors. Fitting yielded values that characterized the nature of magnetization in the sample.

III. RESULTS AND ANALYSIS

One thin film sample and one nanowire array sample were created using electrodeposition. Data from FORC and FMR measurements were fit according to theory. The fits yielded quantities used to characterize magnetic properties of the sample. Results were compared against theory and past studies.

3.1 Thin Film

A nickel thin film was made by pulsed electrodeposition onto gold-coated glass. The thickness was controlled by the total deposition time. This process and related tests are outlined in Appendix B. The thin film was then measured using FORC and FMR techniques.

3.1.1 *Static Magnetometry*

The saturation magnetic moment was $m_s = 7.06 \pm 0.02 \text{ A mm}^2$ at $H_a = 20.0 \text{ A/mm}$. The thin nickel film's average thickness was $184 \pm 1 \text{ nm}$, calculated from m_s , reported M_s [25], and film area. This experimental value showed discrepancy with predicted thickness from deposition tests in Appendix B.

FORC measurement involved 80 reversal curves, shown in FIG. 3.1. FORCs were equally spaced in H_a and H_b with a maximum field $H_a = 23.9 \text{ A/mm}$. The major curve was sloped outside the hysteresis loop. This suggested some of the change in magnetic moment was reversible and would not be accounted for by the Preisach distribution. A few curves were spaced apart inside the hysteresis loop. The curves in the hysteretic region corresponded to different configurations of hysteron states. The few numbers of these curves limited the Preisach distribution's resolution. The other FORCs overlapped with the major curves.

FIG. 3.1. The FORCs of a thin nickel film were measured to analyze magnetic reversal behavior.

The same data were plotted onto the 2D Preisach plane in the (h_b, h_a) basis, shown in FIG. 3.2. The vertical axis represents the different curves in FIG. 3.1. The $m(H_b, H_a)$ plot was more easily comparable to the Preisach distribution and clearly presented FORCs that followed the major curves.

FIG. 3.2. Magnetic moment of the thin film was measured at varying applied field, H_b , ascending from initial applied fields, H_a . The Preisach distribution was calculated from this plot.

The contours in FIG. 3.2 emphasized the asymmetric reversal slopes. Early reversal corresponding to domain nucleation was more gradual. Late reversal corresponding to domain growth was sharper. The subset of measurements that filled in the hysteresis loop in FIG. 3.1 is also shown. Points outside this region overlapped in FIG. 3.1.

The Preisach distribution in FIG. 3.3 was calculated as the mixed derivative of the data in FIG. 3.2 according to Equation (2.1). The Preisach distribution maps hysterons that describe reversal in the sample. The thin film's observed Preisach distribution was a sharp peak with h_c near the sample's overall coercivity. The boomerang-shaped tails in this thin film sample were not as pronounced as in others.

FIG. 3.3. The Preisach distribution was calculated from FORC data of a thin film and plotted in the (h_c, h_u) basis. The distribution was a sharp peak, typical for thin films.

The integrated volume under the distribution yielded a value for the saturation moment, according to Equation (1.3). The total moment from the Preisach distribution, $m_s = 5.35 \text{ A mm}^2$, was less than m_s of the major curve. The Preisach distribution accounted for 76% of the magnetic material undergoing reversal, which is related to the slope near saturation in FIG. 3.1. This ratio varied among thin film samples, which could be a topic of future study.

The boomerang fit based on the moving Preisach model allowed modeling hysterons with respect to the internal field. The fit function defined a surface on the Preisach plane, plotted as contours in FIG. 3.4 against the experimental distribution for comparison. The boomerang distribution did fit the data successfully, giving evidence for an effective magnetizing interaction. The fit variable results in TABLE 3.1 estimated the size and

position of the intrinsic hysteron distribution and the effective magnetizing field. Variables are defined at the end of Section 2.2.5.

FIG. 3.4. The boomerang function, defined in the (h_b, h_a) basis, was fit to the thin film's experimental Preisach distribution. Hystérons were modeled as a larger intrinsic distribution in the presence of a magnetizing field. The fit variable results are in TABLE 3.1.

TABLE 3.1. Thin film boomerang FORC fit results

variable	V [A mm ²]	$\tilde{h}_{c0}$ [A/mm]	$\tilde{h}_{u0}$ [A/mm]	$\tilde{\Delta}_{\text{avg}}$ [A/mm]	$\tilde{\Delta}_r$	H_{ds} [A/mm]
fit value	5.09	5.67	0.187	1.35	0.523	1.68
st. err.	0.09	0.01	0.009	0.02	0.007	0.03

The fit distribution matched the data reasonably considering the limited resolution. The effective distribution accurately approximated the observed distribution's peak center

and width, determined by $\tilde{h}_{c0}$, $\tilde{h}_{u0}$. The volume under the fit distribution was within 5% of the volume under the experimental distribution, meaning most of the hysterons were accounted for.

Zero magnetizing field would have simply resulted in the intrinsic hysteron distribution. The significantly non-zero result for H_{ds} illustrated that the experimental Preisach distribution's shape did not simply follow a 2D Gaussian function. The moving Preisach model provides significantly improved shape in modeling the observed distribution shape. The fit magnetizing field was small compared to the coercivity. This was reasonable considering the expected demagnetizing factor for a plane sample was $N_{\parallel} = 0$.

A single magnetic domain was a region of uniform magnetization. The large perpendicular demagnetizing factor, $N_{\perp} = 1$, constrained magnetization to the sample plane. However, magnetization was free to rotate through the in-plane axis perpendicular to the applied field as illustrated in FIG. 1.3. Therefore, a thin film domain might not have behaved like a two-state hysteron with square hysteresis as assumed by the Preisach model.

The intrinsic distribution center was closer to $h_u = 0$ than the effective distribution center was. An isolated nanomagnet was expected to have hysteresis with inversion symmetry, or $h_u = 0$. Therefore, the $\tilde{h}_{u,0}$ result supported the moving Preisach model theory.

The average coercive field, $\tilde{h}_{c,0}$, represents the coercivity of memory units in the film corrected for the effective magnetizing field. Coercivity estimated by the classic Preisach model is a 7% under-estimate according to Equation (2.5)c and Equation (2.5)d. Coercivity from major curve measurement was 4.85 ± 0.05 A/mm, a 5% under-estimate. The measured coercivity was about 3 times that of bulk cold-worked nickel or 10 times

that of annealed nickel [38]. Coercivity could have been related to the size of grains in the film [10].

3.1.2 *Magnetization Dynamics*

FMR measurement and analysis were performed on the thin film sample with in-plane applied field. Applied field was swept downward from 600 A/mm. The frequency was varied between 1 GHz and 20 GHz in 1 GHz steps. The data at some frequencies, especially low frequencies, were removed due to a lack of discernible resonance signal within the measured field range and step size.

Waveguide correction was performed with respect to the measurement of a blank waveguide. In FIG. 3.5, the complex S-parameters were plotted on the complex plane for each frequency and connected into a curve. The correction successfully removed the group delay and some magnitude attenuation of the transmission S-parameters, S_{12} and S_{21} . The reflection was moderately improved. Removing most of the effects of the waveguide improved the accuracy of calculated susceptibility at the sample's position.

FIG. 3.5. The measured complex S-parameters of a thin film, $[S^{Meas}]$, were compared with those of blank measurement, $[S^{Bkgnd}]$. The derivative of phase with respect to frequency (points along the curves) was large due to the waveguide length. After waveguide correction, $[S^{Corr}]$, the average trend of complex phase was more constant.

The fits were performed for each frequency. Each fit in this section is illustrated using 15 GHz as an example of typical fitting. The magnitude of transmission, $|S_{21}|$, was fit to estimate the resonant field and perform windowing, shown in FIG. 3.6. The resonance signal's amplitude was 1% to 5% of the non-magnetic offset in S_{21} . At 15 GHz, resonance was found to be near 243 A/mm. The field was windowed to a range of 309 A/mm, or four times the full width at half-maximum magnitude (FWHM).

FIG. 3.6. The thin film's magnitude of S_{21} for each frequency (15.0 GHz shown here) was fit to find an initial estimate of the resonant field. The result provided a refined window for data and improved accuracy of the linear background subtraction.

The non-magnetic linear component of S_{χ} was fit, shown in FIG. 3.7. The linear component was subtracted to obtain the sample's magnetic susceptibility. The relative closeness of the fit near resonance shows the previous fit's resonant field value was accurate. The fit linear background was influenced by small errors in resonant field and FWHM, which were kept constant. The non-magnetic offset in the susceptibility calculation for 15 GHz was $-0.1372 - i 0.1278 \pm .0003$.

FIG. 3.7. The linear background was subtracted by fitting with fixed resonant field and FWHM.

The resulting data in FIG. 3.8 were the theoretical susceptibility of the nanowire array multiplied by an unknown geometric factor. The linear background subtraction removed the non-magnetic contributions. The 15 GHz curve had a resonant field of 240.2 ± 0.6 A/mm and a FWHM of 83 ± 2 A/mm.

FIG. 3.8. The corrected complex susceptibility was fit to find resonant field and FWHM for each frequency, 15 GHz in this case.

The susceptibility fit had small but significant errors for fields above and below resonance. Imperfect linear background subtraction or small differences in curve shape could cause this. The fit amplitude, phase, and linewidth contained error due to mismatched fit and data curve shapes. However, the resonant field was reliable since the fit residuals seemed to be symmetric about the resonant field. The root-mean-square residuals were between 5% and 12% of the resonance amplitudes for each frequency.

The fit resonant field and linewidth were weighted by their standard error and were fit versus frequency according to the theoretical trend for a thin film with in-plane applied field. The data and fit curves are shown in FIG. 3.9 and FIG. 3.10. The fit values are in TABLE 3.2. The data for frequencies 10 GHz and 12 GHz were removed as outliers in FWHM based on Chauvenet's criterion for outliers [39]. The relative root mean square

residuals were 2% for resonant field and 9% for linewidth. Neither fit had residual trends, so the FMR measurements were successfully modeled by the derived fit equations.

FIG. 3.9. The fit in-plane resonant field versus frequency data for the thin film sample were fit according to theory. TABLE 3.2 shows the fit results.

FIG. 3.10. The in-plane field's FWHM versus frequency data for the thin film sample were fit according to theory. TABLE 3.2 shows the fit results.

TABLE 3.2. Thin film in-plane FMR fit results

variable	M_s [A/mm]	α	g	Δ_{H0} [A/mm]	H_k [A/mm]
fit value	420	0.057	2.2	40	-8
st. err.	110	0.006	0.2	3	5

The field offset, H_k , was included in the derivation to account for any additional in-plane internal fields, such as crystalline anisotropy. The fit results provided no evidence for such fields within measurement uncertainty since H_k was found to be small relative to M_s . The susceptibility fit at low frequencies resulted in large relative standard errors due to limited applied field resolution and relatively small amplitude.

The Gilbert damping constant, α was found to be on the order of 0.05. This agreed with the cited result for a nickel film of similar thickness, $\alpha = 0.043$ [31]. The Gilbert

damping constant is a quality factor of resonance that depends on the material properties and average quality [34].

The in-homogeneous line-broadening, Δ_{H0} , was a variable that accounted for local variation in resonant field across the sample [34]. Possible nano-scale causes included material defects or grain boundary effects [34]. Saturation magnetization might also have been non-uniform on a larger scale [34]. This value was found to be non-zero and on the order of typical linewidth. The linewidth component from in-homogeneous line broadening was greater than or equal to the component from Gilbert damping from 1 GHz to 20 GHz.

The saturation magnetization, M_s , had a large relative standard error. Qualitatively, M_s affects the fit curvature, which is typically more prominent at low frequencies. More measurements, better resolution, and improved fitting at low frequencies would have yielded smaller M_s standard error. The M_s fit result showed rough agreement with the cited value for bulk nickel, $M_s = 521.7 \pm 0.3$ A/mm [25].

The effective saturation (de)magnetizing field was assumed to be proportional to saturation magnetization. Fit results from FORC and FMR were combined to calculate the effective demagnetizing factor, $N_{||} = -H_{ds}/M_s = -0.0040 \pm 0.0011$.

Alternatively, the demagnetizing factor can be found from FORC fitting alone. The saturation magnetization of a thick enough film can be approximated by the bulk value for nickel [25]. With this assumption, FORC fitting yielded $N_{||} = -0.00322 \pm 0.00006$.

The expected demagnetizing factor for a plane was zero, which was supported by the resulting magnitude being much less than one. The small effective magnetizing field in the thin film provided an explanation for the Preisach distribution's boomerang shape.

3.2 Nanowire Array

Nanowire length was controlled by timed DC electrodeposition of nickel into nano-pore templates as described in Appendix B. The average nanowire length was

estimated to be $4.6 \pm 0.8 \mu\text{m}$ from measured saturation moment, sample area, template porosity, and cited M_s for nickel [31].

3.2.1 Static Magnetometry

The VSM measured 118 FORCs in the applied field range of $\pm 120 \text{ A/mm}$, shown in FIG. 3.11. The nanowire sample had saturation moment $m_s = 12.61 \pm 0.01 \text{ A mm}^2$ at $H_a = 100 \text{ A/mm}$.

FIG. 3.11. The FORCs were measured for a $4.6 \mu\text{m}$ nanowire array to analyze magnetic reversal.

The major curve was more constant than the film major curve was in FIG. 3.1. This shows that almost all of the magnetic material exhibited hysteresis. Secondly, more FORCs filled in the major loop than the film FORCs did in FIG. 3.1. This allowed better Preisach distribution resolution since the distribution size was large relative to coercivity.

