Coded Modulation for Satellite Broadcasting

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Coded Modulation for Satellite Broadcasting

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Abstract

In this paper, three-level block coded 8-PSK modulations, suitable for satellite broadcasting of digital TV signals, are presented. A design principle to achieve unequal error protection is introduced. The coding scheme is designed in such a way that the information bits carrying the basic definition TV signal have a lower error rate than the high definition information bits. The large error coefficients, normally associated with standard mapping by set partitioning, are reduced by considering a nonstandard partition of an 8-PSK signal set. The bits-to-signal mapping induced by this partition allows the use of suboptimal low-complexity soft-decision decodings of binary block codes. Parallel operation of the first and second stage decoders is possible, for high data rate transmission. Furthermore, there is no error propagation from the first-stage decoder to the second-stage decoder.

1 Introduction

In satellite broadcasting of digital high definition TV (HDTV) over the Ka band (21/30 GHz), rain causes severe attenuation. An efficient digital transmission system must be designed to provide a gradual degradation of the received signal. All of the previously proposed schemes have been based on either time-sharing or nonuniform signal sets. In this paper, it is proposed to combine coding and modulation in such a way that the required graceful degradation is achieved by error control coding. Subsets of signal sequences, of increasing minimum squared Euclidean distances, are associated with information bits of increasing importance level (or decreasing image definition).

Rain attenuation may be interpreted as the concatenation of two Gaussian channels. The first channel corresponds to clear sky conditions and the first and second to rainy conditions. A receiver in a clear sky region can recover the full high definition TV signal, while a receiver in a rainy region must be able to recover at least the basic (lower information rate) definition TV signal. This degraded Gaussian broadcast channel was studied by Cover [1], who showed by random coding arguments that capacity can be achieved by superimposing information:

Code sequences in correspondence to the least important part (the high definition TV component) are clustered into clouds. Each coded signal sequence in correspondence to a most important message part (the basic definition TV component) is associated with a cloud. The mapping of source coded bits to coded signal sequences is made in such a way that the minimum distance between signals in different clouds is larger than the minimum distance between signals within a cloud. This is an unequal error protection (UEP) coding scheme [2].

Coded modulation approaches for the terrestrial broadcasting of HDTV signals have been reported in [3, 4, 5]. All of them, however, deal with rectangular (M-QAM type) signal sets. To the best of our knowledge, no UEP coded modulation scheme for satellite broadcasting of HDTV signals, for which constant amplitude modulation (M-PSK type) is required, other than trivial time sharing is known, except for our previous work [6].

A closely related paper is [7], where an inverse set partitioning strategy was introduced to effectively reduce the large error coefficients associated with Ungerboeck’s partitions, as well as to achieve graceful degradation. In this paper, this partitioning strategy is used to construct block coded 8-PSK modulations for degraded Gaussian broadcast channels. Upper bounds on the probability of a bit errors will show that indeed the name inverse is appropriate. The partitioning reduces the contribution of the error coefficients exponentially. It will be assumed that the HDTV signals are produced by a hierarchical source encoder (such as MPEG-2), and that an outer Reed-Solomon code is used. In this way, the target channel bit error rate (BER), necessary to achieve images of good quality, may be set equal to $10^{-5}$.

The paper is organized as follows. In section 2, block coded modulations (BCM) based on multilevel coding [8] are overviewed. A design principle for achieving UEP is proposed. A nonstandard mapping-by-set partitioning of an 8-PSK signal set is used to construct good three-level coded 8-PSK modulation schemes with UEP. In section 3, it is shown that the partition used drastically reduces the average number of nearest neighbor sequences. The proposed schemes have reduced decoding complexity. The decoders in the first and second stages operate on received signals projected in one dimension (the in-phase and
quadrature components of the received signal sequence, respectively). By orthogonality, these two decoders are independent and therefore can operate in parallel. Finally, in section 4, conclusions of this work are presented.

2 Block coded 8-PSK modulation for UEP

Imai and Hirakawa [8] proposed a technique for constructing coded modulation schemes using binary block codes. For coded M-level modulation systems, codewords of M binary block codes are used to select labels of signal points. The resulting signal sequences form a block modulation code over Euclidean space. This block coding scheme is said to be an M-level coded modulation. Over a satellite channel, constant amplitude (M-PSK) type of modulation is preferred. In this paper, three-level coded 8-PSK modulation schemes are considered.

