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Finding Optimal Reduct for Rough Sets by Using a Decision Tree Learning Algorithm

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Finding Optimal Reduct for Rough Sets by Using a Decision Tree Learning Algorithm

A Project Report

Presented to

The Faculty of the Department of Computer Science
San Jose State University

by

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Abstract

Rough Set theory is a mathematical theory for classification based on structural analysis of relational data. It can be used to find the minimal reduct. Minimal reduct is the minimal knowledge representation for the relational data. The theory has been successfully applied to various domains in data mining. However, a major limitation in Rough Set theory is that finding the minimal reduct is an NP-hard problem. C4.5 is a very popular decision tree-learning algorithm. It is very efficient at generating a decision tree. This project uses the decision tree generated by C4.5 to find the optimal reduct for a relational table. This method does not guarantee finding a minimal reduct, but test results show that the optimal reduct generated by this approach is equivalent or very close to the minimal reduct.
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1 Introduction

Data mining can be defined as “The nontrivial extraction of implicit, previously unknown, and potentially useful information from data” (Frawley et al. 1992). Data mining is used by all sorts of organizations and businesses to identify trends across large sets of data. Once these patterns are identified they can be used to enhance decision making in a number of areas.

Data mining is quite pervasive in today’s society and greatly affects the way we are marketed to by corporate America. Advertisers and marketers depend so heavily on data mining and the trend analysis that it provides, that it is hard to imagine life without it. The information harnessed via data mining techniques govern our buying patterns, the food we eat, the medicine we take and so many other facets of our daily lives. For example, one large North American chain of supermarkets discovered through the mining of their data that beer and diapers were often sold together. They used this information to strategically place the two items in close proximity to one another within their stores and saw sales of both climb. Without mining data the greatest of marketers would have had a tough time discovering such a correlation. Another and perhaps more practical application comes from the medical field. Historical patient data is often mined in conjunction with disease information to determine correlations between demonstrated symptoms and disease. Such applications of data mining are able to diagnose and treat
diseases earlier. It is these sorts of applications that make data mining so important to our society today.

Rough Set theory as introduced by Zdzislaw Pawlak in 1982 can be used to create a framework for data mining. Rough Set theory provides many good features, such as representing knowledge as an equivalence class, generating rules from a relational data table and finding the minimal reduct from a relational data table. Rough Set theory is an elegant theory when applied to small data sets because it can always find the minimal reduct and generate minimal rule sets. However, according to (Komorowski et al, 1998), the general solution for finding the minimal reduct is NP-hard. An NP-hard problem is defined a problem that cannot be solved in polynomial time. In other words as data sets grow large in both dimension and volume, finding the minimal reduct becomes computationally impossible.

The C4.5 algorithm is an extension of ID3 algorithm, a classification algorithm used to generate decision trees. C4.5 enhanced ID3 by better attribute selection method, handling missing values, continuous attributes. The C4.5 tree-induction algorithm is capable of generating small decision trees quite quickly. However, it is not necessarily the best solution, as it does not always generate the smallest decision tree possible. In cases where multiple attributes have the same gain ratio, C4.5 is not guaranteed to choose the attribute that will generate the smallest sub-tree. By failing to choose the best attribute to build the decision tree, C4.5 leaves room for improvement.

In our project we sought to improve upon the C4.5 algorithm by applying the Greedy Algorithm to attributes with the same gain ratio. By applying the Greedy
Algorithm we are always able to generate equal or smaller decision trees given the same data set when compared to the original C4.5 algorithm. By combining Rough Set theory and the output decision trees of our modified C4.5 algorithm we are able to find the Optimal Reduct.

The advantage of this algorithm is that finding the Optimal Reduct unlike the minimal reduct is not an NP-Hard problem. Therefore generating the optimal rule set for large data sets become computationally possible using our algorithm. Our results prove that generating the Optimal Reduct is exponentially more attainable than the minimal reduct given the same computing resources. Our tests also show that the Optimal Reduct only slightly increases the total number of rules as compared with the minimal reduct. Given these two findings, the Optimal Reduct is a practical solution given the NP-hard problem faced by Rough Set theory when dealing with large data sets.
2 Rough Sets and Relational Table

Rough Set theory is a popular theory in data mining (Pawlak, 1982). Unlike the traditional database, which focuses on storing and retrieving data, Rough Set theory focus on analyzing and finding the useful trends, patterns and information from the data. The main techniques are set approximation and the reduct generation. The following introduces the definitions and concepts of Rough Set theory.

2.1 Preliminaries

Definition 2.1: Decision System

A Decision System can be described as $DS=(U, A)$, where $U$ is a nonempty finite set of objects and $A$ is a nonempty finite set of attributes. The elements is called objects. For every $a \in A$, $V_a$ is the allowed attribute values.

The following demonstrates a decision system via a simple hiring example. Suppose a department in a company needs to hire some people. A criterion is used to select the candidates for onsite interviews. There are eight total candidates and they are to be evaluated on four different sets of attributes (Education Level, Years of Experience, Oracle technology, Java technology). The criterion has one decision attribute, On-site interview. The rows represent objects or people in this case, and the columns represent attribute value pairs for each of the objects.

<table>
<thead>
<tr>
<th>Candidates</th>
<th>Education</th>
<th>Experience</th>
<th>Oracle technology (For example: ADF)</th>
<th>Java technology</th>
<th>On-site interview</th>
</tr>
</thead>
<tbody>
<tr>
<td>Candidate 1</td>
<td>Bachelor</td>
<td>Senior (&gt; 3 years)</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Candidate 2</td>
<td>High School</td>
<td>Senior (&gt; 3 years)</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Candidate 3</td>
<td>High School</td>
<td>Junior</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Candidate 4</td>
<td>Ph.D.</td>
<td>Junior</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Candidate 5</td>
<td>Bachelor</td>
<td>Junior</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Candidate 6</td>
<td>Master</td>
<td>Senior (&gt; 3 years)</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Candidate 7</td>
<td>High School</td>
<td>Junior</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Candidate 8</td>
<td>Bachelor</td>
<td>Senior (&gt; 3 years)</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

Table 2-1

**Definition 2.2: Equivalence Class**

Let \( x, y \) and \( z \) be the objects in \( U \). If \( xRy = xRx, xRy = yRx \) and \( xRy, yRz, \) then \( xRz \), we call \( R \) the equivalence Relation. The equivalence class is a subset of \( U \), and all the elements in the equivalence class have the equivalence relation with each other.

**Definition 2.3: Indiscernibility Relation**

The indiscernibility relation \( IND(P) \) is a equivalence relation. With every subset of attributes \( P \subseteq A \) in decision system the indiscernibility relation can be defined as follows

\[
IND(P) = \{(x, y) \in U \times U, \text{ for every } a \in P, a(x) = a(y)\}
\]
Elementary set, $U/IND(P)$, means the set of all the equivalence classes in the relation $IND(P)$.

For the decision system given in the Table 2-1, $U/IND(P)$ gives the following result:

$U/IND(\{\text{Education}\}) = \{\{C1, C5, C8\}, \{C2, C3, C7\}, \{C4\}, \{C6\}\}$

$U/IND(\{\text{Experience}\}) = \{\{C1, C2, C6, C8\}, \{C3, C4, C5, C7\}\}$

$U/IND(\{\text{Education, Experience}\}) = \{\{C1, C8\}, \{C2\}, \{C3, C7\}, \{C4\}, \{C5\}, \{C6\}\}$

**Definition 2.4: Set approximation**

*Upper approximation* $\overline{apr}(X)$ is union of all the objects which are possible members of $X$, $X \subseteq U$.

*Lower approximation* $apr(X)$ is union of all the objects, which definitely are members of $X$, $X \subseteq U$.

*Positive Region* of $X$ $POS(X)$ is defined as $apr(X)$.

*Negative region* of $X$ $NEG(X)$ is defined as $U - \overline{apr}(X)$

*Boundary region* of $X$ $BND(X)$ is defined as $\overline{apr}(X) - apr(X)$

The following figure illustrates the positive, negative and boundary regions. Each rectangle represents an equivalence class. From the figure we can see that any element $x \in POS(X)$ belongs to $X$, and any element $x \in NEG(X)$ does not belong to $X$. We call
POS(X) \cup BND(X) upper approximation \textit{apr}(X) of a set X. For any element \( x \in \textit{apr}(X) \), it possible belongs to X.

Rough Set theory is the study of the boundary region. The following are properties of the approximation sets:

1. \( \textit{apr}(X) \supset X \supset \textit{apr}(X) \)
2. \( \text{apr}(U) = \overline{\text{apr}(U)} = U, \text{apr}(\emptyset) = \overline{\text{apr}(\emptyset)} = \emptyset \)

3. \( \text{apr}(X \cup Y) = \overline{\text{apr}(X)} \cup \overline{\text{apr}(Y)} \)

4. \( \text{apr}(X \cap Y) = \overline{\text{apr}(X)} \cap \overline{\text{apr}(Y)} \)

5. \( \text{apr}(X) = -\overline{\text{apr}(U-X)} \)

6. \( \text{apr}(X) = -\overline{\text{apr}(U-X)} \)

7. \( \text{apr}(X \cap Y) \subseteq \overline{\text{apr}(X)} \cap \overline{\text{apr}(Y)} \)

8. \( \text{apr}(X \cup Y) \supset \overline{\text{apr}(X)} \cup \overline{\text{apr}(Y)} \)

9. \( \text{apr}(X - Y) \supset \overline{\text{apr}(X)} - \overline{\text{apr}(Y)} \)

10. \( \text{apr}(X - Y) \subset \overline{\text{apr}(X)} - \overline{\text{apr}(Y)} \)

11. \( X \subseteq Y \rightarrow \overline{\text{apr}(X)} \subseteq \overline{\text{apr}(Y)} \)

12. \( X \subseteq Y \rightarrow \overline{\text{apr}(X)} \subseteq \overline{\text{apr}(Y)} \)

13. \( \overline{\text{apr}(X)} \cup \overline{\text{apr}(-X)} = -\text{BND}(X) \)

14. \( \overline{\text{apr}(X)} \cap \overline{\text{apr}(Y)} = \text{BND}(X) \)

For the decision system given in the Table 2-1, when just using the attribute “Education” to classify the decision system, for the “Yes” decision, \( X = \{C_1, C_2, C_4, C_5, C_6\} \).

\( U/\text{IND}(\{\text{Education}\}) = \{\{C_1, C_5, C_8\}, \{C_2, C_3 C_7\}, \{C_4\}, \{C_6\}\} \)

\( \text{Apr}(X) = \{C_4, C_6\} \)
$\text{Apr}(X) = \{C1, C2, C3, C4, C5, C6, C7, C8\}$

$\text{POS}_{\text{education}} (X) = \text{apr}(X) = \{C4, C6\}$

$\text{NEG}_{\text{education}} (X) = U - \text{apr}(X) = \emptyset$

$\text{BND}_{\text{education}} (X) = \text{apr}(X) - \text{apr}(X) = \{C1, C2, C3, C5, C7, C8\}$

Decision rules we can derive:

If Education = Ph.D or Education = Master

--> Have on-site interview

If Education = Bachelor or Education = High School

--> Unknown

**Definition 2.5: Dispensable and Indispensable attributes**

An attribute a is dispensable in $P$ if $\text{POS}(P) = \text{POS}(P - \{a\})$, otherwise the attribute a is

*indispensable attribute* in $P$.

For the decision system given in the Table 2-1, when $P = \{\text{Education, Experience, Oracle technology, Java technology}\}$, for “Yes” decision, $\text{POS}_{\{\text{Education, Experience, Oracle, Java}\}} (\{C1, C2, C4, C5, C6\}) = \text{POS}_{\{\text{Education, Experience, Java}\}} (\{C1, C2, C4, C5, C6\})$. For “No” decision, $\text{POS}_{\{\text{Education, Experience, Oracle, Java}\}} (\{C3, C7, C8\}) = \text{POS}_{\{\text{Education, Experience, Java}\}} (\{C3, C7, C8\})$
In this case the attribute Oracle technology is a dispensable attribute.

**Definition 2.6: Reduct**

The **reduct** is a minimal subset of attributes $A$, which classify the universe $U$ into the same set of the equivalence class. For a given attribute set $A$, the reduct is not unique.

For the previous example in Table 2.1, the reduct could be:

\{Education, Experience, Oracle technology\} or \{Education, Experience, Java technology\}

**Definition 2.7: Decision rules**

With every object in the universe $U$ we associate a function $D_x : A \rightarrow V$, the function $D_x$ is a **decision rule**. $A$ represents attributes, and $V$ represents decision value. The minimal decision rules are the minimal set of rules that classify the universe $U$ into the same set of the **equivalence class**.

**3 C4.5**

C4.5 algorithm is based on ID3 algorithm. They are both introduced by Quinlan, J. Ross to generate the decision trees from data (Quinlan, 1993). C4.5 improves on ID3 by using the gain ratio criterion instead of the gain criterion to select an attribute. C4.5 can handle missing values, continuous attribute value ranges and prune the decision trees. A decision tree can be used to sort down a tree from the root to the leaf to classify the instances.
Each node in the tree represents a condition attribute in the decision system. Each leaf represents a decision value in the decision system. For example,

![Decision Tree for Golf Data](image)

**Figure 3-1: Decision Tree for Golf Data**
From this decision tree, golf is played when the outlook is sunny and humidity is equal to or less than 75 degrees. When the outlook is sunny and humidity is greater than 75 degrees, a decision is made not to play golf. When outlook is overcast golf is played. When outlook is rainy and windy golf is not played. Finally, when outlook is rainy and not windy golf is played.

Not all the classification tasks can use C4.5. There are several prerequisites:

- All the data can be described as attribute-value table. This table has fixed collection of properties or attributes.
- The table has predefined classes.
- The class type is discrete.

We have sufficient data.

3.1 Preliminaries

As described in (Quinlan, 1993), if there are n equally probable possible messages, the probability of each message is 1/n. The information conveyed by any one of the messages is $- \log_2 \left( \frac{1}{n} \right)$. For example: if there are 8 messages, then $- \log_2 \left( \frac{1}{8} \right) = 3$. So we need 3 bits to identify each message. If there is a set of cases S and it has $(C_1, C_2, ..., C_k)$ classes, then $freq(C_j, S)$ means the number of cases in S that belongs to $C_j$ and $|S|$ means the number of cases in the set S. Selecting any case randomly from S
that is in \( C_j \) has probability \( \frac{\text{freq}(C_j, S)}{|S|} \) and the information conveyed by this is

\[ -\log_2 \left( \frac{\text{freq}(C_j, S)}{|S|} \right) \] bits. The average amount of information we need to identify the

class of a case in \( T \) is \( \text{Info}(S) = -\sum_{j=1}^{k} \frac{\text{freq}(C_j, S)}{|S|} \times \log_2 \left( \frac{\text{freq}(C_j, S)}{|S|} \right) \)

If we partition \( T \) on the basis of the value of test \( X \) into sets \( T_1, T_2, \ldots, T_n \) then the

information needed to identify the class of a case of \( T \) becomes the weighted average of

the information needed to identify the class of a test of \( T_i \)

\[ \text{Info}_x(T) = \sum_{i=1}^{n} \frac{|T_i|}{|T|} \times \text{Info}(T_i) \]

Consider the quantity \( \text{Gain}(X) \) defined as

\[ \text{Gain}(X) = \text{Info}(T) - \text{Info}_x(T) \]

This represents the difference between the information needed to identify a case of \( T \) and the information needed to identify a case of \( T \) after the value of test \( X \) has been obtained; that is, this is the gain information due to test \( X \).

The gain gave very good results but it has a strong bias to the attribute that has the most different values. From the prediction point of view, to choose this kind of the
attribute is useless. For example, most database tables have the primary key. In these tables, by using the gain, it would like to select the primary key. The primary key is not very helpful for finding the knowledge in data mining. In C4.5, Quinlan suggests to use Gain Ratio.

We can use gain ratio to choose the attribute that has the greatest gain among all attributes and then use this attribute as one node of the tree. By doing this we are able to create a small decision tree.

\[
GainRatio(X) = \frac{Gain(X)}{split(X)}
\]

\[
Split(X) = -\sum_{i=1}^{n} \frac{|T_i|}{|T|} \times \log_2 \left( \frac{|T_i|}{|T|} \right)
\]

Where \((T_1, T_2, ... T_n)\) is the partition of T induced by the value of X.