FIG. 3.12 also shows that more individual measurements were inside the hysteretic region. The contours in FIG. 3.12 indicate that early and late reversal events occurred over similar ranges of field, unlike those from the thin film data in FIG. 3.2. In the discussion in Derivation A.2.1 and Derivation A.2.2, this is attributed to the demagnetizing field providing a negative feedback effect in nanowire reversal.

FIG. 3.12. Magnetic moment of the 4.6 μm nanowire array was measured at varying applied field, H_b , ascending from initial applied fields, H_a . Contours of moment and the hysteretic region allow comparison with FIG. 3.1.

The Preisach distribution, obtained by the mixed derivative of FIG. 3.12, is shown in FIG. 3.13. The distribution had a typical wishbone shape. The IFD had a more prominent amplitude than that of the CFD and was slightly tilted counter-clockwise from the h_u direction. The Preisach density elsewhere seemed to be insignificant given measurement uncertainty. The experimental distribution was integrated numerically to obtain a value for

saturation moment, $V = 12.49 \text{ A mm}^2$. This value was 99.0 % of the value obtained from hysteresis, meaning virtually all magnetic reversal events in the sample were irreversible.

FIG. 3.13. The distribution of hysterons on the Preisach plane was calculated from FORC measurements of a $4.6 \mu\text{m}$ nanowire array. The distribution had a typical “wishbone” shape.

The Preisach distribution was fit using the derived wishbone function, Equation (2.10). The fit distribution was plotted as contours on the Preisach plane in FIG. 3.14, along with the experimental distribution for comparison. The fit was more successful near the IFD

than near the CFD. The effective distribution was a simplified distribution for an intermediate calculation step. The effective distribution followed the fit distribution well for the IFD. The effective CFD did not allow curvature seen in the experimental CFD.

FIG. 3.14. The wishbone fit was applied to the 4.6 μm nanowire array's experimental Preisach distribution. Hysteron were modeled as a smaller intrinsic distribution in the presence of a demagnetizing field. The fit variable results are in TABLE 3.3.

TABLE 3.3. Nanowire array wishbone FORC fit results

variable	V [A mm ²]	$\tilde{h}_{c0}$ [A/mm]	$\tilde{h}_{u0}$ [A/mm]	$\tilde{\Delta}_{\text{avg}}$ [A/mm]	$\tilde{\Delta}_r$	H_{ds} [A/mm]	V_r
fit value	11.0	35.52	-3.1	4.05	1.50	25.20	0.250
st. err.	0.1	0.06	0.1	0.04	0.03	0.02	0.006

The experimental CFD followed the fit CFDs for low h_c . At higher h_c , the experimental CFD was sharper than the fit and stayed strictly above $h_u = 0$. The corrected fit distribution volume suggested that about 10% of hysterons were not accounted for during fitting.

The fit result indicated the observed Preisach distribution can be described by a smaller intrinsic distribution at low h_c , illustrated in FIG. 3.14. The most relevant parameter is the average intrinsic coercivity, $\tilde{h}_{c0} = 35.52 \pm 0.06$ A/mm. If a hysteron represented a single nanowire, this would be the coercivity of the nanowire. The nanowire array's $\tilde{h}_{c0}$ is about 7 times that of the thin film. This can be partly explained by the nanowire's anisotropy, or energy cost of rotating the magnetization through the transverse axis according to the Stoner-Wohlfarth model. In reality, a "curling mode," rather than homogeneous rotation, is likely the mechanism of reversal given the nanowire diameter [1]. Therefore, the high intrinsic coercivity can be explained by the 1D nature of the nanowires.

The demagnetizing field from anti-parallel neighbors caused the apparent coercivity to increase. The anti-parallel state of the array was more resilient to changes in the applied magnetic field than a single nanowire would have been. The demagnetizing field strength can be approximated by the size of the observed wishbone-shaped Preisach distribution, as explained in Derivation A.2.2.

A nanowire could behave like an ideal hysteron. In the Stoner-Wohlfarth model, the nanowire's large transverse demagnetizing factor $N_{tr} \approx 1$ prevented magnetization rotation

to the transverse direction. The nanowire prefers to be saturated either positive or negative. Therefore, a nanowire could be reasonably modeled as a single two-state hysteron.

Variation between nanowires would account for an intrinsic coercive field distribution, $\tilde{\Delta}_c$. The single nanowire's coercivity depends on its length and diameter [30]. The electrodeposition rate of nanowires was assumed to be uniform across the sample when calculating nanowire length. In reality, some nanowires might have been longer than others. The pore diameter was experimentally controlled by the supplier but not verified and might also have varied between sites.

An isolated nanowire's major loop was expected to be centered, represented by a hysteron with $\tilde{h}_u = 0$. The fit $\tilde{h}_{u0}$ was significantly negative but on the order of the intrinsic distribution width. This could have been accounted for by a skewed (not a 2D Gaussian) intrinsic distribution. Zeroing error of the applied field would have also resulted in non-zero $\tilde{h}_{u0}$. The CFD's sharp edge at $h_u = 0$ was evidence against possible zeroing error.

The interaction field width of the intrinsic hysteron distribution, $\tilde{\Delta}_u$, could be explained by variation in neighbor spacing. When imaged by a scanning electron microscope before this study, the pores in the AAO templates were not a perfectly close-packed lattice. Variation in neighbor spacing would have caused local changes in magnetic dipole field. The wishbone fit assumed uniform demagnetizing field for all hysterons, which determined the shift distance in h_u . Therefore, variation in neighbor spacing would not be accounted for by the fit and would result in larger $\tilde{\Delta}_u$.

Overall, the wishbone function provided a successful fit of the data given the derivation's assumptions. The effective and intrinsic distributions were based on Gaussian functions for convenience. The demagnetizing field was assumed to be uniform when applying the moving Preisach model. The successful fit of the wishbone shape suggested that magnetic reversal in the nanowire array was accurately described by the moving Preisach model.

3.2.2

Magnetization Dynamics

FMR measurements and analysis were performed on the nanowire array sample with axial applied field. Like in the thin film measurement, field was swept downward from 600 A/mm. The frequencies with reliable fits were in the range 10.00 GHz to 19.00 GHz. The waveguide correction shown in FIG. 3.15 had a similar degree of success as that of the thin film measurement.

FIG. 3.15. The measured complex S-parameters of a 4.6 μm nanowire array, $[S^{Meas}]$, were compared with those of blank measurement, $[S^{Bkgnd}]$. After waveguide correction, $[S^{Corr}]$, the average trend of complex phase was more constant.

Again, the example fits shown here were performed on the 15 GHz data. The fit of $|S_{21}|$ shown in FIG. 3.16 provided values $H_{res} = 189.9 \pm 0.9$ A/mm and

$\text{FWHM} = 165 \pm 6 \text{ A/mm}$. The resonance signal amplitudes were between 1.2% and 9% of the offset value, similar to the thin film measurements.

FIG. 3.16. The magnitude of S_{21} was fit to find an initial estimate of resonant field. The result provided a refined window for data and improved accuracy of the linear background subtraction.

The linear component was subtracted from S_{χ} using the fit shown in FIG. 3.17. The fit also accounted for the resonance signal for improved accuracy. The background was $-0.2118 - i 0.2889 \pm .0001$.

FIG. 3.17. The linear background was subtracted by fitting with fixed resonant field and FWHM.

The resulting susceptibility was plotted and fit, shown in FIG. 3.18. The fit provided resonant field, $H_0 = 192.4 \pm 0.8 \text{ A/mm}$, and linewidth, $\text{FWHM} = 176 \pm 2 \text{ A/mm}$. In this particular fit, there were noticeable errors away from resonance. This is likely an over-estimate of linewidth due to errors in signal extraction.

FIG. 3.18. The corrected complex susceptibility was fit to find resonant field and FWHM for each frequency.

The susceptibility fit results for resonant field and linewidth were compiled and fit versus frequency, shown in FIG. 3.19 and FIG. 3.20. The fit values are in TABLE 3.4. The frequencies not plotted had unsuccessful fits or no discernible resonance signal. Also, the data for frequencies 11 GHz and 19 GHz were removed as outliers in FWHM based on Chauvenet's criterion for outliers [39]. The relative root mean square residuals were 14% for resonant field and 8% for linewidth.

FIG. 3.19. The fit axial resonant field versus frequency data for the nanowire array sample were fit according to theory. TABLE 3.4 shows the fit results.

FIG. 3.20. The axial field's fit FWHM versus frequency data for the nanowire array sample were fit according to theory. TABLE 3.4 shows the fit results.

TABLE 3.4. Nanowire array axial FMR fit results

variable	$N_{\Delta}M_s$ [A/mm]	α	g	Δ_{H0} [A/mm]
fit value	230	0.14	2.0	70
st. err.	70	0.08	0.3	70

The Gilbert damping constant, α , was found to be on the order of 0.1. This result was larger than that of the thin film. The less coherent resonance could have been due to differences in geometry. A thin film is continuous in the in-plane direction perpendicular to applied field. A nanowire array is discontinuous in the transverse direction, which

would reduce resonance coherence. Large α could have also indicated pinning of magnetization due to defects from electrodeposition or at the lateral surfaces.

The in-homogeneous line-broadening, Δ_{H0} , was similar in magnitude to that of the thin film but had a larger relative standard error. More frequencies or improved fitting would be needed to obtain a precise value.

The magnetization term, $N_{\Delta}M_s$, had large standard error. The fit value was on the order of the expected saturation magnetization. The saturation magnetization was not separable in the theoretical equation. Therefore, the effective axial and transverse demagnetizing factors could not be estimated solely from this method of FMR analysis.

However, the FORC analysis results in TABLE 3.3 also provided an estimate for the axial demagnetizing field. This extra information was included with the FMR result and a known property of demagnetizing factors,

$$N_{tr} + N_{tr} + N_{ax} = 1 \quad (3.1a)$$

$$(N_{tr} - N_{ax})M_s = 230 \pm 70 \text{ A/mm} \quad (3.1b)$$

$$N_{ax}M_s = 25.20 \pm 0.02 \text{ A/mm} . \quad (3.1c)$$

Solving Equation (3.1) yielded estimates for N_{ax} , N_{tr} , and M_s ,

$$N_{ax} = 0.047 \pm 0.012 \quad (3.2a)$$

$$N_{tr} = 0.476 \pm 0.006 \quad (3.2b)$$

$$M_s = 540 \pm 140 \text{ A/mm} . \quad (3.2c)$$

Therefore, the demagnetizing factors of nanowire arrays could be determined by combining FORC and FMR analysis results.

The demagnetizing factor estimates were much closer to that of an infinite wire in Equation (1.10), $N_{ax} = 0$, than to that of an infinite plane, $N_{\perp} = 1$. An exact formula exists

for the demagnetizing factor of an ellipsoid [1]. The nanowire's equivalent ellipsoid had a demagnetizing factor $N_{ax} = 0.0011$.

The experimental N_{ax} was an order of magnitude larger than the theoretical single nanowire value. The magnetic dipole interactions between neighboring nanowires could account for the larger N_{ax} . This claim is also supported by the observed wishbone-shaped Preisach distribution, an effect of demagnetizing field from neighbors as explained in Derivation A.2.2. Therefore, the methods of calculating demagnetizing factors could be used in future studies of arrayed nanomagnets.

IV. CONCLUSION

Preisach distributions were calculated from FORC measurements of a nickel thin film and a nanowire array. The Preisach distribution was successfully fit using equations derived from the moving Preisach model, in which magnetic memory units respond to both the applied field and the (de)magnetizing field. The fit function produced distributions similar to typical observed Preisach distribution shapes, the “wishbone” and “boomerang.” The fits removed (de)magnetizing field effects to find the intrinsic coercivity of magnetic memory units in the system.

FMR fitting provided a value for internal magnetization. The fit equation for resonant field versus frequency was derived by modeling the sample as an infinite plane. The derived equation agreed with the equation by Kittel with an additional term for Gilbert damping [24]. Fit values from thin film fitting agreed with results from literature, validating the fitting procedure [31, 25]. The same derivation methods were used to find an equation to fit the nanowire array resonant field. Resulting fit values were of reasonable magnitude.

Demagnetizing factors were calculated using fit saturation magnetization from FMR fitting and saturation demagnetizing field from FORC fitting. The thin film’s effective in-plane demagnetizing factor was -0.004 ± 0.001 . The negative sign suggested a small effective magnetizing interaction during magnetic reversal. The small magnitude agreed with the expected zero demagnetizing factor.

The nanowire array’s calculated axial demagnetizing factor was 0.042 ± 0.010 . The small magnitude made it similar to the theoretical value of zero for an isolated, infinite wire. The demagnetizing field between nanowires was small compared to saturation magnetization but comparable to the intrinsic coercivity.

4.1 Comparison with Other Studies

This study demonstrated quantitative analysis of FORC measurements. The presence of large interactions between nanowires made the classic Preisach model invalid [5]. The calculated Preisach distribution was treated as a “fingerprint” of magnetic interactions rather than a plot of intrinsic reversal behavior in the material [20, 28]. The Preisach distribution of arrayed structures had commonly been interpreted qualitatively rather than quantitatively.

Pike et al. simulated an intrinsic distribution of non-square hysterons in the presence of a uniform demagnetizing field (like Equation (1.7)) and dipole fields from neighbors (Equation (1.11)) [21]. The simulation parameters were manually fit to an experimental Preisach distribution with success [21]. In contrast, the wishbone fit in this study involved a closed-form function and a nonlinear fit program. The wishbone fit neglected small effects that Pike et al. accounted for. A hysteron simulation program was not needed to analyze the experimental Preisach distribution.