A fundamental issue in the design of a multilevel coded modulation system is the labeling of the signal set over which the component codes operate. Such labeling determines the minimum squared Euclidean distance (MSED) of the modulation code and, more generally, the distance structure of the set of coded sequences, as discussed below.

In what follows, Ungerboeck's well known standard mapping-by-set partitioning [9] is briefly reviewed. A uniform unit-energy 8-PSK signal set is partitioned into three levels. For i = 1, 2, 3, at the i-th partition level, the signal set is divided into two subsets \( S_i(0) \) and \( S_i(1) \), such that the maximum distance, \( \delta_i \), is maximized. A label bit \( b_i \in \{0, 1\} \) is associated with the subset choice, \( S_i(b_i) \), at the i-th partition level. This partition process results in a labeling of the 8-PSK modulation signals. Each 8-PSK signal has a three-bit label \( b_1 b_2 b_3 \), and is denoted by \( s(b_1, b_2, b_3) \). With this standard partition of 8-PSK, the maximum distances are \( \delta_1 = 0.586 \), \( \delta_2 = 2 \), and \( \delta_3 = 4 \).

For \( i = 1, 2, 3 \), let \( C_i \) denote an \((n, k_i, d_i)\) binary linear block code of length \( n \), dimension \( k_i \), and minimum Hamming distance \( d_i \). Let \( \tilde{c}_i = (c_{i1}, c_{i2}, \ldots, c_{in}) \) denote a codeword of \( C_i \). A three-level coded 8-PSK is the following set of 8-PSK signal sequences of length \( n \):

\[
\Lambda \triangleq \{s(c_{1j}, c_{2j}, c_{3j}) : \tilde{c}_i \in C_i, 1 \leq j \leq n\}.
\]

The rate of this coded modulation system, in bits/symbol, is \( R = (k_1 + k_2 + k_3)/n \). It can be shown that its MSED, denoted by \( D \), is given by [8]

\[
D = \min_{1 \leq i \leq 3} \{d_i \delta_i\}.
\]

In this paper, QPSK modulation is used as a reference and therefore it is required that \( R \approx 2 \) bits/symbol.

2.1 UEP design principle

In order to achieve graceful degradation for satellite broadcasting, a modulation code must provide unequal error protection (UEP), as pointed out in section 1. To fulfill this requirement, the following design guideline for three-level coded 8-PSK modulation for UEP is proposed:

For \( i = 1, 2, 3 \), the binary block codes \( C_i \) are selected in such a way that the following inequalities are satisfied:

\[
d_1 \delta_1 \geq d_2 \delta_2 > d_3 \delta_3.
\]

Let \( \tilde{c}_i(\tilde{m}_i) \) be a codeword of \( C_i \), in correspondence to a \( k_i \)-bit message vector \( \tilde{m}_i \), and let \( \tilde{s} = \tilde{s}(\tilde{m}_i) \), \( \tilde{m}_i = (\tilde{m}_1, \tilde{m}_2, \tilde{m}_3) \) and \( \tilde{s}' = \tilde{s}(\tilde{m}') \), \( \tilde{m}' = (\tilde{m}_1', \tilde{m}_2', \tilde{m}_3') \) denote coded 8-PSK signal sequences in \( \Lambda \). The Euclidean separations [10] between sequences of 8-PSK signals at the i-th partition level, for \( i = 1, 2, 3 \), are defined as

\[
s_i = \min \{d(\tilde{s}, \tilde{s}') : \tilde{m}_i \neq \tilde{m}_i', \tilde{m}_j = \tilde{m}_j', j < i\}.
\]

Then \( s_1 = d_1 \delta_1 \), \( s_2 = d_2 \delta_2 \), and \( s_3 = d_3 \delta_3 \). Since \( s_1 \geq s_2 > s_3 \), information messages \( \tilde{m}_1 \) and \( \tilde{m}_2 \) are said to be more protected against channel errors than information message \( \tilde{m}_3 \).

2.2 A three-level coded 8-PSK modulation for UEP

As an example, let \( C_1 \) be a (64, 18, 22) extended BCH code, \( C_2 \) be a (64, 45, 8) extended BCH code, and \( C_3 \) be a (64, 63, 2) parity-check code. The Imai-Hirakawa construction [8] produces a three-level coded 8-PSK modulation scheme with \( R = 1.97 \) and MSED=8. The asymptotic coding gain of this system is 6 dB with respect to uncoded QPSK modulation.