### 3.2 Example

Table 2-1 hiring example is used again to illustrate the C4.5 algorithm.

<table>
<thead>
<tr>
<th>Candidates</th>
<th>Education</th>
<th>Experience</th>
<th>Oracle technology (For example: C4.5)</th>
<th>Java technology</th>
<th>On-site interview</th>
</tr>
</thead>
<tbody>
<tr>
<td>Candidate 1</td>
<td>Bachelor</td>
<td>Senior (&gt; 3 years)</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Candidate 2</td>
<td>High School</td>
<td>Senior (&gt; 3 years)</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Candidate 3</td>
<td>High School</td>
<td>Junior</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Candidate 4</td>
<td>Ph.D.</td>
<td>Junior</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Candidate 5</td>
<td>Bachelor</td>
<td>Junior</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Candidate 6</td>
<td>Master</td>
<td>Senior (&gt; 3 years)</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Candidate 7</td>
<td>High School</td>
<td>Junior</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Candidate 8</td>
<td>Bachelor</td>
<td>Senior (&gt; 3 years)</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

In the table, the columns of the table are named **attributes**. The first four columns are further defined as conditional attributes. The last column is referred to as the decision attribute. The rows of the table are referred to as **objects**. The entries in the table are called **attribute-values**. Then each row of the table can be seen as the information for a particular candidate. From the data in this table we can determine that if a candidate demonstrates an education level of Masters or Ph.D degree then said candidate will be rewarded an on-site interview. The decision tree would appear as follows:

Education = Bachelor:
| Experience = Senior:
Tree 1

In this decision tree, the first attribute is chosen to divide the objects and then second and third attributes are selected until a decision is made whether or not to bring the candidate in for an on-site interview.

When using C4.5, it tries to choose the attribute that generates a small decision tree. The following demonstrates the C4.5 algorithm. There are two classes (Yes and No for On-Site interview), five objects belong to Yes and three objects belong to No.

\[
\text{Info}(T) = -\frac{5}{8} \times \log_2\left(\frac{5}{8}\right) - \frac{3}{8} \times \log_2\left(\frac{3}{8}\right) = 0.954
\]

This represents the average information needed to identify the class of the case in T. After using the “Education” to divide T into four subsets, the result is 0.687

\[
\text{Info}_e(T) = \frac{3}{8} \times \left(-\frac{1}{3} \times \log_2\left(\frac{1}{3}\right) - \frac{2}{3} \times \log_2\left(\frac{2}{3}\right)\right) + \frac{3}{8} \times \left(-\frac{2}{3} \times \log_2\left(\frac{2}{3}\right) - \frac{1}{3} \times \log_2\left(\frac{1}{3}\right)\right) \\
+ \frac{1}{8} \times \left(-\frac{1}{1} \times \log_2\left(\frac{1}{1}\right)\right) + \frac{1}{8} \times \left(-\frac{1}{1} \times \log_2\left(\frac{1}{1}\right)\right) = 0.687
\]
The information gained by this test is 0.266.

\[ Gain(X) = Info(T) - Info_x(T) = 0.954 - 0.686 = 0.266 \]

\[ Split(X) = -\frac{3}{8} \times \log_2 \left( \frac{3}{8} \right) - \frac{3}{8} \times \log_2 \left( \frac{3}{8} \right) - \frac{1}{8} \times \log_2 \left( \frac{1}{8} \right) - \frac{1}{8} \times \log_2 \left( \frac{1}{8} \right) = 1.811 \]

\[ GainRatio(X) = \frac{Gain(X)}{Split(X)} = \frac{0.266}{1.811} = 0.147 \]

Now suppose that, instead of dividing T on the attribute “Education”, we have partitioned it on the attribute “Experience”. After using “Experience” to divide T into two subsets, the similar computation is shown below.

\[ Info_x(T) = \frac{4}{8} \times \left( -\frac{3}{4} \times \log_2 \left( \frac{3}{4} \right) - \frac{1}{4} \times \log_2 \left( \frac{1}{4} \right) \right) + \frac{4}{8} \times \left( -\frac{2}{4} \times \log_2 \left( \frac{2}{4} \right) - \frac{2}{4} \times \log_2 \left( \frac{2}{4} \right) \right) = 0.906 \]

\[ Gain(X) = \inf \ o(T) - \inf \ o_x(T) = 0.954 - 0.906 = 0.048 \]

\[ Split(X) = -\frac{4}{8} \times \log_2 \left( \frac{4}{8} \right) - \frac{4}{8} \times \log_2 \left( \frac{4}{8} \right) = 1 \]

\[ GainRatio(X) = \frac{Gain(X)}{Split(X)} = \frac{0.048}{1} = 0.048 \]

When using “Oracle technology” to divide T, we get the following values
\[ \text{Info}_x(T) = \frac{5}{8} \times \left( -\frac{4}{5} \times \log_2 \left( \frac{4}{5} \right) - \frac{1}{5} \times \log \left( \frac{1}{5} \right) \right) + \frac{3}{8} \times \left( -\frac{2}{3} \times \log_2 \left( \frac{2}{3} \right) - \frac{1}{3} \times \log_2 \left( \frac{1}{3} \right) \right) = 0.795 \]

\[ \text{Gain}(X) = \inf o(T) - \text{Info}_x(T) = 0.954 - 0.795 = 0.159 \]

\[ \text{Split}(X) = -\frac{5}{8} \times \log_2 \left( \frac{5}{8} \right) - \frac{3}{8} \times \log_2 \left( \frac{3}{8} \right) = 0.954 \]

\[ \text{GainRatio}(X) = \frac{\text{Gain}(X)}{\text{Split}(X)} = \frac{0.159}{0.954} = 0.166 \]

When using “Java technology” to divide T, we get the following values.

\[ \text{Info}_x(T) = \frac{7}{8} \times \left( -\frac{5}{7} \times \log_2 \left( \frac{5}{7} \right) - \frac{2}{7} \times \log_2 \left( \frac{2}{7} \right) \right) + \frac{1}{8} \times \left( -\frac{1}{1} \times \log_2 \left( \frac{1}{1} \right) \right) = 0.755 \]

\[ \text{Gain}(X) = \inf o(T) - \text{Info}_x(T) = 0.954 - 0.755 = 0.199 \]

\[ \text{Split}(X) = -\frac{7}{8} \times \log_2 \left( \frac{7}{8} \right) - \frac{1}{8} \times \log_2 \left( \frac{1}{8} \right) = 0.544 \]

\[ \text{GainRatio}(X) = \frac{\text{Gain}(X)}{\text{Split}(X)} = \frac{0.199}{0.544} = 0.365 \]

Based on the calculation, C4.5 chooses “Java Technology” to divide the T first, and the tree becomes:
Java Technology = Yes:

Java Technology = No: No

After using the Java Technology to divide the tree the table looks as follow:

When Java Technology = yes

<table>
<thead>
<tr>
<th>Candidates</th>
<th>Education</th>
<th>Experience</th>
<th>Oracle Technology</th>
<th>On-site interview</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bachelor</td>
<td>Senior</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>High School</td>
<td>Senior</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>3</td>
<td>High School</td>
<td>Junior</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>4</td>
<td>PH.D.</td>
<td>Junior</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>5</td>
<td>Bachelor</td>
<td>Junior</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>6</td>
<td>Master</td>
<td>Senior</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>7</td>
<td>High School</td>
<td>Junior</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

Table 3-1

When Java Technology = no

<table>
<thead>
<tr>
<th>Candidates</th>
<th>Education</th>
<th>Experience</th>
<th>Oracle Technology</th>
<th>On-site interview</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>Bachelor</td>
<td>Senior</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

Table 3-2

The same method is recursively used to calculate the Gain Ratio for each attributes in Table 3-1 and Table 3-2. Finally we get the following decision tree.

Java = No: No (1.0)
Java = Yes:
| Experience = Senior: Yes
| Experience = Junior:
  | Education = Bachelor: Yes
  | Education = High School: No
  | Education = PH.D.: Yes
  | Education = Master: Yes

**Tree 2**

When running the above example through the C4.5 program, we can see the following output.

![Decision Tree Output](image)

**Figure 3-2**

By comparing Tree 1 and Tree 2, we can see that for Tree 1, we manually pick the attribute. In this case we pick the attribute from left to right in the columns. So it picks “Education” to start creating the tree. C4.5 picks “Java Technology” to start creating the tree. We can tell that the tree created by C4.5 has smaller size than the manual process.
By testing C4.5 on other data sets, we see that C4.5 generates smaller decision trees and is very fast.
4 Finding Minimal Rules

Decision rules finding, especially optimal decision rules finding is a crucial task in Rough Set applications. The purpose of this project is to generate the optimal decision rule for Rough Set by using the C4.5 algorithm. Though this method does not guarantee finding the minimal set of rules, it does often find the minimal set. In the cases where it does not find the minimal set it does find sets that number very close to the minimum.

The project is based on C4.5 and adds several options to generate an optimal reduct. The program workflow is shown below:
Start

Read

*.data
*.names

AttName[ ], AttValName[ ], MaxAttVal[ ], MaxAtt, ClassName[ ], MaxClass

Use Feature selection to mark the Atts

Compute Gain Ratio

* Use the greedy algorithm
When there are more than one attribute which have the same best gain ratio.

The T is not empty

Use Greedy Algorithm to select Best Att

Create Node

Decision Tree

Generate Reduct

Apply Threshold

Rules

End
**Step One**: The algorithm uses the feature selection first to remove the dispensable attributes from the decision system. By using the feature selection, the unrelated attribute data will not be considered when building the decision tree from C4.5.

**Step Two**: The algorithm uses this modified version of C4.5 to generate the decision tree. The C4.5 is modified to select the best node attribute when there are multiple attributes with the same gain ratio. For those decision systems that have this specific attribute situation, our decision tree is smaller than the tree generated by the original C4.5. By generating a smaller decision tree we are in turn generating a smaller set of rules in the later steps of the process.

**Step Three**: Convert the generated decision tree to rules format.

**Step Four**: Evaluate each rule to remove the superfluous attribute-values. When using different sequence to evaluate each attribute values in the rule will generate different subset of rules. In the algorithm, every attribute in the decision system is used as the first attribute to evaluate the rule. After the subset of rules being generated, the algorithm chooses the best rule whose length of the rule is the smallest. Here the length of the rule means the number of attribute values the rule needs to classify the objects.

**Step Five**: Remove the duplicate rules. After the rules have been simplified, different rules may cover the same set of objects in the decision system. This process is used to remove the duplicate rules, leaving only the necessary rules for decision table classification.
**Step Six**: Apply the threshold to display the decision rules. When a decision system has hundreds of objects it may generate hundreds of decision rules. In this case, it would be very hard for user to read all the rules. If a user specifies a threshold, the program will just show the rules which are suitable for more objects than the threshold value. User can base on the need to specify this value.

The following method constructs the decision tree used in modified C4.5 with a “divide and conquer” strategy. Each node in the tree is associated with a set of cases. In the beginning, only the root is present and associated with the whole training set. At each node, the following “divide and conquer” algorithm is executed.

```
FormTree(T)
{
    ComputeClassFrequency(T);
    if T belongs to OneClass
    {
        return a leaf;
    }
    create a decision node N;
    ForEach( Attribute A)
    {
        ComputeGainRatio(A);
    }
    N.BestAtt=AttributeWithBestGainRatio;
    if ( there are multiple BestAtts)
    {
        calculate the subtree for each BestAtts;
        N.BestAtt=Att associated with the smallest tree size;
    }
    Child of N = FormTree (T');
    return N;
}
```
For a set of data, it calculates the Class Frequency first. If the data set belongs to one class, the algorithm returns a leaf; otherwise, it creates a decision node N. For each attribute it calculates the Gain Ratio value. The algorithm picks the highest Gain Ratio value attribute as the best attribute. If there are multiple attributes (say set MA) with the same best Gain Ratio value, the algorithm will generate the sub-tree for each attribute in MA and use the attribute that has the smallest sub-tree size as the best attribute. C4.5 will use the best attribute to divide the data set and generate the node.

4.1 Feature selection

Feature selection, also known as subset selection or variable selection, is a process commonly used in data mining. The main idea of feature selection is to choose a subset of the input variables by removing features with little or no predictive information. Feature selection is necessary because sometimes there are too many features and it is computationally infeasible to use all the available features.

In this project, we use the functional dependency concept to evaluate every attribute to see if the attribute is dispensable and mark every attribute accordingly. (A functional dependency is a constraint between two sets of attributes in a relation. Given a relation R, a set of attributes X is said to functionally determine another attribute Y if and only if each X value is associated with at most one Y value, we write as \( X \rightarrow Y \) (S. Krishna, 1992)) The algorithm removes one attribute at a time and checks if the remaining condition attribute can determine the decision attribute. If the remaining attributes \( \rightarrow \) decision attribute, the attribute will be marked as “true”. If the remaining attributes cannot determine decision attribute then the attribute is marked “false”. After
each attribute has been evaluated, C4.5 will check the mark and decide whether it needs
to calculate the gain ratio for this attribute. If the attribute is marked “true”, C4.5 will not
calculate the gain ratio for this attribute and will not use this attribute to build the tree.

The algorithm for the feature selection is shown below.

\[ R = \text{ the set of attributes in CON; } \]
\[ \text{AttUse} = \text{ the mark to indicate if the attribute is dispensable} \]
\[ \text{FeatureSelection(R)} \]
\[ \{ \]
\[ \quad \text{InitAttUse(); // Set the default value to false for all the AttUse[i], means all the} \]
\[ \quad \quad \text{attributes are indispensable.} \]
\[ \quad \text{ForEach (i, 0, MaxAtt)} \]
\[ \quad \{ \]
\[ \quad \quad \text{Boolean flag=checkAttribute(i, R); //Check if the remaining attributes can} \]
\[ \quad \quad \quad \text{// determine the decision value.} \]
\[ \quad \quad \text{if (flag)} \]
\[ \quad \quad \quad \{ \]
\[ \quad \quad \quad \quad \text{AttUse[i]=true; //Means the ith attribute can be cut.} \]
\[ \quad \quad \quad \quad \text{R=R-i-th;} \]
\[ \quad \quad \} \]
\[ \} \]
\[ \text{Boolean checkAttribute(Att, R)} \]
\[ \{ \]
\[ \quad R=R-Att; \]
\[ \quad \text{If(R is consistent)} \]
\[ \quad \{ \]
\[ \quad \quad \text{Return true;} \]
\[ \quad \} \]
\[ \quad \text{else} \]
\[ \quad \{ \]
\[ \quad \quad \text{return false;} \]
\[ \quad \} \]
\[ \} \]

The algorithm checks each attribute in sequence from the first attribute to the last
attribute in the table and determines if the attribute can be removed. This option is mainly
used for data sets that have many condition attributes. This helps simplify the computation for the later stages.

The example, which follows (Pawlak, 1991), refers to an optician's decisions as to whether or not a patient is suitable for contact lenses. The set of all possible decisions is listed in Table 4-1.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
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<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
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<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
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<td></td>
</tr>
<tr>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>11</td>
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<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>
Table 4-1

The table is in fact a decision table in which \(a, b, c, d\) and \(e\) are condition attributes whereas \(f\) is a decision attribute.

The attribute \(f\) represents optician's decisions, which are the following:

1. Hard contact lenses.
2. Soft contact lenses
3. No contact lenses

These decisions are based on some facts regarding the patient, which are expressed by the condition attributes, given below together with corresponding attribute values.

\(a\) - Age

1 - young
2 – pre-presbyopic
3 – presbyopic
b – spectacle
1 – myope
2 – hypermetrope
c – astigmatic
1 – no
2 – yes
d – tear production rate
1 – reduced
2 – normal
e- working environment
1 – computer related
2 – non-computer related

Here we need to use the feature selection to eliminate the unnecessary conditions from a decision table.

Step 1:

Removing attribute a in Table 4-1 will result in the following decision table

<table>
<thead>
<tr>
<th>U</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>...</td>
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<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>17</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>19</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>
From the Table 4-2 we see the following pairs of inconsistent decision rules

b2c2d2e1 -> f1 (row 2)
b2c2d2e1 -> f3 (row 18)

This means that the conditions $b,c,d$ and $e$ cannot determine decision $f$. Thus the attribute $a$ cannot be dropped.