The equations in this study were derived based on descriptions from Gilbert et al. The descriptions were accompanied by simple derivations for hysteron movement, like Equation (A.2a), and hysteron simulations similar to those by Pike et al. [22]. Gilbert et al. also found agreement between demagnetizing field values from finite-element simulation and from experimental IFD heights, which showed a trend with respect to spacing between neighbors [22]. The wishbone fit’s IFD height was approximately the demagnetizing field strength, shown by Equation (A.44a).

Clime et al. also studied the demagnetizing field in arrays. They formulated a non-linear equation from finite element simulations for the dependence of the axial and transverse demagnetizing fields on nanowire length and neighbor separation [30]. They successfully verified their equations with experimental transverse major curves from nanowire arrays of varying nanowire length and composition [30]. Using that function

with values for arrays in this study yielded $N_{ax} = 0.08 \pm 0.02$. Precision was limited by nanowire diameter. The experimental result was about half of the value calculated here but had a similar order of magnitude. The results characterizing the nanowire array roughly agreed with results by Clime et al. Therefore, the FORC fit H_{ds} values in this study were considered to represent the axial demagnetizing field between nanowires in the array.

This study presented a relatively simple method for quantitative analysis of FORC measurements by fitting a non-linear function and combining with FMR analysis. The theory and quantitative results generally agreed with those from other studies. The reasonable results from the thin film sample validated the analysis methods in this study. The FORC fit values were therefore considered to describe the real demagnetizing field and intrinsic magnetic reversal in the nanowire array sample.

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APPENDIX

A. DERIVATIONS

This section includes relevant derivations and technical discussions for equations used in the thesis.

A.1 The Moving Preisach Model

The magnetic field strength inside the sample, $\tilde{H}$, can be expressed as the vector sum of the external field and effective demagnetizing field. Since the demagnetizing field is by convention opposing the applied field,

$$\tilde{H} = H - H_d \quad (1.7)$$

$$= H - NM$$

$$= H - NM_s r, \quad (A.1)$$

where $r = M/M_s \in [-1, +1]$, the normalized magnetization. Since magnetic moment is typically proportional to magnetization for a particular sample, $r = m/m_s$. In this case, r can be directly measured with VSM.

The applied field, H_b , along the descending major curve starting from positive saturation results in magnetization $M_\downarrow(H_a)$. The internal field is

$$\tilde{H}_a = H_a - NM_s r_\downarrow(H_a). \quad (A.2a)$$

The applied field $H = H_b$ along the ascending FORC starting at $H = H_a$ results in magnetization $M_\uparrow(H_a, H_b)$. The internal field is

$$\tilde{H}_b = H_b - NM_s r_\uparrow(H_a, H_b). \quad (A.2b)$$

According to the Moving Preisach model, the demagnetizing field causes a movement of a hysteron on the Preisach plane from its intrinsic position, $(\tilde{h}_a, \tilde{h}_b)$ to an observed

position, (h_a, h_b) . The transformation, $(h_a, h_b) \rightarrow (\tilde{h}_a, \tilde{h}_b)$ can be written as

$$\tilde{h}_a = h_a - NM_s r_{\downarrow}(h_a) \quad (\text{A.3a})$$

$$\tilde{h}_b = h_b - NM_s r_{\uparrow}(h_a, h_b) . \quad (\text{A.3b})$$

The observed Preisach distribution, $\rho(h_a, h_b)$, can be related to a point on an intrinsic Preisach distribution, $\tilde{\rho}(\tilde{h}_a, \tilde{h}_b)$. The transformation $(h_a, h_b) \rightarrow (\tilde{h}_a, \tilde{h}_b)$ is nearly isometric (uniform translation) for nearby points if $r_{\downarrow}$ and $r_{\uparrow}$ are smooth. Approximating the transformation as isometric,

$$\rho(h_a, h_b) \approx \tilde{\rho}(\tilde{h}_a, \tilde{h}_b) . \quad (1.8)$$

The transformation, Equation (A.3), may cause distances to contract or expand. In some cases, the transformation is not one-to-one. The integral over the Preisach density, or the number of hysterons, will not necessarily be preserved in these cases. The Preisach distribution can be roughly corrected by introducing a constant multiplied factor. For similar reasons, observed Preisach distributions can have regions of negative density.

A.2 Preisach Distribution Shapes

The observed Preisach distributions had two standard shapes. The moving Preisach model equations in Derivation A.1 account for these two shapes. The two cases are magnetizing and demagnetizing interactions between hysterons, or negative and positive effective demagnetizing factors, respectively. FIG. 1.6 shows hysteron translations due to interactions. This section explains the directions of those translations using equations in Derivation A.1.

A.2.1 Boomerang Preisach Distribution

The in-plane demagnetizing factor due to shape, $N_{\parallel}$, was expected to be zero for an infinite plane sample (see Equation (1.9)). However, the observed Preisach distribution matched that of a circular intrinsic distribution with an effective negative demagnetizing factor.

The vertical axis, h_a , represents reversal events along the descending major curve, $m_{\downarrow}(H_a)$. The first reversals with $m_{\downarrow} \approx +m_s$, are from hysterons with greatest $\tilde{h}_a$. Equation (A.3)a shows these hysterons move to $h_a = \tilde{h}_a - |N_{\parallel}|M_s$. Therefore, hysterons with greatest $\tilde{h}_a$ shift downward. Likewise, those with least $\tilde{h}_a$ shift upward. The distribution contracts vertically.

The horizontal axis, h_b , represents reversal events along the ascending FORCs, $m_{\uparrow}(H_a, H_b)$. Hysterons with greatest $\tilde{h}_b$ are reversals with $m_{\uparrow} \approx +m_s$, moving the hysteron to $h_b = \tilde{h}_b - |N_{\parallel}|M_s$. The distribution contracts leftward. $m_{\uparrow} \approx -m_s$ is satisfied by hysterons with least $\tilde{h}_a$ and least $\tilde{h}_b$. The lower left corner of the distribution contracts rightward.

The resulting distribution is a L-shaped distribution enclosing the lower-left quadrant. In the (h_c, h_u) basis, the shape of the observed distribution is a boomerang shape pointing up.

A.2.2 *Wishbone Preisach Distribution*

The axial demagnetizing factor due to shape, N_{ax} , is expected to be zero for a single infinite wire, Equation (1.10). In an array, the demagnetizing field from neighboring wires introduces an effective positive axial demagnetizing factor, seen in Equation (1.12).

The vertical axis, h_a , represents reversal events along the descending major curve, $m_{\downarrow}(H_a)$. The first reversals, $m_{\downarrow} \approx +m_s$, are from hysterons with greatest $\tilde{h}_a$. Equation (A.3)a shows these hysterons move to $h_a = \tilde{h}_a + |N_{ax}|M_s$. Therefore, hysterons with greatest $\tilde{h}_a$ move upward. Likewise, those with least $\tilde{h}_a$ move downward. The distribution expands vertically.

The horizontal axis, h_b , represents reversal events along the ascending FORCs, $m_{\uparrow}(H_a, H_b)$. Hysterons with greatest $\tilde{h}_b$ are reversals when $m_{\uparrow} \approx +M_s$, moving the hysteron to $h_b = \tilde{h}_b + |N_{ax}|M_s$. The distribution expands rightward in addition to upward. The condition $m_{\uparrow} \approx -m_s$ is satisfied by hysterons with least $\tilde{h}_a$ and least $\tilde{h}_b$. Some

hystérons in the bottom of the distribution expand leftward if they reverse early, while those that reverse late expand rightward as previously mentioned. The bottom of the distribution splits while the top does not.

The resulting distribution is Λ -shaped. In the (h_c, h_u) basis, the shape of the observed distribution is a “wishbone” shape. The interaction field distribution (IFD) contains hystérons that reverse first (least h_b) for each FORC. The coercive field distribution (CFD) contains hystérons that reverse last (greatest h_b) for each FORC.

Gilbert et al. outlines a similar quantitative description of the wishbone-shaped Preisach distribution, which the above explanations were inspired by [22].

A.3 Thin Film In-plane FMR

The static field was applied along $\hat{x}$ (in-plane), and the dynamic field was along $\hat{y}$ (in-plane) and $\hat{z}$ (perpendicular). A thin film has demagnetizing factors given by Equation (1.9). The dynamic demagnetizing field is,

$$\vec{H}_d(t) = N_d \vec{M}(t) \quad (1.6)$$

$$\vec{H}_d(t) = -N_{\parallel} M_s \hat{x} - N_{\parallel} M_y(t) \hat{y} - N_{\perp} M_z(t) \hat{z} . \quad (A.4)$$

The field and magnetization can be expressed as

$$\vec{H}_{\text{int}}(t) = H \hat{x} - N_{\parallel} M_s \hat{x} - N_{\parallel} M_y(t) \hat{y} - N_{\perp} M_z(t) \hat{z} + \vec{H}(t) \quad (A.5)$$

$$\vec{M}(t) = M_s \hat{x} + \vec{M}(t) . \quad (A.6)$$

Equation (A.5) and Equation (A.6) are the vector expressions describing the setup for thin film FMR measurement.

A.3.1 Susceptibility Equation

Equation (A.5) and Equation (A.6) can substituted into the LLG equation,

$$\frac{d\vec{M}}{dt} = -|\gamma| \vec{M} \times \mu_0 \vec{H}_{\text{int}} + \frac{\alpha}{M_s} \vec{M} \times \frac{d\vec{M}}{dt} . \quad (1.17)$$

Since $|\vec{M}(t)| \ll M_s$ and $|\vec{H}(t)| \ll H_{0x}$, their second-order terms can be ignored [34].

Simplifying terms individually,

$$\begin{aligned}
 \vec{M} \times \vec{H}_{int} &= (M_s \hat{x} + \vec{M}(t)) \times (H \hat{x} - N_{\parallel} M_s \hat{x} - N_{\parallel} M_y(t) \hat{y} - N_{\perp} M_z(t) \hat{z} + \vec{H}(t)) \\
 &\approx M_s \hat{x} \times (-N_{\parallel} M_y(t) \hat{y} - N_{\perp} M_z(t) \hat{z} + \vec{H}(t)) + \vec{M}(t) \times (H \hat{x} - N_{\parallel} M_s \hat{x}) \\
 &= [N_{\parallel} M_s M_y(t) \hat{y} + N_{\perp} M_s M_z(t) \hat{z} - M_s \vec{H}(t) + H \vec{M}(t) - N_{\parallel} M_s \vec{M}(t)] \times \hat{x} \quad (\text{A.7a})
 \end{aligned}$$

$$\begin{aligned}
 \vec{M} \times \dot{\vec{M}} &= [M_s \hat{x} + \vec{M}(t)] \times \dot{\vec{M}}(t) \\
 &\approx M_s \hat{x} \times \dot{\vec{M}}(t) \\
 &= [-M_s \dot{\vec{M}}(t)] \times \hat{x}. \quad (\text{A.7b})
 \end{aligned}$$

Then the full LLG equation, Equation (1.17) for the case of a thin film with in-plane field can be written as

$$\dot{\vec{M}}(t) = \{|\gamma|\mu_0[-N_{\parallel} M_s M_y(t) \hat{y} - N_{\perp} M_s M_z(t) \hat{z} + M_s \vec{H}(t) - H \vec{M}(t) + N_{\parallel} M_s \vec{M}(t)] - \alpha \dot{\vec{M}}(t)\} \times \hat{x}. \quad (\text{A.8})$$

Expressing this vector equation in terms of components,

$$\begin{bmatrix} 0 \\ \dot{M}_y(t) \\ \dot{M}_z(t) \end{bmatrix} = \begin{bmatrix} 0 \\ |\gamma|\mu_0(-N_{\perp} M_s M_z(t) + M_s H_z(t) - H M_z(t) + N_{\parallel} M_s M_z(t)) - \alpha \dot{M}_z(t) \\ |\gamma|\mu_0(-M_s H_y(t) + H M_y(t)) + \alpha \dot{M}_y(t) \end{bmatrix}. \quad (\text{A.9})$$

The dynamic field and magnetization are assumed to be complex exponential functions and in-phase [34],

$$\vec{M}(t) = \begin{bmatrix} 0 \\ M_{0y} e^{i\omega t} \\ M_{0z} e^{i\omega t} \end{bmatrix}; \quad \vec{H}(t) = \begin{bmatrix} 0 \\ H_{0y} e^{i\omega t} \\ H_{0z} e^{i\omega t} \end{bmatrix}. \quad (\text{A.10})$$