For multistage decoding, it is well known [3, 7] that the number of nearest neighbors, \( N_{d_i}^{(w)} \), associated with the first decoding stage is very large if the standard partitioning of [9] is used. For 8-PSK signaling, we have at the first decoding stage \( N_{d_1}^{(1)} = 2d_1 A_{d_1}^{(1)} \), where \( A_{d_1}^{(1)} \) denotes the number of codewords of weight \( w \) in the i-th level component code \( C_i \). Therefore an important reduction in real coding gain (i.e., at BER of \( 10^{-5} \)) will be experienced. For the example above, it can be argued that the 18 information bits encoded at the first level, designed to have the highest error protection level, will suffer a coding gain reduction so large that, at a BER of \( 10^{-5} \), they may become very badly protected and dominate the overall error performance. As a result, decoding errors from the first stage will propagate.

The UEP design principle given by (1) is valid only in the high SNR (asymptotic) region. For practical values of signal-to-noise ratio (BER of \( 10^{-5} \)), however, the error coefficients at each coding level must be taken into account. To improve the performance of three-level coded 8-PSK modulation schemes, a nonstandard set partition can be used to reduce these error coefficients, as explained in the next section.
2.3 Using a nonstandard partition

To overcome the severe reductions in the coding gains of a coded modulation system based on the standard partition, which are caused by large error coefficients, $N_d^*$, nonstandard partitioning must be employed. In this paper, the partition used is similar to one presented previously in [6] for trellis coded modulations (TCM) with UEP.

In Fig. 1, a nonstandard partition of a uniform unit-energy 8-PSK signal set is shown. Each signal point is labeled by a three-bit vector in such a way that the signal points with labels $b_1 = 0$ (resp. $b_1 = 1$) all lie on the left (resp. right) half plane, i.e., $x < 0$ (resp. $x > 0$). Signal points with the second label bit $b_2 = 0$ (resp. $b_2 = 1$) are located on the upper (resp. lower) half plane, i.e., $y > 0$ (resp. $y < 0$). The third label bit $b_3$ selects a signal point within a quadrant indexed by $b_1 b_2$. In this figure, the black dots correspond to points labeled $0b_2 b_3$ and the white dots to points with labels $1b_2 b_3$. The in-trast distances of this partition are all equal $\delta_1 = \delta_2 = \delta_3 = 0.586$. At the $i$-th partition level, the signal points corresponding to $b_i = 0$, or $b_i = 1$, are contained in half planes. For the first and second levels, the error coefficients associated with $d_1 \delta_1$ and $d_2 \delta_2$ respectively are reduced to much less than the number of minimum distance codewords of the corresponding binary component codes. This is explained in the next section. In addition, because the decision boundary at each level is the same as for BPSK (i.e., a line), a three-stage decoder can use computationally efficient soft-decision decodings [11] of the component binary block codes.

3 Error performance: Analysis and simulation results

In decoding the first or second level, the decision variable is just the projection of the received signal sequence in the $x$ or $y$ axis, respectively. Fig. 3 shows a block diagram of a decoder for three-level coded 8-PSK modulation. The decoders for the first and second stages operate on the in-phase and quadrature component of the received signal sequences, $\bar{r}_x$ and $\bar{r}_y$, respectively. Once decisions are made as to the values of the corresponding codewords, $\bar{c}_1$ and $\bar{c}_2$, they are passed on to the third decoding stage. Let $\bar{c}_i = (\bar{c}_{i1}, \bar{c}_{i2}, \ldots, \bar{c}_{i m}) \in C_i$ be the decoded codeword at the $i$-th level, $i = 1, 2$. Before the third-level decoding, each two-dimensional coordinate $(r_{xj}, r_{yj})$ of the received signal $\bar{r} = (\bar{r}_x, \bar{r}_y)$ is projected onto a one dimensional coordinate $r_{xj}', 1 \leq j \leq n$. The values $r_{xj}'$ are the decision variables used by the decoder of $C_3$. The projection depends on the decoded quadrant, which is indexed by the pair $(\bar{c}_{1j}, \bar{c}_{2j})$, $1 \leq j \leq n$, as shown in Table 1. The projected sequence $\bar{r}' = (r_{x2}', r_{x3}', \ldots, r_{xn}')$ is then decoded using a soft-decision procedure for component code $C_3$.

Another advantage of the partition of Fig. 1 is that, by orthogonality of the in-phase and quadrature components, the decoders of the first and second levels can operate independently, as shown in Fig. 3. This results not only in a fast parallel decoding, but also in that there is no error propagation between the first and the second decoding stages.