Step 2:

Similarly removing attribute $b$ we get the following decision table.
From the Table 4-3 we the following pairs of inconsistent decision rules can be seen

a2c2d2e1 -> f1 (row 3)
a2c2d2e1 -> f3 (row 18)
a3c2d2e2 -> f1 (row 4)
a3c2d2e2 -> f3 (row 24)

and

a3c1d2e2 -> f2 (row 9)
a3c1d2e2 -> f3 (row 20)
This means that the conditions $a,c,d$ and $e$ cannot determine decision $f$. Thus the attribute $b$ cannot be dropped.

Step 3:

Similarly, removing attribute $c$ the following decision table is seen.

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
<th>$e$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td></td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>1</td>
<td></td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td></td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>2</td>
<td></td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>1</td>
<td></td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>2</td>
<td></td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>2</td>
<td></td>
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<tr>
<td>10</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
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<td>3</td>
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<tr>
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<td>...</td>
<td>...</td>
</tr>
<tr>
<td>19</td>
<td>3</td>
<td>1</td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>20</td>
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<td></td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>21</td>
<td>3</td>
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<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>22</td>
<td>3</td>
<td>2</td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>
Table 4-4

From the Table 4-4 we see the following pairs of inconsistent decision rules

\[ a_1b_1d_2e_1 \rightarrow f_1 \text{ (row 1)} \]
\[ a_1b_1d_2e_1 \rightarrow f_2 \text{ (row 5)} \]

\[ a_1b_2d_2e_1 \rightarrow f_1 \text{ (row 2)} \]
\[ a_1b_2d_2e_1 \rightarrow f_2 \text{ (row 6)} \]

\[ a_2b_1d_2e_1 \rightarrow f_1 \text{ (row 3)} \]
\[ a_2b_1d_2e_1 \rightarrow f_2 \text{ (row 7)} \]

\[ a_3b_1d_2e_2 \rightarrow f_1 \text{ (row 4)} \]
\[ a_3b_1d_2e_2 \rightarrow f_3 \text{ (row 20)} \]
and
\[ a_3b_2d_2d_2 \rightarrow f_2 \text{ (row 9)} \]
\[ a_3b_2d_2d_2 \rightarrow f_3 \text{ (row 24)} \]

This means that the conditions \( a,b,d \) and \( e \) cannot determine decision \( f \). Thus the attribute \( c \) cannot be dropped.

Step 4:

Similarly removing attribute \( d \) we get the following decision table.
<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
<td>e</td>
<td>f</td>
</tr>
<tr>
<td>---</td>
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</tr>
<tr>
<td>1</td>
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<td>2</td>
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<tr>
<td>3</td>
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<td>1</td>
<td>2</td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
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<td>2</td>
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<td>1</td>
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</tr>
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</tr>
<tr>
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<td>1</td>
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<td></td>
<td>1</td>
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</tr>
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</tr>
<tr>
<td>13</td>
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<td>2</td>
<td>2</td>
<td></td>
<td>1</td>
<td>3</td>
</tr>
<tr>
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<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>15</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td></td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>16</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td></td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>17</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td></td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>20</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td></td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>21</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>22</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>23</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>
From the Table 4-5 we see the following pairs of inconsistent decision rules

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td></td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 4-5

\[
\begin{align*}
\text{a1b1c2e1} & \rightarrow f1 \text{ (row 1)} \\
\text{a1b1c2e1} & \rightarrow f3 \text{ (row 11)} \\
\text{a1b2c2e1} & \rightarrow f1 \text{ (row 2)} \\
\text{a1b2c2e1} & \rightarrow f3 \text{ (row 13)} \\
\text{a2b1c2e2} & \rightarrow f1 \text{ (row 4)} \\
\text{a2b1c2e2} & \rightarrow f3 \text{ (row 21)} \\
\text{a1b1c1e1} & \rightarrow f2 \text{ (row 5)} \\
\text{a1b1c1e1} & \rightarrow f3 \text{ (row 10)} \\
\text{a1b2c1e1} & \rightarrow f2 \text{ (row 6)} \\
\text{a1b2c1e1} & \rightarrow f3 \text{ (row 12)} \\
\text{a2b1c1e1} & \rightarrow f2 \text{ (row 7)} \\
\text{a2b1c1e1} & \rightarrow f3 \text{ (row 14)} \\
\text{a2b2c1e1} & \rightarrow f2 \text{ (row 8)} \\
\text{a2b2c1e1} & \rightarrow f3 \text{ (row 16)} \\
\text{a3b2c1e2} & \rightarrow f2 \text{ (row 9)} \\
\text{a3b2c1e2} & \rightarrow f3 \text{ (row 22)}
\end{align*}
\]
This means that the conditions $a,b,c$ and $e$ cannot determine decision $f$. Thus the attribute $d$ cannot be dropped.

Step 5:
Similarly removing attribute $e$ we get the following decision table.

<table>
<thead>
<tr>
<th>$U$</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
<th>$e$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>14</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>15</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>16</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>
From the Table 4-6 we see no inconsistent decision rule. This means that the conditions $a, b, c$ and $d$ can determine decision $f$. Thus the attribute $e$ can be dropped. In the program I set the $\text{AttUse}[4] = \text{true}$.

From the previous example it only drops one attribute. It seems unnecessary to have the feature selection process. However in cases where there are hundreds of condition attributes, this process is necessary to remove the unrelated attributes. The feature selection process may save a lot of time for future computing processes to generate the optimal decision rules. From the chapter 5, we could see that using feature selection doesn’t result in smaller decision rules. Even though, the most important point is that feature selection can remove the unrelated attributes, which makes the generated decision rules more reasonable.

4.2 user input, randomize attribute selection and best attribute selection
The C4.5 algorithm calculates the gain ratio for each condition attribute. It selects the attribute whose gain ratio is the highest to divide the data set and build the decision tree. C4.5 continues the same procedure to build the tree until all the objects in the set belong to the same class.

If there are multiple attributes, which have the same best gain ratio, C4.5 always selects the first attribute that appears in the table. In this case, no matter how many times you run C4.5 using the same data it will create the same decision tree. The generated decision tree never is the smallest possible.

For example, with data like below (Att1, Att2 and Att3):

<table>
<thead>
<tr>
<th>Att1</th>
<th>Att2</th>
<th>Att3</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Y</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Y</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Y</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>Y</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>Y</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>Y</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>N</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>N</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>N</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>Y</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>Y</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>Y</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>N</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>N</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>N</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>N</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>N</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>N</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>N</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>N</td>
</tr>
</tbody>
</table>
Table 4-7

When we compute the gain ratio for these three attributes, they all have the same gain ratio and C4.5 chooses the first attribute as the dividing point. The following image is generated based on the original C4.5.

![C4.5 decision tree generator output](image.png)

Figure 4-2
We can see that the program selects the first attribute (Att1) to start building the tree. But it doesn't mean that the generated decision tree is the best. In the original C4.5 the position of the attribute appears in the table matters.

### 4.2.1 Randomize attribute selection

When using our enhanced version of C4.5 we randomly choose an attribute where there are multiple attributes with the same best gain ratio. The program saves all the attributes that have the same best gain ratio in an array, and randomly picks one of attributes to divide the data set to generate the decision tree.

In this project we introduced several more attributes to the tree structure in order to determine the best attribute. For the tree record we added the following:

1. int point (Shows how many attributes that have the same best gain ratio )

2. float *BestAttributesValue (Saves the best attribute values when there are multiple attributes having the same gain ratio).

3. Int *BestAttributes (Saves the best attributes when there are multiple attributes having the same gain ratio)

4. int TreeSize (Gives the size of the tree)

The way to save the attributes in the array is described here:

```c
BestVal = -Epsilon;
BestAtt = None;
ForEach(Att, 0, MaxAtt)
{
    if ( Gain[Att] > -Epsilon )
    {
```

```c
```
Val=Gain[Att]/Info[Att];
if (Val==BestVal&&Val>-Epsilon)
{
    Node->BestAttributesValue[Node->point]=Val;
    Node->BestAttributes[Node->point]=Att;
    Node->point++;
}

if (Val > BestVal)
{
    Node->point=0;
    BestAtt = Att;
    BestVal = Val;
    Node->BestAttributesValue[Node->point]=Val;
    Node->BestAttributes[Node->point]=Att;
    Node->point++;
}

In this program all the same gain ratio attributes are saved in Node->BestAttributes. The Node->point gives the number of the same gain ratio attributes. We can use the following piece of code to get the random integer number from 0 to max

short get_random(int max)
{
    int random;
    float x;
    srand((unsigned) time (NULL));
    random=rand();
    x=(float) rand()/RAND_MAX;
    return (int)(x*max);
}

For the above example, if we use the randomize attribute selection, we see the following output:
For this example, the gain ratio for every attribute is listed below.

\[ \text{GainRatio}(\text{Att1}) = 0.099, \quad \text{GainRatio}(\text{Att2}) = 0.099, \quad \text{GainRatio}(\text{Att3}) = 0.099. \]

The project uses the algorithm listed above to randomly select one attribute (Att2) as the node to build the decision tree. The advantage of using this method is that it creates a chance of generating a smaller tree. This randomize selection algorithm could generate different trees each time if there are some attributes which have the same gain ratio.

### 4.2.2 Best Attribute selection

In the project, I use the greedy algorithm to select the best attribute if there are multiple attributes that have the same best gain ratio.
The algorithm goes through all the multiple attributes with the same best gain ratio. It measures the TreeSize of the sub-trees that are built based on these multiple attributes. The program creates a sub-tree of these candidate attributes and compares the size of the sub-tree. It chooses the attribute with the smallest sub-tree. In this case it generates a smaller decision tree than C4.5 alone.

The following is the code to use the greedy algorithm to find the best attribute.

```
BestAtt = Node -> BestAttributes[0];
BestVal = Node -> BestAttributesValue[0];
Node = getTree(Node, BestAtt, BestVal, Fp, Lp, Cases, NoBestClass, BestClass);
TempNode = CopyTree(Node);
ForEach (f, 1, Node->point - 1)
{
    BestAtt = Node -> BestAttributes[f];
    BestVal = Node -> BestAttributesValue[f];
    Node = getTree(Node, BestAtt, BestVal, Fp, Lp, Cases, NoBestClass, BestClass);
    if (Node -> TreeSize < TempNode -> TreeSize)
    {
        TempNode = CopyTree(Node);
    }
}
Node = CopyTree(TempNode);
```

For the above example, if we use the greedy algorithm, we will get the following output.
For this example, the gain ratio for every attribute is listed below.

GainRatio(Att1) = 0.099, GainRatio(Att2)=0.099 and GainRatio(Att3)=0.099.

When using Att1 to build the sub-tree, it generates the sub-tree with size 9. If use Att2 to build the sub-tree, it generates the sub-tree with size 9. If use Att3 to build the sub-tree, it generates the sub-tree with size 7. The program automatically selects the best attribute (Att3) and generates a smaller tree.

This method will not generate a better tree in all situations. It only works in cases where there are attributes with the same gain ratio. The attributes in the data set that have the same gain ratio normally share a similar structure. For example, it has same number of attribute values and has same number of cases that has the same attribute values. In this project, the program uses the decision tree generated by this method as the input to find the Optimal Reduct. When the decision tree is smaller, the generated Optimal Reduct will be smaller too.
4.2.3 User input

The project also gives the user the control to select one of the attributes that have the same best gain ratio.

The following is the code for user to select one attribute

```c
printf("Please choose one attribute: (\n; 
ForEach(e,0,Node->point-1) 
{ 
    printf("%d: %s", e, AttName[Node->BestAttributes[e]]); 
    if(e+1==Node->point) 
    { 
        printf(")\n"); 
    } 
} 
scanf("%d", &att); 
printf("The selected attribute is %d which is %s", att, AttName[Node->BestAttributes[att]]); 
BestAtt=Node->BestAttributes[att]; 
BestVal=Node->BestAttributesValue[att];
```

For the above example, if we use the user input, we will get the following output.

50
Figure 4-5

From here we can see the user has the ability to choose the attribute when there are attributes with the same gain ratio. In the example, there are 3 attributes (Att1, Att2, and Att3) that have the same gain ratio. User selects Att2 to build the tree. The advantage of this method is that it gives the control to user. The user input method is good for the small data sets. It allows the user to give the additional information about which attribute is more important based on user knowledge.

4.3 Finding the decision rules

Decision rules contain all the representative attribute values from the original data set. It has been shown that to find the minimal reduct and the decision rules for a decision system is an NP-hard problem. Here we list the method to find the Optimal Reduct that is used in the project.
Step 1:

Convert the decision tree generated from C4.5 to decision rules. For example, we have a tree like below:

\[
\begin{align*}
\text{Att1}=1: & \\
| \quad \text{Att3}=1: & \text{N} \\
| \quad \text{Att3}=0: & \text{Y} \\
\text{Att1}=0: & \\
| \quad \text{Att2}=0: & \text{Y} \\
| \quad \text{Att2}=1: & \\
| \quad \quad \text{Att3}=1: & \text{Y} \\
| \quad \quad \text{Att3}=0: & \text{N}
\end{align*}
\]

After converting the tree to the rules, the rules in the table would look like the following:

<table>
<thead>
<tr>
<th>Rules</th>
<th>Att1</th>
<th>Att2</th>
<th>Att3</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td>N</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td></td>
<td>0</td>
<td>Y</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>Y</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>N</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td></td>
<td>Y</td>
</tr>
</tbody>
</table>

Table 4-8

Step 2:

Evaluate each rule to remove the superfluous attribute-values and at the end remove the duplicate rules from the rule set. Here is the algorithm used in this method.

\[
\text{ForEach object in the table} \ // \text{For each line in the table}
\{
\text{ForEach (attribute-value)}
\{
\text{drop the attribute-values in sequence;} \ // \text{drop one attribute at a time;}
\text{check the consistency of the table;}
\}
\}
\[ \text{Reduct}[i] = \text{Get the smallest attribute-value sets;} \]
\[ \text{EvaluateTheRule(\text{Reduct}[i])}; // \text{remove the duplicate rules} \]

For example when we evaluate the rule 3 in Table 4-8, here is the steps:

1. Evaluate the rule starting from Att1 and use sequence Att1, Att2 and Att3 to remove the superfluous attribute values. Remove Att1=0 to see if the Att2=1 and Att3=1 could determine Decision = Y in the original decision table, Table 4-7. In this case, it cannot determine decision. So Att1=0 cannot be removed. Remove Att2=1, the Att1=0 and Att3=1 could determine Decision = Y in the original decision table. So Att2=1 could be removed. Remove Att3=1 to see if Att1=0 could determine the Decision. In this case, it cannot. So Att3=1 cannot be removed. Then the rule becomes Att1=0, Att3=1, decision is Y. The length of the rule is 2. The final rule is Att1=0, Att3=1, decision is Y.

2. Evaluate the rule starting from Att2 and use sequence Att2, Att3 and Att1 to remove the superfluous attribute values. Then the rule is still Att1=0, Att3=1, decision is Y. The length of the rule is 2. It is not smaller than the rule generated in the first step. The final rule will not be changed and it is still Att1=0, Att3=1, decision is Y. If the length of the rule generated in this step is smaller than 2, the final rule will be replaced with the rule generated in this step.

3. Evaluate the rule starting from Att3 and use sequence Att3, Att1, and Att2 to remove the superfluous attribute values. The result is the same as in step 2.

4. So the final rule for rule 3 is Att1=0, Att3=1, decision is Y.
This method evaluates the attribute-values of conditions starting from each attribute. For example: If one table has four attributes $a, b, c, d$. The program would evaluate each rule as the following sequence: $a,b,c,d$ and $b,c,d,a$ and $c,d,a,b$ and $d,a,b,c$. The program removes the superfluous attribute-value for each rule and then selects the smallest as the Rule. The following is the screen image for the generated rules.

![Screen image of generated rules](image-url)
From the output of the program it can be observed that it removed some unnecessary attributes before showing the rules. For example, compare to the decision tree, for rule 3, it removed Att2. For rule 4, it removed Att1. For rule 5, it removed Att3.