Substituting into Equation (A.9) and simplifying,

$$\begin{aligned}
 \begin{bmatrix} 0 \\ i\omega M_{0y} \\ i\omega M_{0z} \end{bmatrix} &= \begin{bmatrix} 0 \\ |\gamma|\mu_0(-N_{\perp}M_sM_{0z} + M_sH_{0z} - HM_{0z} + N_{\parallel}M_sM_{0z}) - \alpha i\omega M_{0z} \\ |\gamma|\mu_0(-M_sH_{0y} + HM_{0y}) + \alpha i\omega M_{0y} \end{bmatrix} \\
 &= \begin{bmatrix} N_{\perp}M_sM_{0z} + HM_{0z} - N_{\parallel}M_sM_{0z} + \alpha iH_LM_{0z} + iH_LM_{0y} \\ HM_{0y} + \alpha iH_LM_{0y} - iH_LM_{0z} \end{bmatrix} = \begin{bmatrix} M_sH_{0z} \\ M_sH_{0y} \end{bmatrix} \\
 &= \begin{bmatrix} H + i\alpha H_L & -iH_L \\ iH_L & N_{\Delta}M_s + H + i\alpha H_L \end{bmatrix} \begin{bmatrix} M_{0y} \\ M_{0z} \end{bmatrix} = M_s \begin{bmatrix} H_{0y} \\ H_{0z} \end{bmatrix}, \quad (\text{A.11})
 \end{aligned}$$

where $N_{\Delta} = N_{\perp} - N_{\parallel}$, which is 1 for an infinite film, and $H_L = \omega/|\gamma|\mu_0$, which was coincidentally the result for simple Larmour precession. Inverting the matrix yields the linear relationship,

$$\begin{bmatrix} M_{0y} \\ M_{0z} \end{bmatrix} = \frac{M_s}{D} \begin{bmatrix} N_{\Delta}M_s + H + i\alpha H_L & iH_L \\ -iH_L & H + i\alpha H_L \end{bmatrix} \begin{bmatrix} H_{0y} \\ H_{0z} \end{bmatrix}, \quad (\text{A.12})$$

where D is the determinant,

$$D = (H + i\alpha H_L)(N_{\Delta}M_s + H + i\alpha H_L) + (iH_L)^2.$$

Equation (A.12) shows a frequency-dependent precession by the dynamic magnetization in response to a dynamic applied magnetic field. In FMR, a dynamic magnetic field is produced by a waveguide carrying an RF wave such that the magnetic field is in-plane with the sample. The dynamic susceptibility tensor is defined,

$$M_i(t) = \chi_{ij}H_j(t). \quad (1.18)$$

FMR measurements yield the complex susceptibility for the direction of the dynamic magnetic field, in this case χ_{yy} . From the solution Equation (A.12),

$$\chi_{yy} = \frac{M_s(N_{\Delta}M_s + H + i\alpha H_L)}{(H + i\alpha H_L)(N_{\Delta}M_s + H + i\alpha H_L) + (iH_L)^2}. \quad (\text{A.13})$$

Distributing terms yields the final resonance curve equation,

$$\chi_{yy}(H, \omega) = \frac{M_s(N_\Delta M_s + H) + iM_s\alpha H_L}{H(N_\Delta M_s + H) - (1 + \alpha^2)H_L^2 + i\alpha H_L(N_\Delta M_s + 2H)}. \quad (1.19)$$

This result agreed with the more general equation for χ_{yy} derived in the dissertation by Lei Lu [34]. Other sources that included Gilbert damping dropped terms in the numerator and the α^2 term, which would not have greatly affected fit results [37].

A.3.2 *Resonant Field and Linewidth*

The resonance condition is satisfied when the susceptibility in Equation (1.19) is maximized. The susceptibility is near maximum when the real part of the denominator is zero,

$$\begin{aligned} \text{Re}[D(H_{res})] &= 0 \\ H_{res}(N_\Delta M_s + H_{res}) - (1 + \alpha^2)H_L^2 &= 0 \\ H_L^2 &= \frac{H_{res}(N_\Delta M_s + H_{res})}{1 + \alpha^2}. \end{aligned} \quad (A.14)$$

The resonance condition is commonly solved for frequency as a function of applied field,

$$\omega_{res} = \frac{|\gamma|\mu_0}{\sqrt{1 + \alpha^2}} \sqrt{H(N_\Delta M_s + H)}. \quad (A.15)$$

In this study the resonance field was plotted and fit as a function of frequency,

$$H_{res} = \frac{-N_\Delta M_s + \sqrt{(N_\Delta M_s)^2 + 4H_L^2(1 + \alpha^2)}}{2}. \quad (1.20)$$

The trend of resonant frequency given in Equation (1.20) agreed with that of derivations from angle-dependent FMR [34] and from ignoring the effect of damping [24] when measuring in-plane to a film. FIG. A.1 compares the fit equation, Equation (1.20), with the values obtained numerically from Equation (1.19) using typical physical parameters. Equation (1.20) over-estimated resonant field by $7.3 \pm 0.7\%$ for the physical parameters used.

FIG. A.1. (a) The theoretical resonant field was evaluated numerically from the theoretical susceptibility equation and compared with an approximate, derived resonant field equation used for fitting. (b) The same was done for the FWHM. The fit equation was corrected by including a multiplied factor of $\sqrt{3}$, allowing it to overlap with evaluated data markers on the plot.

The resonance curve's full-width at half-maximum (FWHM), Δ_H , could ideally be approximated from the field such that the denominator's magnitude is half as large as that of resonance. However, high-order terms prevented solving analytically. Instead, the FWHM of theoretical susceptibility, Equation (1.22), was evaluated numerically using typical physical parameters. The linear trend is shown in FIG. A.1. The result was compared to the equation derived in the dissertation by Lei Lu [34],

$$\Delta_H = 2\alpha H_L . \quad (\text{A.16})$$

In real magnetic materials, in-homogeneity across the sample or at grain boundaries results in the superposition of curves with different resonant fields [34]. The result is an additional component to linewidth that is independent of frequency, $\Delta_{H,0}$. The equation used by Lei Lu was,

$$\Delta_H = 2\alpha H_L + \Delta_{H,0} . \quad (\text{A.17})$$

The FWHM of theoretical susceptibility, Equation (1.19), was evaluated numerically. FIG. A.1 compares the numerical results Equation (A.17) for typical physical parameters.

Discrepancy was observed. Equation (A.17) seemed to be corrected after introducing a multiplied factor of $\sqrt{3}$,

$$\Delta_H = 2\sqrt{3}\alpha H_L + \Delta_{H0} , \quad (1.21)$$

also shown in FIG. A.1. This correction factor was the same as the one used with the susceptibility curve fit Equation (2.18) to convert from the width parameter to the FWHM. This was not a coincidence since Equation (1.19) and Equation (2.18) had similar structure and produced similar curve shape, as seen in FIG. 2.10.

A.4 Nanowire Array Axial FMR

The derivation for nanowire array FMR is similar to that of Derivation A.3. The static field was applied along $\hat{z}$ (axial), and the dynamic field was along $\hat{x}$ (transverse) and $\hat{y}$ (transverse). A nanowire array has unknown effective demagnetizing factors between that of a plane (Equation (1.9)) and an isolated nanowire (Equation (1.10)) [30]. The dynamic demagnetizing field is,

$$\vec{H}_d(t) = -N_d[M_s\hat{z} + \vec{M}(t)] \quad (A.18)$$

$$\vec{H}_d(t) = -N_{ax}M_s\hat{z} - N_{tr}M_x(t)\hat{x} - N_{tr}M_y(t)\hat{y} \quad (A.19)$$

$$= -N_{ax}M_s\hat{z} - N_{tr}\vec{M}(t) , \quad (A.20)$$

where $\vec{M}(t)$ is the dynamic, transverse component of $\vec{M}$ caused by microwave excitation.

The total magnetic field and magnetization can be expressed as

$$\vec{H}_{int}(t) = H\hat{z} - N_{ax}M_s\hat{z} - N_{tr}\vec{M}(t) + \vec{H}(t) \quad (A.21)$$

$$\vec{M} = M_s\hat{z} + \vec{M}(t) , \quad (A.22)$$

where H is the applied axial static field, and $\vec{H}(t)$ is the applied transverse RF magnetic field. These are the vector expressions describing the setup for nanowire array FMR measurement.

A.4.1

Susceptibility Equation

The fields can be substituted into the LLG equation,

$$\frac{d\vec{M}}{dt} = -|\gamma|\vec{M} \times \mu_0\vec{H}_{int} + \frac{\alpha}{M_s}\vec{M} \times \frac{d\vec{M}}{dt}. \quad (1.17)$$

Since $|\vec{M}(t)| \ll M_s$ and $|\vec{H}(t)| \ll H$, second-order terms can be ignored [34].

Simplifying terms individually,

$$\begin{aligned} \vec{M} \times \vec{H}_{int} &= [M_s \hat{z} + \vec{M}(t)] \times [H \hat{z} - N_{ax}M_s \hat{z} - N_{tr}\vec{M}(t) + \vec{H}(t)] \\ &\approx M_s \hat{z} \times [-N_{tr}\vec{M}(t) + \vec{H}(t)] + \vec{M}(t) \times [H \hat{z} - N_{ax}M_s \hat{z}] \\ &= [N_{tr}M_s\vec{M}(t) - M_s\vec{H}(t) + H\vec{M}(t) - N_{ax}M_s\vec{M}(t)] \times \hat{z} \end{aligned} \quad (A.23)$$

$$\begin{aligned} \vec{M} \times \dot{\vec{M}} &= [M_s \hat{z} + \vec{M}(t)] \times \dot{\vec{M}}(t) \\ &\approx M_s \hat{z} \times \dot{\vec{M}}(t) \\ &= [-M_s\dot{\vec{M}}(t)] \times \hat{z}. \end{aligned} \quad (A.24)$$

Equation (A.23) and Equation (A.24) are substituted into the full LLG equation,

Equation (1.17). For the case of a nanowire, array axial field can be written as,

$$\dot{\vec{M}}(t) = [-|\gamma|\mu_0(N_{tr}M_s\vec{M}(t) - M_s\vec{h} + H\vec{M}(t) - N_{ax}M_s\vec{M}(t)) + \alpha\dot{\vec{M}}(t)] \times \hat{z}.$$

Expressing this vector equation in terms of components,

$$\begin{bmatrix} \dot{M}_x(t) \\ \dot{M}_y(t) \\ 0 \end{bmatrix} = \begin{bmatrix} |\gamma|\mu_0(-N_{tr}M_sM_y(t) + M_sH_y(t) - HM_y(t) + N_{ax}M_sM_y(t)) + \alpha\dot{M}_y(t) \\ |\gamma|\mu_0(N_{tr}M_sM_x(t) - M_sH_x(t) + HM_x(t) - N_{ax}M_sM_x(t)) - \alpha\dot{M}_x(t) \\ 0 \end{bmatrix}. \quad (A.25)$$

The solutions are assumed to be in-phase complex exponential functions,

$$\vec{M}(t) = \begin{bmatrix} M_{0x}e^{i\omega t} \\ M_{0y}e^{i\omega t} \\ 0 \end{bmatrix}; \quad \vec{H}(t) = \begin{bmatrix} H_{0x}e^{i\omega t} \\ H_{0y}e^{i\omega t} \\ 0 \end{bmatrix}. \quad (A.26)$$

Substituting into Equation (A.25) and simplifying,

$$\begin{bmatrix} i\omega M_{0x} \\ i\omega M_{0y} \\ 0 \end{bmatrix} = \begin{bmatrix} |\gamma|\mu_0(-N_{tr}M_sM_{0y} + M_sH_{0y} - HM_{0y} + N_{ax}M_sM_{0y}) + i\alpha\omega M_{0y} \\ |\gamma|\mu_0(N_{tr}M_sM_{0x} - M_sH_{0x} + HM_{0x} - N_{ax}M_sM_{0x}) - i\alpha\omega M_{0x} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} N_{\Delta}M_s + H - i\alpha H_L & -iH_L \\ iH_L & N_{\Delta}M_s + H - i\alpha H_L \end{bmatrix} \begin{bmatrix} M_{0x} \\ M_{0y} \end{bmatrix} = \begin{bmatrix} M_s \cdot H_{0x} \\ M_s \cdot H_{0y} \end{bmatrix}, \quad (\text{A.27})$$

where $N_{\Delta} = N_{tr} - N_{ax}$ and $H_L = \omega/(|\gamma|\mu_0)$ like in the thin film result. Inverting the matrix in Equation (A.27) yields the linear relationship,

$$\begin{bmatrix} M_{0x} \\ M_{0y} \end{bmatrix} = \frac{M_s}{D} \begin{bmatrix} N_{\Delta}M_s + H - i\alpha H_L & iH_L \\ -iH_L & N_{\Delta}M_s + H - i\alpha H_L \end{bmatrix} \begin{bmatrix} H_{0x} \\ H_{0y} \end{bmatrix} \quad (\text{A.28})$$

$$D = (N_{\Delta}M_s + H - i\alpha H_L)^2 - H_L^2.$$

This gives measured susceptibility,

$$\chi_{yy} = \frac{M_s(N_{\Delta}M_s + H - i\alpha H_L)}{(N_{\Delta}M_s + H - i\alpha H_L)^2 - H_L^2}.$$

This simplifies to

$$\chi_{yy}(H, \omega) = \frac{M_s(H + N_{\Delta}M_s) + i\alpha M_s H_L}{(H + N_{\Delta}M_s)^2 - (1 + \alpha^2)H_L^2 - i2\alpha H_L(H + N_{\Delta}M_s)}. \quad (1.22)$$

The result for nanowire array had the same form as the general equation by Lei Lu [34].

Papers with FMR resonance equations specifically for nanowire arrays were not found.

A.4.2

Resonant Field and Linewidth

Again, resonance was approximated to be when the real part of the denominator was zero.

$$\begin{aligned}
 (H_{res} + N_{\Delta}M_s)^2 - (1 + \alpha^2)H_L^2 &= 0 \\
 H_L^2 &= \frac{(H_{res} + N_{\Delta}M_s)^2}{1 + \alpha^2} \\
 \omega_{res}(H) &= \frac{|\gamma|\mu_0}{\sqrt{1 + \alpha^2}}(H_{res} + N_{\Delta}M_s) \tag{A.29}
 \end{aligned}$$

$$H_{res}(\omega) = H_L \sqrt{1 + \alpha^2} - N_{\Delta}M_s . \tag{1.23}$$

The resulting resonance condition is linear, similar to that of a film with perpendicular anisotropy [37, 34], a commonly studied nanomagnet.