3.1 Bounds on the error performance

With reference to Fig. 1, the distances from the origin to the projected signal points are either $\Delta_1 = \sin(\pi/8)$ or $\Delta_2 = \cos(\pi/8)$. This is shown in Fig. 2. Assuming equally likely messages, the probability that the projection of a signal point is at distance $\Delta_1$ (resp. $\Delta_2$) from the origin is equal to 1/2. Using this simple observation, it is possible to obtain an upper bound on the probability of a bit error at the first, or second, level. At the $i$-th level, $i = 1, 2$, a union bound on the probability of a bit error is,

$$P_{bi} \leq \sum_{w=0}^{n} \frac{w A_w^{(i)}}{n} \sum_{t=0}^{w} \binom{w}{t} f(w, t),$$

(2)

Table 1: Preprocessing of the received signals for third stage decoding

<table>
<thead>
<tr>
<th>$\bar{c}_{1j}$</th>
<th>$\bar{c}_{2j}$</th>
<th>$r_{xj}'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$-(r_{xj} - r_{yj})$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>$-(r_{xj} + r_{yj})$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>$(r_{xj} - r_{yj})$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>$(r_{xj} + r_{yj})$</td>
</tr>
</tbody>
</table>

Figure 1: A nonstandard partition of an 8-PSK signal set

Figure 2: Projections of signal points
From (2), we observe that \( N_{j_1}^{(i)} = 2^{-d_j} A_d^{(i)} \). For a standard partition, \( N_{d_1}^{STD} = 2d_1 A_d^{(i)} \). It becomes evident how the nonstandard partition reduces the effect of the error coefficients at the expense of an Euclidean separation reduction. Hence, at relatively low SNR, the first two levels of a coded modulation based on nonstandard partitioning can even yield a **real coding gain** greater than the asymptotic coding gain.

The probability of a bit error in the third level decoding can be approximately (considering that no errors are made in the first and second levels) upper bounded as

\[
P_{33} \leq \sum_{w=d_3}^{n} \frac{w A_w^{(3)}}{n} \tilde{Q} \left( \sqrt{w \Delta_1^2} \right) .
\]

### 3.2 Simulation results

Computer simulations were performed to evaluate three-level coded 8-PSK modulations, using binary BCH codes of length 64 as component codes. Table 2 summarizes the simulated schemes. All the schemes were selected to have the same rate \( R = 126/64 = 1.96875 \). The BCH codes were decoded using an efficient soft-decision decoding procedure based on ordered statistics [11]. Listed in the table are also the orders of reprocessing needed to achieve practically optimal performance, as defined in [11], and the number of real operations per block of \((k_1 + k_2 + k_3) = 126\) decoded bits. For \( \ell \)-order reprocessing, the number of real operations is \( O((n - k) (\ell)) \) [11]. Also, for BER \( \geq 10^{-5} \), these numbers can be significantly reduced by processing the reduced list decoding proposed in [12] and allowing an error

![Decoder of three-level coded 8-PSK modulation](image)

![Simulation results for scheme S1](image)

![Simulation results for scheme S2](image)

![Simulation results for scheme S3](image)
performance degradation less than 0.1 dB with respect to maximum likelihood decoding. As a reference, a 256-state rate-2/3 8-PSK TCM scheme, of equivalent rate, achieves an asymptotic coding gain of 5.75 dB and requires about 112,896 real operations to decode 126 bits.

Figures 4 to 6 plot the BER versus $E_b/N_0$ for the schemes listed in Table 2. For example, for the first level of scheme S1 (Fig. 4) a real coding gain of about 8.8 dB is achieved at the BER $10^{-5}$, compared to an asymptotic coding gain of 8.02 dB. From the plots it is clear that the bounds (2) and (3) are tight for practical values of $E_b/N_0$.

4 Conclusions

In this paper, three-level coded 8-PSK modulation schemes for UEP, to achieve graceful degradation in satellite broadcasting of digital HDTV, were studied. A design principle for UEP was proposed. Nonstandard set partitions can be used to construct powerful coded modulation schemes for unequal error protection, with reduced error coefficients compared to standard mapping by set partitioning.

Nonstandard partitioning has the following practical advantages over other approaches: (1) the reduction in error coefficients is such that, in the first two levels, the real coding gain at practical values of BER can be made greater than the asymptotic coding gain; (2) parallel decoding of the first and second levels is possible, for fast implementation; and (3) there is no error propagation from the first level decoder to the second level decoder.

References