4.4 Using threshold to generate the rules

For simple decision table, the generated rules are easy to read. For above example it has only five rules, which is very easy to understand. But for big complicated decision table, the generated rules would be too much for user to read. For monk2 example (monk2 has 6 attributes and 169 objects), the program will generate 81 rules, which is very hard for user to read. So we need a rule pruning technique to filter out some rules and make the generated rules understandable. In this project I used the threshold to filter out the rules. Users could base on the knowledge of the data set and set the threshold value. For example, users could ask the program to show the rules that have more than 3 instances. In the monk2 example, after set the threshold to 3, it generates 9 rules, which cover 49 objects.

The following is the output of the program for the monk2 example.
Rule 1:
attribute#3 = 1
attribute#4 = 1
attribute#6 = 1
-> class 0 (11.0)

Rule 2:
attribute#1 = 1
attribute#4 = 1
attribute#6 = 1
-> class 0 (5.0)

Rule 3:
attribute#1 = 1
attribute#3 = 1
attribute#4 = 1
-> class 0 (5.0)

Rule 4:
attribute#3 = 2
attribute#4 = 2
attribute#5 = 1
-> class 1 (5.0)

Rule 5:
attribute#3 = 1
attribute#4 = 2
attribute#5 = 2
attribute#6 = 2
-> class 1 (4.0)

Rule 6:
attribute#2 = 3
attribute#4 = 2
attribute#5 = 2
attribute#6 = 2
-> class 0 (4.0)

Rule 7:
attribute#3 = 1
attribute#4 = 3
attribute#5 = 2
attribute#6 = 2
-> class 1 (4.0)

Rule 8:
attribute#3 = 2
attribute#4 = 3
attribute#5 = 3
attribute#6 = 2
-> class 0 (4.0)

Rule 9:
attribute#3 = 2
attribute#4 = 3
attribute#5 = 4
attribute#6 = 2
-> class 0 (7.0)
5 Result comparison

In the project, four data sets were tested. The following list of data sets serves as examples to illustrate how close the results compare to the minimal reduct generated by Rough Sets. The results are also compared with the Covering Algorithm and LEM2 algorithm in Rough Sets Exploration System (Bazan et al, 2002). The first data set is described in chapter 4.1 Table 4-1. The second data set is the monk3. The third and fourth data sets are monk1 and monk2.

Experiment 1: The Table 4-1 is the Optician’s decisions table that is obtained from Pawlak, Z’s book (Pawlak, 1991). It has 24 objects, 4 condition attributes and 1 decision attribute. As explained in the book, by using the Rough Set theory, the minimal reduct of the table has 9 rules as indicated in (Pawlak, 1991). When using the same data set as the input of this project, it generates 9 rules too. In this case this project generates the minimal reduct for this data set.

Experiment 2: Monk1, Monk2 and Monk3 data set (see Appendix B for the monk3 data set)

The MONK’s problem is a collection of three binary classification problems over a six-attribute discrete domain. This monk3 sample has 122 instances. The covering
algorithm in the Rough Sets Exploration System was used to generate the minimal reduct on the Monk3 data set. It generates 18 rules. But the rules do not cover the whole data set. It misses 28 cases in the data set. When using LEM2 algorithm in the Rough Set Exploration System to generate the minimal reduct, it generates 41 rules that cover the whole data set. When using the monk3 data set as the input of this project, it generates 24 rules that are better than LEM2 algorithm and can cover the whole data set.

The following are the test results from the experiments.

<table>
<thead>
<tr>
<th>Data sets</th>
<th>No. objects</th>
<th>Con atts</th>
<th>Covering Algorithm</th>
<th>LEM2</th>
<th>This project</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monk1</td>
<td>124</td>
<td>6</td>
<td>1</td>
<td>95</td>
<td>45</td>
</tr>
<tr>
<td>Monk2</td>
<td>169</td>
<td>6</td>
<td>30</td>
<td>109</td>
<td>94</td>
</tr>
<tr>
<td>Monk3</td>
<td>122</td>
<td>6</td>
<td>18</td>
<td>28</td>
<td>41</td>
</tr>
</tbody>
</table>

Table 5-1

**Findings:** From the test we can see that using feature selection doesn’t result to a better reduct. But the feature selection is useful to remove the unnecessary attributes to make the generated decision rules more reasonable, because we don’t want to have the rules with unrelated attribute values information.

**Complexity comparison**
In the Rough Sets, to find the minimal reduct or minimal decision rules is an NP-Hard problem. Let \( m \) be the number of objects in the decision system. Let \( n \) be the number of condition attributes in the decision system. For the first object, we need \( 2^n \) steps to find the minimal attribute-values for this rule. For the second object combining with the consideration of the first object, we need \((2^n) \times (2^n) = (2^n)^2\) steps to find the minimal attribute-values for object 1 and object 2. Repeat the same calculation, we can get that complexity for finding the minimal decision rules is \( O((2^n)^m) \)

The following is the calculation for the complexity of this project’s algorithm.

Assume the decision table has \( m \) number of objects, \( n \) number of condition attributes and \( i \) number of maximum allowed different attribute values for each attribute.

- For feature selection, we remove the attributes in sequence to see if the rest of the table is still consistent. It has complexity \( O(mn) \).

- For generating the decision tree, maximum number of leaf can be at most \( m \). The maximum depth of the tree can be \( n \). The maximum number of node in the tree can be \( mn \). The complexity to generate the root node is \( O(mni) \). The time to spend on generating the root node is much longer than other node. So the worst-case time complexity to generate the decision tree is \( O(m^2n^2i) \). If we use the greedy algorithm on generating the decision tree, the worst case is that for generating every node, all the attributes have the same gain ratio. Because the maximum depth of the tree is \( n \), and it has maximum \( n \) attributes. The maximum opportunity
to have this situation is $n^2$. The complexity for generating the decision tree becomes $O(m^2n^i)$

- For the reduct generation, we start from every attribute and evaluate the remaining attributes in sequence to remove the superfluous attribute values. For example we have 3 attributes which are att1, att2 and att3. At first we evaluate the attribute values using sequence att1, att2 and att3. The complexity to get the rules for this case is $O(mn)$. Next time we evaluate the attribute values using sequence att2, att3 and att1. The third time the sequence is att3, att1 and att2. So for one rule to remove the superfluous attribute-values, the complexity is $O(mn^2)$. The maximum rules the decision table can be $m$. So the complexity for reduct generation is $O(m^2n^2)$

- For removing the duplicate rules, the complexity is $O(m^2)$.

The total complexity for generating the Optimal Reduct is $O(mn)+O(m^2n^i)+O(m^2n^2)$

$+O(m^2)=O(m^2n^i)$

For example one decision system has 4 attributes and 4 objects and 2 different attribute values. When using Rough Set to get the minimal decision rules, it needs $(2^4)^4 = 65536$ steps. When using the algorithm described here to find the Optimal Reduct, it spends $4^2 \times 4^4 \times 2 = 8192$ steps. From this example you could see that the complexity from this algorithm is much smaller than from the Rough Sets.
From these experiments, we can seen that this project can quickly find the good reduct which is very close to the minimal reduct or the same as the minimal reduct.
6 Conclusion

Rough Set theory is the first theory on relational database and provides many useful concepts on data mining such as representing knowledge in a clear mathematical manner, deriving rules only from facts present in data and reducing information systems to its simplest form. It is very costly to get the minimal reduct from Rough Sets, which has been shown to be an NP-hard problem.

C4.5 is a classification algorithm and it is used to generate the decision tree. The C4.5 tree-induction algorithm provides a small decision tree and is very fast. However, it does not always produce the smallest tree.

In this project, the C4.5 algorithm was enhanced to use the greedy algorithm to generate the decision tree. By using this method, if the data set has multiple attributes that have the same best gain ratio, the generated decision tree has smaller size than the result from the original C4.5.

I use the decision tree generated by the modified version of C4.5 as input to find an Optimal Reduct for the relational table. By testing the result on many data sets, it has been shown that the project can quickly generate the result that is very close to or the same as the minimal reduct, which Rough Sets generates. By comparing the result with Rough Sets Exploration System’s covering algorithm and LEM2 algorithm, the result from this project is better.
References


[18] Keyun Hu, Lili Diao, Yuchang Lu and Chunyi Shi, Sampling for Approximate Reduct in very Large Datasets, Computer Science Department, Tsinghua University, Beijing 100084, P.R.China.

Appendix A – Source code for this project

**Besttree.c**

```c
#include "defns.i"
#include "types.i"
#include "extern.i"

ItemNo *TargetClassFreq;
Tree *Raw;
extern Tree *Pruned;
ItemNo RuleNo1; /*Number of Rules */

/**********************************************************
/*                                         */
/* Grow and prune a single tree from all data*/
/*                                         */
/**********************************************************/

OneTree()
/*  ---------  */
{
   Tree FormTree(), CopyTree();
   Boolean Prune();
   InitialiseTreeData();
   InitialiseWeights();

   Raw = (Tree *) calloc(1, sizeof(Tree));
   Pruned = (Tree *) calloc(1, sizeof(Tree));

   AllKnown = true;

   InitAttUse();
   if (useFeatureSelection)
   {
      FeatureSelection(Item); //Write to AttUse to determine
      //if the attribute will be
      //used in the later
      //calculation.
   }
```

Raw[0] = FormTree(0, MaxItem);
printf("\n");
PrintTree(Raw[0]);

if (generateReduct)
{
    RuleNo1 = PrintToTable3(Raw[0]); // Convert the tree to table Reduct
    GenerateOptimalRedcut(Reduct); // Through process to delete unnecessary attribute-value
    RemoveDuplicateRules(Reduct);
    PrintReduct(Reduct); // Print out the reduct table
}

/**************************************************
/*                                                */
/* Remove the duplicate Rules from Reduct table */
/*                                                */
/**************************************************
RemoveDuplicateRules(Reduct1)
Description *Reduct1;
{
    int i, m, j;
    Boolean flag;
    Description Row, Row1, Row2;
    ForEach(i, 0, RuleNo1)
    {
        ForEach(m, i + 1, RuleNo1)
        {
            flag = true;
            ForEach(j, 0, MaxAtt)
            {
                Row1 = Reduct1[i];
                Row2 = Reduct1[m];

                if (Row1[j].discr_val != Row2[j].discr_val)
                {
                    flag = false;
                    break;
                }
            }

            if (flag)
            {
                ForEach(j, 0, MaxAtt)
                {
                    Row = Reduct1[i];
                    Row[j].discr_val = -1;
                }
            }
        }
    }
}
Reduct1[m][MaxAtt+2]._cont_val+=Reduct1[i][MaxAtt+2]._cont_val;
break;

PrintReduct(Reduct2)
Description *Reduct2;
{
int i,m;
short tmp=0;
int rulenumber=1;
Boolean NeedtoPrint();

ForEach(i,0,RuleNo1)
{
    if(NeedtoPrint(i)==true)
    {
        printf("Rule %d:",rulenumber);
        rulenumber++;
        ForEach(m,0,MaxAtt)
        {
            tmp=-1;
            tmp=DVal(Reduct2[i],m);
            if(tmp!=-1){

                printf("%s = %s",AttName[m],AttValName[m][tmp]);

            }
        }
        printf("\n          -> class %s ( %.1f )
",ClassName[Reduct2[i][MaxAtt+1]._discr_val],
CVal(Reduct2[i],MaxAtt+2));
    }
}

PrintReductAll
Description *Reduct2;
{
PrintDataset(Dataset)
Description *Dataset;
{
    int i,m;
    short tmp=0;
    int Itemnumber=1;

    ForEach(i,0,MaxItem)
    {
        ForEach(m,0,MaxAtt)
        {
            tmp=DVal(Dataset[i],m);
            printf("\n %s = %s",AttName[m],AttValName[m][tmp]);
        }
        printf("\n-> class %s ( %.1f )",ClassName[Dataset[i][MaxAtt+1]._discr_val],
        CVal(Dataset[i],MaxAtt+2));
        printf("Evaluation flag -> %d", DVal(Dataset[i],MaxAtt+3));
    }
}

PrintOneRow(OneRow)
Description OneRow;
{
    int m,tmp;
    ForEach(m,0,MaxAtt)
    {
        tmp=DVal(OneRow,m);
        printf("\n %s = %s",AttName[m],AttValName[m][tmp]);
    }
    printf("Evaluation flag from one row-> %d", DVal(OneRow,MaxAtt+3));
}

Check if we need to show the rule
If this row has only -1 as the data
this method will return false

Boolean NeedtoPrint(i)
int i;
{
    int m;
    short tmp=0;
    Boolean hasdata;

    hasdata=false;
    ForEach(m,0,MaxAtt)
    {
        tmp=DVal(Reduct[i],m);
        if(tmp!=-1){
            hasdata=true;
        }
    }

    return hasdata;
}

.InitAttUse()
{
    int i;
    AttUse=(Boolean *)malloc(MaxAtt*sizeof(Boolean));
    if (AttUse == NULL)
    {
        printf("Not enough memory for AttUse memory allocation\n");
        exit(1);
    }
    ForEach(i,0,MaxAtt)
    {
        AttUse[i]=false;//By default all the attributes will be necessary
    }
}

 Feature selection for the Item

/*****/
FeatureSelection(Item)
Description *Item;
{
    int i,m;

    Boolean flag,checktheattribute();

    ForEach(i,0,MaxAtt)//Will test the ith attribute
    {
        flag=true;

        ForEach(m,0,MaxItem)
        {
            if(!checktheattribute(i,m))
            {
                flag=false;
                break;
            }
        }

        if(flag)
        {
            AttUse[i]=true;//Means the ith
            //attribute can be cut.
        }
    }
}

PLY

/*********************************************************************************/
/*                                                                           */
/*        This method is used in feature selection                           */
/*        Check each attribute to see if this is                            */
/*        necessary to decide the decision attribute                        */
/*                                                                           */
/*********************************************************************************/
Boolean checktheattribute(AttN, ItemN)
int AttN, ItemN;
{
    int i,m;
    Boolean flag;
    ForEach(i,ItemN+1,MaxItem)
    {
        flag=true;
        ForEach(m,0,MaxAtt)
        {
            if(m!=AttN&&!AttUse[m])
            {
                flag=false;
                break;
            }
        }
    }
    if(flag)
    {
        AttUse[i]=true;//Means the ith
        //attribute can be cut.
    }
}
if (Item[i][m]._discr_val != Item[ItemN][m]._discr_val)
{
    flag=false;
    break;
}
}
if (flag)
{

    if (Item[i][MaxAtt+1]._discr_val != Item[ItemN][MaxAtt+1]._discr_val)
    {
        return false;
    }
}
return true;

/********************************************************/
/*                                                      */
/*     Generate the optimal reduct from the table Reduct */
/*                                                       */
/********************************************************/
GenerateOptimalRedcut(Reduct1)
Description *Reduct1;
{
    Description Process();

    int i,t,m,instance;

    CopyItem(Item);
    Verbosity(5){
        printf("Number of Rules are %d",RuleNo1+1);
    }
    ForEach(i,0,RuleNo1)
    {
        Reduct1[i]=Process(Reduct1[i]);

        if (NeedtoPrint(i))
        {
            instance=EvaluateTheRule(Reduct1[i]);
            if (instance==0) //Cannot find any instance
                //associate with this rule.
                //Needs to delete this rule.
            {
                EmptyRule(Reduct1[i]);
            }
        }
    }
}
if (SizeOfDataSet(Dataset)==0)  
    //So far the rules have  
    //covered all the cases. Need  
    //to stop.
{
    ForEach(t,i+1,RuleNo1)
    {
        ForEach(m,0,MaxAtt+2)
        {
            DVal(Reduct1[t],m)=-1;
        }
    }
    break;
}

EmptyRule(OneRow)  
Description OneRow;
{
    int i;
    ForEach(i,0,MaxAtt+2)
    {
        DVal(OneRow,i)=-1;
    }
}

int EvaluateTheRule(Rule)  
Description Rule;
{
    int i;
    int t;
}
int flag;
int instancecount=0;

ForEach(i,0,MaxItem)
{
    flag=1;

    if(DVal(Dataset[i],MaxAtt+3)==1)  //This row hasn't  
        //been applied to //any  
        //rule
    {
        ForEach(t,0,MaxAtt)
        {
            if(DVal(Rule,t)!=-1)
            {
                if(DVal(Dataset[i],t)!=DVal(Rule,t))
                {
                    flag=0;
                }
            }
        }
        if(flag)
        {
            DVal(Dataset[i],MaxAtt+3)=0;  //0 means that  
                //this row has  
                //the rule  
                //associated.
                instancecount++;
        }
    }
}