A.5

Boomerang Fit

The discussion in Derivation A.2.1 can provide a fit function for boomerang-shaped effective Preisach distributions. The intrinsic Preisach distribution is assumed to be a 2D Gaussian distribution in the (h_a, h_b) basis. The independent variables are the coordinates on the Preisach plane, (h_a, h_b) . The fit variables are the shape and position of the intrinsic distribution and the maximum demagnetizing field, NM_s .

In the Moving Preisach model, the magnetic field equals the sum of the external field (experimentally controlled) and the (de)magnetizing field ($\propto M$). Although expected to be boomerang-shaped, the effective distribution is temporarily modeled as a Gaussian distribution, Equation (2.4).

A Gaussian effective distribution allows the magnetization during a particular hysteron's reversal to be easily predicted from fit function inputs. Since magnetization can be predicted from inputs, the 2D Preisach distribution can be evaluated without experimental values for $M(h_a, h_b)$ from the FORCs themselves.

The Preisach integral, Equation (1.3), applied to a Gaussian distribution results in error-functions,

$$y = A \operatorname{erf} \left[\frac{x - x_0}{\Delta_x \sqrt{2}} \right] + y(x_0) . \quad (\text{A.30})$$

The integral requires knowledge of all hysteron states. In FORC measurements, the states are predicted based on their (h_b, h_a) coordinates and (H_b, H_a) magnetic history, according to FIG. 2.3. In this case, the normalized descending major and ascending FORCs are

$$\begin{aligned} r_{\downarrow}(h_a) &= \frac{M_{\downarrow}(h_a)}{M_s} \\ &= \operatorname{erf} \left[\frac{h_a - h_{a,0}}{\Delta_a \sqrt{2}} \right] \end{aligned} \quad (\text{2.5 a})$$

$$\begin{aligned} r_{\uparrow}(h_a, h_b) &= \frac{M_{\uparrow}(h_a, h_b)}{M_s} \\ &= \frac{1 - r_{\downarrow}}{2} \operatorname{erf} \left[\frac{h_b - h_{b,0}}{\Delta_b \sqrt{2}} \right] + \frac{1 + r_{\downarrow}}{2} . \end{aligned} \quad (\text{2.5 b})$$

For a hysteron with switching fields (h_b, h_a) , the magnetization while flipping down and up are predicted by $M_s r_{\downarrow}(h_a)$ and $M_s r_{\uparrow}(h_b, h_a)$, respectively.

The center of the Gaussian effective distribution transforms to the center of the intrinsic distribution. The transformation is solved for by substituting $(h_{a,0}, h_{b,0})$ into Equation (2.5).

$$\begin{aligned} r_{\downarrow}(h_{a,0}) &= \operatorname{erf} \left[\frac{h_{a,0} - h_{a,0}}{\Delta_a \sqrt{2}} \right] \\ &= 0 \end{aligned} \quad (\text{A.31a})$$

$$\begin{aligned} r_{\uparrow}(h_{a,0}, h_{b,0}) &= \frac{1 - r_{\downarrow}}{2} \operatorname{erf} \left[\frac{h_{b,0} - h_{b,0}}{\Delta_b \sqrt{2}} \right] + \frac{1 + r_{\downarrow}}{2} \\ &= \frac{1}{2} \cdot 0 + \frac{1}{2} \\ &= \frac{1}{2} . \end{aligned} \quad (\text{A.31b})$$

Substituting into the transformation equation, Equation (A.3), and introducing an effective magnetizing field,

$$\tilde{h} = h + NM_s r \quad (\text{A.3})$$

$$h_{a,0} = \tilde{h}_{a,0} \quad (\text{2.5 c})$$

$$h_{b,0} = \tilde{h}_{b,0} - \frac{NM_s}{2} . \quad (\text{2.5 d})$$

The effective distribution's width, Δ_b , can be found by the transformation of the left side of the distribution, $(h_{b,1}, h_{a,1}) \rightarrow (\tilde{h}_{b,1}, \tilde{h}_{a,1})$ where $h_{b,1} = h_{b,0} - \Delta_b$ and $h_{a,1} = h_{a,0}$. This section uses numbers 1-4 for the left, top, right, and bottom of distributions, respectively. Substituting into Equation (2.5),

$$\begin{aligned} r_{\downarrow}(h_{a,1}) &= r_{\downarrow}(h_{a,0}) \\ &= 0 \end{aligned} \quad (\text{A.32a})$$

$$\begin{aligned} r_{\uparrow}(h_{a,1}, h_{b,1}) &= \frac{1 - r_{\downarrow}}{2} \operatorname{erf}\left(\frac{h_{b,0} - \Delta_b - h_{b,0}}{\Delta_b \sqrt{2}}\right) + \frac{1 + r_{\downarrow}}{2} \\ &= \frac{1}{2} \left[1 - \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) \right] . \end{aligned} \quad (\text{A.32b})$$

The translation from the intrinsic to effective distribution, Equation (A.3), for the left sides is

$$\begin{aligned} \tilde{h} &= h + NM_s r \quad (\text{A.3}) \\ h_{a,1} &= \tilde{h}_{a,1} \\ h_{b,1} &= \tilde{h}_{b,1} - \frac{NM_s}{2} \left[1 - \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) \right] \\ &\approx \tilde{h}_{b,1} - 0.159 NM_s . \end{aligned}$$

The left side of the intrinsic hysteron shifts slightly left. The new location relative to the intrinsic center is

$$h_{a,1} = \tilde{h}_{a,0} \quad (\text{A.33a})$$

$$h_{b,1} = \tilde{h}_{b,0} - \tilde{\Delta}_b - \frac{NM_s}{2} \left[1 - \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) \right]. \quad (\text{A.33b})$$

Substituting the location of the effective center, Equation (2.5), allows solving for the effective width,

$$\begin{aligned} h_{b,0} - \Delta_b &= h_{b,1} \\ \tilde{h}_{b,0} - \frac{NM_s}{2} - \Delta_b &= \tilde{h}_{b,0} - \tilde{\Delta}_b - \frac{NM_s}{2} \left[1 - \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) \right] \\ \frac{NM_s}{2} + \Delta_b &= \tilde{\Delta}_b + \frac{NM_s}{2} \left[1 - \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) \right] \\ \Delta_b &= \tilde{\Delta}_b - \frac{NM_s}{2} \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) \\ &\approx \tilde{\Delta}_b - 0.341 NM_s. \end{aligned} \quad (2.5 \text{ e})$$

As mentioned in Derivation A.2.1, the distribution contracts horizontally.

Similarly, the effective distribution's height, Δ_a , can be found by the transformation of the bottom side of the distribution, $(h_{b,4}, h_{a,4}) \rightarrow (\tilde{h}_{b,4}, \tilde{h}_{a,4})$ where $h_{b,4} = h_{b,0}$ and $h_{a,4} = h_{a,0} - \Delta_a$. Substituting into Equation (2.5),

$$\begin{aligned} r_{\downarrow}(h_{a,4}) &= \operatorname{erf}\left(\frac{h_{a,0} - \Delta_a - h_{a,0}}{\Delta_a \sqrt{2}}\right) \\ &= -\operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) \end{aligned} \quad (\text{A.34a})$$

$$\begin{aligned} r_{\downarrow}(h_{a,1}) &= \frac{1 - r_{\downarrow}}{2} \operatorname{erf}\left(\frac{h_{b,0} - h_{b,0}}{\Delta_b \sqrt{2}}\right) + \frac{1 + r_{\downarrow}}{2} \\ &= \frac{1}{2} \left[1 - \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) \right]. \end{aligned} \quad (\text{A.34b})$$

The translation from the intrinsic to effective distribution, Equation (A.3), for the bottom side is

$$\begin{aligned}
 \tilde{h} &= h + NM_s r \\
 h_{a,4} &= \tilde{h}_{a,4} + NM_s \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) \\
 &\approx \tilde{h}_{a,4} + 0.683 NM_s \\
 h_{b,4} &= \tilde{h}_{b,4} - \frac{NM_s}{2} \left[1 - \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) \right] \\
 &\approx \tilde{h}_{b,4} - 0.159 NM_s .
 \end{aligned} \tag{A.3}$$

The bottom side of the intrinsic hysteron shifts up and slightly left. The new location relative to the intrinsic center is

$$h_{a,4} = \tilde{h}_{a,0} - \tilde{\Delta}_a + NM_s \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) \tag{A.35a}$$

$$h_{b,4} = \tilde{h}_{b,0} - \frac{NM_s}{2} \left[1 - \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) \right] \tag{A.35b}$$

$$\approx \tilde{h}_{b,0} - 0.159 NM_s$$

$$\approx h_{b,0} + 0.341 NM_s .$$

Equation (A.35a).1 contradicted the defined value, $h_{b,4} = h_{b,0}$, probably since the observed distribution [boomerang-shaped] was not truly a Gaussian function. This contradiction was ignored. Substituting the location of the effective center, Equation (2.5), allows solving for the effective height,

$$\begin{aligned}
 h_{a,0} - \Delta_a &= h_{a,4} \\
 \tilde{h}_{a,0} - \Delta_a &= \tilde{h}_{a,0} - \tilde{\Delta}_a + NM_s \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) \\
 \Delta_a &= \tilde{\Delta}_a - NM_s \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) \\
 &\approx \tilde{\Delta}_a - 0.683 NM_s .
 \end{aligned} \tag{2.5 f}$$

As mentioned in Derivation A.2.1, the distribution contracts vertically. With this model, large values of magnetizing field may cause the effective width and height to become zero or invert. The fit function inputs should satisfy $\tilde{\Delta}_b > 0.341 NM_s$ and $\tilde{\Delta}_a > 0.683 NM_s$.

The effective Gaussian distribution is now known, so values $r_{\downarrow}(h_a)$ and $r_{\uparrow}(h_b, h_a)$ are solved. The point (h_b, h_a) transforms to $(\tilde{h}_b, \tilde{h}_a)$ using Equation (A.3). Lastly, the function output is “looked up” on $\tilde{\rho}(\tilde{h}_b, \tilde{h}_a)$, the intrinsic Gaussian distribution defined by function inputs. The output evaluates the theoretical observed Preisach distribution when measuring a magnetic thin film, described by a particular Gaussian intrinsic Preisach distribution. The evaluated distribution is Gaussian (insignificant magnetizing field) or boomerang-shaped (significant magnetizing field).

Typically $\tilde{h}_{a,0} \approx -\tilde{h}_{b,0}$ for real samples. It is more insightful to use inputs $\tilde{h}_{c,0}$ ($\approx \tilde{h}_{b,0}$) and $\tilde{h}_{u,0}$ (≈ 0) instead. Also, a reasonable intrinsic distribution estimate is circular, or $\tilde{\Delta}_a \sim \tilde{\Delta}_b$. More insightful inputs are $\tilde{\Delta}_{avg} = \sqrt{\tilde{\Delta}_b \tilde{\Delta}_a}$, sufficient to calculate volume, and $\tilde{\Delta}_r = \tilde{\Delta}_b / \tilde{\Delta}_a$, the aspect ratio. The intrinsic distribution volume is typically over-estimated by a fit ($\tilde{V} > m_s$) since the transformation causes contraction. This is roughly corrected by $m_s \approx \tilde{V} \tilde{\Delta}_{avg} / \Delta_{avg}$.

A.6

Wishbone Fit

The wishbone fit applied to samples with strong demagnetizing field. The Preisach distribution of these samples had two branches, a distribution in the interaction field (IFD), h_u , and a distribution that extends to higher coercive field (CFD), h_c . The derivation for the IFD is straightforward. The derived CFD depends on the IFD result. Therefore, the wishbone fit derivation was split into two parts.

A.6.1

Interaction Field Distribution

A 2D Gaussian intrinsic distribution was used, like in the boomerang-shaped fit outlined in Derivation A.5. Derivation A.2.2 illustrates that the wishbone’s IFD is angled in the (h_b, h_a) basis. The Preisach integral, Equation (1.3), of a tilted 2D Gaussian does

not seem to have a simple analytic solution. Since the IFD is typically long in the h_u direction, the effective distribution is modeled as a 1D Gaussian along a general direction. The distribution is described by its center position, $(h_{b,0}, h_{a,0})$, polar tilt angle, θ , and distance from the center, h_l (negative if $h_a < h_{a,0}$).

The choice of intrinsic distribution model basis has a less noticeable effect in this case than in the boomerang case since hysteron translations are outward. This derivation uses a 2D Gaussian distribution in the (h_c, h_u) basis. Emphasis is placed on h_c , representing the traditionally-studied switching field distribution of particles (or wires) within the sample [28, 10].

Besides the choice of effective distribution model, the wishbone-shaped fit derivation uses the same strategy as that of the boomerang-shaped fit. See Derivation A.5 for more discussion of each step in this derivation. The consequential difference between the wishbone and boomerang cases is whether the material has an effective magnetizing or demagnetizing field. This is accounted for by the sign of the (de)magnetizing factor, N , in the hysteron transformation, Equation (A.3).