CVal(Rule,MaxAtt+2)=instancecount;
return instancecount;

/************************************************************/  
/*                                                       */  
/* Return the number of instances which don't have the rule to classify them */  
/*                                                       */  
/************************************************************/  

int SizeOfDataSet(Dataset1)
Description *Dataset1;
{
    int i,counter=0;
    ForEach(i,0,MaxItem)
    {
        if(DVal(Dataset1[i],MaxAtt+3)==1)
        {
            counter++;
        }
    }
}
return counter;
}
if(best<count)  
{  
    best=count;  
    FinalRow=CopyRow(OneRow);  
}

}  
OneRow=CopyRow(FinalRow);  
return OneRow;

}  

/***********************************************************/  
/*  */  
/* Copy the Items to Dataset. And add one more flag at */  
/* the end of each row to indicate if row has been */  
/* covered by a rule. The flag will be used in */  
/* GenerateOptimalReduct method. 1 means it is not */  
/* covered or not evaluated. */  
/* */  
/*******************************************************************************/  
CopyItem(Item)  
Description *Item;
{  
    int i;  
    Dataset =(Description *)calloc(MaxItem+1, sizeof(Description));  
    ForEach(i,0,MaxItem)  
    {  
        Dataset[i] = (Description) calloc(MaxAtt+4, sizeof(AttValue));  
        Dataset[i]=CopyRow(Item[i]);  
        DVal(Dataset[i], MaxAtt+3)=1;  
    }  
}

/***********************************************************/  
/*  */  
/*              Copy the Row                             */  
/* */  
/******************************************************/  
Description CopyRow(OneRow)  
Description OneRow;
{  
    int i;  
    Description NewRow;  
    NewRow = (Description) calloc(MaxAtt+3, sizeof(AttValue));  
    ForEach (i,0,MaxAtt+1)
DVal(NewRow, i) = OneRow[i]._discr_val;
CVal(NewRow,MaxAtt+2)=CVal(OneRow,MaxAtt+2);
return NewRow;

/**********************************************
/*                                            */
/*   Test each attribute-value to see if */
/*   it would affect the consistency of the */
/*   whole table                          */
/***********************************************/

Boolean checkConsistency(OneRow)
Description OneRow;
{
    int i,m;
    Boolean flag;
    ForEach(i,0,MaxItem)//For Item
    {
        flag=true;
        ForEach(m,0,MaxAtt)
        {
            if(OneRow[m]._discr_val!=-1)
            {
                if(OneRow[m]._discr_val!=Item[i][m]._discr_val)
                {
                    flag=false;
                }
            }
        }
        if(flag)
        {
            if(OneRow[MaxAtt+1]._discr_val!=Item[i][MaxAtt+1]._discr_val&&flag)
            {
                return false;
            }
        }
    }
    return true;
}
/************************************************************
*/
/**********
* Grow and prune TRIALS trees and select the best of them
* /
/**********
/************************************************************

short BestTree()
/*    --------  */
{
    Tree CopyTree(), Iterate();
    Boolean Prune();
    short t, Best=0;
    InitialiseTreeData();
    TargetClassFreq = (ItemNo *) calloc(MaxClass+1, sizeof(ItemNo));

    Raw    = (Tree *) calloc(TRIALS, sizeof(Tree));
    Pruned = (Tree *) calloc(TRIALS, sizeof(Tree));

    /* If necessary, set initial size of window to 20% (or twice
     * the sqrt, if this is larger) of the number of data items,
     * and the maximum number of items that can be added to the
     * window at each iteration to 20% of the initial window size */
    if ( ! WINDOW )
    {
        WINDOW = Max(2 * sqrt(MaxItem+1.0), (MaxItem+1) / 5);
    }
    if ( ! INCREMENT )
    {
        INCREMENT = Max(WINDOW / 5, 1);
    }
    FormTarget(WINDOW);
    /* Form set of trees by iteration and prune */
    ForEach(t, 0, TRIALS-1 )
    {
        FormInitialWindow();
        printf("\n--------\nTrial %d\n--------\n", t);
        Raw[t] = Iterate(WINDOW, INCREMENT);
        printf("\n");
        PrintTree(Raw[t]);
        SaveTree(Raw[t], ".unpruned");
    }
Pruned[t] = CopyTree(Raw[t]);
if ( Prune(Pruned[t]) )
{
    printf("\nSimplified ");
    PrintTree(Pruned[t]);
}
if ( Pruned[t]->Errors < Pruned[Best]->Errors )
{
    Best = t;
}
printf("\n--------\n");
return Best;

/***************************************************************************/
/********* The windowing approach seems to work best when the class **********/
/********* distribution of the initial window is as close to uniform as      **********/
/********* possible. FormTarget generates this initial target distribution,  **********/
/********* setting up a TargetClassFreq value for each class.               **********/
/******************************************************************************/

FormTarget(Size)
/*  -----------  */
ItemNo Size;
{
    ItemNo i, *ClassFreq;
    ClassNo c, Smallest, ClassesLeft=0;
    ClassFreq = (ItemNo *) calloc(MaxClass+1, sizeof(ItemNo));

    /* Generate the class frequency distribution */
    ForEach(i, 0, MaxItem)
    {
        ClassFreq[ Class(Item[i]) ]++;  
    }

    /* Calculate the no. of classes of which there are items */
    ForEach(c, 0, MaxClass)
    {
        if ( ClassFreq[c] )
        {
            ClassesLeft++;
        }
    }
    Smallest = ClassesLeft / MaxClass;
    /* Determine the smallest number of classes to which items can be */
    /* assigned. This is the number of classes to assign the initial */
    /* window to in order to get the target distribution. */
    ForEach(c, 0, MaxClass)
    {  
        if ( ClassFreq[c] )
        {
            if ( ClassFreq[c] < Smallest )
            {
                Smallest = ClassFreq[c];
            }
        }
    }
    /* Set the TargetClassFreq values for each class. */
    ForEach(c, 0, MaxClass)
    {  
        if ( ClassFreq[c] )
        {
            TargetClassFreq[c] = ClassFreq[c] / Smallest;
        }
    }
    /* Calculate the TargetItemFreq values for each item. */
    ForEach(i, 0, MaxItem)
    {  
        TargetItemFreq[i] = ItemFreq[i] / ClassesLeft;
    }
}/
ClassesLeft++; 
} 
else 
{ 
    TargetClassFreq[c] = 0; 
} 
}

while ( ClassesLeft ) 
{
    /*  Find least common class of which there are some items */
    Smallest = -1; 
    ForEach(c, 0, MaxClass) 
    { 
        if ( ClassFreq[c] &&
            ( Smallest < 0 || ClassFreq[c] < ClassFreq[Smallest] ) )
        { 
            Smallest = c; 
        }
    }
    /*  Allocate the no. of items of this class to use in the window */
    TargetClassFreq[Smallest] = Min(ClassFreq[Smallest],
    Round(Size/ClassesLeft));
    ClassFreq[Smallest] = 0;
    Size -= TargetClassFreq[Smallest];
    ClassesLeft--; 
}

free(ClassFreq);

/****************************************** ********** **********
/  Form initial window, attempting to obtain the target class profile  */
/  in TargetClassFreq. This is done by placing the targeted number */
/  of items of each class at the beginning of the set of data items. */
/ ****************************************** ********** **********

FormInitialWindow()
/*  -------------------  */
{
    ItemNo i, Start=0, More;
ClassNo c;
void Swap();
Shuffle();

ForEach(c, 0, MaxClass)
{
    More = TargetClassFreq[c];
    for ( i = Start ; More ; i++ )
    {
        if ( Class(Item[i]) == c )
        {
            Swap(Start, i);
            Start++;
            More--;
        }
    }
}

/*************************************************************************************************
*/
/*
/*    Shuffle the data items randomly
*/
/*

Shuffle()
/* ------ */
{
    ItemNo This, Alt, Left;
    Description Hold;

    This = 0;
    for( Left = MaxItem+1 ; Left ; )
    {
        Alt = This + (Left--) * ((rand()&32767) / 32768.0);
        Hold = Item[This];
        Item[This++] = Item[Alt];
        Item[Alt] = Hold;
    }
}

/*************************************************************************************************
*/
/*
/*    Grow a tree iteratively with initial window size Window and
*/
/*    initial window increment IncExceptions.
*/
/* Construct a classifier tree using the data items in the */
/* window, then test for the successful classification of other */
/* data items by this tree. If there are misclassified items, */
/* put them immediately after the items in the window, increase */
/* the size of the window and build another classifier tree, and */
/* so on until we have a tree which successfully classifies all */
/* of the test items or no improvement is apparent. */
/* */
/* On completion, return the tree which produced the least errors. */
/* */
/* ***********************************************************************/

Tree Iterate(Window, IncExceptions)
/* ------ */
ItemNo Window, IncExceptions;
{
    Tree Classifier, BestClassifier=Nil, FormTree();
    ItemNo i, Errors, TotalErrors, BestTotalErrors=MaxItem+1,
    Exceptions, Additions;
    ClassNo Assigned, Category();
    short Cycle=0;
    void Swap();

    printf("Cycle Tree -----Cases-----");
    printf(" ------------------Errors------------------- \n");
    printf(" size window other");
    printf(" window rate other rate total rate\n");
    printf(" ----- ---- ------ ------ ----- ----- ---- ");
    printf(" ----- ---- ------ ------ ----- ----- ---- \n");
    do
    {
        /* Build a classifier tree with the first Window items */
        InitialiseWeights();
        AllKnown = true;
        Classifier = FormTree(0, Window-1);

        /* Error analysis */
        Errors = Round(Classifier->Errors);

        /* Move all items that are incorrectly classified by the */
        /* classifier tree to immediately after the items in the */
        /* current window. */
        Exceptions = Window;
        ForEach(i, Window, MaxItem)
        {
            Assigned = Category(Item[i], Classifier);
            if ( Assigned != Class(Item[i]) )
            {
                Swap(Exceptions, i);
            }
        }
    }
    "}
Exceptions++;
}
Exceptions -= Window;
TotalErrors = Errors + Exceptions;

/* Print error analysis */
printf("%3d  %7d  %8d  %8d  %8d%5.1f%%  %6d%5.1f%%
", ++Cycle, TreeSize(Classifier), Window, MaxItem-Window+1, Errors, 100*(float)Errors/Window,
Exceptions, 100*Exceptions/(MaxItem-Window+1.001),
TotalErrors, 100*TotalErrors/(MaxItem+1.0));

/* Keep track of the most successful classifier tree so far */
if ( ! BestClassifier || TotalErrors < BestTotalErrors )
{
    if ( BestClassifier ) ReleaseTree(BestClassifier);
    BestClassifier = Classifier;
    BestTotalErrors = TotalErrors;
}
else
{
    ReleaseTree(Classifier);
}

/* Increment window size */
Additions = Min(Exceptions, IncExceptions);
Window = Min(Window + Max(Additions, Exceptions / 2), MaxItem + 1);
while ( Exceptions );
return BestClassifier;

Evaluate(CMInfo, Saved)
/* -------- */
Boolean CMInfo;
short Saved;
{
    ClassNo RealClass, PrunedClass, Category();
    short t;
    ItemNo *ConfusionMat, i, RawErrors, PrunedErrors;
}
if ( CMInfo )
{
    ConfusionMat = (ItemNo *) calloc((MaxClass+1)*(MaxClass+1),
    sizeof(ItemNo));
}

printf("\n");

if ( TRIALS > 1 )
{
    printf("Trial	 Before Pruning           After Pruning\n");
    printf("-----	----------------   ---------------------------\n");
}
else
{
    printf("\t Before Pruning           After Pruning\n");
    printf("\t----------------   ---------------------------\n");
}
printf("\tSize      Errors   Size      Errors   Estimate\n\n");

ForEach(t, 0, TRIALS-1)
{
    RawErrors = PrunedErrors = 0;

    ForEach(i, 0, MaxItem)
    {
        RealClass = Class(Item[i]);
        //PrintTree(Raw[t]);

        if ( Category(Item[i], Raw[t]) != RealClass ) RawErrors++;

        PrunedClass = Category(Item[i], Pruned[t]);

        if ( PrunedClass != RealClass ) PrunedErrors++;

        if ( CMInfo && t == Saved )
        {
            ConfusionMat[RealClass*(MaxClass+1)+PrunedClass]++;
        }
    }

    if ( TRIALS > 1 )
    {
        printf("%4d", t);
    }

    printf("\t%4d  %3d(%4.1f%%)   %4d  %3d(%4.1f%%)    (%4.1f%%)%s
",
        TreeSize(Raw[t]), RawErrors, 100.0*RawErrors /
        (MaxItem+1.0),
        TreeSize(Pruned[t]), PrunedErrors, 100.0*PrunedErrors /
        (MaxItem+1.0),
        100 * Pruned[t]->Errors / Pruned[t]->Items,
        ( t == Saved ? "   <<" : "")};

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if ( CMInfo )
{
    PrintConfusionMatrix(ConfusionMat);
    free(ConfusionMat);
}

build.c

/***************************************************************
**
* Central tree-forming algorithm incorporating all criteria *
** ---------------------------------------------------------
*
** ***************************************************************
**
#include "defns.i"
#include "types.i"
#include "extern.i"

ItemCount
*Weight,  /* Weight[i] = current fraction of item i */
**Freq,    /* Freq[x][c] = no. items of class c with
outcome x */
*ValFreq,  /* ValFreq[x] = no. items with outcome x */
*ClassFreq; /* ClassFreq[c] = no. items of class c */

float
*Gain,     /* Gain[a] = info gain by split on att a */
*Info,     /* Info[a] = potential info of split on att a */
*Bar,      /* Bar[a] = best threshold for contin att a */
*UnknownRate;  /* UnknownRate[a] = current unknown rate for
att a */

Boolean
*Tested,  /* Tested[a] set if att a has already been tested */
MultiVal; /* true when all atts have many values */

/* External variables initialised here */

extern float
*SplitGain, /* SplitGain[i] = gain with att value of item i as
threshold */
*SplitInfo; /* SplitInfo[i] = potential info ditto */

extern ItemCount
*Slicel,   /* Slicel[c] = saved values of Freq[x][c] in
subset.c */
extern Set **Subset; /* Subset[a][s] = subset s for att a */
extern short *Subsets; /* Subsets[a] = no. subsets for att a */

int level;
int att;

/**********************************************************************
*/
/* Allocate space for tree tables */
/***********************************************************************/

InitialiseTreeData()
/* ------------------ */
{
  DiscrValue v;
  Attribute a;

  Tested = (char *) calloc(MaxAtt+1, sizeof(char));
  Gain = (float *) calloc(MaxAtt+1, sizeof(float));
  Info = (float *) calloc(MaxAtt+1, sizeof(float));
  Bar = (float *) calloc(MaxAtt+1, sizeof(float));
  Subset = (Set **) calloc(MaxAtt+1, sizeof(Set *));

  ForEach(a, 0, MaxAtt)
  {
    if (MaxAttVal[a])
    {
      Subset[a] = (Set *) calloc(MaxDiscrVal+1, sizeof(Set));

      ForEach(v, 0, MaxAttVal[a])
      {
        Subset[a][v] = (Set) malloc((MaxAttVal[a]>>3) + 1);
      }
    }
  }
  Subsets = (short *) calloc(MaxAtt+1, sizeof(short));
  SplitGain = (float *) calloc(MaxItem+1, sizeof(float));
  SplitInfo = (float *) calloc(MaxItem+1, sizeof(float));
  Weight = (ItemCount *) calloc(MaxItem+1, sizeof(ItemCount));
}
Freq = (ItemCount **) calloc(MaxDiscrVal+1, sizeof(ItemCount *));
ForEach(v, 0, MaxDiscrVal)
{
    Freq[v] = (ItemCount *) calloc(MaxClass+1, sizeof(ItemCount));
}

ValFreq = (ItemCount *) calloc(MaxDiscrVal+1, sizeof(ItemCount));
ClassFreq = (ItemCount *) calloc(MaxClass+1, sizeof(ItemCount));
Slice1 = (ItemCount *) calloc(MaxClass+2, sizeof(ItemCount));
Slice2 = (ItemCount *) calloc(MaxClass+2, sizeof(ItemCount));

UnknownRate = (float *) calloc(MaxAtt+1, sizeof(float));