Deriving the effective distribution requires expressing a general point, (h_b, h_a) , as a point on the effective distribution. In this case,

$$h_a - h_{a,0} = h_l \sin \theta \tag{A.36a}$$

$$h_b - h_{b,0} = h_l \cos \theta . \tag{A.36b}$$

Equation A.36 also relates distribution widths, Δ_a and Δ_b to Δ_l . The normalized magnetization of descending major and ascending FORCs are

$$\begin{aligned} r_{\downarrow}(h_a) &= \operatorname{erf}\left(\frac{h_l(h_a)}{\Delta_l\sqrt{2}}\right) \\ &= \operatorname{erf}\left[\frac{(h_a - h_{a,0}) \csc \theta}{\Delta_a\sqrt{2} \csc \theta}\right] \\ &= \operatorname{erf}\left(\frac{h_a - h_{a,0}}{\Delta_a\sqrt{2}}\right) \end{aligned} \quad (\text{A.37a})$$

$$\begin{aligned} r_{\uparrow}(h_a, h_b) &= \max\left[r_{\downarrow}(h_a), \operatorname{erf}\left(\frac{h_l(h_b)}{\Delta_l\sqrt{2}}\right)\right] \\ &= \max\left\{r_{\downarrow}(h_a), \operatorname{erf}\left[\frac{(h_b - h_{b,0}) \sec \theta}{\Delta_b\sqrt{2} \sec \theta}\right]\right\} \\ &= \max\left[r_{\downarrow}(h_a), \operatorname{erf}\left(\frac{h_b - h_{b,0}}{\Delta_b\sqrt{2}}\right)\right]. \end{aligned} \quad (\text{A.37b})$$

In this derivation, the IFD is treated as having positive slope in the (h_a, h_b) basis, or $\theta \in [+ \pi/4, 0]$. With this constraint, points along the effective IFD line experience the same magnetization for the descending and ascending flips according to the regions of hysteron states in FIG. 2.3. Therefore, any point on the IFD satisfies

$$r_{\downarrow}(h_a) = r_{\uparrow}(h_a, h_b). \quad (\text{A.38})$$

Substituting into the transformation, Equation (A.3),

$$h_a^{\text{IFD}} = \tilde{h}_a + NM_s r_{\downarrow} \quad (\text{A.39a})$$

$$h_b^{\text{IFD}} = \tilde{h}_b + NM_s r_{\downarrow}. \quad (\text{A.39b})$$

At the effective center, $r_{\downarrow} = 0$, so

$$h_{a,0}^{\text{IFD}} = \tilde{h}_{a,0} \quad (\text{A.40a})$$

$$h_{b,0}^{\text{IFD}} = \tilde{h}_{b,0}. \quad (\text{A.40b})$$

The intrinsic center hysterons do not undergo translation.

The effective distribution width, Δ_l , is found by considering a point on the bottom of the IFD,

$$h_{a,1} = h_{a,0} - \Delta_a \quad (\text{A.41a})$$

$$h_{b,1} = h_{b,0} - \Delta_b . \quad (\text{A.41b})$$

Solving for the translation,

$$\begin{aligned} r_{\downarrow}(h_{a,1}) &= \text{erf}\left(\frac{h_{a,0} - \Delta_a - h_{a,0}}{\Delta_a \sqrt{2}}\right) \\ &= -\text{erf}\left(\frac{1}{\sqrt{2}}\right) . \end{aligned} \quad (\text{A.42})$$

After substitution, the translation to this point is

$$\begin{aligned} \tilde{h} &= h - NM_s r \\ h_{a,1} &= \tilde{h}_{a,1} - NM_s \text{erf}\left(\frac{1}{\sqrt{2}}\right) \\ h_{b,1} &= \tilde{h}_{b,1} - NM_s \text{erf}\left(\frac{1}{\sqrt{2}}\right) , \end{aligned} \quad (\text{A.3})$$

or in the (h_c, h_u) basis,

$$\begin{aligned} h_{c,1} &= \tilde{h}_{c,1} \\ h_{u,1} &= \tilde{h}_{u,1} - NM_s \text{erf}\left(\frac{1}{\sqrt{2}}\right) . \end{aligned}$$

In fact, it can be shown that any point on the effective IFD will translate in the h_u direction. This effective point translates from the intrinsic point,

$$\tilde{h}_{c,1} = \tilde{h}_{c,0} + \tilde{\Delta}_c \quad (\text{A.43a})$$

$$\tilde{h}_{u,1} = \tilde{h}_{u,0} - \tilde{\Delta}_u . \quad (\text{A.43b})$$

The signs of the intrinsic widths correspond to point on the IFD of lowest h_a and were tested for self-consistency between the effective and function output distributions. The

effective position relative to the effective center is

$$h_{c,1} = h_{c,0} + \tilde{\Delta}_c$$

$$h_{u,1} = h_{u,0} - NM_s \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) + \tilde{\Delta}_u .$$

Solving for Δ_a and Δ_b ,

$$\begin{aligned} \Delta_a &= h_{a,0} - h_{a,1} \\ &= (h_{u,0} - h_{c,0}) - (h_{u,1} - h_{c,1}) \\ &= NM_s \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) + \tilde{\Delta}_c + \tilde{\Delta}_u \end{aligned} \tag{A.44a}$$

$$\begin{aligned} \Delta_b &= h_{b,0} - h_{b,1} \\ &= (h_{u,0} + h_{c,0}) - (h_{u,1} + h_{c,1}) \\ &= NM_s \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) - \tilde{\Delta}_c + \tilde{\Delta}_u . \end{aligned} \tag{A.44b}$$

Dividing equation A.44 by equation A.44.1 allows solving for the IFD tilt,

$$\begin{aligned} \tan \theta &= \frac{\Delta_l \cos \theta}{\Delta_l \sin \theta} = \frac{\Delta_a}{\Delta_b} \\ &= \frac{NM_s \operatorname{erf}(1/\sqrt{2}) + \tilde{\Delta}_c + \tilde{\Delta}_u}{NM_s \operatorname{erf}(1/\sqrt{2}) - \tilde{\Delta}_c + \tilde{\Delta}_u} \\ \theta &\in \left[-\frac{\pi}{4}, +\frac{\pi}{4} \right) \end{aligned} \tag{A.45a}$$

$$\begin{aligned} \Delta_l &= \sqrt{\Delta_a^2 + \Delta_b^2} \\ &= \sqrt{[NM_s \operatorname{erf}(1/\sqrt{2}) + \tilde{\Delta}_c + \tilde{\Delta}_u]^2 + [NM_s \operatorname{erf}(1/\sqrt{2}) - \tilde{\Delta}_c + \tilde{\Delta}_u]^2} \\ &= \sqrt{2} \sqrt{[NM_s \operatorname{erf}(1/\sqrt{2}) + \tilde{\Delta}_u]^2 + \tilde{\Delta}_c^2} . \end{aligned} \tag{A.45b}$$

If the demagnetizing field is dominant, the IFD is a distribution in h_u which agrees with the discussion in Derivation A.2.2. If the switching field ($\tilde{h}_c$) distribution is dominant, the IFD is a distribution in h_c which makes intuitive sense. However, Equation (A.38)

required constraining the slope to positive values, so this derivation is valid for

$$\theta \in \left[0, +\frac{\pi}{4}\right), \quad (\text{A.46})$$

so $\tilde{\Delta}_c < NM_s \text{erf}(1/\sqrt{2})$. A fit for the case of large $\tilde{\Delta}_c$ is straightforward but not included in this thesis.

At this stage the effective IFD is known. The observed distribution can be evaluated as outlined at the end of Derivation A.5. The resulting distribution is triangle-shaped. This is similar to the wishbone shape without the gap distinguishing the IFD and CFD.

A.6.2 *Coercive Field Distribution*

As discussed in Derivation A.2.2, the wishbone shape is formed by hysteron translation in either the $+h_b$ (CFD) or $-h_b$ (IFD) directions. This is because the transformation, $(h_a, h_b) \rightarrow (\tilde{h}_a, \tilde{h}_b)$ given by Equation (A.3), is not one-to-one. The CFD can be introduced to the Derivation A.6.1 result by allowing a second effective distribution with larger h_b than the IFD's h_b range.

The CFD amplitude varies noticeably in experimental Preisach distributions. In other words, the ratio of intrinsic hysterons reversing early versus late in the ascending FORC is inconsistent and should be treated as an unknown variable, V_r . Mathematically, $V_r = V_{CFD}/m_s$, or the ratio between the CFD volume and the total volume of the Preisach distribution. The methods used so far cannot easily predict V_r , so V_r is treated here as an additional fit parameter.

The overall normalized magnetization curves, $r_\downarrow$ and $r_\uparrow$, can be written as the sum of two components,

$$r_\downarrow(h_a) = (1 - V_r) r_\downarrow^{\text{IFD}}(h_a) + V_r r_\downarrow^{\text{CFD}}(h_a) \quad (\text{A.47})$$

$$r_\uparrow(h_a, h_b) = (1 - V_r) r_\uparrow^{\text{IFD}}(h_a, h_b) + V_r r_\uparrow^{\text{CFD}}(h_a, h_b). \quad (\text{A.48})$$

Several simplifications can be made. Firstly, the sample's full descending major curve, $r_\downarrow$, does not depend on h_b and still follows Equation (A.37a). Secondly, the IFD and CFD in

this treatment only differ by shift in h_b . This allows setting

$$r_{\downarrow}(h_a) = r_{\downarrow}^{\text{IFD}}(h_a) = r_{\downarrow}^{\text{CFD}}(h_a) . \quad (\text{A.49})$$

Thirdly, The IFD still represents the first reversals along the ascending curve, so $r_{\downarrow} = r_{\uparrow}^{\text{IFD}}$ for points on the IFD. Therefore the IFD position, width and tilt is unchanged from Derivation A.6.1. Only its amplitude is reduced, by a factor of $1 - V_r$.

Hystérons along the CFD reverse after hystérons in the IFD have reversed during the ascending FORC. Since the CFD hystérons have not yet reversed, $r_{\uparrow}^{\text{CFD}} = r_{\downarrow}^{\text{CFD}}$ for points on the CFD. The total normalized magnetization on the CFD is

$$r_{\uparrow}(h_a^{\text{CFD}}, h_b^{\text{CFD}}) = (1 - V_r) r_{\uparrow}^{\text{IFD}}(h_a^{\text{CFD}}, h_b^{\text{CFD}}) + V_r r_{\downarrow} . \quad (\text{A.50})$$

The CFD center location is such that hystérons translate from the intrinsic center to that point according to Equation (A.3).

$$\tilde{h}_{a,0} = h_{a,0}^{\text{CFD}} - NM_s r_{\downarrow}(h_{a,0}^{\text{CFD}}) . \quad (\text{A.3})$$

Since $r_{\downarrow} = 0$ at the IFD and CFD centers,

$$h_{a,0}^{\text{CFD}} = \tilde{h}_{a,0} \quad (\text{A.51a})$$

$$\tilde{h}_{b,0} = h_{b,0}^{\text{CFD}} - NM_s r_{\uparrow}(h_{a,0}^{\text{CFD}}, h_{b,0}^{\text{CFD}}) \quad (\text{A.3 b})$$

$$\begin{aligned} \frac{h_{b,0}^{\text{CFD}} - \tilde{h}_{b,0}}{NM_s} &= r_{\uparrow}(h_{a,0}^{\text{CFD}}, h_{b,0}^{\text{CFD}}) \\ &= (1 - V_r) r_{\uparrow}^{\text{IFD}}(h_{a,0}^{\text{CFD}}, h_{b,0}^{\text{CFD}}) + V_r(0) . \end{aligned}$$

Substituting the IFD results,

$$\begin{aligned} \frac{h_{b,0}^{\text{CFD}} - \tilde{h}_{b,0}}{NM_s(1 - V_r)} &= \text{erf}\left(\frac{h_{b,0}^{\text{CFD}} - h_{b,0}^{\text{IFD}}}{\Delta_b^{\text{IFD}}\sqrt{2}}\right) \\ \frac{h_{b,0}^{\text{CFD}} - \tilde{h}_{b,0}}{NM_s(1 - V_r)} &= \text{erf}\left[\frac{h_{b,0}^{\text{CFD}} - \tilde{h}_{b,0}}{NM_s\sqrt{2} \text{erf}(1/\sqrt{2}) + (\tilde{\Delta}_u - \tilde{\Delta}_c)\sqrt{2}}\right] . \end{aligned} \quad (\text{A.51b})$$

Equation A.51.1 has no analytic solution for $h_{b,0}^{\text{CFD}}$, so a numeric approximation is needed.

The first step is to introduce a substitute variable,

$$u = \frac{h_{b,0}^{\text{CFD}} - \tilde{h}_{b,0}}{NM_s}$$

$$\frac{u}{1 - V_r} = \text{erf} \left[\frac{u}{\sqrt{2} \text{erf}(1/\sqrt{2}) + (\tilde{\Delta}_u - \tilde{\Delta}_c)\sqrt{2}} \right].$$

In the case $V_r = 0$, $\tilde{\Delta}_u = 0$, and $\tilde{\Delta}_c = 0$ ($\theta^{\text{IFD}} = +\pi/4$),

$$u = \text{erf} \left[\frac{u}{\sqrt{2} \text{erf}(1/\sqrt{2})} \right]. \quad (\text{A.52})$$

A solution is

$$u = \text{erf} \left(\frac{1}{\sqrt{2}} \right) \quad (\text{A.53})$$

$$h_{b,0}^{\text{CFD}} \approx \tilde{h}_{b,0} + NM_s \text{erf} \left(\frac{1}{\sqrt{2}} \right) \quad (\text{A.54})$$

Setting $V_r = 0$ is reasonable since the equation has no non-zero solution for $V_r > 0.1443$.

For comparison, consider a triangle approximating the extent of the observed wishbone distribution. This triangle has corners $(\tilde{h}_{c,0}, \tilde{h}_{u,0} + NM_s)$, $(\tilde{h}_{c,0}, \tilde{h}_{u,0} - NM_s)$, and $(\tilde{h}_{c,0} + NM_s, \tilde{h}_{u,0})$. The center of the triangle's edge corresponding to the CFD center has $h_b = \tilde{h}_{b,0} + NM_s$.

Following similar steps for the high h_c end of the CFD yields the CFD angle. The effective point,

$$h_{a,1}^{\text{CFD}} = h_{a,0}^{\text{CFD}} - \Delta_a^{\text{CFD}}$$

$$h_{b,1}^{\text{CFD}} = h_{b,0}^{\text{CFD}} - \Delta_b^{\text{CFD}},$$

translates from the intrinsic point,

$$\tilde{h}_{c,1} = \tilde{h}_{c,0} + \tilde{\Delta}_c \quad (\text{A.55a})$$

$$\tilde{h}_{u,1} = \tilde{h}_{u,0} - \tilde{\Delta}_u. \quad (\text{A.55b})$$

The normalized magnetization is

$$\begin{aligned}
 r_{\downarrow}(h_{a,1}^{\text{CFD}}) &= r_{\downarrow}^{\text{IFD}}(h_{a,1}^{\text{CFD}}) \\
 &= \text{erf}\left(\frac{h_{a,1}^{\text{CFD}} - \tilde{h}_{a,0}}{\Delta_a^{\text{IFD}}\sqrt{2}}\right) \\
 &= \text{erf}\left(\frac{h_{a,0}^{\text{CFD}} - \Delta_a^{\text{CFD}} - \tilde{h}_{a,0}}{\Delta_a^{\text{IFD}}\sqrt{2}}\right) \\
 &= -\text{erf}\left(\frac{1}{\sqrt{2}}\right)
 \end{aligned} \tag{A.56a}$$

$$\begin{aligned}
 r_{\uparrow}(h_{a,1}^{\text{CFD}}, h_{b,1}^{\text{CFD}}) &= (1 - V_r) r_{\uparrow}^{\text{IFD}}(h_a, h_b) \\
 &= (1 - V_r) \text{erf}\left(\frac{h_{b,1}^{\text{CFD}} - \tilde{h}_{b,0}}{\Delta_b^{\text{IFD}}\sqrt{2}}\right) \\
 &= (1 - V_r) \text{erf}\left(\frac{h_{b,0}^{\text{CFD}} - \Delta_b^{\text{CFD}} - \tilde{h}_{b,0}}{\Delta_b^{\text{IFD}}\sqrt{2}}\right) \\
 &= (1 - V_r) \text{erf}\left[\frac{NM_s \text{erf}(1/\sqrt{2}) - \Delta_b^{\text{CFD}}}{NM_s\sqrt{2} \text{erf}(1/\sqrt{2}) + (\tilde{\Delta}_u - \tilde{\Delta}_c)\sqrt{2}}\right].
 \end{aligned} \tag{A.56b}$$

Solving for the translation,

$$\begin{aligned}
 \tilde{h}_{a,1} &= h_{a,1}^{\text{CFD}} - NM_s r_{\downarrow}(h_{a,1}^{\text{CFD}}) \\
 \tilde{h}_{a,0} - \tilde{\Delta}_u - \tilde{\Delta}_c &= h_{a,0}^{\text{CFD}} - \Delta_a^{\text{CFD}} + NM_s \text{erf}\left(\frac{1}{\sqrt{2}}\right) \\
 \Delta_a^{\text{CFD}} &= \tilde{\Delta}_u + \tilde{\Delta}_c + NM_s \text{erf}\left(\frac{1}{\sqrt{2}}\right) \\
 \tilde{h}_{b,1} &= h_{b,1}^{\text{CFD}} - NM_s r_{\uparrow}(h_{a,1}^{\text{CFD}}, h_{b,1}^{\text{CFD}}) \\
 \tilde{h}_{b,0} - \tilde{\Delta}_u + \tilde{\Delta}_c &= h_{b,0}^{\text{CFD}} - \Delta_b^{\text{CFD}} - NM_s (1 - V_r) \text{erf}\left[\frac{NM_s \text{erf}(1/\sqrt{2}) - \Delta_b^{\text{CFD}}}{NM_s\sqrt{2} \text{erf}(1/\sqrt{2}) + (\tilde{\Delta}_u - \tilde{\Delta}_c)\sqrt{2}}\right] \\
 \Delta_b^{\text{CFD}} - \tilde{\Delta}_u + \tilde{\Delta}_c &= NM_s \text{erf}(1/\sqrt{2}) - NM_s (1 - V_r) \text{erf}\left[\frac{NM_s \text{erf}(1/\sqrt{2}) - \Delta_b^{\text{CFD}}}{NM_s\sqrt{2} \text{erf}(1/\sqrt{2}) + (\tilde{\Delta}_u - \tilde{\Delta}_c)\sqrt{2}}\right] \\
 \frac{NM_s \text{erf}(1/\sqrt{2}) - \Delta_b^{\text{CFD}} - \tilde{\Delta}_u + \tilde{\Delta}_c}{NM_s (1 - V_r)} &= \text{erf}\left[\frac{NM_s \text{erf}(1/\sqrt{2}) - \Delta_b^{\text{CFD}}}{NM_s\sqrt{2} \text{erf}(1/\sqrt{2}) + (\tilde{\Delta}_u - \tilde{\Delta}_c)\sqrt{2}}\right].
 \end{aligned} \tag{A.57}$$

Like before, a substitute variable can be introduced

$$u = \frac{NM_s \operatorname{erf}(1/\sqrt{2}) - \Delta_b^{\text{CFD}}}{NM_s}$$

$$\frac{u + \tilde{\Delta}_u - \tilde{\Delta}_c}{1 - V_r} = \operatorname{erf} \left[\frac{u}{\sqrt{2} \operatorname{erf}(1/\sqrt{2}) + (\tilde{\Delta}_u - \tilde{\Delta}_c) \sqrt{2}/(NM_s)} \right].$$

Setting $V_r = 0$, $\tilde{\Delta}_u = 0$, and $\tilde{\Delta}_c = 0$ ($\theta^{\text{IFD}} = 0$),

$$u = \operatorname{erf} \left[\frac{u}{\sqrt{2} \operatorname{erf}(1/\sqrt{2})} \right],$$

which has the previously mentioned solution,

$$u = \operatorname{erf} \left(\frac{1}{\sqrt{2}} \right) \quad (\text{A.53})$$

$$\frac{NM_s \operatorname{erf}(1/\sqrt{2}) - \Delta_b^{\text{CFD}}}{NM_s} = \operatorname{erf} \left(\frac{1}{\sqrt{2}} \right)$$

$$\Delta_b^{\text{CFD}} = 0. \quad (\text{A.57b})$$

The CFD is expected to be a distribution in h_a .

At this point, the effective CFD is known. Any point can be transformed to the corresponding intrinsic point on the intrinsic distribution, defined by the function inputs.

Unlike the boomerang fit, an extra step is needed to account for the V_r input.

Excluding the final V_r adjustment results in the correct distribution shape but with inaccurately scaled amplitudes. As mentioned at the beginning of this section, V_r is needed since two points on the observed distribution transform to the same intrinsic point (not one-to-one). Conceptually, V_r prevents double-counting of hysterons. The function output should be the weighted sum of the evaluated IFD and CFD,

$$\rho = (1 - V_r) \rho_{\text{IFD}} + V_r \rho_{\text{IFD}}$$

$$= (1 - V_r) \tilde{\rho}(\tilde{h}_a, \tilde{h}_{b,\text{IFD}}) + V_r \tilde{\rho}(\tilde{h}_a, \tilde{h}_{b,\text{CFD}}), \quad (\text{A.58a})$$

where $(\tilde{h}_{b,\text{IFD}}, \tilde{h}_a)$ is within the intrinsic distribution when (h_b, h_a) is within the IFD. The transformations to obtain IFD and CFD amplitudes independently are

$$\tilde{h}_a = h_a - NM_s r_{\downarrow} \quad (\text{A.58b})$$

$$\tilde{h}_{b,\text{IFD}} = h_b - NM_s r_{\uparrow, \text{ifd}} \quad (\text{A.58c})$$

$$\tilde{h}_{b,\text{CFD}} = h_b - NM_s r_{\uparrow, \text{cfid}} , \quad (\text{A.58d})$$

where $r_{\uparrow, \text{ifd}}$ and $r_{\uparrow, \text{cfid}}$ are substituted values for $r_{\uparrow}$, which are different from $r_{\uparrow}^{\text{IFD}}$ and $r_{\uparrow}^{\text{CFD}}$.

The total normalized moment, $r_{\uparrow}$, needs to be re-formulated to scale the IFD and CFD amplitudes separately. The IFD is isolated by effectively making CFD hysteron states constant by setting an effective CFD normalized magnetization, $r_{\uparrow, \text{eff}}^{\text{CFD}} = r_{\downarrow}$. When calculating the IFD, $r_{\uparrow}$ is replaced by

$$r_{\uparrow, \text{ifd}} = (1 - V_r) r_{\uparrow}^{\text{IFD}}(h_a, h_b) + V_r r_{\downarrow}(h_a) . \quad (\text{A.59})$$

The CFD is isolated by effectively keeping IFD hysteron states constant by setting an effective IFD normalized magnetization, $r_{\uparrow, \text{eff}}^{\text{IFD}} = r_{\uparrow}^{\text{IFD}}(h_a, h_{b,0}^{\text{CFD}})$. When calculating the CFD, $r_{\uparrow}$ is replaced by

$$r_{\uparrow, \text{cfid}} = (1 - V_r) r_{\uparrow}^{\text{IFD}}(h_a, h_{b,0}^{\text{CFD}}) + V_r r_{\uparrow}^{\text{CFD}}(h_a, h_b) . \quad (\text{A.60})$$

The resulting distribution is wishbone-shaped, with IFD and CFD that would theoretically line up with the estimated 1D distributions.

However, noticeable errors in calculated distribution shape occurred when including the V_r amplitude scaling step. The calculated shape was different from the more accurate shape obtained by excluding the V_r amplitude scaling step. Also, the effective fields did not accurately approximate the function's output distribution.

First, inputting typical values often split the CFD into two distributions. The CFD shape seemed to be corrected by setting $r_{\uparrow}^{\text{CFD}} = +\text{erf}(1/\sqrt{2})$, constant instead of letting it

vary in the range $(-1, +1)$. This theoretically makes $h_{b,0}^{\text{CFD}}$ greater than its ideal value by about the width of the CFD. The CFD width also increases slightly.

Second, a small amplitude CFD, $V_r < 0.4$, caused the IFD to split into two distributions. This may be related to the resulting triangle-shaped distribution shape in Derivation A.6.1. Some experimental Preisach distributions had apparently small CFD amplitude and a dominant single IFD. The split IFD seemed to be corrected by setting a minimum V_r when calculating shape. Then the original input V_r was used during final V_r amplitude scaling. This theoretically changes the CFD shape from the ideal shape slightly.

Third, the effective CFD tilt was more self-consistent after setting $\Delta_b^{\text{CFD}} = \tilde{\Delta}_c - \tilde{\Delta}_u$ instead of zero. This expression would be obtained by including these terms in the u substitution. However, this change did not affect the function output since $r_{\uparrow}^{\text{CFD}}$ was set to be constant. These corrections were implemented in the wishbone-shaped fit function to make the effective distribution a better approximation of the function output, which is its primary function. Although these corrections do not come directly from derivation, they provide practical improvements in overall effective distribution shape for more accurate fit results.

A.7

Waveguide Correction

The waveguide was modeled as networks in series, as shown in FIG. 2.9. The equivalent S_{11} of two networks in series, $[S^X]$ and $[S^Y]$, is derived here for demonstration. The derivations for other S-parameters were done in a similar fashion, but they were excluded for brevity.

A.7.1

Reflection of Networks in Series

As illustrated in FIG. 2.8, S_{11} is the reflection of power at port 1 when port 2 is load-matched, or has no incoming power,

$$\begin{aligned} S_{11} &= \left. \frac{b_1}{a_1} \right|_{a_2=0} \\ &= \frac{b_1^X}{a_1^X} . \end{aligned} \quad (2.12)$$

Substituting the tensor equation for $[S^X]$,

$$\begin{aligned} &= \frac{S_{1j}^X a_j^X}{a_1^X} \\ &= \frac{S_{11}^X a_1^X}{a_1^X} + \frac{S_{12}^X a_2^X}{a_1^X} . \end{aligned} \quad (A.61)$$

The outgoing power of $[S^X]$ is incoming power of $[S^Y]$, and vice-versa. Substituting this relationship,

$$\begin{aligned} &= S_{11}^X + \frac{S_{12}^X b_1^Y}{a_1^X} \\ &= S_{11}^X + \frac{S_{12}^X S_{1j}^Y a_j^Y}{a_1^X} \\ &= S_{11}^X + \frac{S_{12}^X (S_{11}^Y a_1^Y + S_{12}^Y a_2^Y)}{a_1^X} \\ &= S_{11}^X + \frac{S_{12}^X (S_{11}^Y b_2^X + S_{12}^Y a_2)}{a_1^X} \\ &= S_{11}^X + S_{12}^X S_{11}^Y \frac{b_2^X}{a_1^X} . \end{aligned} \quad (A.62)$$

The ratio b_2^X/a_1^X is evaluated separately,

$$\begin{aligned}
 \frac{b_2^X}{a_1^X} &= \frac{S_{21}^X a_1^X + S_{22}^X a_2^X}{a_1^X} \\
 &= S_{21}^X + S_{22}^X \frac{b_1^Y}{a_1^X} \\
 &= S_{21}^X + S_{22}^X \frac{S_{11}^Y a_1^Y + S_{12}^Y a_2^Y}{a_1^X} \\
 \frac{b_2^X}{a_1^X} &= S_{21}^X + S_{22}^X S_{11}^Y \frac{b_2^X}{a_1^X} \\
 \frac{b_2^X}{a_1^X} (1 - S_{22}^X S_{11}^Y) &= S_{21}^X \\
 \frac{b_2^X}{a_1^X} &= \frac{S_{21}^X}{1 - S_{22}^X S_{11}^Y} .
 \end{aligned} \tag{A.63}$$

Substituting Equation (A.63) into Equation (A.62),

$$S_{11} = S_{11}^X + S_{11}^Y \frac{S_{12}^X S_{21}^X}{1 - S_{22}^X S_{11}^Y} .$$

Qualitatively, reflection of incoming power occurs simply at $[S^X]$ or at $[S^Y]$ through numerous paths of internal reflection described by a geometric series.