/* Check whether all attributes have many discrete values */
MultiVal = true;
if ( ! SUBSET )
{
    for ( a = 0 ; MultiVal && a <= MaxAtt ; a++ )
    {
        if ( SpecialStatus[a] != IGNORE )
        {
            MultiVal = MaxAttVal[a] >= 0.3 * (MaxItem + 1);
        }
    }
}

/**************************************************************************/
/* Initialise the weight of each item */
/**************************************************************************/

InitialiseWeights()
/* ----------------- */
{
    ItemNo i;

    ForEach(i, 0, MaxItem)
    {
        Weight[i] = 1.0;
    }
}

/**************************************************************************/

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/* Build a decision tree for the cases Fp through Lp: */
/* - It only evaluates the attributes which have been marked necessary after feature selection. */
/* - If user didn't use the feature selection, all the attributes will be evaluated. */

/* - Depend on users options, it builds the decision tree based on the randomly choose the attribute, */
/* user input or best attribute when there are some attributes having the same gain ratio */

/*****************************************************************************/

Tree FormTree(Fp, Lp)
    /* --------- */
    ItemNo Fp, Lp;
    {
        ItemNo i, Kp, Ep, Group();
        ItemCount Cases, NoBestClass, KnownCases, CountItems();
        float Factor, BestVal, Val, AvGain=0, Worth();
        Attribute Att, BestAtt, Possible=0;
        ClassNo c, BestClass;
        Tree Node, Leaf(), SubTree(), TempNode, CopyNode();
        DiscrValue v;
        Boolean PrevAllKnown;

        short a=0, b=0, get_random();
        int atest=0, f, d=10;
        ItemNo e;
        Tree ChildNode;

        Cases = CountItems(Fp, Lp);

        /* Generate the class frequency distribution */
        ForEach(c, 0, MaxClass)
        {
            ClassFreq[c] = 0;
        }
        ForEach(i, Fp, Lp)
        {
            ClassFreq[ Class(Item[i]) ] += Weight[i];
        }

        /* Find the most frequent class */
        BestClass = 0;
        ForEach(c, 0, MaxClass)
        {
            if ( ClassFreq[c] > ClassFreq[BestClass] )
            {
                BestClass = c;
            }
        }

    }
\{ 
    BestClass = c;
\}
NoBestClass = ClassFreq[BestClass];

Node = Leaf(ClassFreq, BestClass, Cases, Cases - NoBestClass);

/* If all cases are of the same class or there are not enough cases to divide, the tree is a leaf */
if ( NoBestClass == Cases || Cases < 2 * MINOBS )
{
    return Node;
}

Verbosity(1)
    printf("\n%d items, total weight %.1f\n", Lp - Fp + 1, Cases);

/* For each available attribute, find the information and gain */
ForEach(Att, 0, MaxAtt)
{
    Gain[Att] = -Epsilon;

    if(!AttUse[Att])
    //  Check if the attribute has been marked as unnecessary during the feature selection
    {
        if ( SpecialStatus[Att] == IGNORE ) continue;
        if ( MaxAttVal[Att] )
        {
            /* discrete valued attribute */
            if ( SUBSET && MaxAttVal[Att] > 2 )
            {
                EvalSubset(Att, Fp, Lp, Cases);
            } else
            if ( !Tested[Att] )
            {
                EvalDiscreteAtt(Att, Fp, Lp, Cases);
            }
        } else
        {
            /* continuous attribute */

            EvalContinuousAtt(Att, Fp, Lp);
        }
    }
}

/* Update average gain, excluding attributes with very many values */
/* So if one attribute has different values more than 30% of the all the items, ignore it*/
if ( Gain[Att] > -Epsilon &&
    ( MultiVal || MaxAttVal[Att] < 0.3 * (MaxItem + 1) ) )
{
    Possible++;  
    AvGain += Gain[Att];
}

/* Find the best attribute according to the given criterion */
BestVal = -Epsilon;
BestAtt = None;
AvGain  = ( Possible ? AvGain / Possible : 1E6 );

Verbosity(2)
{
    if ( AvGain < 1E6 ) printf("\taverage gain %.3f\n", AvGain);
}

ForEach(Att, 0, MaxAtt)
{
    if ( Gain[Att] > -Epsilon )
    {
        // Val = Worth(Info[Att], Gain[Att], AvGain);
        Val=Gain[Att]/Info[Att];
        //This will display every rule even it has only one case
        if (getBest||chooseRandom||chooseA)
        {
            Verbosity(2)
            {
                printf("\n1. Val is %f and BestVal is %f",Val,BestVal);
            }
            if (Val==BestVal&&Val>-Epsilon)
            {
                Node->BestAttributesValue[Node->point]=Val;
                Node->BestAttributes[Node->point]=Att;
                Node->point++;
                Verbosity(5){
                    printf("\n There are some attributes which have the same gain info.");
                }
            }
        }
    } else
    {
        if ( Val > BestVal )
        {
            Node->point=0;
            BestAtt  = Att;
            BestVal = Val;
            if (getBest||chooseRandom||chooseA) { 

Node->BestAttributesValue[Node->point]=Val;
Node->BestAttributes[Node->point]=Att;
Node->point++;
}
}

Verbosity(5){
    printf("\nThere are %d best attributes ",Node->point);
}

// Randomly get the attribute when there are some
// attributes having the same gain ratio.
if (Node->point>1&&chooseRandom)
{
    b=get_random(Node->point);
    Verbosity(5)
    {
        printf("\n The random is %d",b);
    }
    BestAtt=Node->BestAttributes[b];
    BestVal=Node->BestAttributesValue[b];
}

// User input to choose the attribute when there are some
// attributes having the same gain ratio
if (Node->point>1&&chooseA)
{
    printf("\nPlease choose one attribute: (");
    ForEach(e,0,Node->point-1)
    {
        printf("%d: %s, ",e,AttName[Node->BestAttributes[e]]);
        if (e+1==Node->point)
        {
            printf(" )\n");
        }
    }
    scanf("%d", &att);
    printf("\nThe selected attribute is %d which is %s",att,
    AttName[Node->BestAttributes[att]]);
    BestAtt=Node->BestAttributes[att];
    BestVal=Node->BestAttributesValue[att];
}

    /*  Decide whether to branch or not  */

// Get the best attribute when there are some
if (Node->point>1&&getBest)
{
    if (BestAtt!=None)
    {
        BestAtt=Node->BestAttributes[0];
        BestVal=Node->BestAttributesValue[0];
    }

    ChildNode=CopyTree(SubTree(Node,BestAtt, BestVal, Fp, Lp, Cases, NoBestClass, BestClass));

    TempNode=CopyTree(ChildNode);
    ReleaseTree(ChildNode);
    Verbosity(5){
        PrintTree(TempNode);
    }

    Each(f, 1, Node->point-1)
    {
        Verbosity(5){
            printf("Select the \%d attribute as the best Attribute, which is \"%s\", f, AttName[Node->BestAttributes[f]]); // The BestAttribute cannot be reaccess, because it's reallocated before and reassign the data.
        }
        BestAtt=Node->BestAttributes[f];
        BestVal=Node->BestAttributesValue[f];

        ChildNode=CopyTree(SubTree(Node,BestAtt, BestVal, Fp, Lp, Cases, NoBestClass, BestClass));

        Verbosity(5){
            PrintTree(ChildNode);
        }

        // compare the treesize for the subtree.
        if (ChildNode->TreeSize<TempNode->TreeSize)
        {
            // pick the smallest subtree based on the selected attribute.
            TempNode=CopyTree(ChildNode);
        }
        ReleaseTree(ChildNode);
    }

    // Found the smallest subtree based on all the attributes which having the same gain ratio.
    Node=CopyTree(TempNode);
    ReleaseTree(TempNode);
} else // didn't use getBest option.
{
Node=SubTree(Node, BestAtt, BestVal, Fp, Lp, Cases, NoBestClass, BestClass);

Verbosity(5){
    PrintTree(Node);
}
}

return Node;

******************************************************************************
******
*/
/*

Build a subtree for the Node:

- if all cases are of the same class, the subtree is a leaf and so
   the leaf is returned labelled with this class

- for each attribute, calculate the potential information provided
   by a test on the attribute (based on the probabilities of each
   case having a particular value for the attribute), and the gain
   in information that would result from a test on the attribute

(based on the probabilities of each case with a particular
value for the attribute being of a particular class)

- on the basis of these figures, and depending on the current
  selection criterion, find the best attribute to branch on.  Note: this version will not allow a split on an attribute
  unless two or more subsets have at least MINOBS items.

- try branching and test whether better than forming a leaf

******************************************************************************

Tree SubTree(Node, BestAtt, BestVal, Fp, Lp, Cases, NoBestClass, BestClass)

Attribute BestAtt;
float BestVal;


ItemNo Fp, Lp;
Tree Node;
ItemCount Cases, NoBestClass;
ClassNo BestClass;

{

ItemCount KnownCases;
ItemNo Ep, i, Kp;
float Factor;
Boolean PrevAllKnown;
DiscrValue v;

if ( BestAtt != None )
{
  Verbosity(1)
  {
    printf("\n\tbest attribute %s", AttName[BestAtt]);
    if ( ! MaxAttVal[BestAtt] )
    {
      printf(" cut %.3f", Bar[BestAtt]);
    }
    printf(" inf %.3f gain %.3f val %.3f\n", 
      Info[BestAtt], Gain[BestAtt], BestVal);
  }

  /* Build a node of the selected test */
  if ( MaxAttVal[BestAtt] )
  {
    /* Discrete valued attribute */
    if ( SUBSET && MaxAttVal[BestAtt] > 2 )
    {
      SubsetTest(Node, BestAtt);
    }
    else
    {
      DiscreteTest(Node, BestAtt);
    }
  }
  else
  {
    /* Continuous attribute */

    ContinTest(Node, BestAtt);
  }

  /* Remove unknown attribute values */
  PrevAllKnown = AllKnown;

  Kp = Group(0, Fp, Lp, Node) + 1;
  if ( Kp != Fp ) AllKnown = false;
  KnownCases = Cases - CountItems(Fp, Kp-1);
  UnknownRate[BestAtt] = (Cases - KnownCases) / (Cases + 0.001);

"
Verbosity(1)
{
    if ( UnknownRate[BestAtt] > 0 )
    {
        printf("\tunknown rate for %s = %.3f\n",
            AttName[BestAtt], UnknownRate[BestAtt]);
    }
}

/* Recursive divide and conquer */
++Tested[BestAtt];

Ep = Kp - 1;
Node->Errors = 0;

ForEach(v, 1, Node->Forks)
{
    Ep = Group(v, Kp, Lp, Node);
    if ( Kp <= Ep )
    {
        Factor = CountItems(Kp, Ep) / KnownCases;

       .ForEach(i, Fp, Kp-1)
        {
            Weight[i] *= Factor;
        }

        Node->Branch[v] = FormTree(Fp, Ep);
        Node->Errors += Node->Branch[v]->Errors;
        Group(0, Fp, Ep, Node);
       .ForEach(i, Fp, Kp-1)
        {
            Weight[i] /= Factor;
        }
    }
    else
    {
        Node->Branch[v] = Leaf(Node->ClassDist, BestClass, 0.0, 0.0);
    }
}
--Tested[BestAtt];
AllKnown = PrevAllKnown;

/* See whether we would have been no worse off with a leaf */
if ( Node->Errors >= Cases - NoBestClass - Epsilon )
{
    Verbosity(1)
    printf("Collapse tree for %d items to leaf %s\n",
        Lp - Fp + 1, ClassName[BestClass]);
}
Node->NodeType = 0;

else
{
    Verbosity(1)
    printf("\no sensible splits %.1f/%.1f\n",
            Cases, Cases - NoBestClass);
}

Verbosity(5)
    printf("The Node's Size is %d. And the tested attribute is: %d",Node->TreeSize, Node->Tested);
    Node->TreeSize=TreeSize(Node);
    return Node;
}

/*****************************/
/*
/*  Get the random number for the array */
/*
/*****************************/

short get_random(int max)
{
    short tt=1;
    int random;
    float x;
    srand((unsigned)time(NULL));
    random=rand();
    x=(float)rand()/RAND_MAX;
    tt=(int)(x*max);
    return tt;
}

/*****************************/
/*
/*  Group together the items corresponding to branch V of a test */
/* and return the index of the last such */
/* Note: if V equals zero, group the unknown values */
/*****************************/

ItemNo Group(V, Fp, Lp, TestNode)
{
    DiscrValue V;
    ItemNo Fp, Lp;
    Tree TestNode;
    
    ItemNo i;
Attribute Att;
float Thresh;
Set SS;
void Swap();

Att = TestNode->Tested;

if ( V )
{
    /* Group items on the value of attribute Att, and depending 
    on the type of branch */

    switch ( TestNode->NodeType )
    {
        case BrDiscr:
            ForEach(i, Fp, Lp)
            {
                if ( DVal(Item[i], Att) == V ) Swap(Fp++, i);
            }
            break;

        case ThreshContin:
            Thresh = TestNode->Cut;
            ForEach(i, Fp, Lp)
            {
                if ( CVal(Item[i], Att) <= Thresh ) Swap(Fp++, i);
            }
            break;

        case BrSubset:
            SS = TestNode->Subset[V];
            ForEach(i, Fp, Lp)
            {
                if ( In(DVal(Item[i], Att), SS) ) Swap(Fp++, i);
            }
            break;

        else
        {
            /* Group together unknown values */

            switch ( TestNode->NodeType )
            {
                case BrDiscr:
                case BrSubset:
                    ForEach(i, Fp, Lp)
                    {
                        if ( ! DVal(Item[i], Att) ) Swap(Fp++, i);
                    }
                    break;
            }
        }
    }
}
case ThreshContin:
    ForEach(i, Fp, Lp)
    {
        if ( CVal(Item[i], Att) == Unknown ) Swap(Fp++, i);
    }
    break;
}
return Fp - 1;

ItemCount CountItems(Fp, Lp)
/*    ----------  */
ItemNo Fp, Lp;
{
    register ItemCount Sum=0.0, *Wt, *LWt;
    if ( AllKnown ) return Lp - Fp + 1;
    for ( Wt = Weight + Fp, LWt = Weight + Lp ; Wt <= LWt ; )
    {
        Sum += *Wt++;
    }
    return Sum;
}

void Swap(a,b)
/*    ----  */
ItemNo a, b;
{
    register Description Hold;
    register ItemCount HoldW;
}
trees.c

/***************************************************************
*****/
/**
 /**  Routines for displaying, building, saving and restoring trees
 /**  -------------------------------------------------------------
 /**
 /*****************************************************************/

#include "defns.i"
#include "types.i"
#include "extern.i"
#include <stdlib.h>

#define Tab "|   "
#define TabSize 4
#define Width 80 /* approx max width of printed trees */

short Subtree; /* highest subtree to be printed */
Tree Subdef[100]; /* pointers to subtrees */
FILE *TRf = 0, *fopen(); /* file pointer for tree i/o */
char Fn[500]; /* file name */

extern Boolean PRINTTREE;
extern String ModelSuffix;

char suffixbuf[256];

#define Inc1 2048
Description TempRule;
ItemNo RuleNumber=0;

/**************************************************************/
/*                                                        */
/*    Save the Tree as table format                        */
/*    For one path from the tree root to leaf,             */
/*    if it didn't mention a node, the node               */
/*    in the table would have value of -1                 */
/*                                                        */
/**************************************************************/
ItemNo PrintToTable3(T)
    Tree T;
{
    Boolean isWrited();
    DiscrValue v,MaxV;
    ItemNo i=0, ItemSpace1=0;
    T->Writed=isWrited(T);
    
    MaxV=T->Forks;
        //Assign the space for Reduct.
    if ( i >= ItemSpace1 )
    {
        if ( i >= ItemSpace1 )
        {
            ItemSpace1 += Inc1;
            Reduct = (Description *) realloc(Reduct, ItemSpace1*sizeof(Description));
        }
        else
        {
            Reduct = (Description *) malloc((ItemSpace1=Inc1)*sizeof(Description));
        }
    }
    while(!T->Writed)
    {
        TempRule = (Description) calloc(MaxAtt+2, sizeof(AttValue));
        initRule(TempRule);
        ShowBranch3(T);
        //printf("\n");
        RuleNumber++;
    }
    return RuleNumber-1;
}