A.7.2

Waveguide Transmission

The equivalent network of two networks in series is given by,

$$S_{11} = S_{11}^X + S_{11}^Y \frac{S_{12}^X S_{21}^X}{1 - S_{22}^X S_{11}^Y} \tag{A.64a}$$

$$S_{12} = \frac{S_{12}^X S_{12}^Y}{1 - S_{22}^X S_{11}^Y} \tag{A.64b}$$

$$S_{21} = \frac{S_{21}^X S_{21}^Y}{1 - S_{22}^X S_{11}^Y} \tag{A.64c}$$

$$S_{22} = S_{22}^Y + S_{22}^X \frac{S_{12}^Y S_{21}^Y}{1 - S_{22}^X S_{11}^Y} . \tag{A.64d}$$

Small reflection was assumed, and the second-order reflection term, $S_{22}^X S_{11}^Y$, was ignored, simplifying to

$$S_{11} = S_{11}^X + S_{11}^Y S_{12}^X S_{21}^X \quad (\text{A.65a})$$

$$S_{12} = S_{12}^X S_{12}^Y \quad (\text{A.65b})$$

$$S_{21} = S_{21}^X S_{21}^Y \quad (\text{A.65c})$$

$$S_{22} = S_{22}^Y + S_{22}^X S_{12}^Y S_{21}^Y . \quad (\text{A.65d})$$

Reflection coefficients, S_{ii} , are nearly additive in series while transmission coefficients, S_{ij} , are multiplicative. The waveguide, $[S^{Bkgnd}]$, was modeled as two networks in series, as shown in FIG. 2.9b: $[S^A]$ and $[S^B]$. The waveguide with the sample, $[S^{Meas}]$, was modeled as three networks in series, as shown in FIG. 2.9a: $[S^A]$, $[S^{Corr}]$, and $[S^B]$. The desired corrected S matrix representing the sample was $[S^{Corr}]$.

The equivalent series network can be written for the measured S_{21} , representing transmission.

$$S_{21}^{Bkgnd} = S_{21}^A S_{21}^B \quad (\text{A.66a})$$

$$S_{21}^{Meas} = S_{21}^A S_{21}^{Corr} S_{21}^B . \quad (\text{A.66b})$$

The contribution from the waveguide can simply be removed by division,

$$S_{21}^{Corr} = S_{21}^{Meas} / S_{21}^{Bkgnd} . \quad (2.13 \text{ c})$$

The equation for S_{12} has the same structure.

A.7.3

Waveguide Reflection

The equivalent series network for the measured reflection is

$$S_{11}^{Bkgnd} = S_{11}^A + S_{11}^B S_{12}^A S_{21}^A$$

$$S_{11}^{Meas} = S_{11}^A + S_{11}^{Corr} S_{12}^A S_{21}^A + S_{11}^B S_{12}^A S_{21}^A S_{12}^{Corr} S_{21}^{Corr} .$$

In order to solve for S_{11}^{Corr} , the smaller internal reflection terms were neglected by setting $S_{22}^A = S_{11}^B = 0$. Another restriction is $S_{ij}^A = S_{ji}^B$, which is reasonable considering the waveguide has mirror symmetry.

$$S_{11}^{Bkgnd} = S_{11}^A \quad (\text{A.67a})$$

$$S_{11}^{Meas} = S_{11}^A + S_{11}^{Corr} \sqrt{S_{12}^{Bkgnd} S_{21}^{Bkgnd}}. \quad (\text{A.67b})$$

This yields a calculation for the corrected S parameter,

$$S_{11}^{Corr} = \frac{S_{11}^{Meas} - S_{11}^{Bkgnd}}{\sqrt{S_{12}^{Bkgnd} S_{21}^{Bkgnd}}}. \quad (2.13 \text{ a})$$

The equation for S_{22} has the same structure.

APPENDIX

B. ELECTRODEPOSITION METHODS

Nickel magnetic samples were made using wet chemistry techniques. Nickel metal was deposited onto a substrate from a chemical solution. Deposition rate was tested and controlled. The amount of nickel deposited therefore depended on deposition time.

B.1

Setup

Electrodeposition is a chemical reaction that uses electric current to control the reduction of metallic ions onto a conductive surface. The setup illustrated in FIG. B.1 involved an anode, cathode (sample), and an electrolyte solution.

FIG. B.1. (a) Electrodeposition was performed using two electric probes submerged in an electrolyte solution. Nickel metal was deposited onto the cathode. Expanded layer diagrams are shown for (b) the thin film substrate and (c) the nanowire array template holder.

Nickel cations in solution, Ni_2^+ , diffused to a conductive, negatively-charged cathode. The cathode donated two electrons to each cation. Nickel cations reduced and adsorbed onto the cathode. The Ni conducted electrons for more nickel to grow outward. Secondary reactions involving cations, pH buffers, additives also occurred. The anode was a platinum wire, coiled to have relatively large and distributed surface area within a small volume of the solution for sufficient anode efficiency [40]. The cathode was the substrate for the magnetic sample.

Electrodeposition occurred onto any conductive surface on the cathode that was in contact with the solution. A water-tight seal was needed to isolate the electrical contacts between the substrate and the holder. The cathode was custom-made by InRedox to hold the circular nano-pore templates [41].

When depositing thin films, the sample was held by an alligator clip soldered to a metal rod, illustrated in FIG. B.1a. The clip held a copper strip taped to the conductive side of the substrate, illustrated in FIG. B.1b. The substrate was a glass slide coated on one side with gold. The conductive surface and copper strip were sealed using Kapton tape. Nickel was only deposited onto the exposed rectangular region of gold, producing a rectangular thin film with controlled size.

When depositing nanowire arrays, the sample was encased in a specialized template holder made by InRedox, illustrated in FIG. B.1c. The template holder provided a water-tight seal around conductive contacts. Nickel was only deposited onto the exposed gold surface inside the alumina nano-pores.

A 100 mL beaker with a magnetic stirrer was placed on a heating pad. The anode and cathode were held vertically and submerged into opposite sides of the beaker. The exposed substrate was oriented toward the platinum anode with at least 2 cm separation.

B.2

Recipe

The deposition solution provided metallic ions for reduction into solid metal. Different types of solutions are used for different qualities and appearance of the metal [42]. A simple nickel watts solution was used. The recipe came from a study by Galicia-Aguilar et al. [43]. The solution contained

- 300 g/L NiSO_4
- 60 g/L NiCl
- 35 g/L H_3BO_3
- 1.2 g/L Saccharine.

The solution had $\text{pH } 2.8 \pm 0.2$ and temperature $55 \pm 5^\circ\text{C}$ [43]. These concentrations were within the typical range for a basic nickel watts solution [42] [40].

Saccharine was included as an additive for better deposition quality. Saccharine is a grain refinement additive that slows down crystal growth at the beginning of the deposition [44]. This supposedly provided time for nucleation of more small crystals, allowing more complete cathode surface coverage and pore-filling [44].

After several depositions, the solution's pH would drift below pH 2.5, which was a lower limit observed for good-quality thin films. Therefore, nanowire samples were only deposited using recently-mixed watts solutions.

Direct current (DC) deposition involved supplying constant current into the anode and out of the cathode. The current density was chosen to be 0.2 mA/mm^2 . The exposed substrate area was measured using dial calipers with 0.4 mm^2 precision before electrodeposition to determine the necessary applied current. The film thickness was estimated by terminating the electrodeposition at a chosen time with 2 s precision. At the chosen time, current was set to zero, the sample was removed, and the sample was immediately rinsed with de-ionized water to remove any electrolyte solution.

Pulsed deposition with current waveform shown in FIG. B.2 provided advantages over DC deposition. Pulsed deposition was reported to provide better film quality by allowing more diffusion time, reducing byproduct concentration at the surface [45]. The cyclic deposition also reportedly provided more consistent deposition rate [45]. A reduced major loop coercivity, H_c , was observed in films deposited with this method, which suggested improved material quality.

FIG. B.2. Pulsed electrodeposition was performed using 10 ms current pulses at 20 mA/s. Voltage lagged behind current.

Deposition occurred during a 10 ms pulse with current density 0.20 mA/mm^2 . Deposition was halted using a 2 ms negative-current pulse. The negative-current pulse was reported to have a smoothing effect due to nickel dissolving back into solution [45]. Current was set to zero for 80 ms to facilitate diffusion. Voltage was observed to have a low-pass behavior, caused by either capacitance between electrodes or effects of the chemical reaction. Current was a more direct metric for deposition rate since they both related to the rate of electrons leaving the cathode.

B.3 Deposition Testing

Deposition rate was determined using quartz crystal microbalance (QCM) measurements. A thin film was deposited onto a quartz disc with gold contacts. The disc undergoes driven mechanical vibration. The resonant frequency varies linearly with mass per unit area according to a known formula. This allows a direct measurement of deposition rate of nickel thin films.

DC electrodeposition with current density of 0.2 mA/mm^2 resulted in a measured deposition rate of $5.60 \pm 0.15 \text{ nm/s}$. The theoretical deposition rate calculated from Faraday's law of electrolysis was 5.419 nm/s . Undesired reactions that accept electrons, like hydrogen gas production, could result in reaction efficiency less than 100%.

Pulsed electrodeposition with peak current density of 0.200 mA/mm^2 and 9% duty cycle resulted in a deposition rate of $0.545 \pm 0.015 \text{ nm/s}$. Comparing the two rates, the effective deposition time was $10.7 \pm 0.4 \text{ ms}$ per pulse. This was slightly higher than the applied current pulse time of $10.0 \pm 0.1 \text{ ms}$.

QCM measurements were only possible for thin film deposition. It was assumed that nanowires were deposited at nearly the same rate as thin films, and deposition was timed accordingly. Current was reduced to account for the reported $11 \pm 2 \%$ porosity of the templates.

B.4 Sample Details

The thin nickel film was made using pulsed deposition as described above onto a gold film on glass. The film was square-shaped, $73.5 \pm 0.5 \text{ mm}^2$ in area, and foil-like in appearance. Gold film lift-off suggested internal stress in the film. Average thickness estimated from m_s was $184 \pm 2 \text{ nm}$. The predicted thickness from deposition time was $330 \pm 20 \text{ nm}$, showing a discrepancy and need for further deposition testing. The film was selected from about 10 measured samples since it had reasonably good appearance and was the only one made with pulsed deposition.

The nanowire array sample was made using DC deposition as described into a template with 80 nm diameter pores and $11 \pm 2\%$ porosity, or ratio of the transverse area filled by magnetic material. The nanowire array was circular, $48.1 \pm 0.5 \text{ mm}^2$ in area, and black in color. Average nanowire length estimated from m_s was $4.6 \pm 0.8 \mu\text{m}$, with precision limited by the reported porosity. The array was selected from 5 measured samples since it was one of the first samples made and had medium nanowire length. The other samples were made to have various nanowire lengths from $0.5 \mu\text{m}$ to $20 \mu\text{m}$ and were made from the same batch of templates. FORC analysis was performed, but the results were not reported in this thesis.

APPENDIX

C. VSM MEASUREMENT PROGRAM

A program and user interface for VSM measurements in this study were created using LabVIEW. The interface had options to run various types of magnetic sweep measurements in addition to FORC. During measurement, the program sent a value command for the desired applied field, H_{des} . The instrument ramped the applied current while reporting the magnetic moment, m , and the applied field, H , to the program. Single-sampling mode waited until H was close enough to H_{des} and then measured m and H in a single shot. Results could be averaged multiple times for reduced measurement uncertainty.

Multi-sampling mode used the observed m and H values during the ramp to H_{des} with a small overshoot. Then a weighted linear fit interpolated $m(H_{des})$. This mode reduced typical relative measurement uncertainty from 0.5 % to 0.2 % with similar measurement time compared to single-sampling mode. This was the mode used for measurements discussed in this thesis.

For each field, the magnetic moment was observed until

- the applied field was reasonable settled
- enough moment measurements were taken
- enough time had passed
- or the applied field reached a threshold.

These parameters were adjustable by a “Speed” sliding control. Higher speeds purposely included an over-shoot of the desired field to avoid waiting for the field to settle. Since values of H_a needed to be extreme value of field, field over-shoot was disabled for the first fields of each FORC reached from positive saturation.

Measurement results were saved as a text file. The file included measurement details and a table of applied magnetic field and measured magnetic moment. In multi-sampling

mode, the field values were the desired value, and magnetic moment values were interpolated.

The LabVIEW program's front panel had measurement plots, numerical displays, options, and a run-time log. It provided a variety of magnetic measurement routines.

FIG. C.1 is a screenshot of the program's front panel.

FIG. C.1. A custom LabVIEW program was used to perform VSM measurements. The program's front panel is shown here.