/**************************************************************/
/*                                                        */
/*    Initial the TempRule and give the                   */
/*    */
/*        -1 as default value                       */
/*                                                   */
/*                                                   */
/**------------------------------------------------*/
/** initRule(TempRule) */
/** Description TempRule; */
/** { */
/**     int i; */
/**     ForEach(i,0,MaxAtt+1) */
/**     { */
/**         DVal(TempRule,i)=-1; */
/**     } */
/** } */
/**------------------------------------------------*/
/** */
/** */
/** */
/** */
/** Recursively assign the data for Reduct */
/** table based on the tree data */
/** */
/**------------------------------------------------*/
/** */
/** */
/** ShowBranch3(T) */
/** Tree T; */
/** { */
/**     Attribute Att; */
/**     DiscrValue i,v,MaxV; */
/**     int getNotWritedBranch(),BranchNo, Which(),temp; */
/**     MaxV=T->Forks; */
/**     ForEach(v,1,MaxV) */
/**     { */
/**         Att=T->Tested; */
/**         if (T->Forks) */
/**         { */
/**             BranchNo=getNotWritedBranch(T); */
/**             if (BranchNo!=0) */
/**             { */
/**                 printf("%s = %s:", AttName[Att], */
/**                     AttValName[Att][BranchNo]); */
/**             } */
/**         } */
/**         else */
/**         { */
/**             temp=Which(AttValName[Att][BranchNo],AttValName */
/**                 [Att],1,MaxAttVal[Att]); */
/**             DVal(TempRule,Att)=temp; */
/**         } */
/**     } */
/** } */
/**------------------------------------------------*/
ShowBranch3(T->Branch[BranchNo]);
}
T->Writed=isWited(T);
else
{
    printf("( %s )", ClassName[T->Leaf]);
}
break;
if (!T->Forks) {
    DVal(TempRule,MaxAtt+1)=T->Leaf;
    CVal(TempRule,MaxAtt+2)=T->Items;
    if (T->Items <= THRESHOLD) //The threshold
    {
        initRule(TempRule);
    }
    Reduct[RuleNumber]=TempRule;
    T->Writed=true;
}
int getNotWritedBranch(T)
Tree T;
{
    DiscrValue v;
    ForEach(v,1,T->Forks)
    {
        if (!T->Branch[v]->Writed)
        {
            return v;
        }
    }
    return 0;
}
int getNotWritedBranch(T)
Tree T;
{
    DiscrValue v;
    ForEach(v,1,T->Forks)
    {
        if (!T->Branch[v]->Writed)
        {
            return v;
        }
    }
    return 0;
}
Boolean isWited(T)
Tree T;
{
    DiscrValue v,MaxV;
    Boolean flag=true;
    if (!T->Forks) {
        return false;
```c
}
else {
    MaxV = T->Forks;
    ForEach(v, 1, MaxV)
    {
        if (!T->Branch[v]->Writed)
        {
            flag = false;
        }
    }
    return flag;
}

/*************************************************************************/
/* */
/* Display entire decision tree T */
/* */
/*************************************************************************/
PrintTree(T)
/* ----------- */
Tree T;
{
    short s;
    if (PRINTTREE)
    {
        Subtree = 0;
        printf("Decision Tree");
        if (TARGETCOLUMN != 0) printf(" for target %s", ClassAttr);
        printf(":
");
        Show(T, 0);
        printf("\n");
        ForEach(s, 1, Subtree)
        {
            printf("\n\nSubtree [S%d]\n", s);
            Show(Subdef[s], 0);
            printf("\n");
        }
        printf("\n");
    }
}

/*************************************************************************/
Show(T, Sh)
/* ---- */
Tree T;
short Sh;
{
    DiscrValue v, MaxV;
    short MaxLine()
    
    if ( T->NodeType )
    {
        /* See whether separate subtree needed */
        if ( T != Nil && Sh && Sh * TabSize + MaxLine(T) > Width )
        {
            if ( Subtree < 99 )
                { Subdef[+Subtree] = T;
                  printf("[S%d]", Subtree);
                }
            else
                { printf("[S??]");
                }
        }
        else
        {
            MaxV = T->Forks;
            /* Print simple cases first */
            ForEach(v, 1, MaxV)
            { if ( ! T->Branch[v]->NodeType )
                { ShowBranch(Sh, T, v);
                }
            }
            /* Print subtrees */
            ForEach(v, 1, MaxV)
            { if ( T->Branch[v]->NodeType )
                { ShowBranch(Sh, T, v);
                }
            }
        }
    }
}
printf(" %s (%.1f", ClassName[T->Leaf], T->Items);
if ( T->Errors > 0 ) printf("/%.1f", T->Errors);
printf(")");
}

ShowBranch(Sh, T, v)

short Sh;
Tree T;
DiscrValue v;
{
DiscrValue Pv, Last;
Attribute Att;
Boolean FirstValue;
short TextWidth, Skip, Values=0, i;

Att = T->Tested;

switch ( T->NodeType )
{
    case BrDiscr:
        Indent(Sh, Tab);
        printf("%s = %s:", AttName[Att], AttValName[Att][v]);
        break;
    case ThreshContin:
        Indent(Sh, Tab);
        printf("%s %s %g ",
                AttName[Att], ( v == 1 ? "<=" : "" ), T->Cut);
        if ( T->Lower != T->Upper )
        {
            printf("[%g,%g]", T->Lower, T->Upper);
        }
        printf(":");
        break;
    case BrSubset:

/* Count values at this branch */

ForEach(Pv, 1, MaxAttVal[Att])
{
    if ( In(Pv, T->Subset[v]) )
    {
        Last = Pv;
        Values++;
    }
}

if ( ! Values ) return;

Indent(Sh, Tab);

if ( Values == 1 )
{
    printf("%s = %s:", AttName[Att], AttValName[Att][Last]);
    break;
}

printf("%s in {", AttName[Att]);
FirstValue = true;
Skip = TextWidth = strlen(AttName[Att]) + 5;

ForEach(Pv, 1, MaxAttVal[Att])
{
    if ( In(Pv, T->Subset[v]) )
    {
        if ( ! FirstValue &&
            TextWidth + strlen(AttValName[Att][Pv]) + 11 > Width
        )
        {
            Indent(Sh, Tab);
            ForEach(i, 1, Skip) putchar(' '); 
            TextWidth = Skip;
            FirstValue = true;
        }
        printf("%s%c", AttValName[Att][Pv], Pv == Last ? '}' : ',');
        TextWidth += strlen(AttValName[Att][Pv]) + 1;
        FirstValue = false;
    }
    putchar(':');
}

Show(T->Branch[v], Sh+1);

/*****************************/
/* Find the maximum single line size for non-leaf subtree St. */
/* The line format is */
/* <attribute> <> X.xx:[ <class (<Items>)], or */
/* <attribute> = <DVal>:[ <class> (<Items>)] */
/**********************************************************************
 */
short MaxLine(St)
/**********************************************************************
 */
/* -------- */
Tree St;
{
 Attribute a;
 DiscrValue v, MaxV, Next;
 short Ll, MaxLl=0;

 a = St->Tested;

 MaxV = St->Forks;
 ForEach(v, 1, MaxV)
 { 
  Ll = ( St->NodeType == 2 ? 4 : strlen(AttValName[a][v]) ) + 1;
  /* Find the appropriate branch */
  Next = v;

  if ( ! St->Branch[Next]->NodeType )
  {
   Ll += strlen(ClassName[St->Branch[Next]->Leaf]) + 6;
  }
  MaxLl = Max(MaxLl, Ll);
 }

 return strlen(AttName[a]) + 4 + MaxLl;

}

/*******************************************************
 */
/*******************************************************
 */
Indent(Sh, Mark)
/*******************************************************
 */
/*******************************************************
 */
Indent(Sh, Mark)
/* ------ */
short Sh;
char *Mark;
{
 printf("\n");
 while ( Sh-- ) printf("%s", Mark);
}

106
SaveTree(T, Extension)
/* ---- ---- */
Tree T;
String Extension;
{
    static char *LastExt="";

    if ( strcmp(LastExt, Extension) )
    {
        LastExt = Extension;

        if ( TRf ) fclose(TRf);
        strcpy(Fn, FileName);
        if (ModelSuffix == 0)
            sprintf(suffixbuf,"%s",Extension);
        else
            sprintf(suffixbuf,"%s%s",ModelSuffix,Extension);
        strcat(Fn, suffixbuf);
        #ifdef __DOS__
            if ( ! ( TRf = fopen(Fn, "wb") ) ) Error(0, Fn, " for writing");
        #else
            if ( ! ( TRf = fopen(Fn, "w") ) ) Error(0, Fn, " for writing");
        #endif
       putc(\n", TRf);
        OutTree(T);
        SaveDiscreteNames();
    }
}

SaveDiscreteNames();
OutTree(T)
/* --------- */
Tree T;
{
    DiscrValue v;
    int Bytes;
    StreamOut((char *) &T->NodeType, sizeof(short));
    StreamOut((char *) &T->Leaf, sizeof(ClassNo));
    StreamOut((char *) &T->Items, sizeof(ItemCount));
    StreamOut((char *) &T->Errors, sizeof(ItemCount));
    StreamOut((char *) T->ClassDist, (MaxClass + 1) * sizeof(ItemCount));
    if (T->NodeType)
    {
        StreamOut((char *) &T->Tested, sizeof(Attribute));
        StreamOut((char *) &T->Forks, sizeof(short));
        switch (T->NodeType)
        {
            case BrDiscr:
                break;
            case ThreshContin:
                StreamOut((char *) &T->Cut, sizeof(float));
                StreamOut((char *) &T->Lower, sizeof(float));
                StreamOut((char *) &T->Upper, sizeof(float));
                break;
            case BrSubset:
                Bytes = (MaxAttVal[T->Tested]>>3) + 1;
                ForEach(v, 1, T->Forks)
                {
                    StreamOut((char *) T->Subset[v], Bytes);
                }
                break;
        }
        ForEach(v, 1, T->Forks)
        {
            OutTree(T->Branch[v]);
        }
    }
}

/***************************************************************************/
/* Retrieve entire decision tree with extension Extension */
/***************************************************************************/
Tree GetTree(Extension)
/*   -------  */
   String Extension;
{
   Tree Hold, InTree();
static char *LastExt="";

   if ( strcmp(LastExt, Extension) )
   {
      LastExt = Extension;
      if ( TRf ) fclose(TRf);

      if (ModelSuffix == 0)
         sprintf(suffixbuf,"%s",Extension);
      else
         sprintf(suffixbuf,"%s\%s",ModelSuffix,Extension);

      strcpy(Fn, FileName);
      strcat(Fn, suffixbuf);
      #ifdef __DOS__
      if ( ! ( TRf = fopen(Fn, "rb") ) ) Error(0, Fn, """);
      #else
      if ( ! ( TRf = fopen(Fn, "r") ) ) Error(0, Fn, """);
      #endif
   }

   if ( ! TRf || getc(TRf) == EOF ) return Nil;

   Hold = InTree();
   RecoverDiscreteNames();
   return Hold;
}

/***************************************************************************/
/* Retrieve tree from saved characters                                  */
/***************************************************************************/

Tree InTree()
/*   -------  */
{
   Tree T;
   DiscrValue v;
   int Bytes;

   T = (Tree) malloc(sizeof(TreeRec));
StreamIn((char *) &T->NodeType, sizeof(short));
StreamIn((char *) &T->Leaf, sizeof(ClassNo));
StreamIn((char *) &T->Items, sizeof(ItemCount));
StreamIn((char *) &T->Errors, sizeof(ItemCount));

T->ClassDist = (ItemCount *) calloc(MaxClass+1, sizeof(ItemCount));
StreamIn((char *) T->ClassDist, (MaxClass + 1) * sizeof(ItemCount));

if ( T->NodeType )
{
    StreamIn((char *) &T->Tested, sizeof(Attribute));
    StreamIn((char *) &T->Forks, sizeof(short));

    switch ( T->NodeType )
    {
    case BrDiscr:
        break;

    case ThreshContin:
        StreamIn((char *) &T->Cut, sizeof(float));
        StreamIn((char *) &T->Lower, sizeof(float));
        StreamIn((char *) &T->Upper, sizeof(float));
        break;

    case BrSubset:
        T->Subset = (Set *) calloc(T->Forks + 1, sizeof(Set));

        Bytes = (MaxAttVal[T->Tested]>>3) + 1;
        ForEach(v, 1, T->Forks)
        {
            T->Subset[v] = (Set) malloc(Bytes);
            StreamIn((char *) T->Subset[v], Bytes);
        }
    }

    T->Branch = (Tree *) calloc(T->Forks + 1, sizeof(Tree));
    ForEach(v, 1, T->Forks)
    {
        T->Branch[v] = InTree();
    }
}

return T;

/**********************************************************************
***/

StreamOut(s, n)
/* --------- */
String s;
int n;
{
    while ( n-- ) putc(*s++, TRf);
}

StreamIn(s, n)
/* --------- */
String s;
int n;
{
    while ( n-- ) *s++ = getc(TRf);
}

/*/ Free up space taken up by tree Node */
/**/ ReleaseTree(Node)
/* ------- */
Tree Node;
{
    DiscrValue v;
    if ( Node->NodeType )
    {
        ForEach(v, 1, Node->Forks)
        {
            ReleaseTree(Node->Branch[v]);
        }
        free(Node->Branch);
        if ( Node->NodeType == BrSubset )
        {
            free(Node->Subset);
        }
    }
    if(Node->BestAttributes)
    {
        free(Node->BestAttributes);
    }
}
if (Node->BestAttributesValue)
{
    free(Node->BestAttributesValue);
}
free(Node->ClassDist);
free(Node);

Tree Leaf(ClassFreq, NodeClass, Cases, Errors)
/* ---- */
    ItemCount *ClassFreq;
    ClassNo NodeClass;
    ItemCount Cases, Errors;
{
    Tree Node;

    Node = (Tree) calloc(1, sizeof(TreeRec));

    Node->ClassDist = (ItemCount *) calloc(MaxClass+1, sizeof(ItemCount));
    memcpy(Node->ClassDist, ClassFreq, (MaxClass+1) * sizeof(ItemCount));
    Node->BestAttributesValue = (float *) calloc(MaxAtt+1, sizeof(float));
    Node->BestAttributes=(Attribute *) calloc(MaxAtt+1, sizeof(Attribute));

    Node->NodeType = 0;
    Node->Leaf = NodeClass;
    Node->Items = Cases;
    Node->Errors = Errors;
    Node->TreeSize =0;

    if(Node->ClassDist==NULL||Node==NULL||Node->BestAttributesValue==NULL||Node->BestAttributes==NULL)
    {
        printf("Not enough memory\n");
        exit(1);
    }
    return Node;
}
insert branches in a node

Sprout(Node, Branches)

Tree Node;
DiscrValue Branches;
{
    Node->Forks = Branches;
    Node->Branch = (Tree *) calloc(Branches+1, sizeof(Tree));
}

count the nodes in a tree

TreeSize(Node)

Tree Node;
{
    int Sum=0;
    DiscrValue v;
    if ( Node->NodeType )
    {
        ForEach(v, 1, Node->Forks)
        {
            Sum += TreeSize(Node->Branch[v]);
        }
        return Sum + 1;
    }
}

return a copy of tree T

113
Tree CopyTree(T)
/*   ---------  */
  Tree T;
{
  DiscrValue v;
  Tree New;

  New = (Tree) malloc(sizeof(TreeRec));
  memcpy(New, T, sizeof(TreeRec));

  New->ClassDist = (ItemCount *) calloc(MaxClass+1,
  sizeof(ItemCount));
  memcpy(New->ClassDist, T->ClassDist, (MaxClass + 1) *
  sizeof(ItemCount));

  if ( T->NodeType )
  {
    New->Branch = (Tree *) calloc(T->Forks + 1,
    sizeof(Tree));
    ForEach(v, 1, T->Forks)
    {
      New->Branch[v] = CopyTree(T->Branch[v]);
    }
  }
  New->BestAttributes=(Attribute *)
  calloc(MaxAtt+1,sizeof(Attribute));
  memcpy(New->BestAttributes,T->
  BestAttributes,(MaxAtt+1)*sizeof(Attribute));
  New->BestAttributesValue=(float *) calloc (MaxAtt+1,sizeof
  (float));
  memcpy(New->BestAttributesValue, T->BestAttributesValue,
  (MaxAtt+1)*sizeof(float));

  return New;
}

/**
**  Save attribute values read with "discrete N"
**
**  SaveDiscreteNames()

114
foreach (Att, 0, MaxAtt) {
    if (SpecialStatus[Att] != DISCRETE) continue;

    StreamOut(&MaxAttVal[Att], sizeof(int));

    foreach (v, 1, MaxAttVal[Att]) {
        Length = strlen(AttValName[Att][v]) + 1;
        StreamOut(&Length, sizeof(int));
        StreamOut(AttValName[Att][v], Length);
    }
}

foreach (Att, 0, MaxAtt) {
    if (SpecialStatus[Att] != DISCRETE) continue;

    StreamIn(&MaxAttVal[Att], sizeof(int));

    foreach (v, 1, MaxAttVal[Att]) {
        StreamIn(&Length, sizeof(int));
        AttValName[Att][v] = (char*) malloc(Length);
        StreamIn(AttValName[Att][v], Length);
    }
}
c4.5.c

#include "defns.i"
#include "types.i"
#include <time.h>

/* External data, described in extern.i */
short MaxAtt, MaxClass, MaxDiscrVal = 2;
short IGNOREMVCLASS = 0;

unsigned int NrMVClass = 0;

ItemNo MaxItem;

Description *Item;

Boolean *AttUse;

DiscrValue *MaxAttVal;
char *SpecialStatus;
String *ClassName,
   *ClassAttr,
   *AttName,
   **AttValName,
   ModelSuffix = 0;
   FileName = "DF"

short VERBOSITY = 0,
   TRIALS = 10;
   THRESHOLD = 0;

Boolean GAINRATIO = true,
   SUBSET = false,
   BATCH = true,
   UNSEENS = false,
   PROBTHRESH = false,
   PRINTTREE = true,
   PRUNETREE = true;

   chooseRandom =false;
   chooseA=false;
   getBest=false;
   useFeatureSelection=false;
   generateReduct=false;
   int tempno;

   Description *Reduct;
Description *Dataset;

ItemNo  MINOBJS = 2,
        WINDOW = 0,
        TARGETCOLUMN = 0,
        INCREMENT = 0;

float   CF = 0.25;
Tree    *Pruned;
Boolean AllKnown = true;

char suffixbuf[256];

main(Argc, Argv)
/*  ----  */
int Argc;
char *Argv[];
{
    int o;
    extern char *optarg;
    extern int optind;
    Boolean FirstTime=true;
    short Best, BestTree();

    PrintHeader("decision tree generator");

    /* Process options */
    while ( (o = getopt(Argc, Argv,
    {
        if ( FirstTime && o != 'h')
        {
            printf("\n Options:\n");
            FirstTime = false;
        }

        switch (o)
        {
            case 'f':   FileName = optarg;
                        printf("\tFile stem <%s>\n", FileName);
                        break;
            case 'M':   ModelSuffix = optarg;
                        printf("\tModel name <%s>\n", ModelSuffix);
                        break;
            case 'u':   UNSEENS = true;
        }
    }
}
printf("\tTrees evaluated on unseen cases\n");
break;
case 'p':
    PROBTHRESH = true;
    printf("\tProbability thresholds used\n");
    break;
case 'v':
    VERBOSITY = atoi(optarg);
    printf("\tVerbosity level %d\n", VERBOSITY);
    break;
case 't':
    TRIALS = atoi(optarg);
    printf("\tWindowing enabled with %d trials\n", TRIALS);
    Check(TRIALS, 1, 10000);
    BATCH = false;
    break;
case 'w':
    WINDOW = atoi(optarg);
    printf("\tInitial window size of %d items\n", WINDOW);
    Check(WINDOW, 1, 1000000);
    BATCH = false;
    break;
case 'i':
    INCREMENT = atoi(optarg);
    printf("\tMaximum window increment of %d items\n", INCREMENT);
    Check(INCREMENT, 1, 1000000);
    BATCH = false;
    break;
case 'g':
    GAINRATIO = false;
    printf("\tGain criterion used\n");
    break;
case 's':
    SUBSET = true;
    printf("\tTests on discrete attribute groups\n");
    break;
case 'm':
    MINOBJS = atoi(optarg);
    printf("\tSensible test requires 2 branches with >=%d cases\n", MINOBJS);
    Check(MINOBJS, 1, 1000000);
    break;
case 'c':
    CF = atof(optarg);
    printf("\tPruning confidence level %g\n", CF);
    Check(CF, 0, 100);
    CF /= 100;
    break;
case 'S':
    PRINTTREE = false;
    printf("\tsuppress printing of decision tree(s)\n");
    break;
case 'O':
    PRUNETREE = false;
    printf("\tsuppress pruning (Overfit)\n");
    break;
case 'T':
    TARGETCOLUMN = atoi(optarg);
    printf("\tTarget attribute in column %d\n", TARGETCOLUMN);
    break;
case 'I':
    IGNOREMVCLASS = atoi(optarg);
    if (IGNOREMVCLASS == 0)
        printf("\tStandard handling of MVs as class values\n");
    else if (IGNOREMVCLASS == 1)
        printf("\tIgnore cases with MV as class value\n");
else if (IGNOREMVCLASS == 2)
    printf("\tSubstitute first class label instead of MVs\n");
else {
    printf("%s not a valid argument for option -I \n", optarg);
    exit(1);
}
break;
case 'h': printf("usage: %s [options]\n", Argv[0]);
print(" [-g] [-s] [-m<n>] [-c<n>] [-S] [0] [-T<n>] [-M<name>] [-I<n>] [-h]\n");
printf(" -f<fstem>: filestem (DF)\n");
printf(" -r        randomly choose attribute \n");
printf(" -A        use user input to choose the attribute (dont)\n");
printf(" -B        choose the best node (dont)\n");
printf(" -F        use feature selection (dont)\n");
printf(" -R        generate reduct (dont)\n");
printf(" -H        use threshold for pruning the reduct \n");
printf(" -b:       windowing disabled (this is the default)\n");
printf(" -u:       evaluate on unseens in <fstem>.test (false)\n");
printf(" -p:       use probability thresholds \n");
printf(" -v<n>:    verbosity level (0)\n");
printf(" -t<n>:    windowing with n trials (10 if enabled)\n");
printf(" -w<n>:    windowing size (20% of training if windowing enabled)\n");
printf(" -i<n>:    windowing increment (20% of training if windowing enabled)\n");
printf(" -g:       use gain criterion (dont, use gain ratio)\n");
printf(" -s:       use value subsets (dont)\n");
printf(" -m<n>:    minimum cases in 2 branches (2)\n");
printf(" -c<n>:    confidence (25)\n");
printf(" -S:       suppress printing of trees (dont suppress)\n");
printf(" -O:       overfit, do not prune (prune)\n");
printf(" -T<n>:    target is attribute n, use changed namesfile (last attr)\n");
printf(" -M<name>:  create <fstem><name>.tree/.unpruned ('')\n");
printf(" -I<n>:    n=0: treat MV in class as error\n");
printf(" n=1: ignore records w/ MV in class\n");
printf(" n=2: substitute first class label (treat as error)\n");
printf(" -h: show this help info\n");
exit(1);

break;

case 'H': THRESHOLD = atoi(optarg);
printf("\tThe threshold for pruning the reduct is %d", THRESHOLD);
break;

case 'r': chooseRandom=true;
printf("\tRandomly choose attribute when there are some attributes having the same gain ratio\n");
break;

case 'A': chooseA=true;
printf("\tUser can choose the attribute when there are some attributes having the same gain ratio\n");
break;

case 'B': getBest=true;

printf("\tGet the best Node if there are some attributes having the same gain ratio\n");
break;

case 'F': useFeatureSelection=true;
printf("\tUse the Feature Selection method to mark the unnecessary attributes \n");
break;

case 'R': generateReduct=true;
printf("\tNeed to generate reduct for the output of the C4.5 \n");
break;

case '?': printf("\tunrecognised option (use -h for help)\n");
exit(1);
}

/* Initialise */

GetNames();
GetData(".data");

printf("\nRead %d cases (%d attributes) from %s.data\n", MaxItem+1, MaxAtt+1, FileName);
if (IGNOREMVCLASS == 1)
printf("\nIgnored %d records with '?' as class label\n", NrMVClass);
else if ((IGNOREMVCLASS == 2) || (NrMVClass > 0))
printf("\nReplaced '?' in %d cases with first class label\n", NrMVClass);

/* Build decision trees */

if ( BATCH )
{
    TRIALS = 1;
    OneTree();
Best = 0;
}
else
{
    Best = BestTree();
}

scanf("%d", &tempno);
exit(0);
}

extern.i

/**********************
/**
/**    Global data for C4.5
/**        ---------------------
/**
/**    ****************************
/**
extern short MaxAtt,           /* max att number */
    MaxClass,           /* max class number */
    MaxDiscrVal;        /* max discrete values for any att */

extern ItemNo MaxItem;          /* max data item number */
extern Description *Item;       /* data items */
extern DiscrValue *MaxAttVal;   /* number of values for each att */
extern char *SpecialStatus;     /* special att treatment */

extern Description *Reduct;
extern Boolean *AttUse;
extern Description *Dataset;

extern String *ClassName,      /* class names */
    *ClassAttr,           /* Name of class attribute */
    *AttName,            /* att names */
    **AttValName,        /* att value names */
    FileName;           /* family name of files */

extern Boolean AllKnown;       /* true if there have been no splits
                                on atts with missing values above
                                the current position in the tree */
extern short VERBOSITY, /* verbosity level (0 = none) */
        TRIALS;  /* number of trees to be grown */
extern Boolean GAINRATIO, /* true=gain ratio, false=gain */
        SUBSET,  /* true if subset tests allowed */
        BATCH,  /* true if windowing turned off */
        UNSEENS, /* true if to evaluate on test data */
        PROBTHRESH, /* true if to use soft thresholds */
        PRUNETREE; /* true if trees should be pruned */
        chooseRandom; /*true if randomly choose the attribute 
if there are attributes which have the same gain info*/
        chooseA; /*true if user can select attribute when 
there are attributes which have the same gain info */
        getBest; /*Get the best Node if there are some 
attributes have the same gain info*/
        generateReduct;/* true to generate the reduct for the 
output of the C4.5 */
        useFeatureSelection; /* true to use the Feature 
selection method to mark the unnecessary attributes */
extern ItemNo MINOBSJS, /* minimum items each side of a cut */
        WINDOW, /* initial window size */
        TARGETCOLUMN, /* to be used instead of last col */
        INCREMENT; /* max window increment each iteration */
extern float CF; /* confidence limit for tree 
pruning */

types.i

typedef char Boolean, *String, *Set;
typedef int ItemNo; /* data item number */
typedef float ItemCount; /* count of (partial) items */
typedef short ClassNo, /* class number, 0..MaxClass */
    DiscrValue; /* discrete attribute value (0 = ?) */
typedef short Attribute; /* attribute number, 0..MaxAtt */

typedef union _attribute_value
{
    DiscrValue _discr_val;
    float _cont_val;
}
AttValue, *Description;

#define CVal(Case,Attribute)   Case[Attribute]._cont_val
#define DVal(Case,Attribute)   Case[Attribute]._discr_val
#define Class(Case)           Case[MaxAtt+1]._discr_val

#ifdef __GNUC__
#include<float.h>
#define Unknown  FLT_MIN
#else
#define Unknown  1.1E-38   /* unknown value for continuous attribute */
#endif

#define BrDiscr 1 /* node types: branch */
#define ThreshContin 2 /* threshold cut */
#define BrSubset 3 /* subset test */

typedef struct _tree_record *Tree;
typedef struct _tree_record
{
    short NodeType; /* 0=leaf 1=branch 2=cut 3=subset */
    ClassNo Leaf; /* most frequent class at this node */
    ItemCount Items, /* no of items at this node */
        *ClassDist, /* class distribution of items */
    Errors; /* no of errors at this node */
    Attribute Tested; /* attribute referenced in test */
    short Forks; /* number of branches at this node */
    float Cut, /* threshold for continuous attribute */
    Lower, /* lower limit of soft threshold */
    Upper; /* upper limit ditto */
    *Subset; /* subsets of discrete values */
    Tree *Branch; /* Branch[x] = (sub)tree for outcome x */
    int point; /* Shows how many attributes have the same gain info */
    float *BestAttributesValue; /* save the best attribute value */
    Attribute *BestAttributes; /* save the best attribute when */
    /* there are multiple attributes having the same gain info */
    int TreeSize; /* Give the size of the tree */
    Boolean Writed; /* Show if the Node has been printed */
}

TreeRec;

#define IGNORE 1 /* special attribute status: do not use */
#define DISCRETE 2 /* ditto: collect values as data read */
typedef short RuleNo;                /* rule number */

typedef struct TestRec *Test;

struct TestRec
{
    short NodeType;          /* test type (see tree nodes) */
    Attribute Tested;       /* attribute tested */
    short Forks;            /* possible branches */
    float Cut;              /* threshold (if relevant) */
    Set *Subset;            /* subset (if relevant) */
};

typedef struct CondRec *Condition;

struct CondRec
{
    Test CondTest;         /* test part of condition */
    short TestValue;       /* specified outcome of test */
};

typedef struct ProdRuleRec PR;

struct ProdRuleRec
{
    short Size;             /* number of conditions */
    Condition *Lhs;         /* conditions themselves */
    ClassNo Rhs;            /* class given by rule */
    float Error,           /* estimated error rate */
    Bits;                  /* bits to encode rule */
    ItemNo Used,           /* times rule used */
    Incorrect;             /* times rule incorrect */
};

typedef struct RuleSetRec RuleSet;

struct RuleSetRec
{
    PR *SRule;              /* rules */
    RuleNo SNRules,        /* number of rules */
    *SRuleIndex;           /* ranking of rules */
    ClassNo SDefaultClass; /* default class for this ruleset */
};

Appendix B – Data set of Monk3 Sample

Monk3.names

0, 1       | | classes

124
attribute#1 : 1, 2, 3
attribute#2 : 1, 2, 3
attribute#3 : 1, 2
attribute#4 : 1, 2, 3
attribute#5 : 1, 2, 3, 4
attribute#6 : 1, 2

monk3.data

1,1,1,1,1,2,1
1,1,1,1,2,1,1
1,1,1,1,2,2,1
1,1,1,1,3,1,0
1,1,1,1,4,1,0
1,1,1,2,1,1,1
1,1,1,2,2,2,1
1,1,1,2,4,2,0
1,1,2,1,2,2,1
1,1,2,1,4,2,0
1,1,2,2,2,2,1
1,1,2,2,4,1,0
1,1,2,2,4,2,0
1,1,2,3,1,1,1
1,1,2,3,1,2,1
1,1,2,3,3,1,1
1,1,2,3,3,2,1
1,2,1,1,3,1,1
1,2,1,2,2,1,1
1,2,1,2,2,2,1
1,2,1,3,1,1,1
1,2,1,3,2,1,1
1,2,1,3,2,2,1
1,2,1,3,3,1,1
1,2,1,3,3,2,1
1,3,1,1,2,1,0
1,3,1,1,4,1,0
| 3,1,1,3,4,2,0 | 3,1,2,1,2,1,1 |
| 3,1,2,2,3,2,1 | 3,1,2,2,4,2,0 |
| 3,1,2,3,1,1,1 | 3,2,1,1,2,2,1 |
| 3,2,1,1,4,1,0 | 3,2,1,2,3,1,1 |
| 3,2,1,3,1,2,1 | 3,2,2,1,2,2,1 |
| 3,2,2,1,3,2,1 | 3,2,2,2,1,2,1 |
| 3,2,2,2,3,2,1 | 3,2,2,3,1,1,1 |
| 3,2,2,3,3,2,1 | 3,2,3,1,4,1,0 |
| 3,3,1,1,3,2,1 | 3,3,1,1,4,1,1 |
| 3,3,1,2,4,2,0 | 3,3,1,3,1,2,1 |
| 3,3,1,3,2,2,0 | 3,3,1,3,4,1,0 |
| 3,3,2,1,1,1,0 | 3,3,2,1,1,2,0 |
| 3,3,2,2,2,2,0 | 3,3,2,2,3,2,0 |
| 3,3,2,3,1,1,0 | 3,3,2,3,2,0,0 |
| 3,3,2,3,3,2,0 | 3,3,2,3,4,2,0 |

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